

# Mass decomposition of the nucleon

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### Outline

#### Quark mass and proton mass





# The decompositions of proton mass

### **Results and further challenges**





### Scale independent quark mass contribution <sup>0.3</sup> global fit

$$m \to \frac{m\langle N | \bar{\psi}\psi | N \rangle}{\langle N | N \rangle}$$

$$= m \frac{\partial m_N}{\partial m} \simeq \frac{m_\pi}{2} \frac{\partial m_N}{\partial m_\pi}$$





- Such a quantity can be obtain by either the direct matrix element calculation, or the derivative of the nucleon mass on the quark mass;
- Or extracted from the  $\pi N$  scattering experiments with  $\chi PT$ .
- But it is just ~50 MeV for three light quarks.

# Scale independent quark mass contribution



 $\sigma_{\pi N} = \langle H_m(u) + H_m(d) \rangle = 45.9(7.4)(2.8) \text{ MeV}$ 

 $f_{s^N} M_N = \langle H_m(s) \rangle = 40.2(11.7)(3.5) MeV$ 

YBY, et.al. ¿QCD Collaboration, PRD94(2016)054503

 $m_{u/d} (2 \text{ GeV}) = 3.36(4) \text{ MeV}$ 

 $m_s(2 \text{ GeV}) = 92(1) \text{ MeV}$ 

http://flag.unibe.ch/2019/Quark%20masses



$$m_N = \langle -T_{\mu\mu} \rangle = \langle m\bar{\psi}\psi + \gamma_m m\bar{\psi}\psi + \frac{\beta(g)}{2g}F^2 \rangle$$

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### **QCD Energy momentum Tensor**

The gauge-invariant, symmetric QCD energy momentum tensor (EMT) in the Euclidean space is given by:

$$T_{\mu\nu} = \frac{1}{4} \bar{\psi} \gamma_{(\mu} \overleftrightarrow{D}_{\nu)} \psi + F_{\mu\rho} F^{\rho}_{\ \nu} - \frac{1}{4} g_{\mu\nu} F^2$$
$$D_{\mu} = \partial_{\mu} + igA_{\mu}, A_{(\mu}B_{\nu)} = A_{\mu}B_{\nu} + A_{\nu}B_{\mu}, \overleftrightarrow{D}_{\mu} = D_{\mu} - \overleftarrow{D}_{\mu}$$

For the nucleon with momentum p, its matrix element of QCD EMT satisfies:

$$\langle T_{\mu\nu} \rangle = \begin{pmatrix} -E & ip_i \\ ip_i & \frac{p_i p_j}{E} \end{pmatrix}_{\overline{p_i \to 0}}^{\overline{p_i \to 0} \begin{pmatrix} -m_N & 0 \\ 0 & 0 \end{pmatrix} }$$
 The rest frame   
 
$$\begin{pmatrix} -E & ip_i \\ p_i p_j \end{pmatrix}_{\overline{p_z \to \infty}}^{\overline{p_i \to 0} \begin{pmatrix} -p_z & ip_z \\ ip_z & p_z \end{pmatrix}$$
 The infinite momentum frame

#### Trace anomaly in Dim. Reg.

Under the dimensional regularization, the QCD EMT can be decomposed into the trace part and the traceless part:

$$\begin{split} T_{\mu\nu} &= \frac{1}{4} \bar{\psi} \gamma_{(\mu} \overleftrightarrow{D}_{\nu)} \psi + F_{\mu\rho} F_{\nu}{}^{\rho} - \frac{1}{4} g_{\mu\nu} F^2 \\ &= \left( T_{\mu\nu} - \frac{g_{\mu\nu}}{d} T^{\alpha}_{\alpha} \right) + \frac{g_{\mu\nu}}{d} T^{\alpha}_{\alpha} \equiv \bar{T}_{\mu\nu} + \hat{T}_{\mu\nu}, \end{split}$$

where  $T^{\alpha}_{\alpha} = m\bar{\psi}\psi - 2\epsilon \frac{F^2}{4} + \mathcal{O}(\epsilon^2)$ . See Y. Hatta, et.al., JHEP12(2018)008 as an example

After the renormalization,

$$m\bar{\psi}\psi = \left(m\bar{\psi}\psi\right)_R, \ F^2 = -\frac{1}{\epsilon}\left(\frac{\beta_R}{g_R}F_R^2 + 2\gamma_m^R \left(m\bar{\psi}\psi\right)_R\right) + \mathcal{O}(\epsilon^0),$$

And then,

$$T^{\alpha}_{\alpha} = (1 + \gamma^R_m) \left( m \bar{\psi} \psi \right)_R + \frac{\beta_R}{2g_R} F^2_R.$$

#### The trace less part of EMT

Let us go back to the ME of the traceless EMT:

$$\frac{\langle P \mid \bar{T}^{q,g}_{\mu\nu} \mid P \rangle}{\langle P \mid P \rangle} = A^{q,g} \frac{P_{\mu}P_{\nu} + \frac{1}{d}g_{\mu\nu}m_{N}^{2}}{P_{0}},$$
  
where  $\bar{T}^{q}_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi - \frac{1}{d}g_{\mu\nu}m\bar{\psi}\psi, \ \bar{T}^{g}_{\mu\nu} = F_{\mu\rho}F_{\nu}{}^{\rho} - \frac{1}{d}g_{\mu\nu}$ 

The Lorentz quark/gluon momentum fraction A can be obtained in any frame, likes the rest frame:

$$\frac{\langle P \,|\, \bar{T}^{q,g}_{\mu\nu} \,|\, P \rangle}{\langle P \,|\, P \rangle}_{P_{x,y,z}=0,} = \frac{d-1}{d} A^{q,g} m_N,$$

or on the light-cone as:

$$\frac{\langle P | \bar{T}^{q,g}_{++} | P \rangle}{\langle P | P \rangle} = A^{q,g}P_{+}, \quad \text{where} \quad \bar{T}^{q}_{++} = \frac{1}{2}\bar{\psi}\gamma_{+}\overleftrightarrow{D}_{+}\psi, \quad \bar{T}^{g}_{++} = F_{+\rho}F_{+}^{\rho}.$$

The trace terms are omitted as  $P_+ >> m_N$ 

# The decompositions of the QCD EMT



Thus one can have the following Ji's decomposition of the nucleon mass (the energy in the rest frame) :

$$\begin{split} m_{N} &= \langle T_{44} \rangle_{P_{x,y,z}=0, d \to 4} = \langle \bar{T}_{44}^{q} \rangle + \langle \bar{T}_{44}^{g} \rangle + \frac{1}{4} (1 + \gamma_{m}) \langle H_{m} \rangle + \frac{\beta}{8g} \langle F^{2} \rangle \\ &= \langle \bar{\psi} \gamma_{4} \overrightarrow{D}_{4} \psi \rangle + \langle \bar{T}_{44}^{g} \rangle + \frac{1}{4} \gamma_{m} \langle H_{m} \rangle + \frac{\beta}{8g} \langle F^{2} \rangle \\ &= \langle \sum_{i} \bar{\psi} \gamma_{i} \overrightarrow{D}_{i} \psi \rangle + \langle \bar{T}_{44}^{g} \rangle + \langle H_{m} \rangle + \frac{1}{4} \gamma_{m} \langle H_{m} \rangle + \frac{\beta}{8g} \langle F^{2} \rangle \end{split}$$

Xiangdong Ji, PRL 74(1995)1071

Or the following decomposition of EMT following the structure of perfect fluid,

$$\langle P | T_{i,\mu,\nu} | P \rangle = \frac{\langle P | P \rangle}{2E} \left( 2P_{\mu}P_{\nu}\langle x \rangle_{i} - 2m_{N}g_{\mu\nu}\bar{p}_{i} \right), \ \bar{p}_{i} = (-\langle x \rangle_{i} + \langle H_{m,i} \rangle)/4.$$

C. Lorce, EPJC78(2018)120

Ji's decomposition of proton mass (the proton energy in the rest frame) Proton 0 0  $M = -\langle T_{44} \rangle = \langle H_E \rangle + \langle H_m \rangle + \langle H_g \rangle + \frac{1}{4} \langle H_a \rangle,$ Quarks  $M = -\langle \hat{T}_{44} \rangle = \langle H_m \rangle + \langle H_a \rangle$ Xiangdong Ji, PRL 74(1995)1071 With  $\langle H_m(u,d,s) \rangle / M_N = 9(2)\%$ The quark  $H_m = \sum_{u,d,s\cdots} \int d^3x \, m \, \overline{\psi} \psi,$ **YBY**, et.al. *x*QCD Collaboration, mass PRD94(2016)054503 The QCD anomaly The total energy Gauge Invariant and scale independent  $H_a = H_a^a + H_m^\gamma,$ The glue combinations.  $H_E = \sum_{u,d,s...} \int d^3x \ \overline{\psi} (\vec{D} \cdot \vec{\gamma}) \psi,$ The quark energy anomaly  $H_{g}^{a} \;=\; \int d^{3}x\; rac{-eta(g)}{g}(E^{2}+B^{2}),$  $H_m^{\gamma} = \sum \int d^3x \gamma_m m \overline{\psi} \psi.$  $H_g = \int d^3x \ \frac{1}{2} (B^2 - E^2),$ The glue field energy The quark mass anomaly

YBY, et. al.,  $\chi$ QCD collaboration, Phys. Rev. D 91(2015)074516

### Proton mass decomposition The QCD anomaly

Then we have

$$\begin{split} M &= -\langle T_{44} \rangle = \langle H_q \rangle + \langle H_g \rangle + \langle H_a^a \rangle + \langle H_m^\gamma \rangle \\ &= \langle H_E \rangle + \langle H_m \rangle + \langle H_g \rangle + \frac{1}{4} \langle H_a \rangle, \end{split}$$

 $M = -\langle \hat{T}_{44} \rangle = \langle H_m \rangle + \langle H_a \rangle,$ 

The QCD anomaly  $H_a = H_g^a + H_m^\gamma$ , The glue anomaly  $H_g^a = \int d^3x \, \frac{-\beta(g)}{g} (E^2 + B^2),$  $H_m^\gamma = \sum_{\substack{u,d,s \cdots \\ \mathbf{L} \mathbf{h} \mathbf{h}} \int d^3x \, \gamma_m m \, \overline{\psi} \psi.$ 

- The joint contribution of the QCD anomaly can be deduced from the quark mass term, with the sum rule above.
- The total QCD anomaly is renormalization scheme/scale independent.
- ·  $H_a/4M_N = 23(1)\%$

### Proton mass decomposition

### The quark/gluon energy



$$M = -\langle \hat{T}_{44} 
angle = \langle H_m 
angle + \langle H_a 
angle,$$

• The quark/glue energy can be deduced from the momentum fraction,

- The renormalization of the quark momentum fraction is much more trivial, which is just mixed with the glue one.
- It is more straightforward to obtain the quark/ glue momentum fraction first, and convert it to the quark/glue energy.

The total energy $H_E = \sum_{u \in I} \int d^3x \ \overline{\psi} (\vec{D} \cdot \vec{\gamma}) \psi,$ 

The quark energy

$$H_g = \int d^3x \; \frac{1}{2} (B^2 - E^2),$$

The glue field energy

### Gluon renormalization

- Calculate the renormalization factor of the glue EMT non-perturbatively on a ~5 fm box will require ~30,000,000 configurations to make the uncertainty to be ~0.01;
- Taking the localization of the correlations between the glue fields/ operators into account, the uncertainty can be reduced by a factor ~200;
- Use reasonable computer resource (~1M CPU hours) to increase the statistics, the ~0.01 uncertainty goal can be obtained with 365 configurations.

**YBY,** et. al.,  $\chi$ QCD collaboration, PRD98(2018) 074506



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### Proton mass decomposition

### Comparing the momentum fractions

Protor

Quarks



- Direct calculation of the quark/glue momentum fraction with non-perturbative renormalization and normalization.
- Trace anomaly contribution deduced by the direct calculation of the quark scalar condensate in nucleon, based on the sum rule





# The next challenge: Trace anomaly under the Lattice Regularizations

- Scheme 1: Define the exact EMT under the lattice regularization and then the trace anomaly can be obtained automatically
- Scheme 2: Renormalize the ME of F<sup>2</sup>
   in the RI/MOM scheme and convert to the MS-bar scheme, then one can use the MS-bar beta function.
- Scheme 3: Calculate the ME of F<sup>2</sup> of both the nucleon and pion, then normalize the nucleon case with the pion case



### Possible experiments on the anomaly

 The near-threshold γ+N->J/psi+N photo production cross section would be sensitive to the form factor of the trace anomaly...

D. Kharzeev, Proc. Int. Sch. Phys. Fermi 130 (1996), 105

- ...if is Q2 dependence is similar to that of the traceless part of EMT.
   Y. Hatta and D. L. Yang, PRD98(2018)074003
- Such an assumption can be checked with Lattice QCD.



## Summary

- Lattice QCD provides a systematic way to investigate the decomposition of the nucleon mass and also QCD EMT;
- It is crucial to investigate the trace anomaly with the regularization other than the dim. reg., especially the Lat. reg.;
- The Lattice result of the trace anomaly will be available in the near future, for both the proton and pion.