Excited light baryons from quark-gluon-level calculations

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Quantum Chromodynamics in its non-perturbative regime

Emergence

Low-level rules producing high-level phenomena with enormous apparent complexity

Start from the QCD Lagrangian:

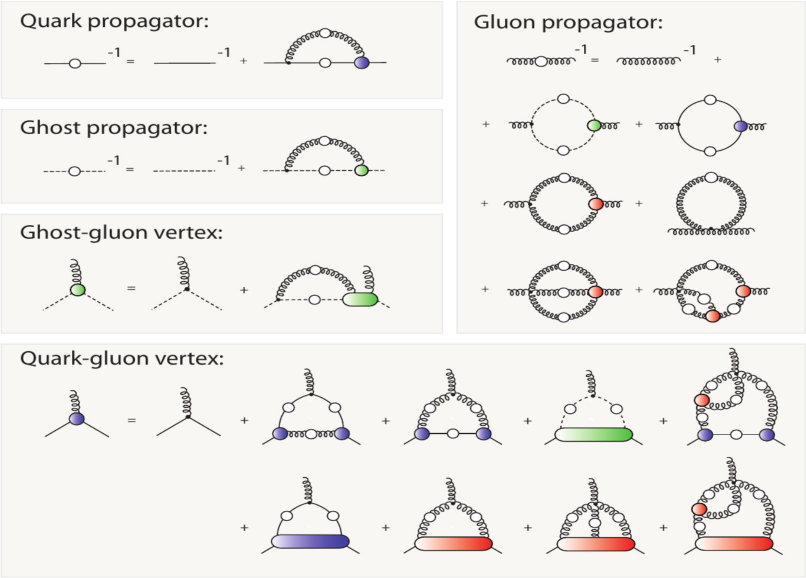
$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a + \frac{1}{2\xi}(\partial^\mu A_\mu^a)^2 + \partial^\mu \bar{c}^a \partial_\mu c^a + g\, f^{abc} (\partial^\mu \bar{c}^a) A_\mu^b c^c + \text{quarks}$$

Lattice, DSEs, ...

And obtain:

- Dynamical generation of fundamental mass scale in pure Yang-Mills (gluon mass).
- Quark constituent masses and chiral symmetry breaking.
- Bound state formation: mesons, baryons, glueballs, hybrids, multiquark systems...
- Signals of confinement.

Emergent phenomena *could be associated with dramatic, dynamically driven changes in the analytic structure of* **QCD's Green functions** *(propagators and vertices).*



Off-shell Green's (correlation) functions

☞ Even though they are:

- Gauge dependent.
- Renormalization point and scheme dependent.

☞ However:

- They capture characteristic features of the underlying dynamics, both perturbative and non-perturbative.
- When appropriately combined they give rise to physical observables.

Theory tool based on Dyson-Schwinger equations

☞ Interesting features:

- Inherently non-perturbative but, at the same time, captures the perturbative behavior $\rightarrow$ accommodates the full range of physical momenta.
- Cover smoothly the full quark mass range, from the chiral limit to the heavy-quark domain.

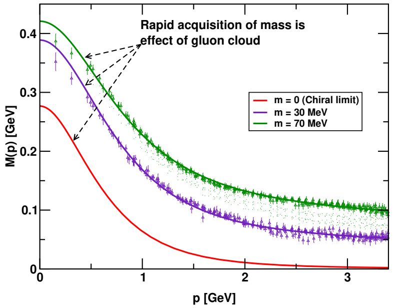
☞ Main caveats:

- Truncation of the infinite system of coupled non-linear integral equations that preserves the underlying symmetries of the theory.
- No expansion parameter $\rightarrow$ no formal way of estimating the size of the omitted terms $\leftrightarrow$ the projection of higher Green's functions on the lower ones is small.

☞ Dressed-quark propagator in Landau gauge:

$$S^{-1}(p) = Z_2(i\gamma\cdot p + m) + \Sigma(p) = \left(\frac{Z(p^2)}{i\gamma\cdot p + \mathbf{M(p^2)}}\right)^{-1}$$

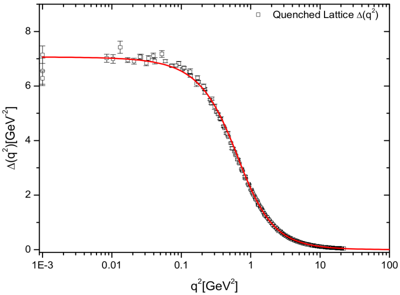
- Mass generated from the interaction of quarks with the gluon-medium.
- Light quarks acquire a HUGE constituent mass.
- Responsible of the 98% of proton's mass, the large splitting between parity partners, . . .



☞ Dressed-gluon propagator in Landau gauge:

$$i\Delta_{\mu\nu} = -iP_{\mu\nu}\mathbf{\Delta(q^2)}, \quad P_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu/q^2$$

- An inflexion point at $p^2 > 0$.
- Breaks the axiom of reflexion positivity.
- Gluon mass generation $\leftrightarrow$ Schwinger mechanism.

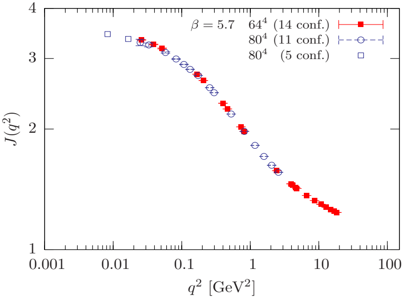


Non-perturbative QCD: Ghost saturation and three-gluon-vertex suppression

☞ Dressed-ghost propagator in Landau gauge:

$$G^{ab}(q^2) = \delta^{ab}\,\frac{\mathbf{J(q^2)}}{q^2}$$

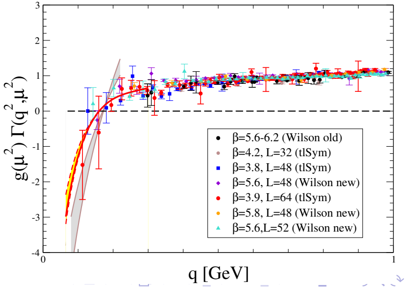
- No power-like singular behavior at $q^2 \to 0$.
- Good indication that $J(q^2)$ reaches a plateau.
- Saturation of ghost's dressing function.



☞ Three-gluon vertex form factor in Landau gauge: ($\propto$ the tree-level tensor structure)

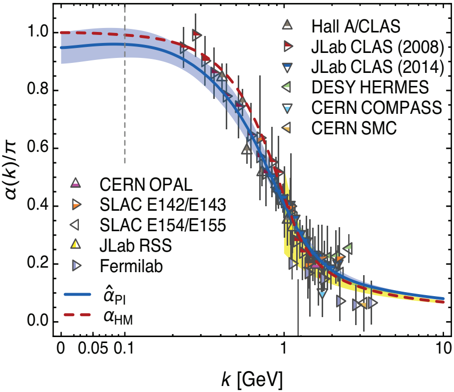
$$\Gamma^{asym}_{T,R}(q^2) \overset{q^2\to 0}{\sim} F(0)\left[\frac{\partial}{\partial q^2}\Delta_R^{-1}(q^2) - C_1(r^2)\right]$$

- Appearance of (longitudinally coupled) massless poles.
- Suppression of the form factor in the so-called asymmetric momentum configuration.
- Plausible zero-crossing.



Non-perturbative QCD: Saturation at IR of process-independent effective-charge

D. Binosi *et al.*, Phys. Rev. D96 (2017) 054026.
A. Deur *et al.*, Prog. Part. Nucl. Phys. 90 (2016) 1-74.



- Data = running coupling defined from the Bjorken sum-rule.

$$\int_0^1 dx \left[g_1^p(x, k^2) - g_1^n(x, k^2) \right] = \frac{g_A}{6} \left[1 - \frac{1}{\pi} \alpha_{g_1}(k^2) \right]$$

- Curve determined from combined continuum and lattice analysis of QCD's gauge sector (massless ghost and massive gluon).

- The curve is a running coupling that does NOT depend on the choice of observable.
  - No parameters.
  - No matching condition.
  - No extrapolation.

- It predicts and unifies an enormous body of empirical data via the matter-sector bound-state equations.

- Perturbative regime:

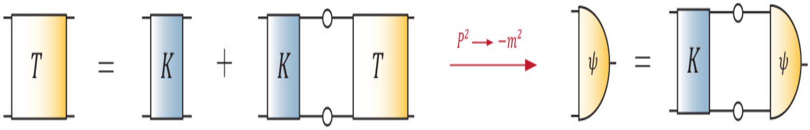
$$\alpha_{g_1}(k^2) = \alpha_{\overline{\text{MS}}}(k^2) \left[1 + 1.14 \alpha_{\overline{\text{MS}}}(k^2) + \ldots \right]$$

$$\hat{\alpha}_{\text{PI}}(k^2) = \alpha_{\overline{\text{MS}}}(k^2) \left[1 + 1.09 \alpha_{\overline{\text{MS}}}(k^2) + \ldots \right]$$

Extraction of hadron properties from poles in $q\bar{q}$, qqq, $qq\bar{q}\bar{q}$... scattering matrices

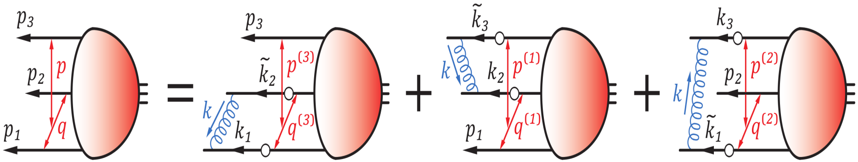
Use **scattering equation** (inhomogeneous BSE) to obtain T in the first place: $T = K + KG_0T$

Homogeneous BSE for **BS amplitude:**



☞ Baryons. A 3-body bound state problem in quantum field theory:

Faddeev equation in rainbow-ladder truncation



Faddeev equation: Sums all possible quantum field theoretical exchanges and interactions that can take place between the three dressed-quarks that define its valence quark content.

non-relativistic

relativistic

Mesons: $P = (-1)^{L+1}$

~~$P = (-1)^{L+1}$~~

S	L	J^{PC}
0	0	0^{-+}
1	0	1^{--}
0	1	1^{+-}
1	1	0^{++}

Bethe,Salpeter, Llewelyn-Smith 1950ies

$$\Gamma_\pi(P,p) = \gamma_5 \left[F_1(P,p) + F_2(P,p) i\not{P} + F_3(P,p) pP i\not{p} + F_4(P,p)[\not{p}, \not{P}]\right]$$

s-wave (F_1, F_2)

p-wave (F_3, F_4)

Baryons: $P = (-1)^{L}$

~~$P = (-1)^{L}$~~

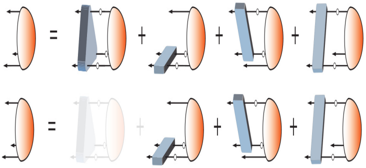
S	L	J^P
$1/2$	0	$1/2^+$
$3/2$	2	

J^P	total	s-wave	p-wave	d-wave	f-wave
$1/2^+$	64	8	36	20	
$3/2^+$	128	4	36	60	28

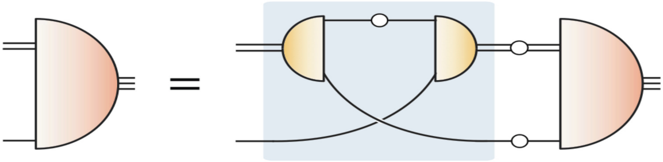
The attractive nature of quark-antiquark correlations in a colour-singlet meson is also attractive for $\bar{3}_c$ quark-quark correlations within a colour-singlet baryon

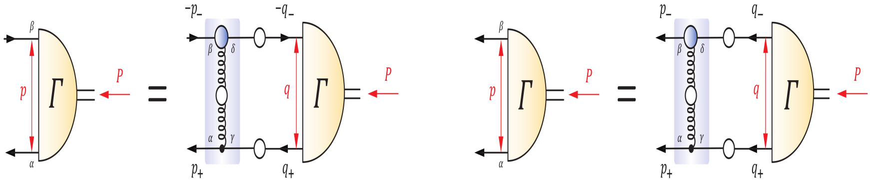
☞ Diquark correlations:

- A tractable truncation of the Faddeev equation.
- In $N_c = 2$ QCD: diquarks can form colour singlets and are the baryons of the theory.
- In our approach: Non-pointlike colour-antitriplet and fully interacting.



Diquark-quark approximation:





☞ Owing to properties of charge-conjugation, a diquark with spin-parity J^P may be viewed as a partner to the analogous J^{-P} meson:

$$\Gamma_{q\bar{q}}(p;P) = - \int \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu\, S(q+P)\Gamma_{q\bar{q}}(q;P)S(q)\,\frac{\lambda^a}{2}\,\gamma_\nu$$

$$\Gamma_{qq}(p;P)C^\dagger = -\frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu\, S(q+P)\Gamma_{qq}(q;P)C^\dagger S(q)\,\frac{\lambda^a}{2}\,\gamma_\nu$$

☞ Whilst no pole-mass exists, the following mass-scales express the strength and range of the correlation:

$$m_{[ud]_{0^+}} = 0.7-0.8\,\text{GeV}, \quad m_{\{uu\}_{1^+}} = 0.9-1.1\,\text{GeV}, \quad m_{\{dd\}_{1^+}} = m_{\{ud\}_{1^+}} = m_{\{uu\}_{1^+}}$$

☞ Diquark correlations are soft, they possess an electromagnetic size:

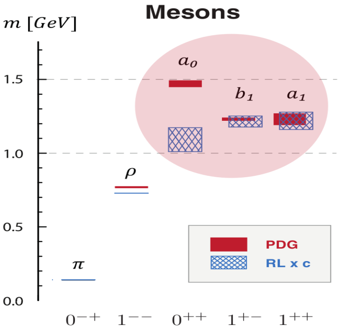
$$r_{[ud]_{0^+}} \gtrsim r_\pi, \qquad r_{\{uu\}_{1^+}} \gtrsim r_\rho, \qquad r_{\{uu\}_{1^+}} > r_{[ud]_{0^+}}$$

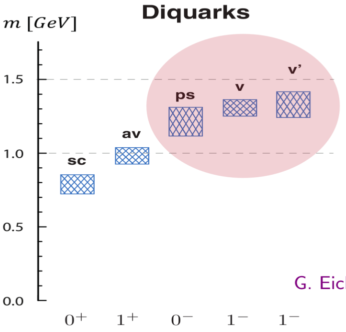
Octet and decuplet baryons

	[nn]	{nn}	[ns]	{ns}	{ss}
N	●	●			
Δ		●			
Λ	●		●	●	
Σ		●	●	●	
Ξ			●	●	●
Ω					●

Other baryons as parity partners

- $[I=0, J^P=0^+]$: Isoscalar-scalar.
- $[I=1, J^P=1^+]$: Isovector-pseudovector.
- $[I=0, J^P=0^-]$: Isoscalar-pseudoscalar.
- $[I=0, J^P=1^-]$: Isoscalar-vector.
- $[I=1, J^P=1^-]$: Isovector-vector.

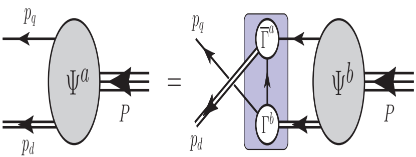




G. Eichmann *et al.*

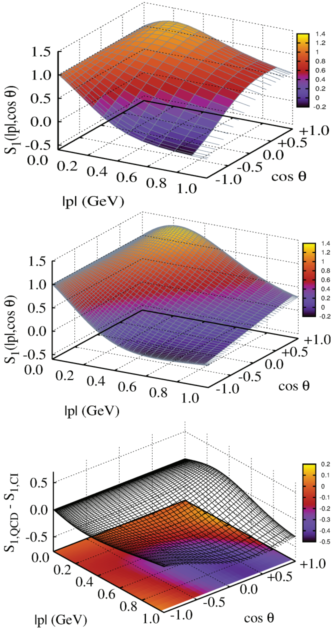
The quark+diquark structure of the nucleon

Faddeev equation in the quark-diquark picture



Dominant piece in nucleon's eight-component Poincaré-covariant Faddeev amplitude: $s_1(|p|, \cos\theta)$

- There is strong variation with respect to both arguments in the quark+scalar-diquark relative momentum correlation.
- Support is concentrated in the forward direction, $\cos\theta > 0$. Alignment of p and P is favoured.
- Amplitude peaks at $(|p| \sim M_N/6, \cos\theta = 1)$, whereat $p_q \sim p_d \sim P/2$ and hence the *natural* relative momentum is zero.
- In the anti-parallel direction, $\cos\theta < 0$, support is concentrated at $|p| = 0$, i.e. $p_q \sim P/3$, $p_d \sim 2P/3$.

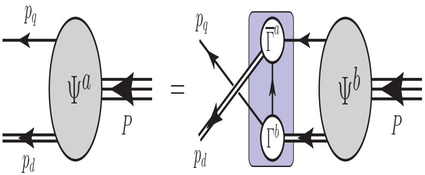


Shu-Sheng Xu *et al.*, Phys. Rev. D92 (2015) 114034

The quark+diquark structure of any baryon

☞ A baryon can be viewed as a **Borromean bound-state**, the binding within which has two contributions:

- Formation of tight diquark correlations.
- Quark exchange depicted in the shaded area.



☞ The exchange ensures that diquark correlations within the baryon are fully dynamical: no quark holds a special place.

☞ The rearrangement of the quarks guarantees that the baryon's wave function complies with Pauli statistics.

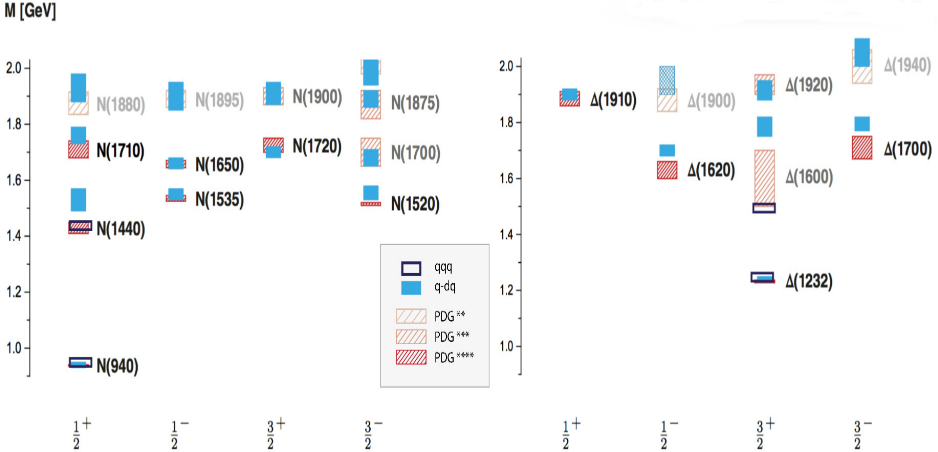
☞ Modern diquarks are different from the old static, point-like diquarks which featured in early attempts to explain the so-called missing resonance problem.

☞ The number of states in the spectrum of baryons obtained is similar to that found in the three-constituent quark model, just as it is in today's LQCD calculations.

☞ Modern diquarks enforce certain distinct interaction patterns for the singly- and doubly-represented valence-quarks within the baryon.

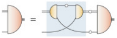
Three-quark cf. quark-diquark

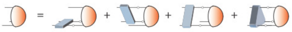
G. Eichmann *et al.*, Phys. Rev. D94 (2016) 094033;
Few Body Syst. 58 (2017) 81;
Prog. Part. Nucl.Phys. 91 (2016) 1-100



- Spectrum in one to one agreement with experiment.
- Correct level ordering (without coupled-channels effects).
- Three-body agrees with quark-diquark where applicable.

level of complexity →





			I) NJL/contact interaction	II) Quark-diquark model	III) DSE (RL)		IV) DSE (bRL)
up/down	$P=+$	N,Δ masses	✓	✓	✓	✓	✓
		N,Δ em. FFs	✓	✓	✓	✓	
		$N\to\Delta\gamma$	✓	✓	✓	✓	
	$P=+$	N^*,Δ^*masses	✓	✓	✓	✓	
		$\gamma N\to N^*/\Delta^*$	✓	✓			
	$P=-$	N^*,Δ^*masses	✓	✓	✓	✓	
		$\gamma N\to N^*/\Delta^*$					
strange		ground states	✓	✓	✓	✓	
		excited states	✓	✓	✓	✓	
		em. FF			✓	✓	
		TFFs				✓	
c/b		ground states	✓	✓		✓	
		excited states		✓		✓	
			Cloet, Thomas, Roberts, Segovia, Chen, et al.	Oettel, Alkofer, Bloch, Roberts, Segovia, Chen, et al.	Eichmann, Alkofer, Krassnigg, Nicmorus, Sanchis-Alepuz, CF	Eichmann, Alkofer, Sanchis-Alepuz, CF, Qin, Roberts	Sanchis-Alepuz, Williams, CF

An example of diquark content

$\Lambda - \Sigma$ mass splitting

Whilst the Λ and Σ are associated with the same combination of valence-quarks, their spin-flavor wave functions are different.

$\Downarrow$

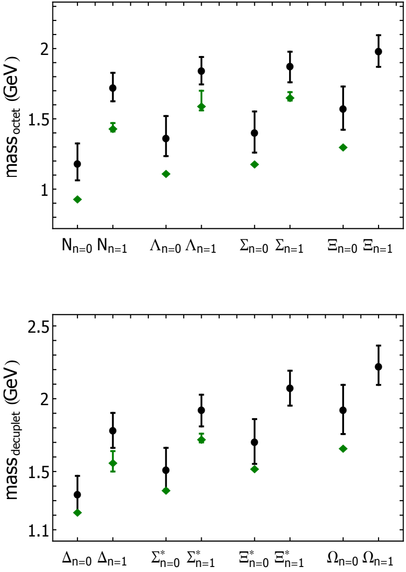
Λ contains more of the (lighter) scalar diquark correlations than Σ

$$u_\Lambda = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{2}\, s[ud]_{0^+} \\ d[us]_{0^+} - u[ds]_{0^+} \\ d\{us\}_{1^+} - u\{ds\}_{1^+} \end{bmatrix} \leftrightarrow \begin{bmatrix} s_\Lambda^1 \\ s_\Lambda^{[2,3]} \\ a_\Lambda^{[6,8]} \end{bmatrix} \;;\quad u_\Sigma = \begin{bmatrix} u[us]_{0^+} \\ s\{uu\}_{1^+} \\ u\{us\}_{1^+} \end{bmatrix} \leftrightarrow \begin{bmatrix} s_\Sigma^2 \\ a_\Sigma^4 \\ a_\Sigma^6 \end{bmatrix}$$

	N	Λ	Σ	Ξ	Δ	Σ^*	Ξ^*	Ω
The.	1.19(13)	1.37(14)	1.41(14)	1.58(15)	1.35(12)	1.52(14)	1.71(15)	1.93(17)
Exp.	0.94	1.12	1.19	1.31	1.23	1.38	1.53	1.67
The.	1.73(10)	1.85(09)	1.88(11)	1.99(11)	1.79(12)	1.93(11)	2.08(12)	2.23(13)
Exp.	1.44(03)	$1.51^{+0.10}_{-0.04}$	1.66(03)	-	1.57(07)	1.73(03)	-	-

C. Chen *et al.*, accepted by Phys. Rev. D, arXiv:nucl-th/1901.04305

- The computed masses are uniformly larger than the corresponding empirical values.
- The quark-diquark kernel omits all resonant contributions associated with meson-baryon final state interactions, which typically generate a measurable reduction.
- The Faddeev equations analyzed to produce the results should be understood as producing the dressed-quark core of the bound state, not the completely dressed and hence observable object.



C. Chen *et al.*, accepted by Phys. Rev. D, arXiv:nucl-th/1901.04305

L content	$N_{n=0}$	$N_{n=1}$	$\Lambda_{n=0}$	$\Lambda_{n=1}$	$\Sigma_{n=0}$	$\Sigma_{n=1}$	$\Xi_{n=0}$	$\Xi_{n=1}$
S,P,D	1.19	1.73	1.37	1.85	1.41	1.88	1.58	1.99
$-,P,D$	–	–	–	–	–	–	–	–
$S,-,D$	1.24	1.71	1.40	1.83	1.42	1.84	1.59	1.97
$S,P,-$	1.20	1.74	1.37	1.85	1.41	1.89	1.58	1.99
$S,-,-$	1.24	1.71	1.40	1.83	1.42	1.84	1.59	1.97

L content	$\Delta_{n=0}$	$\Delta_{n=1}$	$\Sigma^*_{n=0}$	$\Sigma^*_{n=1}$	$\Xi^*_{n=0}$	$\Xi^*_{n=1}$	$\Omega_{n=0}$	$\Omega_{n=1}$
S,P,D,F	1.35	1.79	1.52	1.93	1.71	2.08	1.93	2.23
$-,P,D,F$	–	–	–	–	–	–	–	–
$S,-,D,F$	1.36	1.75	1.52	1.90	1.71	2.06	1.93	2.22
$S,P,-,F$	1.35	1.82	1.52	1.95	1.71	2.09	1.93	2.24
$S,P,D,-$	1.35	1.79	1.52	1.93	1.71	2.08	1.93	2.23
$S,-,-,-$	1.35	1.80	1.52	1.93	1.71	2.08	1.93	2.23

- No state is generated by the Faddeev equation unless S-wave components are contained in the wave function.
- This observation provides support in quantum field theory for the constituent quark model classification of these systems.
- The P- and D-wave components play a measurable role in, respectively, octet and decuplet baryons.

Angular momenta of the octet and decuplet (I)

L content	$N_{n=0}$	$N_{n=1}$	$\Lambda_{n=0}$	$\Lambda_{n=1}$	$\Sigma_{n=0}$	$\Sigma_{n=1}$	$\Xi_{n=0}$	$\Xi_{n=1}$
S,P,D	1.19	1.73	1.37	1.85	1.41	1.88	1.58	1.99
$-,P,D$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$S,-,D$	1.24	1.71	1.40	1.83	1.42	1.84	1.59	1.97
$S,P,-$	1.20	1.74	1.37	1.85	1.41	1.89	1.58	1.99
$S,-,-$	1.24	1.71	1.40	1.83	1.42	1.84	1.59	1.97

L content	$\Delta_{n=0}$	$\Delta_{n=1}$	$\Sigma^*_{n=0}$	$\Sigma^*_{n=1}$	$\Xi^*_{n=0}$	$\Xi^*_{n=1}$	$\Omega_{n=0}$	$\Omega_{n=1}$
S,P,D,F	1.35	1.79	1.52	1.93	1.71	2.08	1.93	2.23
$-,P,D,F$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$S,-,D,F$	1.36	1.75	1.52	1.90	1.71	2.06	1.93	2.22
$S,P,-,F$	1.35	1.82	1.52	1.95	1.71	2.09	1.93	2.24
$S,P,D,-$	1.35	1.79	1.52	1.93	1.71	2.08	1.93	2.23
$S,-,-,-$	1.35	1.80	1.52	1.93	1.71	2.08	1.93	2.23

- ☞ No state is generated by the Faddeev equation unless S-wave components are contained in the wave function.
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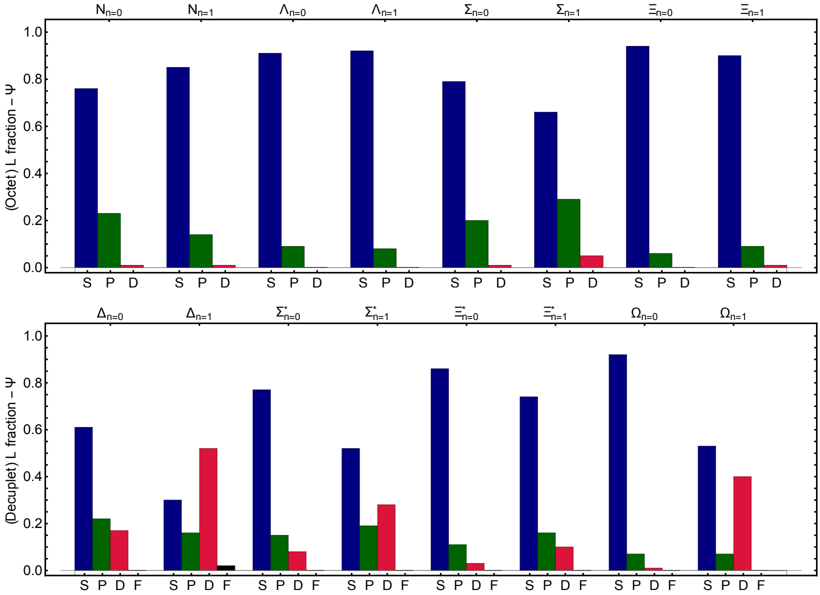
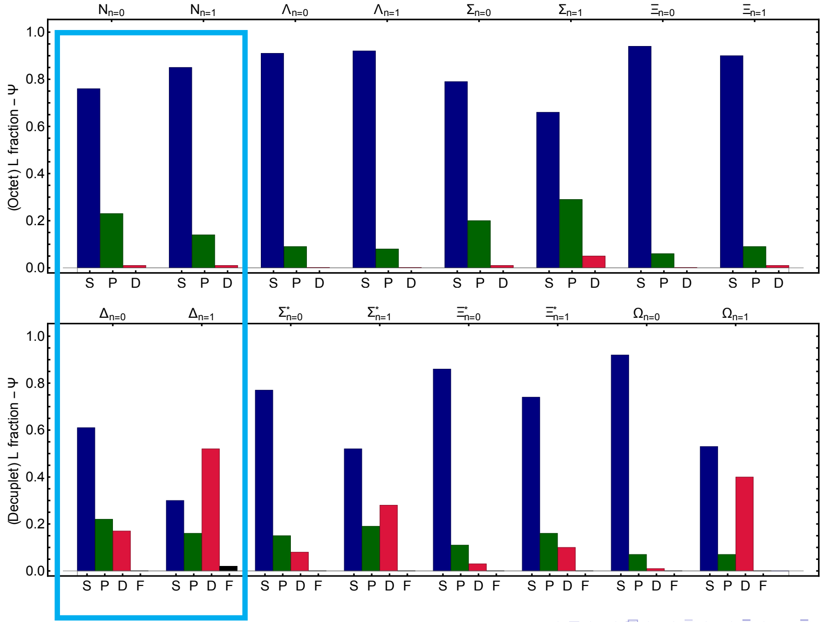
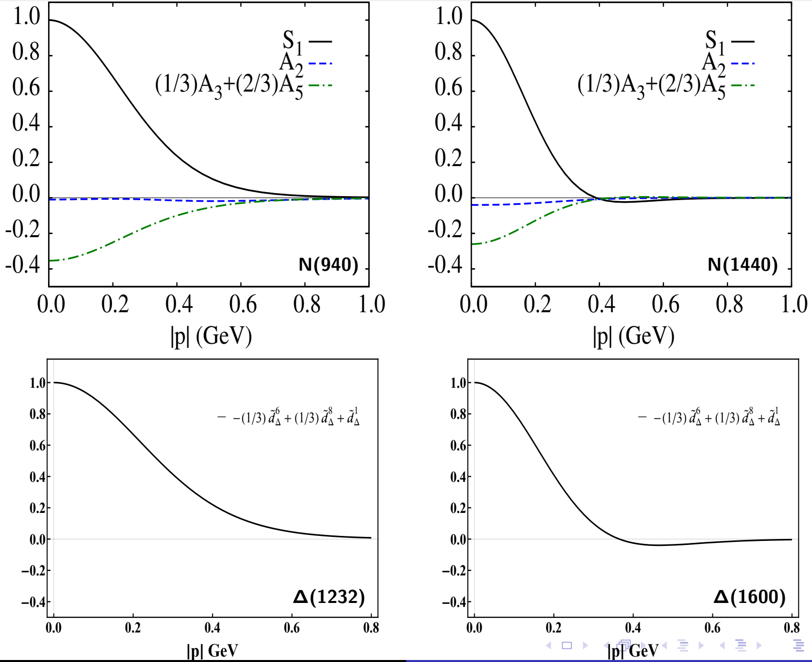
Angular momenta of the octet and decuplet (II)



Angular momenta of the octet and decuplet (II)



Radial excitations in quantum field theory



Wave function decomposition: $N(1440)$ cf. $\Delta(1600)$

	$N(940)$	$N(1440)$	$\Delta(1232)$	$\Delta(1600)$
scalar	62%	62%	–	–
pseudovector	29%	29%	100%	100%
mixed	9%	9%	–	–
S-wave	0.76	0.85	0.61	0.30
P-wave	0.23	0.14	0.22	0.15
D-wave	0.01	0.01	0.17	0.52
F-wave	–	–	~ 0	0.02

$N(1440)$

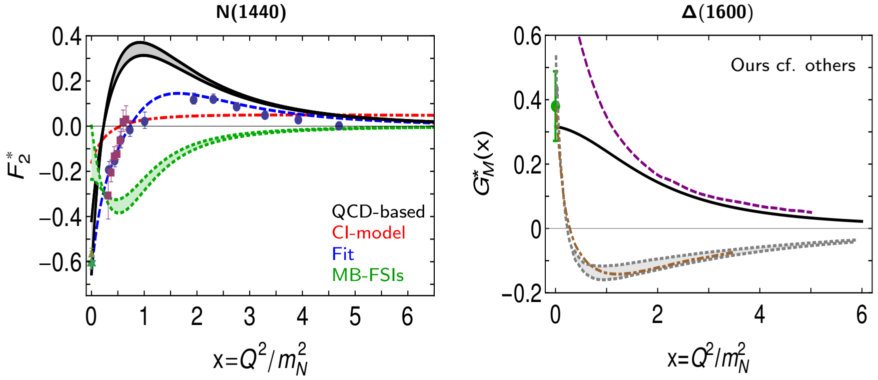
- Roper's diquark content are almost identical to the nucleon's one.
- It has an orbital angular momentum composition which is very similar to the one observed in the nucleon.

$\Delta(1600)$

- $\Delta(1600)$'s diquark content are almost identical to the $\Delta(1232)$'s one.
- It shows a dominant $\ell = 2$ angular momentum component with its S-wave term being a factor 2 smaller.

The presence of all angular momentum components compatible with the baryon's total spin and parity is an inescapable consequence of solving a realistic Poincaré-covariant Faddeev equation

Consequences on, e.g., EM trasition form factors



☞ Observations:

N(1440): Our calculation agrees quantitatively in magnitude and qualitatively in trend with the data on $x \gtrsim 2$.

N(1440): The mismatch between our prediction and the data on $x \lesssim 2$ is due to meson cloud contribution.

Δ(1600): It is positive defined in the whole range of photon momentum and decreases smoothly with larger Q^2-values → D-wave dominance.

J. Segovia *et al.*, Phys. Rev. Lett. 115 (2015) 171801, arXiv:nucl-th/1504.04386
Y. Lu *et al.*, Phys. Rev. D100 (2019) 034001, arXiv:nucl-th/1904.03205

Parity partners at a glance (I)

- The $N(940)$ and $N(1440)$ are primarily S-wave in nature, since they are not supported by the Faddeev equation unless S-wave components are contained in the wave function.

- The $N(1535)$ and $N(1650)$ are essentially P-wave in character, since they are not supported by the Faddeev equation unless P-wave components are contained in the wave function.

- These observations provide (again) support in quantum field theory for the constituent-quark model classifications of these systems.

L content	N_0^+	N_1^+	N_0^-	N_1^-
S, P, D	1.19	1.73	1.83	1.91
$-, P, D$	–	–	1.89	1.98
$S, -, D$	1.24	1.71	–	–
$S, P, -$	1.20	1.74	1.83	1.91
$S, -, -$	1.24	1.71	–	–
$-, P, -$	–	–	1.90	1.98

Masses of the quark core against values determined for the meson-undressed bare-excitations

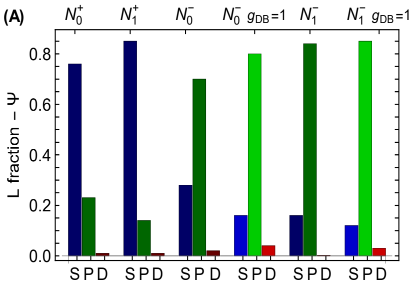
	m_N	$m_{N(1440)}^{1/2^+}$	$m_{N(1535)}^{1/2^-}$	$m_{N(1650)}^{1/2^-}$
herein	1.19	1.73	1.83	1.91
M_B^0		1.76	1.80	1.88

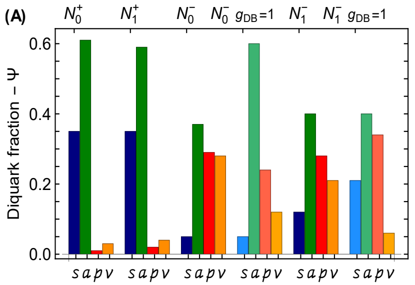
L. Ya *et al.*, Phys. Rev. C96 (2017) no.1, 015208, arXiv:nucl-th/1705.03988.
C. Chen *et al.*, Phys. Rev. D97 (2018) 034016, arXiv:nucl-th/1711.03142.

- Again, the two lightest $1/2^+$ doublets are predominantly S-wave in character, whereas the negative parity states are chiefly P-wave. In all cases the D-wave components are negligible.

- $N(940)$ and $N(1440)$ are mostly constituted by scalar and pseudovector diquarks and $g_{DB} < 1$ has little impact on the nucleon and Roper, so we do not draw $g_{DB} = 1$ results.

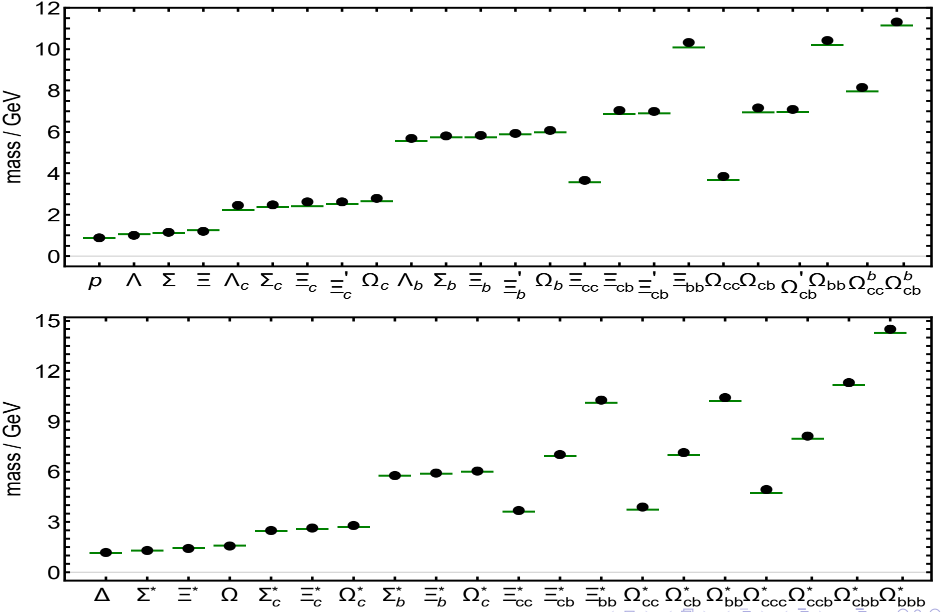
- $g_{DB} < 1$ has a significant effect on the structure of the negative parity baryons, it increases both the effective energy-cost (mass) of positive parity diquarks and the fraction of pseudoscalar- and vector-diquarks they contain.





C. Chen *et al.*, Phys. Rev. D97 (2018) 034016, arXiv:nucl-th/1711.03142.

Pei-Lin Yin *et al.*, Phys. Rev. D100 (2019) 034008, arXiv:nucl-th/1903.00160



☞ Take home messages:

- Baryon spectrum: fair agreement with experiment!
- Results for up/down, strange and heavy quarks.
- Three-body vs diquark-quark: good agreement.

☞ QFT combined with quark-diquark picture:

- The running of the strong coupling constant which is expressed in e.g. the momentum dependence of the dressed-quark mass produces DCSB.
- DCSB and its correct implementation produces pions as well as nonpointlike and fully-dynamical diquark correlations inside baryons.
- The Faddeev kernel ensures that every valence-quark participates actively in all diquark correlations to the fullest extent allowed by kinematics and symmetries.
- Poincaré covariance demands the presence of dressed-quark orbital angular momentum in the baryon.