Dispersive techniques for processes $\gamma^*\gamma^* \to \pi\pi,\ \pi\eta$

Igor Danilkin

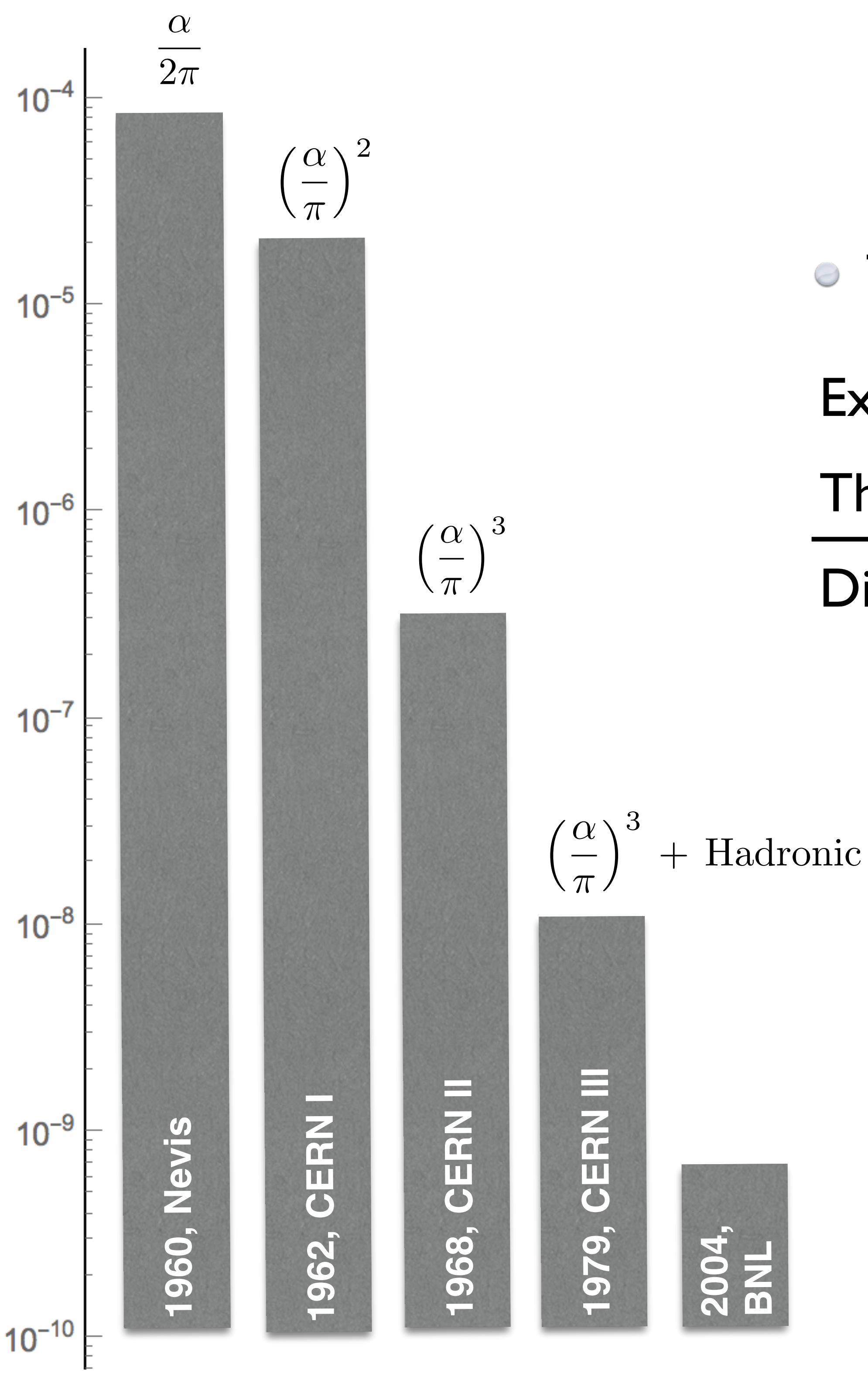
August 20, 2019, Gulin, China

SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

JOHANNES GUTENBERG UNIVERSITÄT MAINZ
JG|U

Motivation

- Accuracy



- Magnetic moment of the muon

$$\vec{\mu} = \frac{Q}{2m}\, g\, \vec{S}$$

- Anomalous part

$$a_\mu = \frac{(g-2)_\mu}{2}$$

- Today

Experiment (BNL 2004)	$a_\mu^{\mathrm{exp}} = 0.0011659209(6)$
Theory (Standard model)	$a_\mu^{\mathrm{th}} = 0.0011659183(5)$
Difference	$\sim (3-4)\,\sigma$

Hagiwara et al. (2011)

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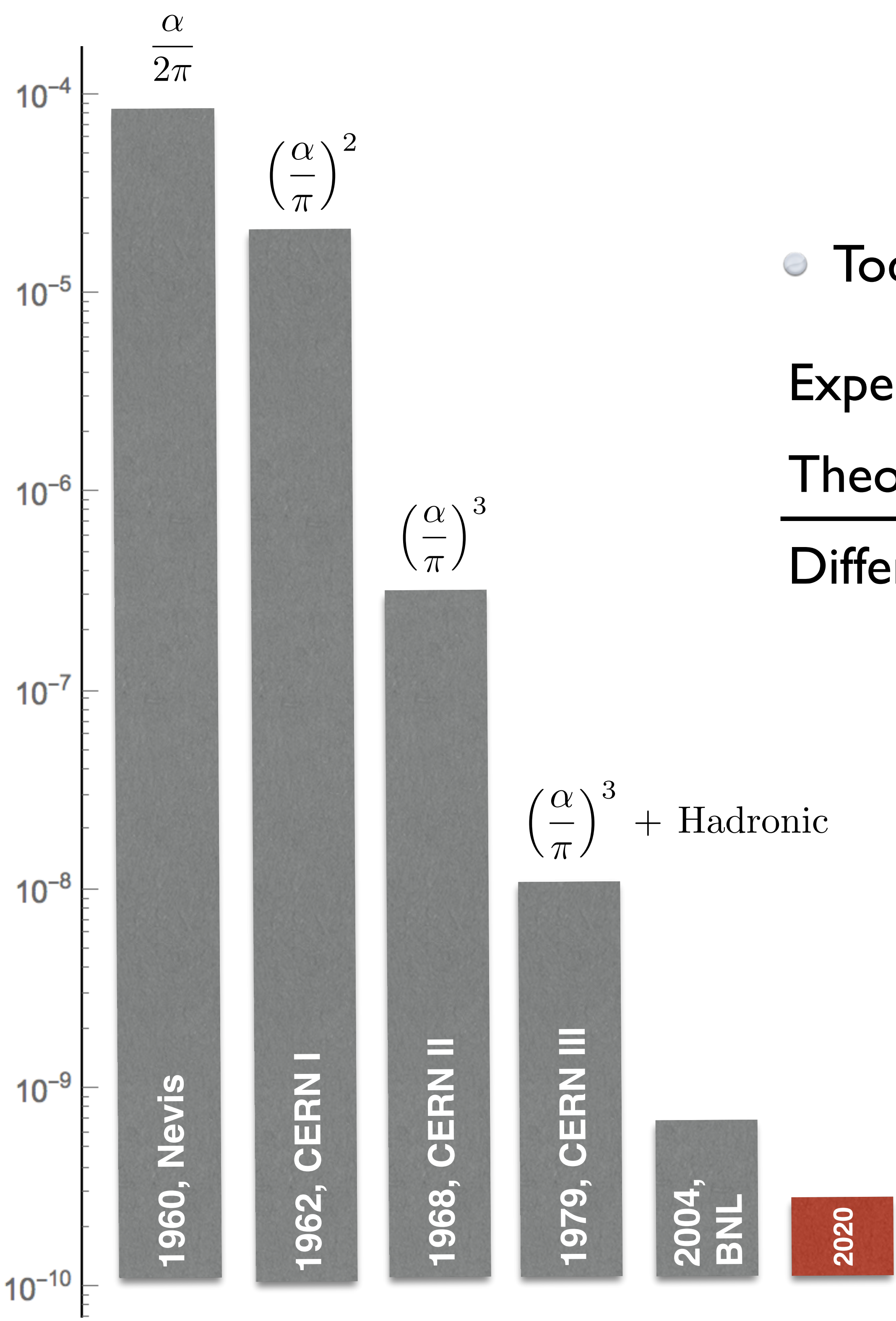

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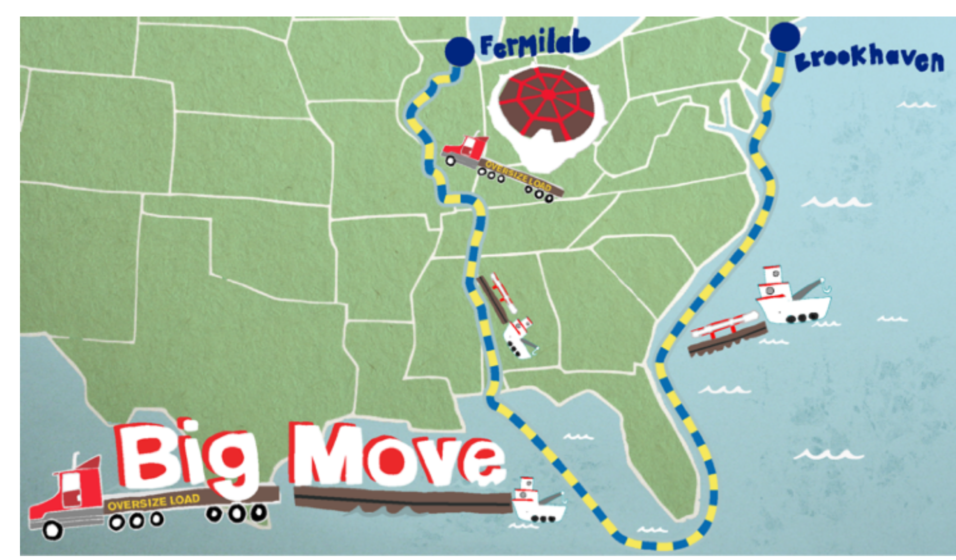
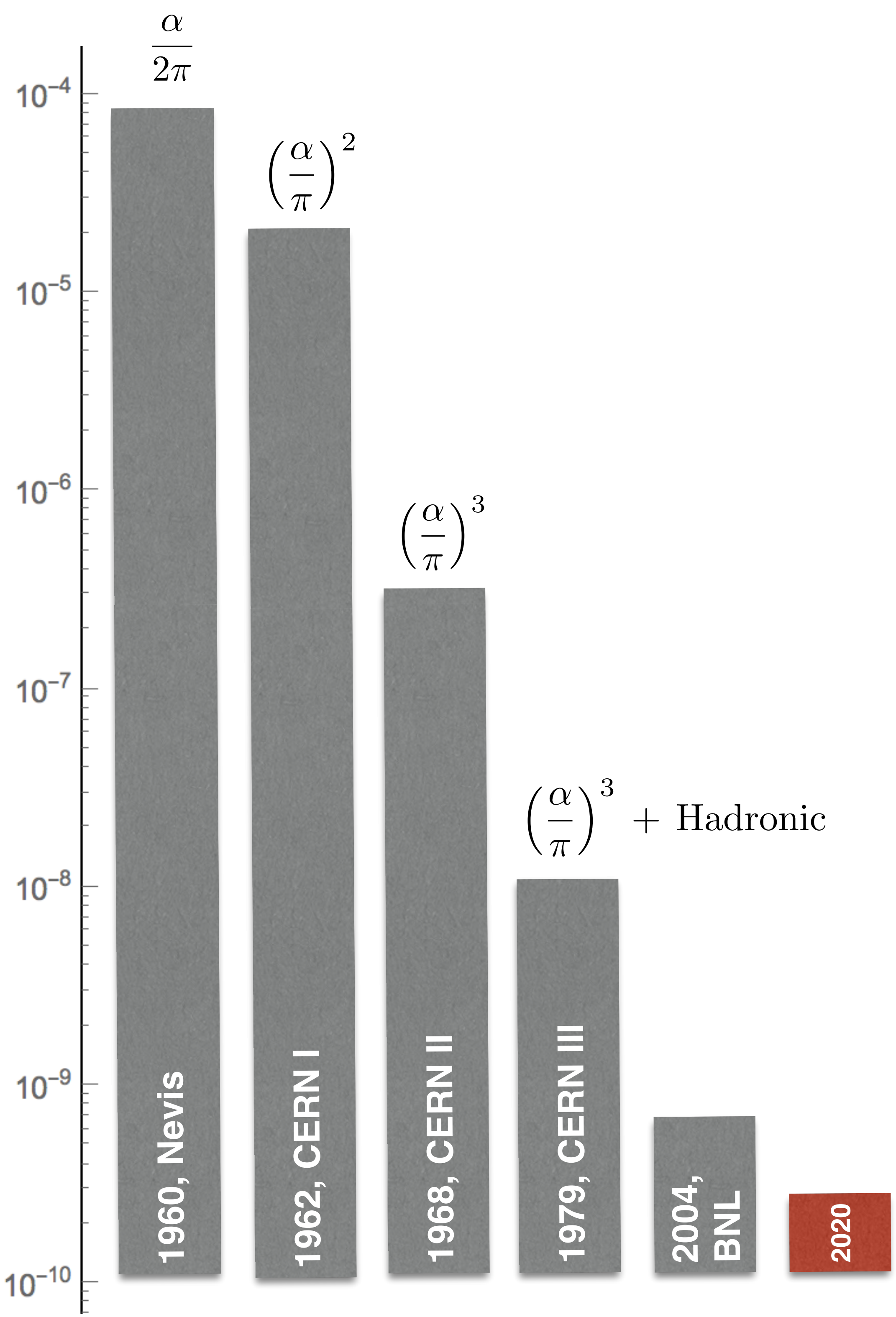
$$\Delta a_\mu^{\text{exp}} = 6.3 \times 10^{-10}$$
$$\to 1.6 \times 10^{-10}$$

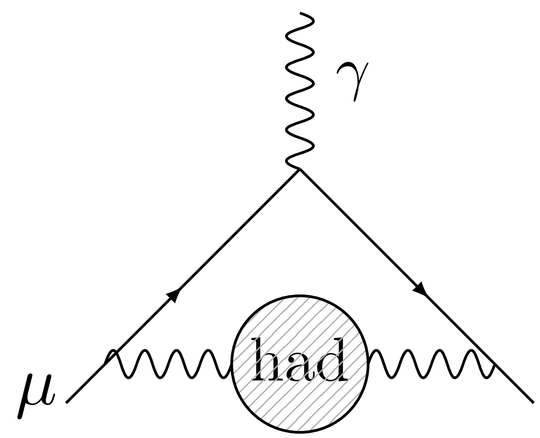


History of achieved accuracy

- Accuracy



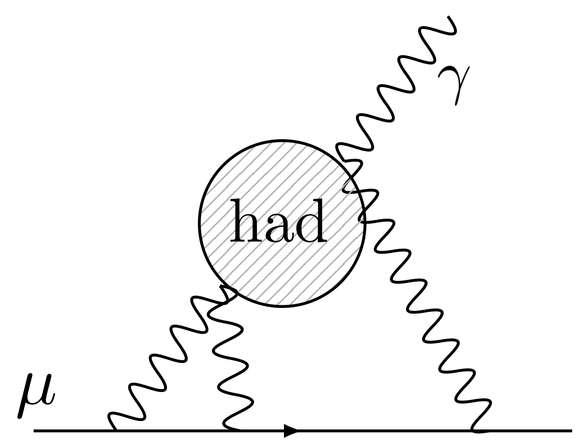
- Hadronic corrections



- hadronic vacuum polarization (LO + NLO)

$$a_\mu^{\text{had, VP}} = 685.1(4.3) \times 10^{-10}$$

Hagiwara et al. (2011)



- hadronic light-by-light scattering

$$a_\mu^{\text{had, LbL}} = 10.2(3.9) \times 10^{-10}$$

Prades et al. (2009)
Jegerlehner, Nyffeler (2009, 2015)

$$\Delta a_\mu^{\text{exp}} = 6.3 \times 10^{-10}$$
$$\rightarrow 1.6 \times 10^{-10}$$

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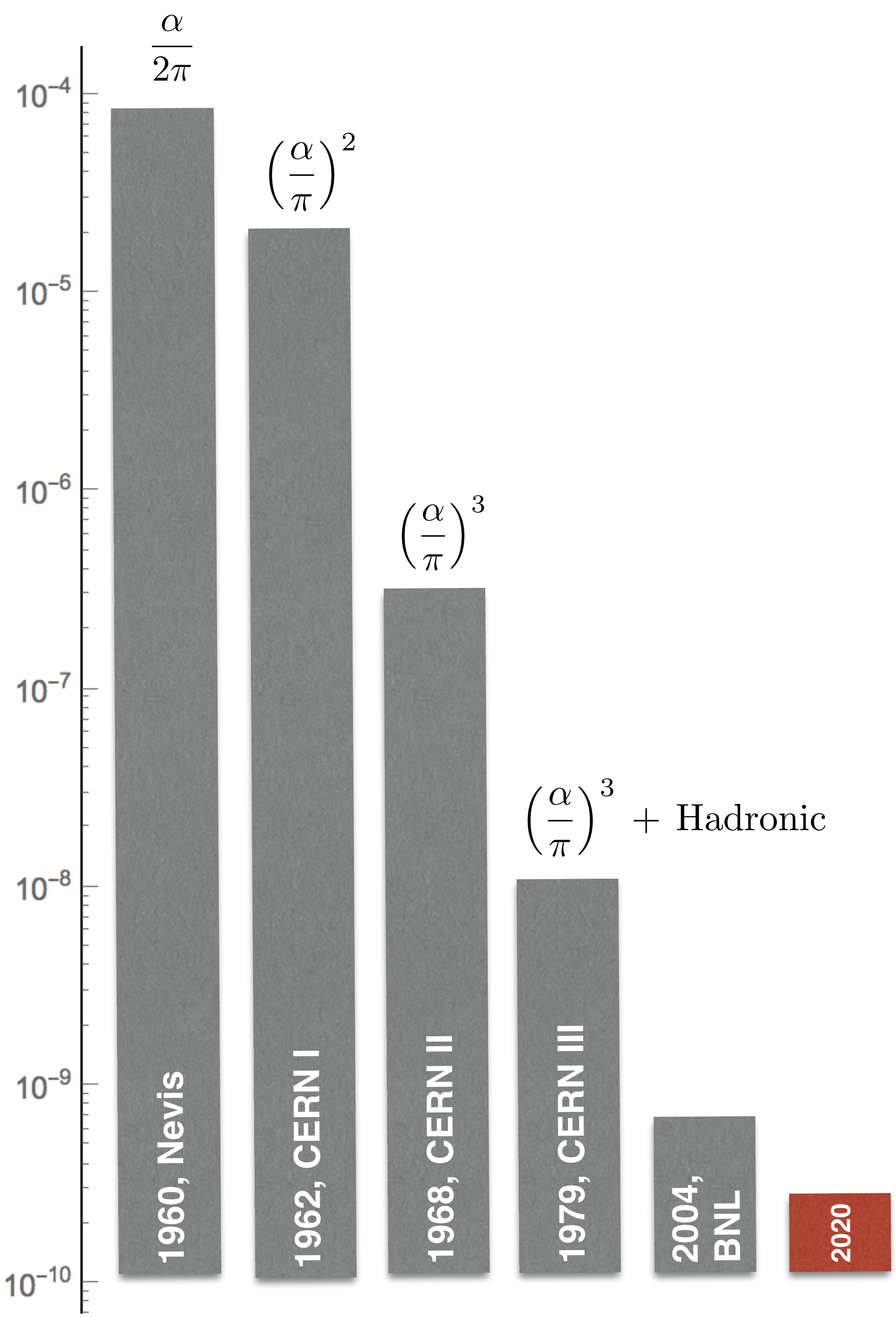

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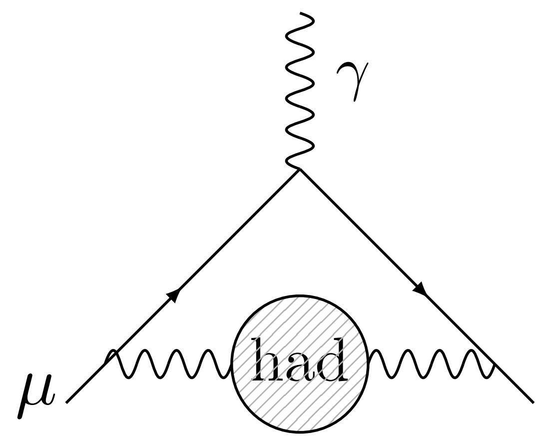

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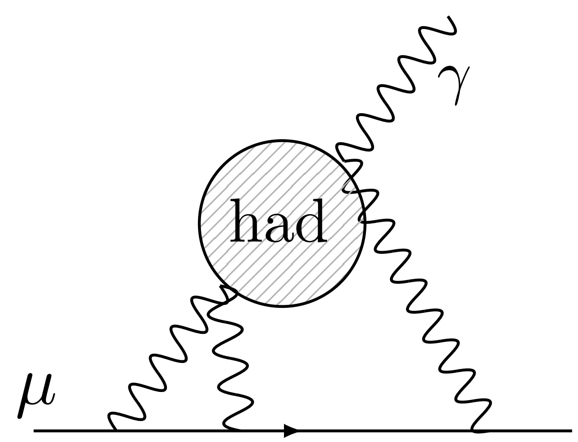
Hagiwara et al. (2011)
Keshavarzi et al. (2018)



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$$8.7(1.3) \times 10^{-10}$$

Prades et al. (2009)
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I.D, Redmer, Vanderhaeghen (2019)

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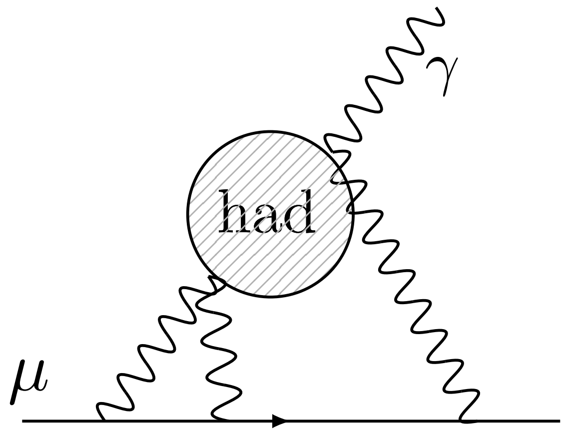
White paper (ongoing)

QCD contribution to (g-2)



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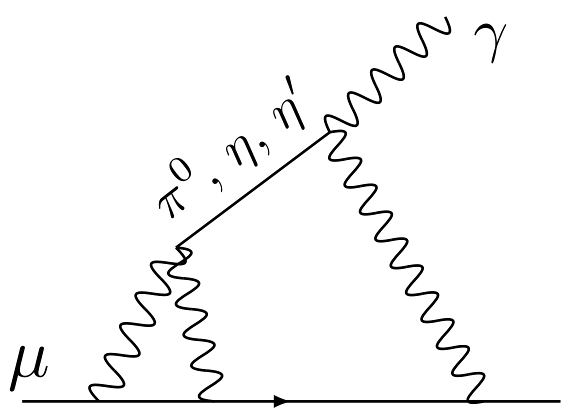

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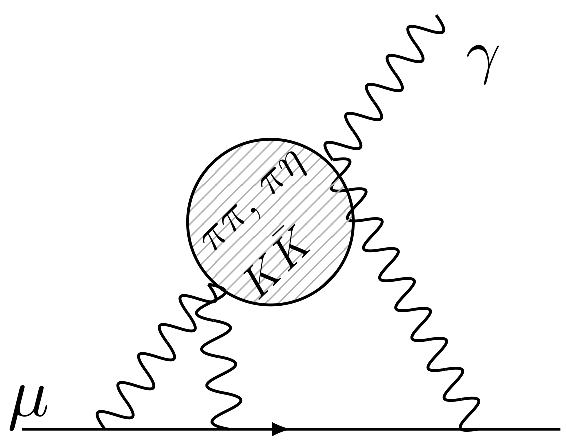
Relies on measurements of **TFF** $\pi^0\gamma^*\gamma^{(*)}, \eta\gamma^*\gamma^{(*)}, \ldots$ to reduce the model dependence



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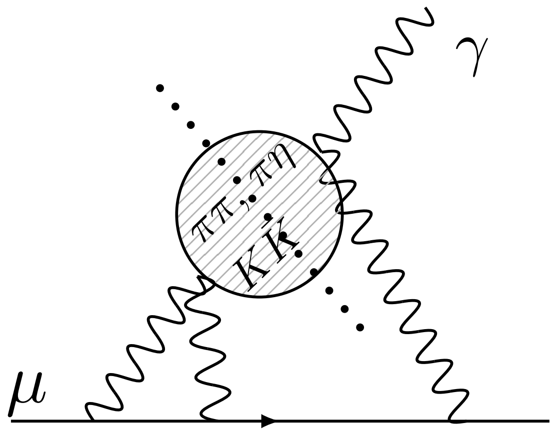
Dispersive analysis for ππ, πη, … loops is needed

…

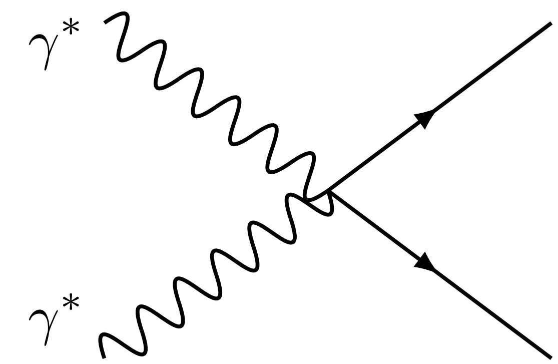
Multi-meson production



Important ingredient:

$$\gamma^*\gamma^* \to \pi\pi, \pi\eta, \ldots$$

$q^2 = -Q^2 < 0$ space-like γ^*



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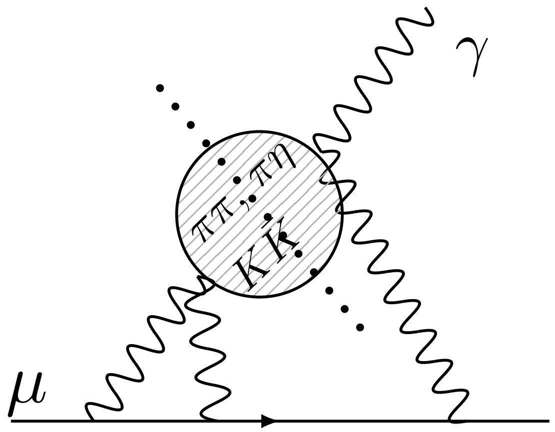

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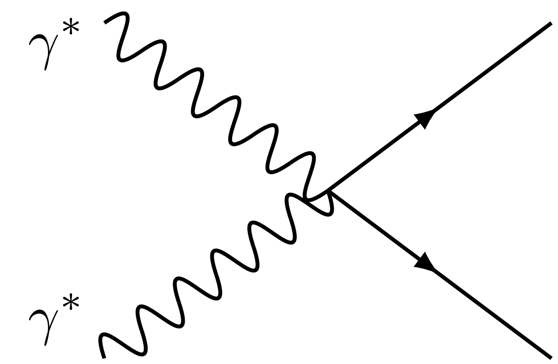
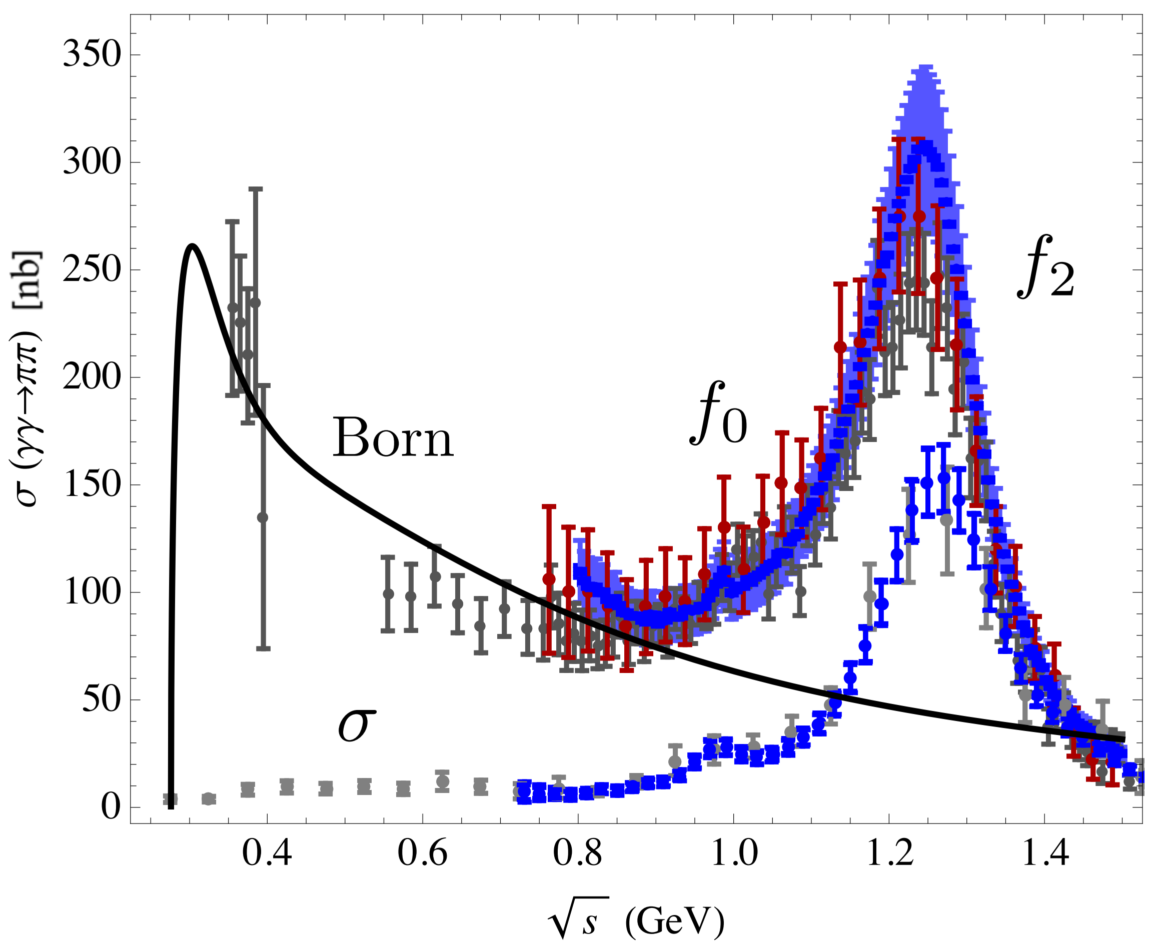
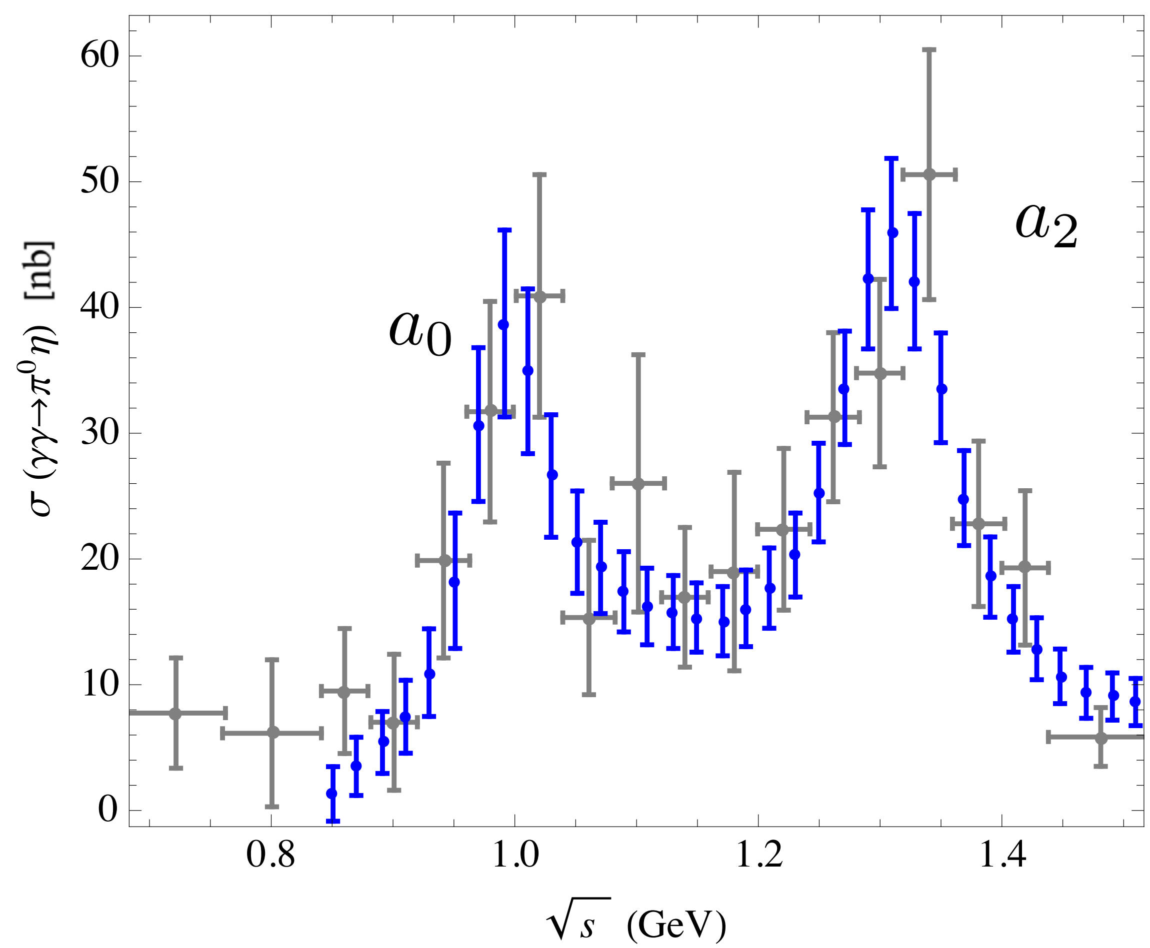


$\gamma\gamma \to \pi^+\pi^-, \pi^0\pi^0$



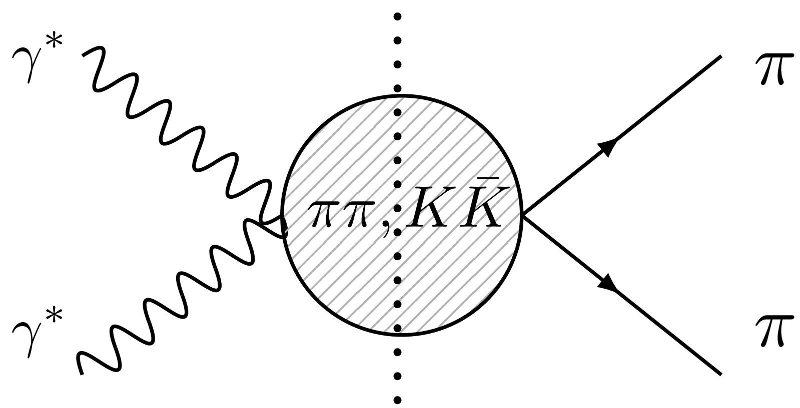
MarkII('90), CELLO ('92), Crystal Ball ('90), Belle ('07,'09)
Crystal Ball ('90), Belle ('09)

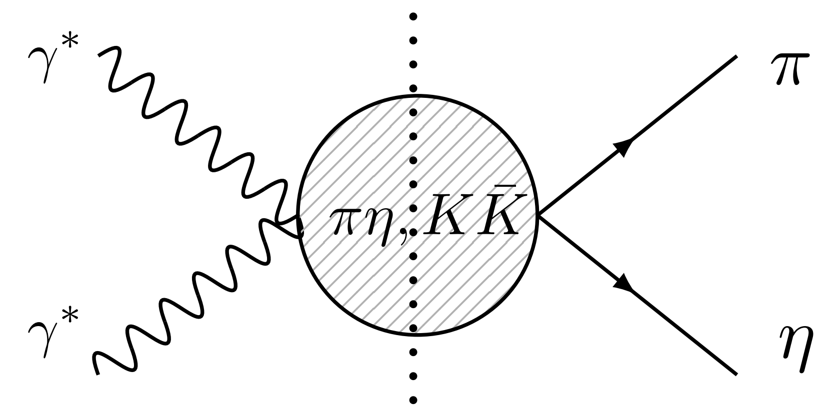
$\gamma\gamma \to \pi^0\eta$



+ Belle (2016) $\gamma\gamma^* \to \pi^0\pi^0$, $Q^2 > 3.5$ GeV^2

Inelastic contributions





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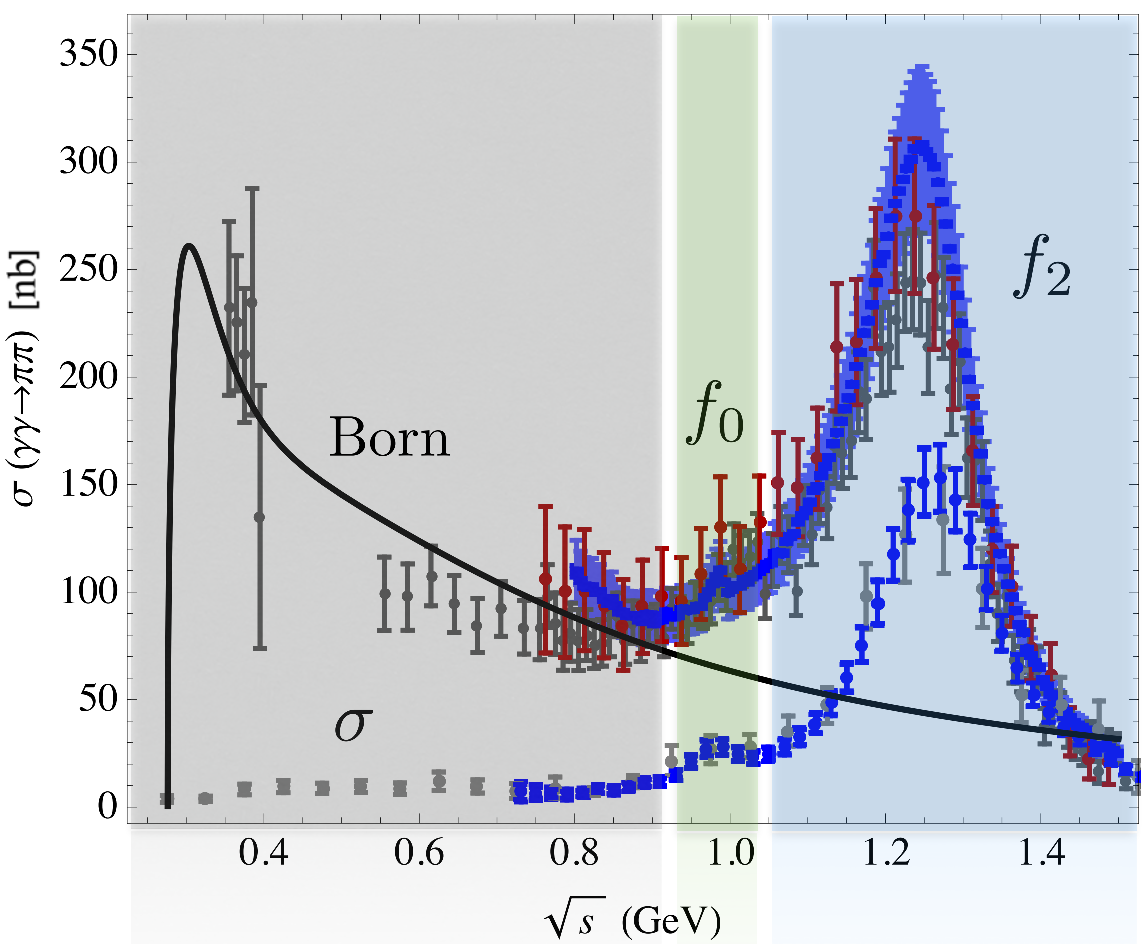

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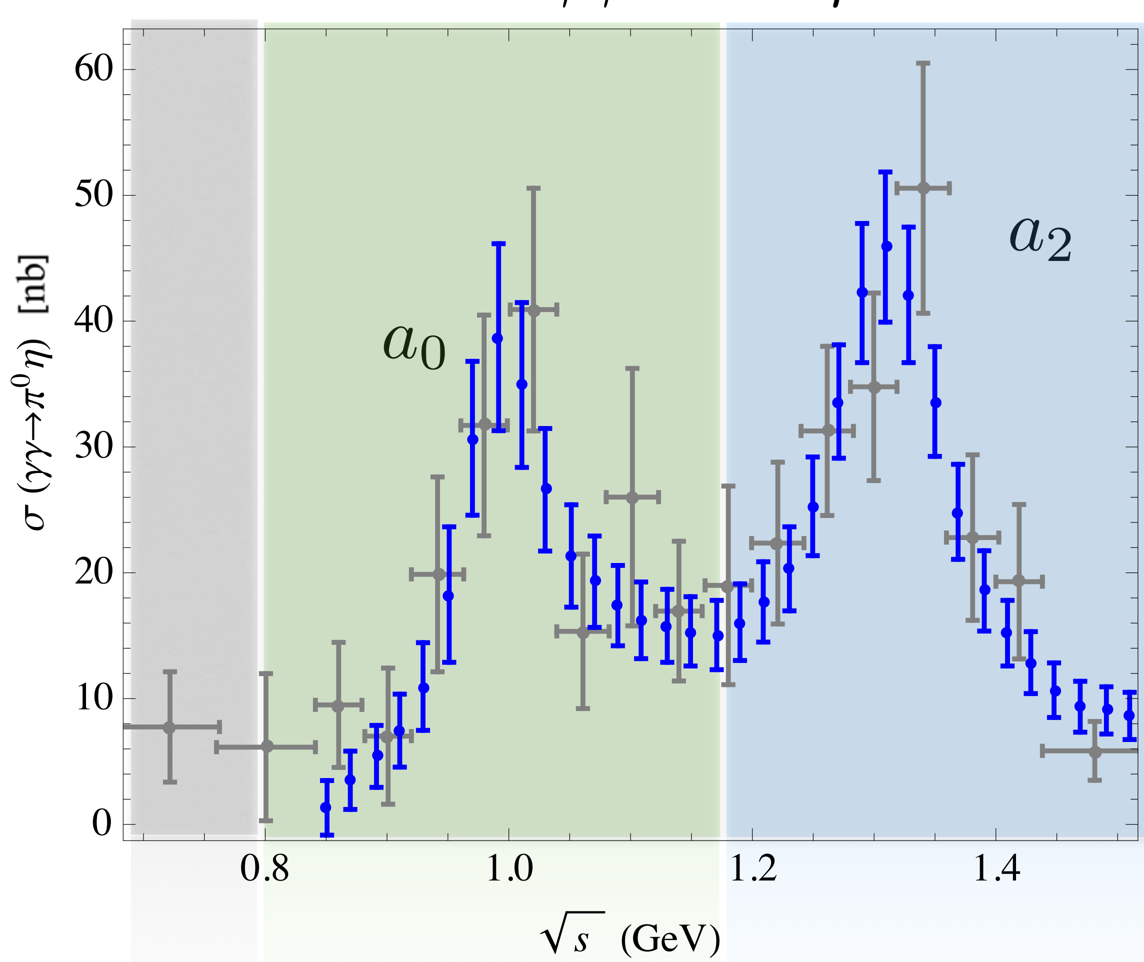
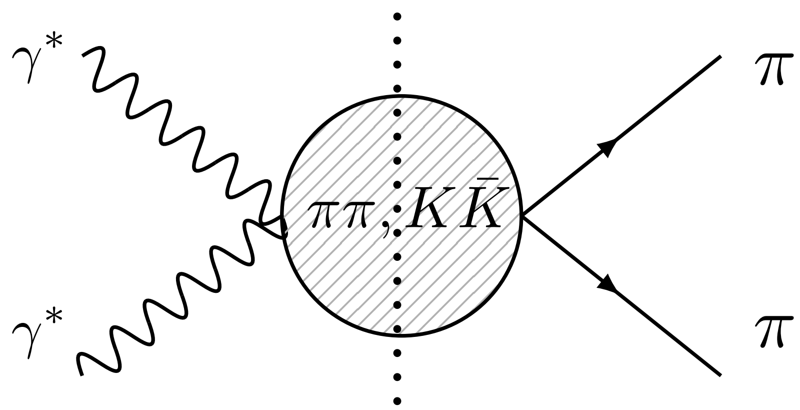
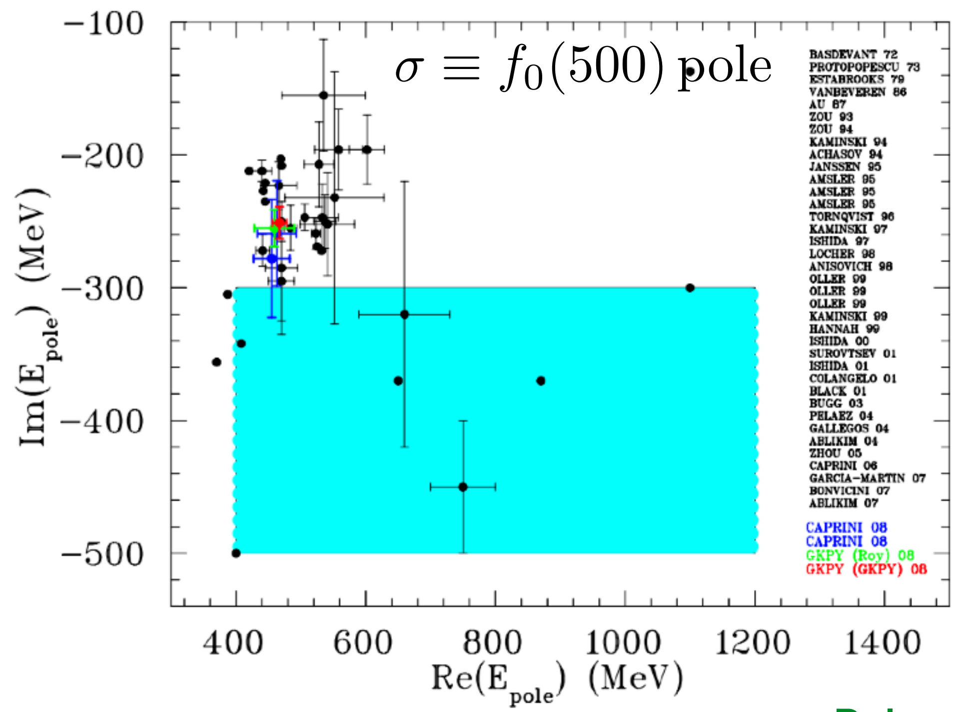


Right-hand cuts (hadronic input)





Pelaez (2016)

$$\sqrt{s^{\mathrm{II}}_{\mathrm{Roy}}} = \left(449^{+22}_{-16}\right) \pm i\,(275 \pm 12)\ \mathrm{MeV}$$

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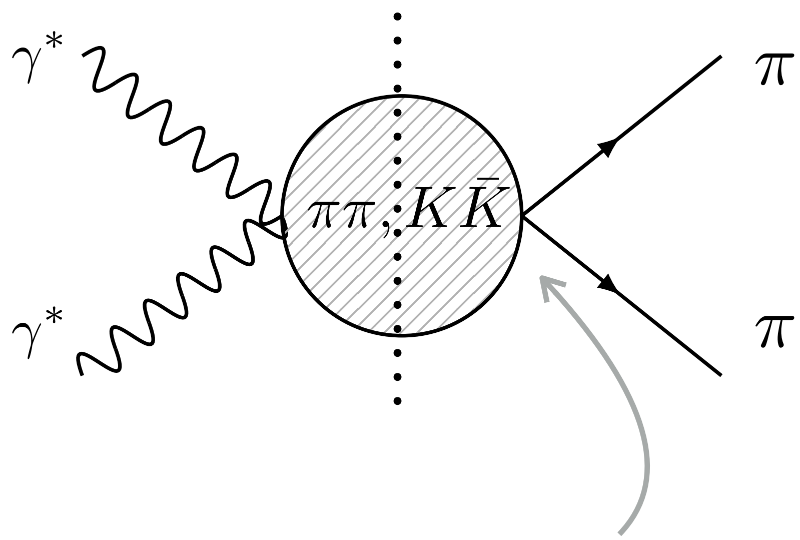

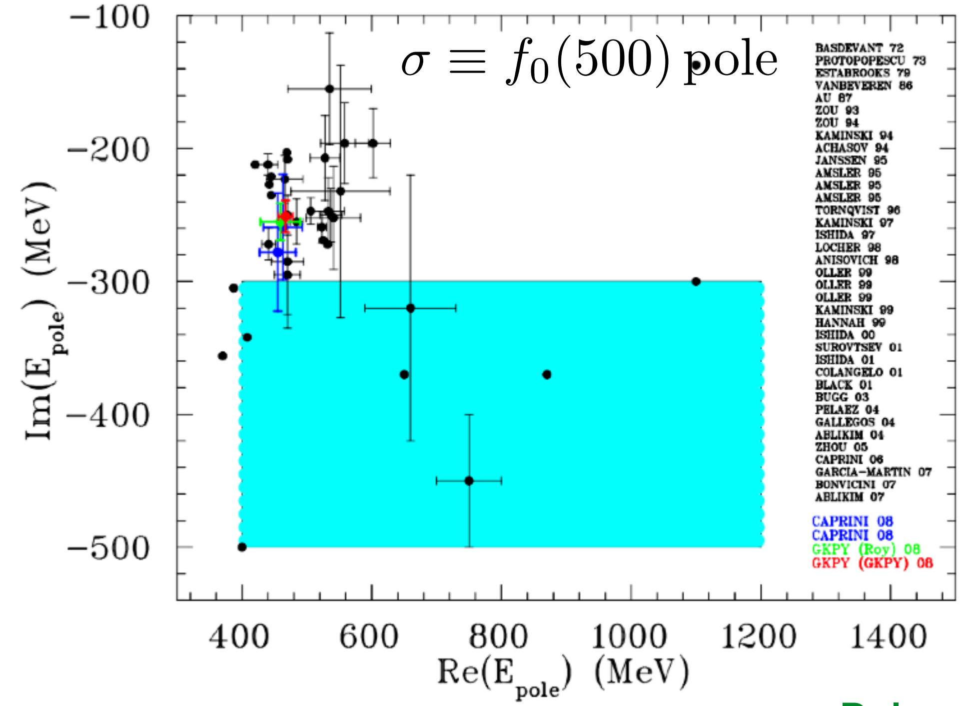

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- Coupled-channel Omnès formalism

$$t(s) = U(s) + \int_R \frac{ds'}{\pi} \frac{\rho(s')\,|t(s')|^2}{s' - s}$$

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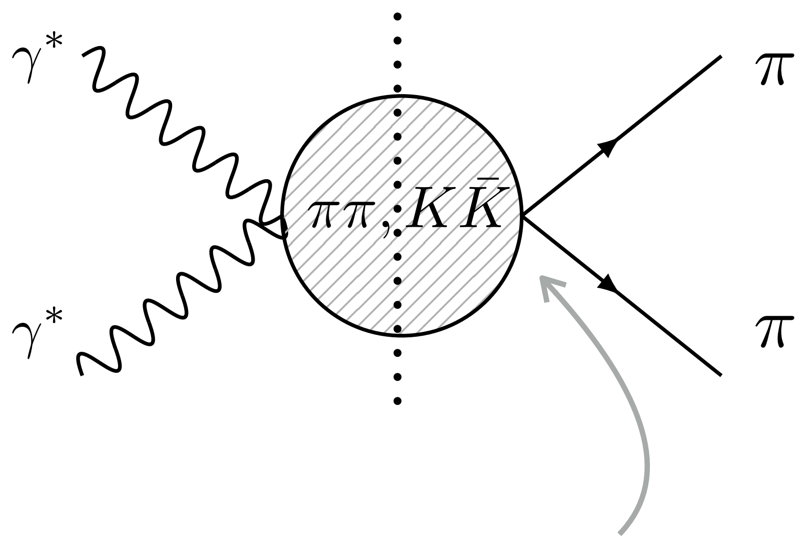

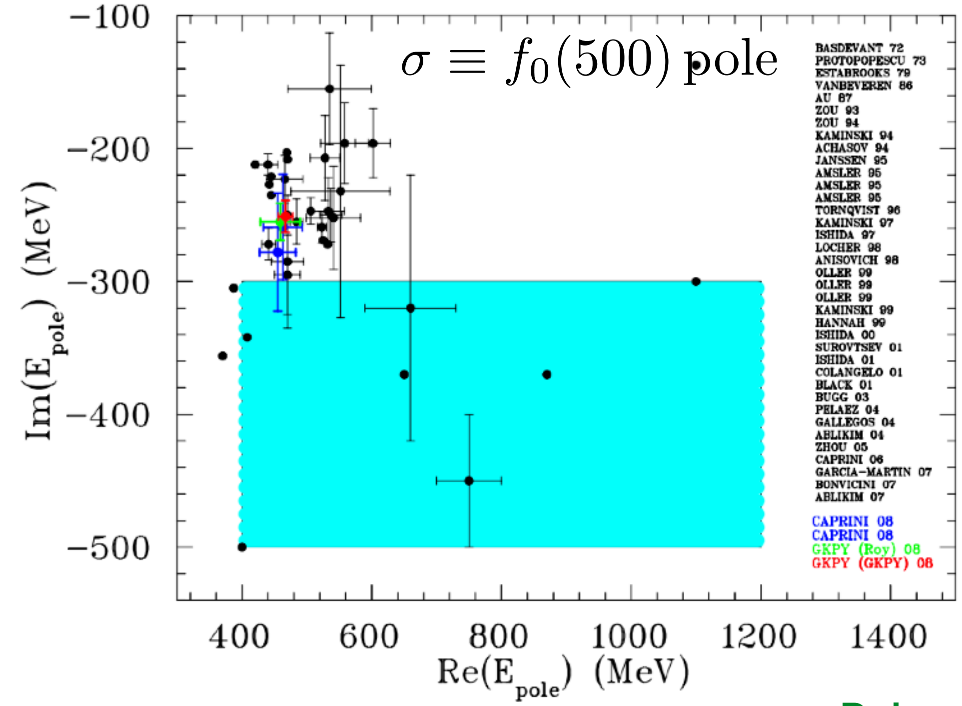

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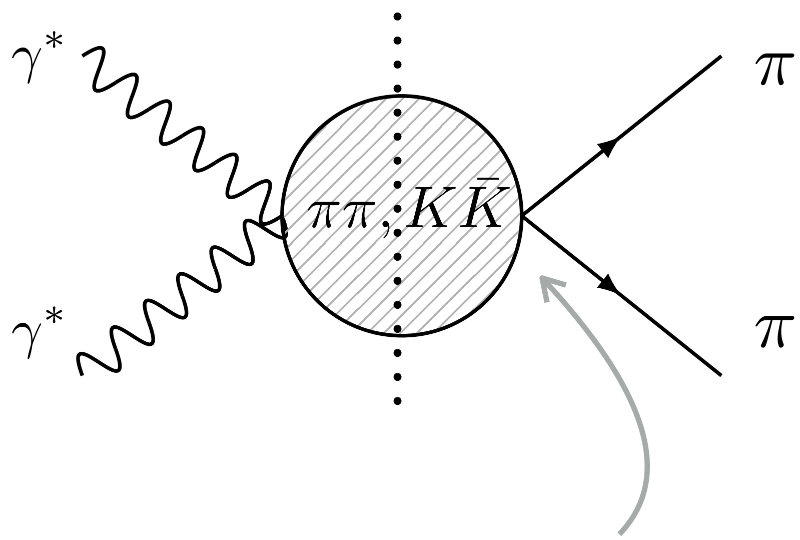

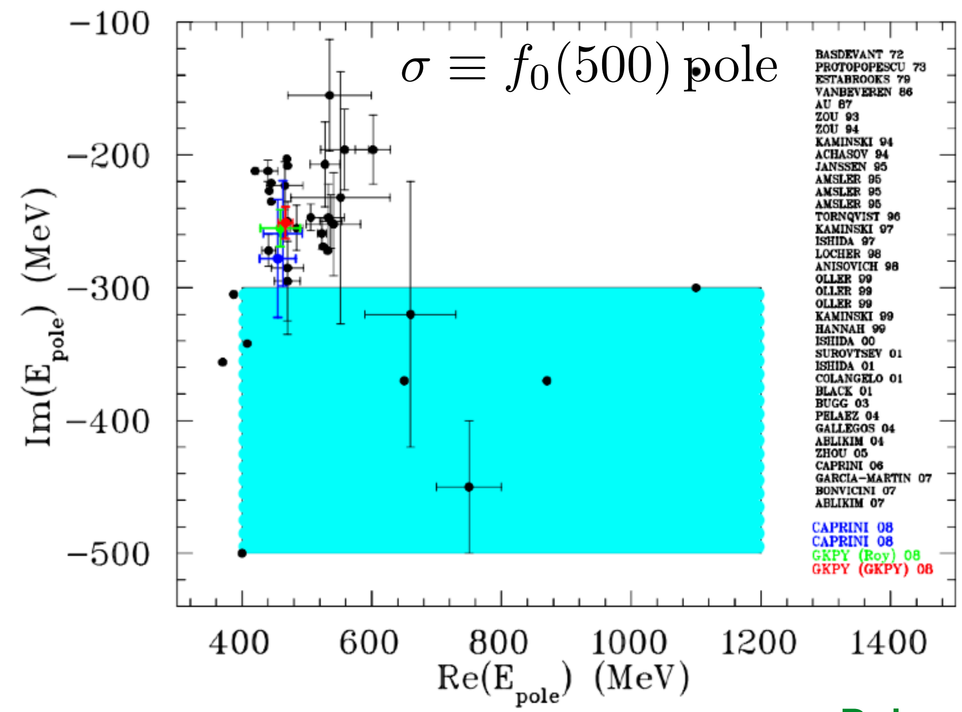

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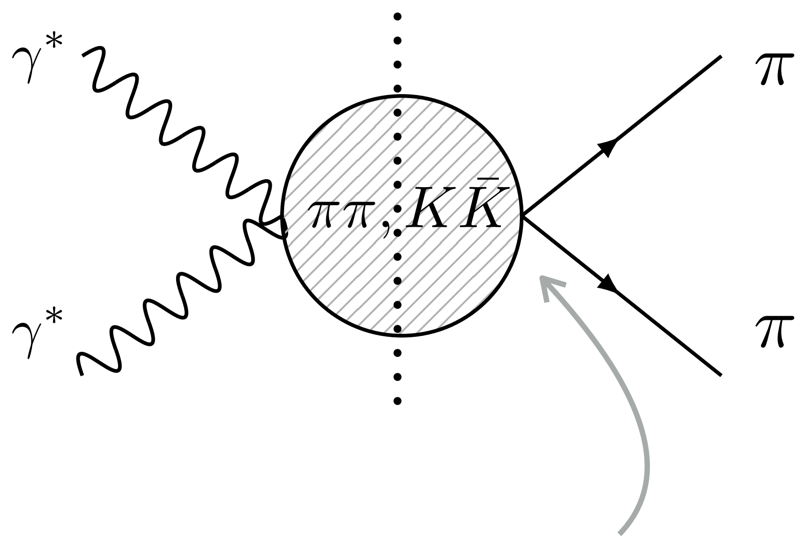

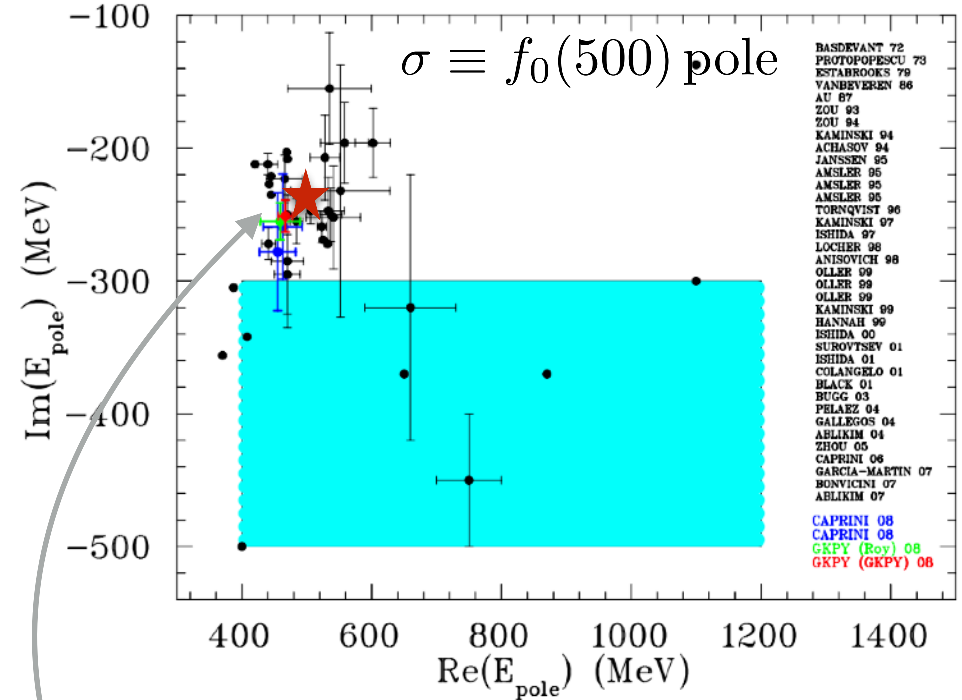

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$$\sqrt{s^{II}_{Roy}} = (449^{+22}_{-16}) \pm i\,(275 \pm 12)\ \text{MeV}$$
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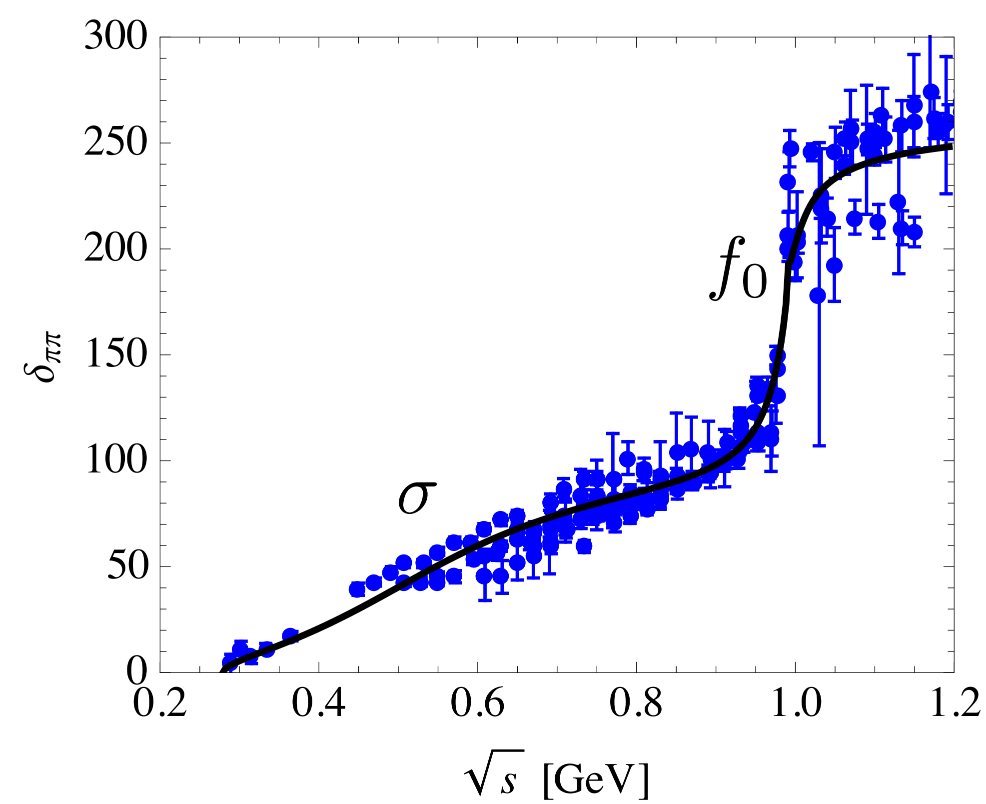

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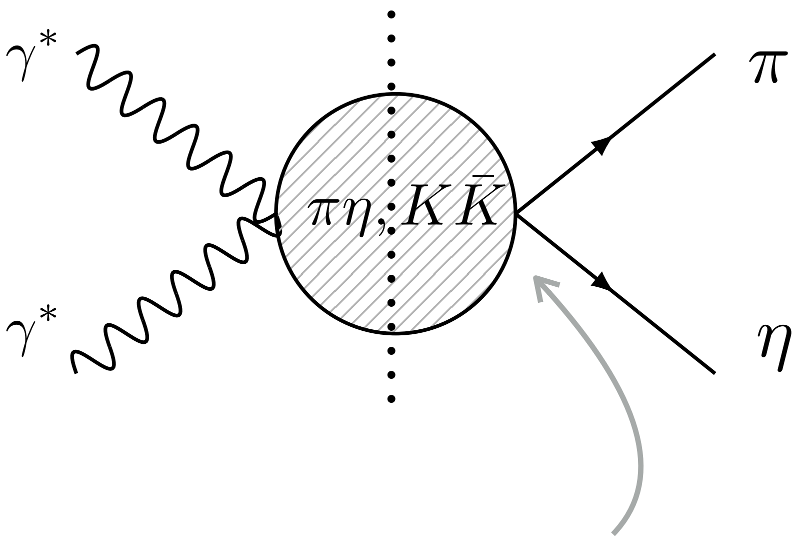

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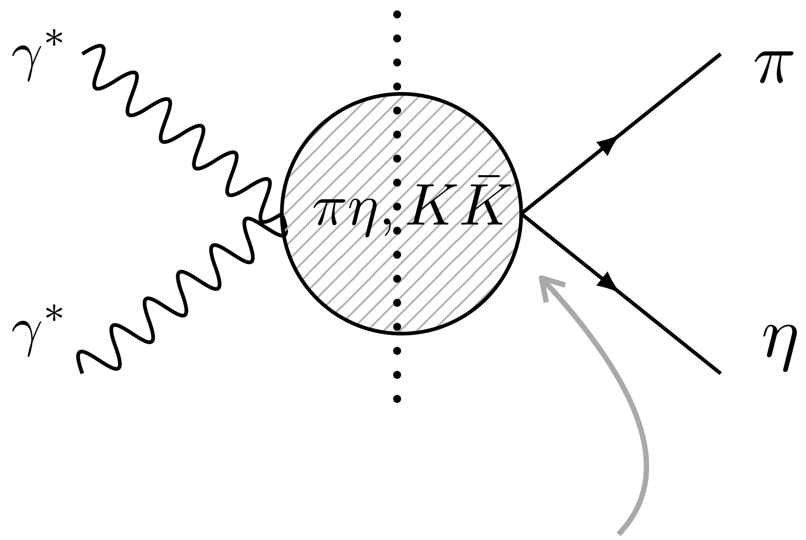

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I.D., Gil, Lutz (2011, 2013)

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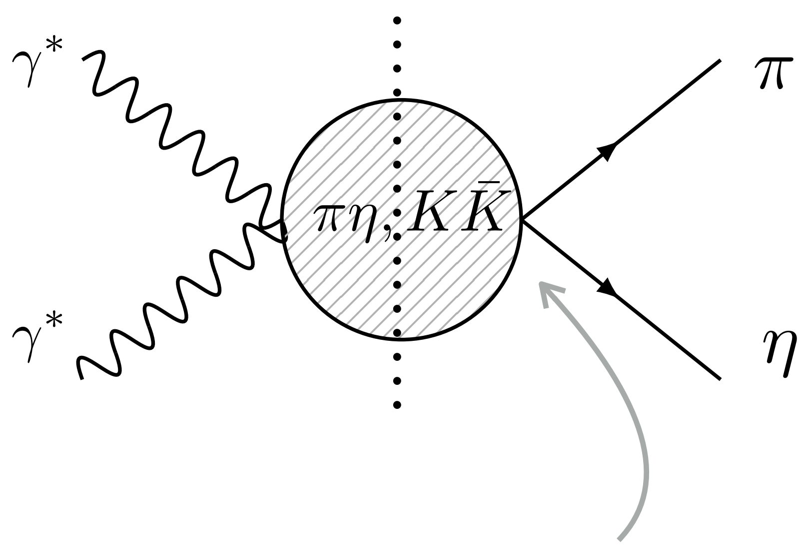

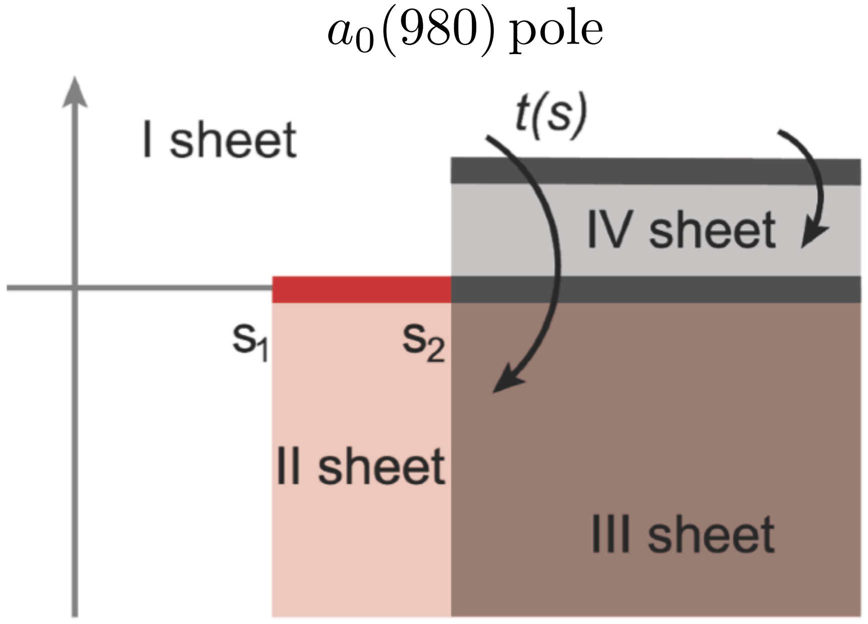

$$\sqrt{s_{a_0}^{\rm IV}} = (1.12^{-0.07}_{+0.02}) \pm \frac{i}{2}\,(0.28^{+0.08}_{-0.13})\ \text{GeV}$$

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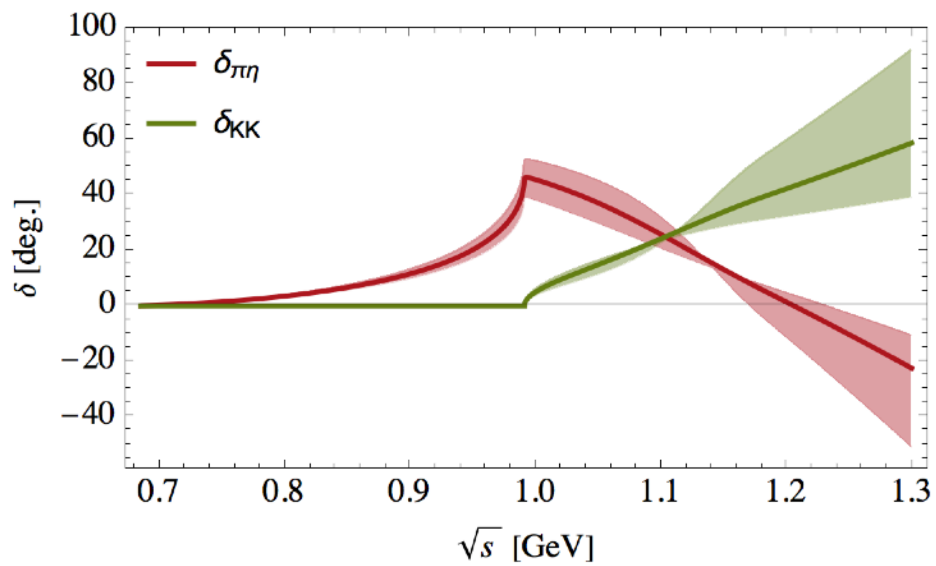
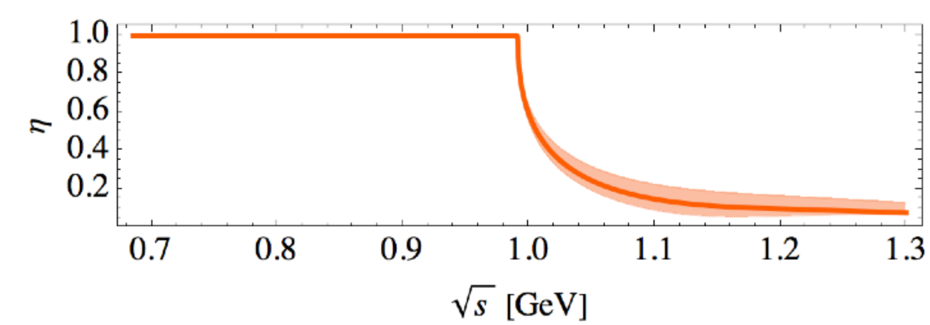
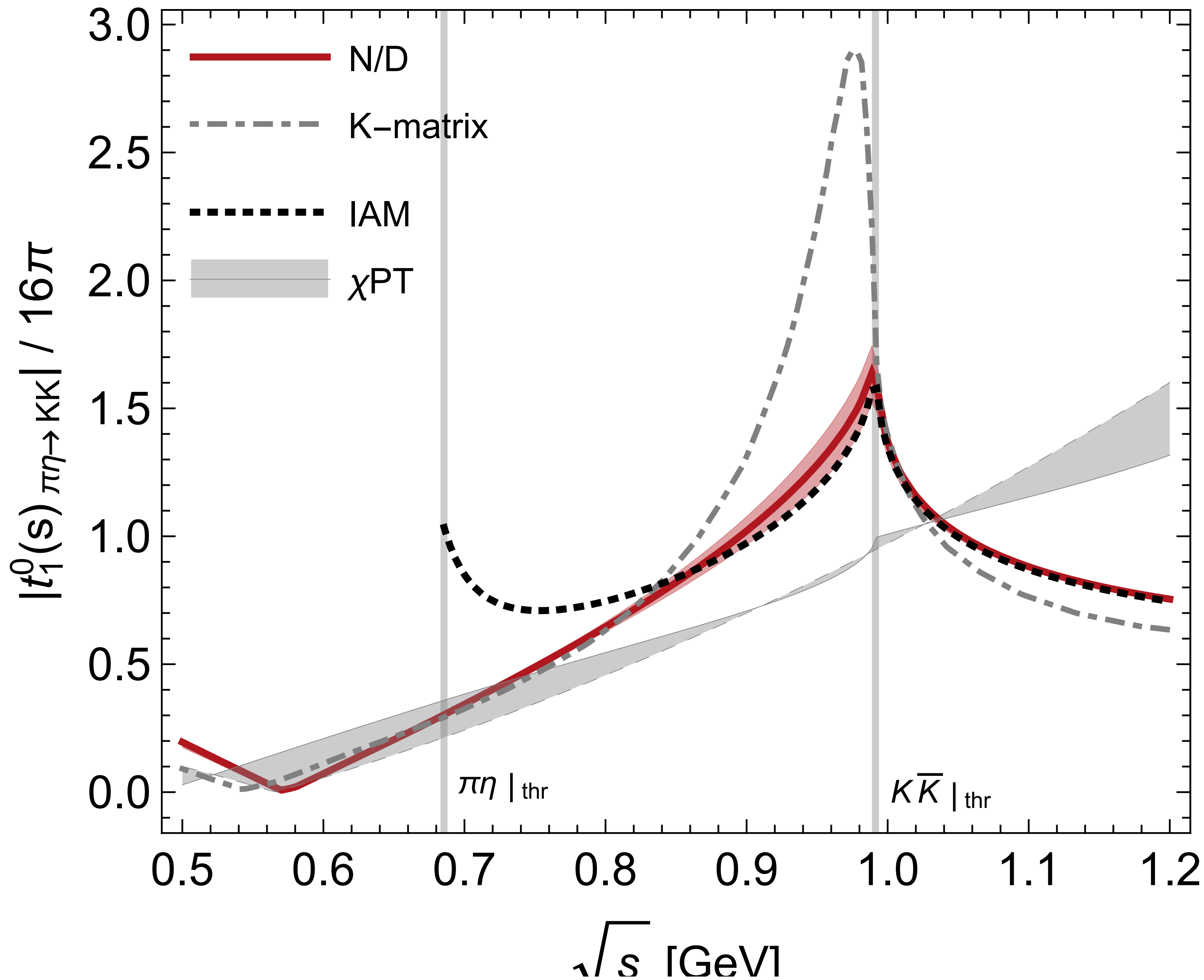




cf. also HadSpec Coll. (2016)

Scattering amplitude $\pi\eta \to KK$



N/D
I.D., Gil, Lutz (2013), I.D., Deineka, Vanderhaeghen (2017)

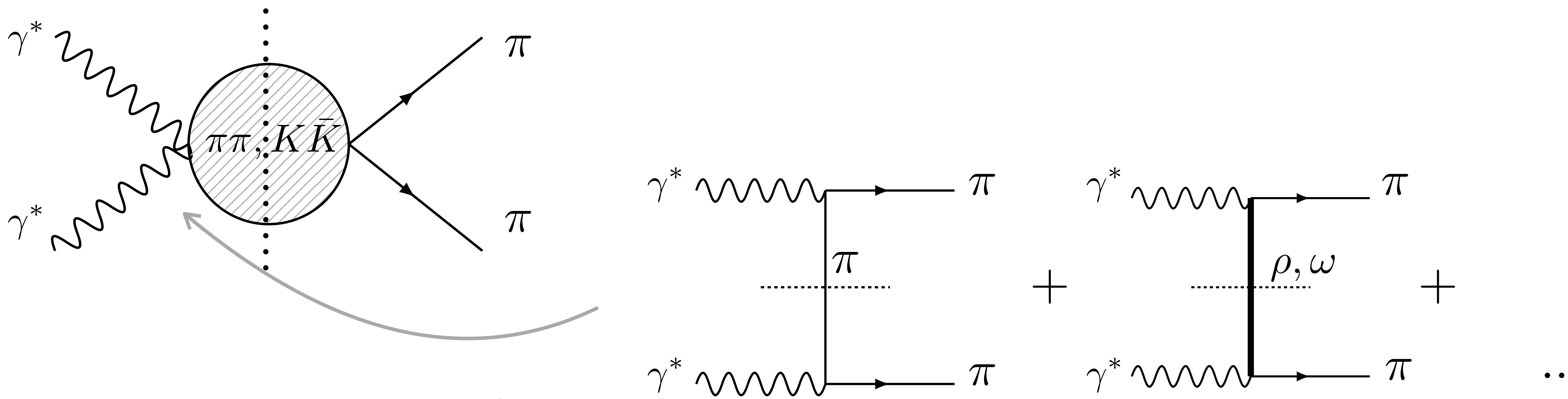
K-matrix
Albaladejo et. al. (2017)

Inverse Amplitude Method (IAM)
Gomez Nicola et.al. (2002)

Chiral Perturbation Theory
Gasser et. al. (1985)

- First lattice analysis for m_π=391 MeV HadSpec Coll. (2016)
- Chiral extrapolation of the lattice results Zhi-Hui Guo et. al. (2017)

Left-hand cuts (pion pole)



- Left-hand cuts requires knowledge from $\gamma^*\pi\pi,\ \gamma^*\pi\omega,\ \gamma^*\pi\rho$ transition form factors

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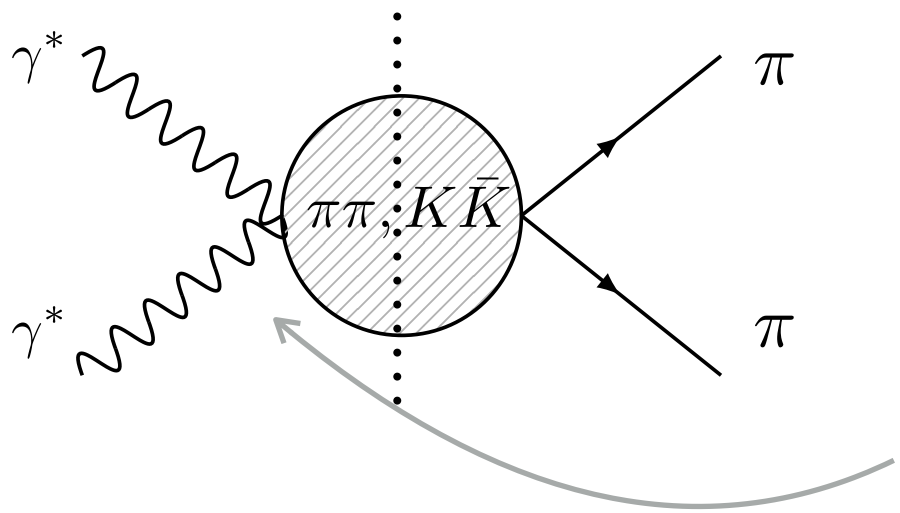

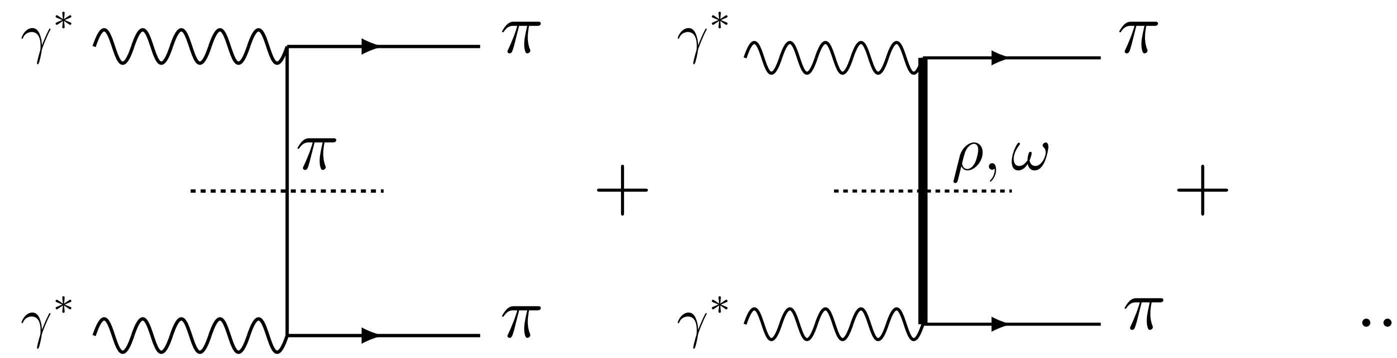

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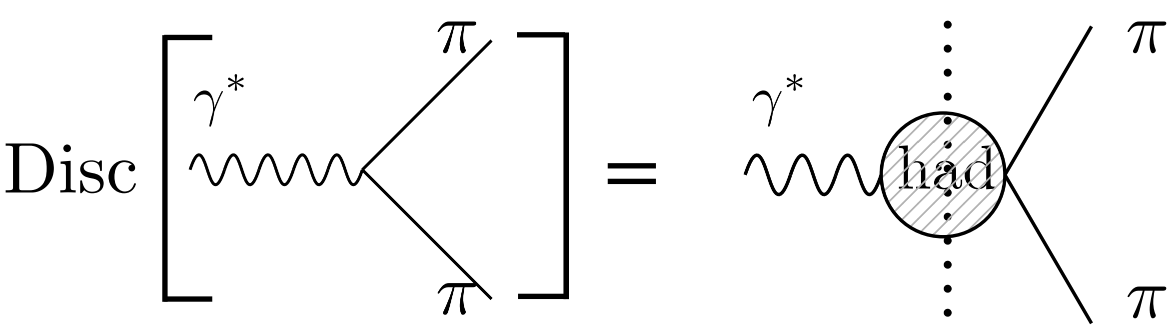
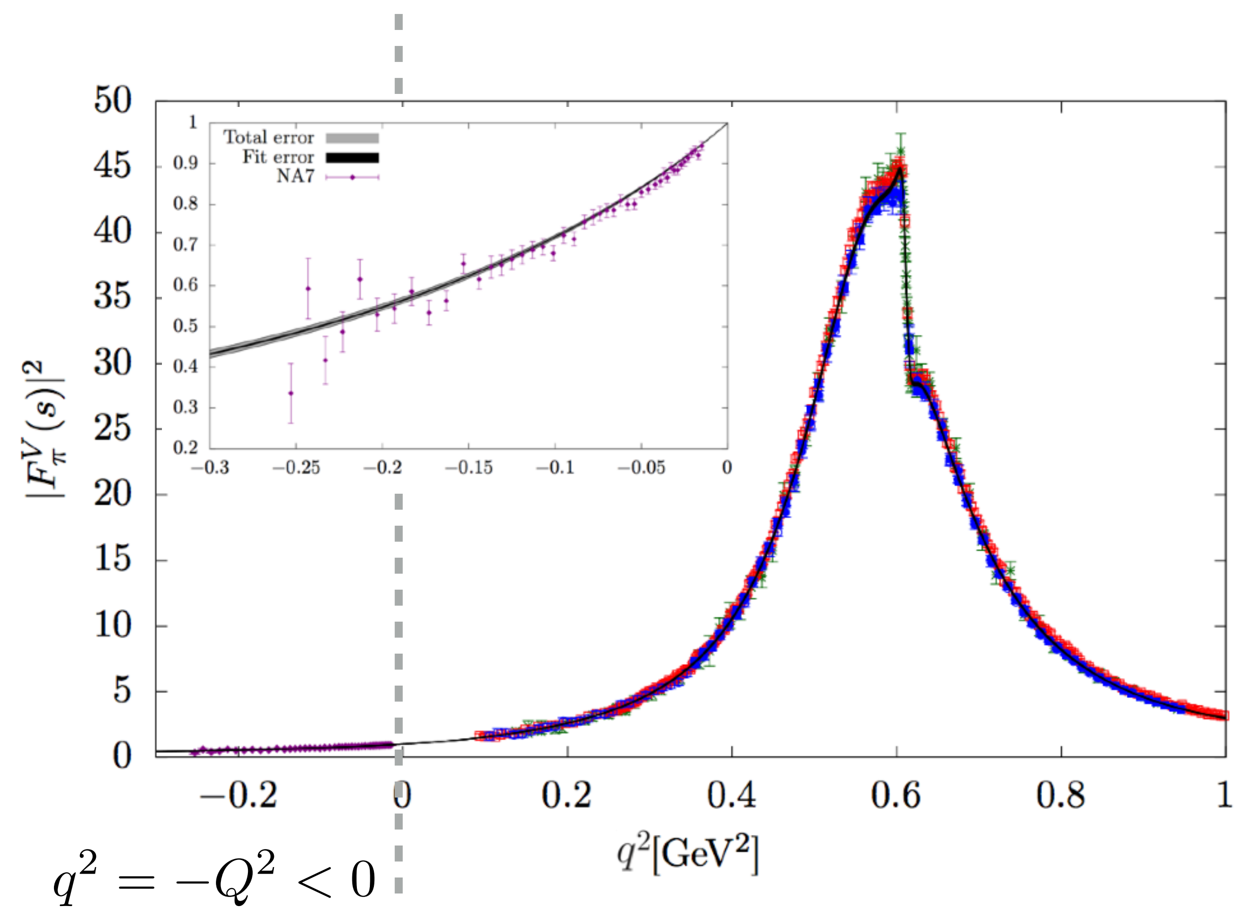
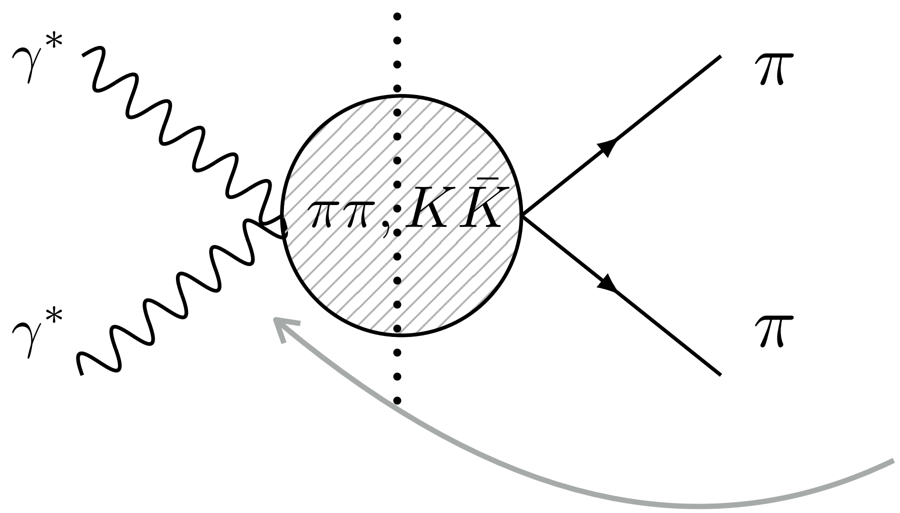


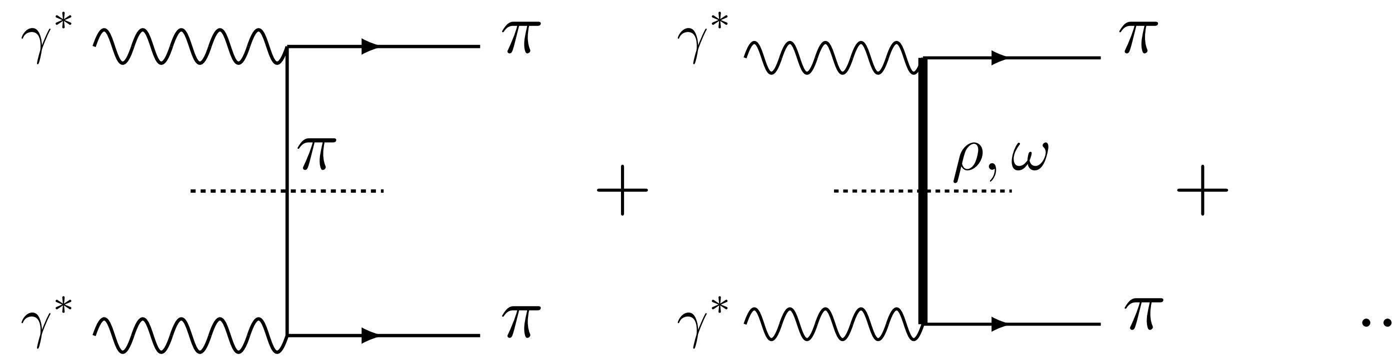
- Input from data: monopole TFF works well for space like region



Colangelo et al. (2019)

Left-hand cuts (vector poles)





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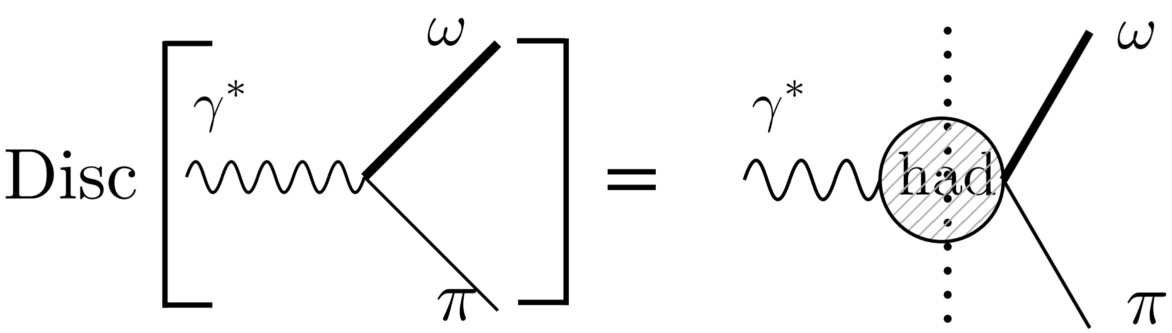
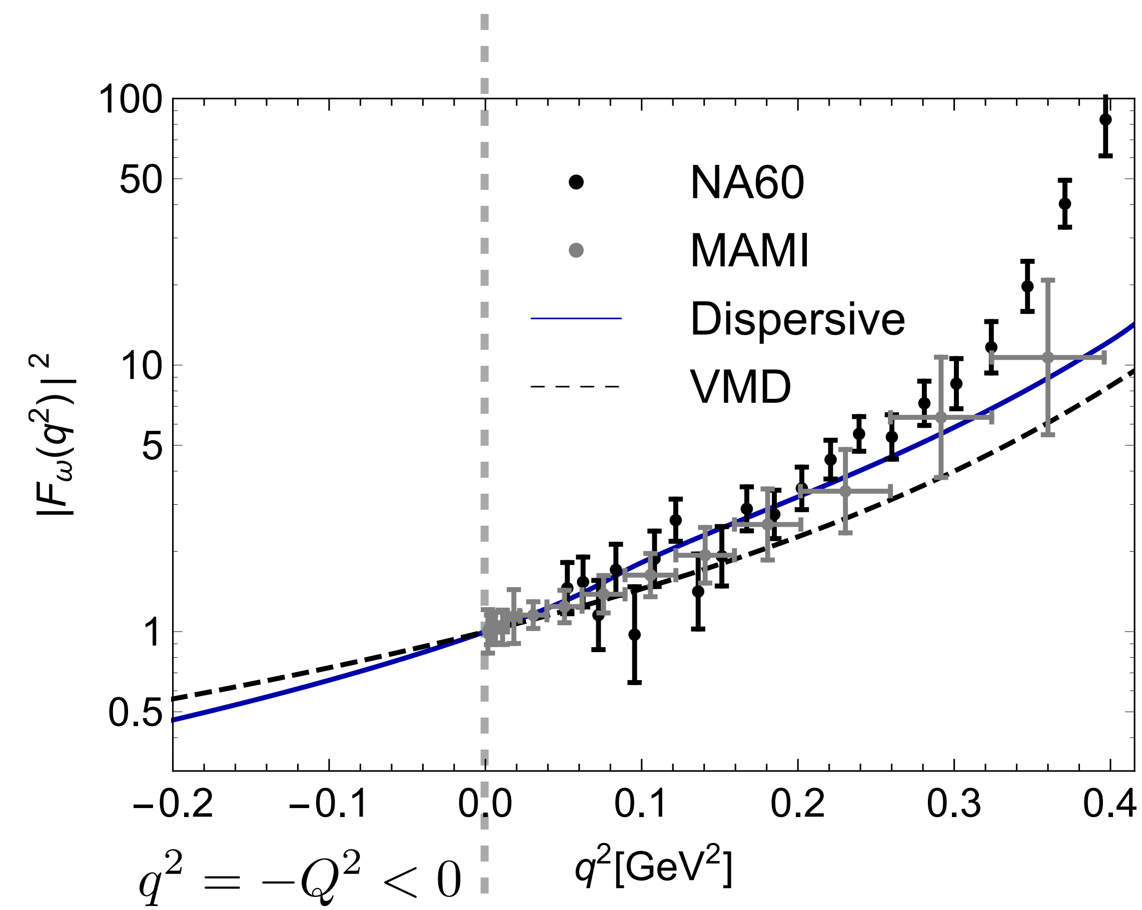
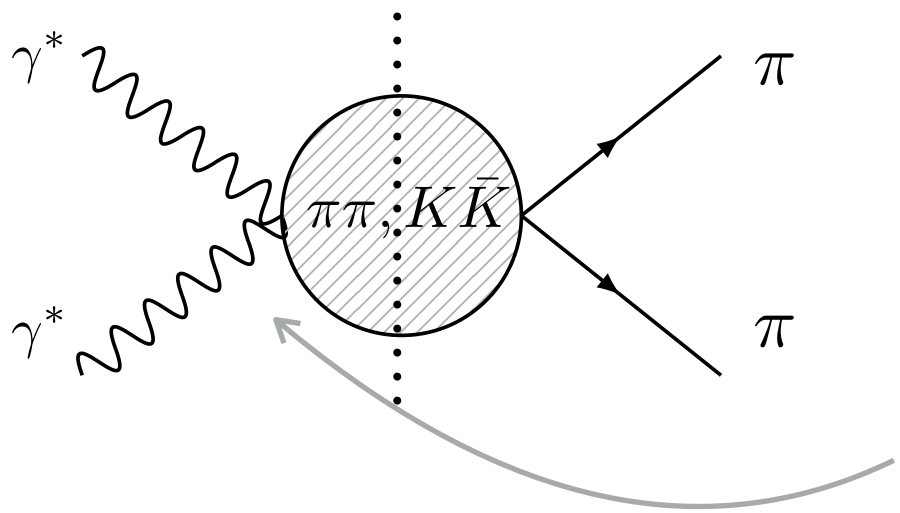


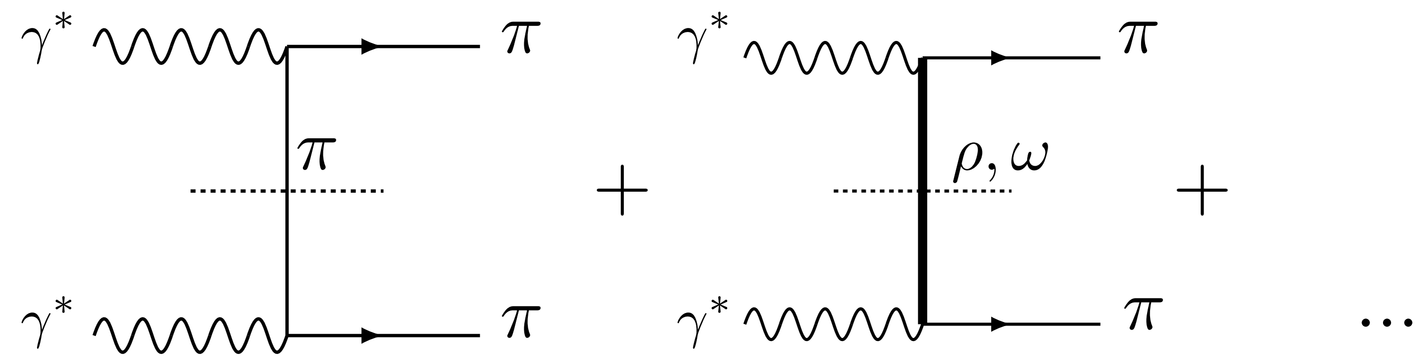


- Input from: $\omega \to 3\pi$

Khuri-Treiman (1960)
Schneider et al. (2012)
I.D. & JPAC (2015)

Left-hand cuts (vector poles)





- Left-hand cuts requires knowledge from $\gamma^*\pi\pi,\ \gamma^*\pi\omega,\ \gamma^*\pi\rho$ transition form factors

- Fitted parameter is the coupling: $g_{V\to\pi\gamma} \simeq C_{\rho^{\pm,0}\to\pi^{\pm,0}\gamma} \simeq \frac{1}{3} C_{\omega\to\pi^0\gamma} \overset{\text{PDG}}{=} 0.37(2)\ \text{GeV}^{-1}$

$g_{V\to\pi\gamma} = 0.33\ \text{GeV}^{-1}$ PDG (2018)

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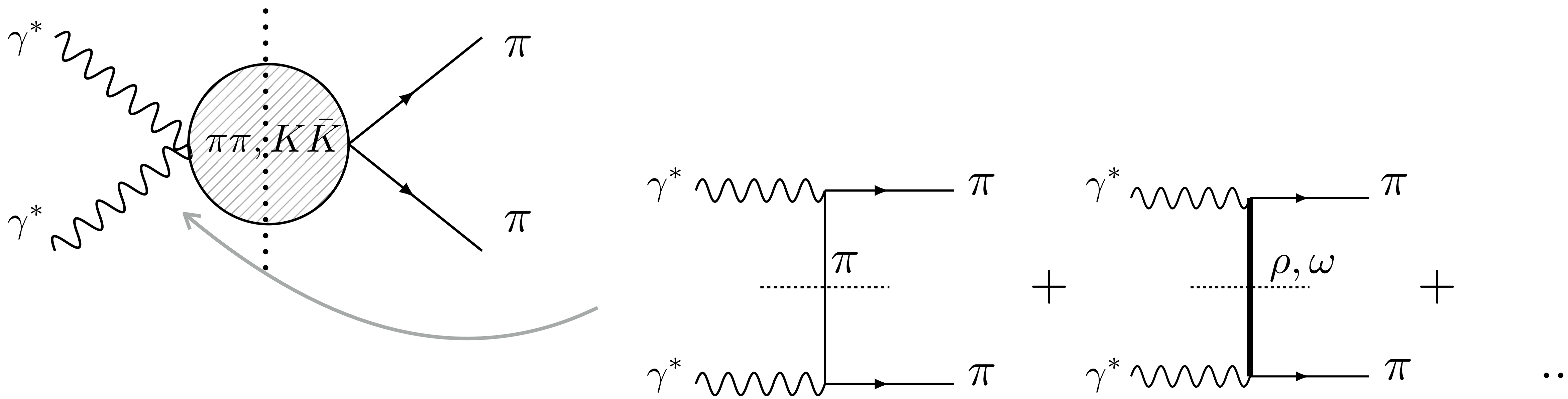

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$$g_{V\to\pi\gamma} = 0.33\ \text{GeV}^{-1}$$

PDG (2018)

- Left-hand cuts: "anomalous thresholds" for large virtualities $Q_1^2 Q_2^2 > (M_V^2 - m_\pi^2)^2$

Hoferichter, Stoffer (2019)
Mandelstam (1960)

Kinematic constraints

- Helicity amplitudes

$$H_{\lambda_1,\lambda_2} = \epsilon_\mu(\lambda_1)\epsilon_\nu(\lambda_2) \sum_{i=1}^{5} F_i(s,t)\, L_i^{\mu\nu}$$

Tarrach (1975)
Drechsel, Metz et al (1998)

where $\lambda_{1,2} = \pm 1, 0$ are photon helicities
(minimal basis for Born subtracted amplitudes)

Low et al. (1954)

- p.w. helicity amplitudes suffer from kinematic constraints

$$h^{(J)}_{\lambda_1\lambda_2} = \int \frac{d\cos\theta}{2}\, d^J_{\lambda_1-\lambda_2,0}(\theta)\, H_{\lambda_1\lambda_2}$$

$$A^{(J)}_n = \frac{1}{(p\,q)^J} \int \frac{d\cos\theta}{2}\, P_J(\theta)\, F_n(s,t)$$ ⟵ object free of kinematic constraints

Lutz et al. (2010, 2014)

- Unconstrained basis for Born subtracted p.w. amplitudes

$$\bar{h}^{(J)}_i \equiv h^{(J)}_i - h^{(J),\mathrm{Born}}_i$$

$$\bar{h}^{(J)}_i = K_{ij}\, \bar{h}^{(J)}_j \qquad j \equiv \lambda_1\lambda_2 = \{++,\, +-,\, +0,\, 0+,\, 00\}$$

K_{ij} is 5×5 matrix

Dispersion relation

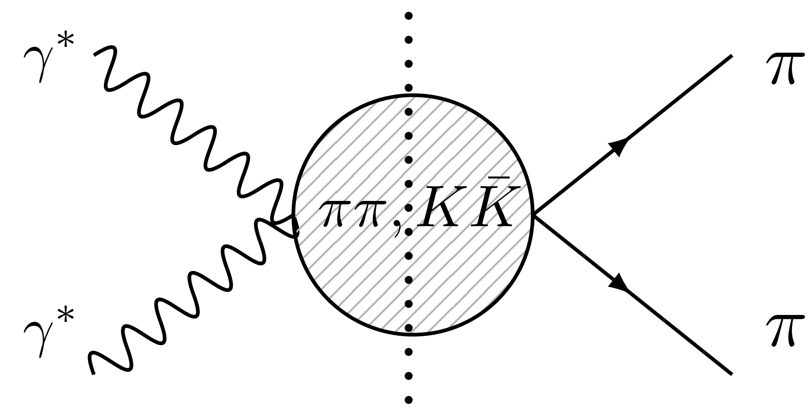
- Unsubtracted dispersion relation for kinematically unconstrained p.w. amplitudes

$$\bar{h}_i^{(J)} = \int_{-\infty}^{0} \frac{ds'}{\pi} \frac{\mathrm{Disc}\,\bar{h}_i^{(J)}(s')}{s'-s} + \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\mathrm{Disc}\,\bar{h}_i^{(J)}(s')}{s'-s}$$

Garcia-Martin et. al (2010)
Hoferichter et. al. (2011,19)
Dai et al. (2014)
Moussallam (2013)

- Omnès solution of the unitarity relation

$$\mathrm{Disc}\,h_i^{(J)} = h_i^{(J)}\,\rho\,t_{\pi\pi}^{(J)*}$$
$$\mathrm{Disc}\,\Omega^{(J)} = \Omega^{(J)}\,\rho\,t_{\pi\pi}^{(J)*} \quad \Big|_{s>4m_\pi^2}$$

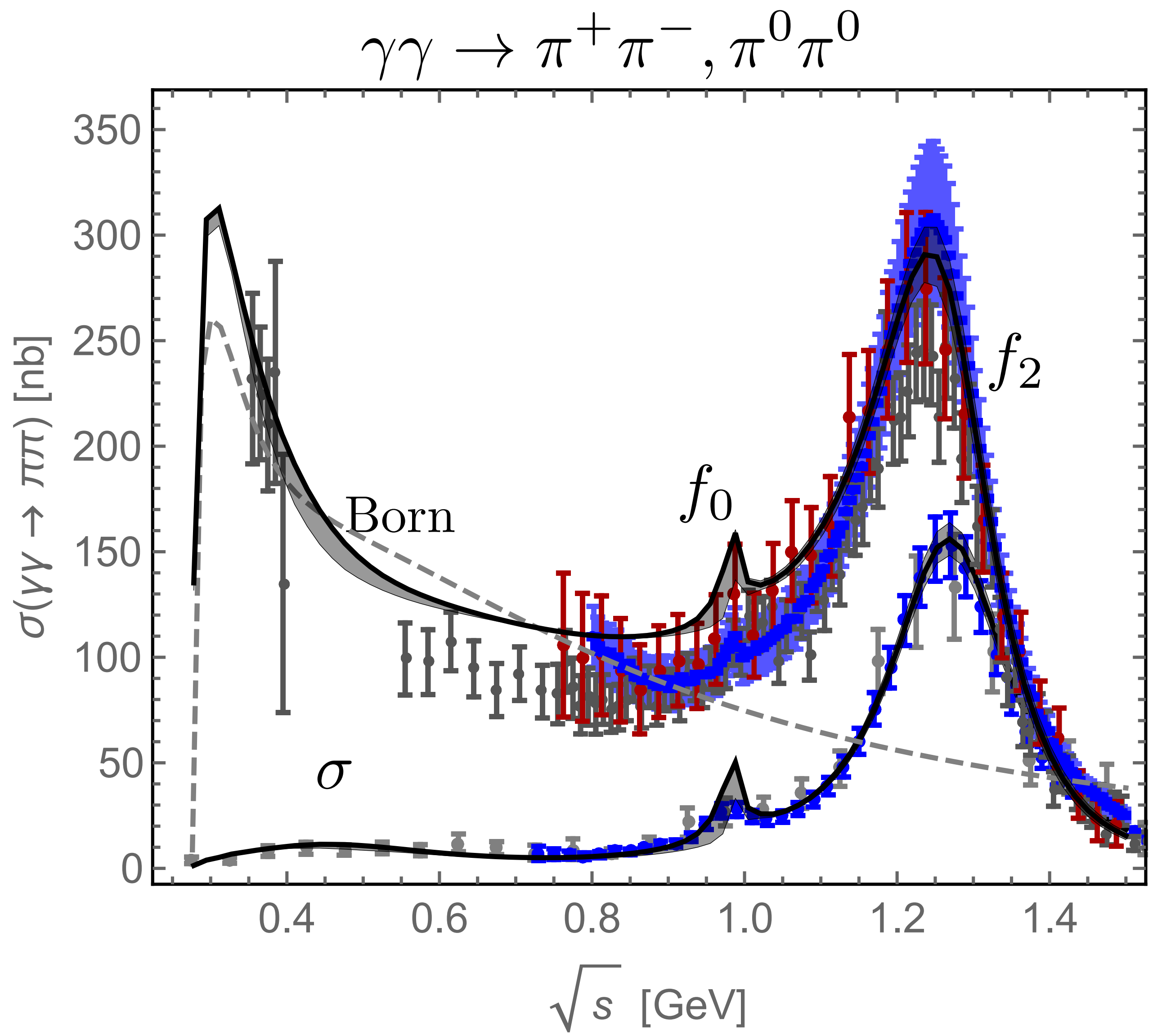


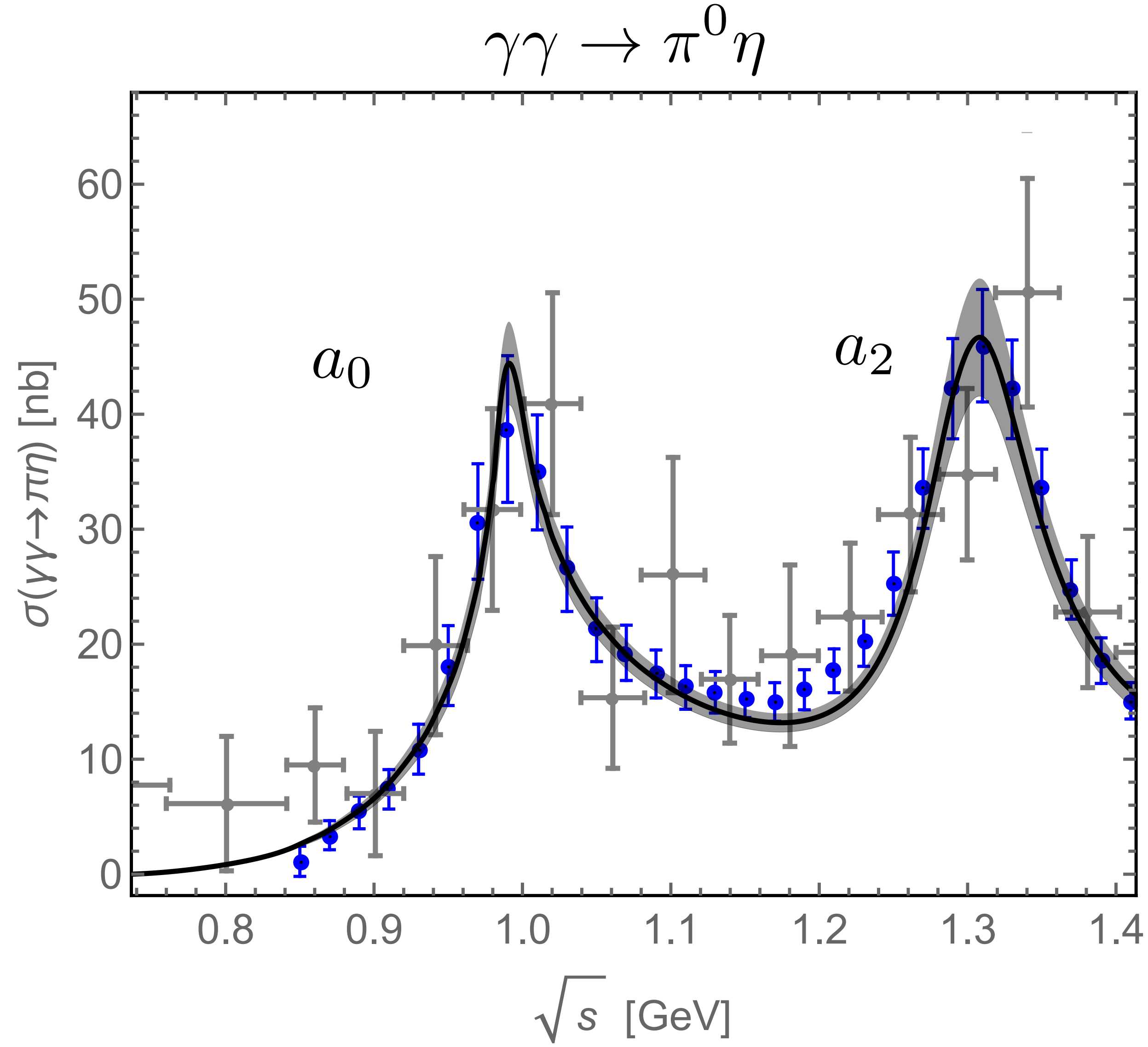
leads to

$$h_i^{(J)} = h_i^{(J),\mathrm{Born}} + \Omega^{(J)} \left(\int_{-\infty}^{0} \frac{ds'}{\pi} \frac{\mathrm{Disc}(\bar{h}_i^{(J)}(s'))\,\Omega^{(J)}(s')^{-1}}{s'-s} - \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{h^{(J),\mathrm{Born}}(s')\,\mathrm{Im}\Omega^{(J)}(s')^{-1})}{s'-s} \right)$$

Omnès (1958)
Muskhelishvili (1953)

Results for real photons



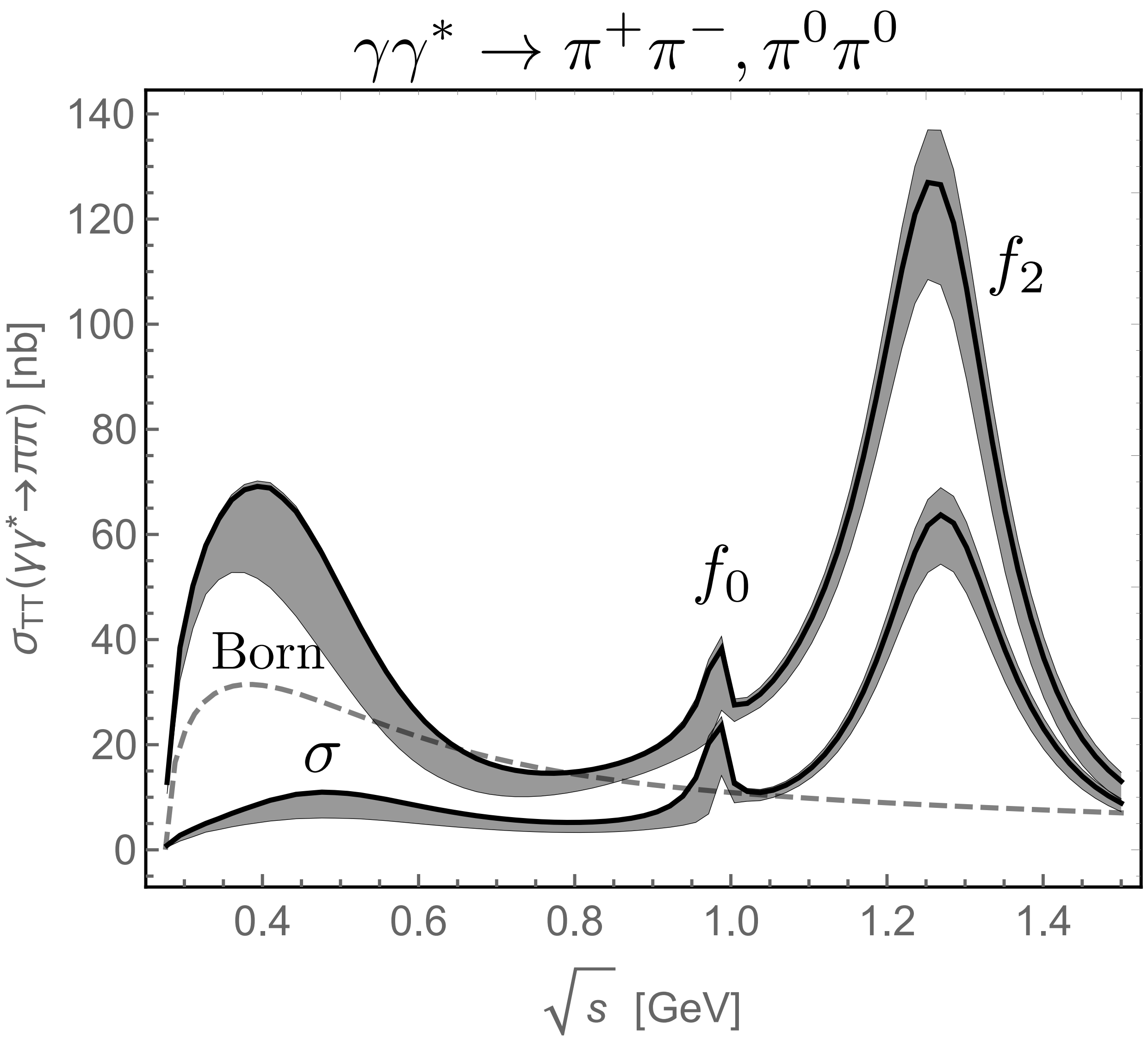


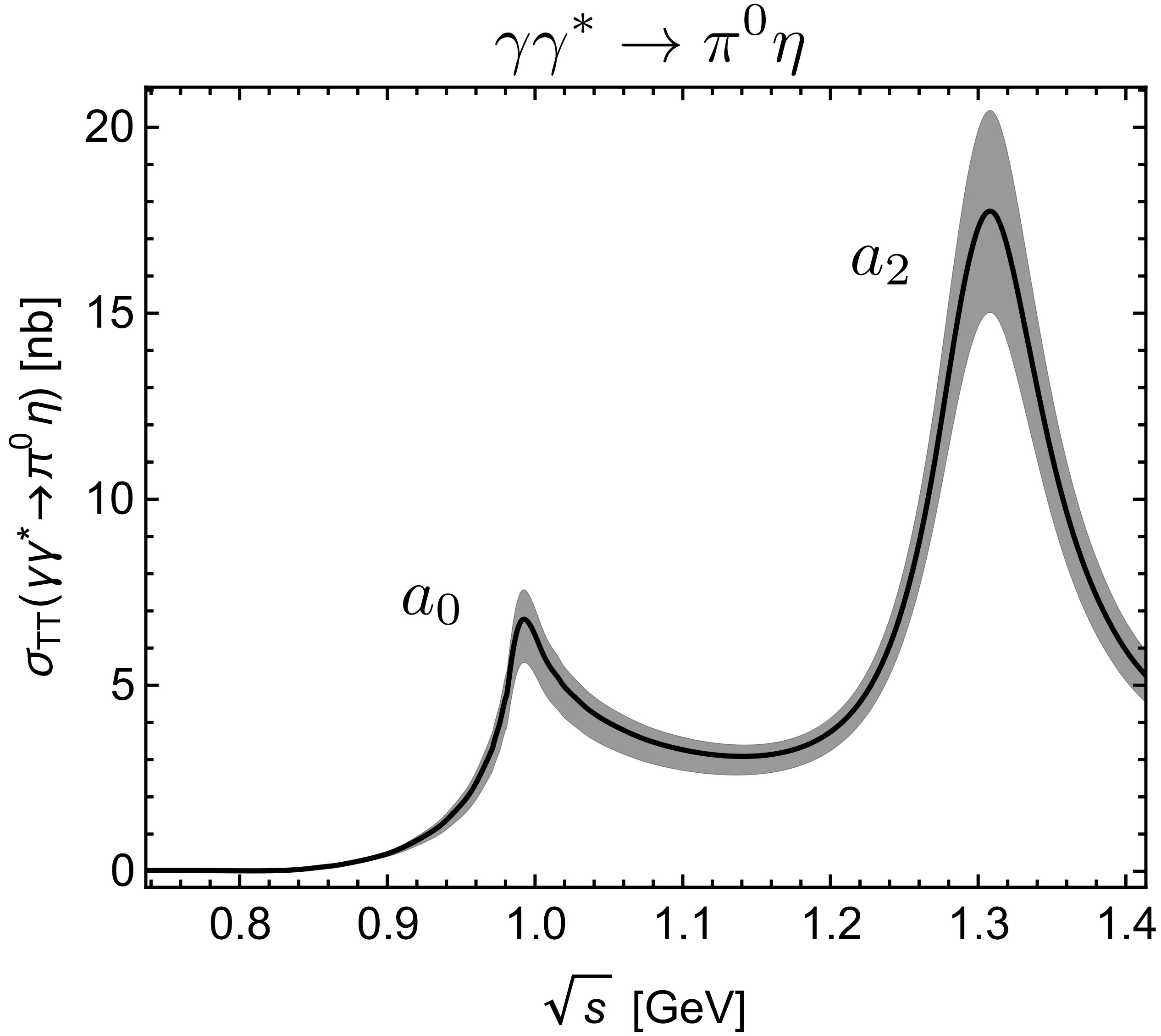
- Coupled-channel dispersive treatment of $f_0(980)$ and $a_0(980)$ is crucial
- $f_2(1270)$ described dispersively through Omnès function
- $a_2(1320)$ described as a Breit Wigner resonance

I.D., Deineka, Vanderhaeghen (2017, 2018)

cf. also Dai et al. (2014)
Hoferichter et. al. (2011,19)
Garcia-Martin et. al (2010)

Results for single virtual photon (Q^2=0.5)

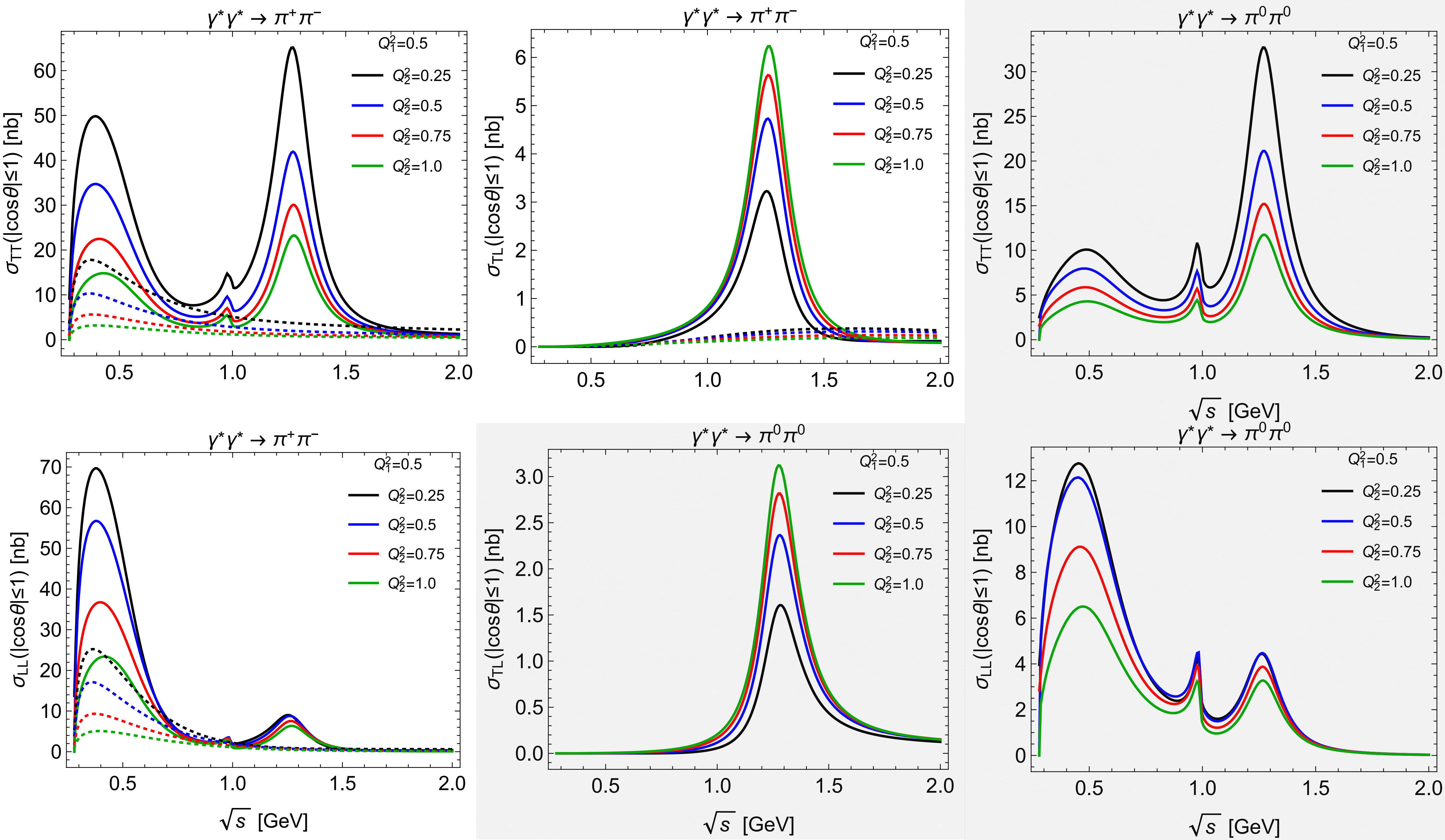




- Single tagged BESIII data for $\pi^+\pi^-$, $\pi^0\pi^0$ in range $0.1\ \mathrm{GeV}^2 < Q^2 < 2\ \mathrm{GeV}^2$ under analysis. It will validate left-hand cuts approximations.

I.D., Deineka, Vanderhaeghen (2018)
cf. also Moussallam (2013)
Hoferichter Stoffer (2019)

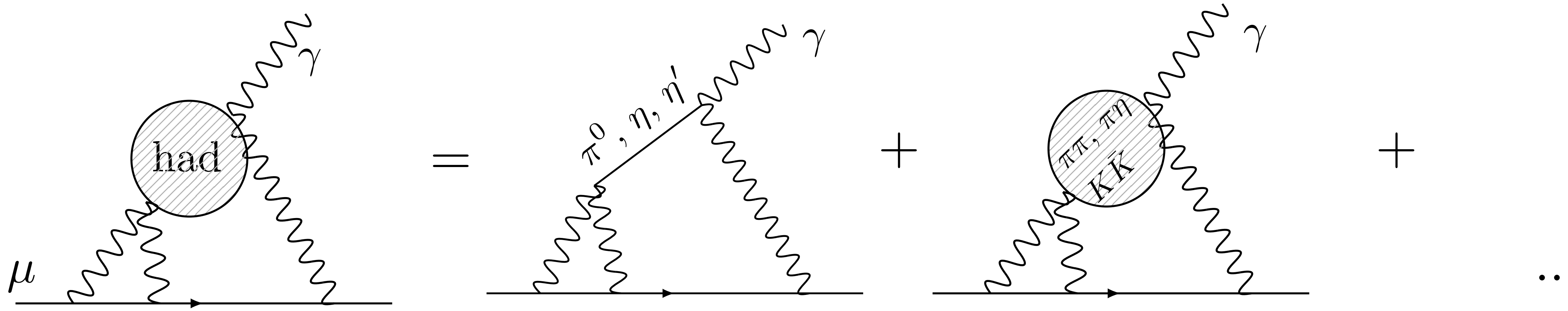
Results for double virtual photons



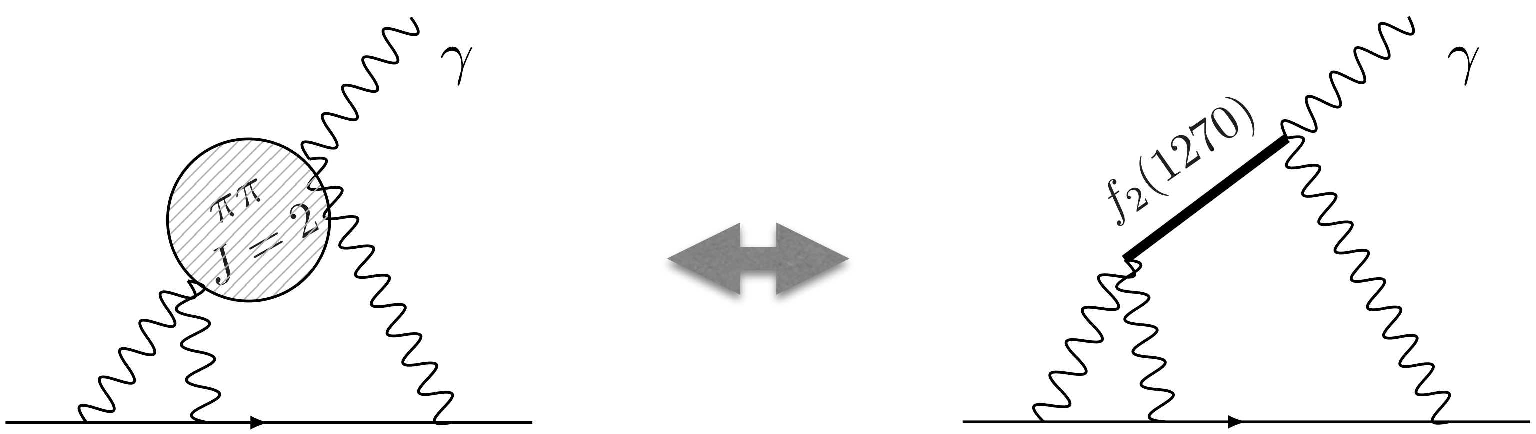
$$\frac{d\sigma_{TT}}{d\cos\theta} \sim |H_{++}|^2 + |H_{+-}|^2, \quad \frac{d\sigma_{TL}}{d\cos\theta} \sim |H_{+0}|^2, \quad \frac{d\sigma_{LL}}{d\cos\theta} \sim |H_{00}|^2$$

I.D., Deineka, Vanderhaeghen (2019)
cf. also Hoferichter Stoffer (2019)

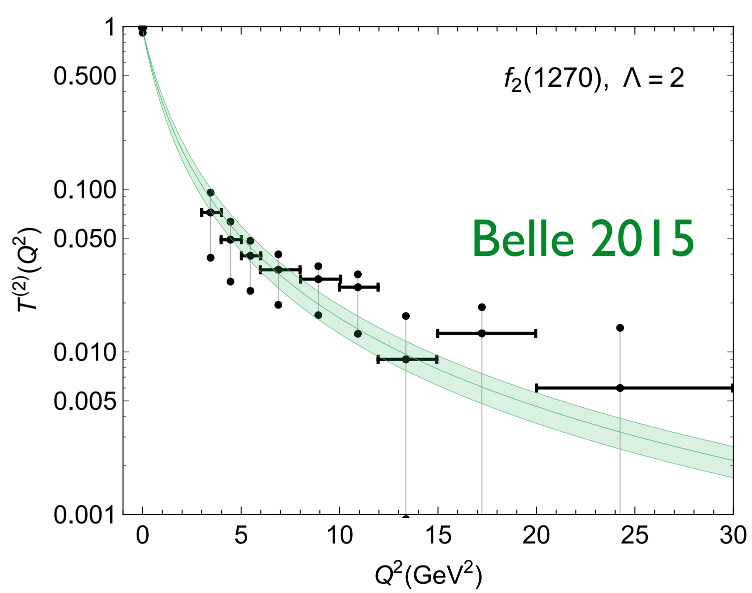
Effective resonance description



- One needs to compare



ongoing work



Pascalutsa, Pauk
Vanderhaeghen (2012)

I.D., Vanderhaeghen (2016)

$$a_\mu = (0.05 \pm 0.01) \times 10^{-10}$$

Summary and Outlook

- new a_μ Fermilab and J-Parc experiments ongoing:
  aim: factor 4 improvement in experimental value

- Need to take into account $f_0(500)$, $f_0(980)$, $a_0(980)$, $f_2(1270)$ … and non resonant contributions in a dispersive approach to (g-2)

$$a_\mu^{\pi\text{-box}} + a_{\mu,J=0}^{\pi\pi,\pi\text{-pole LHC}} = -2.4(1) \times 10^{-10}$$

Colangelo et al. (2014-2017)

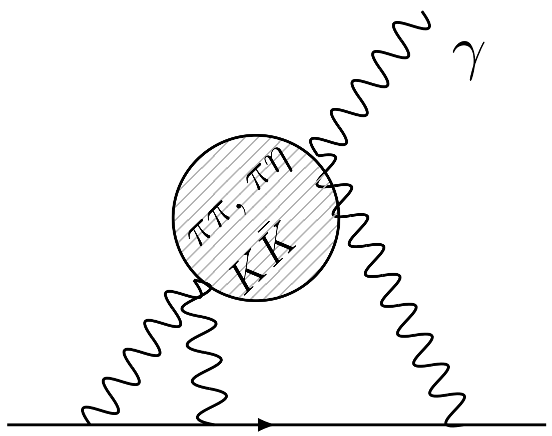
- Main ingredients: $\gamma^*\gamma^*\to\pi\pi$, $\pi\eta$ and (KK work in progress). Can be used in different (g-2) dispersive approaches.

- Need to implement high energy asymptotic on TFF ($\gamma^*\pi\omega,\ \gamma^*\pi\rho$) used as input in dispersive formalism

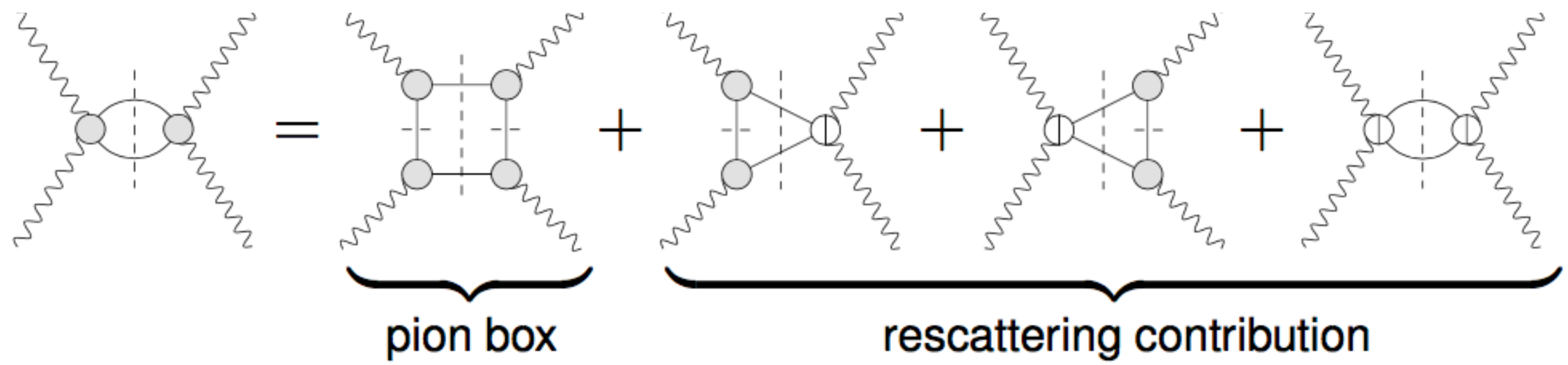
- It is important to validate dispersive treatment of $\gamma\gamma^*\to\pi\pi$, $\pi\eta$, KK… with upcoming BESIII data

Thank you!

Multi-meson contribution to (g-2)



- Pioneering dispersive analyses for ππ loop contribution to a_μ



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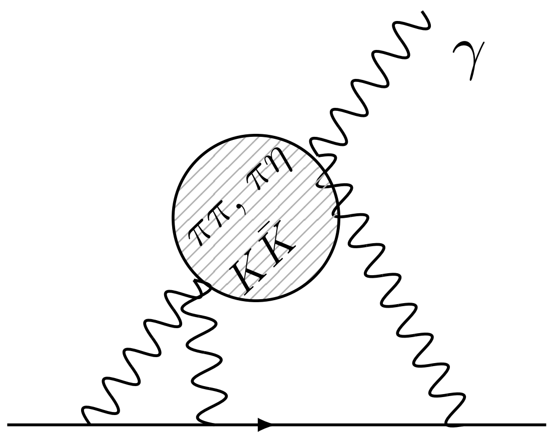
Colangelo et al. (2014-2017)

Multi-meson contribution to (g-2)



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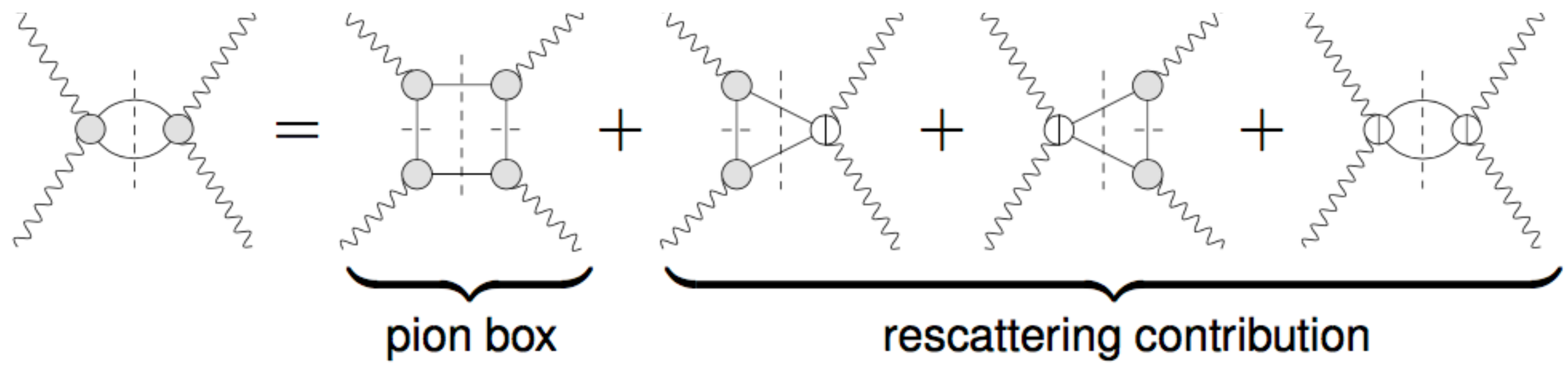

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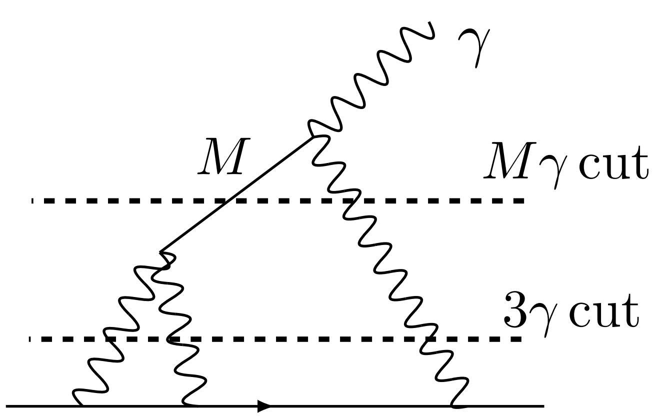
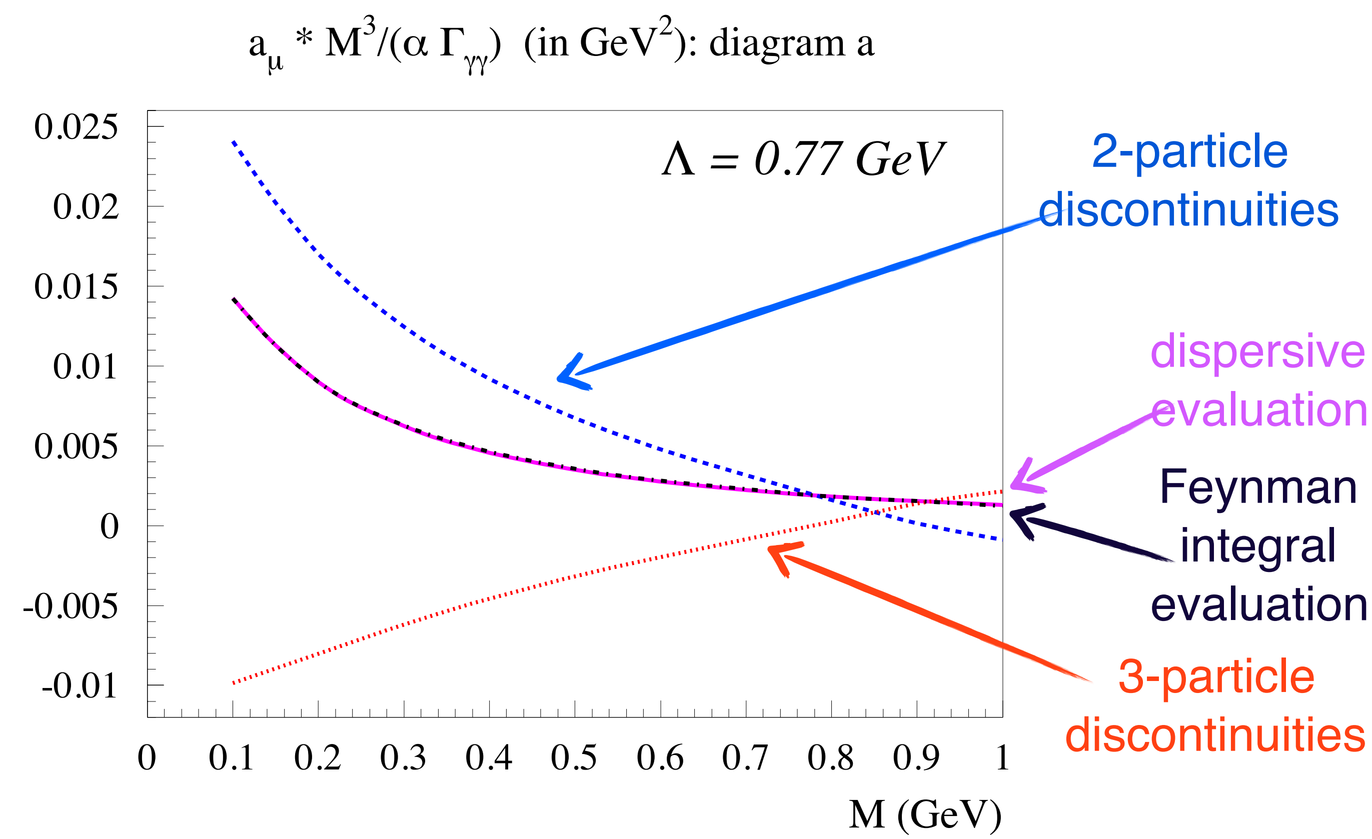
Colangelo et al. (2014-2017)

- Dispersive analysis for muon Pauli Form Factor

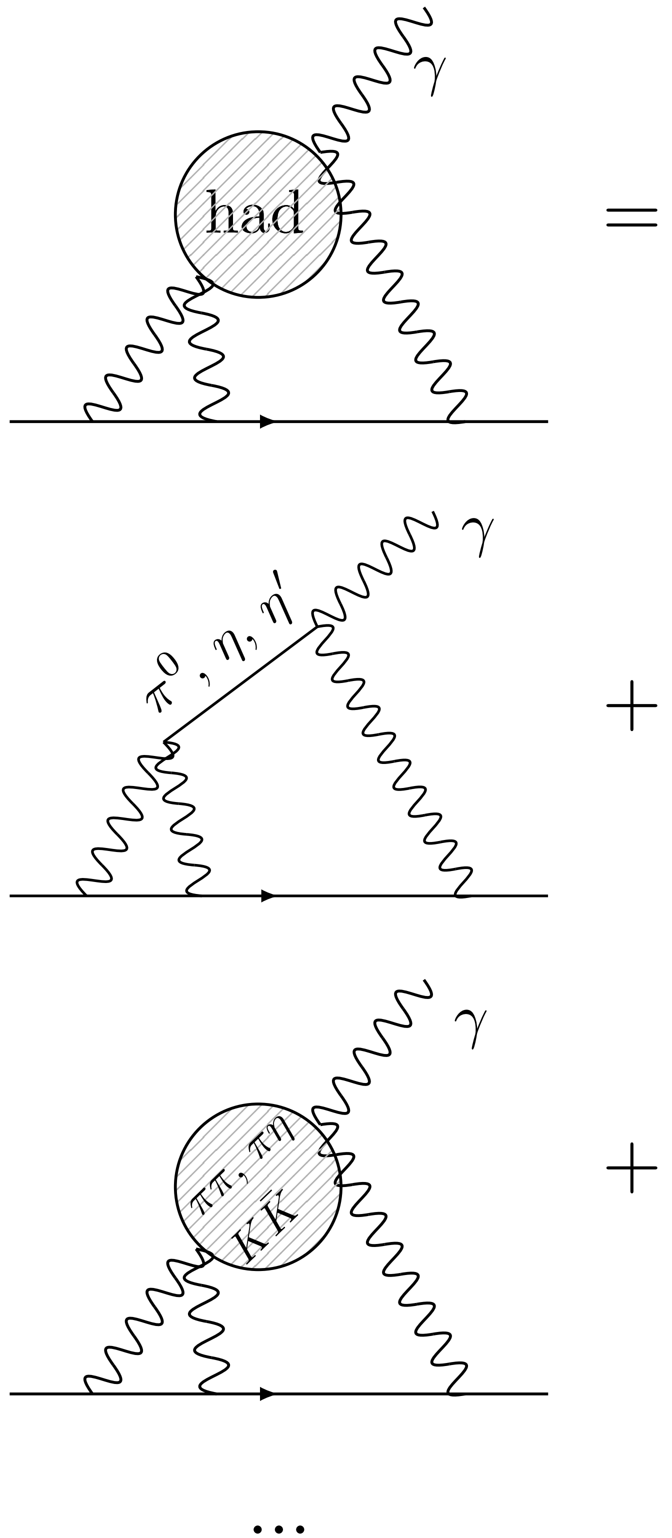


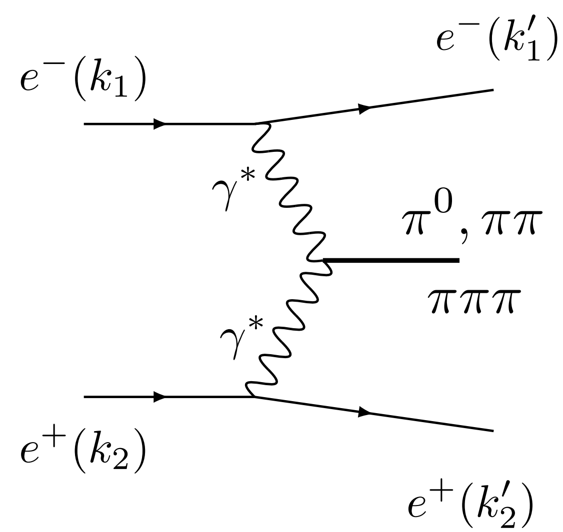


Pauk, Vanderhaeghen (2014)

Observables in experiment

$q^2 > 0$	time-like γ^*
$q^2 = -Q^2 < 0$	space-like γ^*





$\gamma\gamma^*\pi^0$, $\gamma\gamma^*\eta$, $\gamma\gamma^*\eta'$

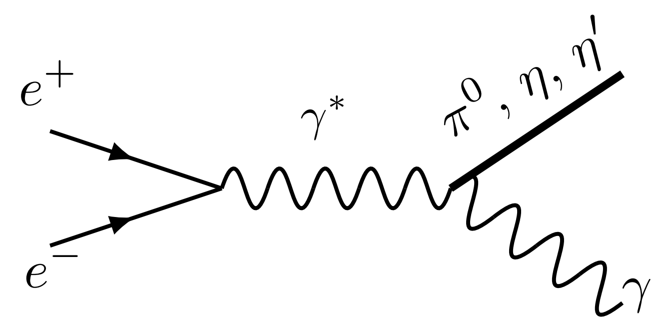
CLEO, BaBar, Belle, BESIII, ...

$\gamma\gamma \to \pi\pi, \pi\eta, K\bar{K}$

Mark II, CELLO, Crystal Ball, Belle

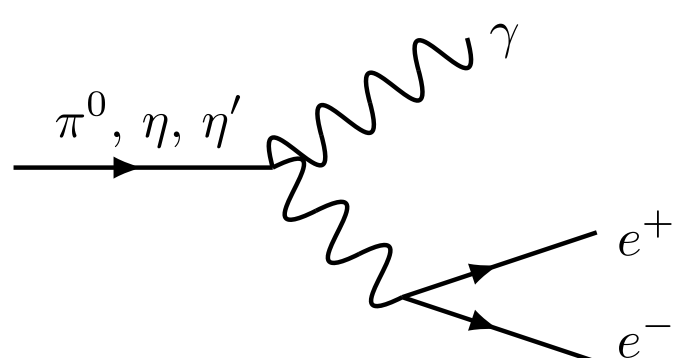
$\gamma\gamma^* \to \pi\pi$

BESIII



$\gamma\gamma^*\pi^0$, $\gamma\gamma^*\eta$

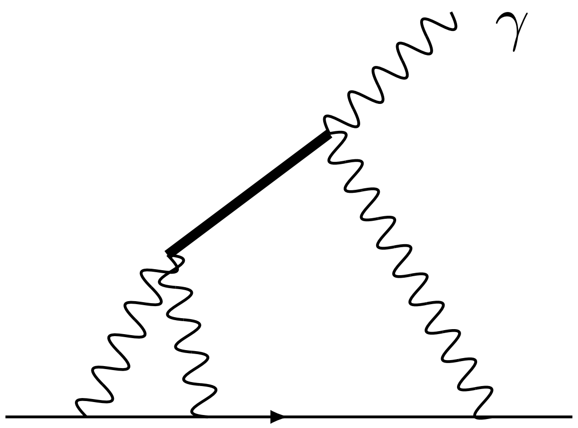
SND, CMD-2, BESIII, ...



$\gamma\gamma^*\pi^0$, $\gamma\gamma^*\eta$, $\gamma\gamma^*\eta'$

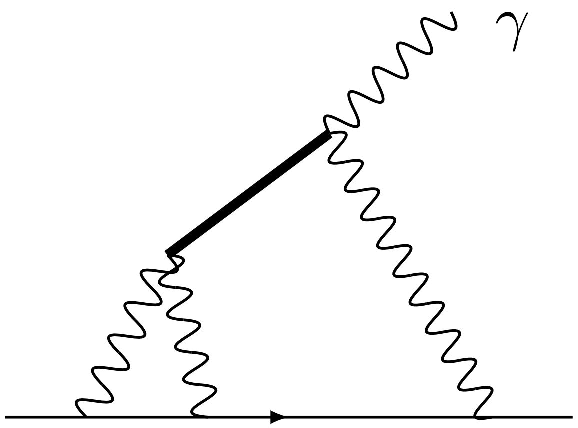
KLOE, MAMI/A2

Meson pole contributions to (g-2)



$$a_\mu^{LbL} = \lim_{k\to 0} ie^6 \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4}\, T^{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2)\, \Pi_{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2)$$
$$\frac{1}{q_1^2}\,\frac{1}{q_2^2}\,\frac{1}{(k-q_1-q_2)^2}\,\frac{1}{(p+q_1)^2-m^2}\,\frac{1}{(p'-q_2)^2-m^2}$$

Meson pole contributions to (g-2)



Lepton tensor: well known

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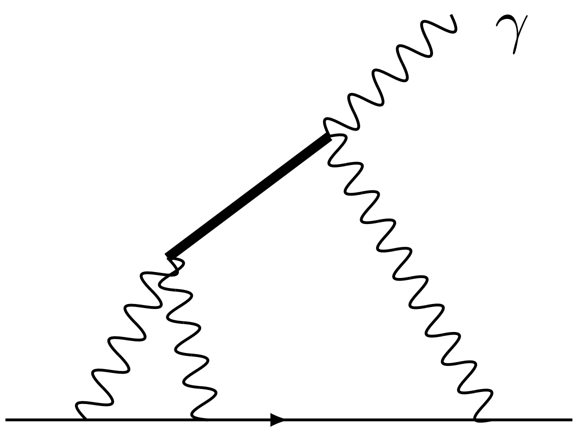

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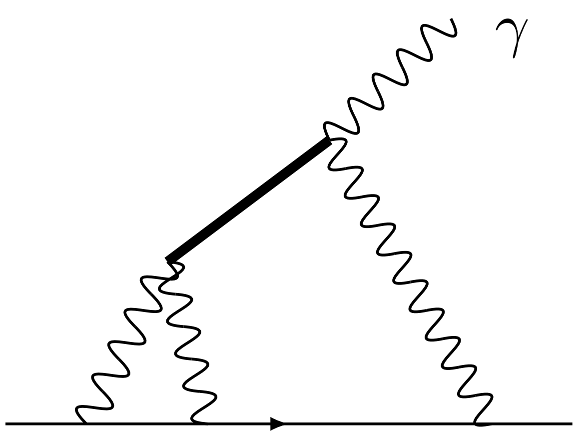

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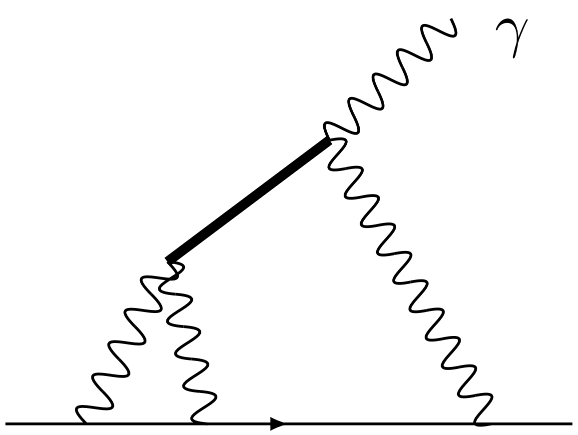
Results (excluding low energy region):

$$a_\mu[f_2(1270), f_2(1565)] = (0.1 \pm 0.01) \times 10^{-10}$$

Meson pole contributions to (g-2)



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Results (excluding low energy region):

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New evaluation of axial vector contributions (satisfying Landau-Yang theorem)

$$a_\mu[f_1(1285), f_1(1420)] = (0.64 \pm 0.20)\times 10^{-10}$$
$$= (0.75 \pm 0.27)\times 10^{-10}$$

Pauk, Vdh (2013)
Jegerlehner (2015)

$$\delta a_\mu^{exp} = 1.6 \times 10^{-10}$$

FNAL, J-PARC experiments