Coupled-channel effects in Heavy Hadrons

18th International Conference on Hadron Spectroscopy and Structure (HADRON2019)

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Overview

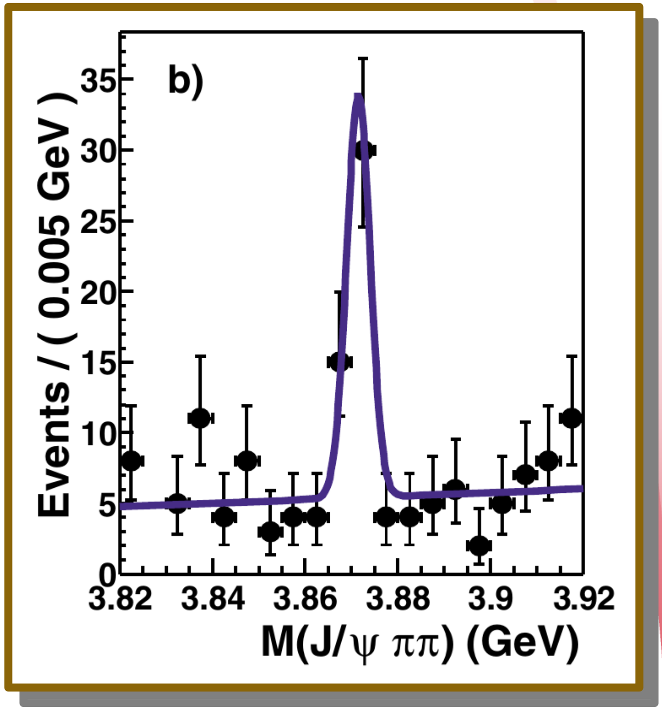
- *Symmetry breaking effects*
  - *Isospin breaking*
    - *Isospin breaking for the X(3872)*
    - *Isospin breaking for pentaquark*
  - *HQSS and HFS breaking*
- *Threshold cusps*
- *Triangle singularities*

The X(3872) now $\chi_{c1}(3872)$

Mass very close to the $D^{*0}\,\bar{D}^0$

$$B \equiv m_{X(3872)} - m_{D^{*0}} - m_{\bar{D}^0} = 1.1^{+0.6+0.1}_{-0.4-0.3}\ \text{MeV} = (0.00 \pm 0.18)\ \text{MeV}$$

Belle 2003



From a ω Belle arXiv: hep-ex/0505037

Isospin violating decay

$$\frac{\mathcal{B}(X(3872)\to\pi^+\pi^-\pi^0 J/\psi)}{\mathcal{B}(X(3872)\to\pi^+\pi^- J/\psi)} = 1.0 \pm 0.4(\text{stat}) \pm 0.3(\text{syst})$$

From a ρ^0 CDF Phys. Rev. Lett. 96, 102002

Isospin breaking scale

$$m_{D^{*+}} + m_{D^-} - m_{D^{*0}} - m_{\bar{D}^0} = 8.2 \pm 0.1\ \text{MeV} \gg B$$

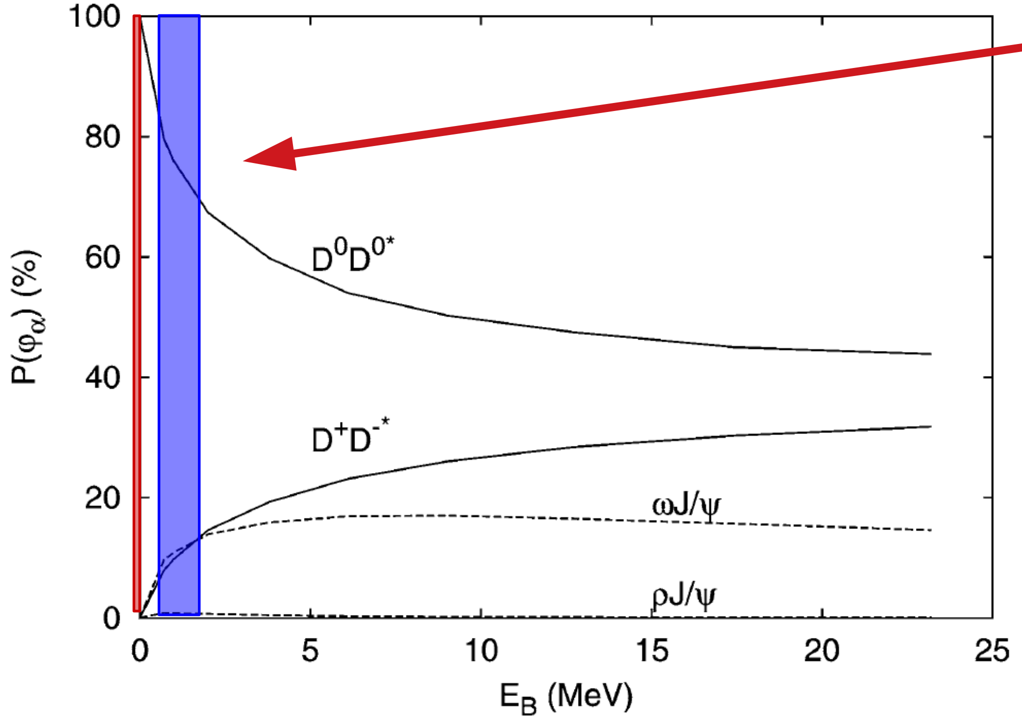
$$\frac{m_{D^+} - m_{D^0}}{m_{D^+} + m_{D^0}} \sim 0.13\%$$

$$\frac{m_{D^{*+}} - m_{D^{*0}}}{m_{D^{*+}} + m_{D^{*0}}} \sim 0.08\%$$

Isospin violation

E.S. Swanson / Physics Letters B 588 (2004) 189–195

Big isospin violation on the Wave function for the long range



$$\psi(r) \xrightarrow{r\to\infty} e^{-\gamma r}$$

$$\gamma^{-1}_{D^0 D^{*0}} = 3.4 - 5.8\,\mathrm{fm}$$

$$\gamma^{-1}_{D^+ D^{*-}} = 1.4 - 1.5\,\mathrm{fm}$$

$$\gamma^{-1}_{D^0 D^{*0}} = 10.6 - \infty\,\mathrm{fm}$$

$$\gamma^{-1}_{D^+ D^{*-}} = 1.55 - 1.57\,\mathrm{fm}$$

$$B \equiv m_{X(3872)} - m_{D^{*0}} - m_{\bar{D}^0} = 1.1^{+0.6+0.1}_{-0.4-0.3}\ \mathrm{MeV}$$

$$= (0.00 \pm 0.18)\,\mathrm{MeV}$$

$$m_{D^{*+}} + m_{D^-} - m_{D^{*0}} - m_{\bar{D}^0} = 8.2 \pm 0.1\ \mathrm{MeV} \gg B$$

Isospin violation

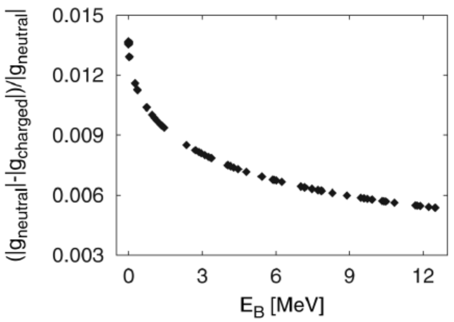
Short range is different → *confusion about isospin violation*

D. Gamermann and E. Oset, Phys. Rev. D 80, 014003 (2009). QFT treatment

Less than 1% isospin violation

Interpreted as probabilities

$$g_{D^+D^{*-}} = 2982$$
$$g_{D^0\bar{D}^{*0}} = 3005$$



Small isospin breaking effect $\mathcal{R}_{\rho/\omega} \sim 3.2\%$

$$\frac{\mathcal{B}(X(3872)\to\pi\pi J/\psi)}{\mathcal{B}(X(3872)\to\pi\pi\pi J/\psi)} = \mathcal{R}_{\rho/\omega}\frac{\int_0^\infty q\mathcal{S}(s,m_\rho,\Gamma_\rho)\theta(m_X-m_{J/\psi}-\sqrt{s})ds}{\int_0^\infty q\mathcal{S}(s,m_\omega,\Gamma_\omega)\theta(m_X-m_{J/\psi}-\sqrt{s})ds}\times\frac{\mathcal{B}_\rho}{\mathcal{B}_\omega}$$

Final result compatible with experimental data

Phase space effect due to large differences on the widths

Isospin violation

This was clarified in D. Gamermann et al., Phys. Rev. D 81, 014029 (2010).

The coupling is

$$g^2 = \lim_{E\to E_\alpha}(E-E_\alpha)T = \left|v\int d^3k\langle\vec{k}|\alpha\rangle\right|^2 = -\left(\frac{dG}{dE}\right)^{-1}_{E=E_\alpha}$$

Which can be related with the wave function at the origin

$$gG(E_\alpha) = (2\pi)^{3/2}\psi(0)$$

A short range amplitude will be given by

Isospin factors

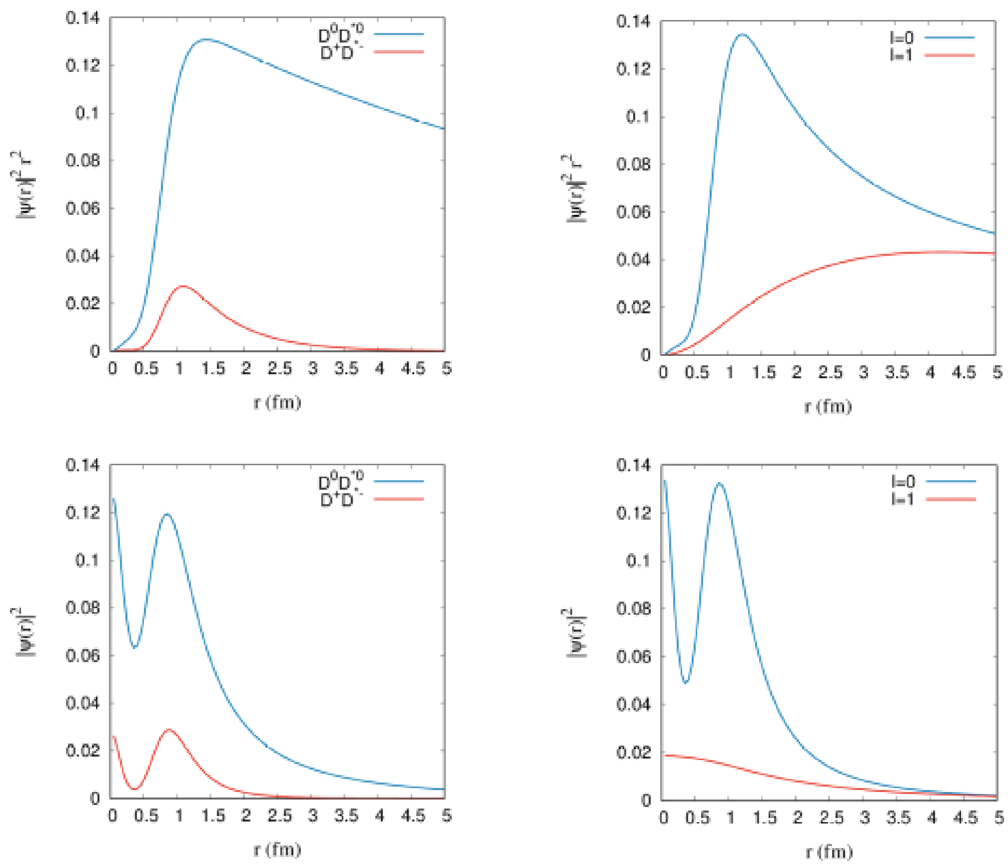
$$\mathcal{M} = g_1G_{11}(E_\alpha)F_1 + g_2G_{22}(E_\alpha)F_2 = (2\pi)^{3/2}(\psi_1(0)F_1 + \psi_2(0)F_2)$$

Short range observables will show small isospin violation

Long range observables will show large isospin violation

Isospin conserving interactions

Isospin violation



Wave functions in the CQM

For an isospin conserving decay

$$F_2 = -F_1$$

and

$$\begin{aligned} \mathcal{M} &= (2\pi)^{3/2}(\psi_1(0)F_1 + \psi_2(0)F_2) \\ &\sim \psi_{I=0}(0) \Rightarrow |\mathcal{M}|^2 \sim |\psi_{I=0}(0)|^2 \end{aligned}$$

Short range observables will show small isospin violation

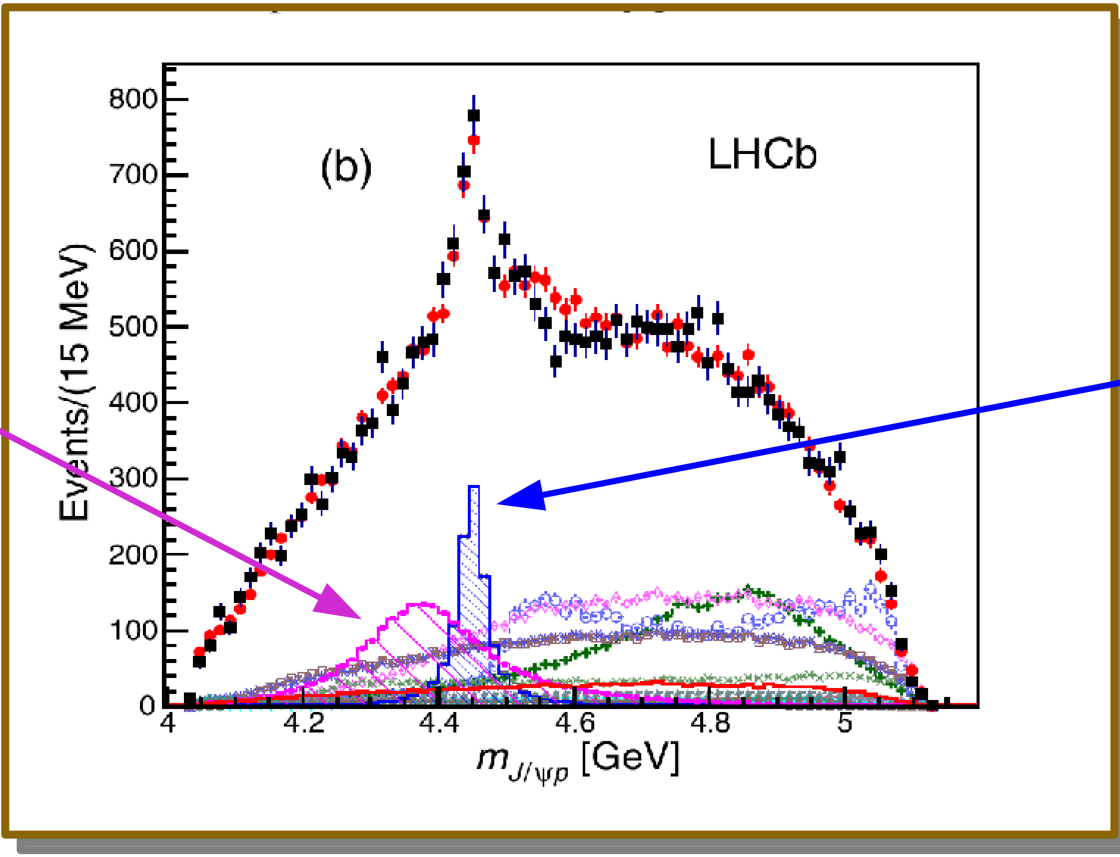
Long range observables will show large isospin violation

Isospin conserving interactions

Pentaquark states

Measured by LHCb in 2015 PRL 115, 072001 (2015)

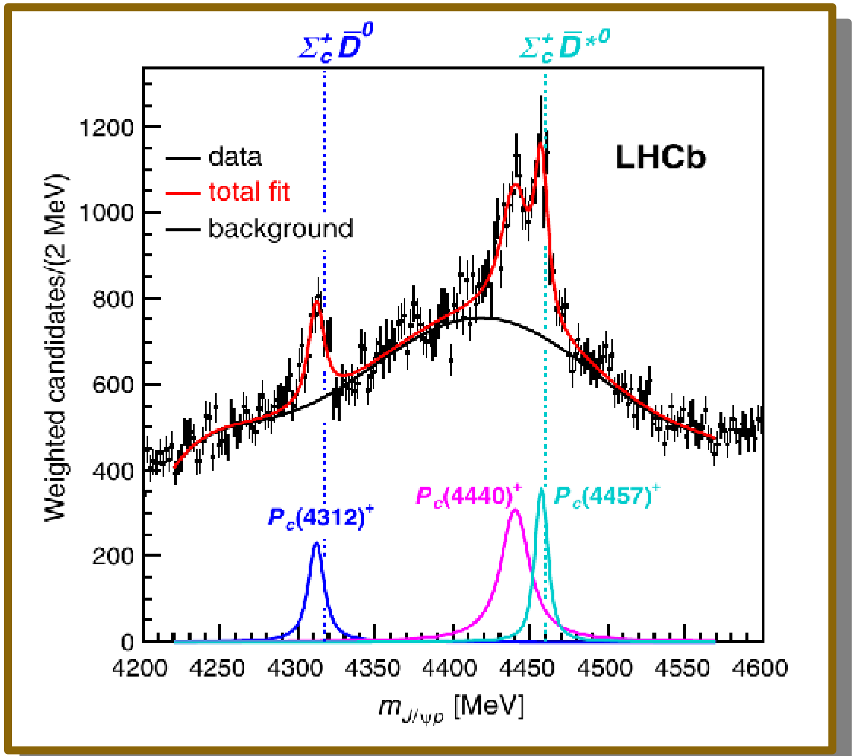
$P_c(4380)^+$



$P_c(4450)^+$

Pentaquark states

New pentaquarks, LHCb PRL 122, 222001 (2019)



States close to thresholds

Isospin violation on Pc(4457) decays

F.-K. Guo et al., Phys. Rev. D 99, 091501 (2019).

$$
\begin{aligned}
M_{P_c(4457)^+} &= 4457.3 \pm 0.6^{+4.1}_{-1.7} \\
\Gamma_{P_c(4457)^+} &= 6.4 \pm 2.0^{+5.7}_{-1.9} \\
M_{\Sigma_c^+} + M_{\bar{D}^{*0}} &= 4459.8 \pm 0.4\,\mathrm{MeV} \Rightarrow B_1 = 2.5^{+1.8}_{-4.2}\,\mathrm{MeV} \\
M_{\Sigma_c^{++}} + M_{D^{*-}} &= 4464.23 \pm 0.15\,\mathrm{MeV} \Rightarrow B_2 = 6.9^{+1.8}_{-4.1}\,\mathrm{MeV}
\end{aligned}
$$

In analogy to the X(3872)

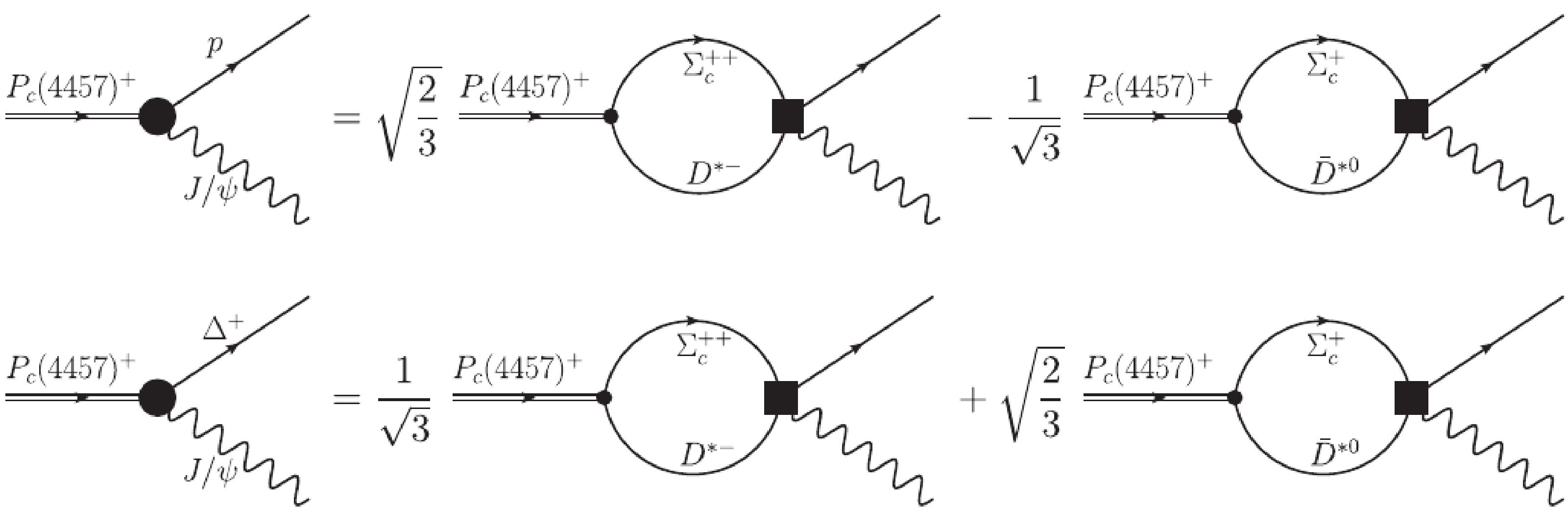
$$
\begin{aligned}
|\Sigma_c \bar{D}^*; I = 1/2, I_3 = 1/2\rangle &= \sqrt{\frac{2}{3}}|\Sigma_c^{++} D^{*-}\rangle - \sqrt{\frac{1}{3}}|\Sigma_c^{+} \bar{D}^{*0}\rangle \\
|\Sigma_c \bar{D}^*; I = 3/2, I_3 = 1/2\rangle &= \sqrt{\frac{1}{3}}|\Sigma_c^{++} D^{*-}\rangle + \sqrt{\frac{2}{3}}|\Sigma_c^{+} \bar{D}^{*0}\rangle
\end{aligned}
$$

Isospin violation on Pc(4457) decays

$$
\begin{aligned}
V_{11} &\equiv \langle \Sigma_c^{++} D^{*-} | V | \Sigma_c^{++} D^{*-} \rangle = \frac{2}{3} V_{1/2} + \frac{1}{3} V_{3/2} \\
V_{22} &\equiv \langle \Sigma_c^{+} \bar{D}^{*0} | V | \Sigma_c^{+} \bar{D}^{*0} \rangle = \frac{1}{3} V_{1/2} + \frac{2}{3} V_{3/2} \\
V_{12} &\equiv \langle \Sigma_c^{++} D^{*-} | V | \Sigma_c^{+} \bar{D}^{*0} \rangle = \frac{\sqrt{2}}{3} (V_{3/2} - V_{1/2}) \\
G_1 &\equiv G_{\Sigma_c^{++} D^{*-}} \\
G_2 &\equiv G_{\Sigma_c^{+} \bar{D}^{*0}}
\end{aligned}
$$

Isospin conserving interactions

$$
\boxed{
\begin{aligned}
T^{-1} &= V^{-1} - G = 0 \\
\frac{g_{++,-}}{g_{+,0}} &= \frac{2V_{1/2} + V_{3/2} - 3V_{1/2}V_{3/2}G_2}{-\sqrt{2}(V_{1/2} - V_{3/2})} \sim -\sqrt{2}
\end{aligned}
}
$$



Isospin violation on Pc(4457) decays

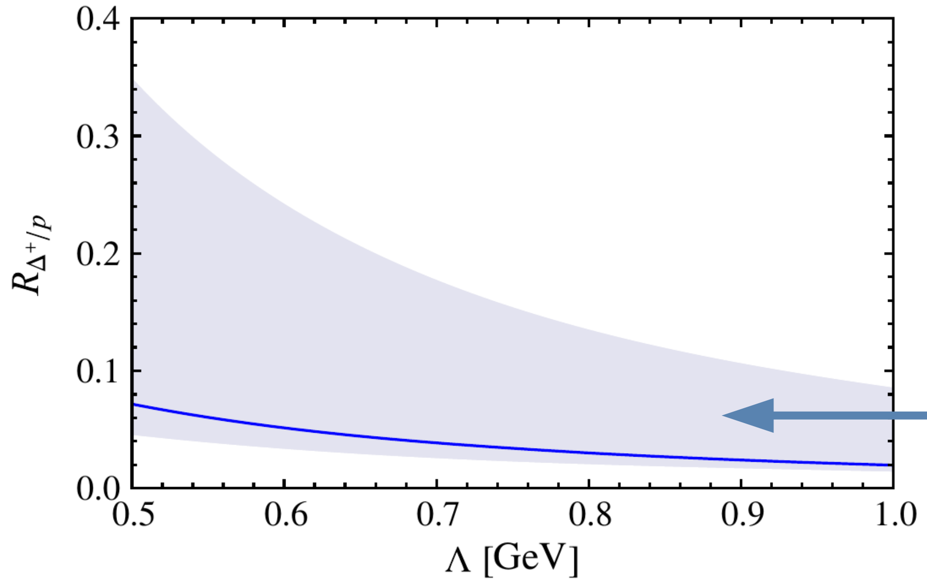
Again

$$\psi_i^{\Lambda}(r=0) = -2\mu_i g_i \int \frac{d^3q}{(2\pi)^3} \frac{e^{-\frac{2q^2}{\Lambda^2}}}{\gamma_i^2 + q^2} \equiv g_i G_i^{\Lambda}$$

and

$g_{++,-} \sim -\sqrt{2} g_{+,0}$

$$\frac{|\mathcal{A}(P_c(4457)^+ \to J/\psi \Delta^+)|}{|\mathcal{A}(P_c(4457)^+ \to J/\psi p)|} = \sqrt{10} \left| \frac{\psi_{3/2}^{\Lambda}(r=0)}{\psi_{1/2}^{\Lambda}(r=0)} \right| = 2\sqrt{5} \left| \frac{G_{++,-}^{\Lambda} - G_{+,0}^{\Lambda}}{2G_{++,-}^{\Lambda} + G_{+,0}^{\Lambda}} \right|$$



$$R_{\Delta^+/p} = \frac{|\mathcal{B}(P_c(4457)^+ \to J/\psi \Delta^+)|}{|\mathcal{B}(P_c(4457)^+ \to J/\psi p)|}$$

Uncertainties coming from pentaquark mass

HQSS and HFS breaking

Heavy Quarks Spin Symmetry (HQQS) and Heavy Flavor Symmetry (HFS) are good approximate symmetries of QCD.

Slightly broken in the heavy-light

$$\frac{M_{D^*} - M_D}{M_{D^*} + M_D} \sim 3.6\%$$

And heavy-heavy sectors

$$\frac{M_{J/\psi} - M_{\eta_c}}{M_{J/\psi} + M_{\eta_c}} \sim 1.8\%$$

$$\frac{M_{\chi_{c2}(1P)} - M_{\chi_{c0}(1P)}}{M_{\chi_{c2}(1P)} + M_{\chi_{c0}(1P)}} \sim 2.0\%$$

- *HQSS The interaction does not depend on the heavy quark spin*
- *HFS The interaction of charm and bottom is the same*

HQSS and HFS predictions

- *HQSS implies the DD*(1⁺⁺) interaction the same as D*D*(2⁺⁺)*

$$\boxed{V(D\bar{D}^*\,1^{++}) \;=\; V(D^*\bar{D}^*\,2^{++})} \qquad X(3872)(1^{++}) \Rightarrow X(4012)(2^{++})$$

Not found

J. Nieves and M. Pavón Valderrama, Phys. Rev. D 86, 056004 (2012).
Baru, V. et al., A.V., EPJ Web Conf. 137, 06002 (2017).

- *HFS implies interaction between $D^{(*)}D^{(*)}$ the same as $B^{(*)}B^{(*)}$*

$$\boxed{V(B^{(*)}\bar{B}^{(*)}) \;=\; V(D^{(*)}\bar{D}^{(*)})} \qquad X(3872)(1^{++}) \Rightarrow X_b(1^{++}), X_b(2^{++})$$

C. Hidalgo-Duque et al., Phys. Rev. D 87, 076006 (2013).

Not found by
- *CMS S. Chatrchyan et al., Physics Letters B 727, 57 (2013).*
- *ATLAS G. Aad et al., Physics Letters B 740, 199 (2015).*
- *Belle X. H. He et al., Phys. Rev. Lett. 113, 142001 (2014).*

HQSS and HFS breaking

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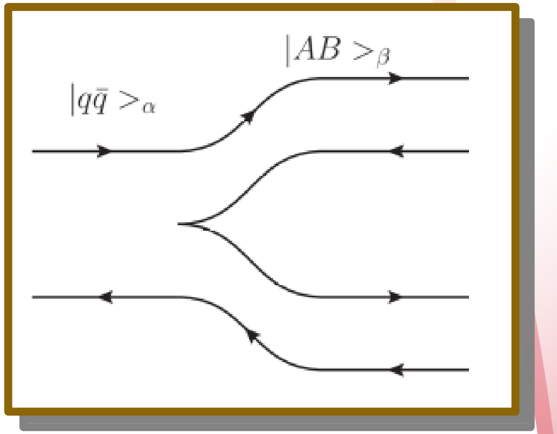
OZI allowed decays couples one meson and two meson states strongly

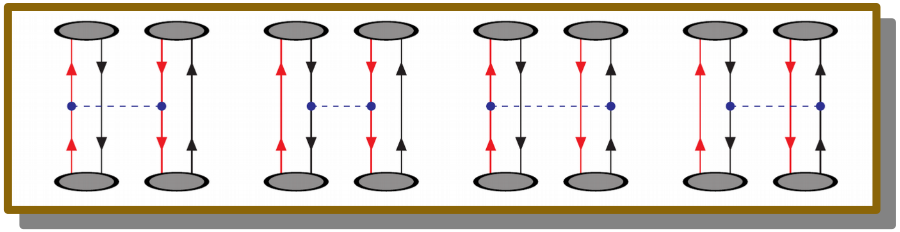
$$c\bar{c} \rightarrow (c\bar{n})(n\bar{c})$$



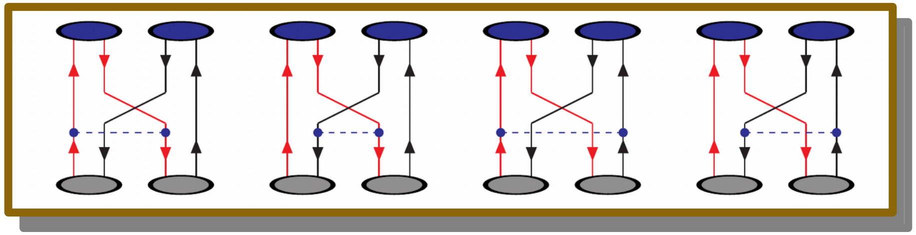
The quark model is appropriate for studying these effects:

- *The 3P0 model gives a microscopic model for these couplings. Fitted to strong decays*
- *The quark-quark interaction gives the wave functions of the mesons. Gives the form factors.*
- *The Resonanting group method gives the interaction between mesons using the quark-quark interaction and meson wave functions.*
- *The Chiral quark model introduces pion-exchange interactions.*
- *Rearrangement process produces OZI suppressed decays*

Interaction



Rearrangement processes



The X(3872)

- $^3S_1$ and $^3D_1$ DD^* partial waves included.
- Coupling to 1^{++} ground and first excited $c\bar{c}$ states with bare masses within the model: $c\bar{c}(1^3P_1) \rightarrow M = 3503{,}9\ MeV$ $c\bar{c}(2^3P_1) \rightarrow M = 3947{,}4\ MeV$ and
- Isospin breaking $M_{D^\pm} + M_{D^{*\pm}} \neq M_{D^0} + M_{D^{*0}}$

Parameter free calculation.

$M\ (MeV)$	$c\bar{c}(1^3P_1)$	$c\bar{c}(2^3P_1)$	D^0D^{*0}	$D^\pm D^{*\mp}$	Assignment
3937	0 %	79 %	7 %	14 %	
3863	1 %	30 %	46 %	23 %	$\rightarrow X(3872)$
3467	95 %	0 %	2,5 %	2,5 %	

- Isospin probabilities: $\mathcal{P}_{I=0} = 66\,\%$, $\mathcal{P}_{I=1} = 3\,\%$, $\mathcal{P}_{c\bar{c}} = 30\,\%$.
- Fine tune $^3P_0$ γ strength parameter to E_{bind}. $\mathcal{P}_{I=0} \sim 70\,\%$, $\mathcal{P}_{I=1} \sim 23\,\%$, $\mathcal{P}_{c\bar{c}} \sim 7\,\%$
- P.G. Ortega, J. Segovia, DRE, F. Fernández, Phys. Rev. D 81 (2010)
- P.G. Ortega, DRE, F. Fernández, J. Phys. G 40 (2013)
- M. Takizawa, S. Takeuchi, PTEP 9 (2013) at hadron level

The X(3872) analogs

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Including only open-charm and open-bottom channels

- *No X(4012) is found*
- $X_b(1^{++})$ *is close to bind and* $X_b(2^{++})$ *is bounded*

DRE et al., AIP Conf. Proc. 1735, 060006 (2016);

More refined calculation including other channels gives the same conclusion for the charmonium sector

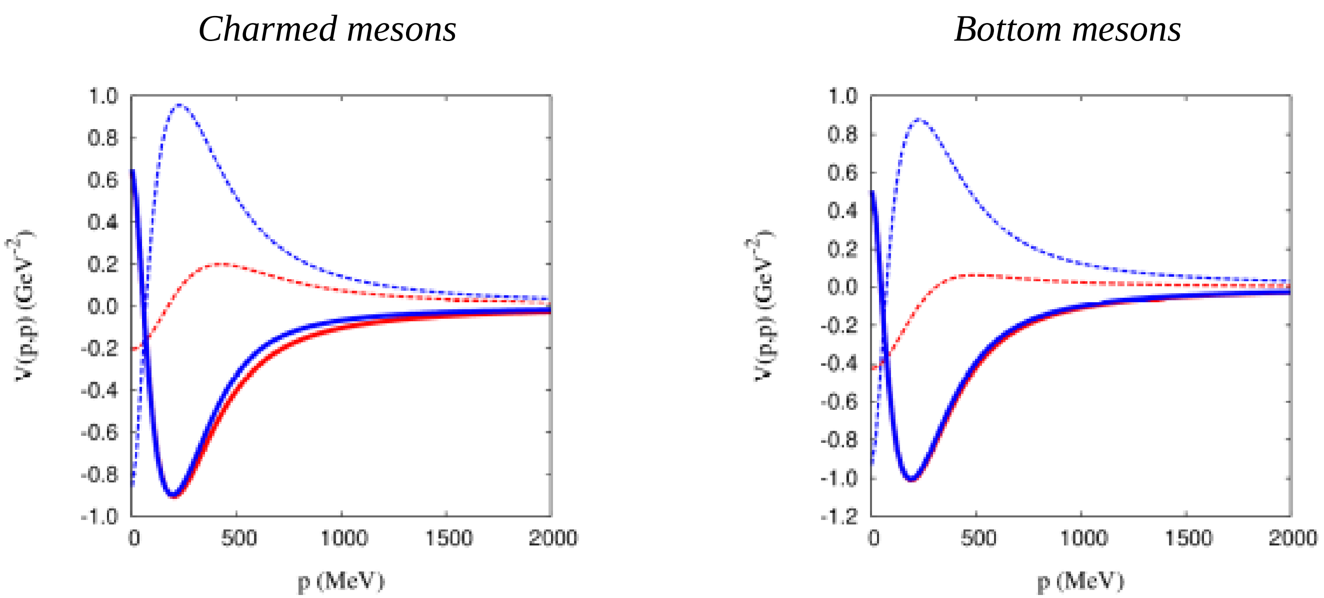
P.G. Ortega et al., Phys.Lett. B778 (2018) 1

Why this discrepancies with HQSS and HFS?

HQSS and HFS

Heavy Quark Spin Symmetry and Heavy Flavor Symmetry is fulfilled by the model

$$\frac{2}{\sqrt{3}}\langle D^*D^*(0^{++})|H_I|DD(0^{++})\rangle \quad = \quad \langle DD(0^{++})|H_I|DD(0^{++})\rangle - \langle D^*D^*(0^{++})|H_I|D^*D^*(0^{++})\rangle$$



$\langle DD(0^{++})|H|DD(0^{++})\rangle$

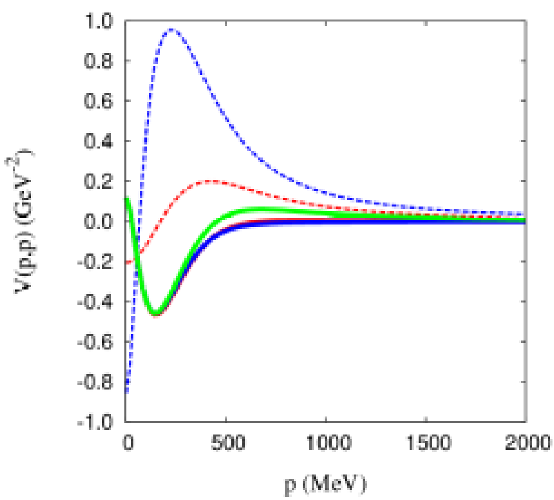
$\langle D^*D^*(0^{++})|H|D^*D^*(0^{++})\rangle$

HQSS and HFS

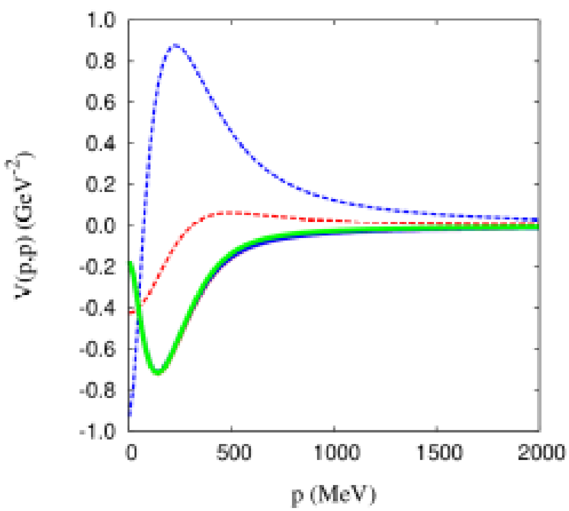
Heavy Quark Spin Symmetry and Heavy Flavor Symmetry is fulfilled by the model

$$\begin{aligned}\langle DD^*(1^{++})|H_I|DD^*(1^{++})\rangle &= \langle D^*D^*(2^{++})|H_I|D^*D^*(2^{++})\rangle \\ &= \frac{3}{2}\left[\langle DD(0^{++})|H_I|DD(0^{++})\rangle - \frac{1}{3}\langle D^*D^*(0^{++})|H_I|D^*D^*(0^{++})\rangle\right]\end{aligned}$$

Charmed mesons



Bottom mesons



$\langle DD(0^{++})|H|DD(0^{++})\rangle$

$\langle D^*D^*(0^{++})|H|D^*D^*(0^{++})\rangle$

HQSS and HFS breaking

$$\frac{M_{D^*}-M_D}{M_{D^*}+M_D}\sim 3.6\%$$ *but* $$M_{D^*}-M_D\sim 141\text{MeV}\Rightarrow M_{D^*D^*}-M_{DD}\sim 282\text{MeV}$$

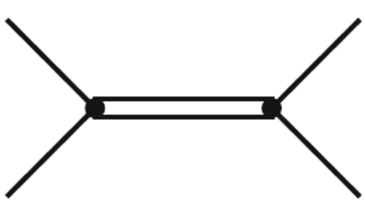
$$\frac{M_{\chi_{c2}(1P)}-M_{\chi_{c0}(1P)}}{M_{\chi_{c2}(1P)}+M_{\chi_{c0}(1P)}}\sim 2.0\%\qquad M_{\chi_{c2}(1P)}-M_{\chi_{c0}(1P)}\sim 141\text{MeV}\Rightarrow M_{\chi_{c2}(2P)}-M_{\chi_{c0}(2P)}\leq 141\text{MeV}$$

D. Ebert et al., The Eur. Phys. J. C 71, 1825 (2011) $M_{\chi_{c2}(2P)}-M_{\chi_{c0}(2P)}\sim 80\,\text{MeV}$

S wave states:

- $0^{++}\Rightarrow D\bar{D}, D^*\bar{D}^*$
- $1^{++}\Rightarrow D\bar{D}^*$
- $2^{++}\Rightarrow D^*\bar{D}^*$

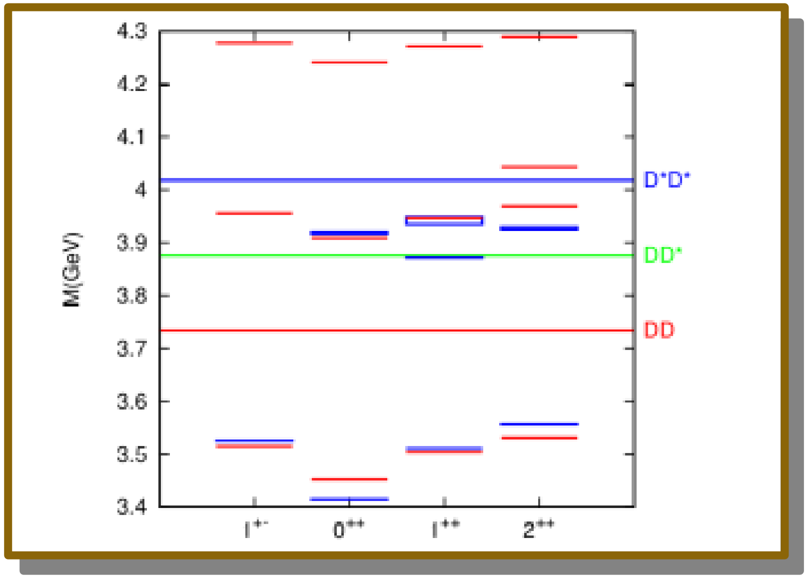
1^3P_0	0^{++}	3413	$\chi_{c0}(1P)$	3414.75(31)
1^3P_1	1^{++}	3511	$\chi_{c1}(1P)$	3510.66(7)
1^3P_2	2^{++}	3555	$\chi_{c2}(1P)$	3556.20(9)
1^1P_1	1^{+-}	3525	$h_c(1P)$	3525.41(16)
2^3P_0	0^{++}	3870		
2^3P_1	1^{++}	3906		
2^3P_2	2^{++}	3949	$\chi_{c2}(2P)$	3927.2(2.6)
2^1P_1	1^{+-}	3926		



$$\sim\quad G^0_{c\bar{c}}=\frac{1}{E-\mathring{m}_{c\bar{c}}}\begin{cases}<0 & \mathring{m}_{c\bar{c}}>E\\ >0 & \mathring{m}_{c\bar{c}}<E\end{cases}$$ *Attraction* / *Repulsion*

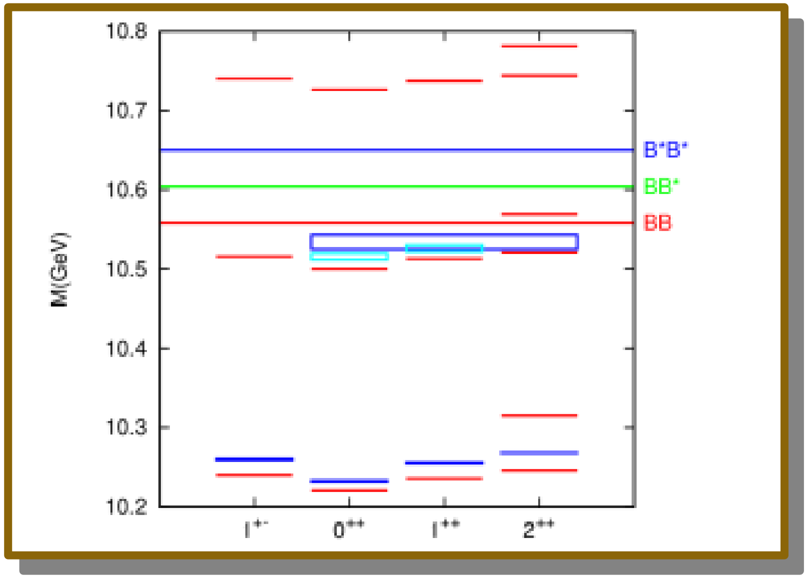
HQSS and HFS breaking

Charmonium



$D\bar{D}^*$ $D^*\bar{D}^*$

Bottomonium



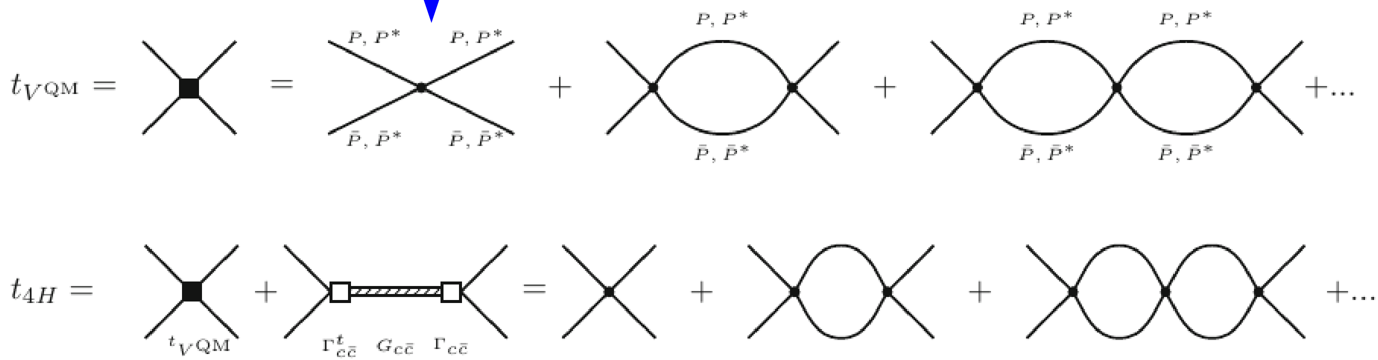
$B\bar{B}^*$ $B^*\bar{B}^*$

HQSS and HFS breaking

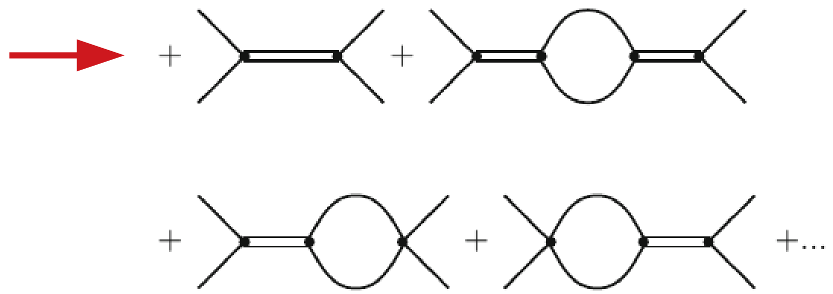
E. Cincioglu et al., Eur. Phys. J. C 76, 576 (2016)

A systematic study of this effect has been performed at hadron level

HQSS Hamiltonian for mesons



Coupling with cc mesons is included



HQSS breaking

Mass of the X(3872) is fixed varying d and C_{0X} at the same time

d [fm$^{1/2}$]	C_{0X} [fm^2]	$g^{X(3872)}_{D\bar{D}^*}$ [GeV$^{-1/2}$]	$\tilde{X}_{X(3872)}$	$(m_{\chi_{c1}}, \Gamma_{\chi_{c1}})$ [MeV]	$g^{\chi_{c1}}_{D\bar{D}^*}$ [GeV$^{-1/2}$]	$\lvert\tilde{X}_{\chi_{c1}}\rvert$	$\tilde{Z}_{\chi_{c1}}$
0.	−0.789	0.90	1	(3906.0,0)	0.	0.	1.
0.05	−0.774	0.89	0.98	(3906.6, 1.9)	$0.01+0.16\,i$	0.02	$0.99+0.01\,i$
0.1	−0.731	0.87	0.92	(3908.2, 7.9)	$0.03+0.31\,i$	0.06	$0.96+0.05\,i$
0.15	−0.659	0.83	0.84	(3910.5, 19.2)	$0.07+0.44\,i$	0.14	$0.92+0.11\,i$
0.20	−0.559	0.78	0.75	(3912.4, 37.8)	$0.14+0.56\,i$	0.23	$0.87+0.19\,i$
0.25	−0.429	0.73	0.66	(3912.0, 67.0)	$0.24+0.65\,i$	0.36	$0.82+0.31\,i$
0.30	−0.271	0.68	0.57	(3903.9, 112.8)	$0.38+0.73\,i$	0.55	$0.77+0.50\,i$
0.35	−0.084	0.63	0.49	(3864.5, 185.2)	$0.63+0.85\,i$	>1	$0.70+1.01\,i$
d^{crit}	0.000	0.61	0.47	(3798.3, 209.4)	$0.93+1.09\,i$	>1	$0.53+2.12\,i$
0.375	0.020	0.61	0.46	(3754.4, 186.4)	$1.21+1.37\,i$	>1	$0.29+3.66\,i$
0.3775	0.031	0.61	0.46	(3701.6, 93.5)	$2.19+2.39\,i$	>1	$-0.44+12.27\,i$
0.40	0.132	0.59	0.43	(3827.1, 0) at SRS	0.96	$\tilde{X}_{\chi_{c1}}<0$	2.07
0.45	0.376	0.55	0.37	(3850.9,0) at SRS	0.63	$\tilde{X}_{\chi_{c1}}<0$	1.52
0.5	0.649	0.51	0.32	(3858.4,0) at SRS	0.51	$\tilde{X}_{\chi_{c1}}<0$	1.36
1.0	4.963	0.29	0.11	(3869.7, 0) at SRS	0.21	$\tilde{X}_{\chi_{c1}}<0$	1.08
2.0	22.217	0.15	0.03	(3871.3, 0) at SRS	0.10	$\tilde{X}_{\chi_{c1}}<0$	1.02
$d \gg d^{\text{crit}}$	$\sim \frac{d^2}{\mathring{m}_{\chi_{c1}}-M_X}$	$\mathcal{O}(1/d)$	$\mathcal{O}(1/d^2)$	$(M_X-\mathcal{O}(\frac{1}{d^2}), 0)$ at SRS	$\mathcal{O}(1/d)$	$\tilde{X}_{\chi_{c1}}=-\mathcal{O}(\frac{1}{d^2})$	$1+\mathcal{O}(\frac{1}{d^2})$

The molecular component of the X(3872) decreases

χ_{c2}(2P) becomes a resonance that goes to SRS

HQSS breaking

Mass of the X(3872) is fixed varying d and C_{0X} at the same time

d [fm$^{1/2}$]	$\tilde{X}_{X(3872)}$	$g^{\chi_{c2}}_{D^*\bar{D}^*}$ [GeV$^{-1/2}$]	$\tilde{X}_{\chi_{c2}}$	$\mathring{m}_{\chi_{c2}}$ [MeV]	$M_{X_2} - 2M_{D^*} - i\frac{\Gamma_{X_2}}{2}$ [MeV]	$g^{X_2}_{D^*\bar{D}^*}$ [GeV$^{-1/2}$]	$\tilde{X}_{X_2}$
0.	1	0.0	0.0	3927.2	-5.6	0.97	1.
0.05	0.98	0.27	0.01	3927.8	-4.5	0.90	0.996
0.10	0.92	0.51	0.02	3929.6	-1.8	0.67	0.991
0.15	0.84	0.69	0.04	3932.2	-0.0 at SRS	$-0.12\,i$	>1
0.20	0.75	0.82	0.05	3935.2	-6.4 at SRS	$-0.76\,i$	>1
0.22	0.71	0.86	0.06	3936.4	-21.2 at SRS	$-1.24\,i$	>1
0.25	0.66	0.90	0.06	3938.3	$-28.3 - \frac{72.9}{2}\,i$	$0.23 - 0.65\,i$	$0.47 + 0.32\,i$
0.30	0.57	0.95	0.07	3941.2	$-31.2 - \frac{162.8}{2}\,i$	$0.03 + 0.67\,i$	$0.48 - 0.04\,i$
0.35	0.49	0.96	0.07	3943.8	$-59.5 - \frac{312.6}{2}\,i$	$0.30 + 0.71\,i$	$0.52 - 0.39\,i$

The X(4012) disappears

$$V^{2^{++}}_{\rm eff}(E=2M_{D^*}) = V^{1^{++}}_{\rm eff}(E=M_X) + d^2 \times \left(\frac{(2M_{D^*}-M_X)-(\mathring{m}_{\chi_{c2}}-\mathring{m}_{\chi_{c1}})}{(2M_{D^*}-\mathring{m}_{\chi_{c2}})(\mathring{m}_{\chi_{c1}}-M_X)}\right)$$

Repulsion

HFS breaking

1^{++}

d [fm$^{1/2}$]	$\tilde{X}_{X(3872)}$	$g^{\chi_{b1}}_{B\bar{B}^*}$ [GeV$^{-1/2}$]	$\tilde{X}_{\chi_{b1}}$	$\mathring{m}_{\chi_{b1}}$ [MeV]	$E_{X_b} - M_B - M_{B^*}$ [MeV]	$g^{X_b}_{B\bar{B}^*}$ [GeV$^{-1/2}$]	$\tilde{X}_{X_b}$
0.	1	0.0	0.0	10512.1	-65.9	2.30	1.
0.05	0.98	0.98	0.09	10515.0	-60.7	2.04	0.91
0.10	0.92	1.46	0.20	10521.4	-47.6	1.55	0.80
0.15	0.84	1.59	0.24	10527.8	-30.8	1.11	0.77
0.20	0.75	1.57	0.23	10532.6	-13.1	0.69	0.80
0.25	0.66	1.49	0.21	10536.1	-0.1	0.16	0.96
0.30	0.57	1.40	0.18	10538.5	$4.9 - \frac{68.2}{2}\,i$	$0.05 - 0.26\,i$	$0.43 + 0.16\,i$
0.35	0.49	1.29	0.16	10540.2	$44.8 - \frac{181.4}{2}\,i$	$0.12 + 0.28\,i$	$0.55 - 0.21\,i$

2^{++}

d [fm$^{1/2}$]	$\tilde{X}_{X(3872)}$	$g^{\chi_{b2}}_{B^*\bar{B}^*}$ [GeV$^{-1/2}$]	$\tilde{X}_{\chi_{b2}}$	$\mathring{m}_{\chi_{b2}}$ [MeV]	$E_{X_{b2}} - 2M_{B^*}$ [MeV]	$g^{X_{b2}}_{B^*\bar{B}^*}$ [GeV$^{-1/2}$]	$\tilde{X}_{X_{b2}}$
0.	1	0.0	0.0	10522.1	-66.2	2.31	1.
0.05	0.98	0.69	0.02	10523.4	-62.5	2.17	0.98
0.10	0.92	1.20	0.06	10526.9	-52.3	1.82	0.94
0.15	0.84	1.50	0.10	10531.3	-37.2	1.37	0.91
0.20	0.75	1.64	0.11	10535.7	-19.4	0.90	0.90
0.25	0.66	1.67	0.12	10539.5	-3.1	0.41	0.93
0.30	0.57	1.64	0.11	10542.5	$-18.0 - \frac{37.4}{2}\,i$	$0.16 - 0.28\,i$	$0.46 + 0.75\,i$
0.35	0.49	1.59	0.11	10545.0	$27.1 - \frac{195.1}{2}\,i$	$0.09 + 0.28\,i$	$0.57 - 0.15\,i$

Conclusion: the repulsion from the χ_{bJ}(3P) states is not so important and the two states could be found

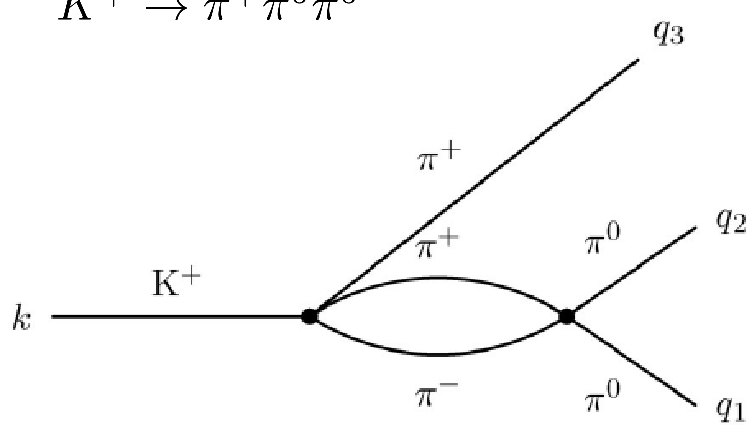
Threshold cusp

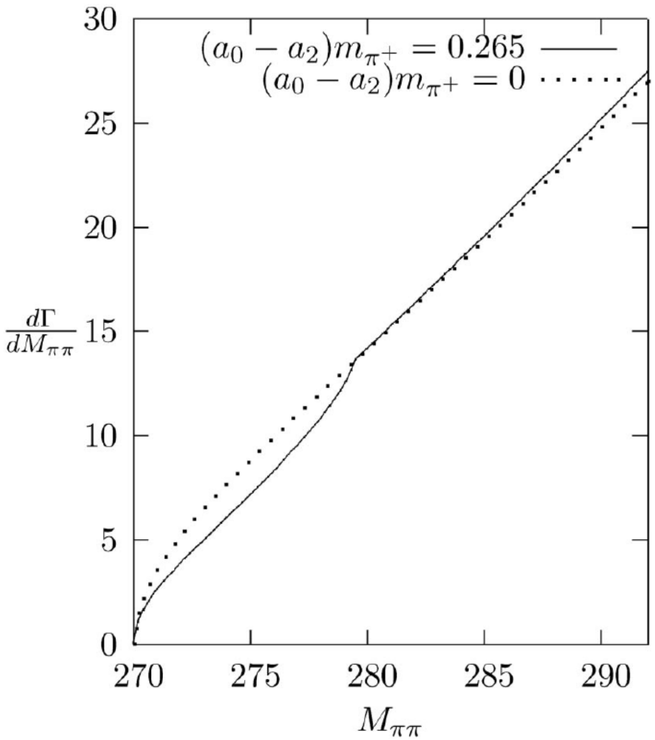
Discontinuity of the amplitude due to the opening of a threshold

N. Cabibbo, Phys. Rev. Lett. 93, 121801 (2004)

Proposal to extract (a_0-a_2) $\pi\pi$ scattering lengths due to the threshold cusp produce by the charge exchage reaction.

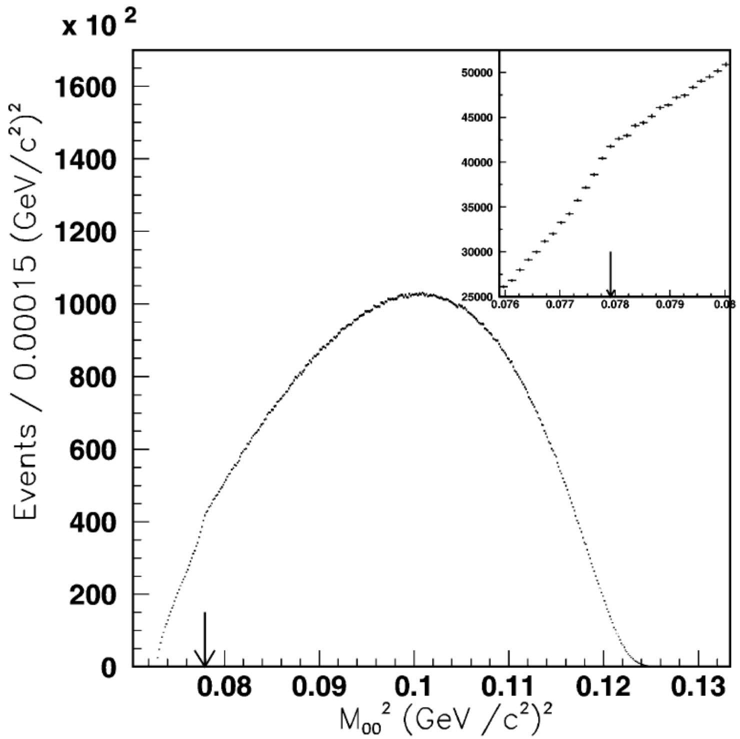
$K^+ \to \pi^+\pi^0\pi^0$





Threshold cusp in ππ

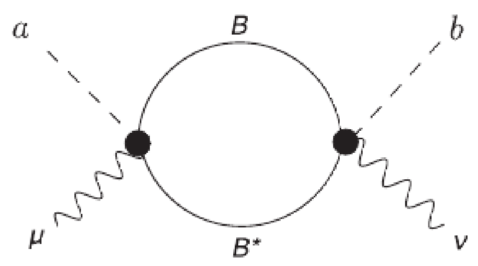
J. Batley et al., Physics Letters B 633, 173 (2006)



$$(a_0 - a_2)m_{\pi^+} = 0.268 \pm 0.010(\text{stat}) \pm 0.004(\text{syst})$$

The Zc and Zb as threshold cusps

E. Swanson, Phys. Rev. D 91, 034009 (2015)



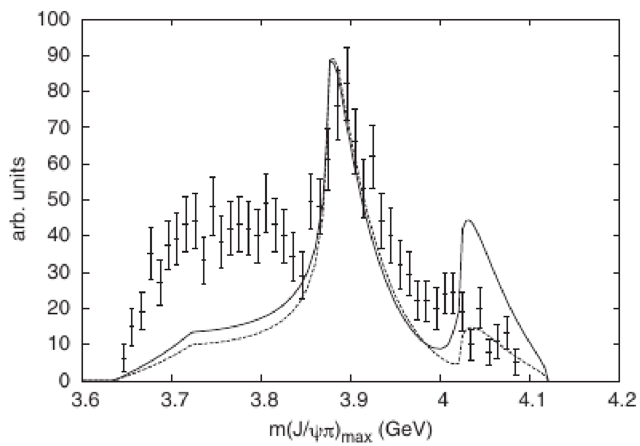
$$\mathrm{Im}\Pi_{\alpha\beta}(s) = \sum_i k_i^{1+\ell_{\alpha i}+\ell_{\beta i}} F_{\alpha i}(s) F_{\beta i}(s)$$

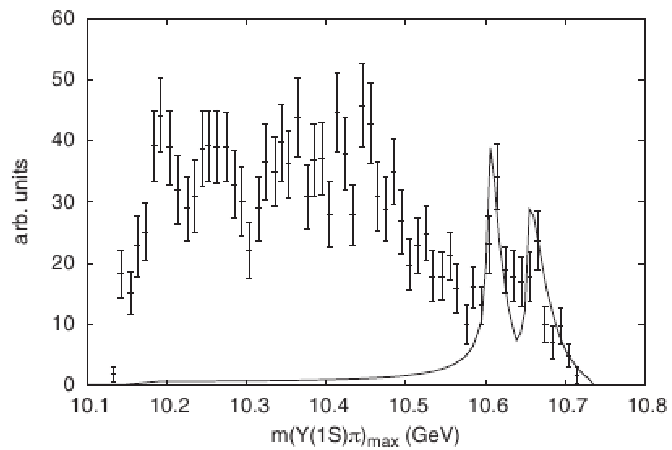
$$\Pi_{\alpha\beta}(s) = \frac{1}{\pi}\int_{s_{\mathrm{th}}}^{\infty} ds' \frac{\mathrm{Im}\Pi_{\alpha\beta}(s')}{s'-s-i\epsilon}.$$

$$\overline{|\mathcal{M}_{\alpha\beta}|^2} \propto |\Pi_{\alpha\beta}(s)|^2.$$

$e^+e^- \to \pi\pi J/\psi$

$\Upsilon(5S) \to \Upsilon(1S)\pi\pi$





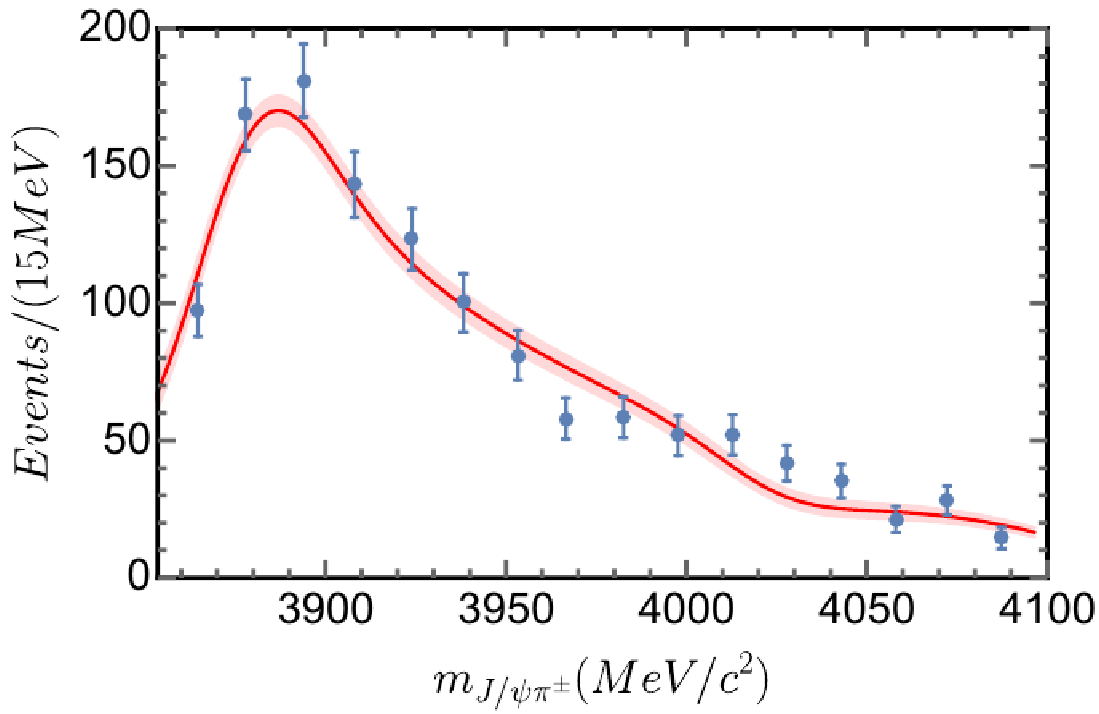
F.-K. Guo et al., Phys. Rev. D 91, 051504R (2015) *One-loop approximation is not justified and nearby poles appear*

Zc states in the CQM

P.G. Ortega et al, Eur.Phys.J. C79 (2019) no.1, 78

$$e^+e^- \to \pi^+\pi^- J/\psi$$

$$\sqrt{s} = 4.26 \text{ GeV}$$



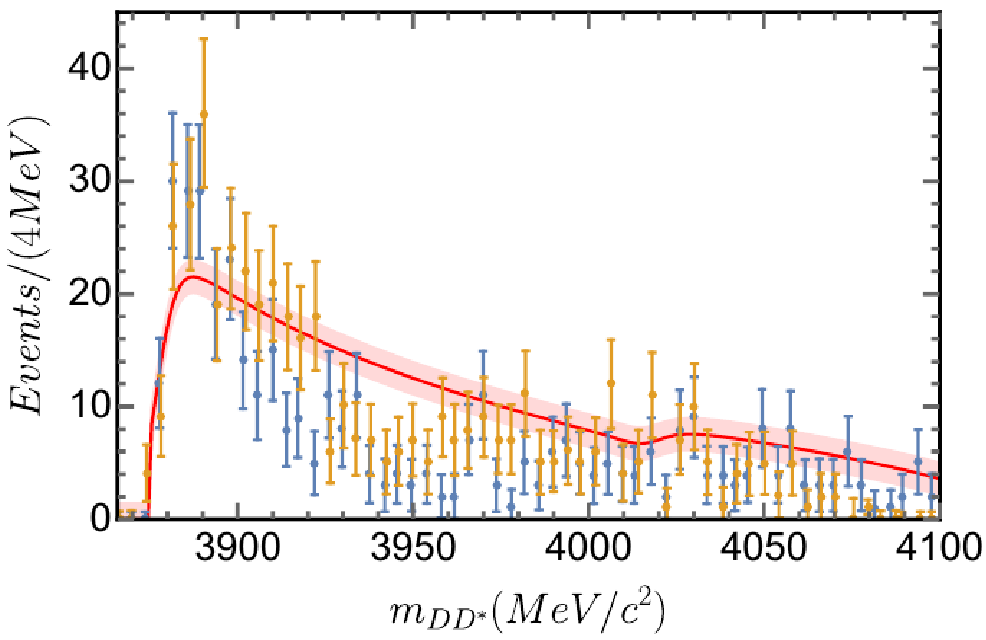
Data from BESIII Phys. Rev. Lett. 119, 072001 (2017)

Zc states in the CQM

P.G. Ortega et al, Eur.Phys.J. C79 (2019) no.1, 78

$$e^+e^- \to \pi^\pm (D\bar{D}^*)^\mp$$

$$\sqrt{s} = 4.26 \text{ GeV}$$



Data from BESIII Phys. Rev. D 92, 092006 (2015)

Zc states in the CQM

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P.G. Ortega et al, Eur.Phys.J. C79 (2019) no.1, 78

Calculation	$Z_c(3900)$ pole	RS	$Z_c(4020)$ pole	RS
$D\bar{D}^*$	$3871.37 - 2.17\,i$	(S)	-	-
$D\bar{D}^* + D^*\bar{D}^*$	$3872.27 - 1.85\,i$	(S,F)	$4014.16 - 0.10\,i$	(S,S)
$\rho\eta_c + D\bar{D}^*$	$3871.32 - 0.00\,i$	(S,S)	-	-
$\rho\eta_c + D\bar{D}^* + D^*\bar{D}^*$	$3872.07 - 0.00\,i$	(S,S,F)	$4013.10 - 0.00\,i$	(S,S,S)
$\pi J/\psi + \rho\eta_c + D\bar{D}^* + D^*\bar{D}^*$	$3871.74 - 0.00\,i$	(S,S,S,F)	$4013.21 - 0.00\,i$	(S,S,S,S)

Calculation	$Z_c(3900)$	type
This work	3871.74	virtual
F. Aceti et al.	$3878 - 23\,i$	resonance
M. Albaladejo et al.	$3894 \pm 6 \pm 1 - 30 \pm 12 \pm 6\,i$	resonance
	$3886 \pm 4 \pm 1 - 22 \pm 6 \pm 4\,i$	resonance
	$3831 \pm 26^{+7}_{-28}$	virtual
	$3844 \pm 19^{+12}_{-21}$	virtual
Y. Ikeda et al.	$3709 \pm 94 - 183(46)\,i$	virtual
	$3748 \pm 76 - 157(32)\,i$	virtual
	$3686 \pm 56 - 44(27)\,i$	virtual
J. He et al.	$3876 - 5\,i$	resonance
Calculation	**$Z_c(4020)$**	**type**
This work	4013.21	virtual
F. Aceti et al.	$(3990 - 4000) - 50\,i$	bound/virtual

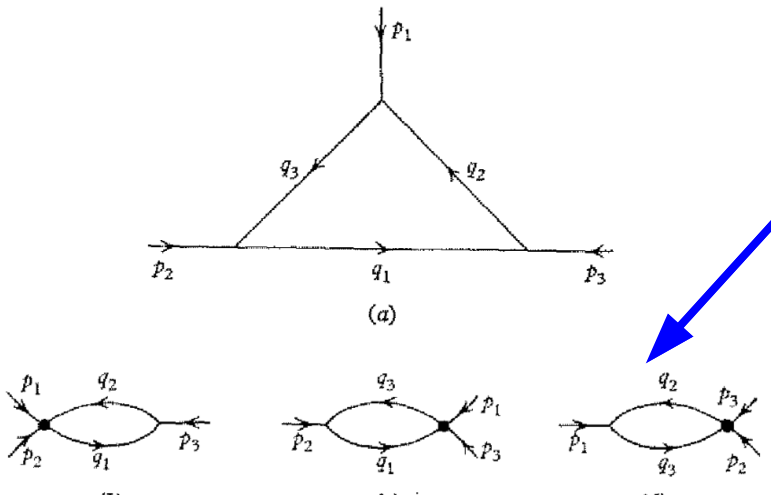
Triangle singularities

Singularities of a triangle diagram that can be not poles, but kinematical effects

$$\int \frac{d^4k}{(2\pi)^4} \frac{B}{(q_1^2 - m_1^2)(q_2^2 - m_2^2)(q_3^2 - m_3^2)}$$

Singularities are given by the Landau equations

$$\boxed{\begin{aligned} 1 + 2\mu_1\mu_2\mu_3 &= \mu_1^2 + \mu_2^2 + \mu_3^2 \\ \mu_i &= \frac{m_j^2 + m_k^2 - p_i^2}{2m_j m_k} \end{aligned}}$$



- *Normal threshold singularities*
- *Anomalous threshold singularities*
- *The width of the states makes the singularity move from the physical region*
- *Since they are kinematical effects they don't need to be present in all channels as in the case of poles*

The X(3872) binding energy

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F.-K. Guo, Phys. Rev. Lett. 122, 202002 (2019)

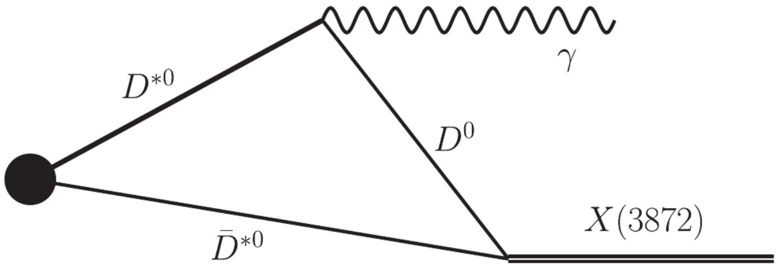
Proposal to measured the X(3872) binding energy with the triangle singularity
Prove the long range properties of the X(3872)

$$I(E_{X\gamma}) = \frac{1}{E_\gamma}\left[\arctan\left(\frac{c_2 - c_1}{2b\sqrt{c_1}}\right) + \arctan\left(\frac{c_1 - c_2 + 2b^2}{2b\sqrt{c_2 - b^2}}\right)\right]$$

Invariant mass of the X and γ

$$\begin{aligned} b &= m_{D^{*0}} E_\gamma / (m_{D^0} + m_{D^{*0}}) \\ c_1 &= m_{D^{*0}}(2m_{D^{*0}} - E_{X\gamma} - i\Gamma_{D^{*0}}) \\ c_2 &= \mu_{D^{*0}D^0}\left[2(m_{D^{*0}} + m_{D^0} + E_\gamma - E_{X\gamma}) + E_\gamma^2/m_{D^0} - i\Gamma_{D^{*0}}\right] \end{aligned}$$

The triangle singularity is close to threshold



$$\begin{aligned} E_{X\gamma}^{TS} &= 2m_{D^{*0}} + \frac{x^2}{2m_{D^0}} + \mathcal{O}\left(\frac{x^3}{m_{D^0}^2}\right) \\ x &= m_{D^{*0}} - m_{D^0} - 2\sqrt{-m_{D^0}\delta + \delta} \end{aligned}$$

Binding energy of the X(3872)

The X(3872) binding energy

The line shape

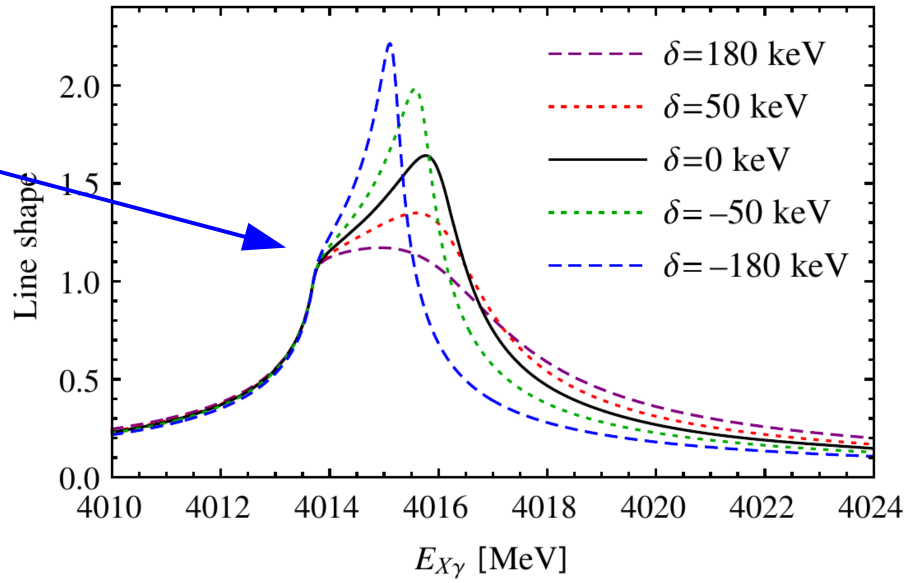
$$F(E_{X\gamma}) = \frac{|I(E_{X\gamma})|^2}{|I(2m_{D^{*0}})|^2} \frac{E_\gamma^3}{[(4m_{D^{*0}}^2 - m_X^2)/(4m_{D^{*0}}^2)]^3}$$

Strongly depends on the binding energy

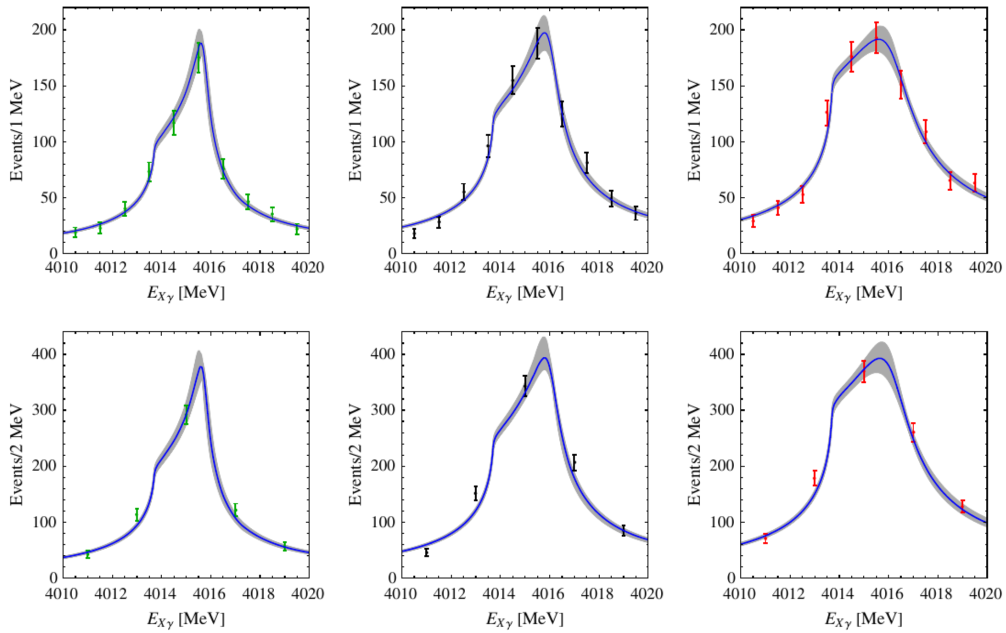
Cusp fixed

Peak fixed at $E_{X\gamma}^{\mathrm{TS}}$

δ (keV)	$E_{X\gamma}^{\mathrm{TS}}$ (MeV)
-180	$4015.2 - i0.1$
-50	$4015.7 - i0.2$
0	$4016.0 - i0.4$



The X(3872) binding energy



Montecarlo simulation to extract The binding energy of the X(3872)

TABLE I. Results on δ (in units of keV) from fitting to the synthetic data sets generated using three different input values of $\delta_{\rm in}$: -50 keV, 0 keV, and 50 keV.

Data set *A*	$\delta_{\rm in} = -50$ keV (127 events)	$\delta_{\rm in} = 0$ keV (164 events)	$\delta_{\rm in} = 50$ keV (192 events)
10 bins	-24^{+24}_{-28}	11^{+31}_{-20}	22^{+41}_{-23}
5 bins	-17^{+24}_{-27}	30^{+64}_{-29}	40^{+67}_{-31}
Data set *B*	$\delta_{\rm in} = -50$ keV (626 events)	$\delta_{\rm in} = 0$ keV (831 events)	$\delta_{\rm in} = 50$ keV (1006 events)
10 bins	-47^{+13}_{-16}	-1^{+13}_{-11}	63^{+34}_{-24}
5 bins	-48^{+15}_{-19}	-4^{+11}_{-10}	53^{+38}_{-25}
Data set *C*	$\delta_{\rm in} = -50$ keV (3133 events)	$\delta_{\rm in} = 0$ keV (4027 events)	$\delta_{\rm in} = 50$ keV (5015 events)
10 bins	-53^{+7}_{-8}	-2 ± 5	55^{+13}_{-11}
5 bins	-52^{+7}_{-8}	-2^{+7}_{-6}	61^{+17}_{-14}

Summary

Symmetry breaking effects

- *Small symmetry breaking effects in thresholds can induce interesting breaking effects*
  - *Isospin violating decays: X(3872) and Pc(4457)*
  - *Deviations from HQSS and HFS*

Threhold cusps

- *Threshold openings induce enhancements in cross sections*
- *Big effects without nearby poles are debatable*

Triangle singularities

- *Not present in all channels*
- *Can be used to measured the X(3872) binding energy*

Thank you for your attention