Three-body System in a Box

Jin-Yi Pang

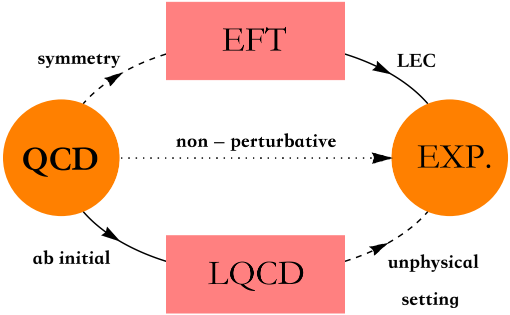
Helmholtz-Institut für Strahlen- und Kernphysik, Universität Bonn

Hadron 2019 August 17 , 2019

Collaborate with *M. Döring, J. J. Wu*

1. *Introduction*
2. *Lattice QCD and 3-body Effective Field Theory*
3. *Application on 3-body System*
4. *Summary and Outlook*

- *Hadron Physics*
  - *Quantum ChromoDynamics*
  - *Hadron Spectroscopy and Strong Decay Process, e.g.,* $N^*(1440) \to N\pi\pi$

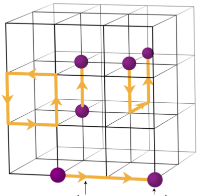


- *Effective Field Theory*

- *Lattice Quantum Chromodynamics*

$$\mathscr{L}_{\text{QCD}} = \sum_i \bar{\psi}_i (i\not{D}_\mu - m_i)\psi_i - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}$$

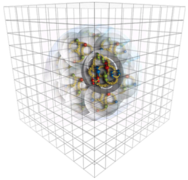
where quarks' flavor $i = u, d, s, c, b$ and gluon field $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$



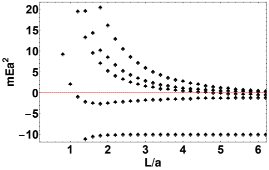
Input: lattice spacing a, lattice size L

mesons, e.g., π, K, D_s, B_s

Build operators for specific system



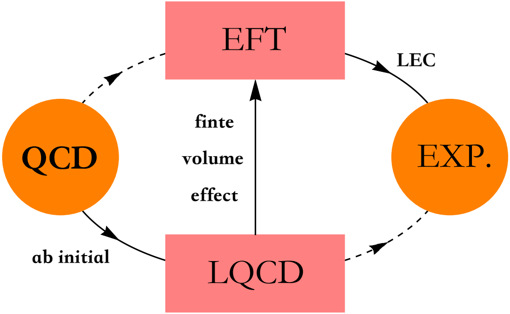
Output: Finite volume spectrum



Relation to experiment?

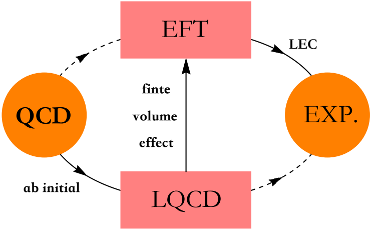
ρ $(\pi\pi)$, Roper Resonance ($N\pi$ and $N\pi\pi$), $\cdots$

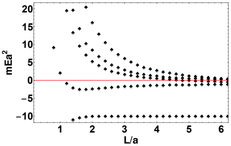
- *From QCD to Experiment* *(Extract 3-body Physical Observable from LQCD)*



- *Efimov State in a Box and 3-body Force* arXiv:1706.07700
- *Transparent Theoretical Framework for LQCD 3-body Simulation* arXiv:1707.02176
- *3-body State in Cubic Representation, i.e.,* $A_1^+, E^+, T_1^-, \cdots$ arXiv:1802.03362
- *Energy Shift of 3-body State and 3-body Physical Observable* arXiv:1902.01111
- *Triton in a Box (including Higher Partial Wave, e.g.,* $S-D$ *mixing in Deuteron)*
- *Twisted Boundary Condition, Relativistic Kinematics*
- *Roper Resonance,* $N^*(1440) \to N\pi$ *and* $N\pi\pi$

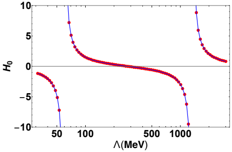
- *From QCD to Experiment*





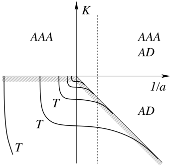
3-body Finite Volume Spectrum

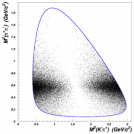
???



3-body Force in EFT

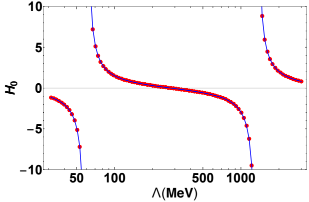
???



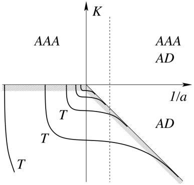


3-body Observables

3-body EFT and 3-body Observables



3-body Force in EFT

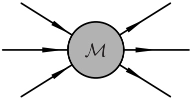


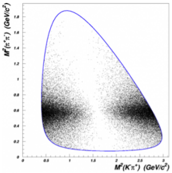
Efimov States: 3-body Bound States

- *Effective Theory of 3-body System*

$$\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_I^{(\text{2-body})} + \mathscr{L}_I^{(\text{3-body})}$$

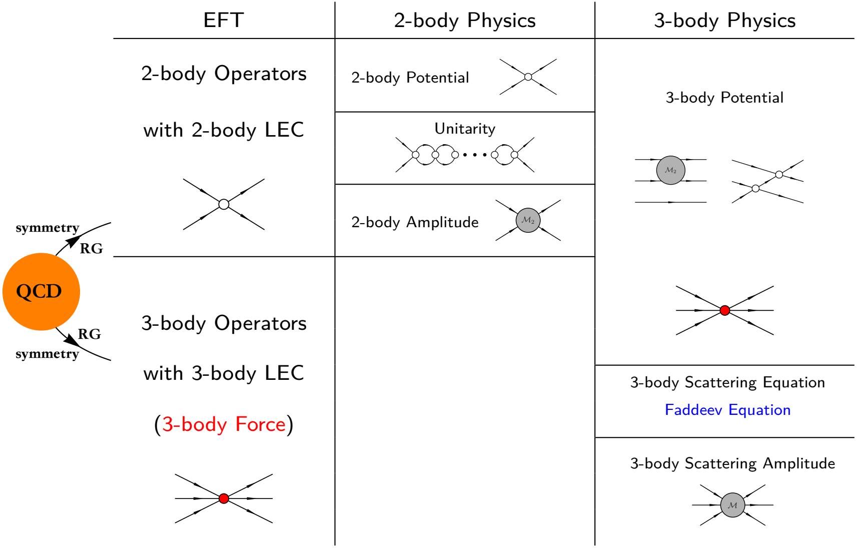
- *3-body Scattering Amplitude*





Dalitz Plot for 3-body Decay

Effective Field Theory for 3-body System



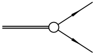
universitätbonn

EFT	2-body Physics	3-body Physics

EFT

2-body Operators

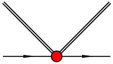
with 2-body LEC



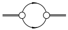
3-body Operators

with 3-body LEC

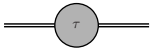
(3-body Force)



2-body Physics

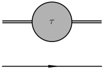
Dimer Self-Energy 

Unitarity

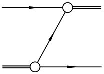
Dimer Propagation 

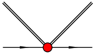
3-body Physics

Dimer-Spectator Propagation



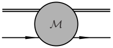
Particle-Dimer Potential

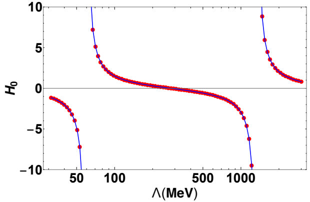




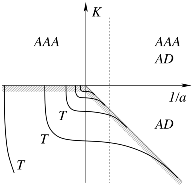
Particle-Dimer Scattering Equation

Particle-Dimer Scattering Amplitude



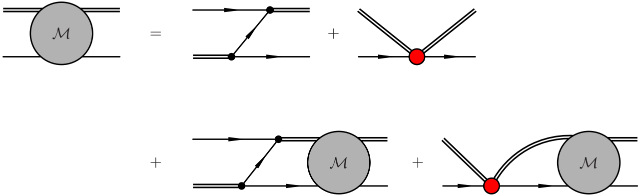


3-body Force in EFT



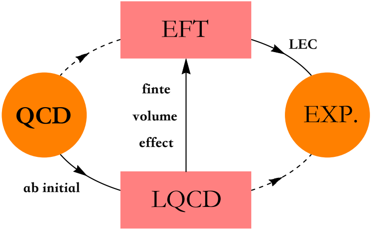
Efimov States: 3-body Bound States

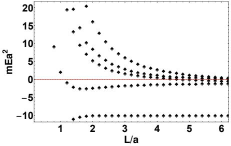
- *Particle-Dimer Scattering Equation*



- *Projection and Cut-off Regularization (Relative Momentum between Dimer and Spectator)*

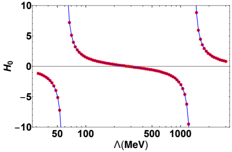
- *From QCD to Experiment*





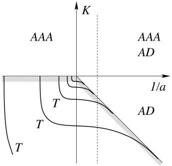
3-body Finite Volume Spectrum

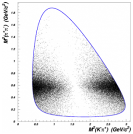
??? ⟷



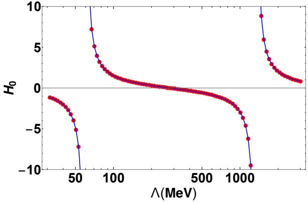
3-body Force in EFT

✓✓ ⟷

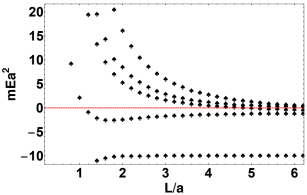




3-body Observables

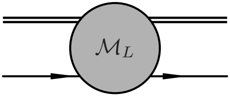


3-body Force in EFT



3-body Lattice Spectrum

- *Scattering Amplitude in a Box*



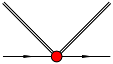
- *Quantization Condition*

EFT

3-body Operators

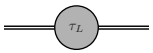
with 3-body LEC

(3-body Force)



2-body Physics

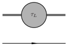
Dimer Propagation



$$= \frac{1}{p \cot\delta - S_F}$$

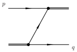
3-body Physics

Dimer-Spectator

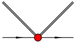


$$= \frac{1}{p_* \cot\delta - S_F(p_*)}$$

Potential



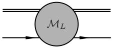
$$= \frac{1}{p^2 + q^2 + \mathbf{pq} - mE}$$



$$= \frac{H(\Lambda)}{\Lambda^2}$$

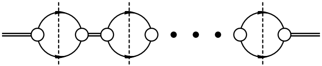
Particle-Dimer Scattering Equation in a Box

Particle-Dimer Scattering Amplitude in a Box

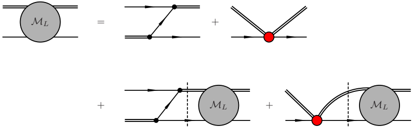


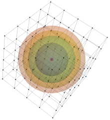
- *Finite Volume Correction* *(Discrete Momentum in a Box)*

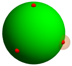
- *Particle-Dimer Scattering Equation in a Box*
  - *Dimer Propagation in a Box*



  - *Scattering Equation in a Box*



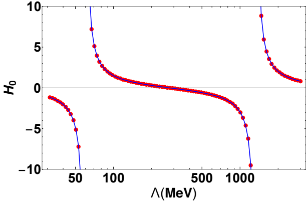




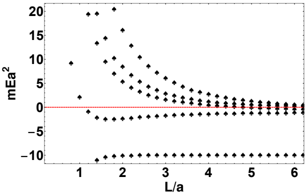
  - *Projection by Cubic Symmetry*

Rotaional Symmetry $\rightarrow$ Partial Wave Expansion

Cubic Symmetry $\rightarrow$ Cubic Irreps. Expansion (Shells and Irreps A_1, A_2, E, T_1, T_2)



3-body Force in EFT

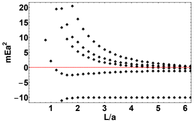


3-body Lattice Spectrum

- *Projected Quantization Condition*

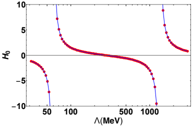
$$\det\left(\tau_L^{-1}(r;E)\delta_{sr}\delta_{ij} - \frac{8\pi}{L^3} Z_{ij}^{(\Gamma)}(s,r;E)\right) = 0$$

universitätbonn



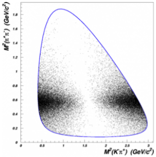
3-body Finite Volume Spectrum

✓✓



3-body Force in EFT

✓✓

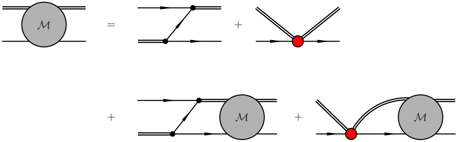


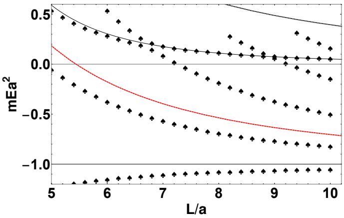
3-body Observables

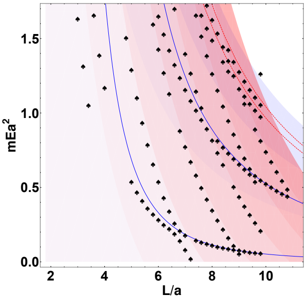
- *Quantization Condition*

$$\det\left(\tau_L^{-1}(r;E)\delta_{sr}\delta_{ij} - \frac{8\pi}{L^3}Z_{ij}^{(\Gamma)}(s,r;E)\right) = 0$$

- *Particle-Dimer Scattering Equation in the Infinite Volume*







- *Avoided Level Crossing* *(3-body Threshold and Particle-Dimer Threshold)* [arXiv:1809.07350]

- *Energy Shifts*

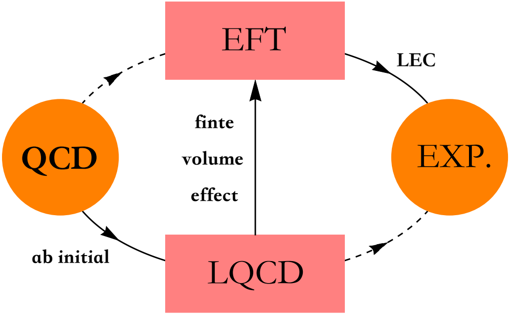
  [arXiv:1902.01111]

Ground State: $\kappa^2 = \frac{g_0}{L}\left(1 + \frac{g_1}{L} + \frac{g_2}{L^2} + \frac{g_3}{L^3}\ln\frac{mL}{2\pi} + \frac{g_4}{L^3} + \cdots\right)$

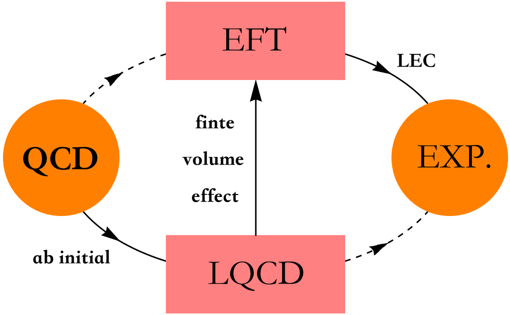
Excited State: $\kappa^2 - 1 = \frac{h_0}{L}\left(1 + \frac{h_1}{L} + \frac{h_2}{L^2} + \frac{h_3}{L^3}\ln\frac{mL}{2\pi} + \frac{h_4}{L^3} + \cdots\right),$

where $g_4 = \left(-5.159159617 + 6\pi\left(\frac{r}{a}\right) - 8\pi\left(\frac{\hat{\mathscr{M}}}{a^2}\right)\right)a^3$

$$h_4 = \left(-172.001650 + 83.745841\left(\frac{r}{a}\right) - \frac{27}{5}\times 8\pi\left(\frac{\hat{\mathscr{M}}}{a^2}\right)\right)a^3 .$$



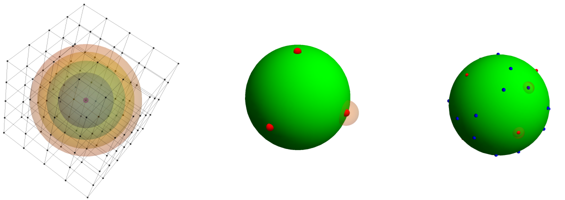
- *Build 3-body Operators and Obtain 3-body Finite Volume Spectrum from LQCD Simulation*
- *Quantization Condition: 3-body Finite Volume Spectrum $\to$ 3-body Force in Effective Theory*
- *Particle-Dimer Scattering Equation: 3-body Force $\to$ 3-body Physical Observables*



- *Triton in a Box (including Higher Partial Wave, e.g., $S-D$ mixing in Deutron)*
- *Twisted Boundary Condition, Relativistic Kinematics*
- *Roper Resonance, $N^*(1440) \to N\pi$ and $N\pi\pi$*

Thank you for your attention!

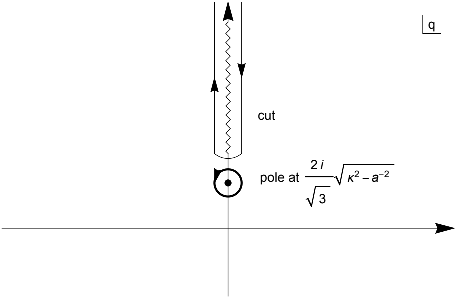
- *Shell Structure* Shell is a set of momenta with the same $|\mathbf{p}|$, which can be obtained from reference momentum $\mathbf{p}_0$, $\mathbf{p} = g\mathbf{p}_0, \quad g \in O_h$. The momenta unrelated by the O_h, but having $|\mathbf{p}| = |\mathbf{p}'|$, belong to the different shells.



- *Cubic Irreps. Expansion*

$$f(\mathbf{p}) \quad = f(g\mathbf{p}_0(s)) = \sum_{\Gamma,ij} f_{ij}^{(\Gamma)}(s)\, T_{ij}^{(\Gamma)}(g). \tag{1}$$

$$\sum_{\mathbf{p}} f(\mathbf{p}) = \underbrace{\sum_{s}}_{\text{different shells}} \underbrace{\frac{\vartheta_s}{G}\sum_{g}}_{\text{orientations inside shell } s} f(g\mathbf{p}_0(s)). \tag{2}$$



- *Energy Shifts of Bound States*

$$\Delta E = 8\pi \int \frac{d^3 q}{(2\pi)^3}\phi^\dagger(\mathbf{q}) \sum_{\mathbf{n}\neq 0} e^{i\mathbf{n}\mathbf{q}L}\tau(\mathbf{q})\phi(\mathbf{q}) + \cdots \qquad (3)$$

  - *Regular Wave Function* $\phi \sim$ *const.*
  - *Cut and Pole of* $\tau(\mathbf{q};E) = \frac{1}{-a^{-1}+\sqrt{\frac{3}{4}\mathbf{q}^2 - mE - i\varepsilon}}$

The energy of the scattering states vanishes in the infinite volume limit. We quote their finite volume energy E in terms of the quantity $\kappa^2 = L^2 mE/(2\pi)^2$.

The energy shift of the ground state (which resides in the A_1^+ irrep) is:

$$\kappa^2 = \frac{g_0}{L}\left(1 + \frac{g_1}{L} + \frac{g_2}{L^2} + \frac{g_3}{L^3}\ln\frac{mL}{2\pi} + \frac{g_4}{L^3} + \cdots\right), \tag{4}$$

with

$$\begin{aligned}
g_0 &= \frac{3}{\pi}\, a, \\
g_1 &= 2.837297480\, a, \\
g_2 &= 9.725330808\, a^2, \\
g_3 &= 8\pi\left(\frac{2\sqrt{3}}{\pi} - \frac{8}{3}\right) a^3, \\
g_4 &= \left(-5.159159617 + 6\pi\left(\frac{r}{a}\right) - 8\pi\left(\frac{\hat{\mathscr{M}}}{a^2}\right)\right) a^3.
\end{aligned} \tag{5}$$

The energy shift of the 1st excited state in the A_1^+ irrep is

$$\kappa^2 - 1 = \frac{h_0}{L}\left(1 + \frac{h_1}{L} + \frac{h_2}{L^2} + \frac{h_3}{L^3}\ln\frac{mL}{2\pi} + \frac{h_4}{L^3} + \cdots\right), \tag{6}$$

with

$$\begin{aligned}
h_0 &= \frac{10}{\pi}\,a\,,\\
h_1 &= 0.279070\,a\,,\\
h_2 &= \left(8.494802 + \frac{7\pi^2}{5}\left(\frac{r}{a}\right)\right)a^2\,,\\
h_3 &= \frac{27}{5}\times 8\pi\left(\frac{2\sqrt{3}}{\pi} - \frac{8}{3}\right)a^3\,,\\
h_4 &= \left(-172.001650 + 83.745841\left(\frac{r}{a}\right) - \frac{27}{5}\times 8\pi\left(\frac{\hat{\mathscr{M}}}{a^2}\right)\right)a^3\,.
\end{aligned} \tag{7}$$