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北京航空航天大學
BEIHANG UNIVERSITY

Recent progress in the construction of a covariant chiral nuclear force

Lisheng Geng (耿立升) @ Beihang U.

1603.07802 · 1611.08475 · 1612.08482 · 1802.04433 · 1809.03199 · 1812.03005 · 1812.03799

In collaboration with:

Beihang: *Yang Xiao, Jun-Xu Lu, Kai-Wen Li, Jie Song*

PKU/TMU/Bochum: *Jie Meng, Peter Ring, Xiu-Lei Ren*

SCU: *Bing-Wei Long*

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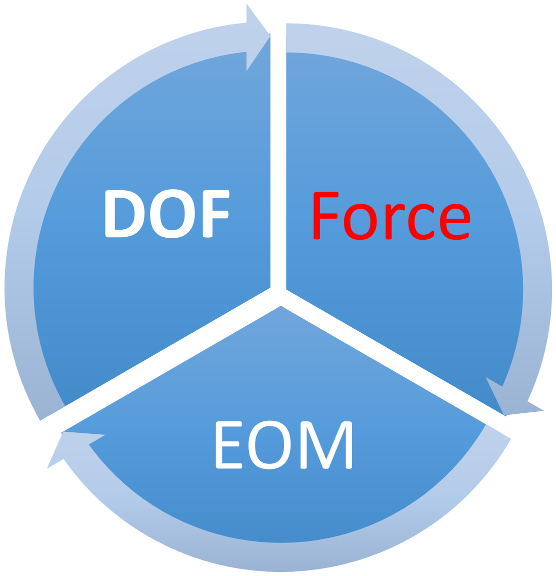
Why (bare) nuclear forces

Understanding static and dynamic properties of nuclear systems in terms of bare nucleon-nucleon (NN, NNN) forces is one of the ultimate goals of nuclear theory

Quark/gluon
Nucleon
Nucleon/meson



Microscopic
Phenomenological
Two-body/three-body

Non-relativistic/**relativistic**
Mean field/shell model/***ab initio***

One of the most difficult problems

in the history of mankind

SCIENTIFIC AMERICAN, September 1953

What Holds the Nucleus Together?

by Hans A. Bethe

In the past quarter century physicists have devoted a huge amount of experimentation and mental labor to this problem – probably more man-hours than have been given to any other scientific question in the history of mankind.

Hans Bethe
Nobel Prize in Physics
1967

A brief account of the long history

Advantages of ChEFT

① Closer link with QCD
② Systematic/order-by-order improvements possible
③ Consistent descriptions of two/three/four body interactions on the same footing

1935 Yukawa

Meson theo.

1950-60

Pheno.

1974 Wilson

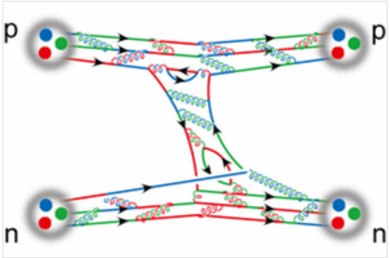
Lattice QCD

1990/91 Weinberg

HB ChEFT

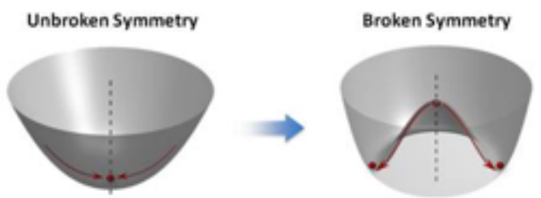
2007-

Lattice QCD



Why chiral （effective field theory)

☐ Chiral perturbation theory—low energy EFT of QCD



- Because of **quark confinement and asymptotic freedom**, low energy QCD can not be solved perturbatively
- Maps quark (u, d, s) dof’ s to those of the asymptotic states, hadrons
- **Perturbative formulation of low energy QCD in powers of the external momenta and the light quark masses, by utilizing chiral symmetry and its breaking pattern (the third feature of QCD)**

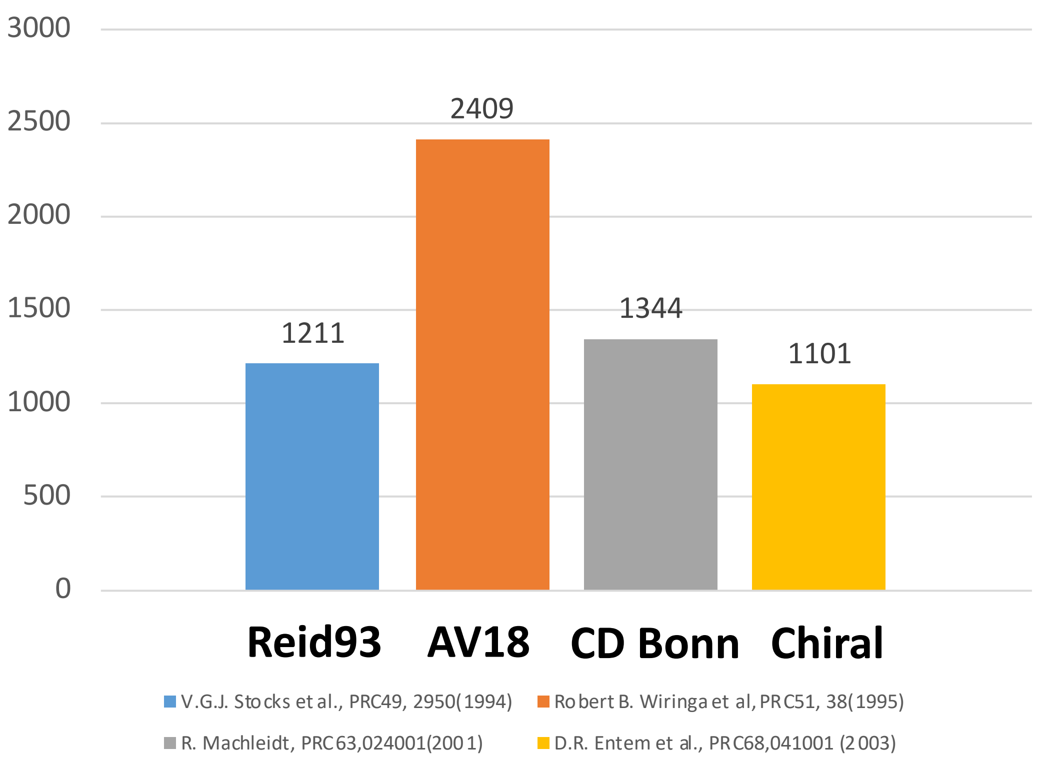
☐ Development—Trilogy

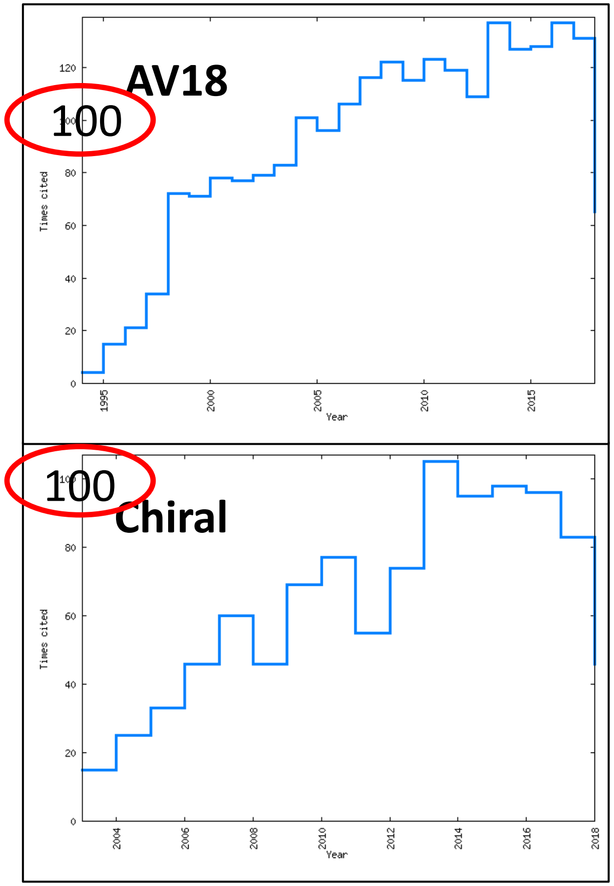
- 1979, pion-pion, Weinberg
- 1989, to the one-baryon sector, Gasser, Sainio, Svarc
- 1990/91, to NN, Weinberg—**very successful**

Steven Weinberg
Nobel Prize in Physics in 1979

Reviews of Modern Physics 81(2009)1773； Physics Reports 503(2011)1

Why (bare) nuclear forces

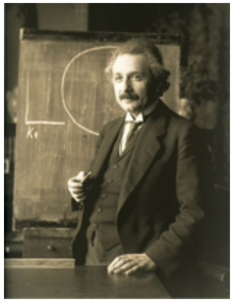




Yearly citation about 100 times

Why covariant/relativistic

- **Lorentz invariance is one of the most important symmetries of nature**.
- Include kinematical and dynamical relativistic corrections self-consistently and simultaneosly
- Relativistic approaches successful in explaining **fine structures**
  - Atomic and molecular systems, why gold is yellow
  - Nuclear system：**spin-orbit splitting, pseudospin symmetry**
  - One-baryon sector：magnetic moments、masses、sigma terms


Einstein


Dirac


Mayer


Jensen


Arima

What it is not

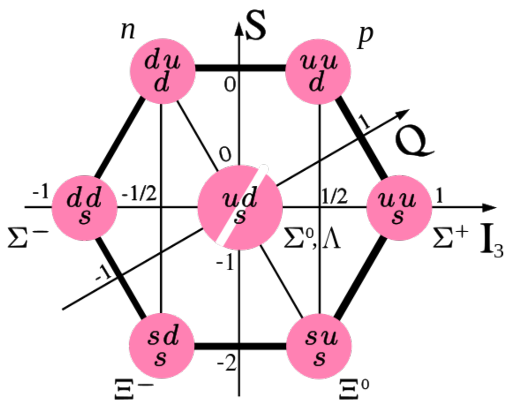
- ***Not** to challenge or replace the successful Weinberg chiral nuclear force*
- ***Not** to compete with phenomenological forces in terms of agreement with NN phaseshifts*

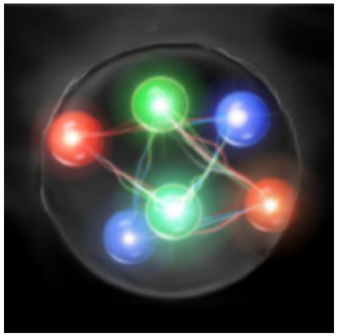
Our purpose 1: Exploration

- Provide a high precision covariant chiral nuclear force that can be used as inputs for ab initio nuclear structure and reaction studies in a covariant setting
- Explore the consequence of such a covariant framework in solving some long-standing open questions, such as RG invariance, Ay puzzle, …

Our purpose 2: from NN to BB

- ☐ **Nucleon-nucleon**
- ☐ **Hyperon-nucleon**
- ☐ **Hyperon-hyperon**



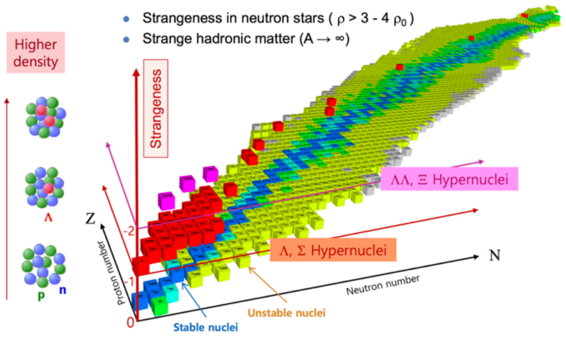


A bound H-dibaryon?

Inoue PRL 106 (2011) 162002

Neutron star

Lonardoni PRL 114 (2015) 092301

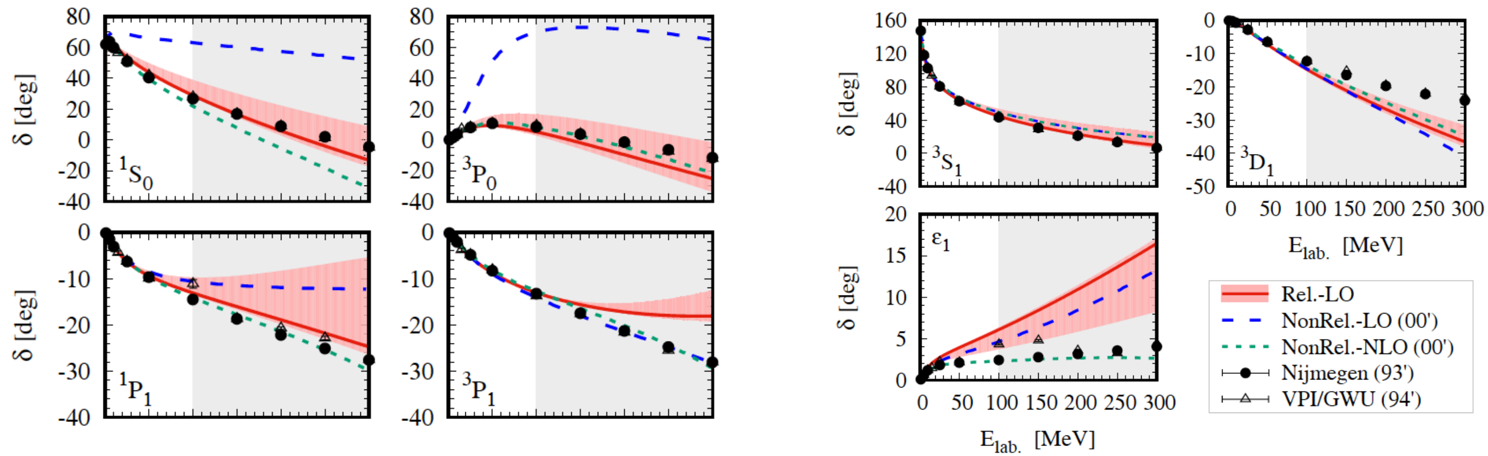


Three D nuclear chart

Kaneta M, Tohoku University, Japan)

Exploratory/feasibility studies

① **Leading order relativistic chiral nucleon-nucleon interaction, 1611.08475**

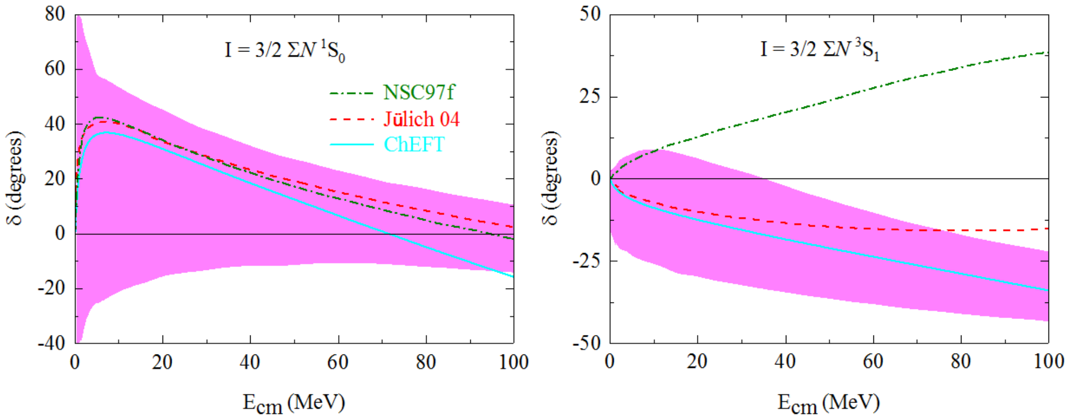


② **Leading order relativistic hyperon-nucleon interactions in chiral effective field theory, 1612.08482**

36 YN data	Weinberg's approach		Covariant ChEFT	NSC97f$
No. of LECs	5 (LO*)	23 (NLO#)	12 (LO)	29
χ^2	28.3	16.2	16.7	16.7

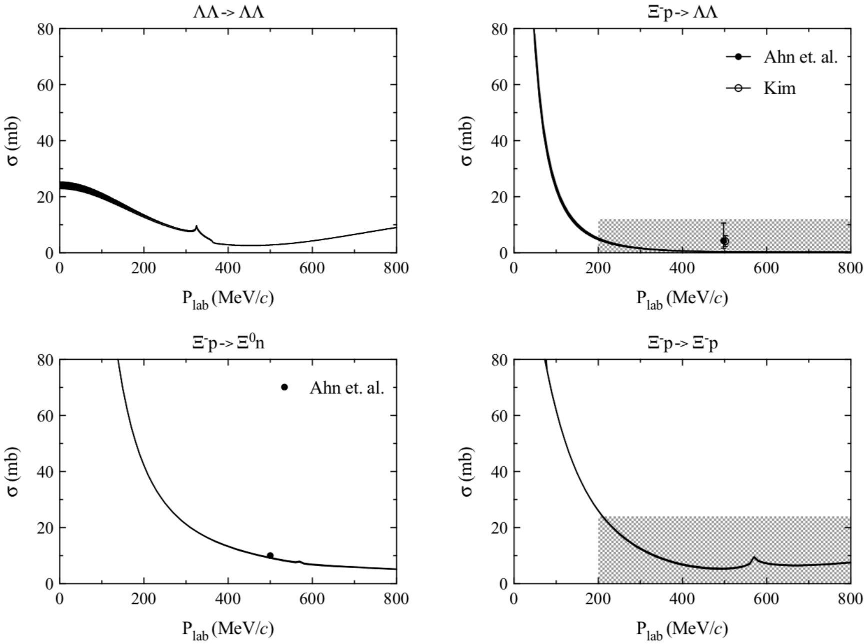
*Polinder NPA799 (2006) 244 #Haidenbauer NPA915 (2013) 24 $Rijken PRC 59 (1999) 21

Exploratory/feasibility studies

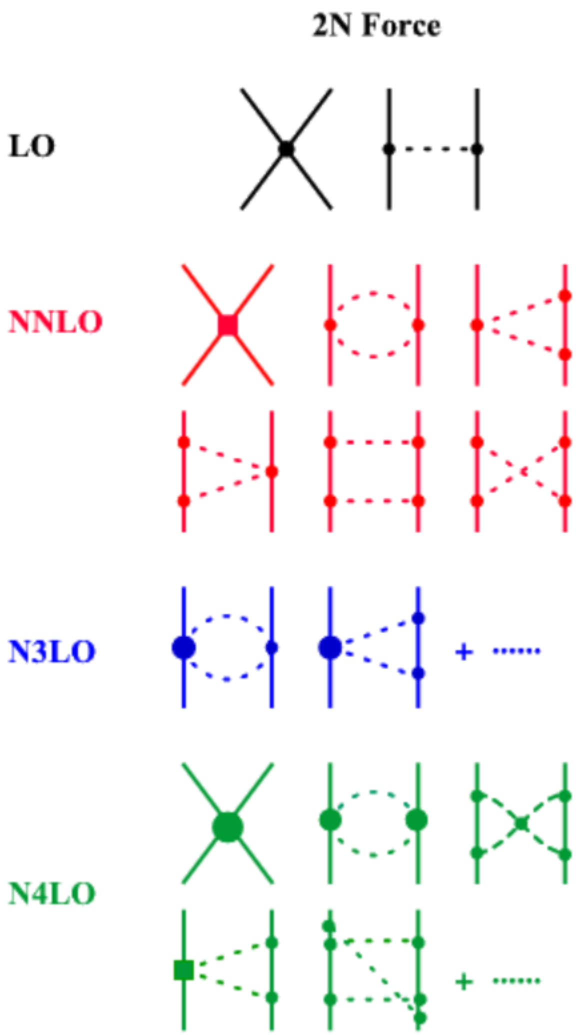


③ **Strangeness S=−1 hyperon-nucleon interactions: Chiral effective field theory versus lattice QCD, 1802.04433**

④ **Strangeness S=−2 baryon-baryon interactions in relativistic chiral effective field theory, 1809.03199**



Going to higher orders—challenging



Three key inputs

1) **Four nucleon (baryon) vertices, 1812.03005**
2) **Meson-baryon vertices, 1812.03799**
3) *Two-pion exchanges, to come soon*

3: How to construct non-relativistic Lagrangians

- **Basic requirement:** A scalar.
- **Building blocks:** $N, N^\dagger, \sigma, \vec{\nabla}, \overleftarrow{\nabla}$.
- **Symmetry constraint:** Even ∇ operators (Parity).
- **Power counting rules:**
  - $\vec{\nabla} \sim O(p^1), \overleftarrow{\nabla} \sim O(p^1), \sigma \sim O(p^0), N \sim O(p^0), N^\dagger \sim O(p^0)$.
- **Linear relations** (center-of-mass frame):

$$a, b, c, d \in \{\sigma, \vec{\nabla}, \overleftarrow{\nabla}\}$$

1. $a \cdot b = b \cdot a, a \times b = -b \times a$,
2. $a \cdot b \times c = -b \cdot a \times c = -c \cdot b \times a,$
3. $a \times (b \times c) = b(a \cdot c) - c(a \cdot b),$
4. $(a \times b) \cdot (c \times d) = (a \cdot c)(b \cdot d) - (a \cdot d)(b \cdot c)$,
5. $(N^\dagger N)(N^\dagger \vec{\nabla} N) \equiv -(N^\dagger \vec{\nabla} N)(N^\dagger N)$,
6. $(N^\dagger N)(N^\dagger \overleftarrow{\nabla} N) \equiv -(N^\dagger \overleftarrow{\nabla} N)(N^\dagger N)$.

3: Non-relativistic Lagrangians (NNLO)

O_S	$(N^\dagger N)(N^\dagger N)$	O_{11}	$(N^\dagger \overrightarrow{\nabla} \cdot \overleftarrow{\nabla} N)(N^\dagger \overrightarrow{\nabla} \cdot \overleftarrow{\nabla} N)$
O_T	$(N^\dagger \boldsymbol{\sigma} N) \cdot (N^\dagger \boldsymbol{\sigma} N)$	O_{12}	$i\,(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\nabla} \times \overleftarrow{\nabla} N)(N^\dagger \overrightarrow{\nabla}^2 N) + \text{h.c.}$
O_1	$(N^\dagger N)(N^\dagger \overrightarrow{\nabla}^2 N) + \text{h.c.}$	O_{13}	$i\,(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\nabla} \times \overleftarrow{\nabla} N)(N^\dagger \overrightarrow{\nabla} \cdot \overleftarrow{\nabla} N)$
O_2	$(N^\dagger N)(N^\dagger \overrightarrow{\nabla} \cdot \overleftarrow{\nabla} N)$	O_{14}	$(N^\dagger \sigma^j N)(N^\dagger \sigma^j \overrightarrow{\nabla}^4 N) + \text{h.c.}$
O_3	$i\,(N^\dagger \boldsymbol{\sigma} N) \cdot (N^\dagger \overrightarrow{\nabla} \times \overleftarrow{\nabla} N)$	O_{15}	$(N^\dagger \sigma^j \overrightarrow{\nabla} \cdot \overleftarrow{\nabla} N)(N^\dagger \sigma^j \overrightarrow{\nabla}^2 N) + \text{h.c.}$
O_4	$(N^\dagger \sigma^j N)(N^\dagger \sigma^j \overrightarrow{\nabla}^2 N) + \text{h.c.}$	O_{16}	$(N^\dagger \sigma^j \overrightarrow{\nabla}^2 N)(N^\dagger \sigma^j \overleftarrow{\nabla}^2 N)$
O_5	$(N^\dagger \sigma^j N)(N^\dagger \sigma^j \overrightarrow{\nabla} \cdot \overleftarrow{\nabla} N)$	O_{17}	$(N^\dagger \sigma^j \overrightarrow{\nabla} \cdot \overleftarrow{\nabla} N)(N^\dagger \sigma^j \overrightarrow{\nabla} \cdot \overleftarrow{\nabla} N)$
O_6	$(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\nabla} N)(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\nabla} N) + \text{h.c.}$	O_{18}	$(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\nabla} N)(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\nabla} \overrightarrow{\nabla}^2 N) + \text{h.c.}$
O_7	$(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\nabla} N)(N^\dagger \boldsymbol{\sigma} \cdot \overleftarrow{\nabla} N)$	O_{19}	$(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\nabla} N)(N^\dagger \boldsymbol{\sigma} \cdot \overleftarrow{\nabla} \overrightarrow{\nabla}^2 N) + \text{h.c.}$
O_8	$(N^\dagger N)(N^\dagger \overrightarrow{\nabla}^4 N) + \text{h.c.}$	O_{20}	$(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\nabla} N)(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\nabla} \overrightarrow{\nabla} \cdot \overleftarrow{\nabla} N) + \text{h.c.}$
O_9	$(N^\dagger \overrightarrow{\nabla}^2 N)(N^\dagger \overrightarrow{\nabla} \cdot \overleftarrow{\nabla} N) + \text{h.c.}$	O_{21}	$(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\nabla} N)(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\nabla} \overleftarrow{\nabla}^2 N) + \text{h.c.}$
O_{10}	$(N^\dagger \overrightarrow{\nabla}^2 N)(N^\dagger \overleftarrow{\nabla}^2 N)$	O_{22}	$(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\nabla} N)(N^\dagger \boldsymbol{\sigma} \cdot \overleftarrow{\nabla} \overrightarrow{\nabla} \cdot \overleftarrow{\nabla} N)$

2+7+15 terms up to NNLO, consistent with the known Weinberg chiral nuclear force

3: How to construct cov. Lagrangians

□ Symmetry Constraints

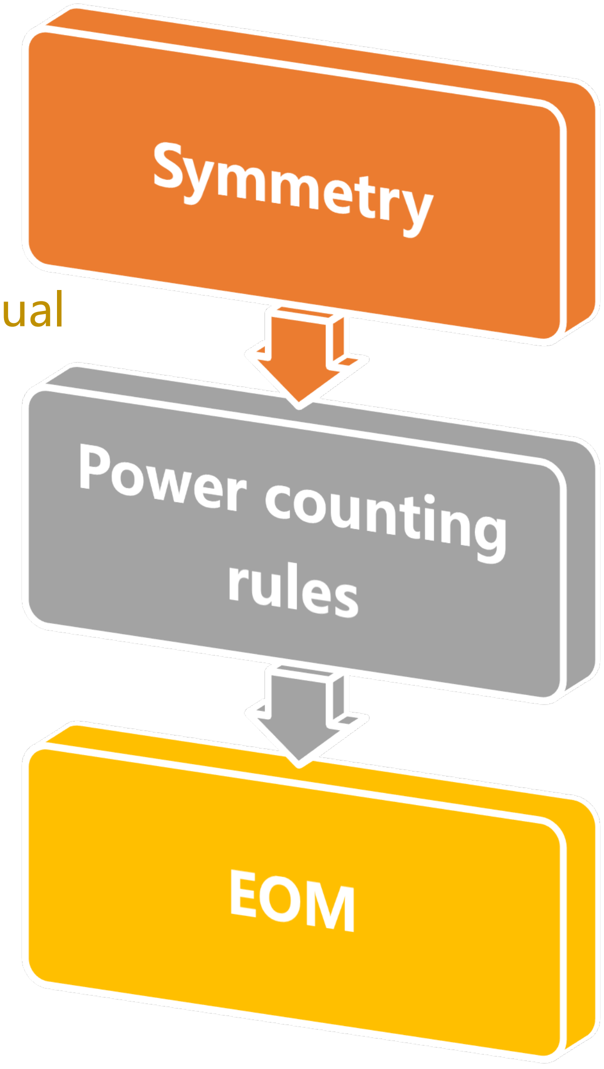
- ✓ Lorentz invariance: α, β, γ … …
- ✓ Chiral symmetry: matter field $\psi \rightarrow K\psi K^{\dagger}$, NGB as usual
- ✓ Hermitian conjugation: add an appropriate "i".
- ✓ Parity and Charge conjugation symmetries:
- ✓ Time inverse symmetry: CPT theory.

□ How to raise chiral order ?

→**Power counting rules**

□ How to deal with redundant terms ?

→**Equation of motion (EOM)**



3: Symmetry requirements

- Building blocks (Dirac matrices & partial derivatives)

	$\mathbb{1}$	γ_5	γ_μ	$\gamma_5\gamma_\mu$	$\sigma_{\mu\nu}$	$\epsilon_{\mu\nu\rho\sigma}$	$\overleftrightarrow{\partial}_\mu$	∂_μ
$\mathcal{P}$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$+$
$\mathcal{C}$	$+$	$+$	$-$	$+$	$-$	$+$	$-$	$+$
h.c.	$+$	$-$	$+$	$+$	$+$	$+$	$-$	$+$
$\mathcal{O}$	0	1	0	0	0	$-$	0	1

- General form of a Lagrangian term

$$\boxed{\frac{1}{(2m)^{N_d}}\left(\bar{\psi}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\ldots\Gamma_A\psi\right)\partial^{\lambda}\partial^{\mu}\ldots\left(\bar{\psi}i\overleftrightarrow{\partial}^{\sigma}i\overleftrightarrow{\partial}^{\tau}\ldots\Gamma_B\psi\right),}$$

Note $\overleftrightarrow{\partial}^{\alpha} = \bar{\psi}(\overrightarrow{\partial}^{\alpha} - \overleftarrow{\partial}^{\alpha})\psi$ vs. $\partial^{\alpha} = \partial^{\alpha}(\bar{\psi}\Gamma\psi)$

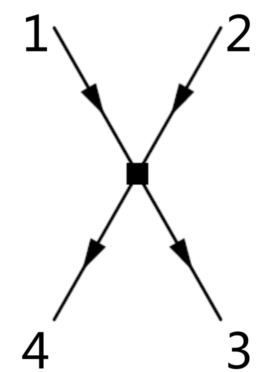
3: Power counting rules

$$\frac{1}{(2m)^{N_d}}\left(\bar{\psi} i\overleftrightarrow{\partial}^{\alpha} i\overleftrightarrow{\partial}^{\beta}\ldots\Gamma_A\psi\right)\partial^{\lambda}\partial^{\mu}\ldots\left(\bar{\psi} i\overleftrightarrow{\partial}^{\sigma} i\overleftrightarrow{\partial}^{\tau}\ldots\Gamma_B\psi\right)$$

N_d is the number of ,$\overleftrightarrow{\partial}=\overrightarrow{\partial}-\overleftarrow{\partial}$

- Nucleon field: $\psi = \binom{p}{n} \sim O(p^0)$, Nucleon mass: $m \sim O(p^0)$,
- Dirac matrices: $\Gamma \in \{1, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu} \sim O(p^0), \gamma_5 \sim O(p^1)\}$
- **Covariant derivative:** $\partial(\bar{\psi}\Gamma\psi) \sim O(p^1)$, $(\bar{\psi}\overleftrightarrow{\partial}\psi) \sim O(p^0)$**, except**

$(\bar{\psi}\sigma_{\mu\nu}\psi)(\bar{\psi}\overleftrightarrow{\partial}^{\mu}\Gamma\psi) \sim O(p^1)$, $(\bar{\psi}\gamma_5\gamma_\mu\psi)(\bar{\psi}\overleftrightarrow{\partial}^{\mu}\Gamma\psi) \sim O(p^1)$



- **Treatment for covariant derivative:**

$$\widetilde{O}^{(n)}_{\Gamma_A\Gamma_B} = \frac{1}{(2m)^{2n}}(\bar{\psi}\, i\overleftrightarrow{\partial}^{\mu_1}\, i\overleftrightarrow{\partial}^{\mu_2}\ldots i\overleftrightarrow{\partial}^{\mu_n}\,\Gamma_A^{\alpha}\,\psi)\,(\bar{\psi}\, i\overleftrightarrow{\partial}_{\mu_1}\, i\overleftrightarrow{\partial}_{\mu_2}\cdots i\overleftrightarrow{\partial}_{\mu_n}\,\Gamma_{B\alpha}\,\psi)$$

-Expansion of such structure:

-up to $\boldsymbol{O(q^2)}$: $\boldsymbol{n = 0, 1;}$

-up to $\boldsymbol{O(q^4)}$: $\boldsymbol{n = 0, 1, 2.}$

$$\boxed{\frac{[(p_1+p_3)\cdot(p_2+p_4)]^n}{(2m)^{2n}}} \Leftrightarrow \boxed{\left[1+\frac{(s-4m^2)-u}{4m^2}\right]^n}$$

3: Reduction using equation of motion (EOM)

- **Equation of motion :** $\not{D}B = \gamma^{\mu} D_{\mu} B = -\mathrm{i} M_0 B + \mathcal{O}(q)$
- Beyond the obvious replacements one can bring terms that do not containing $\not{D}B$ into a form where they do. *Annals Phys., 283:273, (2000)*

$$\boxed{-2im(\bar{\psi}\Gamma\psi) \approx 2(\bar{\psi}\Gamma \times \gamma_{\lambda}\partial^{\lambda}\psi) = (\bar{\psi}\Gamma'_{\lambda}\overleftrightarrow{\partial}^{\lambda}\psi) + \partial^{\lambda}(\bar{\psi}\Gamma''_{\lambda}\psi),}$$

TABLE II. Decomposition of the Dirac matrix products $\Gamma \times \gamma_{\lambda}$ into charge conjugation even (Γ'_{λ}) and charge conjugation odd (Γ''_{λ}) parts [43].

Γ	Γ'_{λ}	Γ''_{λ}
$\mathbb{1}$	γ_{λ}	0
γ_{μ}	$g_{\mu\lambda}1$	$-i\sigma_{\mu\lambda}$
γ_5	0	$\gamma_5\gamma_{\lambda}$
$\gamma_5\gamma_{\mu}$	$\frac{1}{2}\epsilon_{\mu\lambda\rho\tau}\sigma^{\rho\tau}$	$g_{\mu\lambda}\gamma_5$
$\sigma_{\mu\nu}$	$\epsilon_{\mu\nu\lambda\tau}\gamma_5\gamma^{\tau}$	$-i(g_{\mu\lambda}\gamma_{\nu} - g_{\nu\lambda}\gamma_{\mu})$
$\epsilon_{\mu\nu\rho\tau}\gamma^{\tau}$	$\epsilon_{\mu\nu\rho\lambda}1$	$g_{\mu\lambda}\gamma_5\sigma_{\nu\rho} + g_{\rho\lambda}\gamma_5\sigma_{\mu\nu} + g_{\nu\lambda}\gamma_5\sigma_{\rho\mu}$
$\epsilon_{\mu\nu\rho\tau}\gamma_5\gamma^{\tau}$	$g_{\mu\lambda}\sigma_{\nu\rho} + g_{\rho\lambda}\sigma_{\mu\nu} + g_{\nu\lambda}\sigma_{\rho\mu}$	$\epsilon_{\mu\nu\rho\lambda}\gamma_5$
$\epsilon_{\mu\nu\rho\alpha}\sigma^{\alpha}_{\tau}$	$\gamma_5\gamma_{\rho}(g_{\lambda\nu}g_{\mu\tau} - g_{\lambda\mu}g_{\nu\tau}) + \gamma_5\gamma_{\nu}(g_{\lambda\mu}g_{\rho\tau} - g_{\lambda\rho}g_{\mu\tau}) + \gamma_5\gamma_{\mu}(g_{\lambda\rho}g_{\nu\tau} - g_{\lambda\nu}g_{\rho\tau})$	$ig_{\lambda\tau}\epsilon_{\mu\nu\rho\alpha}\gamma^{\alpha} - i\epsilon_{\mu\nu\rho\lambda}\gamma_{\tau}$
$\frac{i}{2}\epsilon_{\mu\nu\rho\tau}\sigma^{\rho\tau} = \gamma_5\sigma_{\mu\nu}$	$\frac{1}{i}(g_{\mu\lambda}\gamma_5\gamma_{\nu} - g_{\nu\lambda}\gamma_5\gamma_{\mu})$	$\epsilon_{\mu\nu\lambda\rho}\gamma^{\rho}$

3: Covariant NN contact Lagrangians (N2LO)

$\widetilde{O}_1$	$(\bar\psi\psi)(\bar\psi\psi)$
$\widetilde{O}_2$	$(\bar\psi\gamma^\mu\psi)(\bar\psi\gamma_\mu\psi)$
$\widetilde{O}_3$	$(\bar\psi\gamma_5\gamma^\mu\psi)(\bar\psi\gamma_5\gamma_\mu\psi)$
$\widetilde{O}_4$	$(\bar\psi\sigma^{\mu\nu}\psi)(\bar\psi\sigma_{\mu\nu}\psi)$
$\widetilde{O}_5$	$(\bar\psi\gamma_5\psi)(\bar\psi\gamma_5\psi)$
$\widetilde{O}_6$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^\alpha\psi\right)\left(\bar\psi\gamma_5\gamma_\alpha i\overleftrightarrow{\partial}_\mu\psi\right)$
$\widetilde{O}_7$	$\frac{1}{4m^2}\left(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^\alpha\psi\right)\left(\bar\psi\sigma_{\mu\alpha} i\overleftrightarrow{\partial}_\nu\psi\right)$
$\widetilde{O}_8$	$\frac{1}{4m^2}\left(\bar\psi i\overleftrightarrow{\partial}^\mu\psi\right)\partial^\nu\left(\bar\psi\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_9$	$\frac{1}{4m^2}(\bar\psi\sigma^{\mu\alpha}\psi)\partial_\alpha\partial^\nu\left(\bar\psi\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_{10}$	$\frac{1}{4m^2}(\bar\psi\psi)\partial^2(\bar\psi\psi)$
$\widetilde{O}_{11}$	$\frac{1}{4m^2}(\bar\psi\gamma^\mu\psi)\partial^2\left(\bar\psi\gamma_\mu\psi\right)$
$\widetilde{O}_{12}$	$\frac{1}{4m^2}(\bar\psi\gamma_5\gamma^\mu\psi)\partial^2\left(\bar\psi\gamma_5\gamma_\mu\psi\right)$
$\widetilde{O}_{13}$	$\frac{1}{4m^2}(\bar\psi\sigma^{\mu\nu}\psi)\partial^2\left(\bar\psi\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_{14}$	$\frac{1}{4m^2}\left(\bar\psi i\overleftrightarrow{\partial}^\alpha\psi\right)\left(\bar\psi i\overleftrightarrow{\partial}_\alpha\psi\right)-\widetilde{O}_1$
$\widetilde{O}_{15}$	$\frac{1}{4m^2}\left(\bar\psi\gamma^\mu i\overleftrightarrow{\partial}^\alpha\psi\right)\left(\bar\psi\gamma_\mu i\overleftrightarrow{\partial}_\alpha\psi\right)-\widetilde{O}_2$
$\widetilde{O}_{16}$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^\alpha\psi\right)\left(\bar\psi\gamma_5\gamma_\mu i\overleftrightarrow{\partial}_\alpha\psi\right)-\widetilde{O}_3$
$\widetilde{O}_{17}$	$\frac{1}{4m^2}\left(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^\alpha\psi\right)\left(\bar\psi\sigma_{\mu\nu} i\overleftrightarrow{\partial}_\alpha\psi\right)-\widetilde{O}_4$

O_S	$(N^\dagger N)(N^\dagger N)$
O_T	$(N^\dagger\boldsymbol{\sigma}N)\cdot(N^\dagger\boldsymbol{\sigma}N)$
O_1	$(N^\dagger N)(N^\dagger\overrightarrow{\nabla}^2N)+\text{h.c.}$
O_2	$(N^\dagger N)(N^\dagger\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)$
O_3	$i\,(N^\dagger\boldsymbol{\sigma}N)\cdot(N^\dagger\overrightarrow{\nabla}\times\overleftarrow{\nabla}N)$
O_4	$(N^\dagger\sigma^jN)(N^\dagger\sigma^j\overrightarrow{\nabla}^2N)+\text{h.c.}$
O_5	$(N^\dagger\sigma^jN)(N^\dagger\sigma^j\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)$
O_6	$(N^\dagger\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}N)(N^\dagger\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}N)+\text{h.c.}$
O_7	$(N^\dagger\boldsymbol{\sigma}\cdot\overleftarrow{\nabla}N)(N^\dagger\boldsymbol{\sigma}\cdot\overleftarrow{\nabla}N)$

Relativistic: 17

vs.

Non-relativistic: 9

3: Comparison with other works (N2LO)

	Terms	Procedure	Advantage	Disadvantage
L.Girlanda [1]	36	① n=0,1 $(\frac{1}{(2m)^{2n}}(\bar{\psi}\partial^{\mu_1}\ldots\partial^{\mu_n}\psi)(\bar{\psi}\partial_{\mu_1}\ldots\partial_{\mu_n}\psi))$	A complete set of *NN* contact Lagrangians	**Not minimal**
Stefan Petschauer [2]	25 (*NN* case)	① n=0,1,2; ② Apply EOM; ③ Ignore Lagrangians with $\partial^\mu(\bar{\psi}\sigma_{\mu\nu}\psi)$ cause they claim it contribute to higher order $O(p^1)$ and can be subsumed in higher order Lagrangians	Contains less terms compared with [1]	**Not complete**
Our work	**17**	① n=0,1; ② Apply EOM; ③ Include Lagrangians with $\partial^\mu(\bar{\psi}\sigma_{\mu\nu}\psi)$ cause it contains unique Lorentz structure	A **complete and minimal** set of *NN* contact Lagrangians	

[1] *PRC81 (2010) 034005* [2] *NPA916 (2013) 1*

3: Covariant NN contact Lagrangians (N4LO)

$\widetilde{O}_1$	$(\bar{\psi}\psi)(\bar{\psi}\psi)$	$\widetilde{O}_{21}$	$\frac{1}{16m^4}\left(\bar{\psi}i\overleftrightarrow{\partial}^{\mu}\psi\right)\partial^2\partial^{\nu}\left(\bar{\psi}\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_2$	$(\bar{\psi}\gamma^{\mu}\psi)\left(\bar{\psi}\gamma_{\mu}\psi\right)$	$\widetilde{O}_{22}$	$\frac{1}{16m^4}\left(\bar{\psi}\sigma^{\mu\alpha}\psi\right)\partial^2\partial_{\alpha}\partial^{\nu}\left(\bar{\psi}\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_3$	$(\bar{\psi}\gamma_5\gamma^{\mu}\psi)\left(\bar{\psi}\gamma_5\gamma_{\mu}\psi\right)$	$\widetilde{O}_{23}$	$\frac{1}{16m^4}\left(\bar{\psi}\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^{\beta}\partial_{\nu}\left(\bar{\psi}\sigma_{\alpha\beta}i\overleftrightarrow{\partial}_{\mu}\psi\right)$
$\widetilde{O}_4$	$(\bar{\psi}\sigma^{\mu\nu}\psi)\left(\bar{\psi}\sigma_{\mu\nu}\psi\right)$	$\widetilde{O}_{24}$	$\frac{1}{16m^4}(\bar{\psi}\psi)\partial^4(\bar{\psi}\psi)$
$\widetilde{O}_5$	$(\bar{\psi}\gamma_5\psi)(\bar{\psi}\gamma_5\psi)$	$\widetilde{O}_{25}$	$\frac{1}{16m^4}(\bar{\psi}\gamma^{\mu}\psi)\partial^4\left(\bar{\psi}\gamma_{\mu}\psi\right)$
$\widetilde{O}_6$	$\frac{1}{4m^2}\left(\bar{\psi}\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar{\psi}\gamma_5\gamma_{\alpha}i\overleftrightarrow{\partial}_{\mu}\psi\right)$	$\widetilde{O}_{26}$	$\frac{1}{16m^4}(\bar{\psi}\gamma_5\gamma^{\mu}\psi)\partial^4\left(\bar{\psi}\gamma_5\gamma_{\mu}\psi\right)$
$\widetilde{O}_7$	$\frac{1}{4m^2}\left(\bar{\psi}\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar{\psi}\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\nu}\psi\right)$	$\widetilde{O}_{27}$	$\frac{1}{16m^4}(\bar{\psi}\sigma^{\mu\nu}\psi)\partial^4\left(\bar{\psi}\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_8$	$\frac{1}{4m^2}\left(\bar{\psi}i\overleftrightarrow{\partial}^{\mu}\psi\right)\partial^{\nu}\left(\bar{\psi}\sigma_{\mu\nu}\psi\right)$	$\widetilde{O}_{28}$	$\frac{1}{4m^2}\left(\bar{\psi}\gamma_5 i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar{\psi}\gamma_5 i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_5$
$\widetilde{O}_9$	$\frac{1}{4m^2}(\bar{\psi}\sigma^{\mu\alpha}\psi)\partial_{\alpha}\partial^{\nu}\left(\bar{\psi}\sigma_{\mu\nu}\psi\right)$	$\widetilde{O}_{29}$	$\frac{1}{16m^4}\left(\bar{\psi}\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar{\psi}\gamma_5\gamma_{\alpha}i\overleftrightarrow{\partial}_{\mu}i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_6$
$\widetilde{O}_{10}$	$\frac{1}{4m^2}(\bar{\psi}\psi)\partial^2(\bar{\psi}\psi)$	$\widetilde{O}_{30}$	$\frac{1}{16m^4}\left(\bar{\psi}\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar{\psi}\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\nu}i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_7$
$\widetilde{O}_{11}$	$\frac{1}{4m^2}(\bar{\psi}\gamma^{\mu}\psi)\partial^2\left(\bar{\psi}\gamma_{\mu}\psi\right)$	$\widetilde{O}_{31}$	$\frac{1}{16m^4}\left(\bar{\psi}i\overleftrightarrow{\partial}^{\mu}i\overleftrightarrow{\partial}^{\beta}\psi\right)\partial^{\alpha}\left(\bar{\psi}\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_8$
$\widetilde{O}_{12}$	$\frac{1}{4m^2}(\bar{\psi}\gamma_5\gamma^{\mu}\psi)\partial^2\left(\bar{\psi}\gamma_5\gamma_{\mu}\psi\right)$	$\widetilde{O}_{32}$	$\frac{1}{16m^4}\left(\bar{\psi}\sigma^{\mu\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\partial_{\alpha}\partial^{\nu}\left(\bar{\psi}\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_9$
$\widetilde{O}_{13}$	$\frac{1}{4m^2}(\bar{\psi}\sigma^{\mu\nu}\psi)\partial^2\left(\bar{\psi}\sigma_{\mu\nu}\psi\right)$	$\widetilde{O}_{33}$	$\frac{1}{16m^4}\left(\bar{\psi}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar{\psi}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{10}$
$\widetilde{O}_{14}$	$\frac{1}{4m^2}\left(\bar{\psi}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar{\psi}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_1$	$\widetilde{O}_{34}$	$\frac{1}{16m^4}\left(\bar{\psi}\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar{\psi}\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{11}$
$\widetilde{O}_{15}$	$\frac{1}{4m^2}\left(\bar{\psi}\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar{\psi}\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_2$	$\widetilde{O}_{35}$	$\frac{1}{16m^4}\left(\bar{\psi}\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar{\psi}\gamma_5\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{12}$
$\widetilde{O}_{16}$	$\frac{1}{4m^2}\left(\bar{\psi}\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar{\psi}\gamma_5\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_3$	$\widetilde{O}_{36}$	$\frac{1}{16m^4}\left(\bar{\psi}\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar{\psi}\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{13}$
$\widetilde{O}_{17}$	$\frac{1}{4m^2}\left(\bar{\psi}\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar{\psi}\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_4$	$\widetilde{O}_{37}$	$\frac{1}{16m^4}\left(\bar{\psi}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar{\psi}i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{14}-\widetilde{O}_1$
$\widetilde{O}_{18}$	$\frac{1}{4m^2}(\bar{\psi}\gamma_5\psi)\partial^2(\bar{\psi}\gamma_5\psi)$	$\widetilde{O}_{38}$	$\frac{1}{16m^4}\left(\bar{\psi}\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar{\psi}\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{15}-\widetilde{O}_2$
$\widetilde{O}_{19}$	$\frac{1}{16m^4}\left(\bar{\psi}\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\nu}\psi\right)\partial^2\left(\bar{\psi}\gamma_5\gamma_{\nu}i\overleftrightarrow{\partial}_{\mu}\psi\right)$	$\widetilde{O}_{39}$	$\frac{1}{16m^4}\left(\bar{\psi}\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar{\psi}\gamma_5\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{16}-\widetilde{O}_3$
$\widetilde{O}_{20}$	$\frac{1}{16m^4}\left(\bar{\psi}\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar{\psi}\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\nu}\psi\right)$	$\widetilde{O}_{40}$	$\frac{1}{16m^4}\left(\bar{\psi}\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar{\psi}\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{17}-\widetilde{O}_4$

Relativistic: 40

vs.

Non-relativistic: 24

3: Non-relativistic reduction

- **Why need non-relativistic reduction: self consistency check**
- **Non-relativistic expansion:** $\psi \to N$, expand Lagrangians in terms of **1/m**
  - **Relativistic nucleon field operator:** $\psi(x) = \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{m}{E_p} \tilde{b}_s(\mathbf{p})\, u^{(s)}(\mathbf{p})\, \mathrm{e}^{-ip\cdot x},$
  - **Non-relativistic nucleon field operator:** $N(x) = \int \frac{d\mathbf{p}}{(2\pi)^3} b_s(\mathbf{p})\, \chi_s\, \mathrm{e}^{-ip\cdot x}$
  - **Expansion of field operator**

$$\psi(x) = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{i}{2m} \begin{pmatrix} 0 \\ \boldsymbol{\sigma}\cdot\boldsymbol{\nabla} \end{pmatrix} + \frac{1}{8m^2} \begin{pmatrix} \boldsymbol{\nabla}^2 \\ 0 \end{pmatrix} - \frac{3i}{16m^3} \begin{pmatrix} 0 \\ \boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\boldsymbol{\nabla}^2 \end{pmatrix} + \frac{11}{128m^4} \begin{pmatrix} \boldsymbol{\nabla}^4 \\ 0 \end{pmatrix} \right] N(x) + \mathcal{O}(Q^5)\,.$$

  - **Dirac matrices expressed in term of pauli matrices**

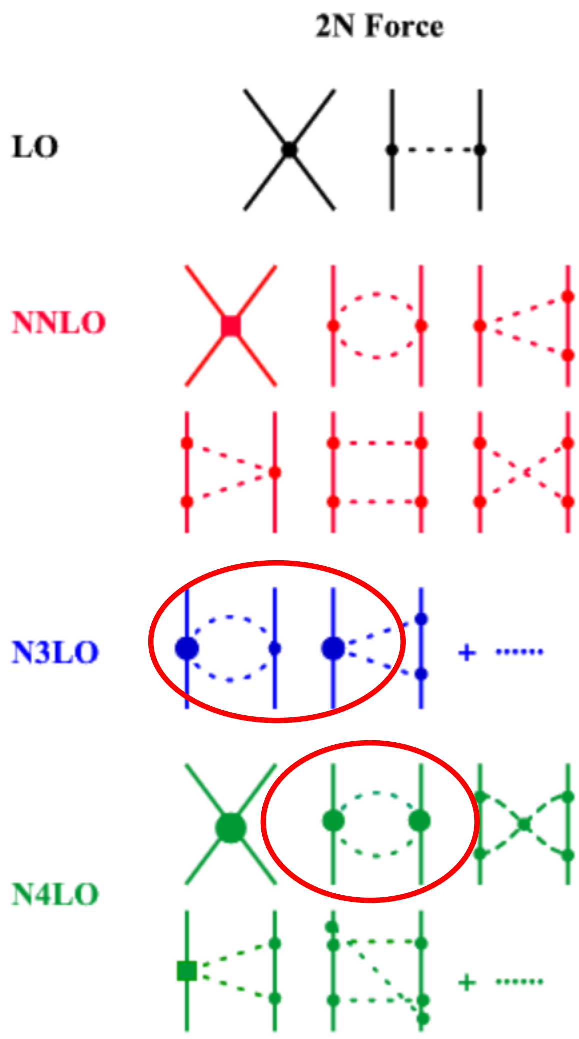
$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]\,.$$

3: Non-relativistic reduction

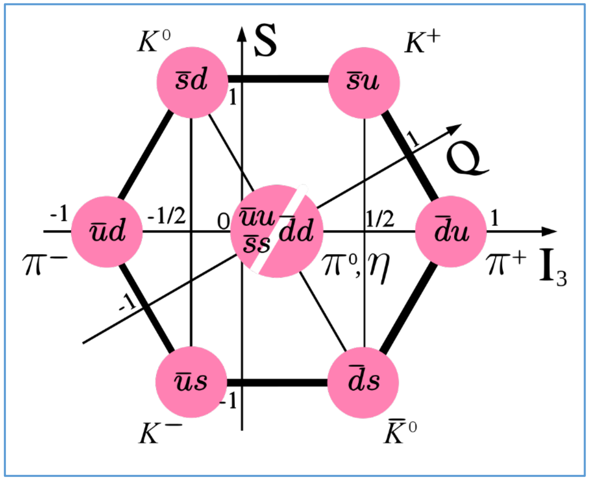
After expansion and keeping only appropriate powers of 1/mN, we can reduce the 40 relativistic terms into the 24(2+7+15) non-relativistic terms

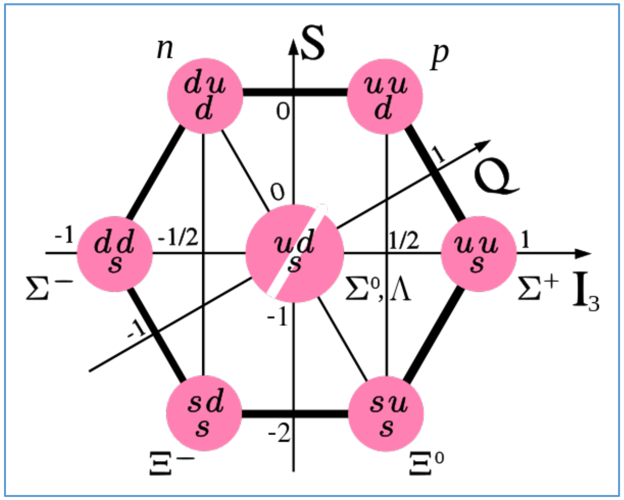
4: Towards N^{4}LO

We will need pion-nucleon inputs



Simultaneous description of meson-baryon scattering

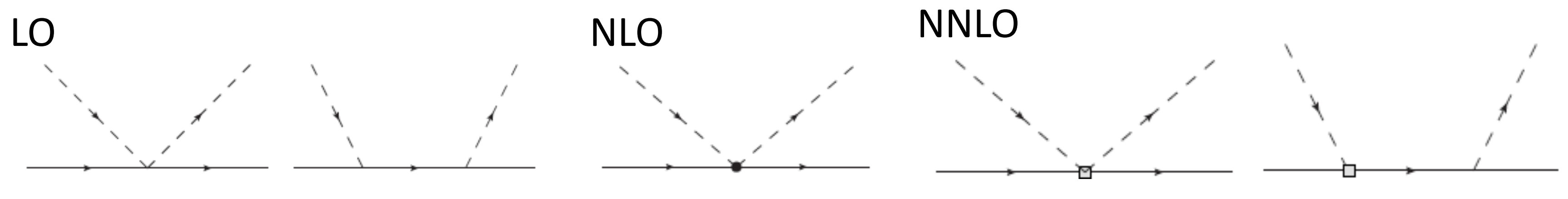




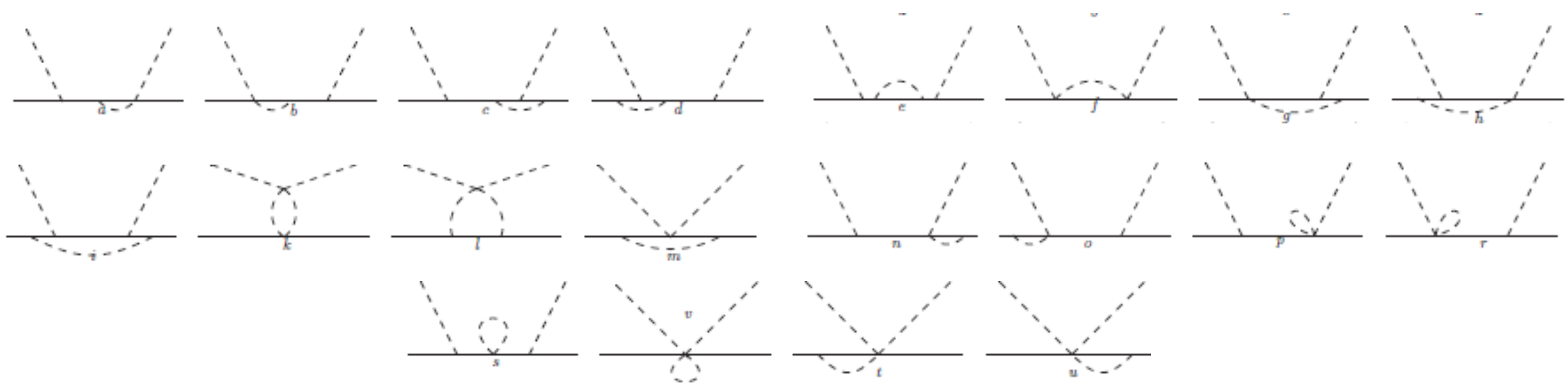
- **Kaon-nucleon(s=1): some data**
- **Pion-nucleon (s=0): plenty of data**
- Kbar-nucleon (s=-1): some data and nonperturbative
- Other channels (s=-2, -3): very limited lattice QCD data

4: Meson-baryon scattering upto N2LO

Tree



Loop



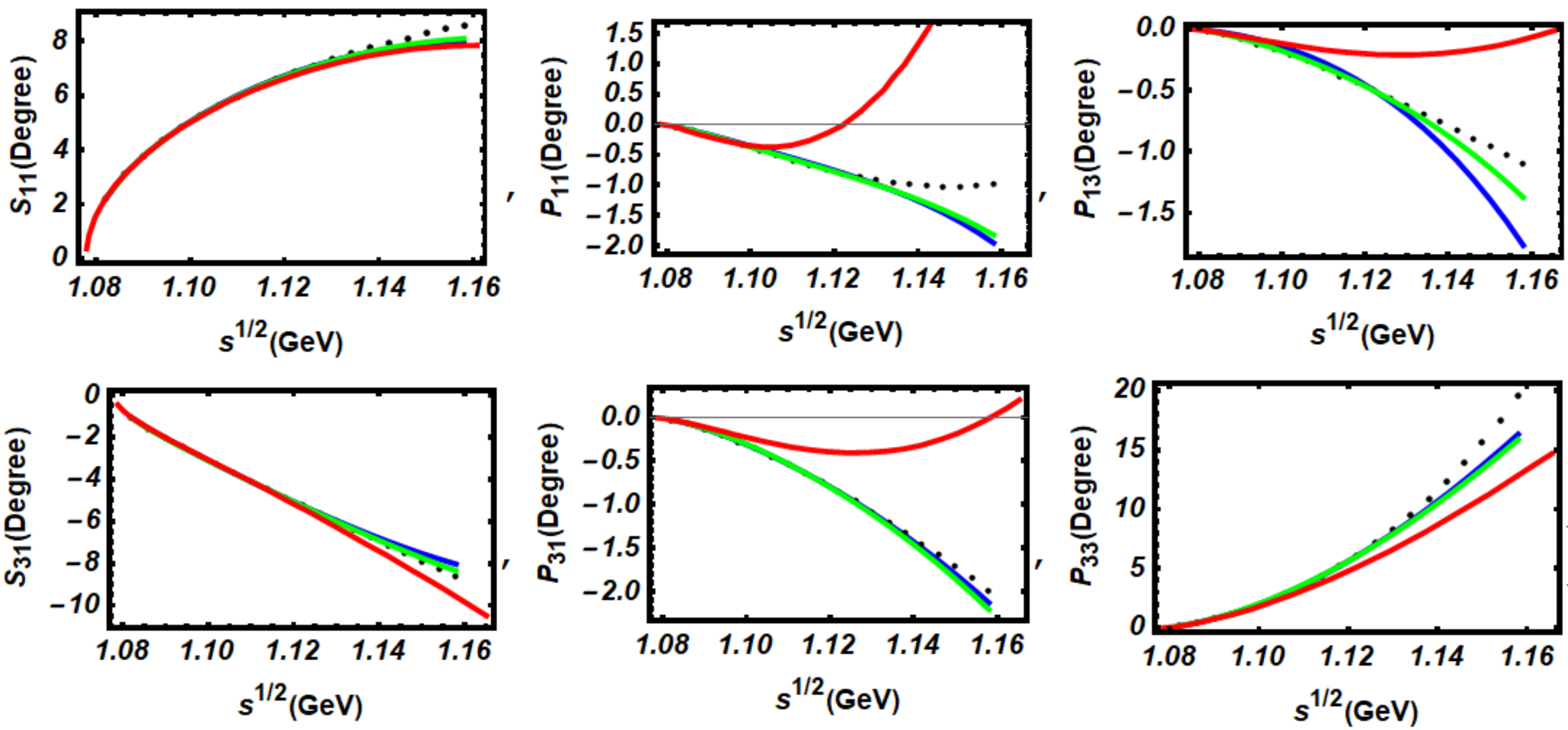
4: Meson-baryon scattering

In total 37 LECs, but decoupled in S=0 & S=1

	πN	$KN_{I=0}$	$KN_{I=1}$
	$b_1 + b_2 + b_3 + 2b_4$	$b_3 - b_4$	$b_1 + b_2 + b_4$
	$b_5 + b_6 + b_7 + b_8$	$2b_6 - b_8$	$2b_5 + 2b_7 + b_8$
	$c_1 + c_2$	$4c_1 + c_3$	$4c_2 + c_3$
24	$2b_0 + b_D + b_F$	$b_0 - b_F$	$b_0 + b_D$
	d_2	$d_1 + d_2 + d_3$	$d_1 - d_2 - d_3$
	d_4	$d_4 + d_5 + d_6$	$d_4 - d_5 + d_6$
	$d_8 + d_{10}$	$d_7 - d_8 + d_{10}$	$d_7 + d_8 + d_{10}$
	d_{49}	$d_{48} + d_{49} + d_{50}$	$d_{48} + d_{49} - d_{50}$

4: Pion-nucleon scattering

$L_{2I,2J}$



Blue: SU3 EOMS **Green: SU2 EOMS Chen.et al. PRD87(2013)054019**

Red: SU3 HB Huang et al. PRD96(2017)016021 **Black dot: EXP**

4: Pion-nucleon scattering

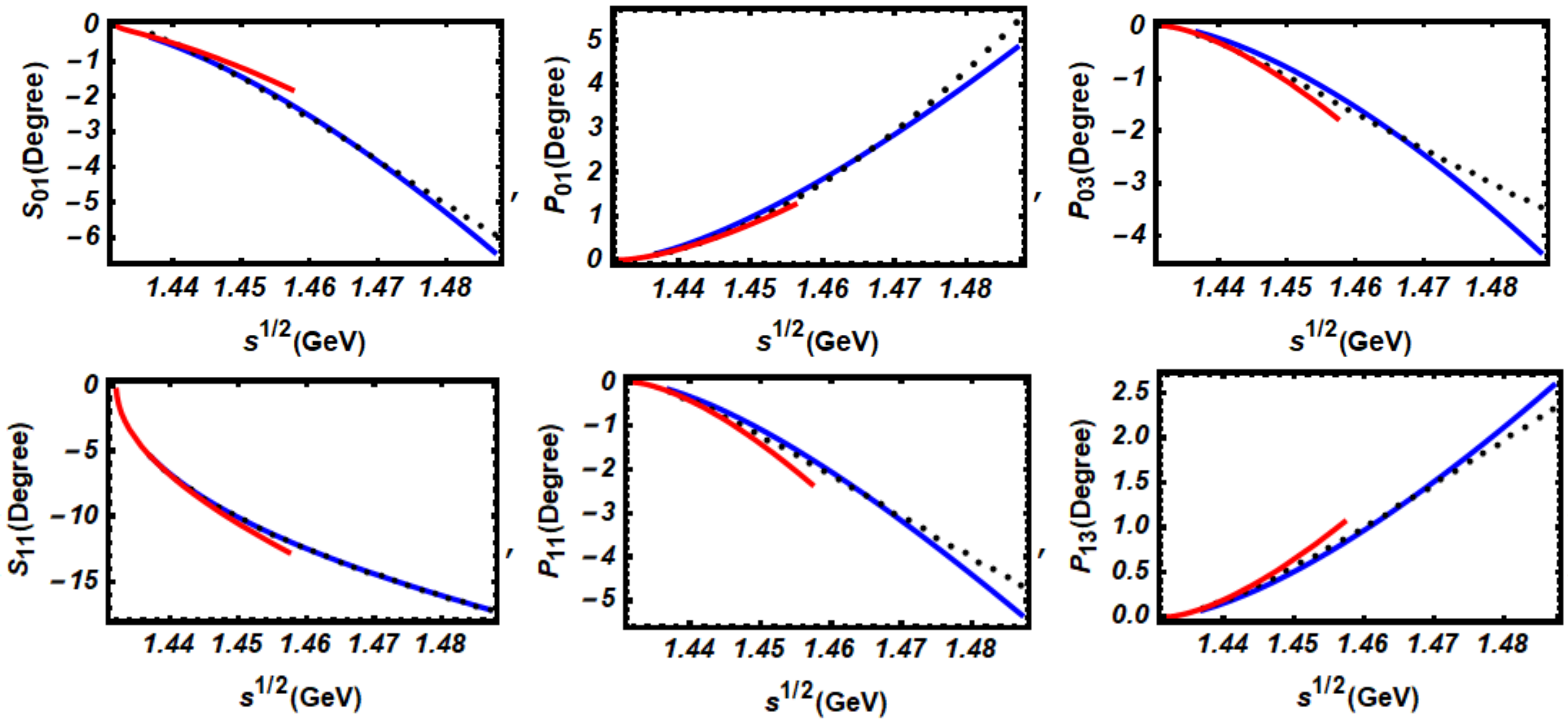
- Pion-nucleon phase shifts only (up to $s^{1/2}$=1.13 GeV)

α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	$\chi^2/d.o.f.$
$-7.64(6)$	$1.42(2)$	$1.34(1)$	$-1.36(6)$	$0.61(2)$	$3.25(6)$	$1.45(3)$	$-0.32(12)$	0.154

4: Kaon-Nucleon scattering

$L_{2I,2J}$

- Isospin=0, χ^2/d.o.f = 1.046
- Isospin=1, χ^2/d.o.f = 0.507



Blue: SU3 EOMS

Red: SU3 HB Huang et al. PRD96(2017)016021

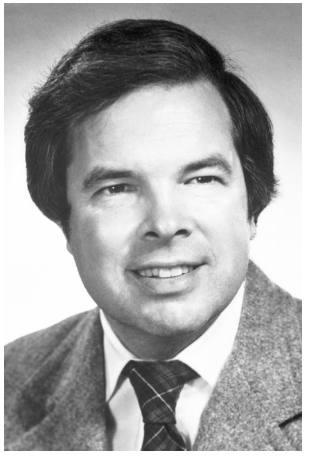
Black dot: EXP

5.1: Renormalization Group Invariance

- **A fundamental feature of EFT** that physics at long-distance scales is **insensitive to** the details at short-distance scales

Peter Lepage

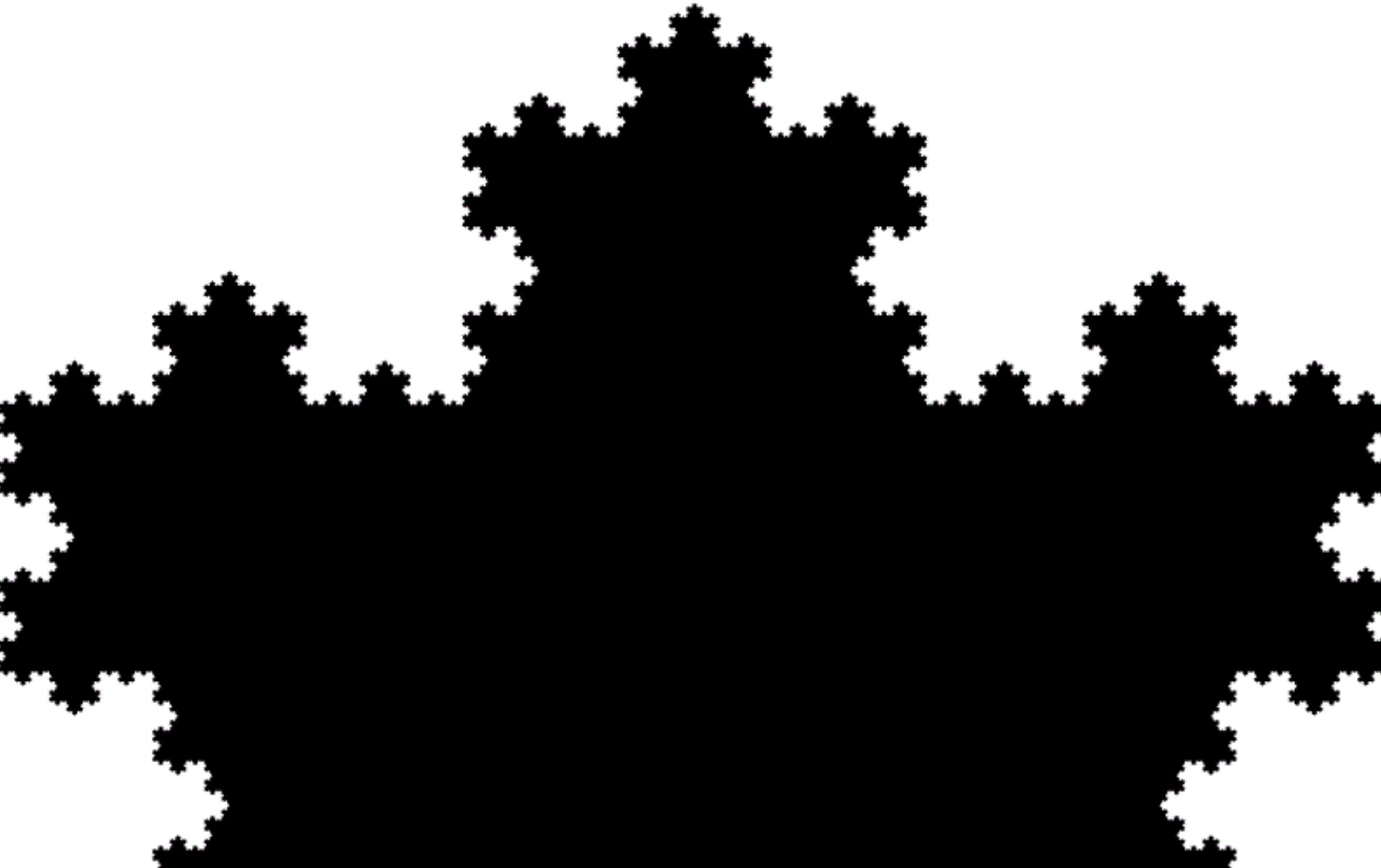
- The NN interaction can be considered properly renormalized **when the calculated observables are independent of the cutoff scale** within the range of validity of the ChEFT



- In the language of **Wilson's renormalization group**, this means that **the LECs must run with the cutoff** scale in such a way that the scattering amplitude becomes RG invariant (RGI)

Kenneth G. Wilson

5.1: RGI vs. self-similarity



5.1: RGI-a highly debated topic

Kaplan

van Kolck

Epelbaum

Machleidt

Key issues

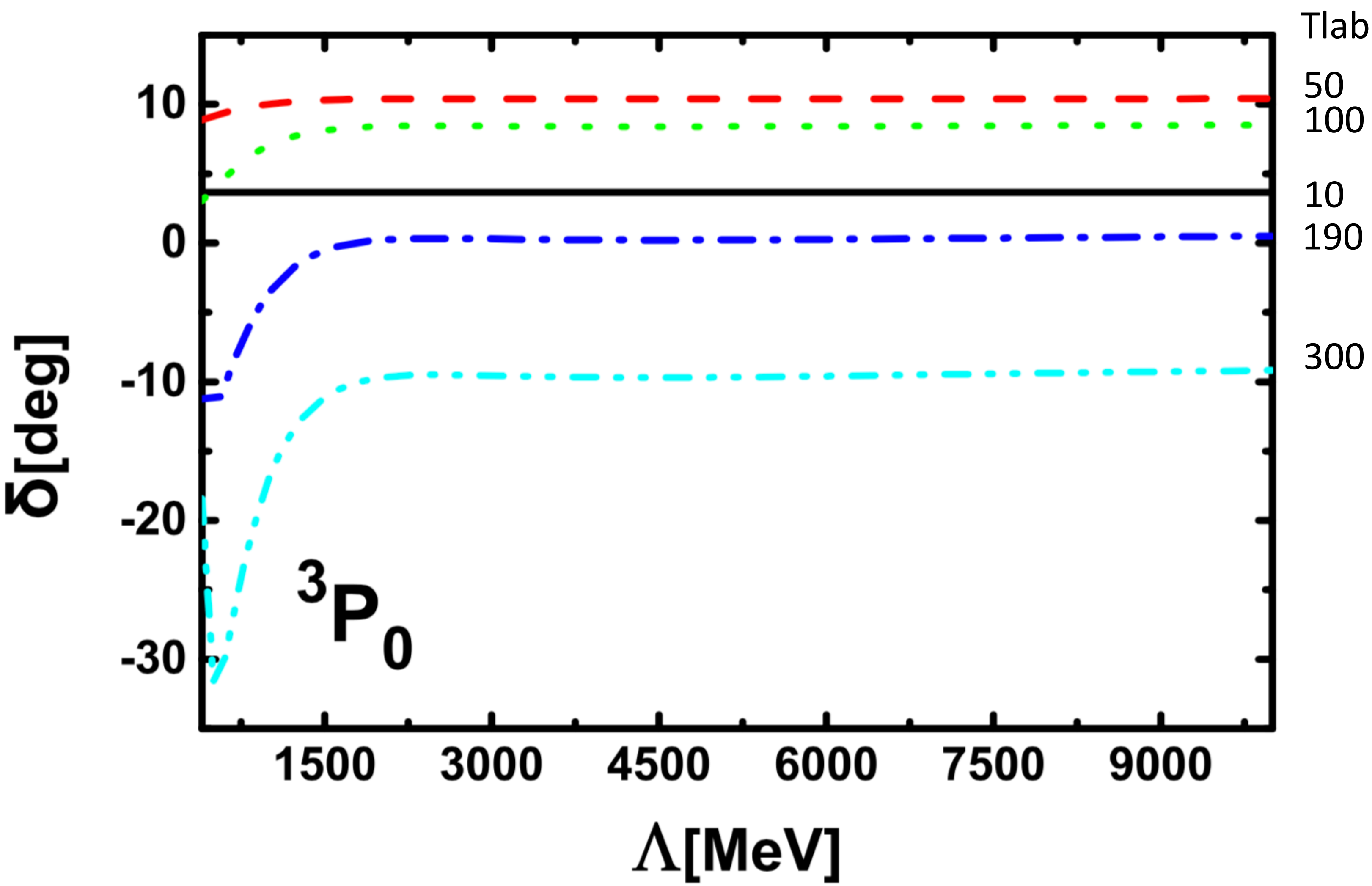
- ☐ Whether RGI is essential?
- ☐ How to implement it?

Key references

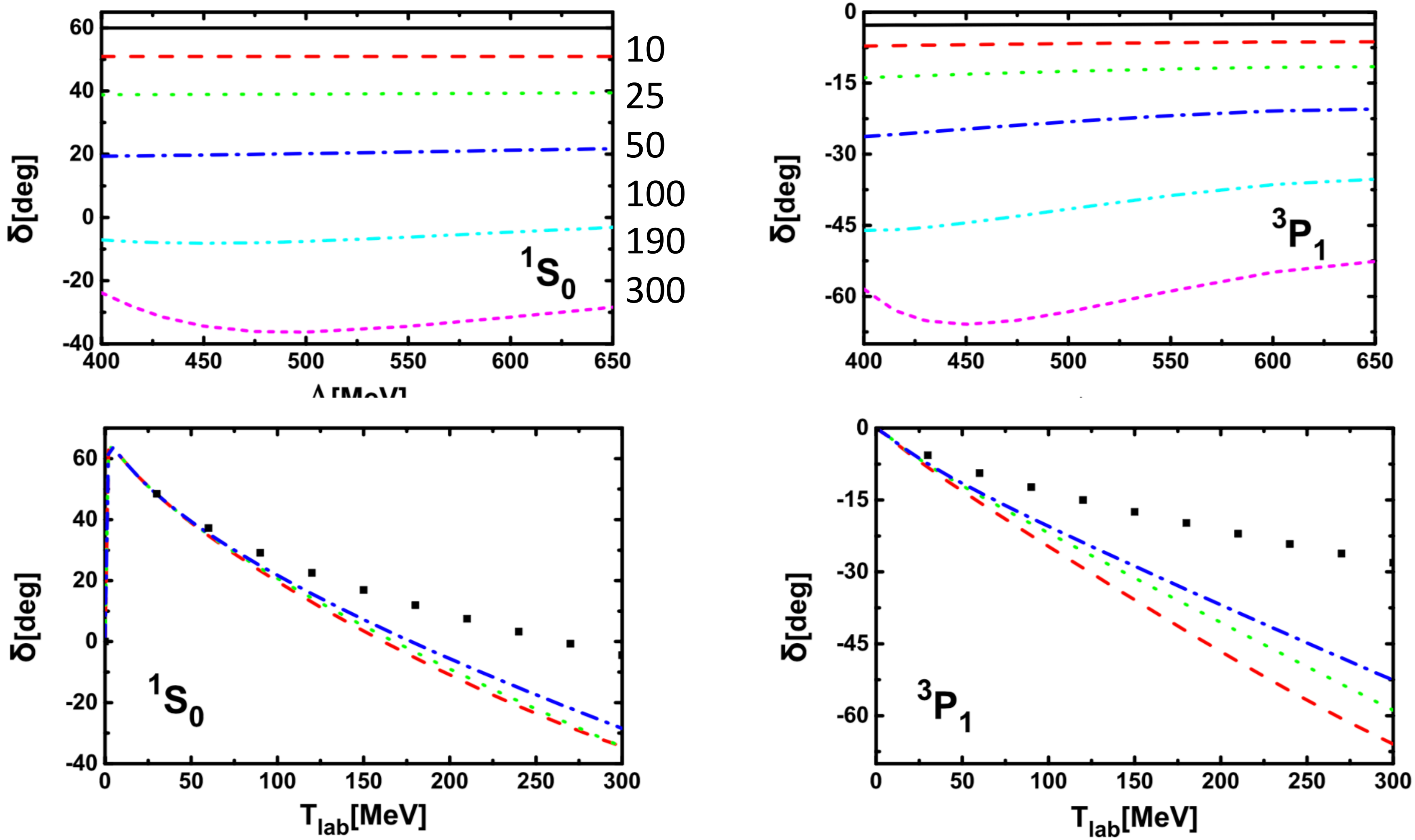
- ✓ *A New expansion for nucleon-nucleon interactions, David B. Kaplan et al., Phys.Lett. B424 (1998) 390*
- ✓ *Towards a perturbative theory of nuclear forces, S.R. Beane et al., Nucl.Phys. A700 (2002) 377*
- ✓ *How to renormalize the Schrodinger equation, P. Lepage,* nucl-th/9706029

5.1: 3P0 issue nicely solved

Chun-Xuan Wang et al.,

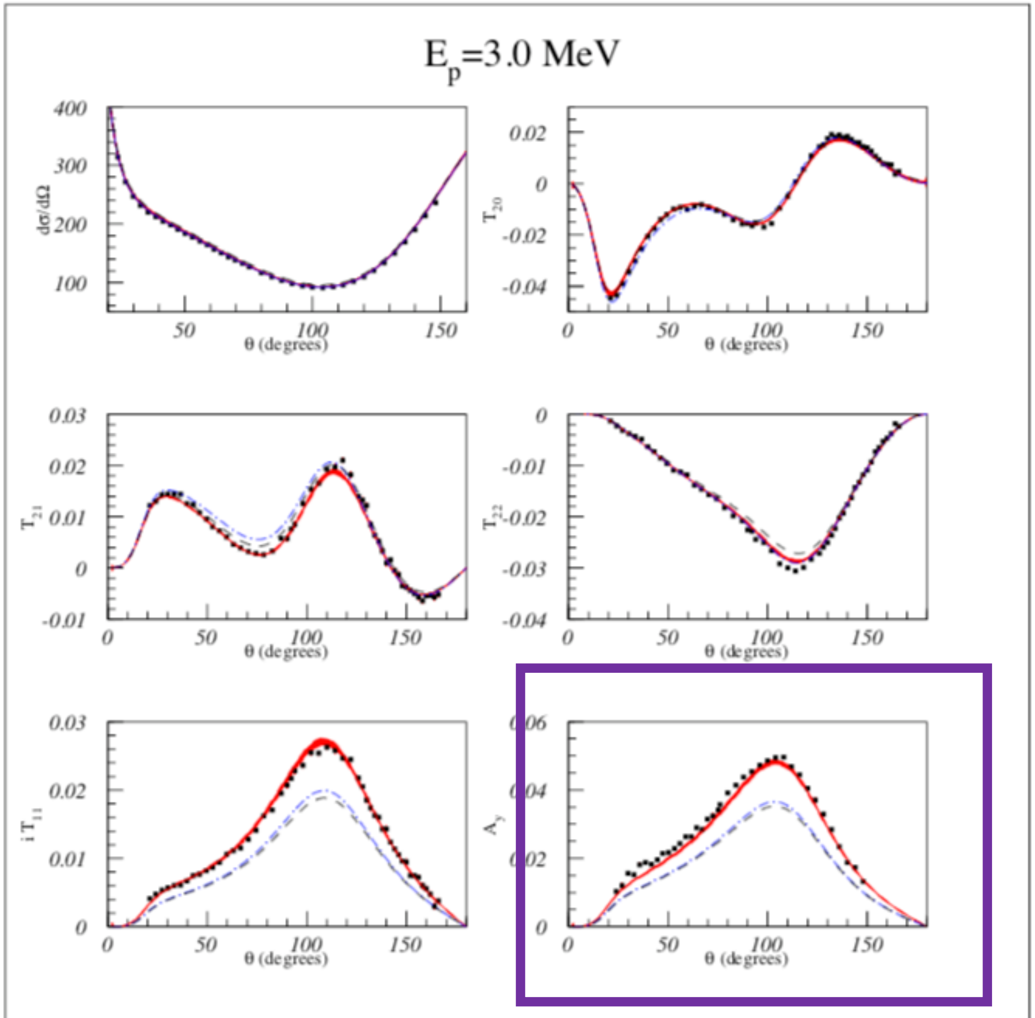


5.1: Interesting correlation 1S0 & 3P1



- Pure contact interactions cannot describe 1S0 – large cutoff limit
- NLO 3P1 in the Weinberg case loses RGI

5.2. Three-body study promising



1811.09398

L. Girlanda, A. Kievsky, M. Viviani and L.E. Marcucci

"Fit results in the leading order of the relativistic counting to a set of cross section and polarization p – d observables at 3 MeV proton energy, for Λ = 200 – 500 MeV (red bands) as compared to the purely two-body AV18 interaction (dashed, black lines) and to the AV18+UIX two- and three-nucleon interaction (dashed-dotted, blue lines). "

Ay puzzle– a 32 year old persistent problem!

Summary and outlook

- Based on the consideration of symmetries and also requested by nuclear structure community, we proposed to build a high-precision nuclear force based on covariant baryon chiral perturbation theory
- Exploratory studies show that the idea is feasible
- Steady progress is being made and the results are encouraging
  - ✓ NLO & NNLO contact terms
  - ✓ NLO meson-baryon scattering
- A high precision covariant chiral nuclear force will come soon, stay tuned

北京航空航天大學
BEIHANG UNIVERSITY

Thanks for your attention！

August 20th , 2019

- Lorentz invariance or relativistic corrections might play an important role in speeding up the convergence of ChEFT

T_{lab} [MeV]	1	50	100	150	200	250	300
P_{cm} [MeV/c]	21.67	153.22	216.68	265.38	306.43	342.60	375.30
P_{cm}/M_N	0.023c	0.16c	0.23c	0.28c	0.33c	0.36c	0.40c

In comparison m_π/m_N=138/939~0.15