Nuclear Science Computing Center at CCNU

2019 Guilin, China HADRON XVIII International Conference on Hadron Spectroscopy and Structure

中国科学院近代物理研究所 Institute of Modern Physics, Chinese Academy of Sciences

Chiral phase transition in (2 + 1)-flavor QCD

Sheng-Tai Li for HotQCD collaboration

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Outline

- Introduction to QCD Phase diagram
- General approach to study phase transition
- Estimators to determine chiral phase transition temperature T_c^0
- T_c^0 estimated in thermodynamic limit, chiral limit and continuum limit
- Conclusion

QCD symmetry & phase transition

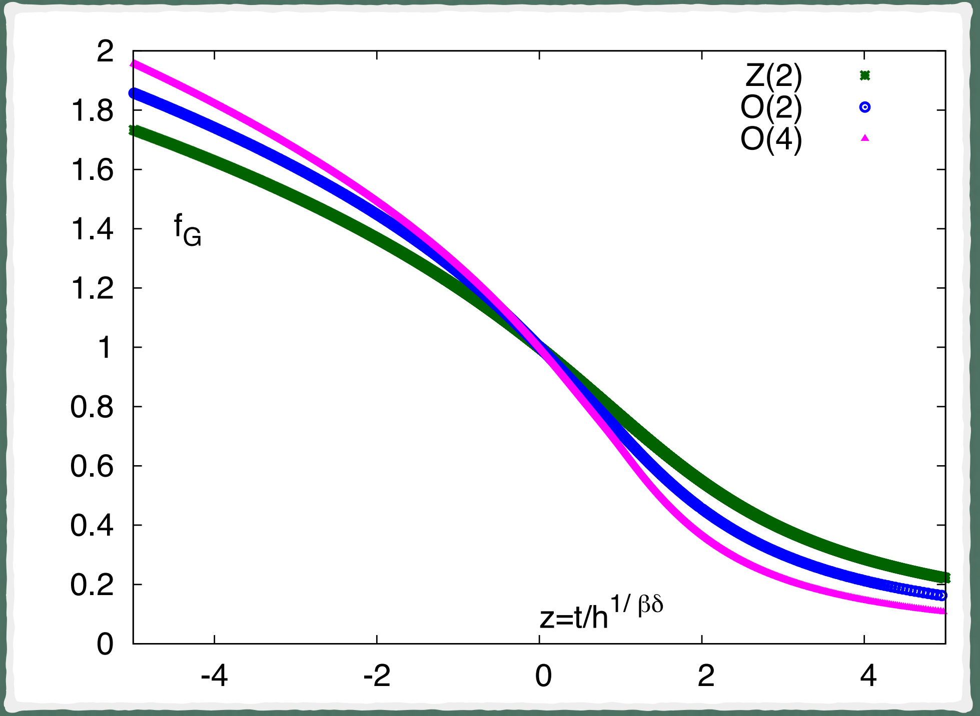
$$\mathcal{L}_{QCD} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \overline{q}\left[i\gamma^{\mu}\left(\partial_{\mu} - igA_{\mu}\right) - m_q\right]q$$

$$SU\left(N_f\right)_L \times SU\left(N_f\right)_R \times U(1)_V \times U(1)_A \quad \text{at } m_q = 0$$

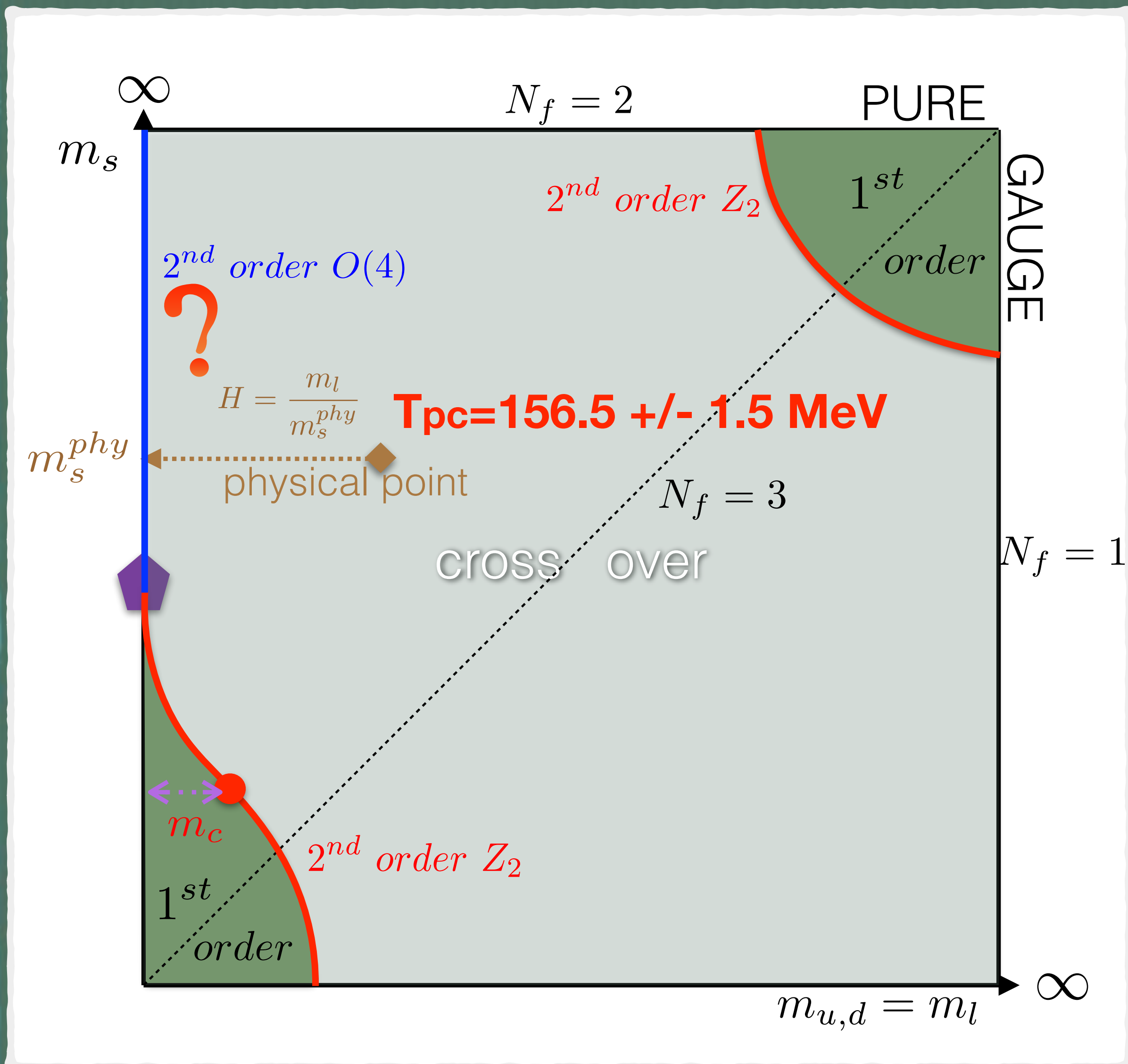
$$SU(2)_L \bigotimes SU(2)_R \simeq O(4)$$

$$\downarrow$$

$$SU(2)_V$$



QCD symmetry & phase transition



Physical point: T_{pc}=156.5 ±1.5MeV

HotQCD Collaboration: A. Bazavov, …, S.T. Li et al., Phys.Lett. B795 (2019) 15-21

The order of chiral phase transition ?

Pisarski R D, Wilczek F, 1984. Phys. Rev. D, 29:338-341

1. **Second order O(4) if $U_A(1)$ is significant broken**

   Butti A, Pelissetto A, Vicari E, 2003, JHEP, 08:029.
   Pisarski R D, Wilczek F, 1984.

2. **First order or Second order if $U_A(1)$ is negligible near T_c^0**

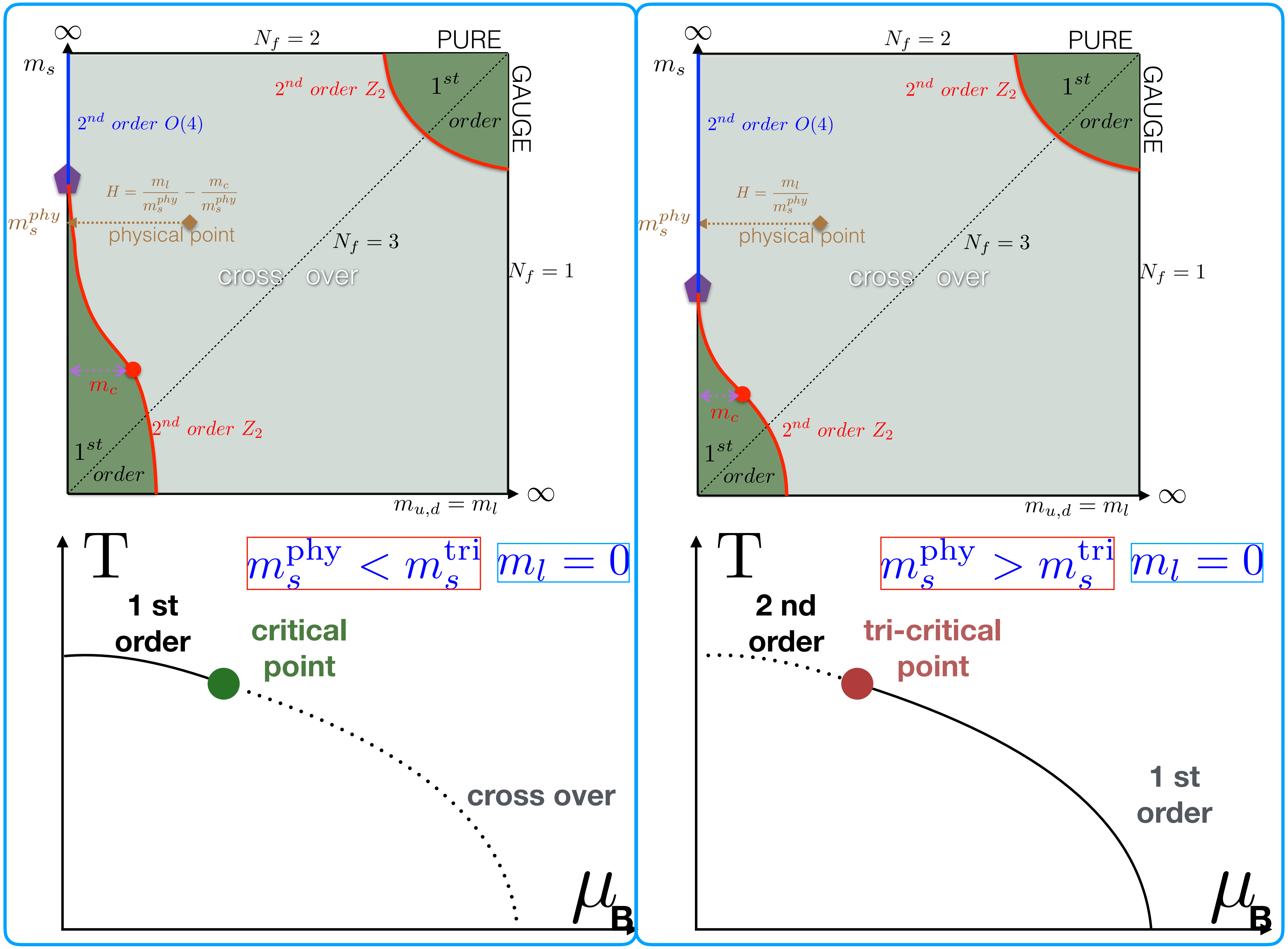
   First order: Pisarski R D, Wilczek F, 1984.

   Second order:
   Grahl M, Rischke D H, 2013. Phys. Rev., D88(5):056014 (FRG)
   Pelissetto A, Vicari E, 2013. Phys. Rev., D88(10):105018

Phase transition temperature about 20-30 MeV smaller than T_{pc}

Berges J, Jungnickel D U, Wetterich C, 1999. Phys. Rev. D, 59:034010.
Pelissetto A, Vicari E, 2013. Phys. Rev., D88(10):105018.

Influence to QCD phase structure at μB>0

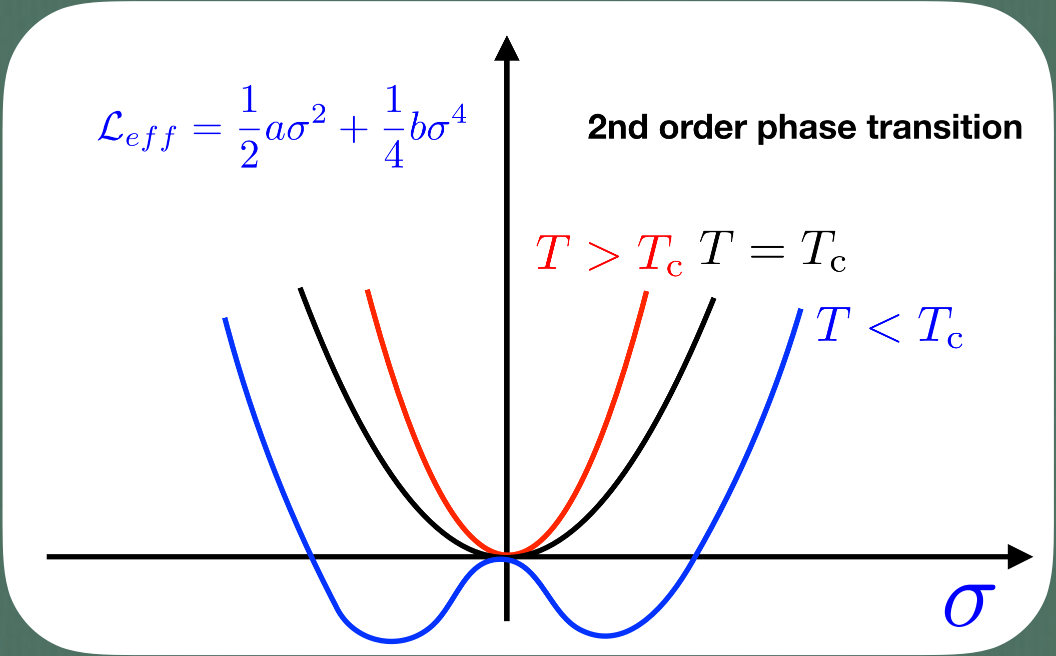


General approaches to study chiral phase transition

Ginzburg-Landau-Wison approach

$$Z = \int [\mathrm{d}\sigma] \exp\left(-\int \mathrm{dx} \mathcal{L}_{eff}(\sigma(\mathrm{x}); K)\right)$$

$$\mathcal{L}_{eff} = \frac{1}{2}(\nabla\sigma)^2 + \sum_n a_n(K)\sigma^n$$



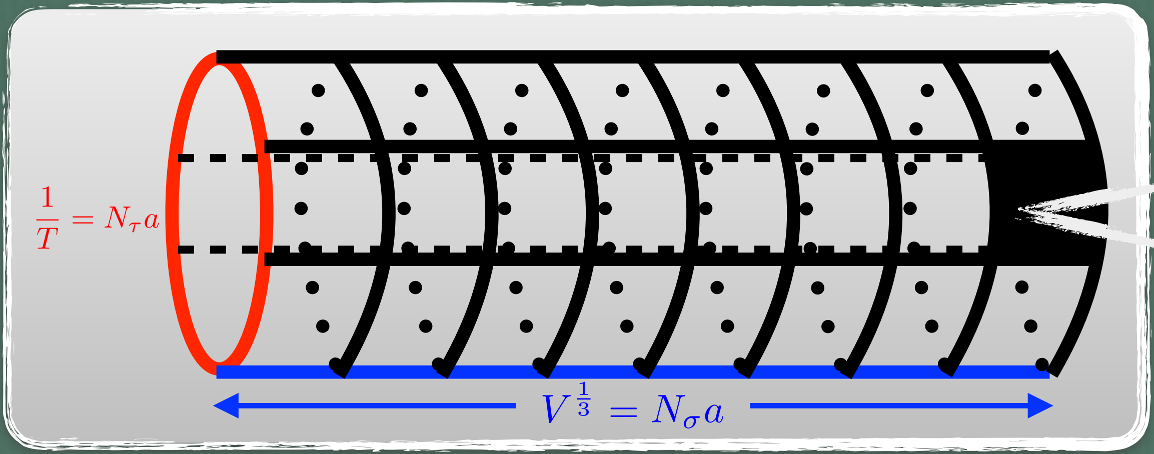
Landau functional of QCD

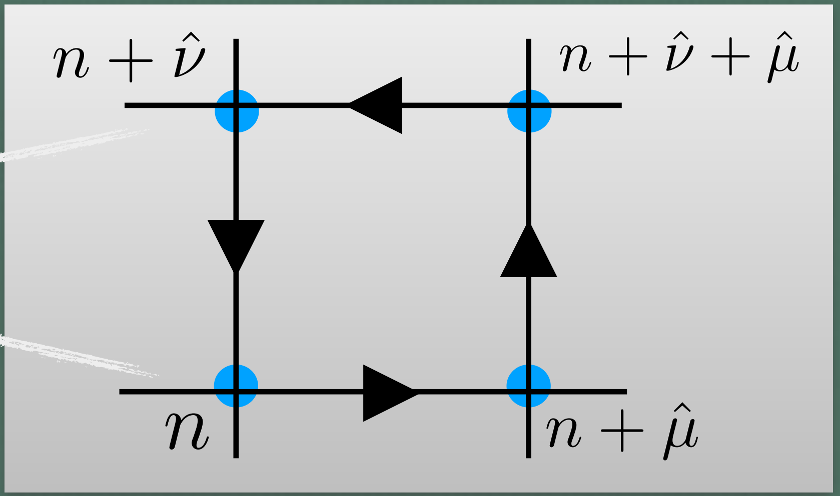
Robert D. Pisarski and Frank Wilczek in 1984

$$\begin{aligned}\mathcal{L}_{eff}^{QCD} &= \frac{1}{2}\,\mathrm{tr}\,\partial\Phi^\dagger\partial\Phi + \frac{a}{2}\,\mathrm{tr}\,\Phi^\dagger\Phi \\ &+ \frac{b_1}{4!}\left(\mathrm{tr}\,\Phi^\dagger\Phi\right)^2 + \frac{b_2}{4!}\,\mathrm{tr}\left(\Phi^\dagger\Phi\right) \\ &- \frac{c}{2}\left(\det\Phi + \det\Phi^\dagger\right) \\ &- \frac{d}{2}\,\mathrm{tr}\,h\left(\Phi + \Phi^\dagger\right)\end{aligned}$$

- **AdS/CFT**
- **FRG**
- **Dyson Schwinger Equation**
- **LQCD**

Lattice QCD





Why chiral phase transition is hard to study on lattice?

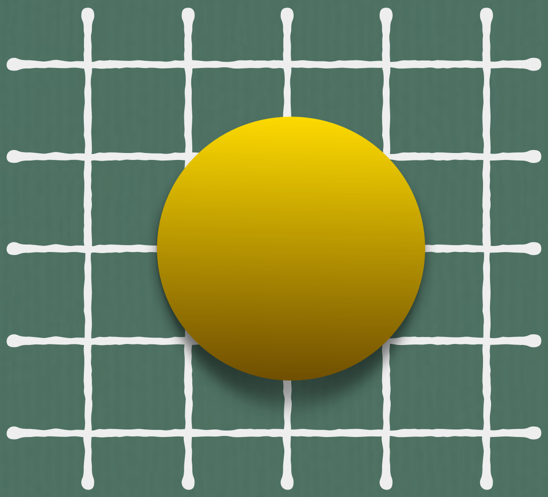
$$\mathcal{Z}(T,V) = \int [DU] \prod_{q=u,d,s} \det M_q[U] e^{-S_G[U]}$$

$12N_\sigma^3 \times N_\tau$

$$\langle \overline{\psi}\psi \rangle_u = \frac{1}{N_\sigma^3 N_\tau} \frac{\partial \ln Z}{\partial \hat{m}_u} = \langle \mathrm{Tr}\, M_u^{-1} \rangle$$

$$C_{op} = k \left(\frac{20\mathrm{MeV}}{\overline{m}}\right)^{[1-2]} \left(\frac{L}{3\mathrm{fm}}\right)^{[4-5]} \left(\frac{0.1\mathrm{fm}}{a}\right)^{[4-6]} \mathrm{Teraflops} \times \mathrm{years}$$

Karl Jansen, PoS LATTICE2008 (2008) 010, arXiv:0810.5634

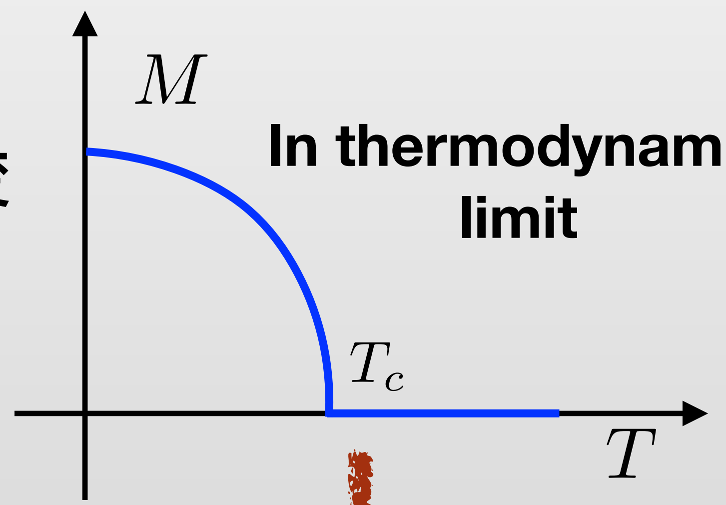


$$N_\sigma a \simeq \frac{\lambda}{2} \propto m_\pi^{-1}$$

Access Chiral limit is very hard !

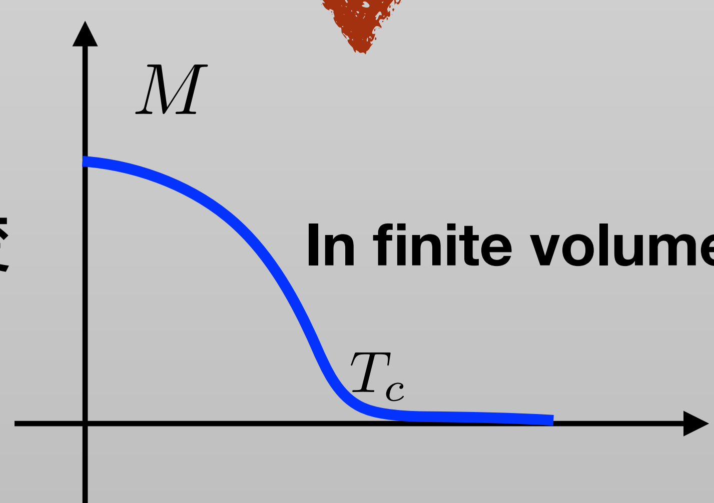
Thermodynamic limit and continuum limit

连续相变



⭐ **Finite size effects complicated**

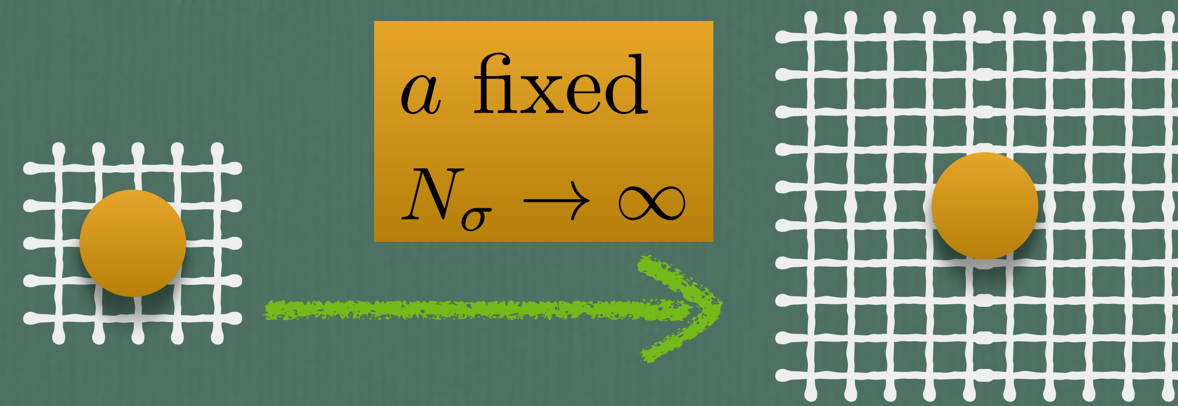
连续相变



Thermodynamic limit

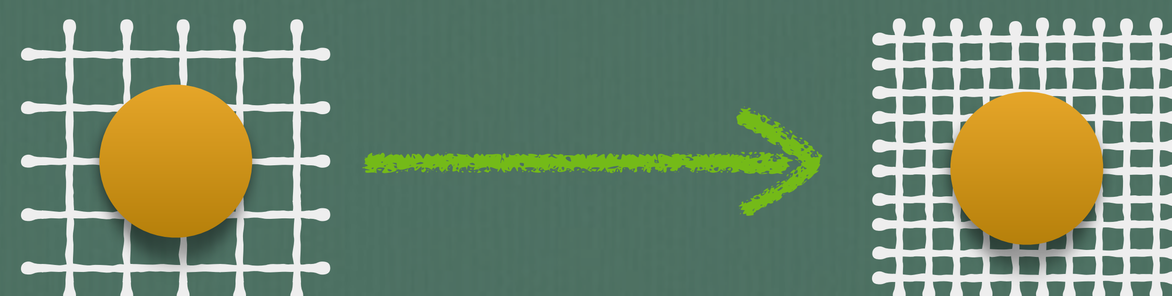
a fixed

$N_\sigma \to \infty$



Continuum limit

$\Lambda \propto 1/a$



$a \to 0$, by $N_\tau \to \infty$

$$V^{\frac{1}{3}} = aN_\sigma = \frac{N_\sigma}{N_\tau T} \text{ fixed}$$

Magnetic equation of state (MEOS)

General theory
Singular part dominate near Tc

Specific for QCD chiral transition

$$M(t,h) = h^{1/\delta} f_G(z) + f_{\mathrm{reg}} = \frac{m_s}{f_K^4}\left[\langle\bar{\psi}\psi\rangle_l - \frac{2m_l}{m_s}\langle\bar{\psi}\psi\rangle_s\right]$$

$$\chi_M(t,h) = \frac{\partial M}{\partial H} = H^{-1} h^{1/\delta} f_\chi(z) + \frac{\partial f_{\mathrm{reg}}}{\partial H} = \frac{m_s^2}{f_K^4}\chi_{\mathrm{subtot}}$$

$$f_{\mathrm{reg}} = a_1(T)H + a_3(T)H^3 + \ldots, \quad a_i(T) = \left(a_{i0} + a_{i1}\frac{T-T_c}{T_c} + a_{i2}\left[\frac{T-T_c}{T_c}\right]^2\right)$$

Scaling variables

$$z = t/h^{1/\beta\delta}$$

$$t = \frac{1}{t_0}\frac{T-T_c^0}{T_c^0}$$

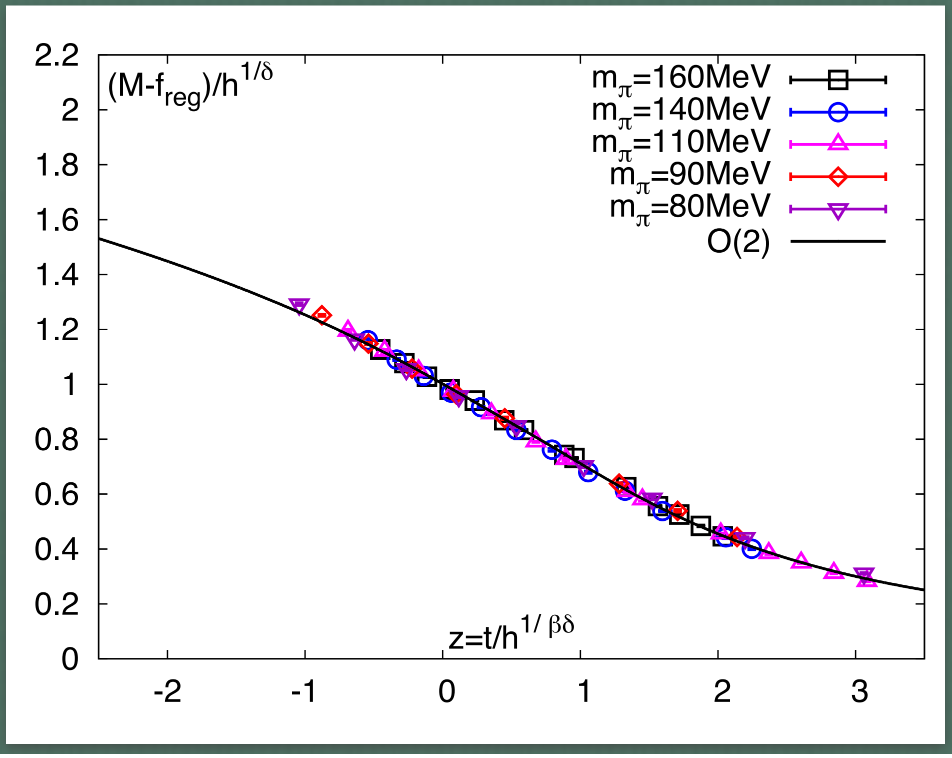
$$h = \frac{H}{h_0} = \frac{1}{h_0}\frac{m_l}{m_s}$$

Universal	Non-universal
$\beta,\ \delta$	t_0, h_0
f_G, f_χ	a_{ij}, T_c^0

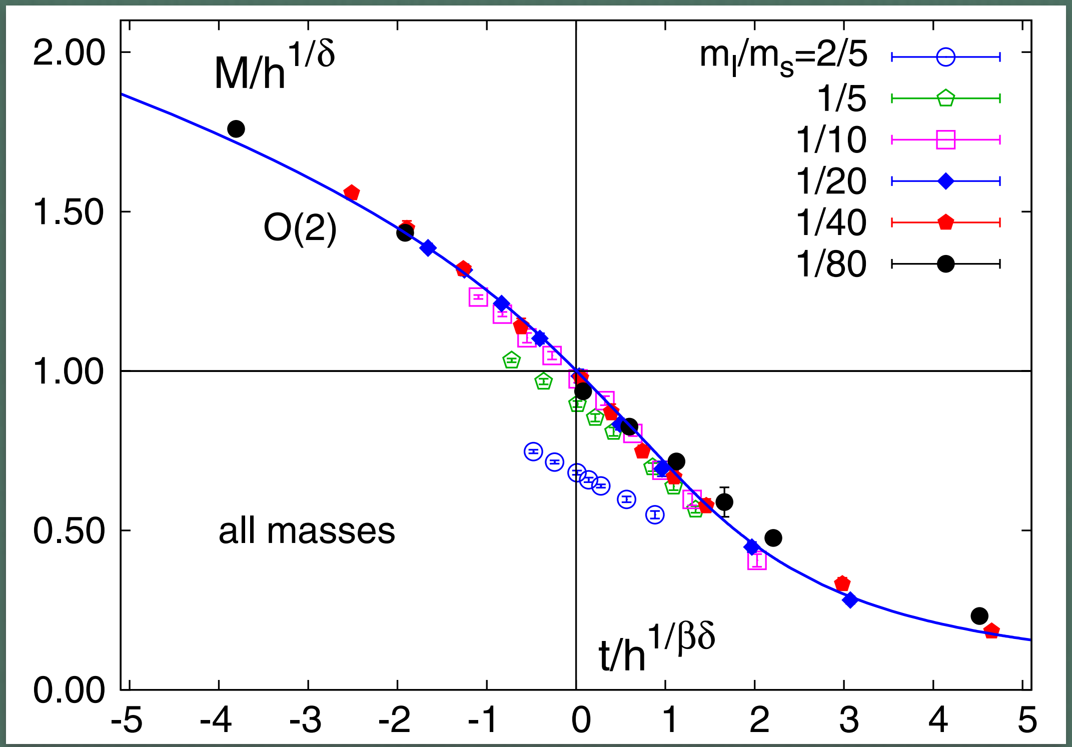
Fundamental quantity of QCD

Problems：
Universality class ?
Finite size effects ?

Magnetic equation of state (MEOS)



Sheng-Tai Li, Heng-Tong Ding, PoS LATTICE2016 (2017) 372



S. Ejiri et al., PHYSICAL REVIEW D 80, 094505 (2009)

Scaling variables

$$z = t/h^{1/\beta\delta}$$

$$t = \frac{1}{t_0}\frac{T - T_c^0}{T_c^0}$$

$$h = \frac{H}{h_0} = \frac{1}{h_0}\frac{m_l}{m_s}$$

Universal	Non-universal
$\beta,\ \delta$	t_0, h_0
f_G, f_χ	a_{ij}, T_c^0

Fundamental quantity of QCD

Problems：

Universality class ?

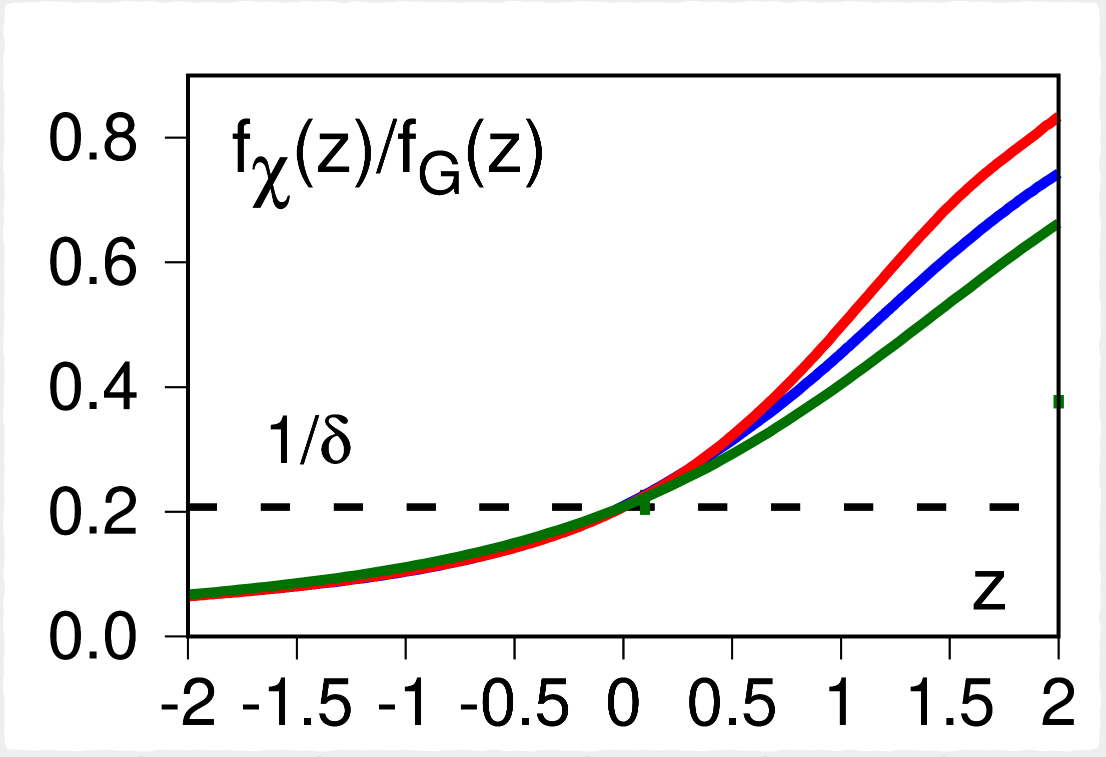
Finite size effects ?

Novel estimators (T_δ and T_{60}) to determine T_c^0

$$\frac{H\chi_M}{M} = \frac{h^{1/\delta} f_\chi(z) + a_1(T)H}{h^{1/\delta} f_G(z) + a_1(T)H}$$

$$\frac{H\chi_M\,(T_\delta, H, L)}{M\,(T_\delta, H, L)} = \frac{1}{\delta}$$

$$\lim_{L\to\infty, H\to 0} T_\delta = T_c^0$$



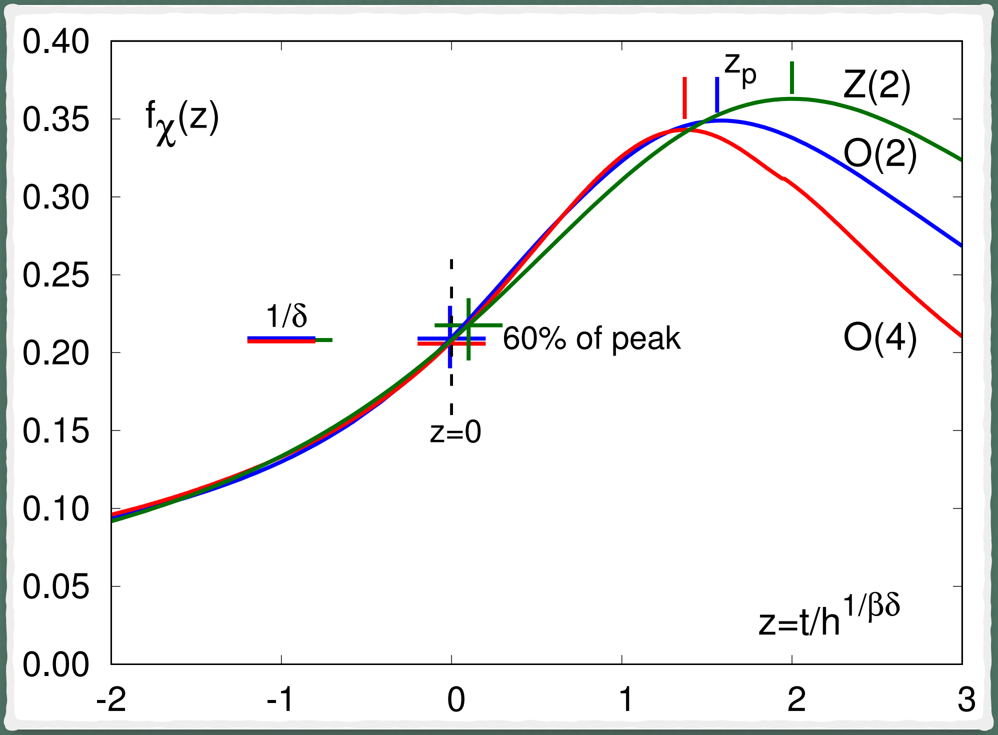
3-d O(4) finite-size scaling functions

$$T_\delta(H,L) = T_c^0 + \frac{H^{1/\beta\delta(1-5.7v)}}{L^{5.7}} B + c_\delta H^{1-1/\delta+1/\beta\delta}, \quad v = 0.7377(41)$$

J. Engels and F. Karsch Phys. Rev. D **90**, 014501 – 1 July 2014

Model	δ	β
Z(2)	4.805	0.3258
O(4)	4.824	0.380
O(2)	4.780	0.349

Novel estimators (T_δ and T_{60}) to determine T_c^0



$$\chi_M\left(T_{60}, H\right) = 0.6\chi_M^{\max}$$

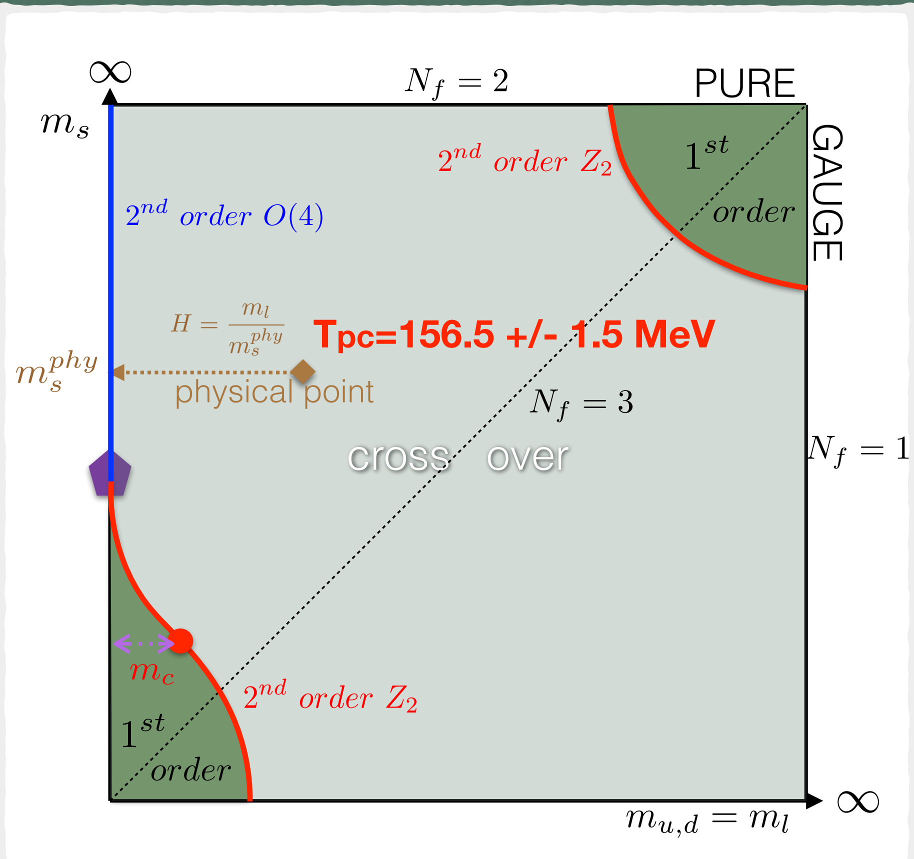
3-d O(4) finite-size scaling functions

$$T_{60}(H,L) = T_c^0 + \frac{H^{1/\beta\delta(1-5.7v)}}{L^{5.7}} B + c_{60} H^{1-1/\delta+1/\beta\delta}, \quad v = 0.7377(41)$$

J. Engels and F. Karsch Phys. Rev. D **90**, 014501 – 1 July 2014

Model	z_p	z_{60}
Z(2)	2.0	0.1
O(4)	1.37	-0.01
O(2)	1.56	-0.009

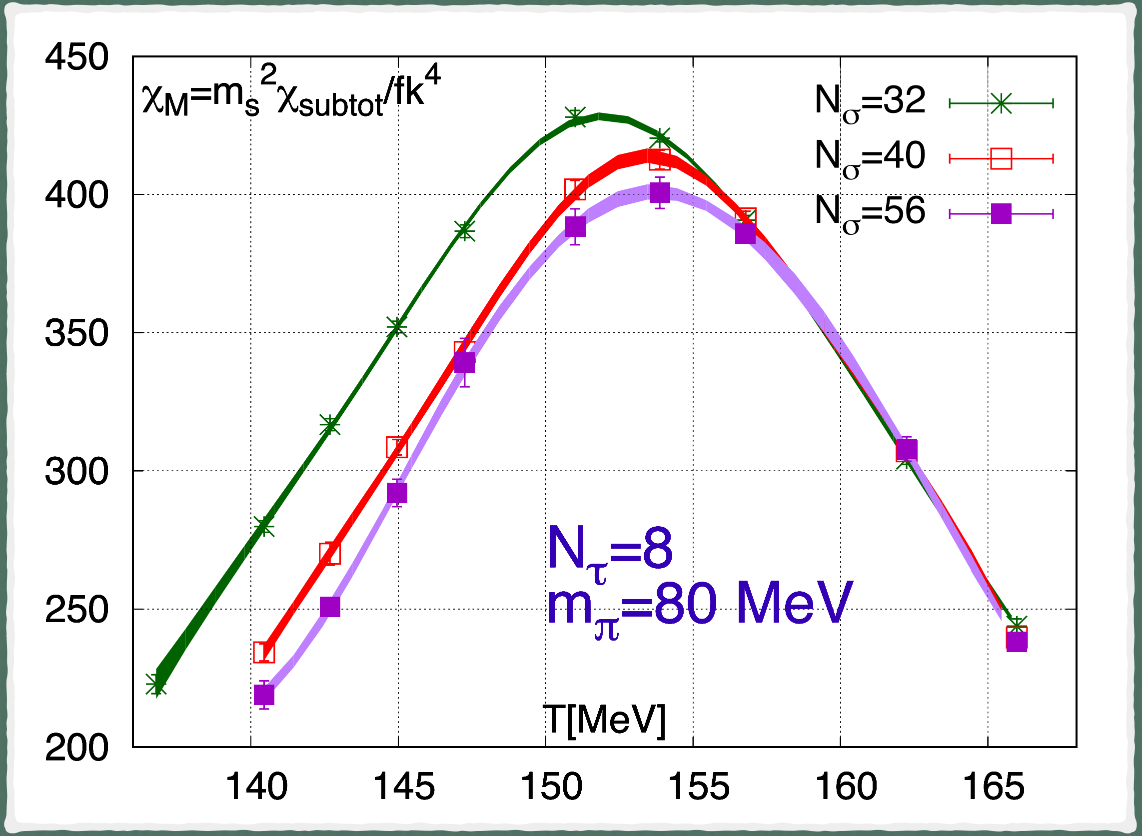
Lattice Setup

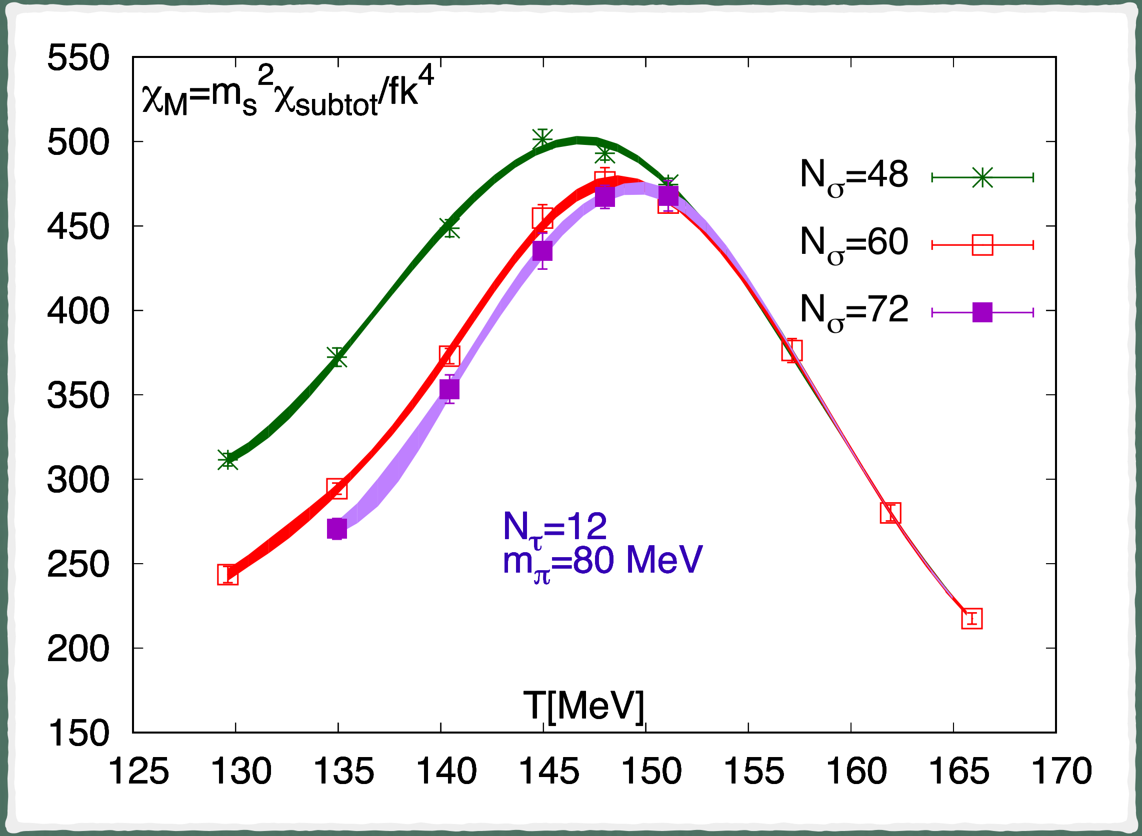


- The strange quark mass is fixed at its physical value
- 4-5 values of quark masses are chosen correspond to 55MeV≤mπ ≤160MeV to approach the chiral limit
- Nτ = 6, 8 ,12 allow us to perform continuum limit
- Three different Ns values are chosen to perform the thermodynamic limit

Nt	$m_s^{\rm phy}/m_l$ window	m_π window	N_σ/N_τ window	Computing source
6	[20, 80]	[80, 160] MeV	[4, 8]	Jefferson Lab
8	[20, 160]	[55, 160] MeV	[4, 7]	CSCS Centro Svizzero di Calcolo Scientifico Swiss National Supercomputing Centre
12	[20, 80]	[80, 160] MeV	[4, 6]	NSC3 Nuclear Science Computing Center at CCNU

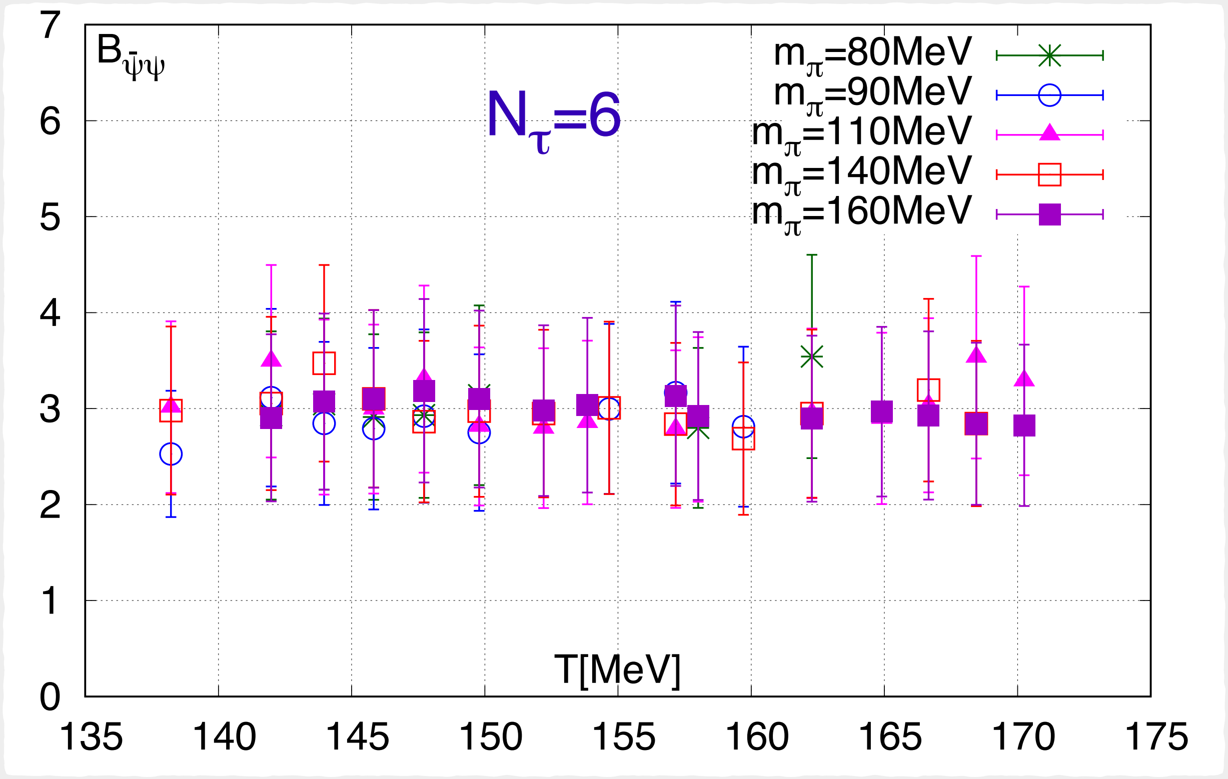
The volume dependence of the chiral susceptibility

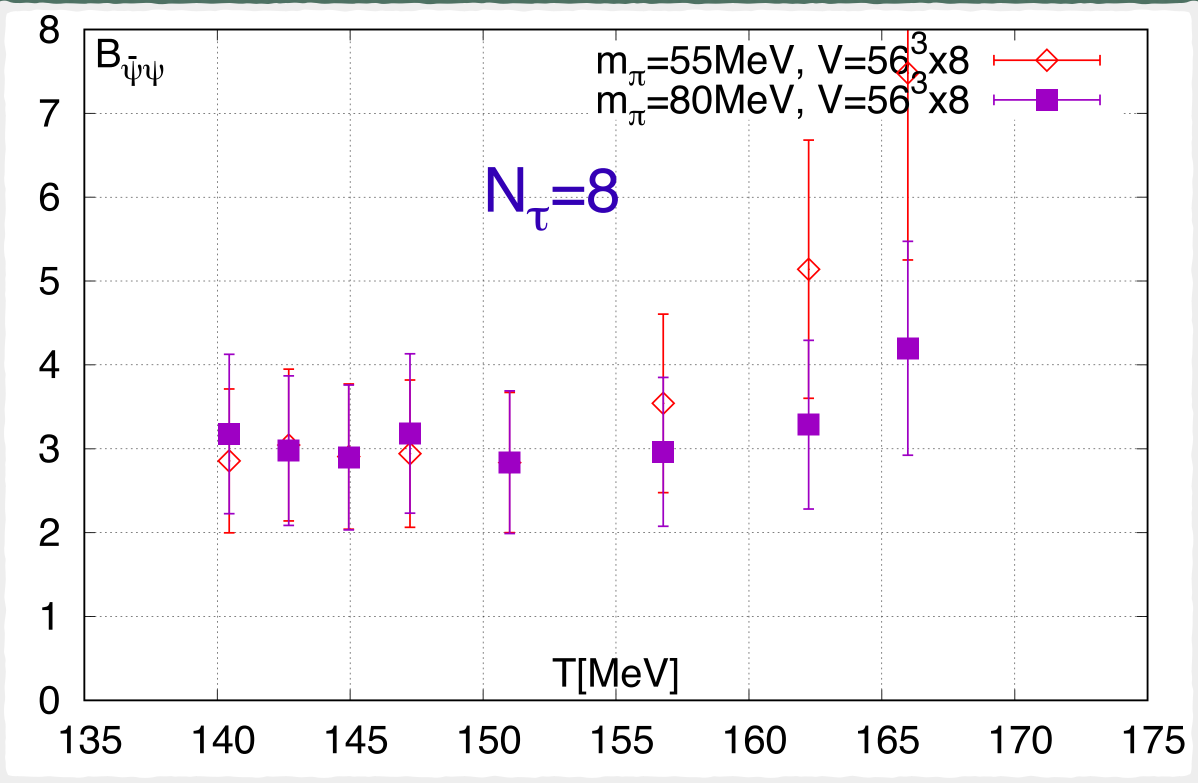




- Chiral susceptibility does not grow linearly in volume
- No evidence for first order phase transition was found in the current pion mass window $m_\pi \geq 80$ MeV

Binder cumulant of chiral order parameter



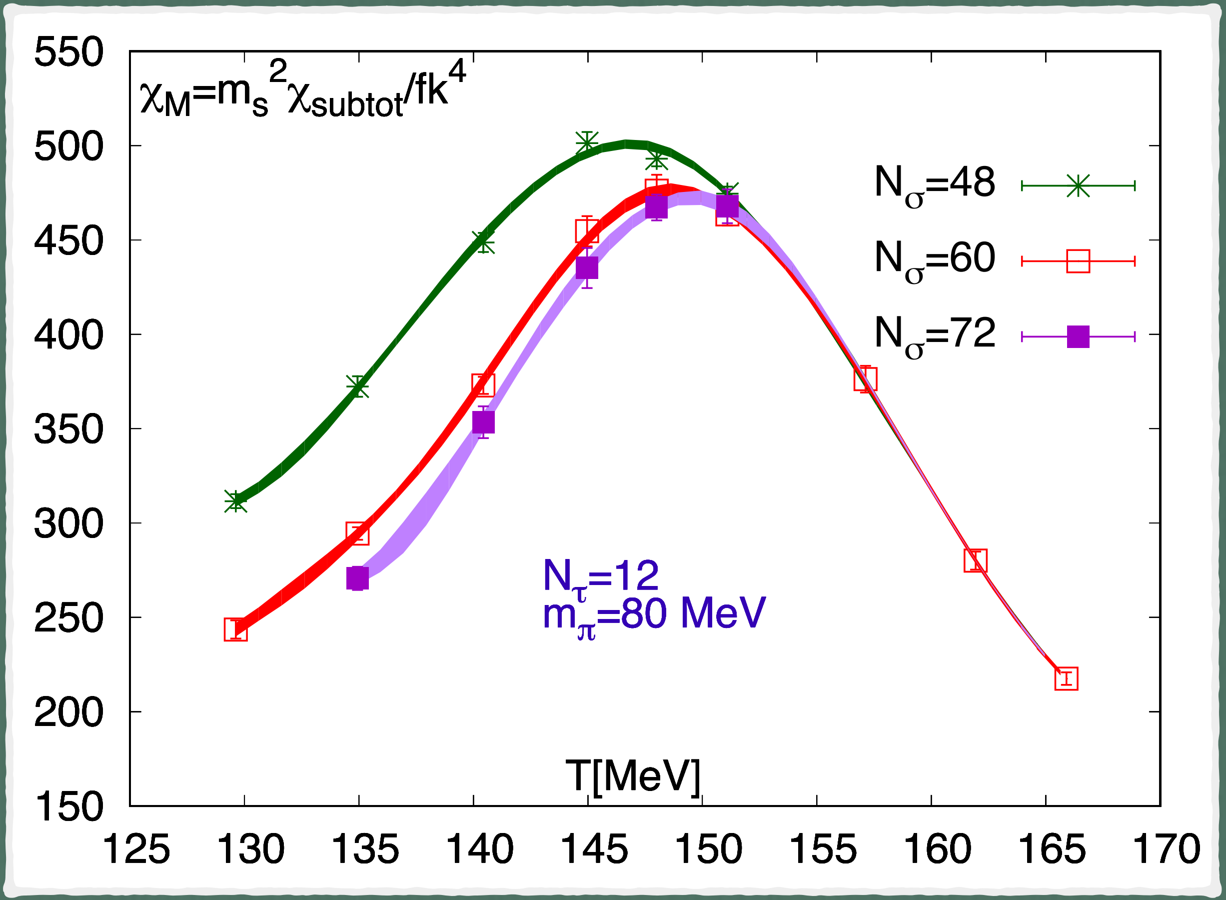


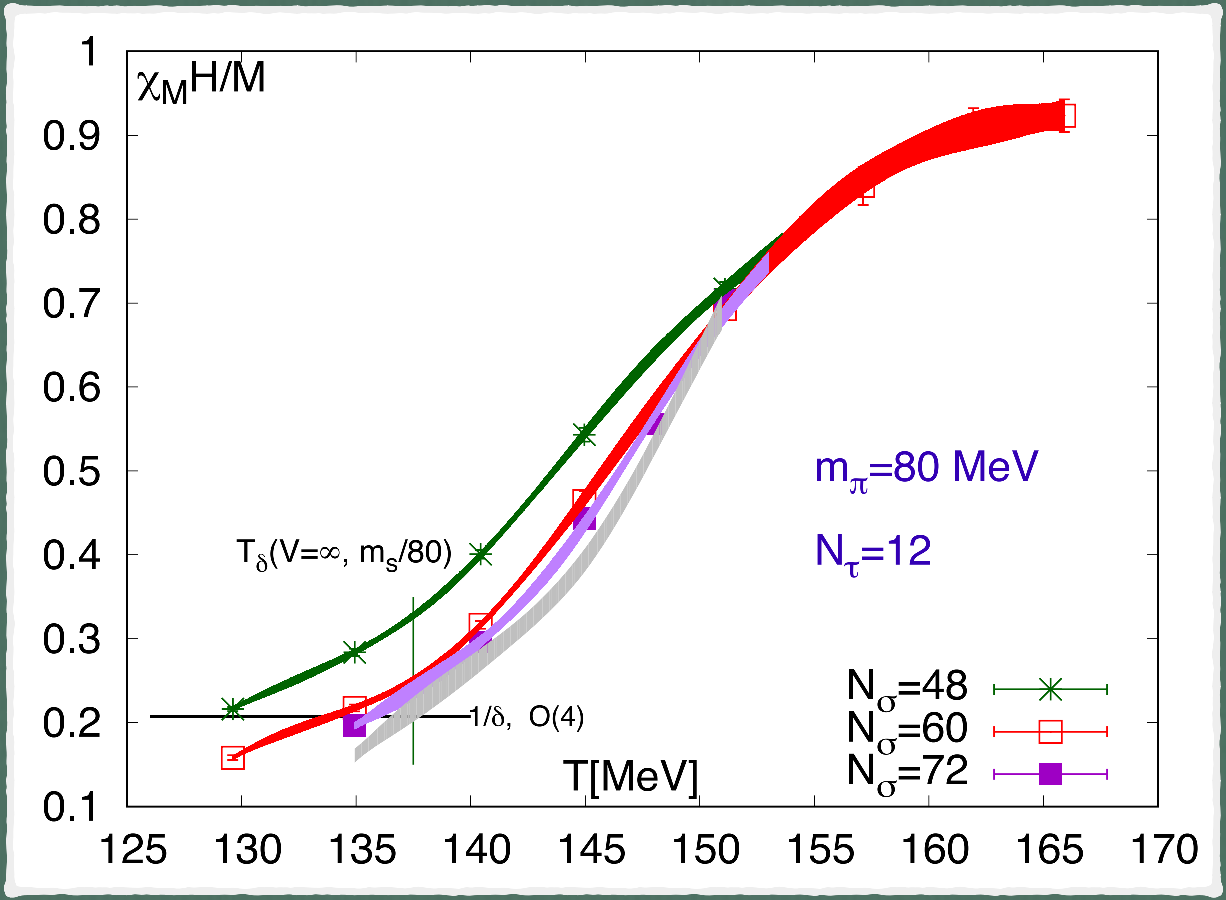
Model	$B_{\bar{\psi}\psi}$
Z(2)	1.604(2)
O(2)	1.242(2)
O(4)	1.092(3)
1 st	1
crossover	3

$$B_X = \langle (X - \langle X \rangle)^4 \rangle / \langle (X - \langle X \rangle)^2 \rangle^2$$

The Binder cumulant suggest the transition is crossover in the pion mass window [55, 160] MeV

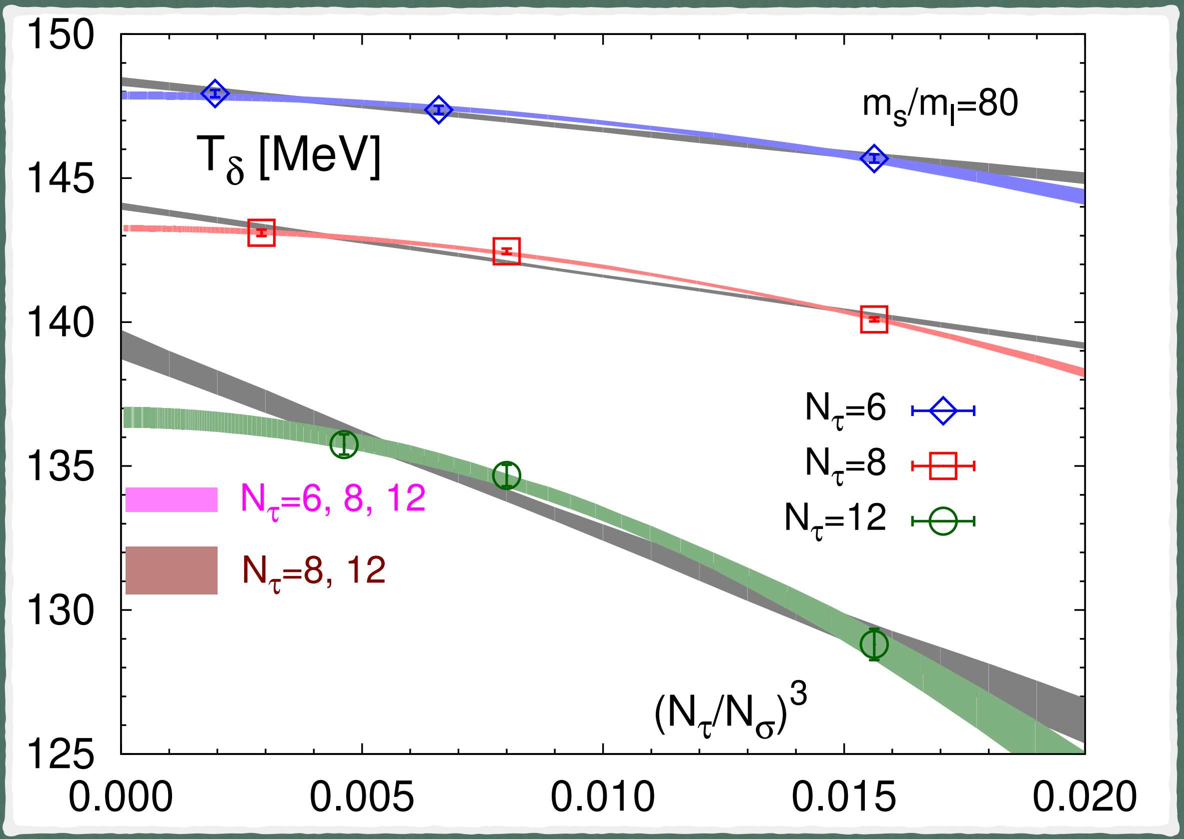
Study T_c^0 by T_{60} and T_δ





$\frac{m_s}{m_l}$	$N_\sigma^3 \times N_\tau$	$T_{\rm pc}$	T_{60}	T_δ
80	48^3 x 12	147.0 ± 0.1	128.7 ± 0.6	128.8 ± 0.4
80	60^3 x 12	148.7 ± 0.1	134.2 ± 0.4	133.9 ± 0.4
80	72^3 x 12	149.5 ± 0.3	136.3 ± 0.5	135.6 ± 0.3

Thermodynamic limit → Continuum limit → Chiral limit



O(4) Thermodynamic limit

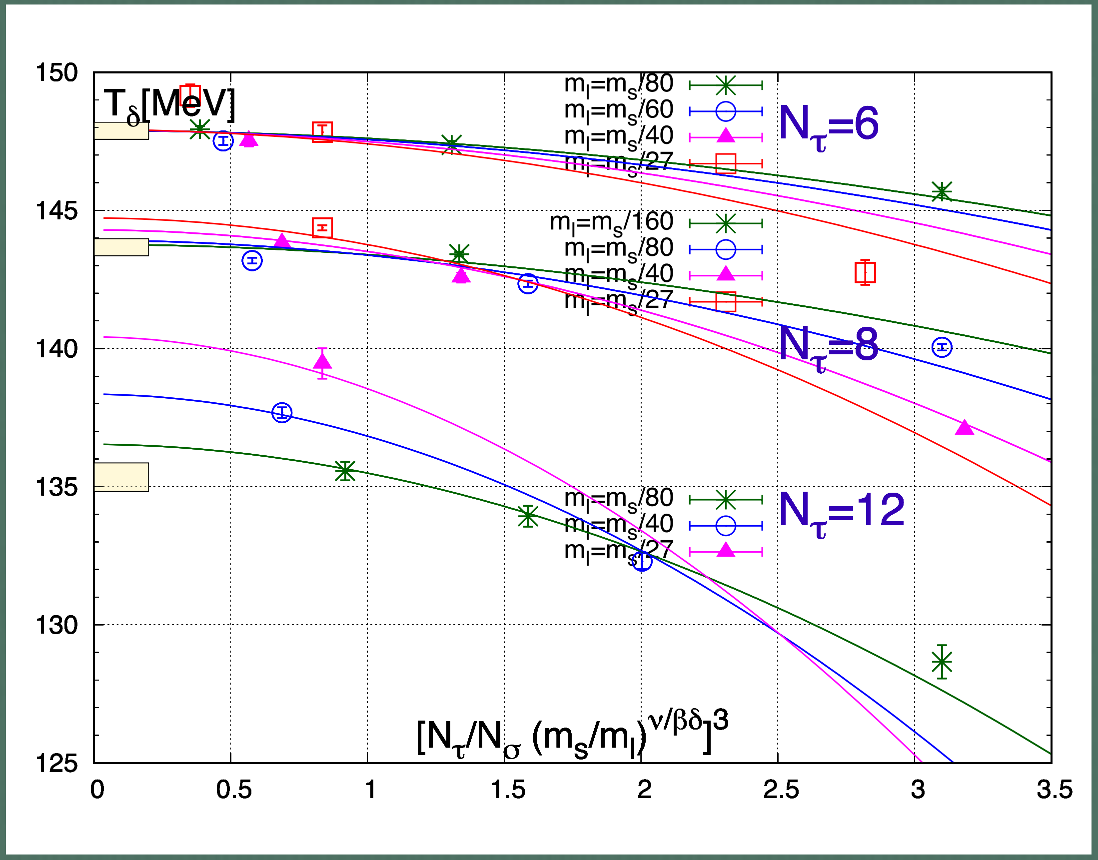
$$T_\delta\left(H = H_0, L\right) = T_c^0\left(H_0\right) + \frac{B'}{L^{5.7}}$$

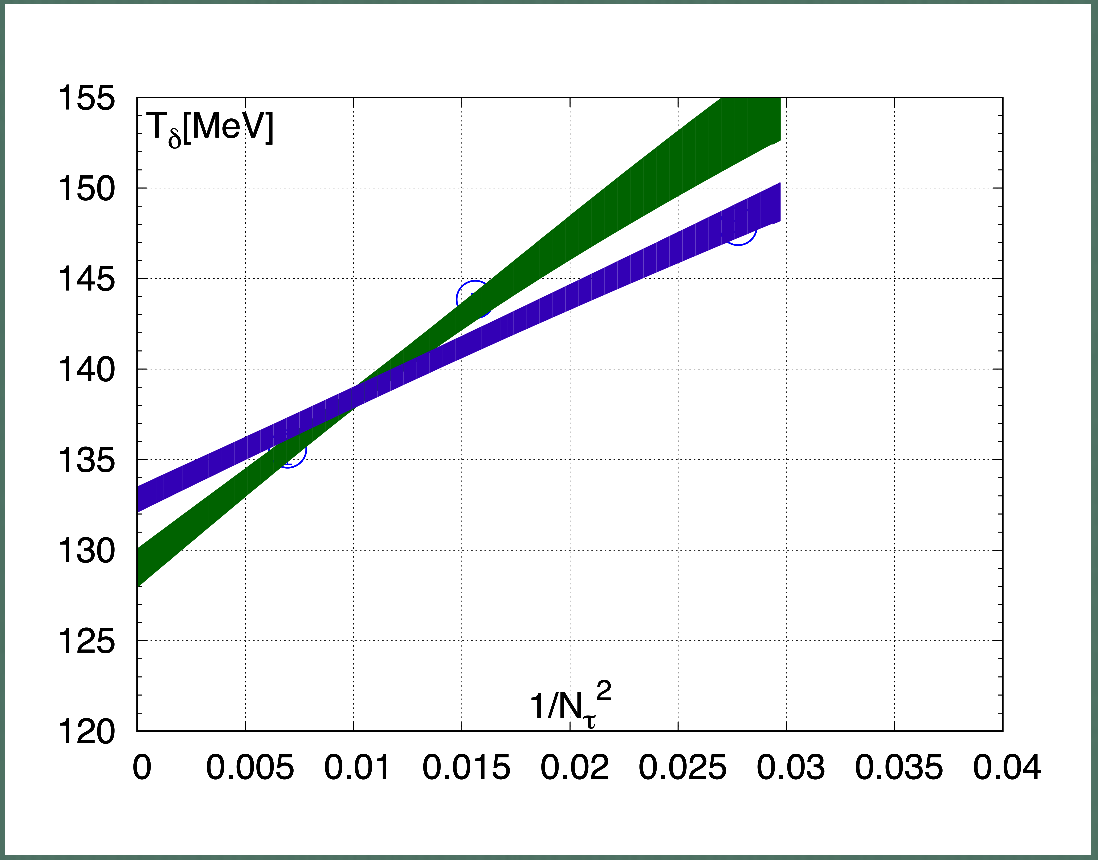
O(4) Chiral limit

$$T_\delta(H, L \to \infty) = T_c^0 + c_\delta H^{1-1/\delta+1/\beta\delta}$$

- ⭐ **T_c^0=131.6 +/- 0.9 MeV, by Nt = 6,8,12**
- ⭐ **T_c^0=128.6 +/- 1.7 MeV, by Nt = 8,12**

Thermodynamic limit+Chiral limit → Continuum limit

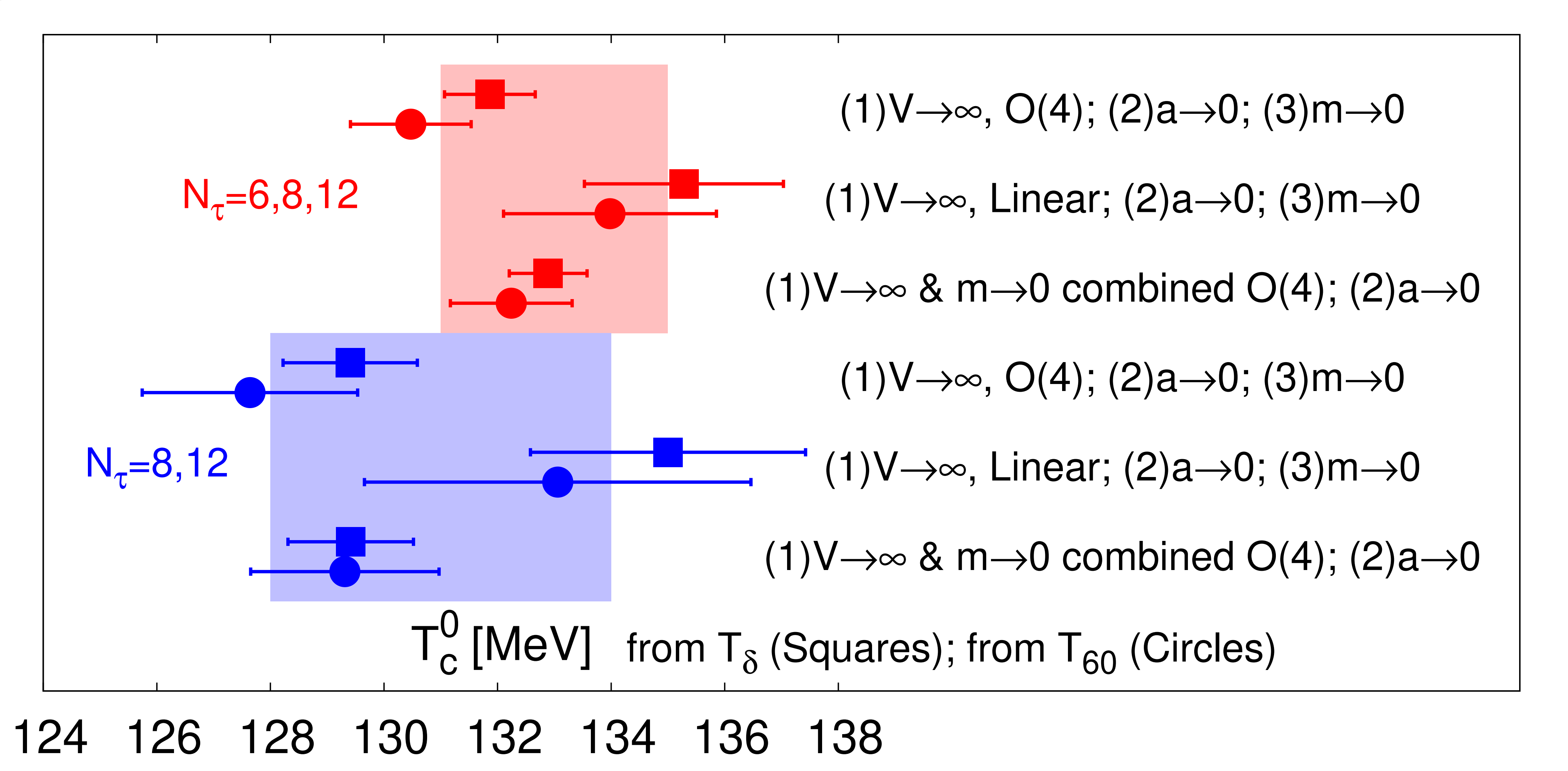




$$T_\delta(H,L) = T_c^0 + \frac{H^{1/\beta\delta(1-5.7v)}}{L^{5.7}}B + c_\delta H^{1-1/\delta+1/\beta\delta}, \quad v = 0.7377(41)$$

- T_c^0=132.8 +/- 0.9 MeV, by Nt = 6,8,12
- T_c^0=128.9 +/- 1.3 MeV, by Nt = 8,12

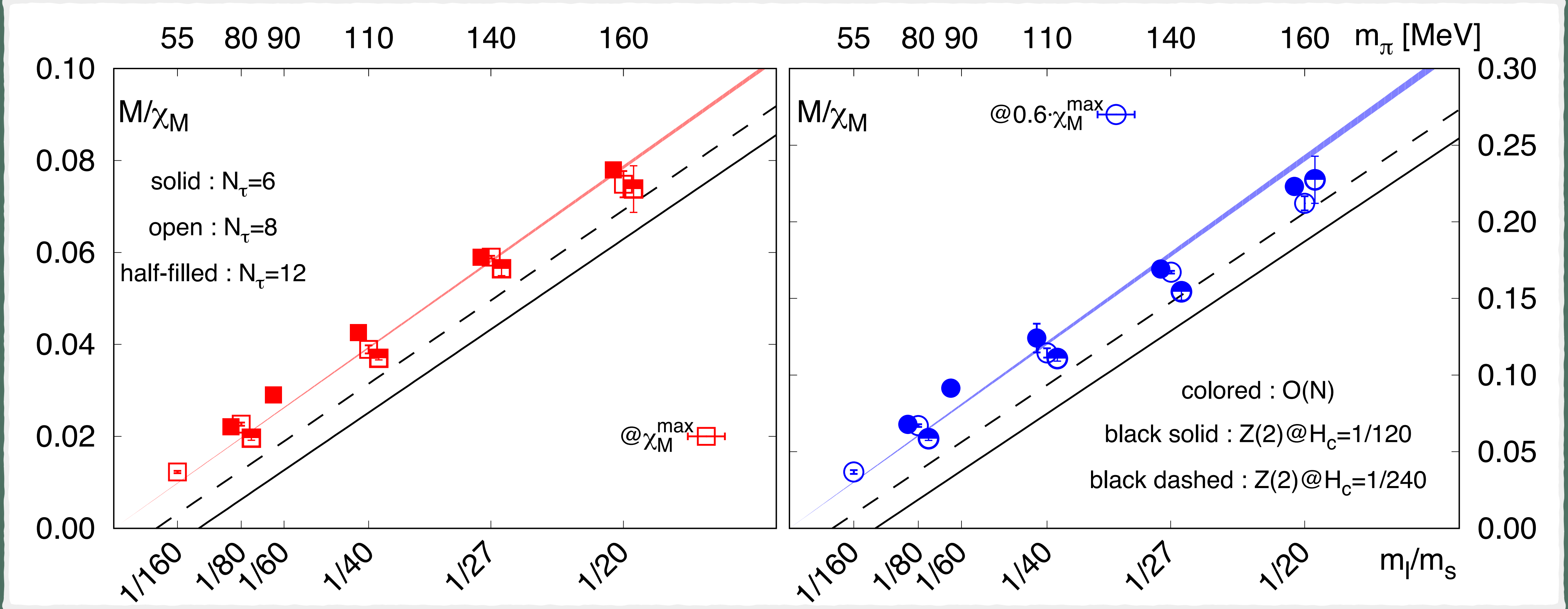
Final estimation of T_c^0



We present our Final estimate: $T_c^0 = 132^{+3}_{-6}$ **MeV**

HotQCD Collaboration: H.T. Ding, …, S.T. Li et al., Phys.Rev.Lett. 123 (2019) 062002

Order of chiral phase transition in the chiral limit



HotQCD Collaboration: Heng-Tong Ding, …, Sheng-Tai Li et al., arXiv:1905.11610, PoS LATTICE (2018) 171

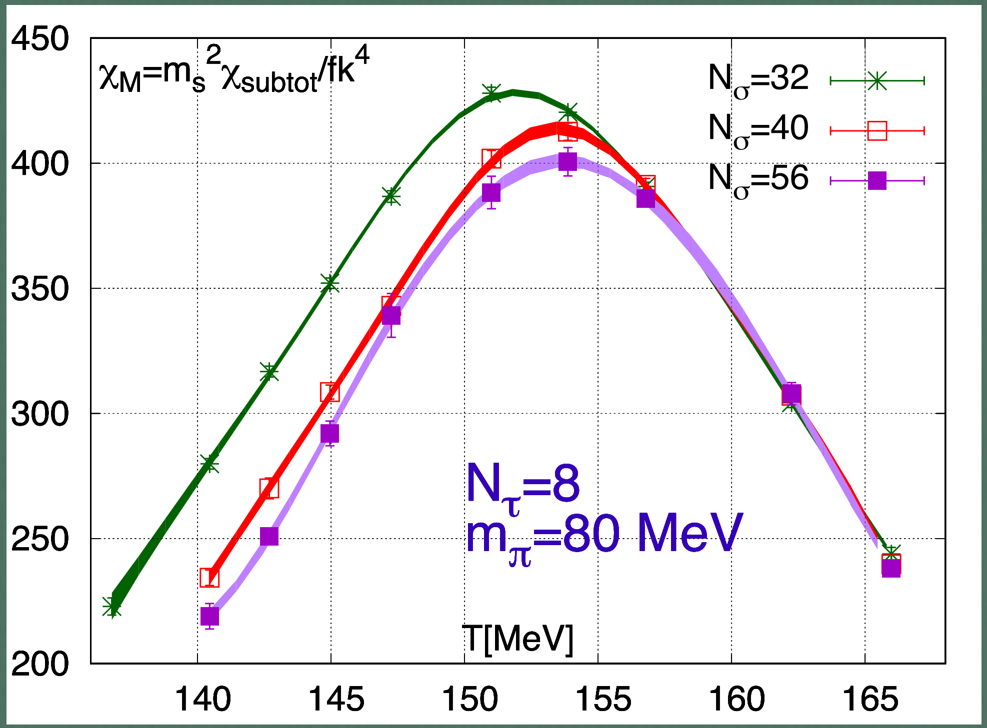
For Z(2) case:

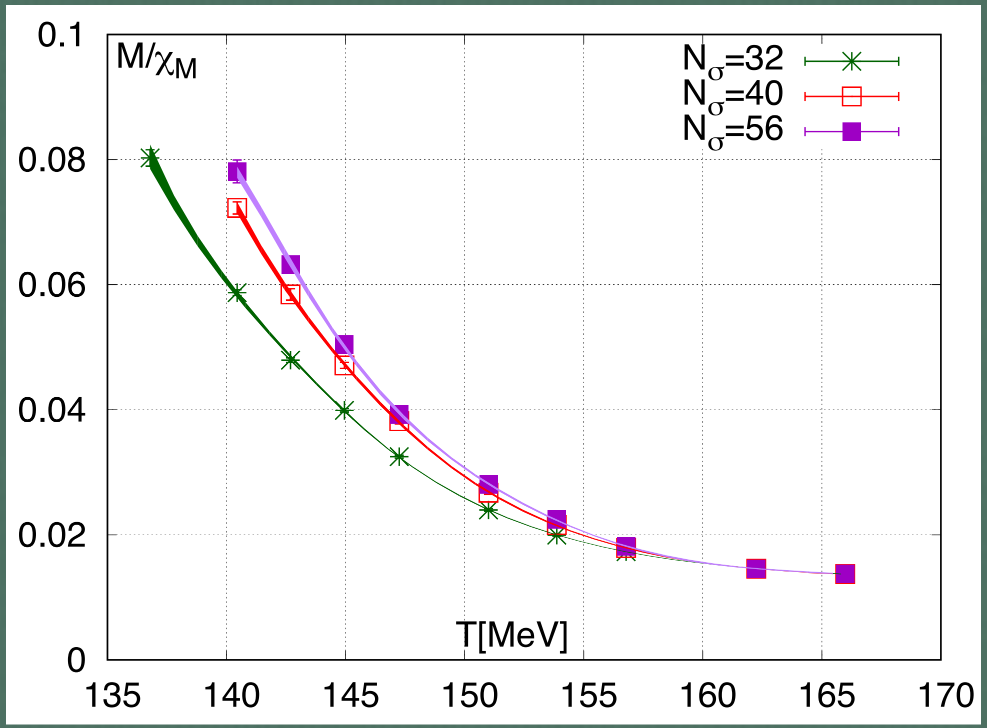
$$\frac{M}{\chi_M} = \frac{m_l - m_c}{m_s} \frac{f_G(z)}{f_\chi(z)}$$

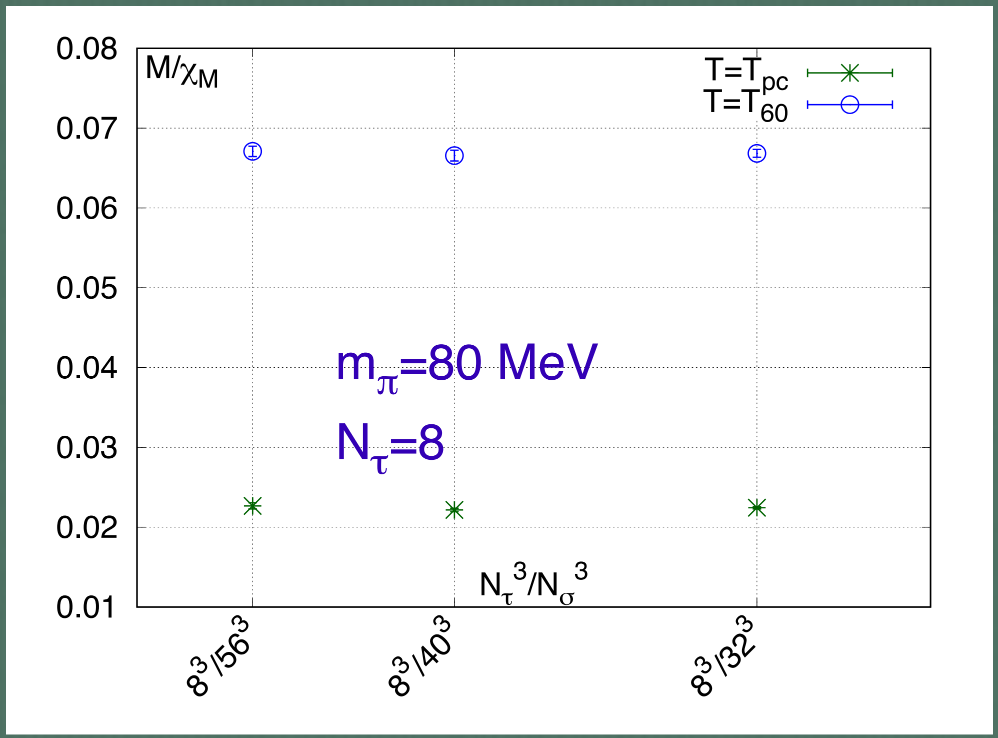
$f_G(z)/f_\chi(z)$ at $z \simeq 0$ and z_p is a number fixed by universality class

☑ **Second order O(4)**

Volume dependence of M/χ_M at T_{pc} and T_{60}







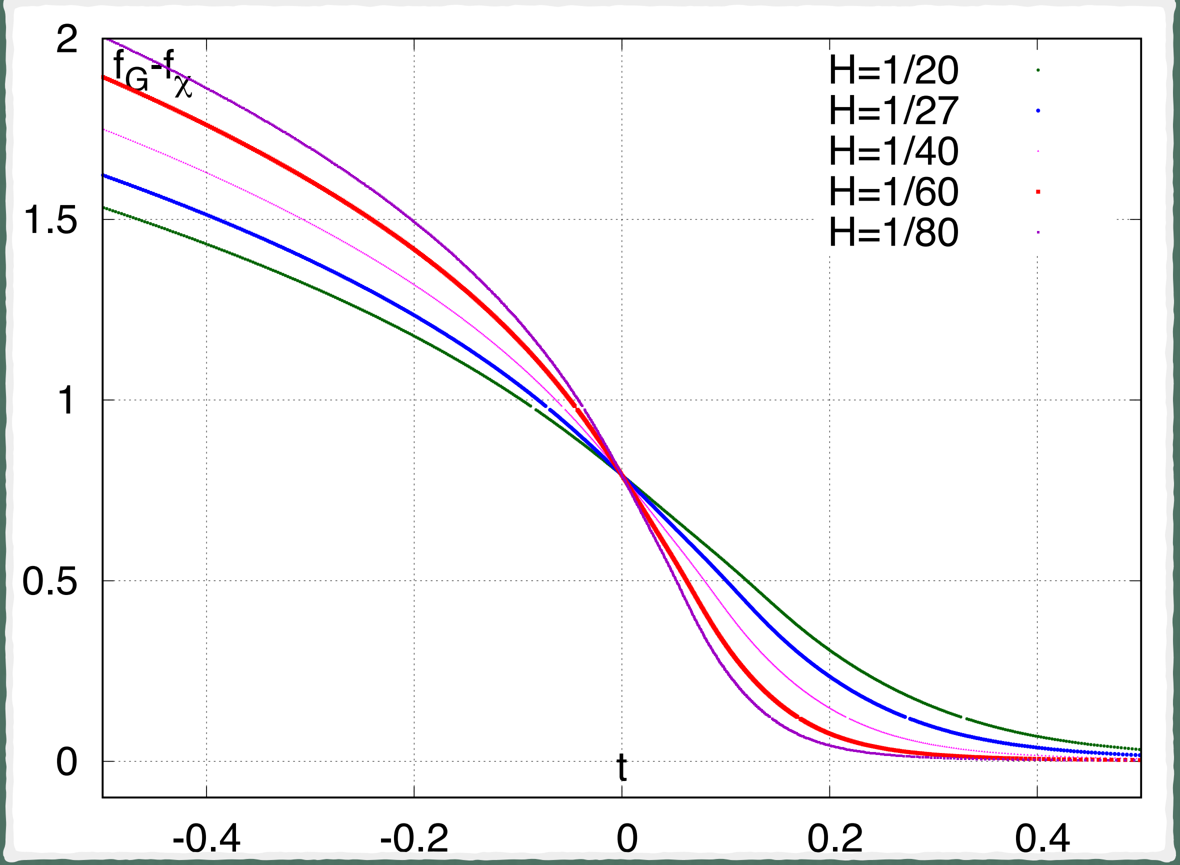
Thanks for your attention !

Backup

Novel estimators $f_G(z)-f_\chi(z)$ to determine T_c^0

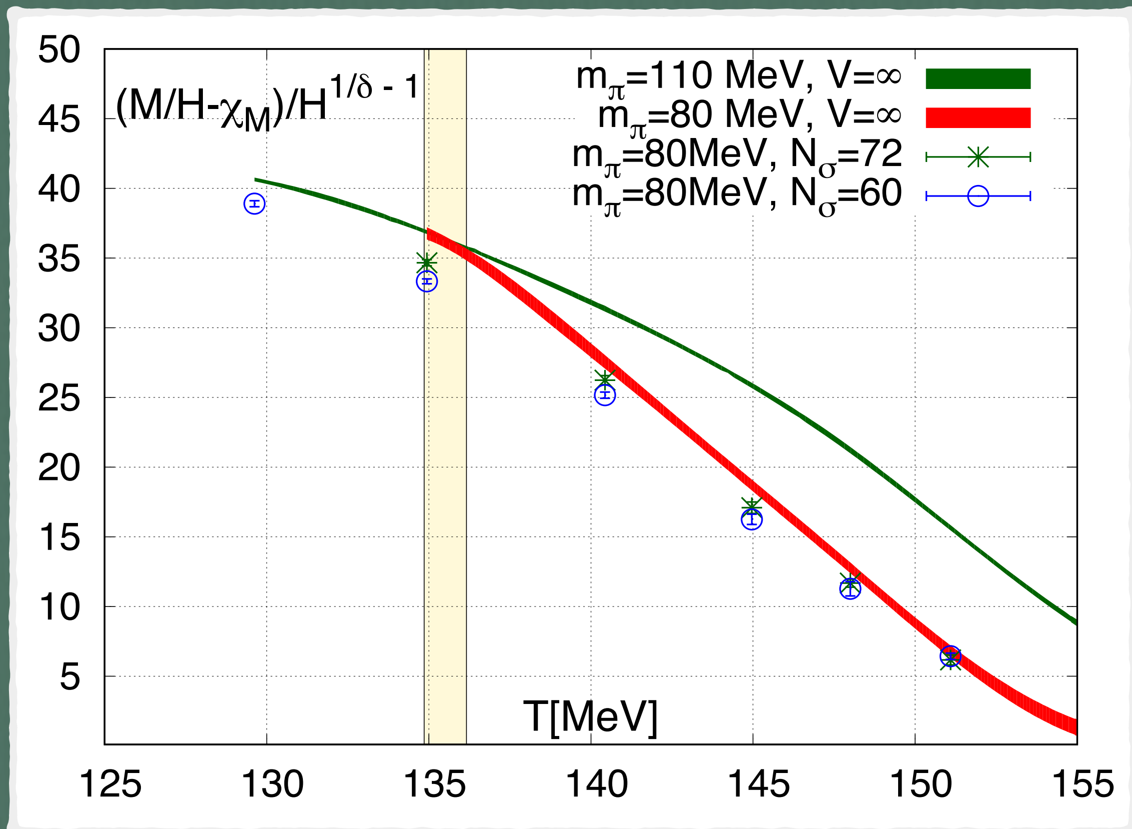
$$M/H - \chi_M = \left(H^{-1}h^{1/\delta}f_G(z) + a_1(T)\right) - \left(H^{-1}h^{1/\delta}f_\chi(z) + a_1(T)\right)$$
$$= H^{-1}h^{1/\delta}\left(f_G - f_\chi\right)$$

$$\left(M/H - \chi_M\right)/H^{1/\delta-1} = \text{constant (at } T_c^0)$$

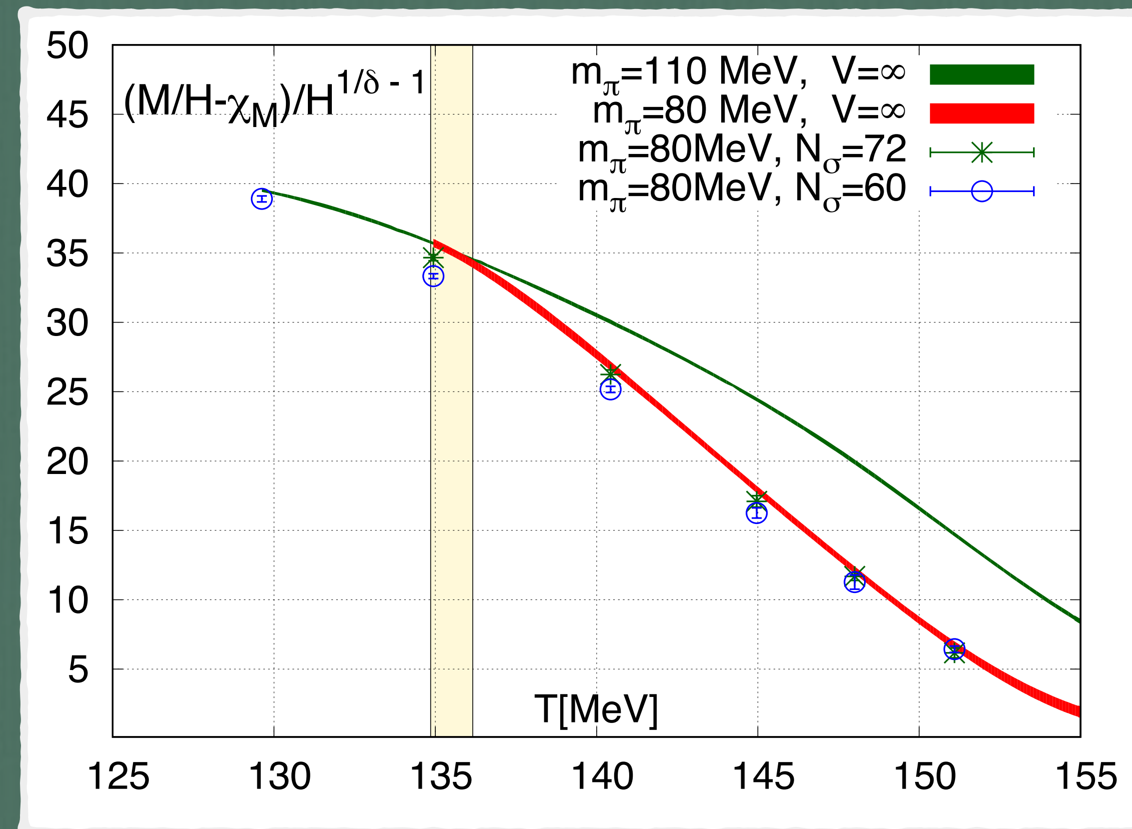


- For O(4), Z(2), 1/δ is only 0.4% difference
- Linear quark mass terms in M are subtracted

Sanity check by looking at $f_G(z)-f_\chi(z)$



Linear thermodynamic limit



O(4) thermodynamic limit

The crossing point located in the T_c^0 range we have estimated