Sea quark unpolarized GPD($\xi = 0$) and Sivers function in proton with chiral Lagrangian

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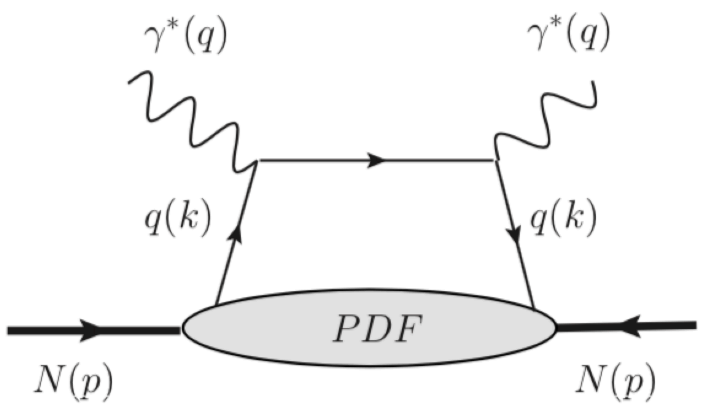
Outline

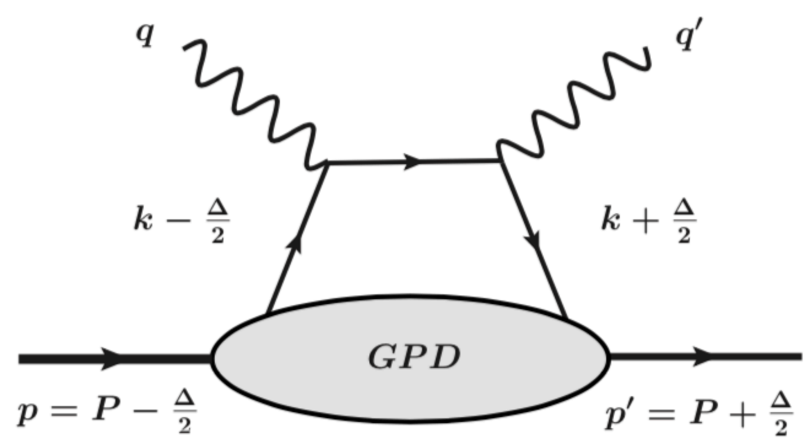
- Generalized parton distribution
  - Convolution formula for GPD
  - Numerical results (GPD, PDF, Form Factor)
- Sivers function
  - Gauge invariant bilocal pion operator
  - Convolution formula for TMD
  - Numerical result (The first momentum of Sivers function)
- Summary and outlook

Generalized parton distribution

Deeply virtual Compton scattering

Forward Compton scattering → Deeply virtual Compton scattering(Ji, PRD55(1997))





Parameters in GPDs:

The square of transfer momentum: $t = \Delta^2 = (p' - p)^2$

Skewness: $\xi = -\frac{\Delta \cdot n}{2P \cdot n} = -\frac{\Delta^+}{2P^+}$

Average parton momentum fraction: $x = \frac{k \cdot n}{P \cdot n} = \frac{k^+}{P^+}$

Unpolarized quark GPDs of nucleon

Unpolarized quark GPDs of nucleon:

$$V^q = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ix(P\cdot z)} \langle p'|\, \bar{q}(-\tfrac{1}{2}z)\, \tilde{\not{n}} q(\tfrac{1}{2}z)\, |p\rangle \Big|_{z=\lambda n}$$

$$= \frac{1}{2P\cdot n}\left[H^q(x,\xi,t)\, \bar{u}(p')\tilde{\not{n}} u(p) + E^q(x,\xi,t)\, \bar{u}(p')\frac{i\sigma^{\beta\alpha}\Delta_\alpha \tilde{n}_\beta}{2m} u(p)\right],$$

Parton distribution function ($\xi = t = 0$):

$$H^q(x>0,0,0) = q(x), \qquad H^q(x<0,0,0) = -\bar{q}(-x)$$

Dirac and Pauli Form factor:

$$F_1^q(t) = \int_{-1}^{1} dx\, H^q(x,\xi,t), \qquad F_2^q(t) = \int_{-1}^{1} dx\, E^q(x,\xi,t)$$

Electromagnetic Form factor:

$$G_E^q = F_1^q(t) - \frac{t}{4m_p^2}F_2^q(t), \qquad G_M^q = F_1^q(t) + F_2^q(t)$$

Convolution formula($\xi = 0$)

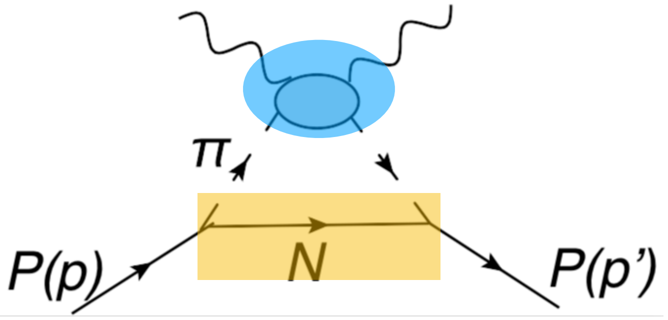
Matrix element for GPDs in quark level:

$$V_q = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix(Pz)} < p'|O_q|p > |_{z=\lambda n}, \quad \text{where} \quad O_q = \bar{q}(-\tfrac{1}{2}z)\,\tilde{\not{n}}\,q(\tfrac{1}{2}z)$$

Match to hadron level

Splitting functions

Sullivan Process



$$V_q = \frac{1}{2}\int_0^1 dy\,\theta(0 \le \frac{x}{y} \le 1)\, q_H^v(\frac{x}{y},0,t) \int \frac{dz^-}{2\pi} e^{iy(Pz)} < p'|O_H|p >$$

where O_H is twist-2 hadron operator, $q_H^v(\frac{x}{y},0,t)$ is quark valance GPD in hadron.

quark valance GPD

One loop Feynman diagrams

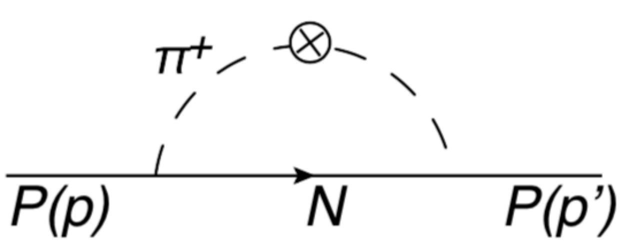
Hadron operator for π^+ : $O_{\pi^+} = \pi^-(\frac{z}{2})\partial^+\pi^+(-\frac{z}{2}) - \partial^+\pi^-(\frac{z}{2})\pi^+(-\frac{z}{2})$

The hadron matrix element can be written as

$$V_H = \int \frac{dz^-}{2\pi} e^{iy(Pz)} < p' | O_{\pi^+} | p > \; = \bar{u}(p') \left\{ \gamma^\mu f(y,0,t) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} g(y,0,t) \right\} u(p),$$

Splitting functions

Rainbow diagram



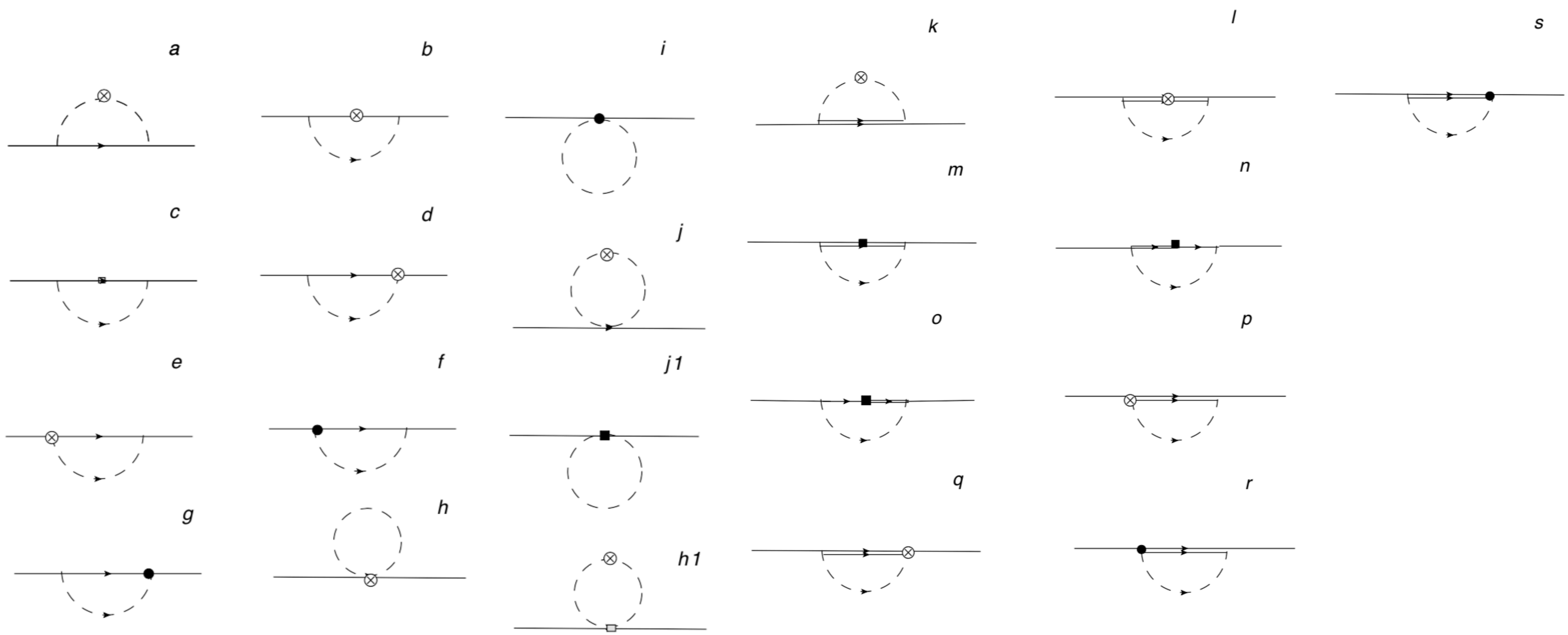
For sea quark GPDs in the proton can be obtained by convolution formalism:

$$H^{\bar{d}}(x,0,t) = \int_x^1 \frac{dy}{y} H^{\bar{d}}_{\pi^+}(\frac{x}{y},0,t) f(y,0,t), \qquad E^{\bar{d}}(x,0,t) = \int_x^1 \frac{dy}{y} H^{\bar{d}}_{\pi^+}(\frac{x}{y},0,t) g(y,0,t)$$

$H^{\bar{d}}_{\pi^+}(\frac{x}{y},0,t)$ is the valance quark GPD in π^+.

To get rid of UV divergence, we use a dipole regulator: $\tilde{F}(k) = \left(\frac{\Lambda^2 - M_\phi^2}{\Lambda^2 - k^2}\right)^2$

Feynman diagrams for splitting functions



One-loop contributions to the proton splitting functions. The double-solid, solid, dashed and are for the octet, decuplet baryons and pseudoscalar mesons, respectively. The rectangle and blackdot represent magnetic and additional interaction vertex. cross represents an operator insertion.

Quark valance GPDs

PDF flavour symmetry(X.G.Wang et.al ,PRD(2016)):

Valance quark in mesons:

$$\bar{s}_{K^0}(x) = \bar{s}_{K^+}(x) = \bar{d}_{\pi^+}(x) = \bar{u}_{\pi^-}(x)$$

Valance quark in Baryons:

$$\begin{aligned} s_\Lambda(x) &= \frac{1}{3}\Big[2u(x) - d(x)\Big] \\ s_{\Sigma^+}(x) &= s_{\Sigma^0}(x) = d(x) \\ s_{K^+}^{(\mathrm{tad})}(x) &= \frac{1}{2}u(x), \quad s_{K^0}^{(\mathrm{tad})}(x) = d(x) \\ s_{\Sigma^{*,0}}(x) &= s_{\Sigma^{*,+}}(x) = \frac{1}{3}\Big[2u(x) - d(x)\Big] \end{aligned}$$

quark disrtibutions for KR diagrams:

$$s_{\Sigma^+}^{(KR)}(x) = s_{\Sigma^0}^{(KR)}(x) = \frac{\Delta d(x)}{F-D}, s_\Lambda^{(KR)}(x) = \frac{2\Delta u(x) - \Delta d(x)}{3F+D},$$

$$s_{\Sigma^{*+}}^{(KR)}(x) = s_{\Sigma^{*0}}^{(KR)}(x) = \frac{2\Delta d(x) - \Delta u(x)}{-2D},$$

Extend to GPDs

GPD flavour symmetry:

Valance quark in mesons:

$$H_{K^+}^{\bar{s}} = H_{K^0}^{\bar{s}} = H_{\pi^+}^{\bar{d}} = H_{\pi^-}^{\bar{u}}(x,0,t)$$

Valance quark in Baryons:

$$\begin{aligned} H_\Lambda^s(x,0,t) &= \frac{1}{3}\Big[2H_p^u(x,0,t) - H_p^d(x,0,t)\Big], \\ E_\Lambda^s(x,0,t) &= \frac{1}{2\kappa_u - \kappa_d}\Big[2E_p^u(x,0,t) - E_p^d(x,0,t)\Big], \end{aligned}$$

....

quark GPDs for KR diagrams:

$$\tilde{H}_\Lambda^{s(\mathrm{KR})}(x,0,t) = \frac{1}{D+3F}\Big[2\tilde{H}_p^u(x,0,t) - \tilde{H}_p^d(x,0,t)\Big],$$

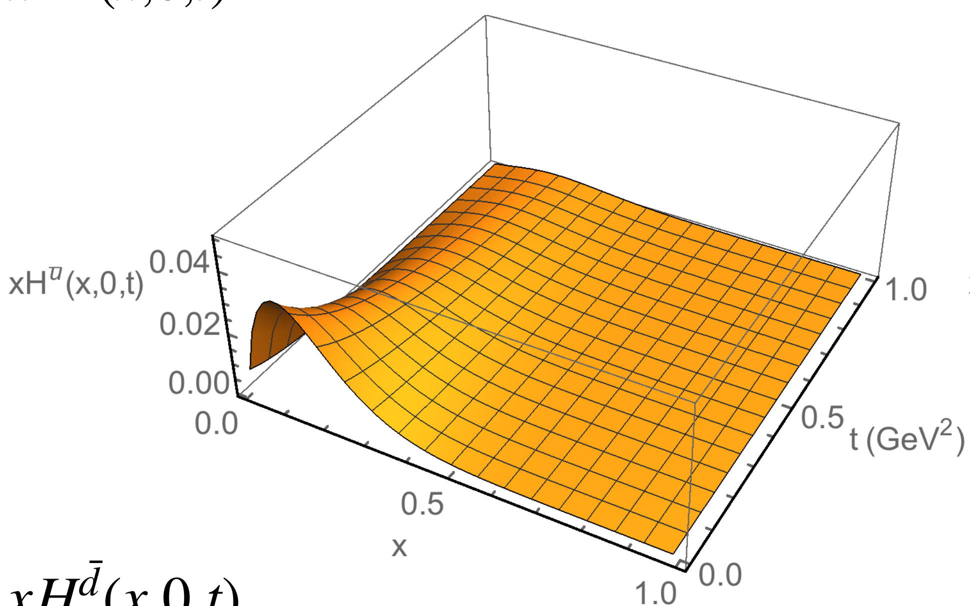
....

quark GPDs for transition diagrams:

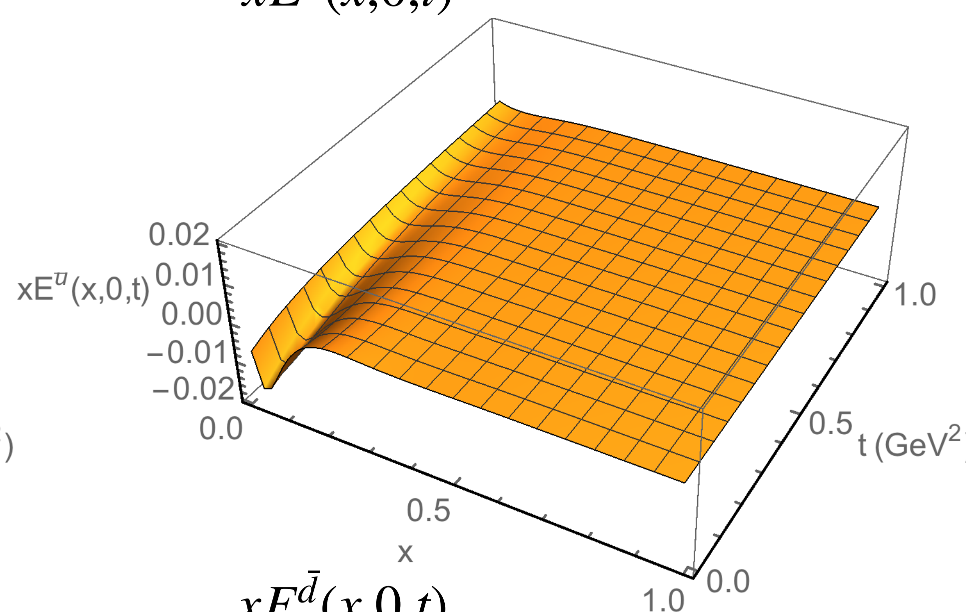
$$E_T^s(x,0,t) = \frac{1}{\kappa_u - 2\kappa_d}\Big[E_p^u(x,0,t) - 2E_p^d(x,0,t)\Big],$$

Numerical results($\xi = 0$)

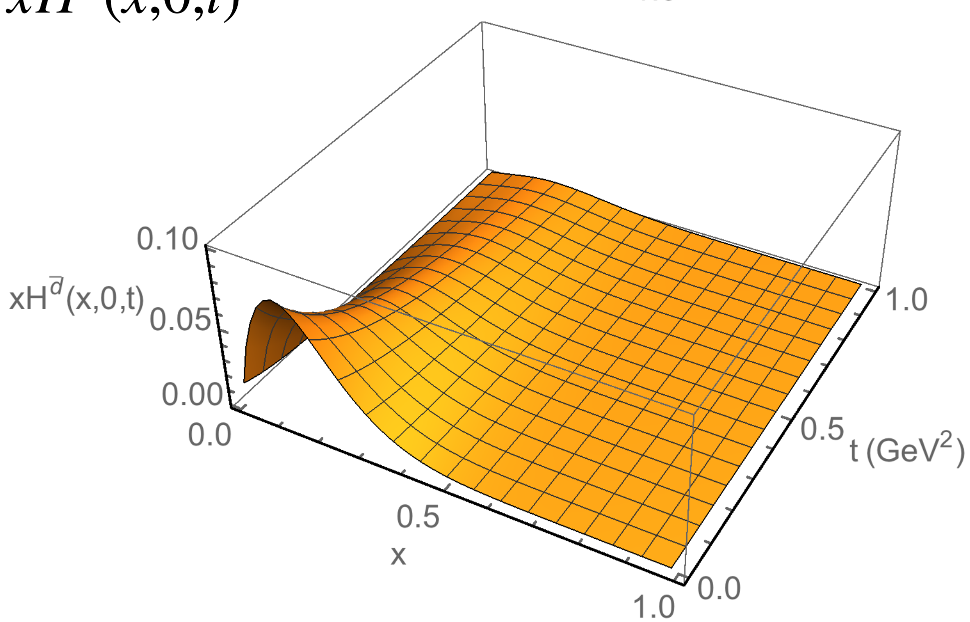
$xH^{\bar{u}}(x,0,t)$:



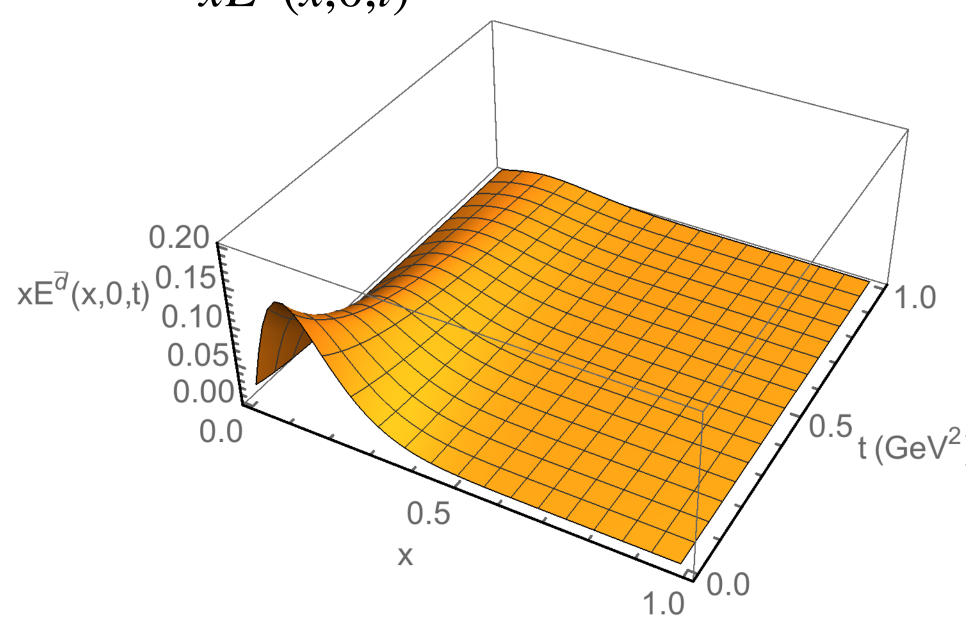
$xE^{\bar{u}}(x,0,t)$



$xH^{\bar{d}}(x,0,t)$

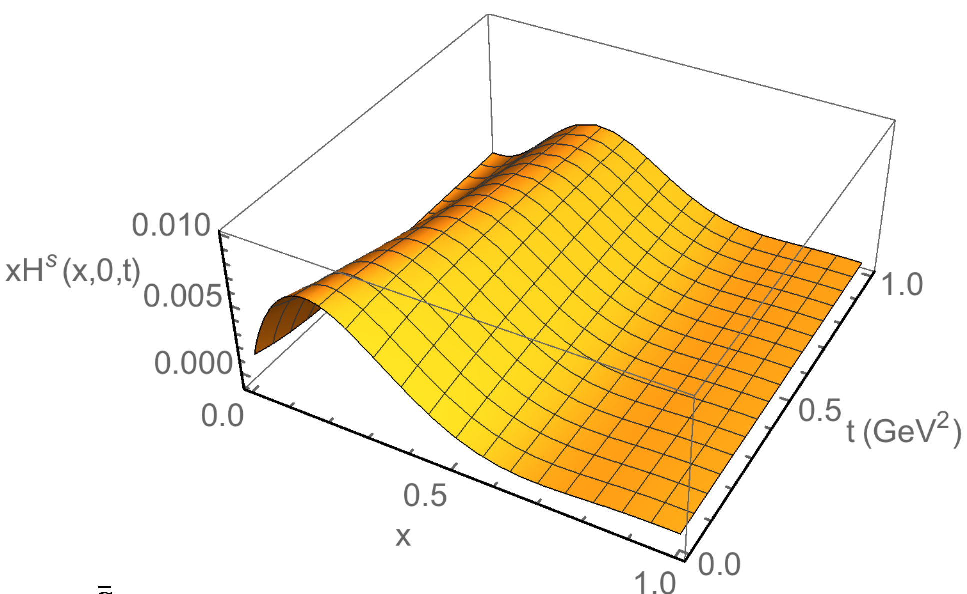


$xE^{\bar{d}}(x,0,t)$

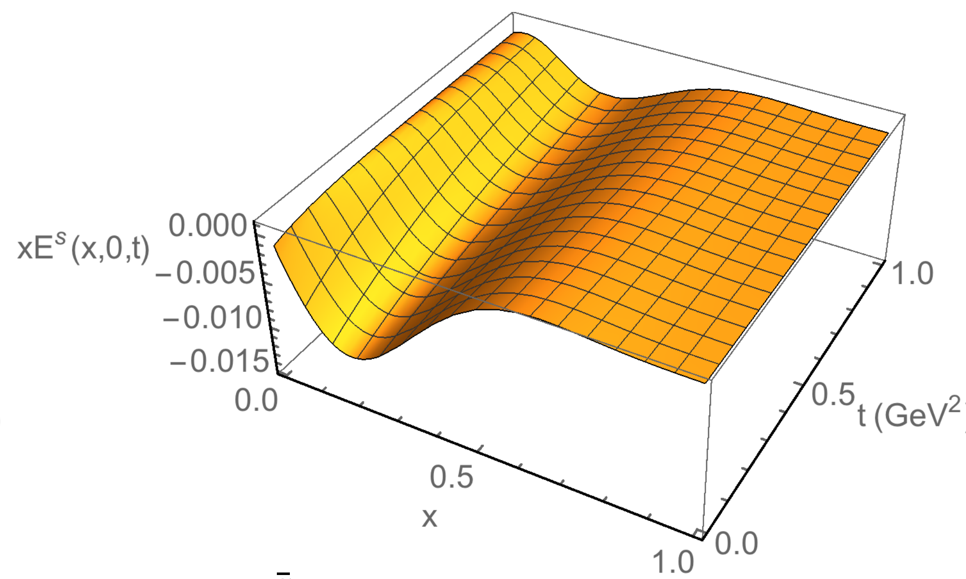


Numerical results($\xi = 0$)

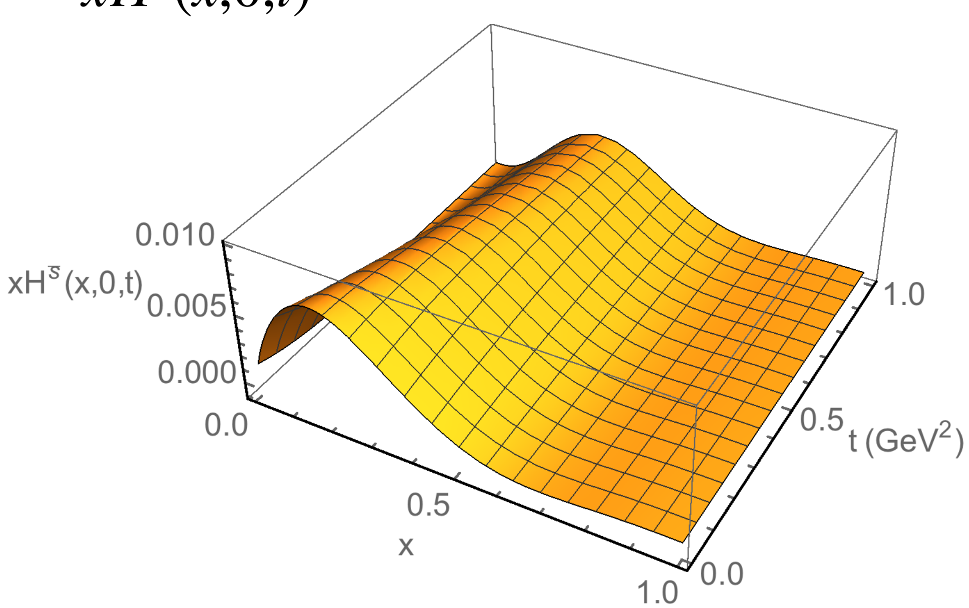
$xH^{S}(x,0,t)$



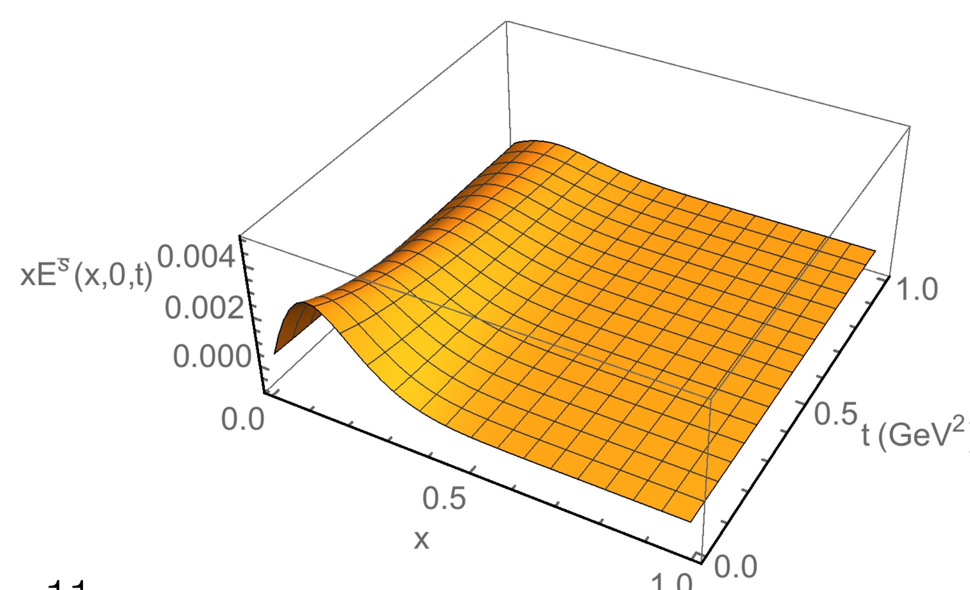
$xE^{S}(x,0,t)$



$xH^{\bar{S}}(x,0,t)$

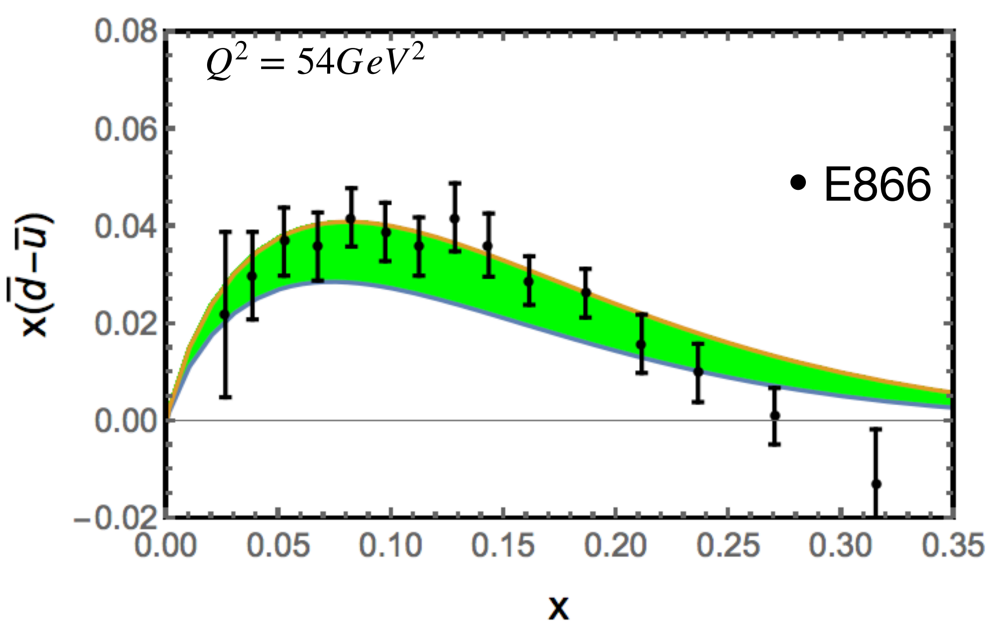


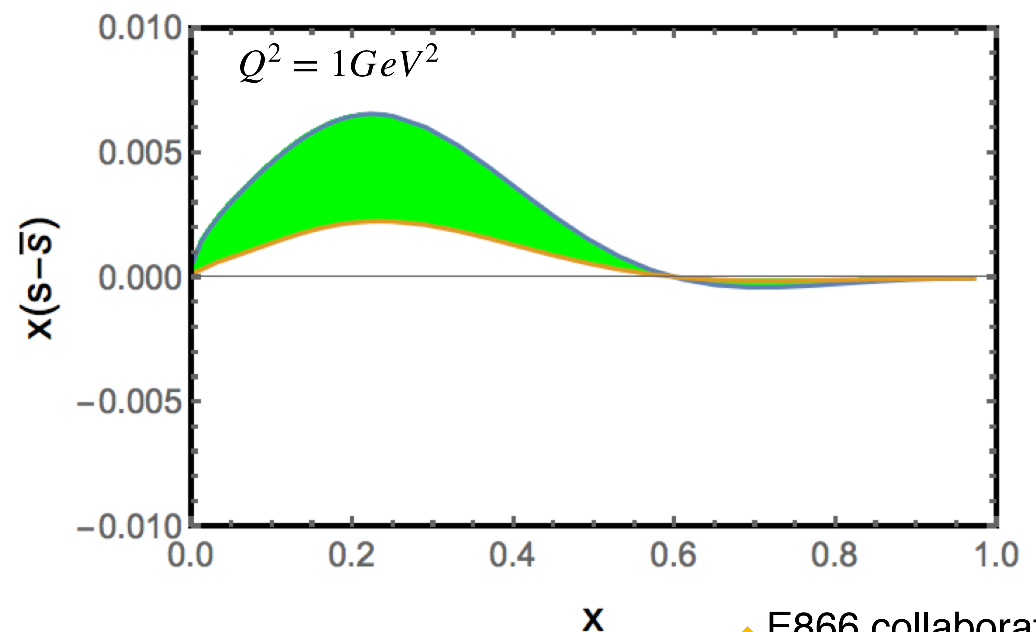
$xE^{\bar{S}}(x,0,t)$



Numerical results(sea quark asymmetry)

$\xi = t = 0 \; (0.9GeV \leq \Lambda \leq 1.1GeV, Q_0^2 = 1GeV^2)$





The lowest momentum:

$$< \bar{d} - \bar{u} > = \int_0^1 (\bar{d}(x) - \bar{u}(x))dx = 0.090 - 0.131,$$

E866 collaboration

$$< \bar{d} - \bar{u} > = 0.118(12)$$

The first momentum for strange quark:

$$< xs^- > = < x(s - \bar{s}) > = \int_0^1 x(s(x) - \bar{s}(x))dx = 0.0007 - 0.0021,$$

NNPDF at Q = 1GeV : $< xs^- > = 0.0009 - 0.0053.$

W. Bentz, et.al,Physics Letters B 693 (2010) 462–466

	$\langle xs^- \rangle$
Mason et al. [8]	0.00196 ± 0.00143
NNPDF [9]	0.0005 ± 0.0086
Alekhin et al. [30]	$0.0013 \pm 0.0009 \pm 0.0002$
MSTW [31]	$0.0016^{+0.0011}_{-0.0009}$
CTEQ [32]	$0.0018^{+0.0016}_{-0.0004}$
This work (Eq. (10))	0.0 ± 0.0020

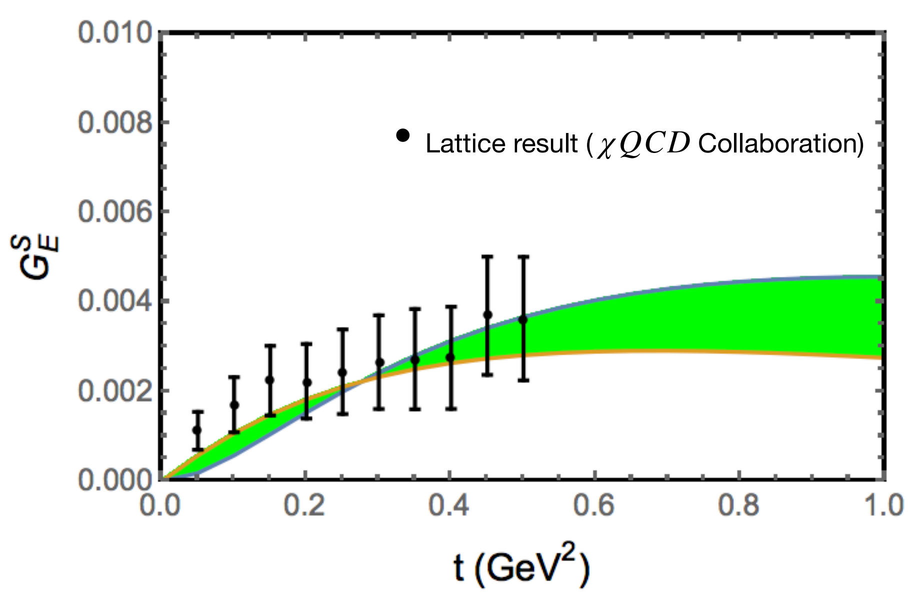
Numerical results(form factors)

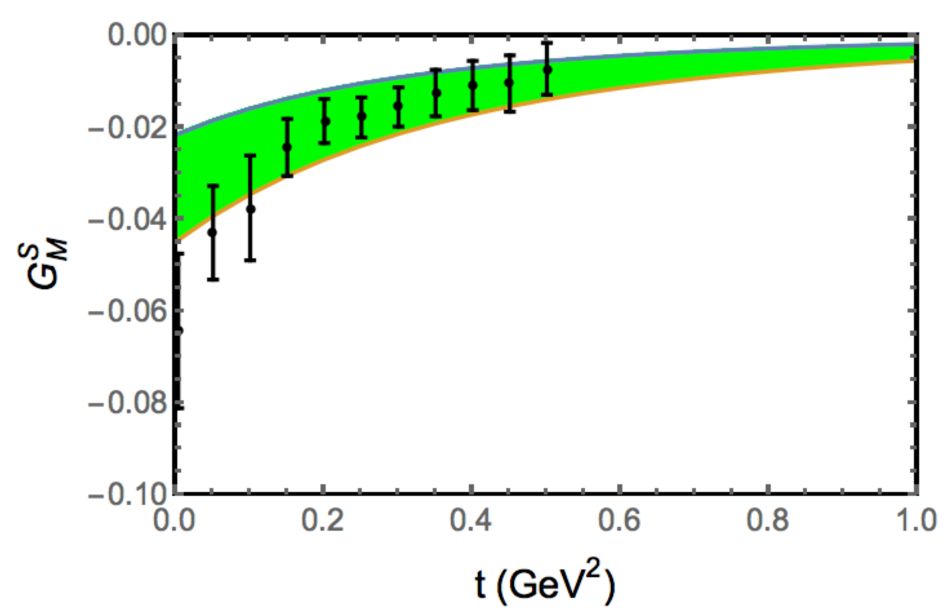
Strange Dirac and Pauli Form factor:

$$F_1^S = \int_0^1 \{H^S(x,0,t) - H^{\bar{S}}(x,0,t)\}dx \qquad F_2^S = \int_0^1 \{E^S(x,0,t) - E^{\bar{S}}(x,0,t)\}dx$$

Strange Electromagnetic Form factor:

$$G_E^S = F_1^S(t) - \frac{t}{4m_p^2}F_2^S(t) \qquad G_M^S = F_1^S(t) + F_2^S(t)$$

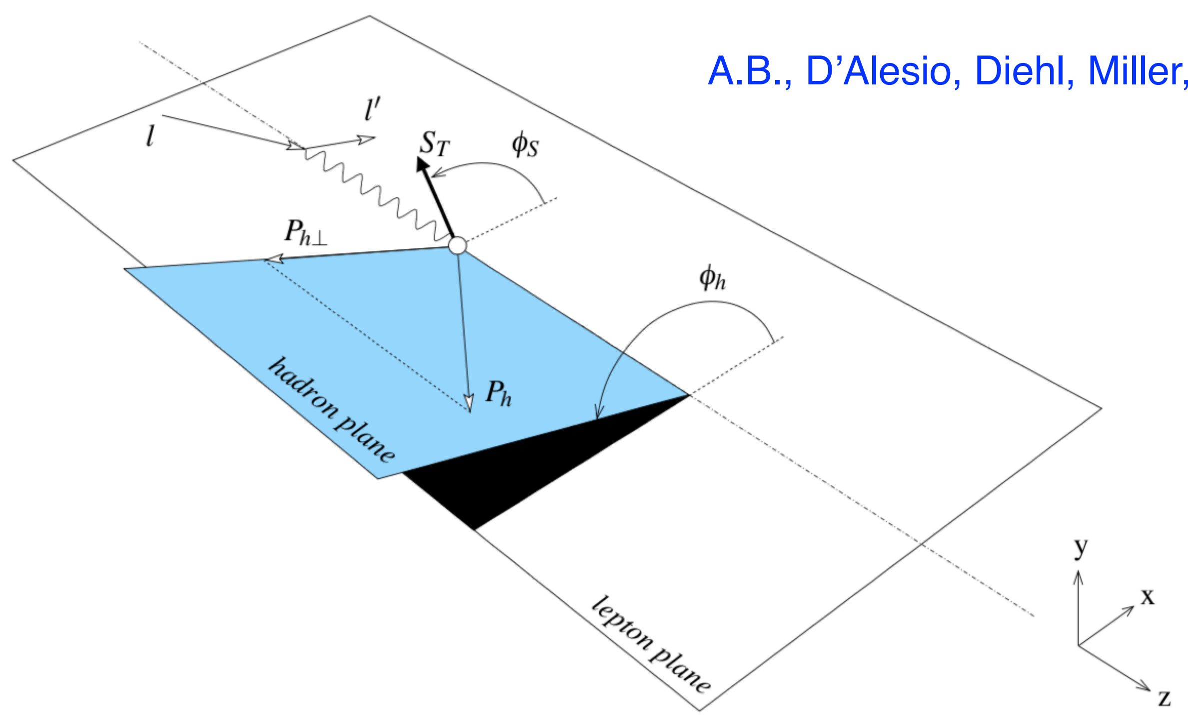




Sivers function

Single spin asymmetry in SIDIS

Semi-inclusive DIS $\big(e(l) + N(P) \rightarrow e(l') + h(P_h) + X\big)$



A.B., D'Alesio, Diehl, Miller, PRD70 (04)

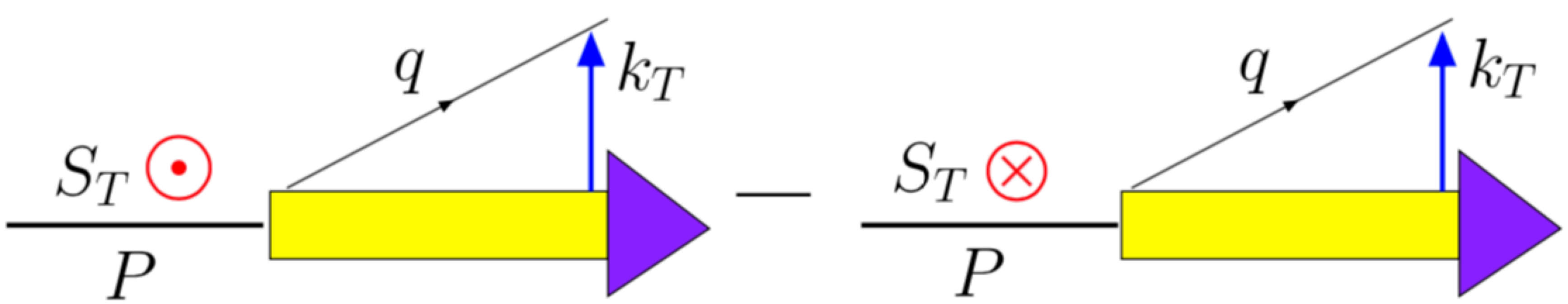
The SIDIS transverse single spin asymmetry can be defined as

$$A_{UT}^{\sin(\phi_h-\phi_S)} = 2\,\frac{\int d\phi_S\, d\phi_h\, [d\sigma^{\uparrow} - d\sigma^{\downarrow}]\,\sin(\phi_h - \phi_S)}{\int d\phi_S\, d\phi_h\, [d\sigma^{\uparrow} + d\sigma^{\downarrow}]}$$

Sivers function

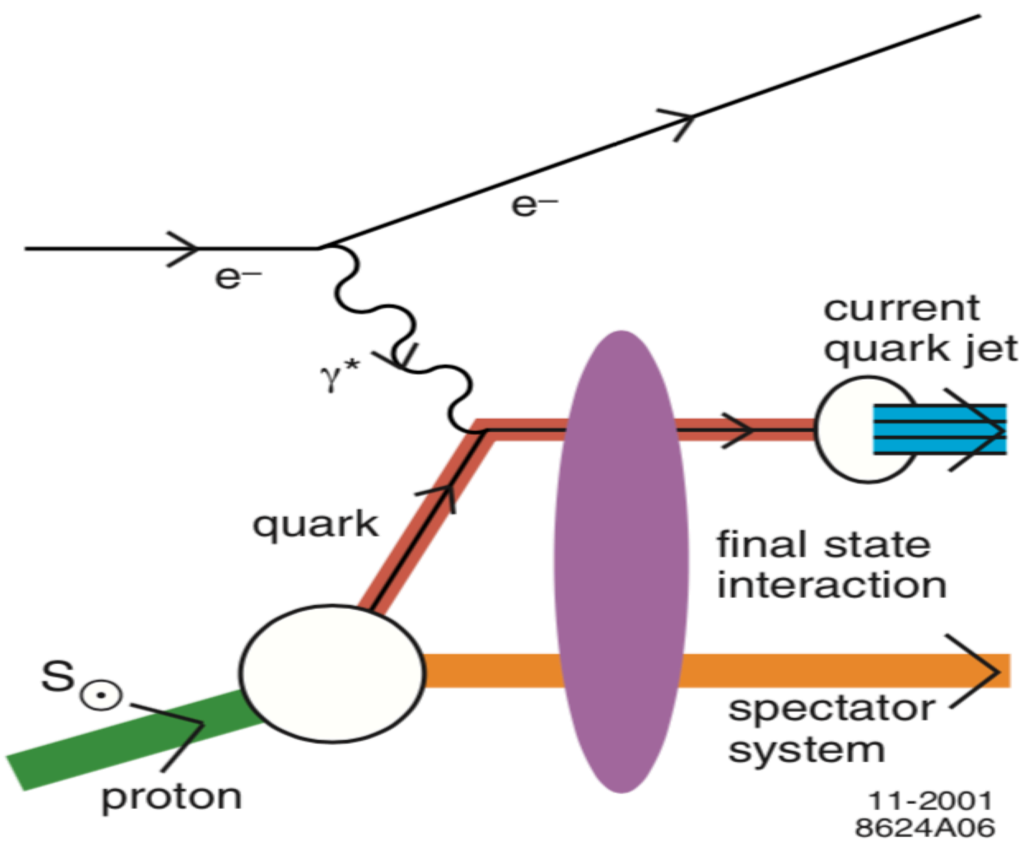
Sivers function is introduced by Sivers (D. Sivers,PRD(1990)) to explain the single spin asymmetry. It describes the difference between the transverse momentum distribution of unpolarized quark in the nucleons which transversely polarized in opposite direction.

Schematic depiction:



Final state interaction

BHS model (S. Brodsky, Dae Sung Hwang, and Ivan Schmidt, 2002)



The Sivers TMD is obtained by considering the one-gluon exchange between active quark and spectator quark.

$$f_{1T}^{\perp} \propto Im\Big(M[\gamma p(J_p^z = \frac{1}{2}) \rightarrow F]^* M[\gamma p(J_p^z = -\frac{1}{2}) \rightarrow F]\Big);$$

and

$$M[\gamma^* p(J_p^z = \pm\frac{1}{2}) \rightarrow F] = M[\gamma^* p(J_p^z = \pm\frac{1}{2}) \rightarrow F]_L e^{i\chi_\pm}$$

Gauge invariant bilocal quark operator

The gauge invariant parton TMD is defined as (Ji and Yuan,PLB543(2002))

$$f_1^q(x,\vec{k}_\perp) - \frac{\epsilon^{ij}k_\perp^i S_\perp^j}{m_p} f_{1T}^{\perp q}(x,\vec{k}_\perp) = \frac{1}{2}\int \frac{d\xi^- d^2\vec{\xi}_\perp}{(2\pi)^3} e^{-ixP^+\xi^- + i\vec{k}_\perp\cdot\vec{\xi}_\perp} \langle P,\vec{S}_\perp \mid \mathcal{O}^q \mid P,\vec{S}_\perp\rangle$$

Where $\mathcal{O}^q = \bar{q}(\xi^-,\vec{\xi}_\perp)\mathcal{L}^\dagger_{\xi_\perp}(\infty,\xi^-)\gamma^+\mathcal{L}_0(\infty,0)q(0,0),$

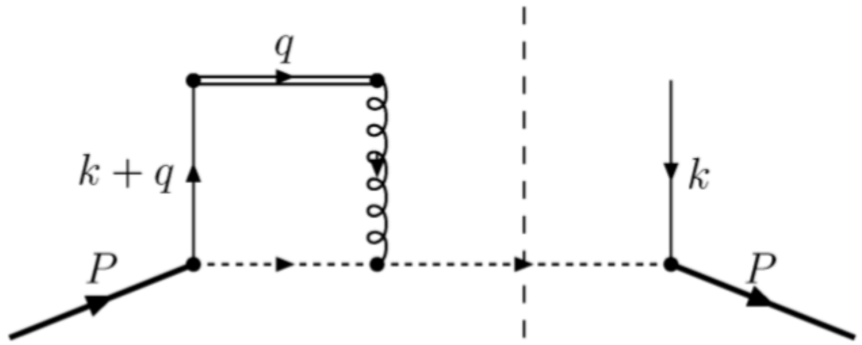
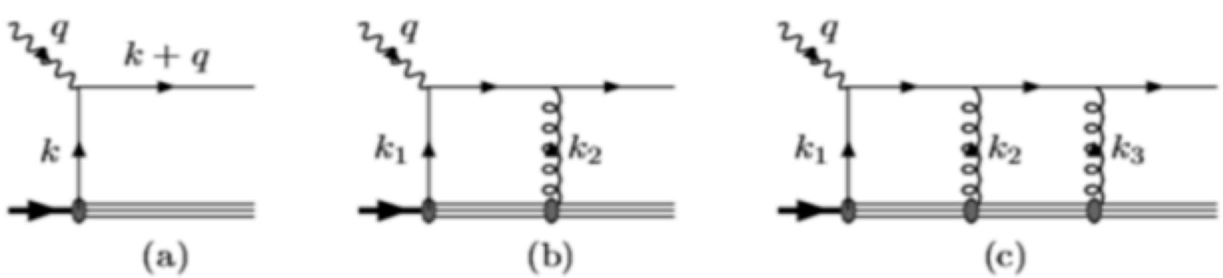


Fig. 1. One-loop contribution to the spin-dependent transverse-momentum distribution in the nucleon.

The gauge link is from final-state inetraction

J. Collins, Phys.Lett. B(2002)
A.V. Belitsky, X. Ji, F. Yuan, Nucl.Phys. B(2002)
X. Ji, F. Yuan, Phys.Lett. B543(2002)
D. Boer, P.J. Mulders and F. Pijlman, Nucl.Phys. B667 (2003)



Distribution function of pion in proton(splitting function)

For π^+, the bilocal operator can be defined as

$$\mathcal{O}^{\pi^+} = i\big[\pi^-(y^-, \vec{y}_\perp)\partial^+\pi^+(0) - \partial^+\pi^-(y^-, \vec{y}_\perp)\pi^+(0)\big]$$

Compare with the quark bilocal operator O_q, the gauge link is missing. It is a Time-reversal even operator and never contributes to T-odd Sivers function, means that the Sivers function of a pion in proton is zero. On the other hand, pion is color singlet, gluon can not be introduced as the gauge link.

Hidden symmetry

The lowest order Lagrangian in CHPT

$$\mathcal{L} = \frac{1}{4} f_\pi^2 \,\mathrm{tr}((D^\mu U)^\dagger (D_\mu U)), \quad U = \exp\left(2i \frac{\pi^a T^a}{f_\pi}\right),$$

The chiral field U transforms as $U \to LUR^+$.

BKUYY, Phys.Rev.Lett. 54 (1985)

the chiral Lagrangian can be written in another form: $\mathcal{L} = f_\pi^2 Tr[\alpha_\mu \alpha^\mu],$

Where α_μ is defined as $\alpha_\mu = \frac{1}{2i}(D_\mu \xi_L \cdot \xi_L^\dagger - D_\mu \xi_R \cdot \xi_R^\dagger).$

Chiral covariant derivatives:

$$D_\mu \xi_{L/R} = \partial_\mu \xi_{L/R} + ig V_\mu \xi_{L/R} + i\xi_{L/R} L_\mu / R_\mu,$$

The Lagrangian is invariant under transformation:

$$\xi_L(x) \to h(x)\xi_L(x) g_L^\dagger, \quad \xi_R(x) \to h(x)\xi_R(x) g_R^\dagger,$$

$$V_\mu(x) \to ih(x)\partial_\mu h(x)^\dagger + h(x) V_\mu(x) h(x)^\dagger, \qquad (4)$$

$$V^\mu = \vec{\rho}^\mu \cdot \vec{\tau} = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 & \rho^+ \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 \end{pmatrix}^\mu .$$

h(x) is the gauge transformation for hidden symmetry.

Gauge invariant bilocal pion operator

Nonlocal action can be written as

He and Wang (1904.06815)

$$\mathcal{S} = \int dx\, dy f_\pi^2 Tr[\alpha_\mu(y) W(y,x) \alpha^\mu(x) W^\dagger(y,x)],$$

$$\alpha_\mu = \frac{1}{2i}(D_\mu \xi_L \cdot \xi_L^\dagger - D_\mu \xi_R \cdot \xi_R^\dagger).$$

$$W(y,x) = Pe^{-ig\int_x^y dz_\nu V^\nu(z)}$$

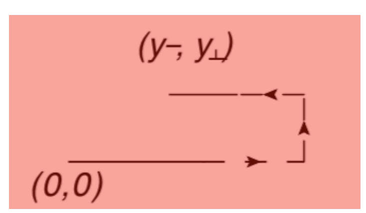
gauge link

$$D_\mu \xi_{L/R} = \partial_\mu \xi_{L/R} + igV_\mu \xi_{L/R} + i\xi_{L/R} L_\mu / R_\mu,$$

Traditional chiral Lagrangian: $\xi_L^\dagger = \xi_R = \xi = \exp(i\pi/f_\pi)$ and $L_\mu = R_\mu = \tau_q v_\mu$, where $\tau_q = \mathrm{diag}(\delta_{qu}, \delta_{qd})$

At the leading order, the current that couples to the external field can be written as

$$\mathcal{J}^\mu_{\bar{d}/\pi^+} = \mathcal{J}^\mu_{u/\pi^+} = i[\pi^-(y)\partial^\mu \pi^+(x) - \partial^\mu \pi^-(y)\pi^+(x)] \times \left[1 - i\sqrt{2}g \int_x^y dz_\nu \rho^{0\nu}(z)\right].$$



Change it to be bilocal opeator

$$\mathcal{O}^{\pi^+} = i\left[\pi^-(y^-, \vec{y}_\perp)\partial^+ \pi^+(0) - \partial^+ \pi^-(y^-, \vec{y}_\perp)\pi^+(0)\right].$$

$$\mathcal{O}^{\pi^+}_{Sivers} = +\sqrt{2}g\left[\pi^-(y^-, \vec{y}_\perp)\partial^+ \pi^+(0) - \partial^+ \pi^-(y^-, \vec{y}_\perp)\pi^+(0)\right]\left\{\int_0^\infty dz^- \rho^{0+}(z^-, 0) + \int_\infty^{y^-} dz^- \rho^{0+}(z^-, \vec{y}_\perp)\right\}.$$

Sivers function of pion in proton

The Sivers distribution function of a pion in proton $f_{1T}^{\pi/p}(z, \overrightarrow{k}_{\perp})$

$$-\frac{\epsilon^{ij} k^i_{\perp\pi} S^j_{\perp}}{m_p} f_{1T}^{\pi/p}(z, \vec{k}_{\perp\pi}) = \frac{1}{2} \int \frac{dy^- d^2\vec{y}_{\perp}}{(2\pi)^3} e^{-i(zP^+ y^- - \vec{y}_{\perp}\cdot\vec{k}_{\perp\pi})} \langle P, \vec{S}_{\perp} \mid \mathcal{O}^{\pi}_{Sivers} \mid P, \vec{S}_{\perp} \rangle$$

One loop diagrams:

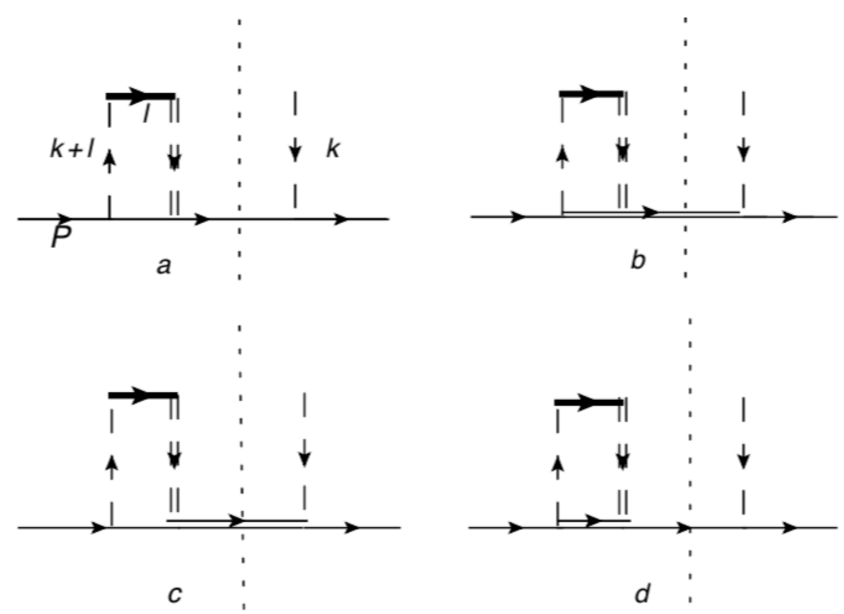


FIG. 1: The Sivers distribution function of a pseudoscalar meson in the nucleon. The solid, dashed, double dashed, and double solid lines are for the octet baryons, pseudoscalar mesons, vector mesons and decuplet baryons respectively. The thick solid line is the eikonal propagator and the dotted line means the on-shell cut.

Final state interaction in hadron level

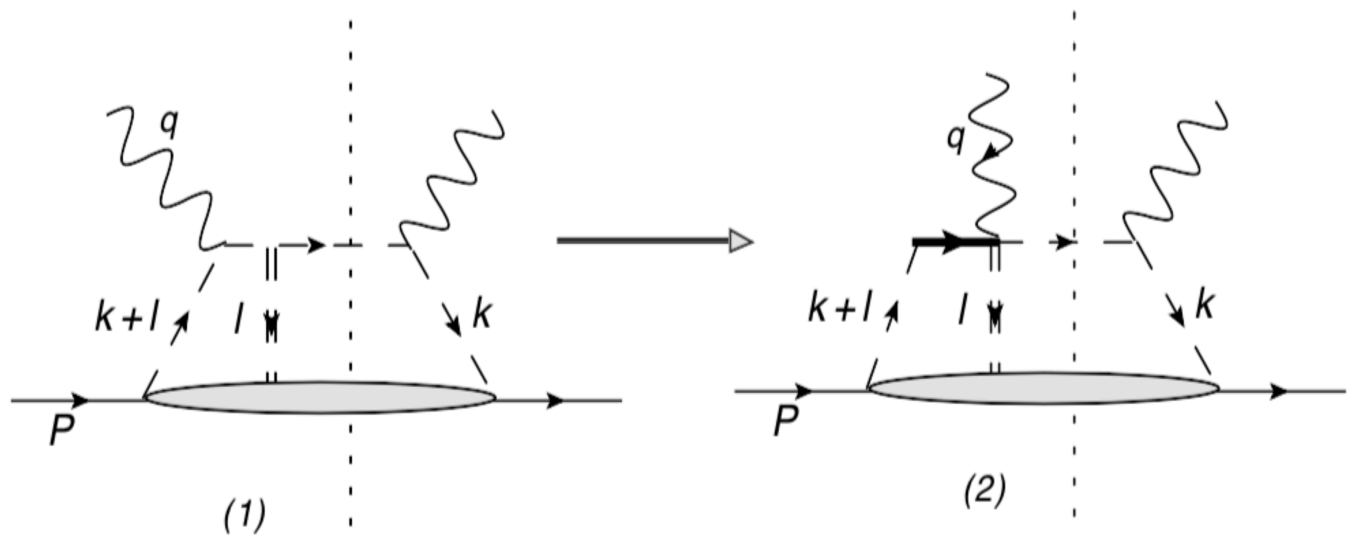


FIG. 2: Final-state interaction in Sullivan process and collinear approximation. The dashed, double dashed and wave lines are for the pseudoscalar mesons, vector mesons and photons respectively. The gray bubble represents baryon, including octet, decuplet and octet-decuplet transition.

In collinear approximation, The $\rho\pi\pi$ vertex and psedoscalar propagator turns into the eikonal propagator approximately as

$$\frac{(2k+2q+l)^-}{(k+q+l)^2-M^2} \approx \frac{(2k+2q+l)^-}{(2k+2q+l)l+i\epsilon} \approx \frac{1}{l^+ + i\epsilon}.$$

Convolution formula

quark Sivers function: $$\frac{\epsilon^{ji}k_\perp^i S_\perp^j}{m_p} f_{1T}^q(x,\vec{k}_\perp) = \frac{1}{2}\int \frac{d\xi^- d^2\vec{\xi}_\perp}{(2\pi)^3} e^{-ixP^+\xi^- + i\vec{k}_\perp\cdot\vec{\xi}_\perp} \langle P,\vec{S}_\perp \mid \mathcal{O}^q \mid P,\vec{S}_\perp\rangle,$$

pion Sivers function: $$-\frac{\epsilon^{ij}k_{\perp\pi}^i S_\perp^j}{m_p} f_{1T}^{\pi/p}(z,\vec{k}_{\perp\pi}) \ = \ \frac{1}{2}\int \frac{dy^- d^2\vec{y}_\perp}{(2\pi)^3} e^{-i(zP^+y^- - \vec{y}_\perp\cdot\vec{k}_{\perp\pi})} \langle P,\vec{S}_\perp \mid \mathcal{O}^\pi_{Sivers} \mid P,\vec{S}_\perp\rangle$$

Convolution formula for TMD:

$$k_\perp^i f_{1T}^{\bar{q}/p}(x,\vec{k}_\perp) = \int d^2\vec{k}_{\perp\pi} \int_x^1 \frac{dz}{z} f_{1v}^{\bar{q}/\pi}(\frac{x}{z}, \vec{k}_\perp - \frac{x}{z}\vec{k}_{\perp\pi}) f_{1T}^{\pi/p}(z,\vec{k}_{\perp\pi}) k_{\perp\pi}^i,$$

unpolarized valance quark TMD distribution in pion

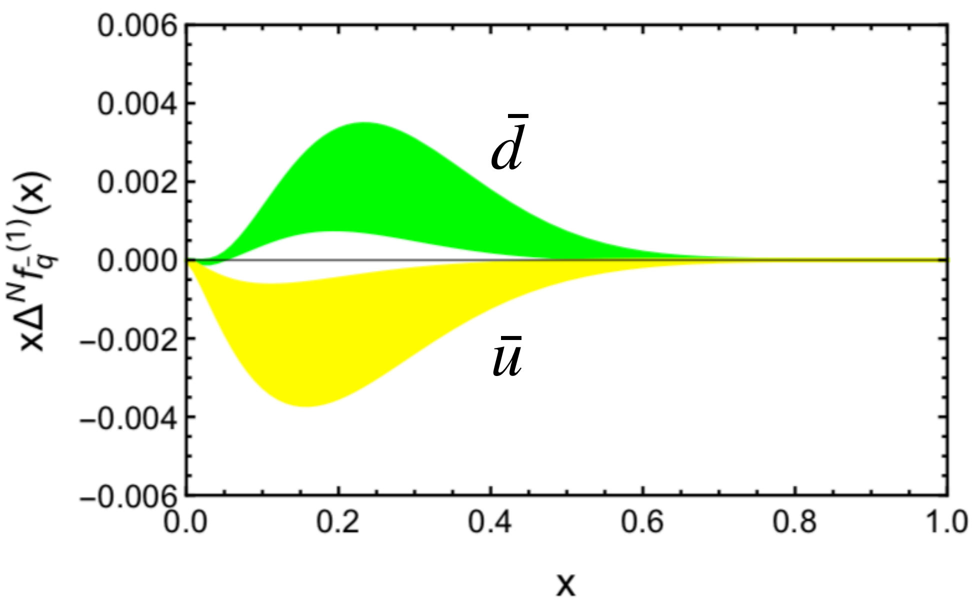
The first moment of Sivers function:

$$\Delta^N f_{\bar{q}}^{(1)}(x) = \int d^2\vec{k}_\perp \frac{-k_\perp^2}{2m_p^2} f_{1T}^{\bar{q}/p}(x,\vec{k}_\perp) = \frac{1}{2m_p^2}\int_1^x d(\frac{x}{z}) f_{1v}^{\bar{q}/\pi}(\frac{x}{z}) \int d^2\vec{k}_{\perp\pi}\vec{k}_{\perp\pi}^2 f_{1T}^{\pi/p}(z,\vec{k}_{\perp\pi}),$$

unpolarized valance quark distribution in pion

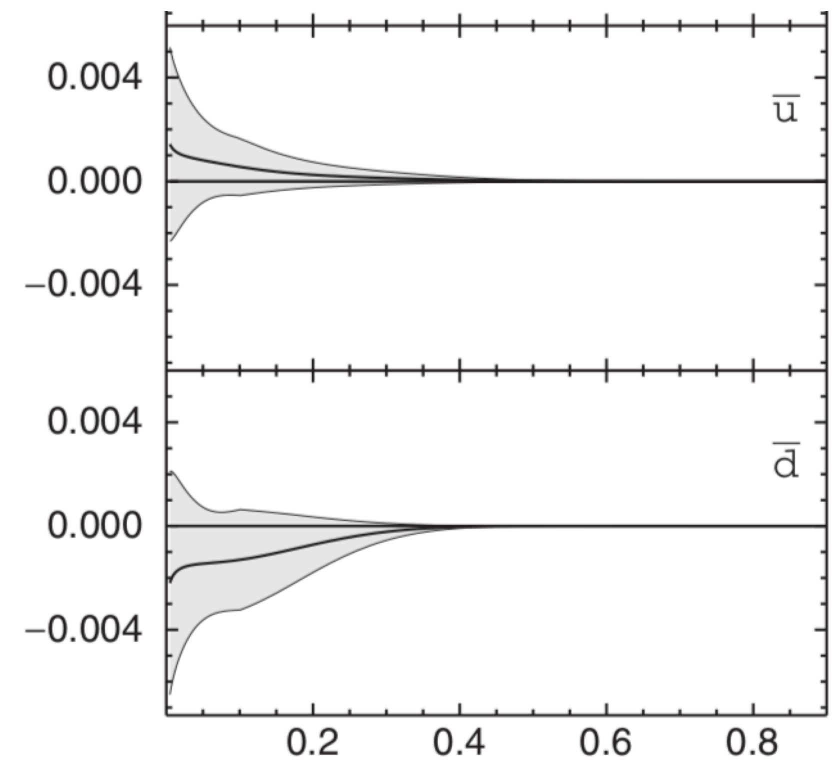
Numerical result

Our result $(0.8GeV \leq \Lambda \leq 1.2GeV)$:



$$\Delta^N f_{\bar{q}}^{(1)}(x) = - \int d^2\vec{k}_\perp \frac{k_\perp^2}{2m_p^2} f_{1T}^{\bar{q}/p}(x, \vec{k}_\perp)$$

A. Bacchetta and M. Radici(Phys. Rev. Lett. 107 (2011) 212001)



$$f_{1T}^{\perp\bar{q}}(x) = \int d^2\vec{k}_\perp \frac{k_\perp^2}{2m_p^2} f_{1T}^{\bar{q}/p}(x, \vec{k}_\perp)$$

Large Nc result(P. V. Pobylitsa arXiv:hep-ph/0301236) :

$$\left(f_{1T}^{\perp(\pm)}\right)_{t_1 t_2} = -\frac{8}{3} I_3(\tau^3)_{t_2 t_1} M^2 N_c V_4^{(\pm)}$$

Summary and Outlook

In summary:

I. We calculate the sea quark zero-skewness GPDs, the asymmetry of sea quark PDF and strange electromagnetic Form factor can be got from GPDs, which are consistent with experimental or Lattice result.

II. We calculate the sea quark Sivers function, the vector meson is introduced as gauge link in hardon level, our result is consistent with the prediction in the large Nc limit and some phenomenological extractions.

In outlook:

I. We can calculate GPDs when skewness is nonzero, polarized GPDs…

II. Maybe we can get a relation between GPDs and TMDs with Chiral Larangian.

Thanks for your attention!

Nonlocal chiral Lagrangian

Local interaction including K meson

$$\mathcal{L}_K{}^{local} = -\int dx \frac{D+3F}{\sqrt{12}f}\bar{p}(x)\gamma^\mu\gamma_5\,\Lambda(x)(\partial_\mu + ie\,\mathscr{A}^s_\mu(x))K^+(x),$$

Corresponding nonlocal Lagrangian

$$\mathcal{L}_K{}^{nl} = -\int dx \int dy \frac{D+F}{\sqrt{12}f}\bar{p}(x)\gamma^\mu\gamma_5\Lambda(x)(\partial_{x,\mu} + i\,e\mathscr{A}_\mu(x))\left(\exp[ie\int_x^y dz_\nu\,\mathscr{A}^\nu(x)]\,k^+(y)F(x-y)\right),$$

The Lagrangian is gauge invariant under the following transformation:

$$K^+(y) \to e^{-i\alpha(y)}K^+(y), \qquad \Lambda(x) \to e^{i\alpha(x)}\Lambda(x), \qquad \mathscr{A}_\mu(x) \to \mathscr{A}_\mu(x) - \frac{1}{e}\partial_\mu\alpha(x),$$

EM currents including K meson:

$$\mathcal{L}^{nor} = -ie\int dx \int dy \frac{D+F}{\sqrt{12}f}\bar{p}(x)\gamma^\mu\gamma_5\Lambda(x)F(x-y)K^+(y)\mathscr{A}_\mu(x),$$

$$\mathcal{L}^{add} = -ie\int dx \int dy \frac{D+F}{\sqrt{12}f}\bar{p}(x)\gamma^\mu\gamma_5\Lambda(x)\partial_{x,\mu}\left(F(x-y)\int_x^y dz_\nu\mathscr{A}^\nu(z)\,K^+(y)\right)$$