Basis Light-front Approach to Hadron Structure

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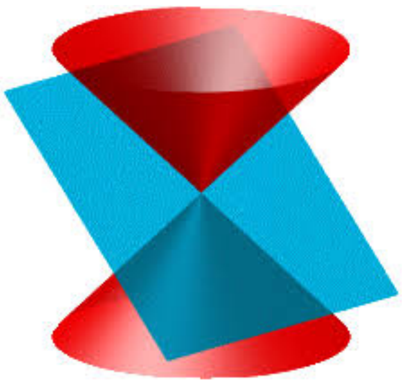


XVIII International Conference on Hadron Spectroscopy and Structure, Guilin China, 08/20/2019

Collaborators

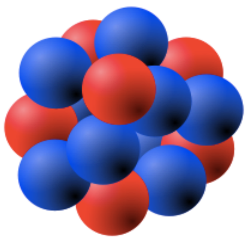
- Institute of Modern Physics, Chinese Academy of Sciences
  - Siqi Xu, Jiangshan Lan, Kaiyu Fu, Hengfei Zhao, Chandan Mondal, Sreeraj Nair
- Iowa State University
  - Shaoyang Jia, Yang Li, Weijie Du, James Vary
- University of Maryland
  - Henry Lamm
- Indian Institute of Technology, Kanpur
  - Dipankar Chakrabarti
- Beihang University
  - Li-Sheng Geng
- University of Washington
  - Gerald Miller

Outline

- What is light-front and why?
  - Relativistic bound states
- Basis Light-front Quantization
  - Many body
  - Rotational symmetry
- Applications:
  - QED: physical electron, positronium
  - QCD: nucleon, light meson, heavy quarkonium

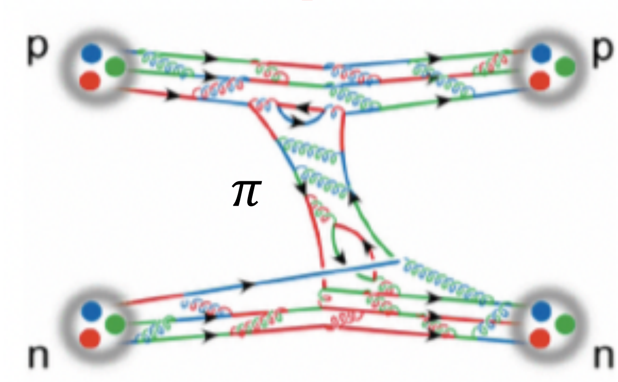
Two Sides of Nuclear Physics

Low Energy

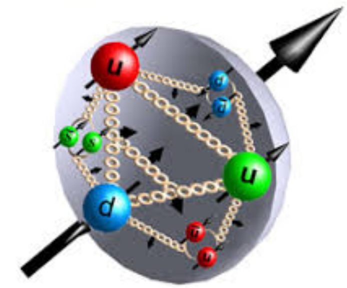


Nucleons & mesons

Mass scale ~ GeV

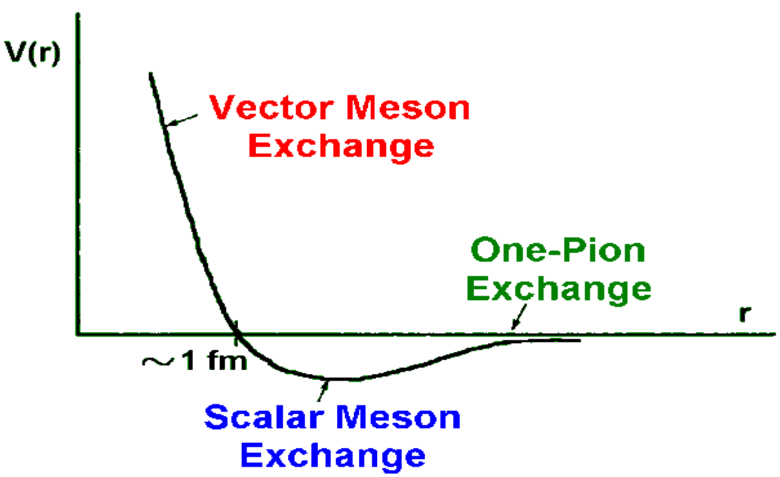


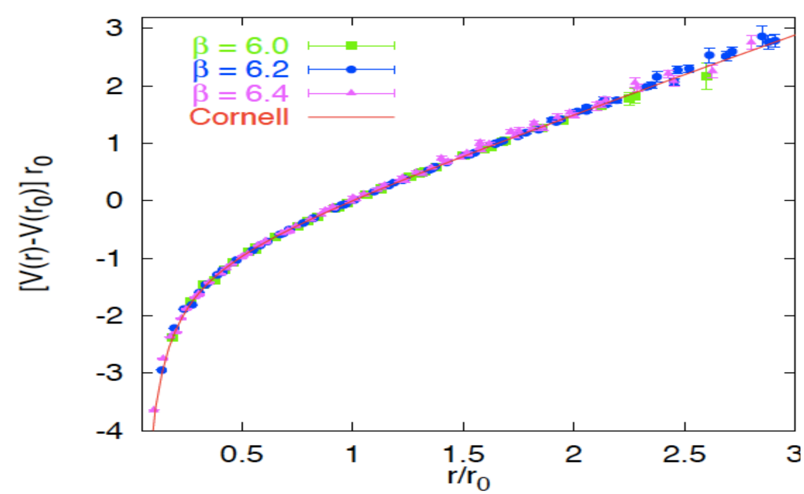
High Energy



Quarks & gluons

Mass scale ~ MeV

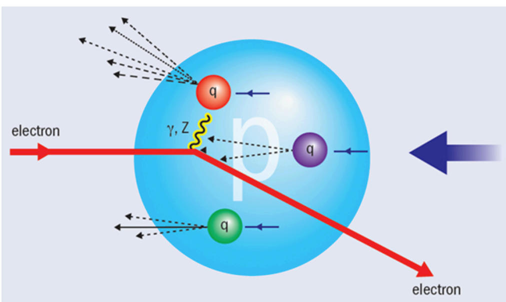


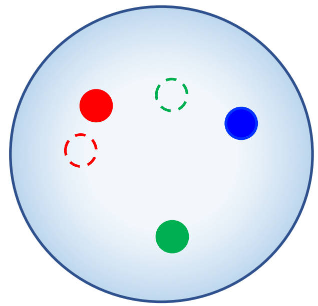


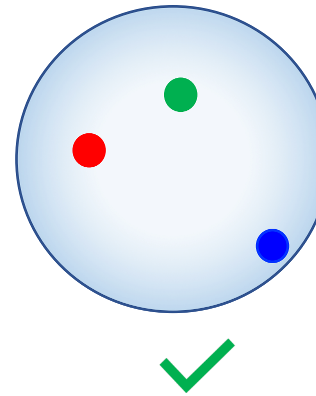
Need frame-independent wave functions

Light-front Time

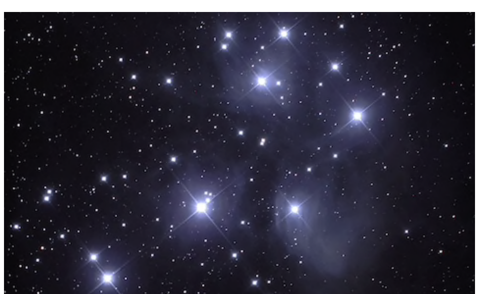
- We measure nucleon structure by virtual photon

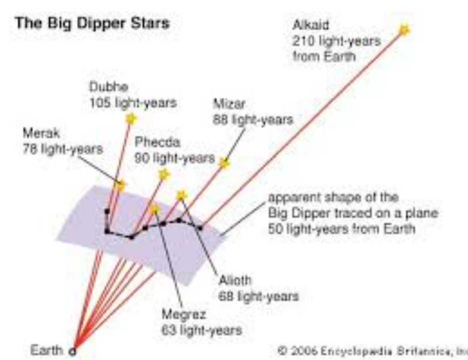






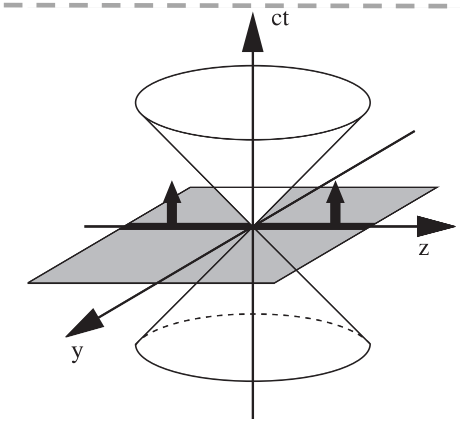
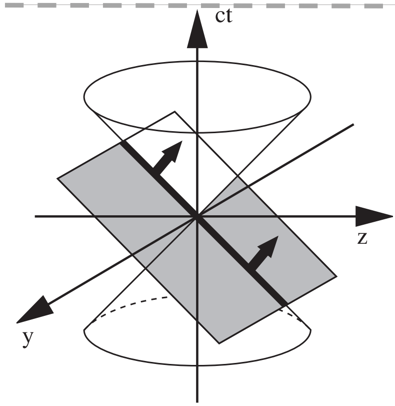
- We "see” the world at fixed light-front time ($t = x^0 + x^3$)





Light-front vs Equal-time Quantization

[Dirac 1949]

equal-time dynamics	V.S.	light-front dynamics
$t \equiv x^0$		$t \equiv x^+ = x^0 + x^3$
		
$i\frac{\partial}{\partial t}\left\vert\varphi(t)\right\rangle = H\left\vert\varphi(t)\right\rangle$ $H = P^0$		$i\frac{\partial}{\partial x^+}\left\vert\varphi(x^+)\right\rangle = \frac{1}{2}P^-\left\vert\varphi(x^+)\right\rangle$ $P^- = P^0 - P^3$
Kinematic generators: $\vec{P}, \vec{J}$		$\vec{P}^{\perp}, P^+, \vec{E}^{\perp}, E^+, J_z$

Why Go To Light-front?

- Boost invariant light-front wave functions
- Simple vacuum = free theory vacuum + zero modes
- Hamiltonian formalism for relativistic systems

$$|\mathrm{proton}\rangle = a|uud\rangle + b|uudg\rangle + \mathrm{c}|uudgg\rangle + d|uudq\bar{q}\rangle + \ldots.$$

$$|\mathrm{pion}\rangle \quad = a|q\bar{q}\rangle \quad + b|q\bar{q}g\rangle \quad + \mathrm{c}|q\bar{q}gg\rangle \quad + d|q\bar{q}q\bar{q}\rangle + \ldots.$$

. . . .

Basis Light-Front Quantization

[Vary *et al.*, 2008]

- Eigenvalue problem for Light-front Hamiltonian

$$P^-|\beta\rangle = P^-_\beta|\beta\rangle$$

Non-perturbative

– P^- : light-front Hamiltonian

– P^-_β : eigenvalue → hadron mass spectrum

– $|\beta\rangle$: eigenvector → light-front wave function

- Observables

$$O \equiv \langle\beta'|\hat{O}|\beta\rangle$$

Basis Construction

1. Fock-space expansion:

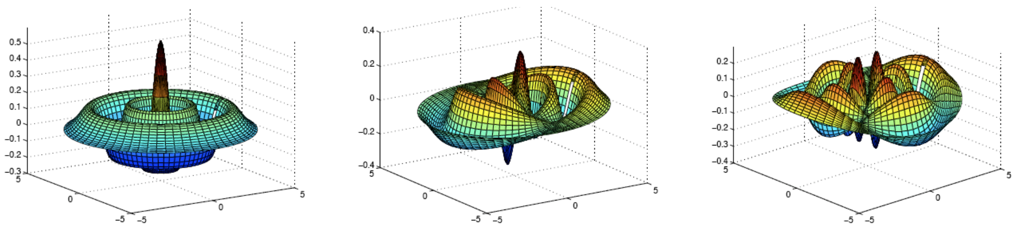
e.g. $|\boldsymbol{e_p}\rangle \quad = a|e\rangle + b|e\gamma\rangle + \mathrm{c}|ee\bar{e}\rangle \ + d|ee\bar{e}\gamma\rangle + \ldots.$

$|\mathbf{Ps}\rangle \quad = a|e\bar{e}\rangle + b|e\bar{e}\gamma\rangle + c|\gamma\rangle + d|e\bar{e}e\bar{e}\rangle \ + \ldots.$

2. For each Fock particle:

- Transverse: 2D harmonic oscillator basis: $\Phi^b_{n,m}(\vec{p}_\perp)$
  labeled by radial (angular) quantum number n (m); scale parameter b

e.g., n=4



m=0 m=1 m=2

- Longitudinal: plane-wave basis, labeled by k
- Helicity: labeled by λ

e.g. $|e\gamma\rangle = |e\rangle \otimes |\gamma\rangle$ with $e = \{n^e, m^e, k^e, \lambda^e\}$ and $\gamma = \{n^\gamma, m^\gamma, k^\gamma, \lambda^\gamma\}$

Basis Truncation Scheme

- Symmetries of Hamiltonian:

  - Net fermion number: $\sum_i n_i^f = N^f$

  - Total angular momentum projection: $\sum_i (m_i + s_i) = J_z$

  - Longitudinal momentum: $\sum_i k_i = K$

- Further truncation:

  - Fock-sector truncation

  - Discretization in longitudinal direction $k_i = \begin{cases} 1, 2, 3.... & \text{bosons} \\ 0.5, 1.5, 2.5 ... & \text{fermions} \end{cases}$

  - "N_{max}" truncation in transverse directions $\sum_i [2n_i + |m_i| + 1] \leq N_{max}$

  UV cutoff $\Lambda \sim b\sqrt{N_{max}}$; IR cutoff $\lambda \sim b/\sqrt{N_{max}}$

Features of BLFQ

- Basis respects (transverse) rotational symmetry
  - Basis states are eigenstates of J_z
- Single-particle basis for many-body system
  - (Anti-)symmetrization of identical particles
- Exact factorization of intrinsic and c.m. motion
  - Harmonic oscillator basis with Nmax truncation
- Harmonic oscillator basis suitable for bound states

Applications to QED

- QED Lagrangian $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m_e)\Psi$
- Derived Light-front Hamiltonian

$(A^+ = 0)$

$$P^- = \int \mathrm{d}^2 x^\perp \mathrm{d}x^- \;\; F^{\mu +}\partial_+ A_\mu + i\bar{\Psi}\gamma^+ \partial_+ \Psi - \mathcal{L}$$

$$= \int \mathrm{d}^2 x^\perp \mathrm{d}x^- \;\; \frac{1}{2}\bar{\Psi}\gamma^+ \frac{m_e^2 + (i\partial^\perp)^2}{i\partial^+}\Psi + \frac{1}{2} A^j (i\partial^\perp)^2 A^j$$

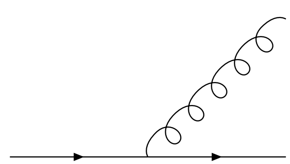
kinetic energy terms

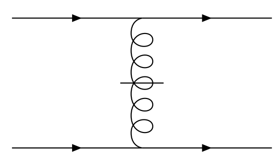
$$+ e j^\mu A_\mu + \frac{e^2}{2} j^+ \frac{1}{(i\partial^+)^2} j^+ + \frac{e^2}{2} \bar{\Psi}\gamma^\mu A_\mu \frac{\gamma^+}{i\partial^+}\gamma^\nu A_\nu \Psi$$

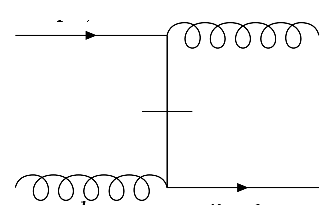
vertex interaction

instantaneous photon interaction

instantaneous fermion interaction



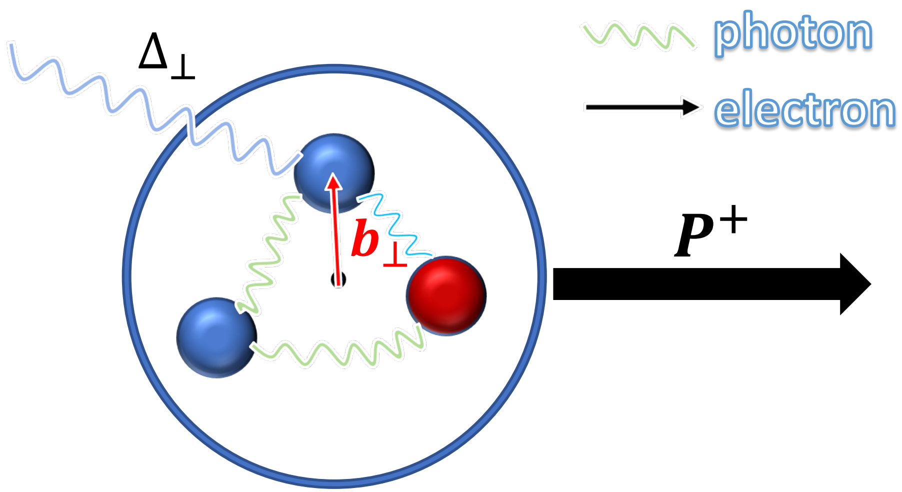




Light-front QCD Hamiltonian

$$
\begin{aligned}
H = &\frac{1}{2}\int d^3x \overline{\widetilde{\psi}}\gamma^+ \frac{(\mathrm{i}\partial^\perp)^2 + m^2}{\mathrm{i}\partial^+}\widetilde{\psi} - A_a^i(\mathrm{i}\partial^\perp)^2 A_{ia} \\
&- \frac{1}{2}g^2 \int d^3x \mathbf{Tr}\left[\widetilde{A}^\mu, \widetilde{A}^\nu\right]\left[\widetilde{A}_\mu, \widetilde{A}_\nu\right] \\
&+ \frac{1}{2}g^2 \int d^3x \overline{\widetilde{\psi}}\gamma^+ T^a \widetilde{\psi} \frac{1}{(\mathrm{i}\partial^+)^2}\overline{\widetilde{\psi}}\gamma^+ T^a \widetilde{\psi} \\
&- g^2 \int d^3x \overline{\widetilde{\psi}}\gamma^+ \left(\frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+ \widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\right)\widetilde{\psi} \\
&+ g^2 \int d^3x \mathbf{Tr}\left(\left[\mathrm{i}\partial^+ \widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+ \widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\right) \\
&+ \frac{1}{2}g^2 \int d^3x \overline{\widetilde{\psi}}\widetilde{\not{A}}\frac{\gamma^+}{\mathrm{i}\partial^+}\widetilde{\not{A}}\widetilde{\psi} \\
&+ g \int d^3x \overline{\widetilde{\psi}}\widetilde{\not{A}}\widetilde{\psi} \\
&+ 2g \int d^3x \mathbf{Tr}\left(\mathrm{i}\partial^\mu \widetilde{A}^\nu \left[\widetilde{A}_\mu, \widetilde{A}_\nu\right]\right)
\end{aligned}
$$

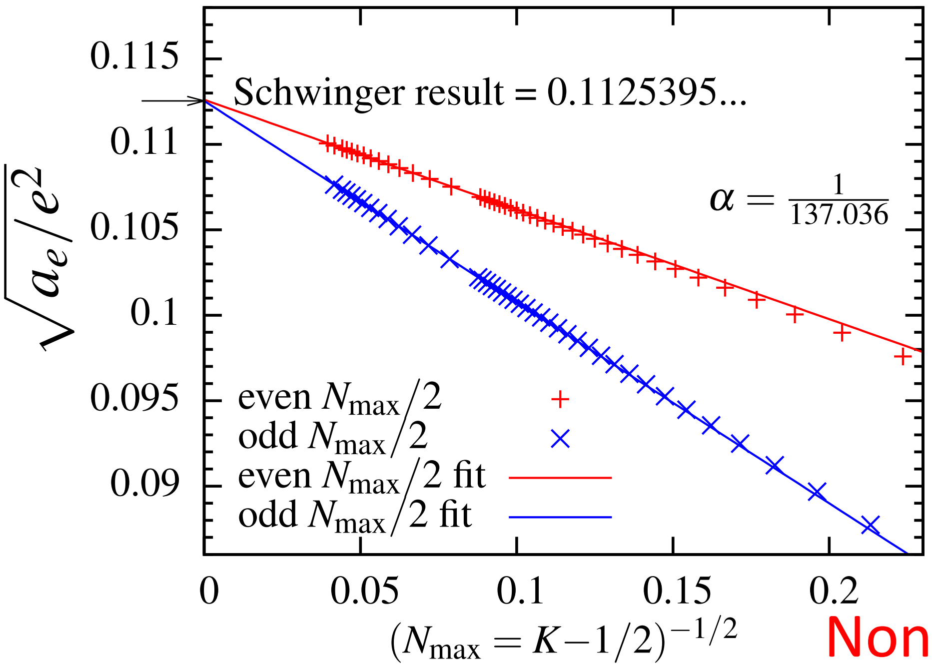
Application to QED (I): Physical Electron

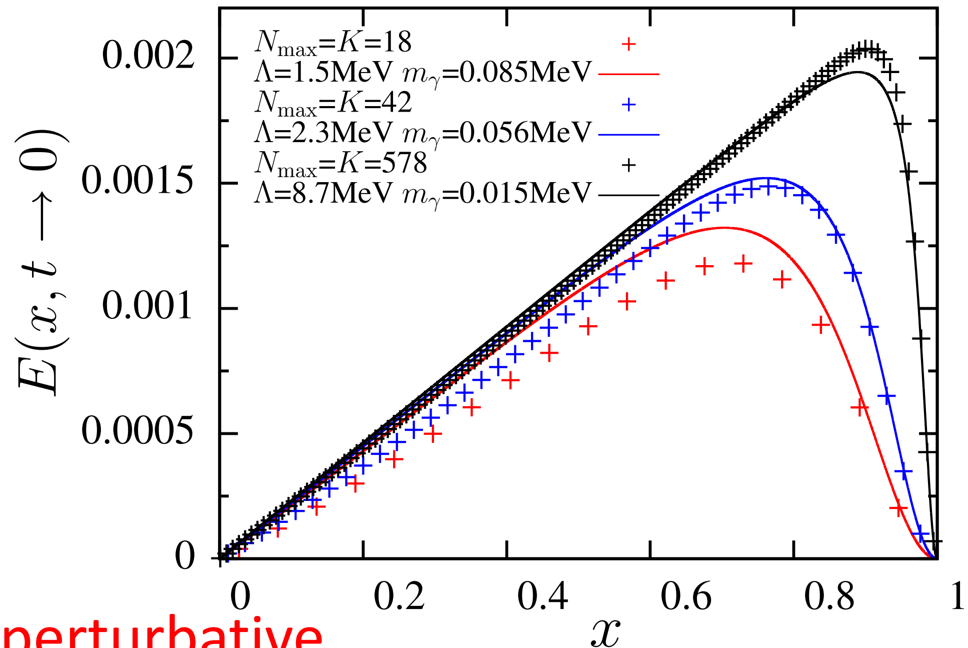


$$|e_{phys}\rangle = a|e\rangle + b|e\gamma\rangle + c|e\gamma\gamma\rangle + d|ee\bar{e}\rangle + \ldots.$$

Electron g-2 & GPD E(x, t) BLFQ vs Perturbation Theory

X. Zhao, H. Honkanen, P. Maris, J. P. Vary, S. J. Brodsky, Phys. Letts. B737, 65 (2014)

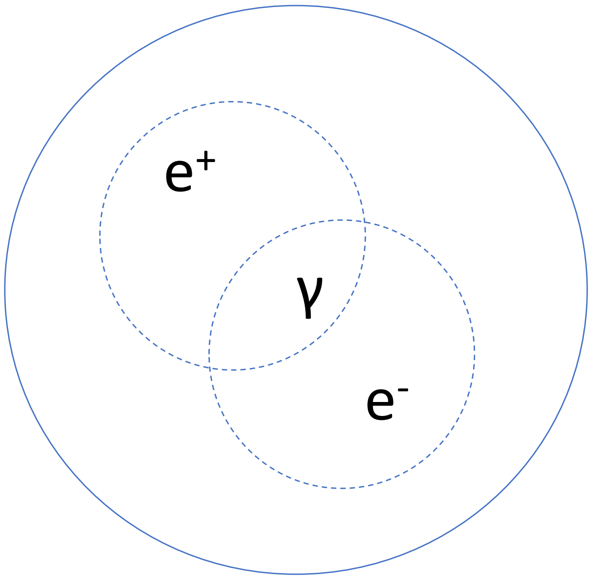




Nonperturbative

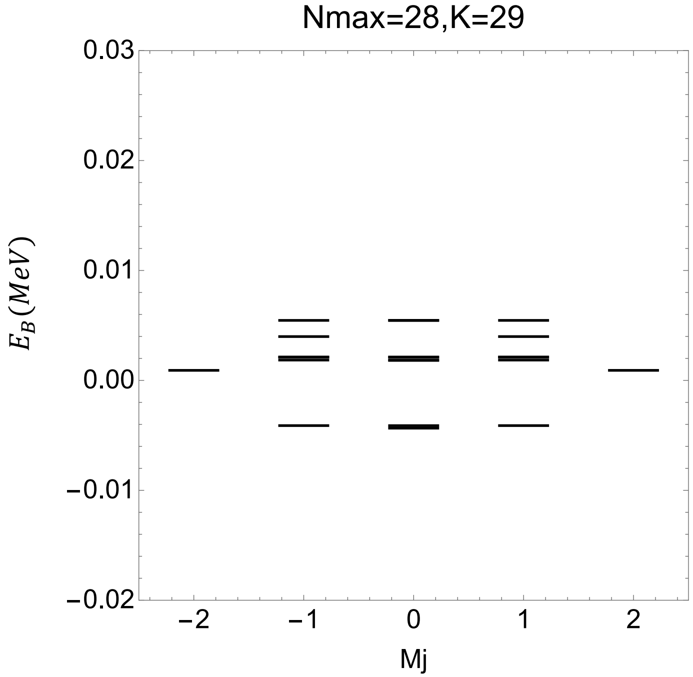
- $\frac{q_1 - iq_2}{2m_e} E(x,0,t=q^2) = \left\langle e^{\uparrow}_{phys}(\vec{q}) \middle| \int dy^- e^{ixP^+y^-/2} \bar{\psi}(0)\gamma^+\psi(y) \middle| e^{\downarrow}_{phys}(0) \right\rangle \Big|_{y^+=0, y_\perp=0}$
- Anomalous magnetic moment $a_e = \int_0^1 E(x, t \to 0) dx$
- Less than 0.1% deviation from Schwinger result for a_e
- Largest calculation with basis dim > 28 billion

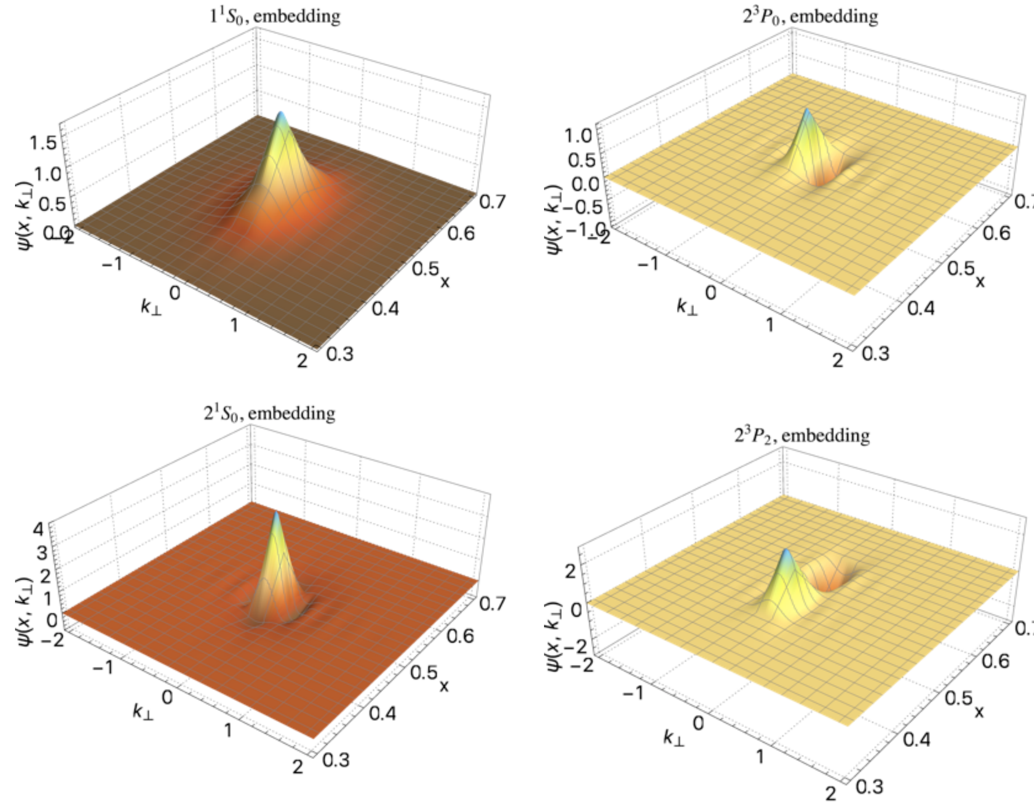
Application to QED (II): Positronium



Energy spectrum and wavefunction

See Kaiyu Fu's talk
Saturday (17) pm



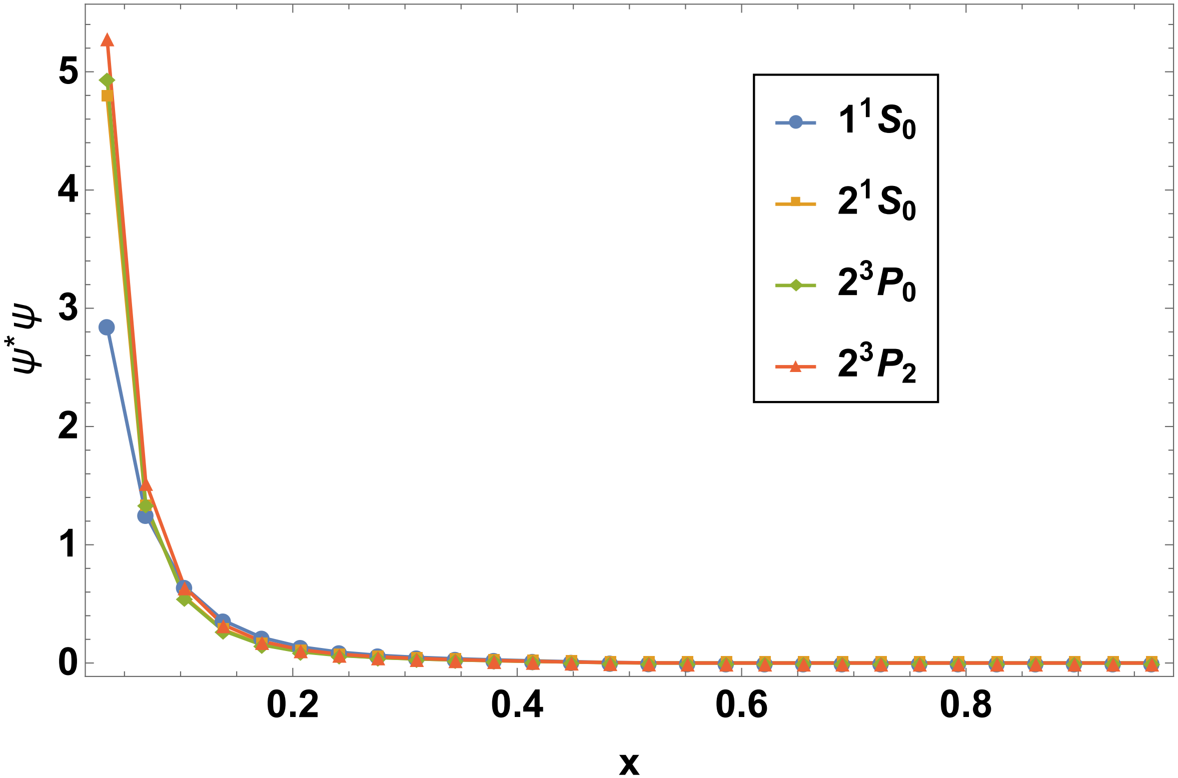


lowest 8 states of Mj=0 : parity and charge conjugation parity agree with hydrogen atom.

[Kaiyu Fu et al, in preparation]

Photon Distribution In Positronium

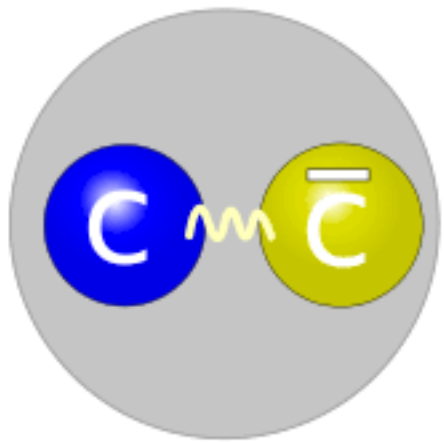
Nmax=28, K=29



[Kaiyu Fu et al, in preparation]

- In excited states photons have larger probability at small-x region
- Photon is massless, so peak is at small-x region

Application to Heavy Quarkonium



Hamiltonian

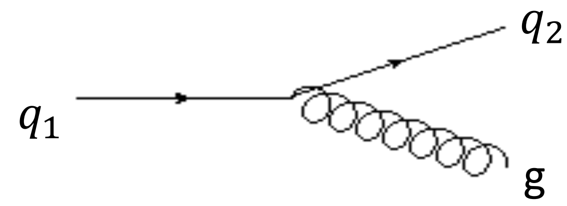
$$P^{-} = H_{free} + H_{conf} + H_{qtoqg} + H_{qqtoqq}$$

1. Kinetic Hamiltonian and confining potentials

$$H_{free} + H_{conf} = \vec{q}_{\perp}^{2} + \kappa^{4}\vec{\xi}_{\perp}^{2} + \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} - \frac{\kappa^4}{\left(m_q + m_{\bar{q}}\right)^2}\partial_x(x(1-x)\partial_x)$$

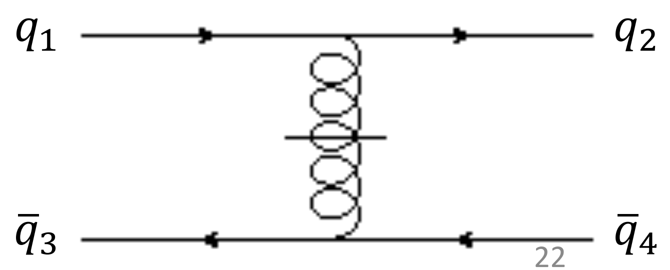
2. Vector coupling vertex

$$P^{-} = 2g\int d^3x\bar{\psi}(x)\gamma^{\mu}T^{\alpha}\psi(x)A_{\mu a}\Big|_{x^+=0}$$



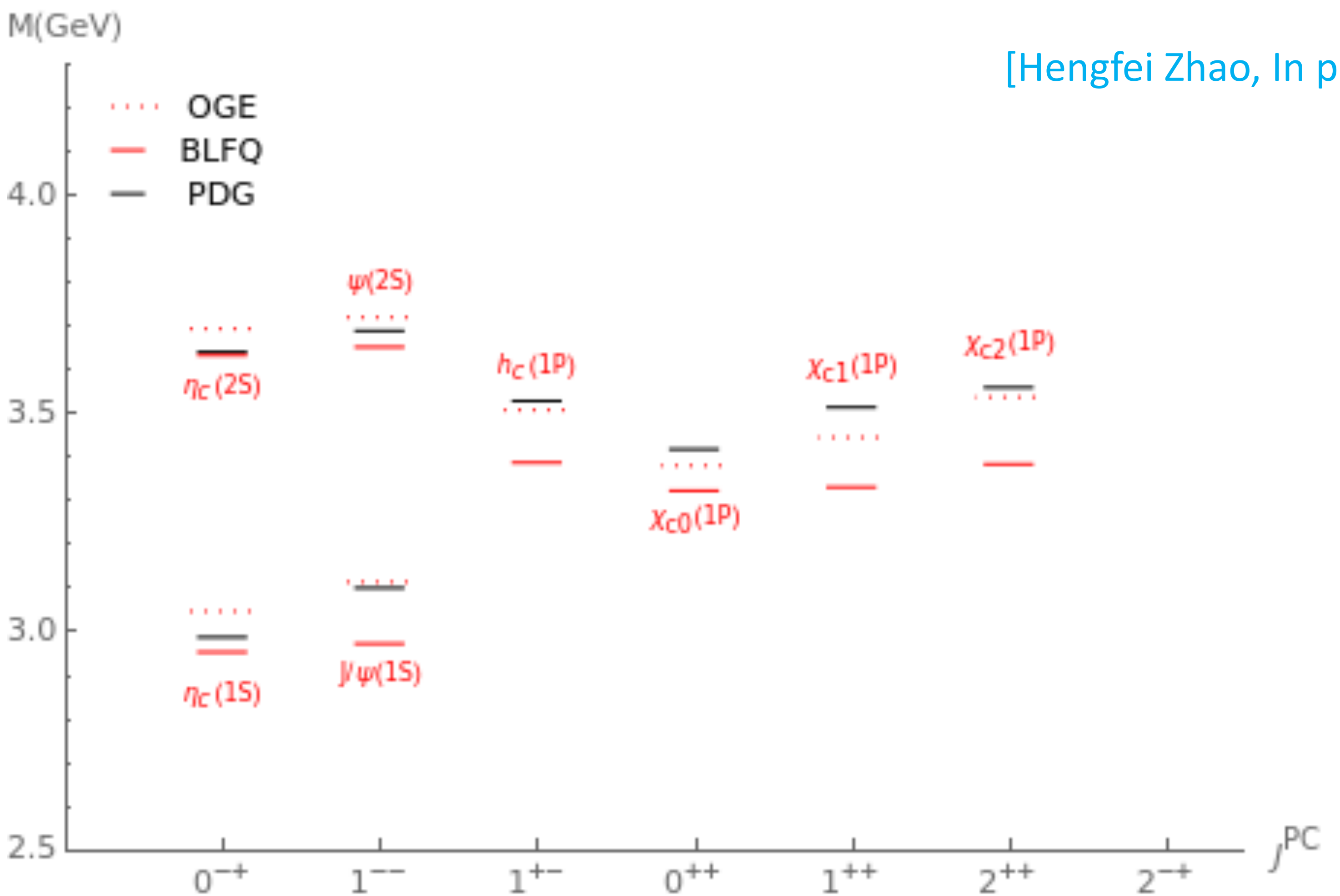
3. Vector coupling with instantaneous gluon

$$P^{-} = g^2\int d^3x\bar{\psi}\gamma^{+}T^{\alpha}\psi\frac{1}{(i\partial^{+})^2}\bar{\psi}\gamma^{+}T^{\alpha}\psi\Big|_{x^+=0}$$



Energy Spectrum

[Hengfei Zhao, In progress]

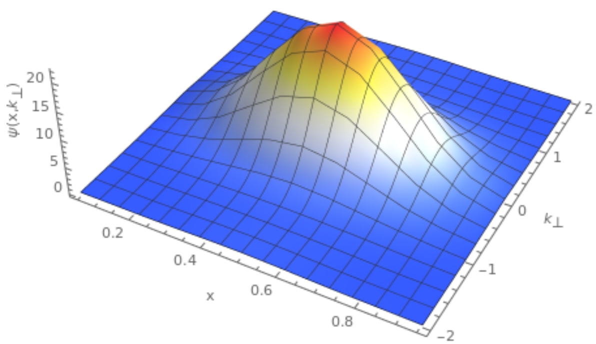
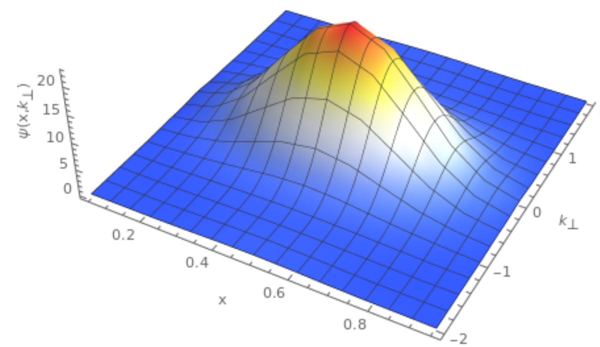
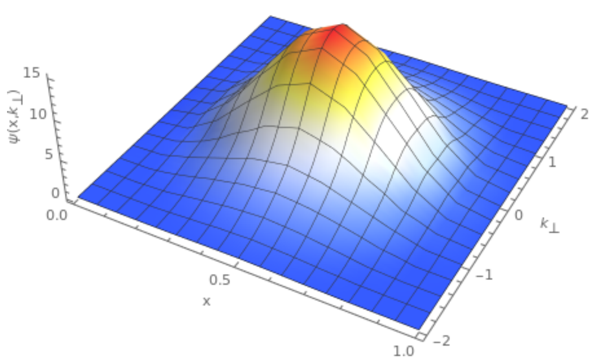
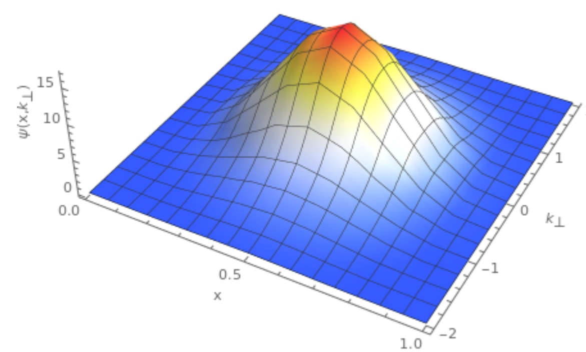


OGE : Yang Li,Maris & Vary ,PRD 17

Kt=11 Nmax=10 Mj=0 me=1.5GeV b=1.64GeV binst=3.2GeV k2l=0.3 k2t=0.1

Wave function

[Hengfei Zhao, In progress]

State	$\eta_c(1S)$	$J/\psi(1S)$
BLFQ		
OGE		

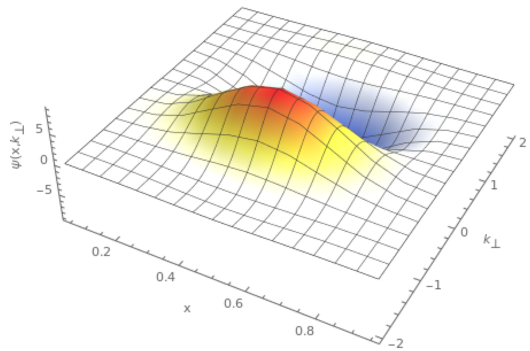
OGE; Li, Maris & Vary ,PRD 17

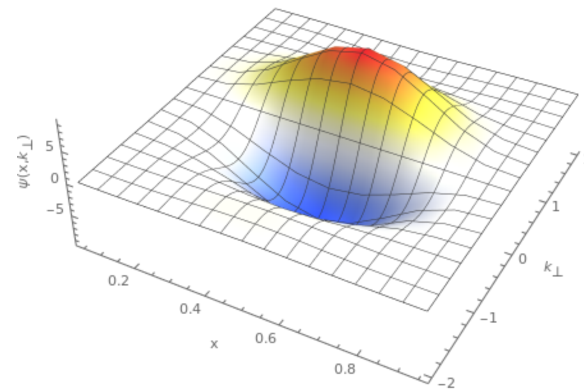
Wave function

[Hengfei Zhao, In progress]

State $\chi_{c0}(1P)$ $\chi_{c1}(1P)$

BLFQ





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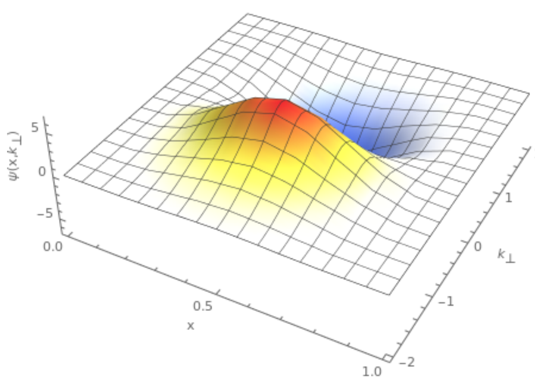
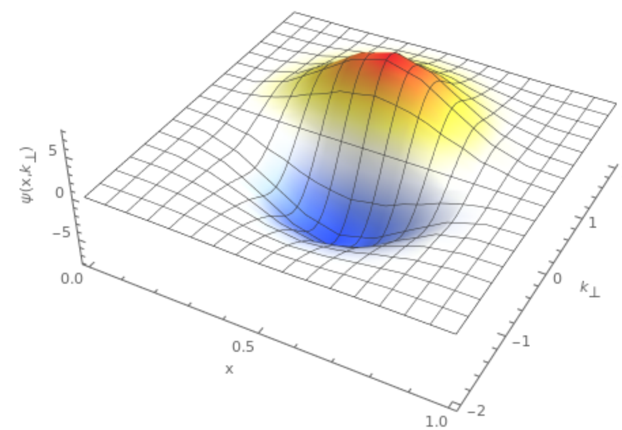
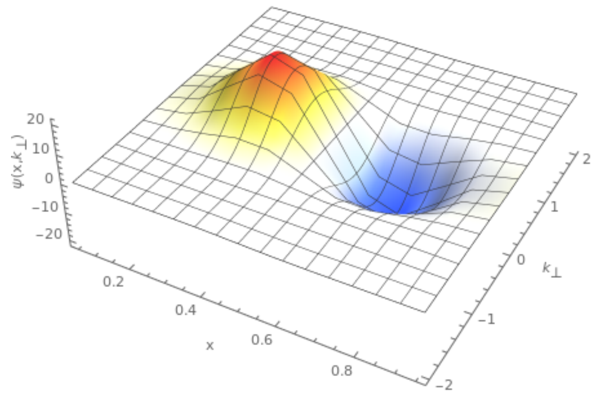
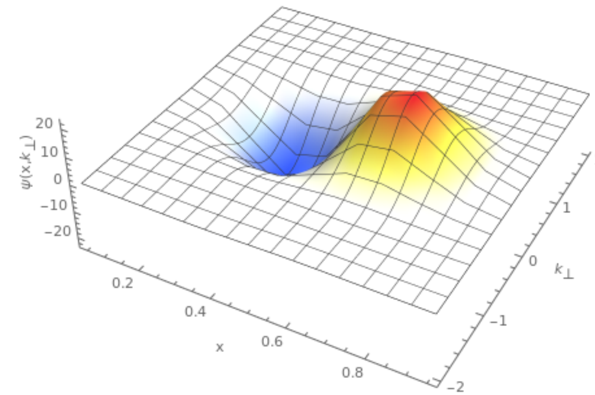
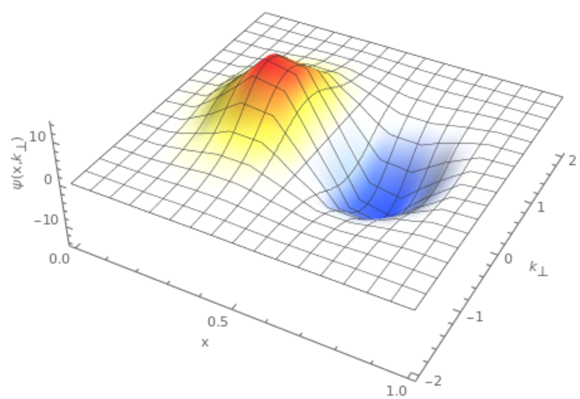
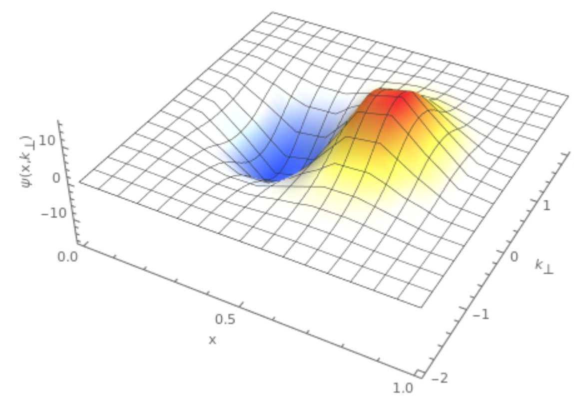




OGE; Li, Maris & Vary ,PRD 17

Wave function

[Hengfei Zhao, In progress]

state	$h_c(1P)$	$\chi_{c2}(1P)$
BLFQ		
OGE		

OGE; Li, Maris & Vary ,PRD 17

Wave function

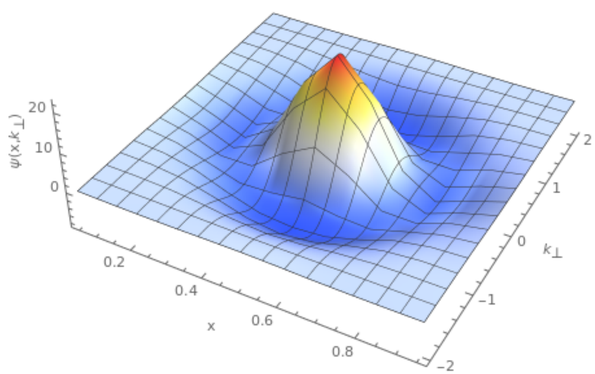
[Hengfei Zhao, In progress]

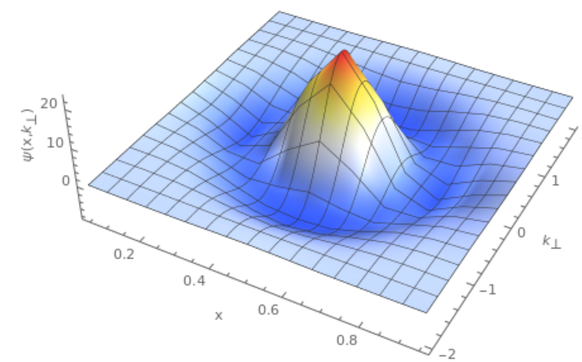
State

$\eta_c(2S)$

$\psi(2S)$

BLFQ





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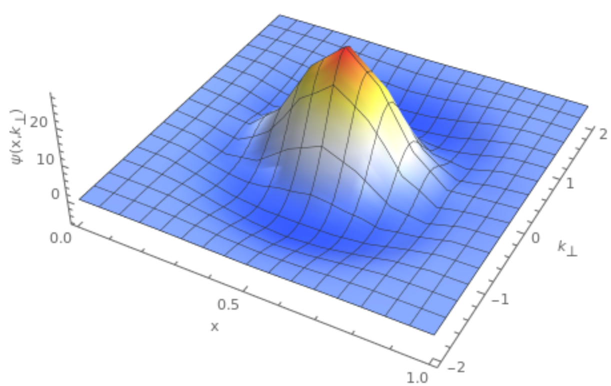
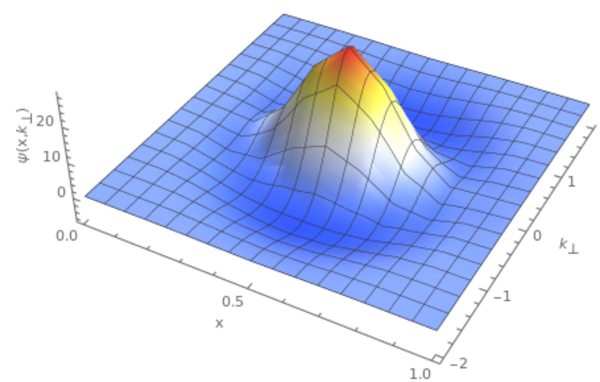




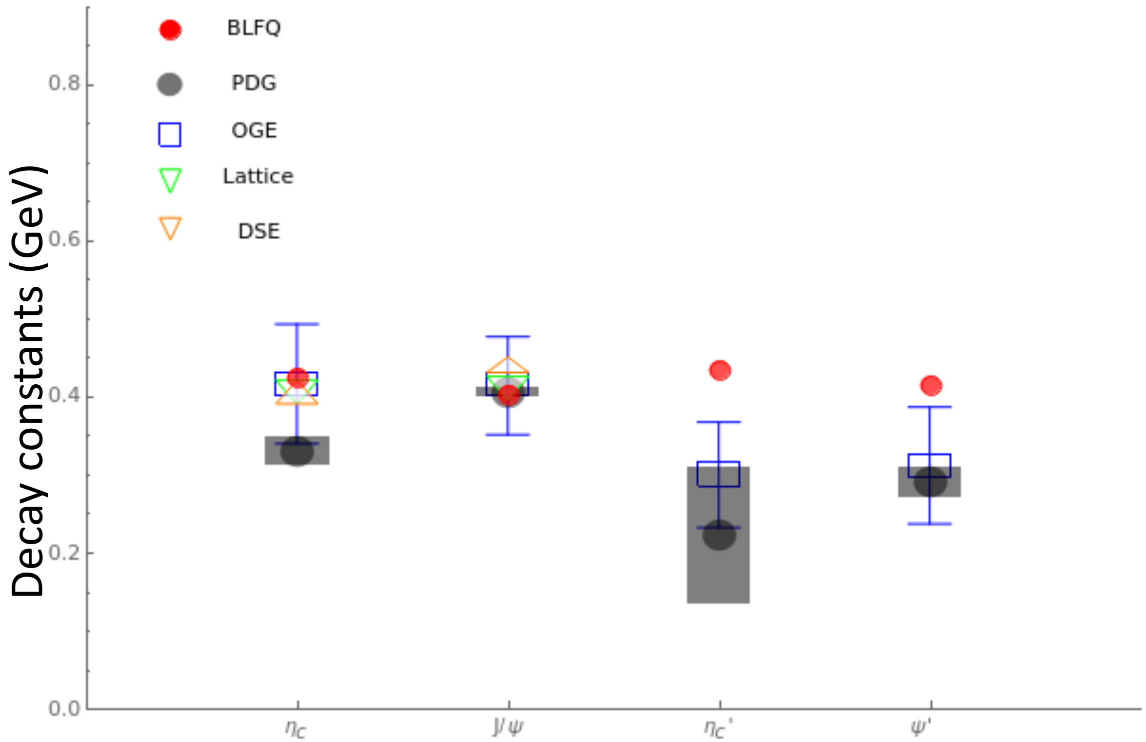
OGE; Li, Maris & Vary ,PRD 17

Decay constants

[Hengfei Zhao, In progress]

Wave function at the origin – probe short-distance physics LFWF representation

$$\frac{f_{P,V}}{2\sqrt{2N_c}} = \int_0^1 \frac{dx}{2\sqrt{x(1-x)}} \int \frac{d^2k_\perp}{(2\pi)^3} \varphi^{\lambda=0}_{\uparrow\downarrow\mp\downarrow\uparrow}(x, \vec{k}_\perp)$$

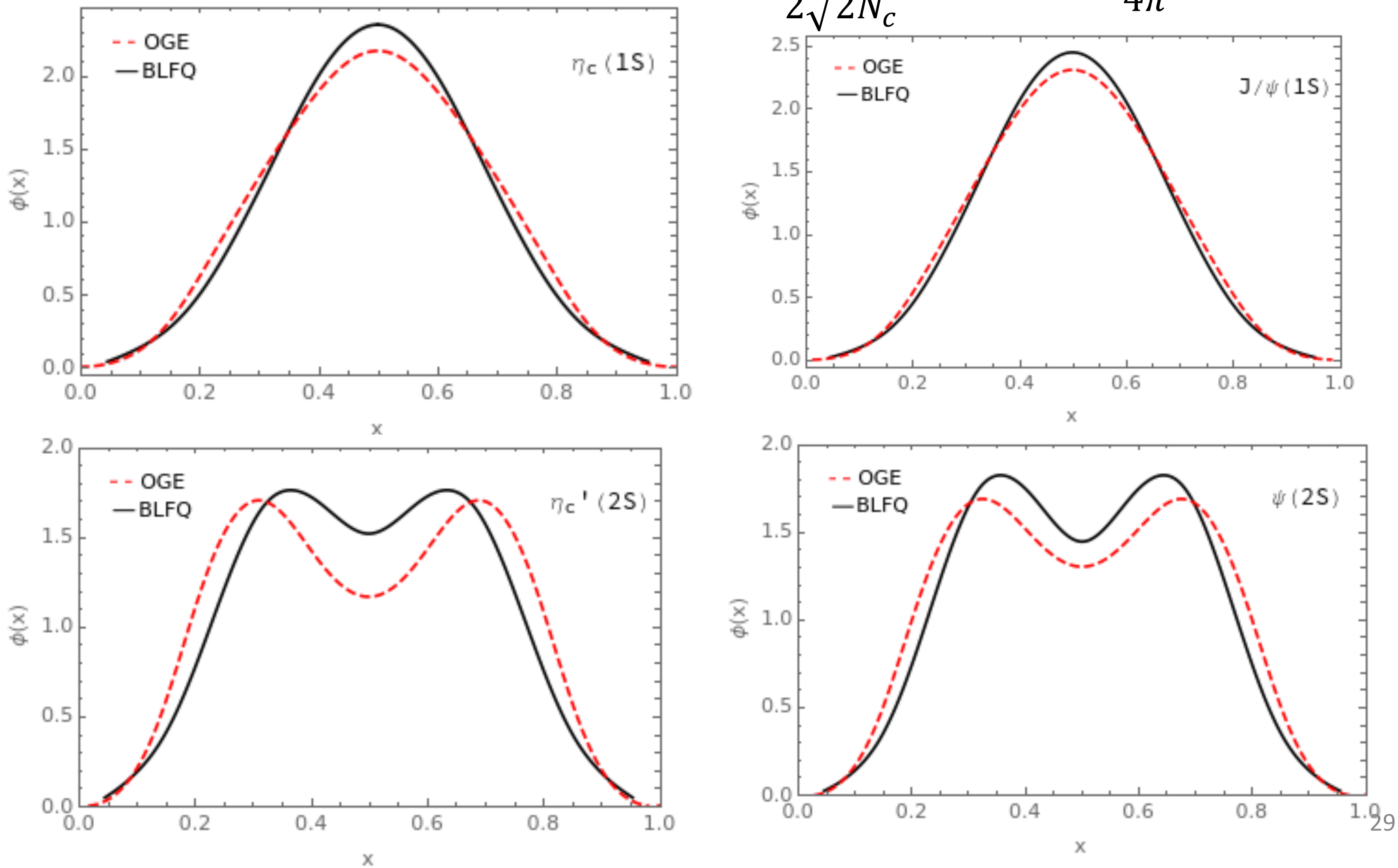


PDA

OGE; Li, Maris & Vary ,PRD 17

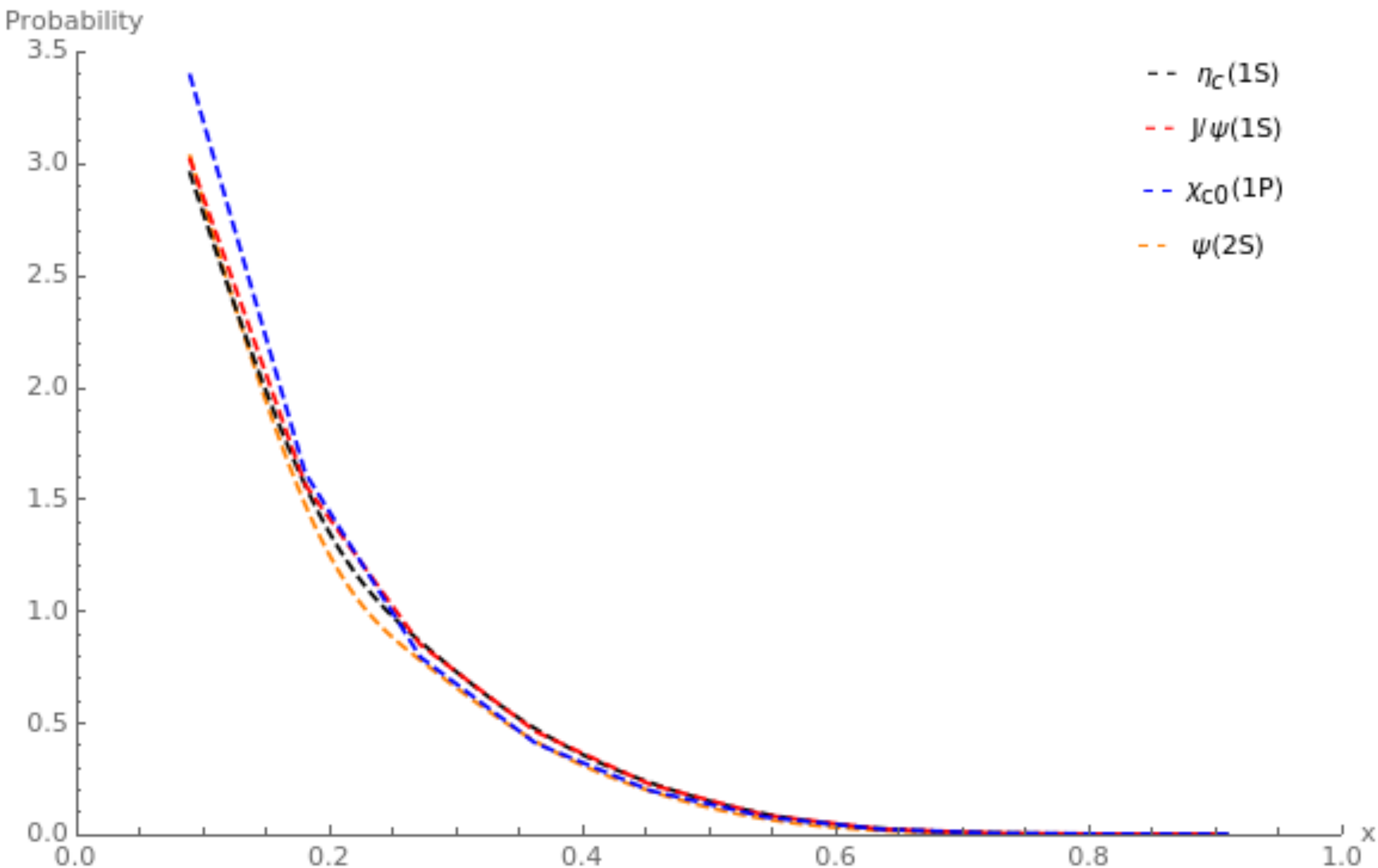
Generalized Van Royen-Weisskopf formula:

$$\frac{f_{P,V}}{2\sqrt{2N_c}}\phi_{P,V}(x;\mu)=\frac{1}{4\pi}\tilde{\psi}^{\lambda=0}_{\uparrow\downarrow\mp\downarrow\uparrow}(x,\vec{b}_\perp=0)$$

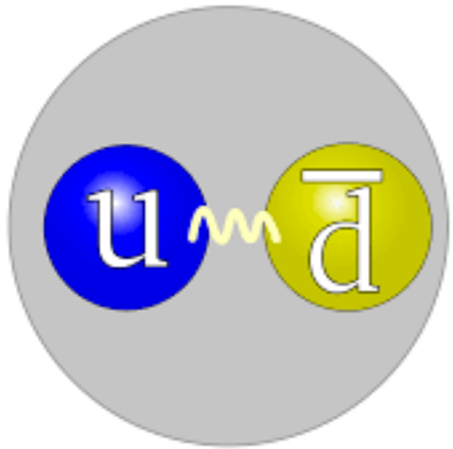


Gluon PDF

[Hengfei Zhao, In progress]



Application to Pion



PDF with QCD Evolution

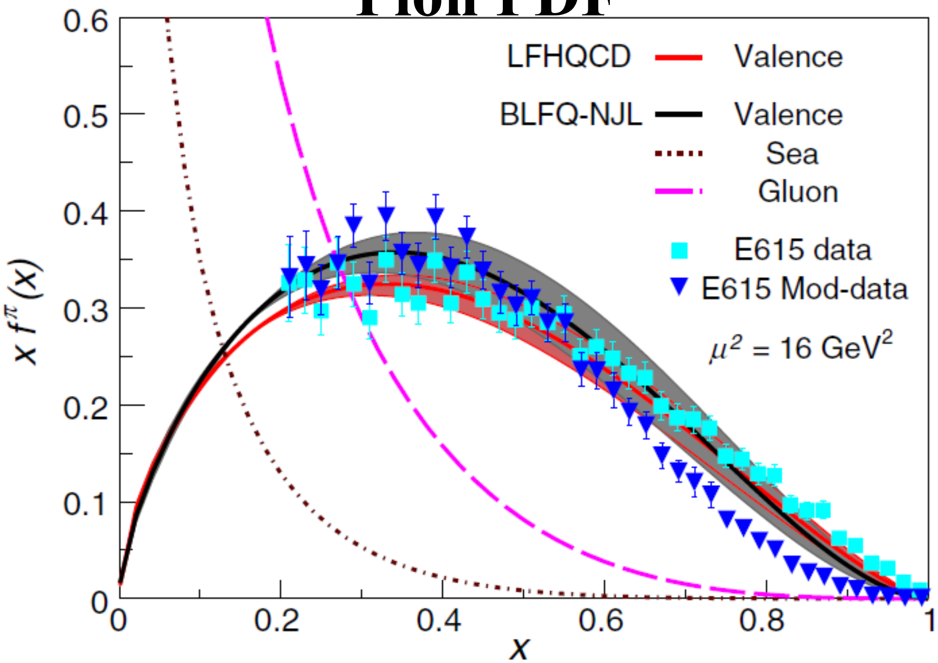
See Jiangshan Lan's talk Sunday (18) pm

See Shaoyang Jia's talk Tuesday (20) pm

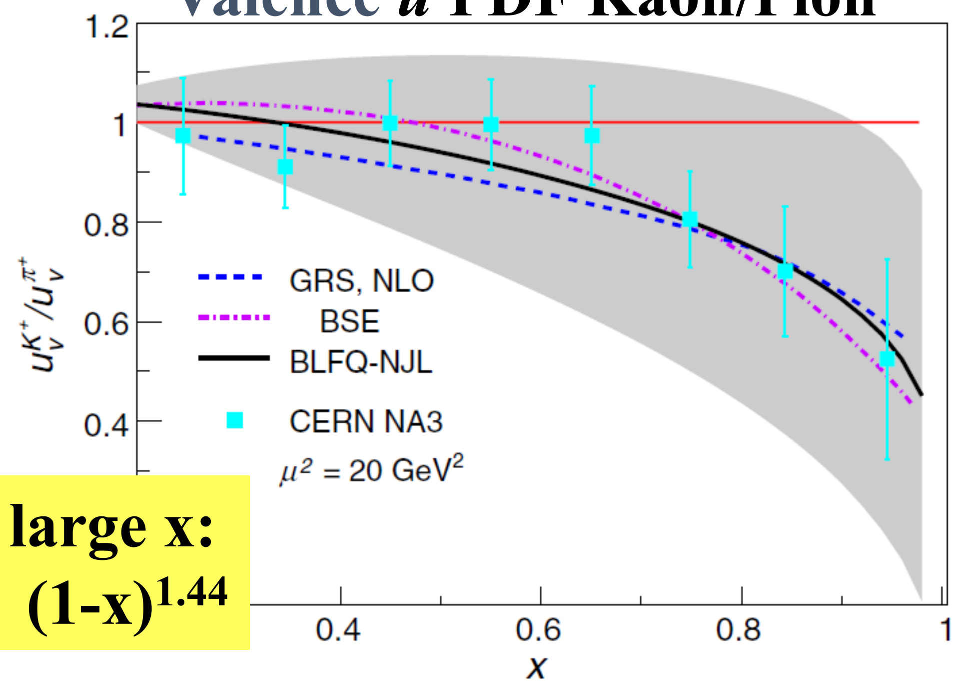
$$H_{LF} = \underbrace{\frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF Kinetic energy}} + \underbrace{\kappa^4 x(1-x)\vec{r}_\perp^{\,2}}_{\text{Transverse}} - \underbrace{\frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\partial_x\big(x(1-x)\partial_x\big)}_{\text{Longitudinal}} + H_{\rm NJL}^{\rm eff}$$

PDF for the valence quark result from the light front wave functions obtain by diagonalizing the effective Hamiltonian.

Pion PDF



Valence *u* PDF Kaon/Pion



large x: $(1\text{-}x)^{1.44}$

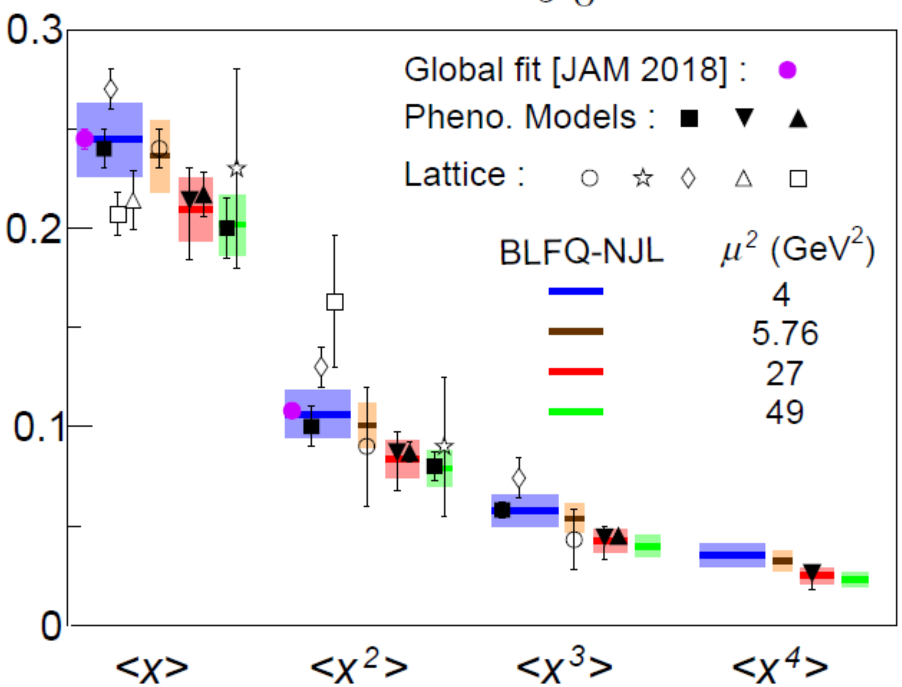
[Lan, Mondal, Jia, Zhao, Vary, PRL122, 172001(2019)]

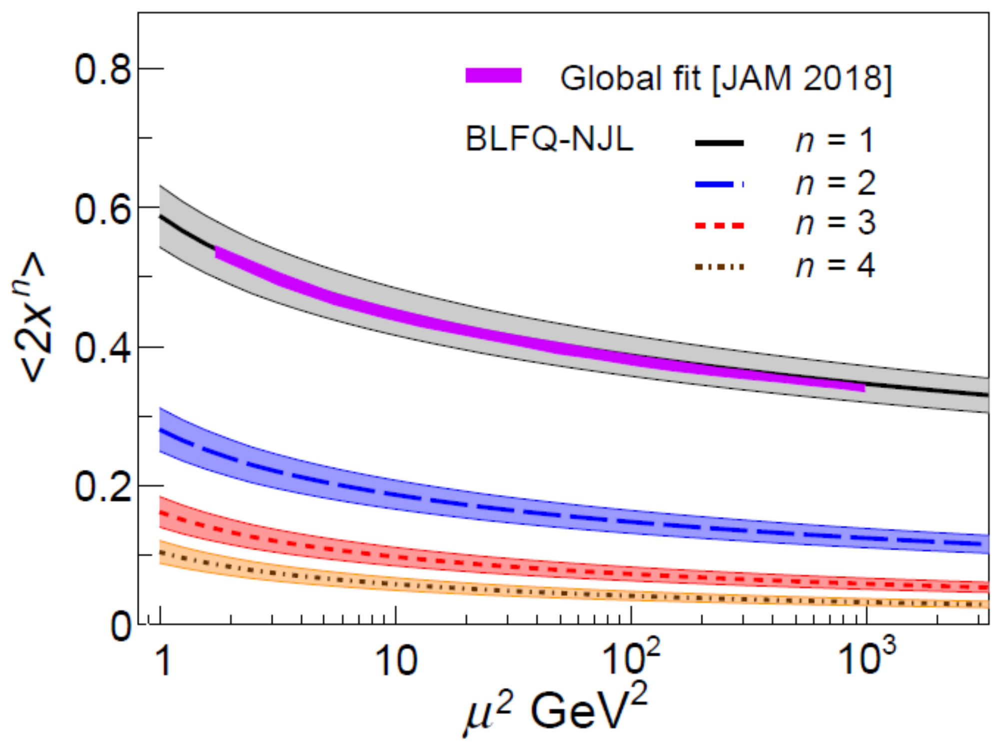
PDF with QCD Evolution

[Lan, Mondal, Jia, Zhao, Vary, arxiv: 1907.01509]

The **moments** of **pion** valence quark PDF:

$$\langle x^n \rangle = \int_0^1 dx\; x^n f_v^{\pi/K}(x, \mu^2),\;\; n = 1, 2, 3, 4.$$





<x> @4 GeV^2	Valence	Gluon	Sea
BLFQ-NJL	**0.489**	**0.398**	**0.113**
[Aguilar *et. al.,* Pion and Kaon Structure at the Electron-Ion Collider]	0.48(3)	0.41(2)	0.11(2)

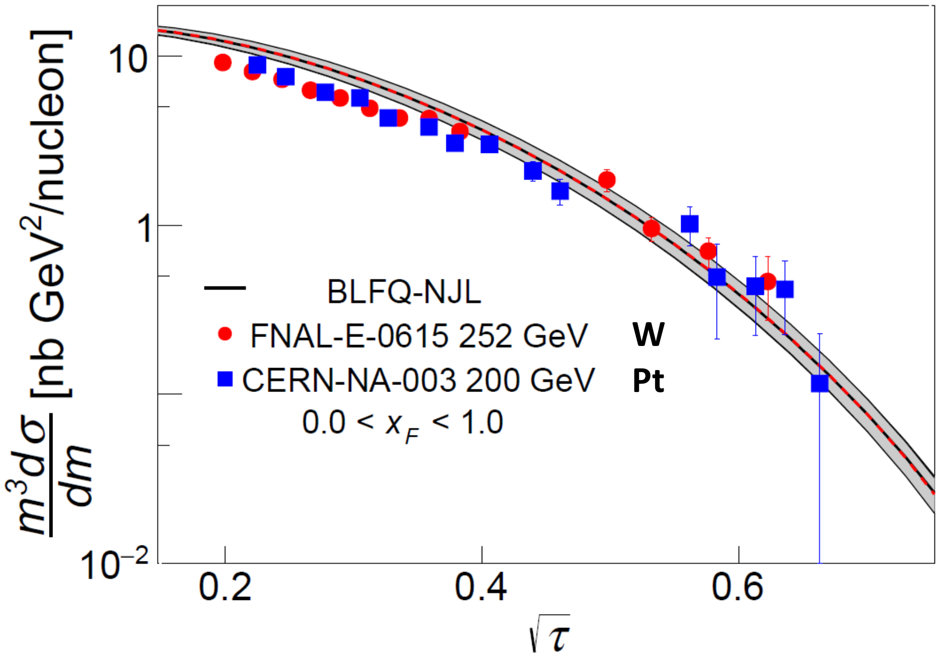
Drell-Yan cross section

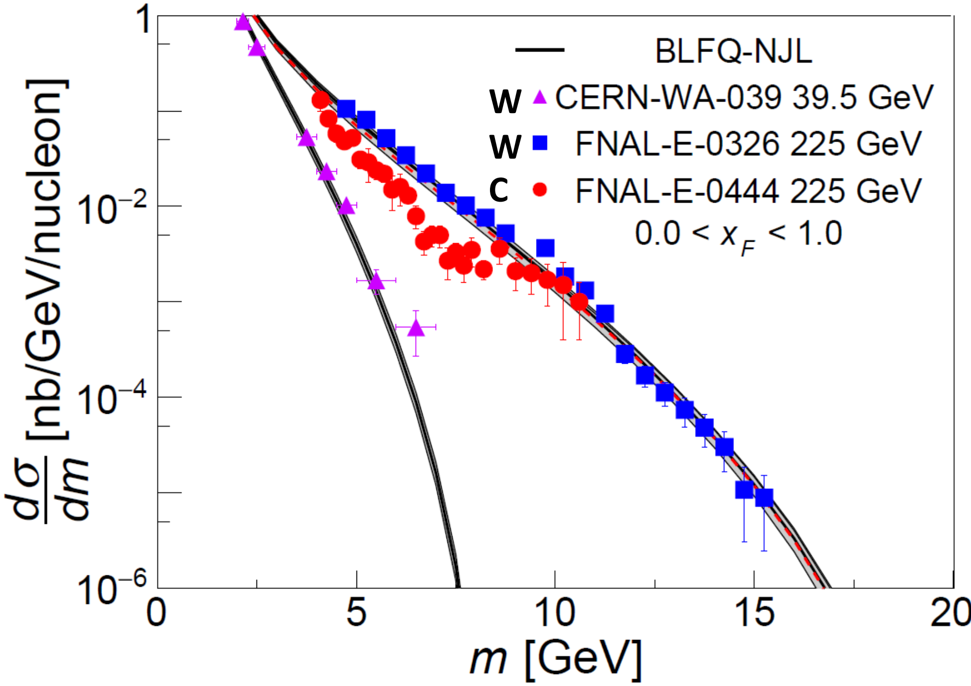
$$\pi^- \ \text{Nucleus} \rightarrow \mu^+ \ \mu^- \ \text{X}$$

[nCTEQ 2015]

[S. D. Drell and T.-M. Yan, PRL (1970)]
[McGaughey et al, Drell-Yan experiment FNAL-E-0772, PRD (1994)]

$$\frac{m^3 d^2\sigma}{dm\, dx_{\rm F}} = \frac{8\pi\alpha^2}{9}\frac{x_1 x_2}{x_1+x_2}\sum_a e_a^2 f_a^{\pi^-}(x_1) f_{\bar{a}}^N(x_2)$$

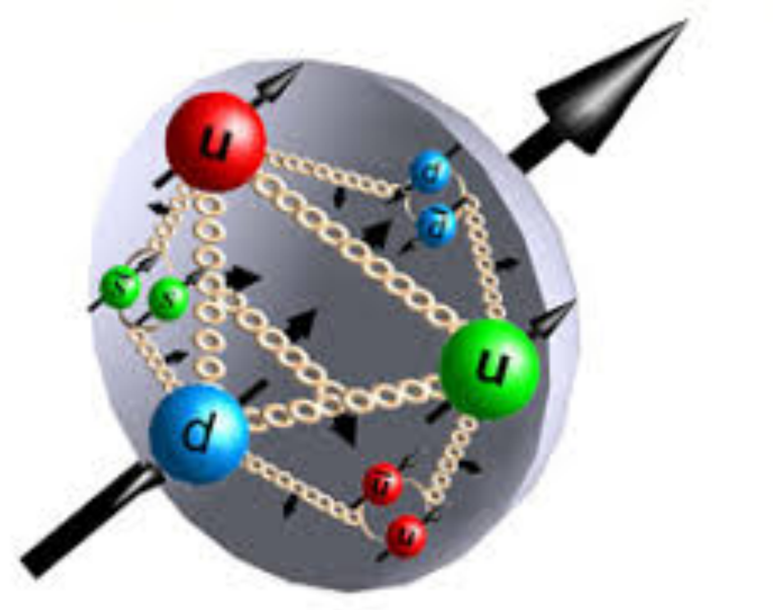




Agree with experimental data (**FNAL E615**, **326**, **444**, & **CERN NA3**, **WA-039**).

[Lan, Mondal, Jia, Zhao, Vary, arxiv: 1907.01509]

Application to Proton



Light-Front Hamiltonian

See Siqi Xu's talk
Tuesday (20) pm

$$P^- = H_{K.E.} + H_{trans} + H_{longi} + H_{OGE}$$

$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

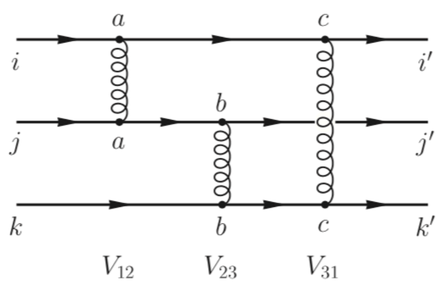
$$H_{trans} \sim \kappa_T^4 r^2$$ -- Brodsky, Teramond arXiv: 1203.4025

$$H_{longi} \sim -\sum_{ij} \kappa_L^4 \partial_{x_i}\left(x_i x_j \partial_{x_j}\right)$$ ---Y Li, X Zhao , P Maris , J Vary, PLB 758(2016)

$$H_{OGE} = -\frac{C_F 4\pi\alpha_s}{Q^2} \sum_{i,j(i<j)} \bar{u}_{s_i'}(k_i')\gamma^\mu u_{s_i}(k_i)\bar{u}_{s_j'}(k_j')\gamma_\mu u_{s_j}(k_j)$$

Color factor : $C_F = -\frac{2}{3}$

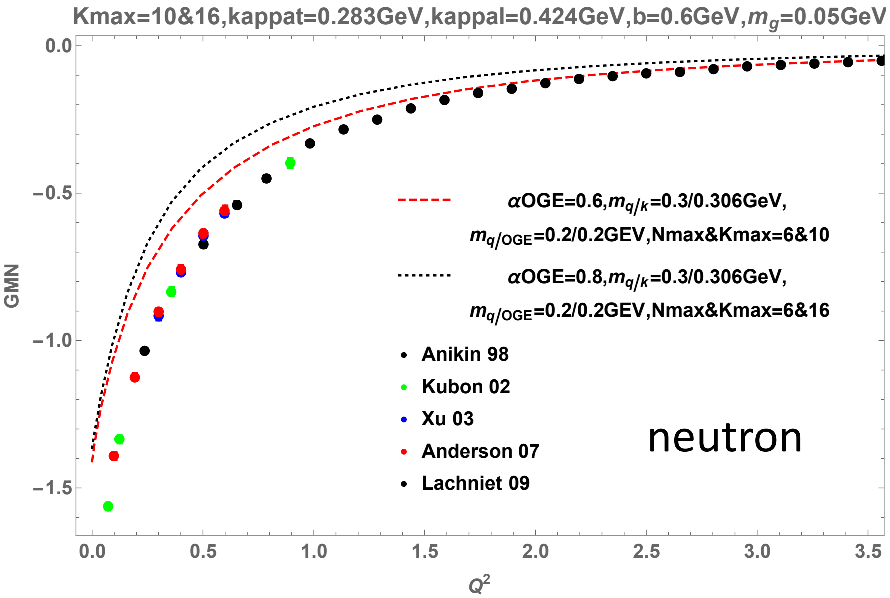
$$|P_{baryon}\rangle = \boxed{|qqq\rangle} + |qqqg\rangle + |qqq\, q\bar{q}\rangle + \cdots\cdots$$

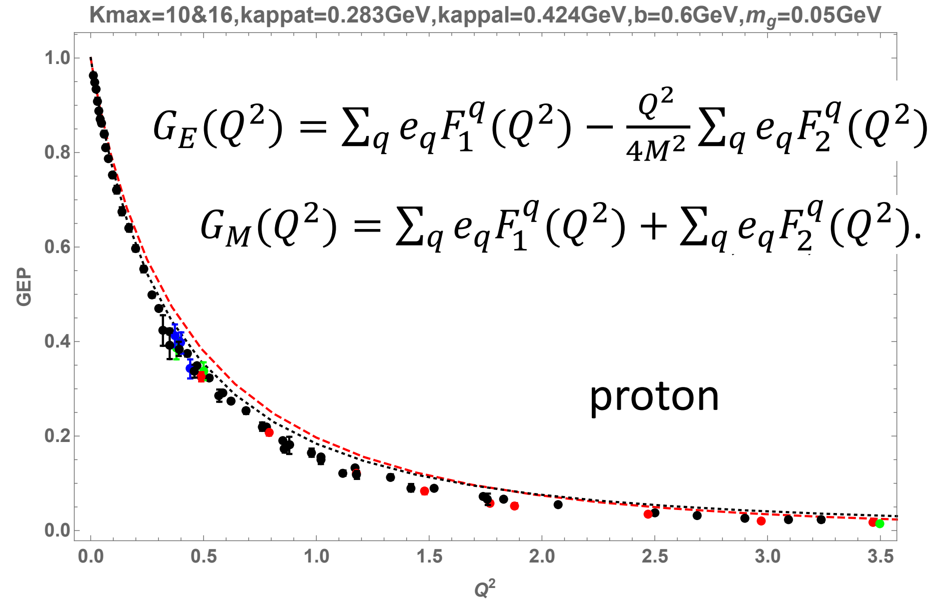


Three active-quark approach

Sach Form Factor

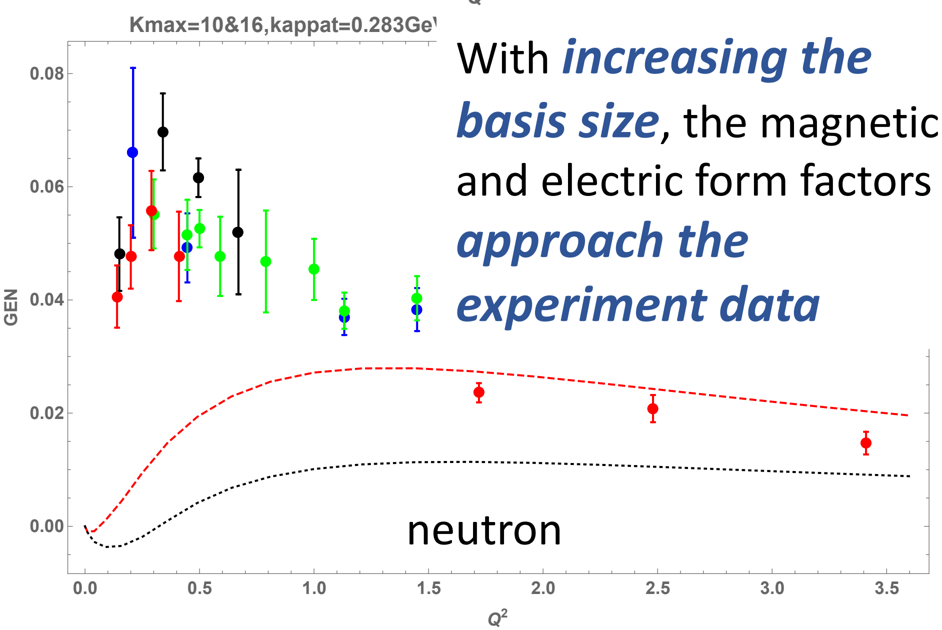
[Work in progress, C. Mondal, Siqi Xu, *et.al*]



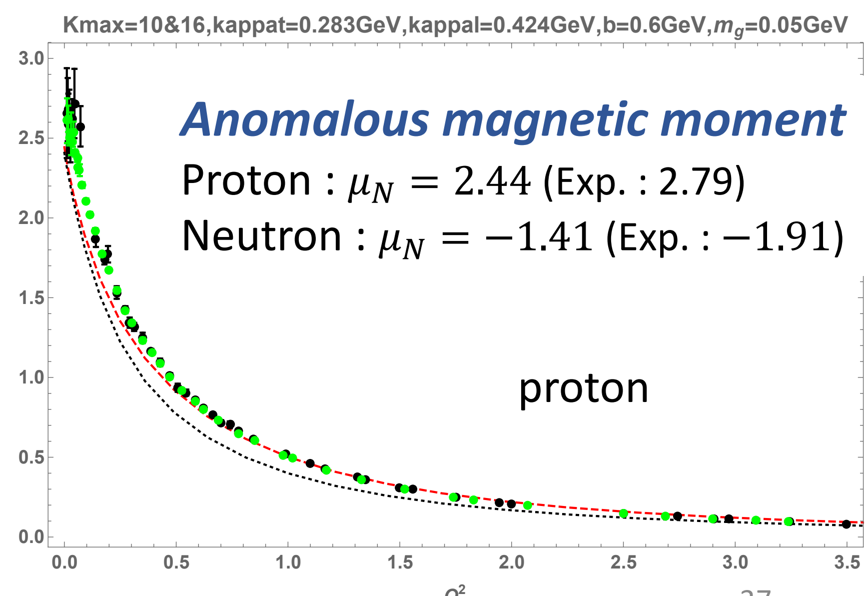


$$G_E(Q^2) = \sum_q e_q F_1^q(Q^2) - \frac{Q^2}{4M^2} \sum_q e_q F_2^q(Q^2)\,,$$

$$G_M(Q^2) = \sum_q e_q F_1^q(Q^2) + \sum_q e_q F_2^q(Q^2).$$



With ***increasing the basis size***, the magnetic and electric form factors ***approach the experiment data***



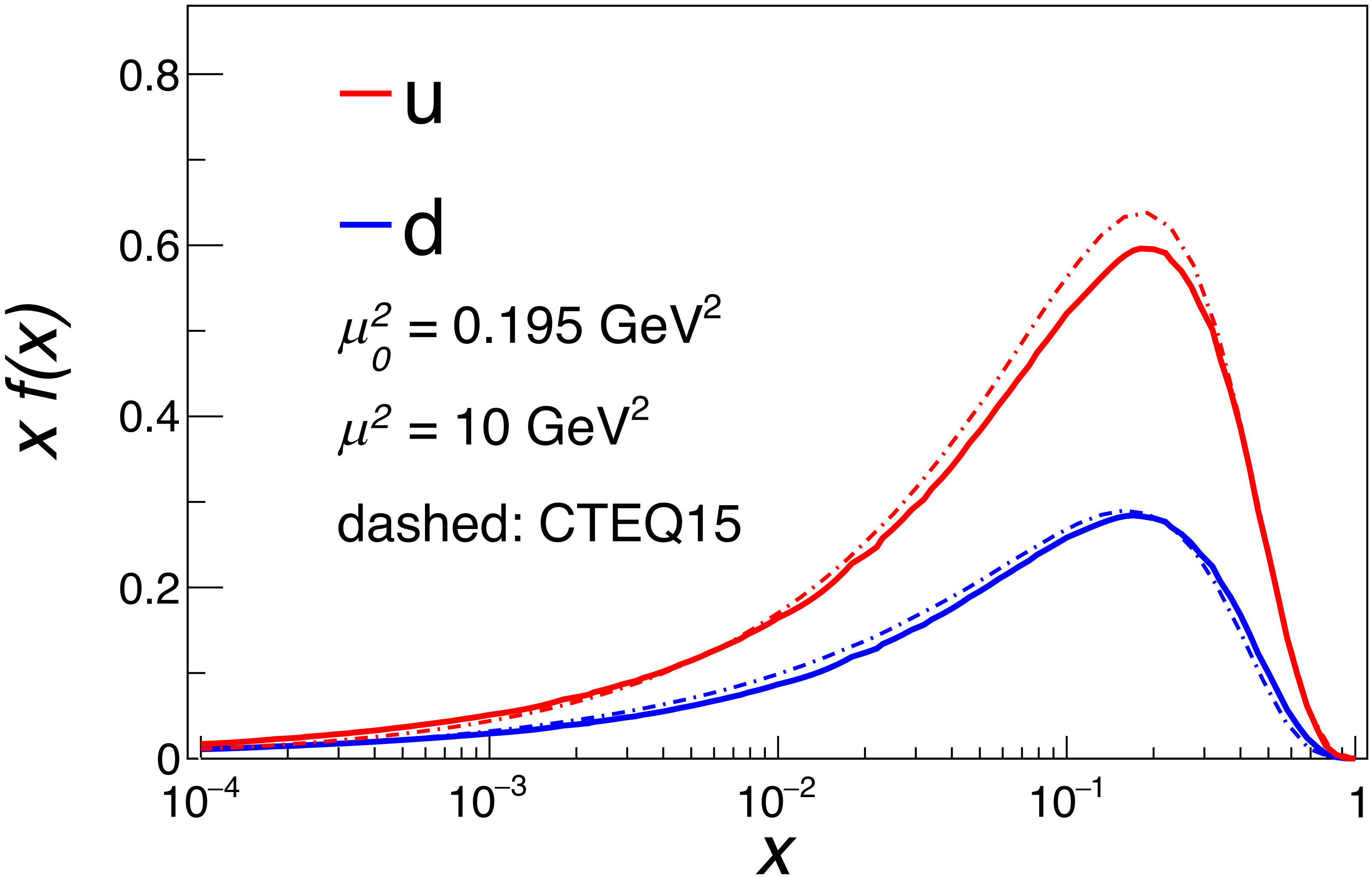
Anomalous magnetic moment

Proton : $\mu_N = 2.44$ (Exp. : 2.79)

Neutron : $\mu_N = -1.41$ (Exp. : -1.91)

Parton Distribution Functions (PDF)

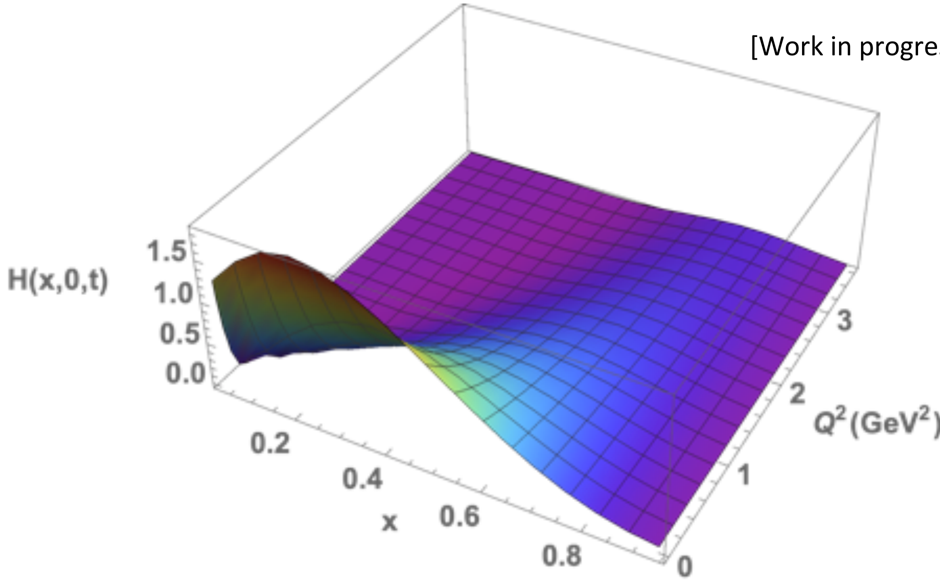
[Work in progress, C. Mondal, Siqi Xu, Jiangshan Lan, *et.al*]

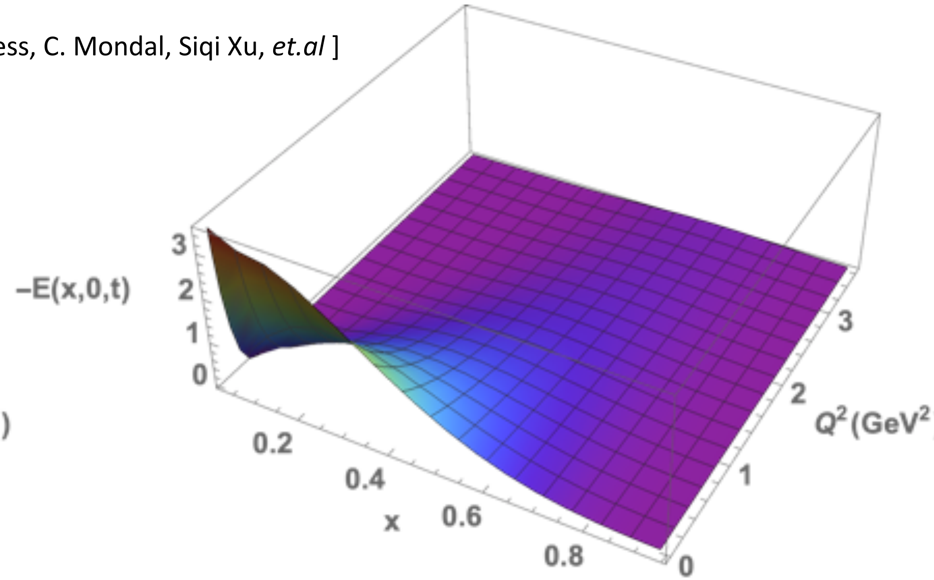


Use the NNLO DGLAP to evolve the PDF. Qualitative behavior is consistent with **the CTEQ 15 PDF**.

Generalized Parton Distribution Functions (GPD)

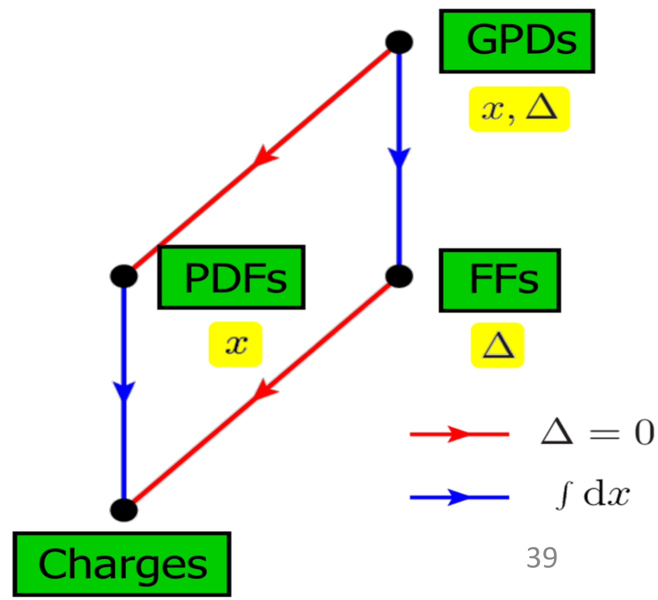
[Work in progress, C. Mondal, Siqi Xu, *et.al*]





With **increasing momentum transfer (t)**, the **peaks of distributions** shift to **larger x**;

$$t = \Delta^2, x = \frac{k^+}{P^+}, \boldsymbol{\zeta = \frac{\Delta^+}{P^+} = 0}$$



Chiral model of nucleon and pion

Relativistic $N\pi$ chiral Lagrangian density

$$\mathcal{L} = \underbrace{\frac{1}{4} f^2 \mathrm{Tr}\Big(\partial_\mu U\ \partial^\mu U^\dagger\Big) + \frac{1}{4} M_\pi^2 f^2 \mathrm{Tr}\Big(U + U^\dagger - 2\Big)}_{\text{pion field}}$$

$$+ \underbrace{\bar{N}\Big\{\gamma_\mu i\partial^\mu - M_N + \frac{1}{1+(\frac{\pi}{2f})^2}\Big[\frac{1}{2f}\gamma_\mu\gamma_5 \vec{\tau}\cdot\partial^\mu\vec{\pi} - (\frac{1}{2f})^2 \gamma_\mu \vec{\tau}\cdot\vec{\pi}\times\partial^\mu\vec{\pi}\Big]\Big\}N}_{\text{N}\pi\ \text{interaction}}$$

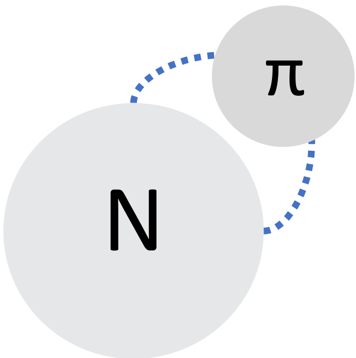
$$U = \frac{1 + i\gamma_5 \vec{\tau}\cdot\vec{\pi}/(2f)}{1 - i\gamma_5 \vec{\tau}\cdot\vec{\pi}/(2f)} = 1 + i\gamma_5 \frac{\vec{\tau}\cdot\vec{\pi}}{f} - \frac{1}{2f^2}\pi^2 + O\Big(\frac{1}{f^3}\Big)$$

f = 93 MeV: pion decay constant
M_π = 137 MeV: pion mass
M_N = 938 MeV: nucleon mass

[Miller, 1997]

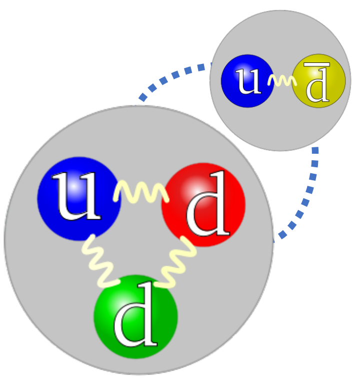
The idea: cloudy-bag model

$$|\text{proton}\rangle = a|N\rangle + b|N\pi\rangle + \mathrm{c}|N\pi\pi\rangle + d|N\bar{N}N\rangle + \ldots.$$



Step: 1

Solve the proton wave function in terms of the nucleon and pion

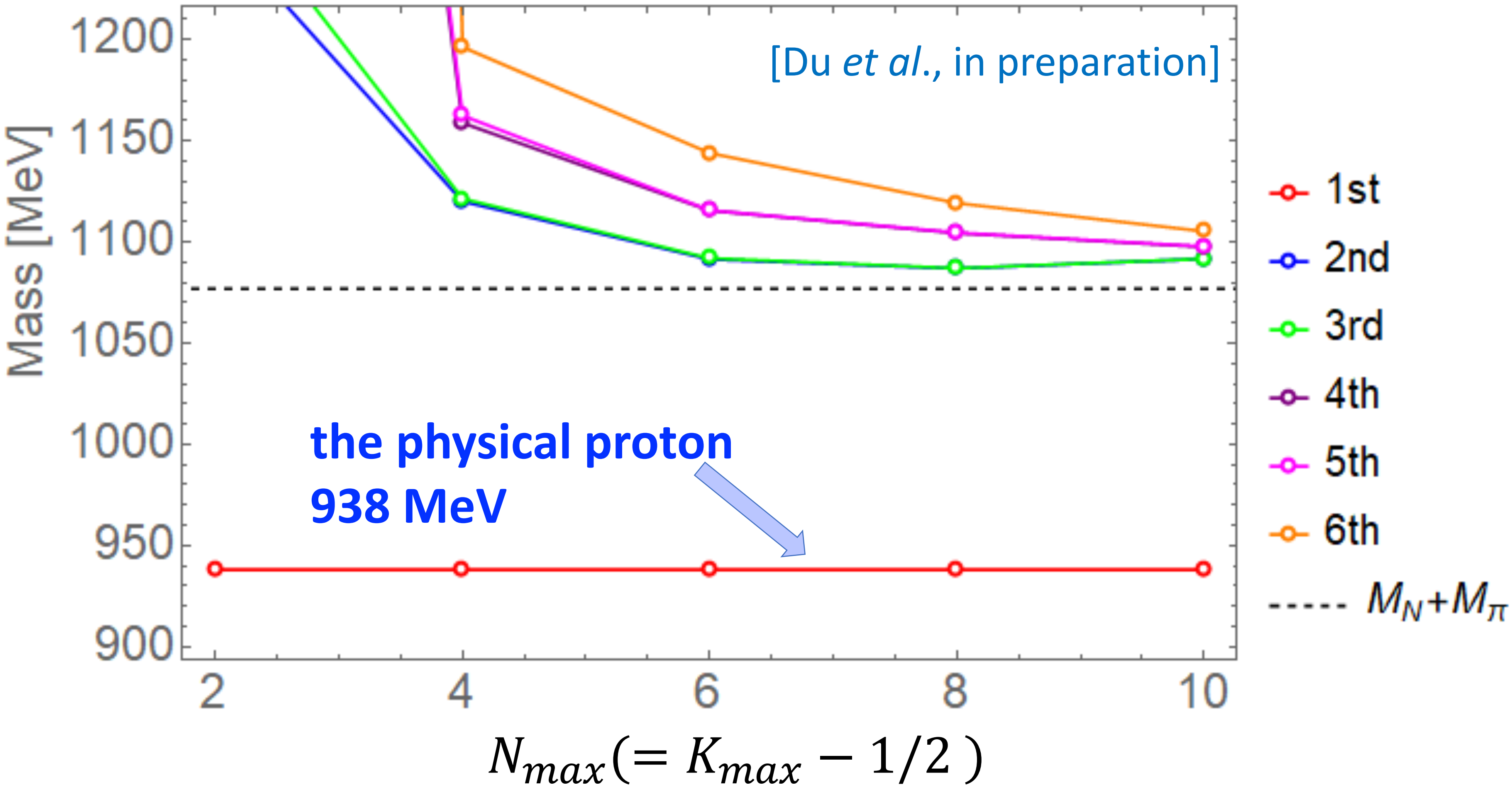


Step: 2

Attach the fundamental degree of freedom: quarks and gluons

$$|\text{proton}\rangle = a|uud\rangle + b|uudg\rangle + \mathrm{c}|uudgg\rangle + d|uudq\bar{q}\rangle + \ldots.$$

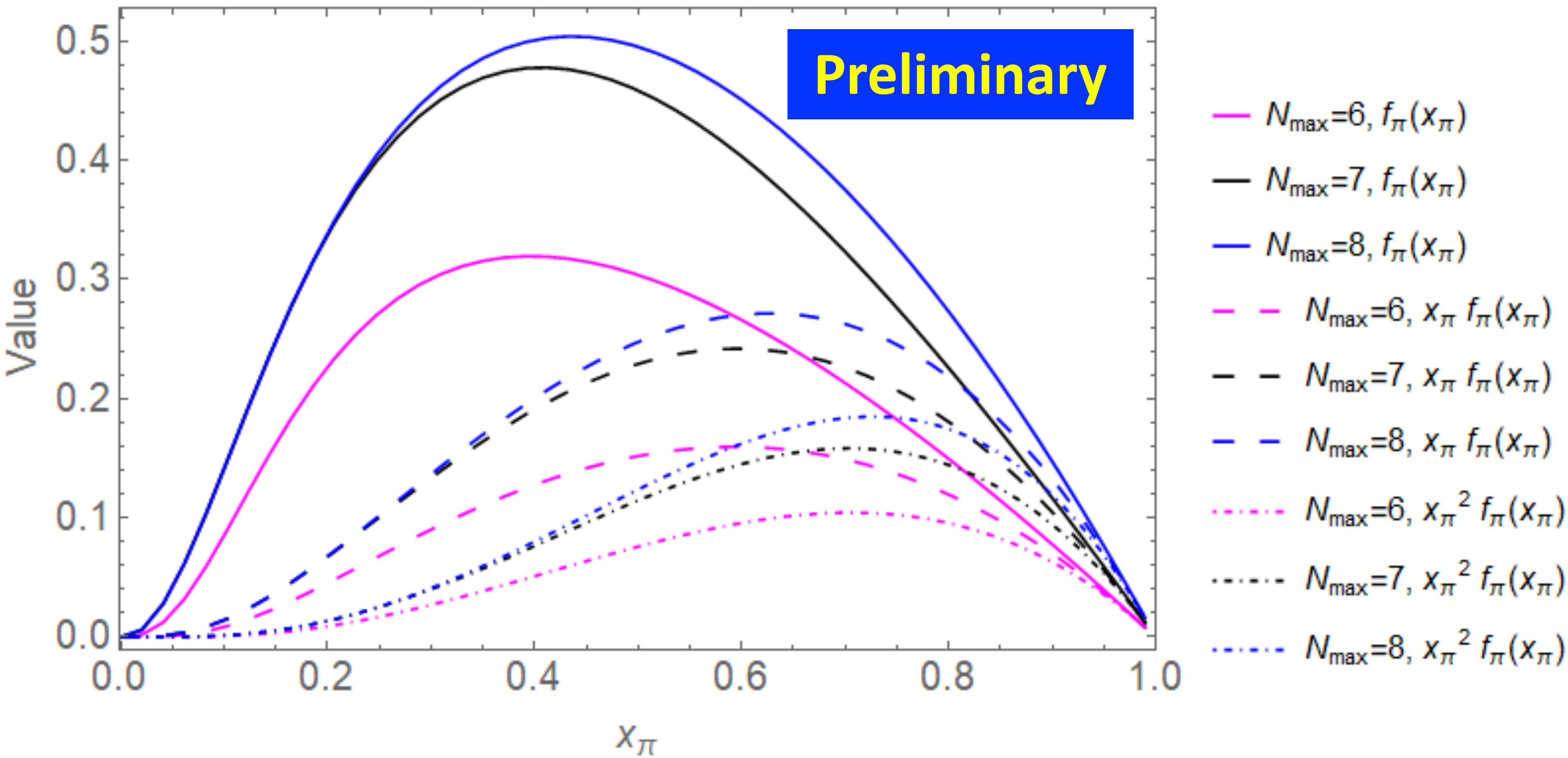
Mass spectrum of the $N\pi$ system



- Fock sector-dependent renormalization applied
- Mass counterterm applied to $|N\rangle$ sector only

[Karmanov et al, 2008, 2012]

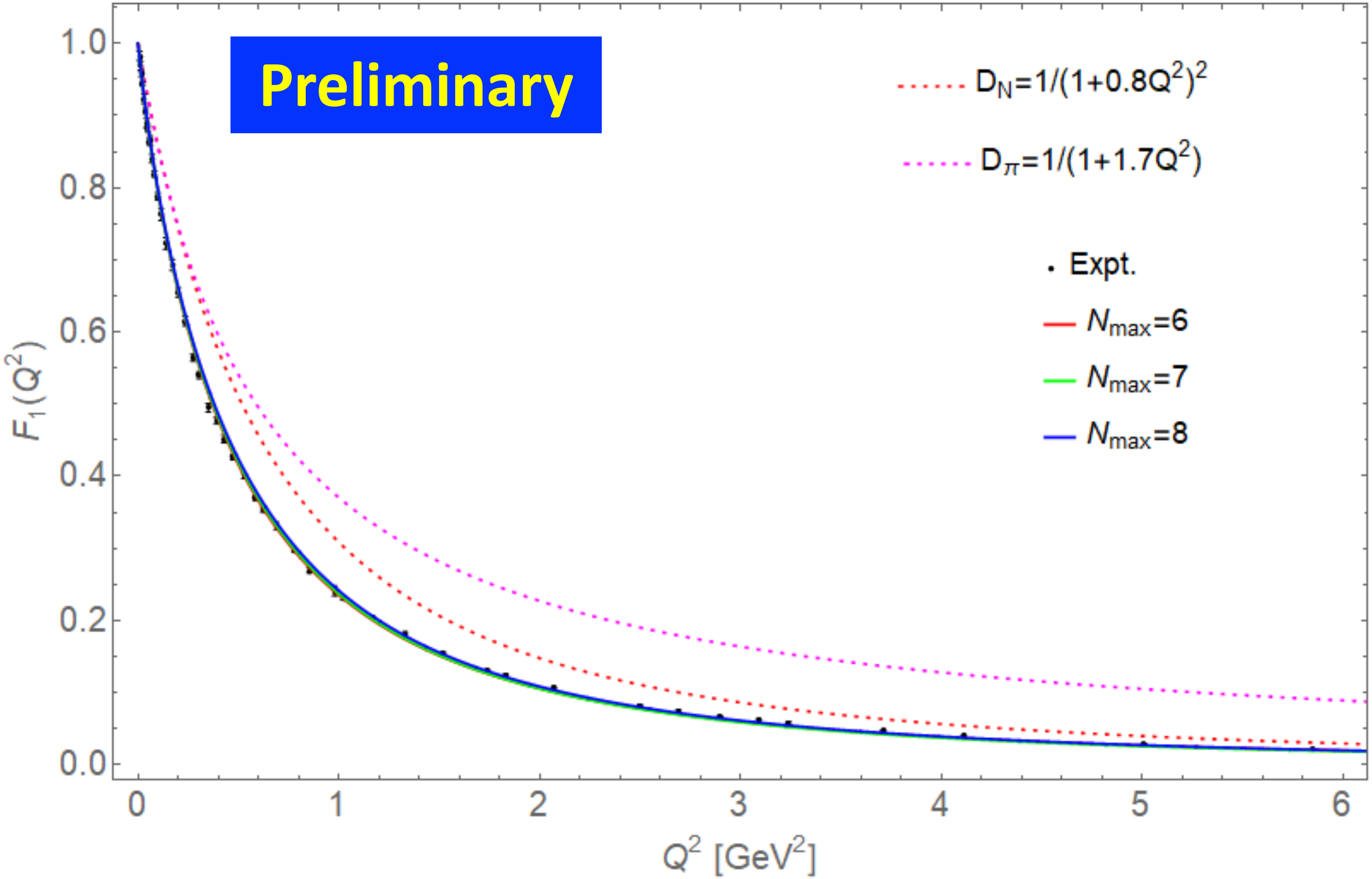
Proton parton distribution function



$$f_\pi(x_\pi) = {\sum}' a_\xi^* a_\xi$$

$$|\xi\rangle = |n_N, m_N, x_N, \lambda_N, t_N; n_\pi, m_\pi, x_\pi, \lambda_\pi, t_\pi\rangle$$

Proton Dirac form factor



Proton Dipole form with constituents' internal structures

$$F_1(\alpha_1,\alpha_2,q^2) = D_1(\alpha_1,q^2)\left[F^p_{1,f}(q^2) + F^{p\pi^0}_{1,f}(q^2)\right] + D_2(\alpha_2,q^2)F^{n\pi^+}_{1,b}(q^2)$$

Summary and Outlook

- Nonperturbative approach for relativistic bound states
- Access to (frame-independent) many-body wave functions
- Straightforward to calculate lightcone observables eg. PDFs
- Can be applied to effective/first-principles interactions
- Systematically improvable by including higher Fock sectors

- Apply to different systems: excited/exotic hadron states
- Apply to different interactions: NJL, Chiral EFT...
- More observables: GPD, TMD, GTMD...
- Apply to first-principles QCD interactions

Thank you for your attention!