

# On decays of $X(3872)$ to $\chi_{cJ}\pi^0$ and $J/\psi\pi^+\pi^-$ in the extended Friedrichs scheme

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arXiv:1904.07509, with Zhiguang Xiao and Meng-ting Yu

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- 5  $X(3872) \rightarrow \chi_{cJ}\pi^0, J/\psi\pi^+\pi^-, J/\psi\pi^+\pi^-\pi^0$

# Motivation

Uniqueness of  $X(3872)$ , right on the  $D^0\bar{D}^{0*}$  threshold

$$M_{X(3872)} = 3871.69 \pm 0.17 \text{ MeV}, \Gamma_{X(3872)} < 1.2 \text{ MeV}$$

Understandings:

- Molecular state
- Tetraquark state
- Mixing state of charmonium and continua
- .....

see Reviews:

H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, Phys. Rept. 639, 1 (2016)

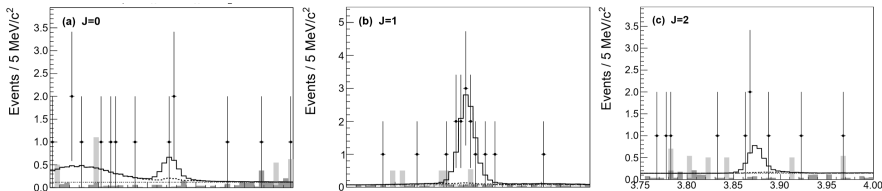
F.-K. Guo, C. Hanhart, U.-G. Meiner, Q. Wang, Q. Zhao, and B.-S. Zou, Rev. Mod. Phys. 90, 015004 (2018)

R. Lebed, R. Mitchell, E. Swanson, Prog.Part.Nucl.Phys. 93 (2017) 143-194

# Experiment efforts

BES  $e^+e^- \rightarrow \gamma X(3872)$  with  $X(3872) \rightarrow \chi_{cJ}\pi^0$ ,  $J=0,1,2$

M. Ablikim et al., Phys.Rev.Lett. 122 (2019) no.20, 202001



**Figure:** Distributions of  $\pi^0\chi_{cJ}$  mass,  $M_{\pi^0\chi_{cJ}}$ , from the process  $e^+e^- \rightarrow \gamma\pi^0\chi_{cJ}$  for (a)  $J=0$ , (b)  $J=1$ , and (c)  $J=2$ . The dashed line is the total background in the fit and includes contributions from events  $\pi^0$  with interchanged  $\gamma_1$  and  $\gamma_2$  and cross-feed among the search channels.

$$\frac{\mathcal{B}(X(3872) \rightarrow \chi_{c1}\pi^0)}{\mathcal{B}(X(3872) \rightarrow J/\psi\pi^+\pi^-)} = 0.88^{+0.33}_{-0.27} \pm 0.10. \quad (1)$$

Belle  $B^+ \rightarrow \chi_{c1}\pi^0 K^+$  V. Bhardwaj et al. Phys. Rev. D 99,(2019) 111101

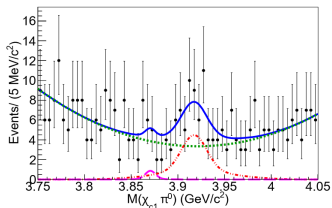


FIG. 2: 1D UML fit to the  $M_{\chi_{c1}\pi^0}$  distribution in the  $-30 \text{ MeV} < \Delta E < 20 \text{ MeV}$  signal region for the  $B^+ \rightarrow (\chi_{c1}\pi^0)K^+$  decay mode. The curves show the  $B^+ \rightarrow X(3872)(\rightarrow \chi_{c1}\pi^0)K^+$  signal (magenta dashed),  $B^+ \rightarrow X(3915)(\rightarrow \chi_{c1}\pi^0)K^+$  signal (red double dotted-dashed), and the background component (green dotted for combinatorial) as well as the overall fit (blue solid). Points with error bar represent the data.

No significant signal of  $X(3872) \rightarrow \chi_{c1}\pi^0$ .

$$\frac{\mathcal{B}(X(3872) \rightarrow \chi_{c1}\pi^0)}{\mathcal{B}(X(3872) \rightarrow J/\psi\pi^+\pi^-)} < 0.97 \quad (2)$$

at 90% confidence level.

# Motivation

S. Dubynskiy and M. B. Voloshin, Phys. Rev., D77, 014013 (2008)

$\Gamma_{\chi_{c0}\pi} : \Gamma_{\chi_{c1}\pi} : \Gamma_{\chi_{c2}\pi} = 0 : 2.7 : 1$  when assuming the  $X(3872)$  as a traditional charmonium state

$\Gamma_{\chi_{c0}\pi} : \Gamma_{\chi_{c1}\pi} : \Gamma_{\chi_{c2}\pi} = 2.88 : 0.97 : 1$  as a four-quark state.

Other calculations through EFT method (as a fourquark state)

S. Fleming and T. Mehen, Phys. Rev., D78, 094019 (2008)

Y.B.Dong et. al., Phys. Rev. D79, 094013 (2009),

S. Fleming and T. Mehen, Phys. Rev., D85, 014016 (2012)

T. Mehen, Phys. Rev., D92, 034019 (2015)

Theoretical calculation of the ratios of  $\chi_{cJ}\pi^0$ ,  $J/\psi\pi^+\pi^-$ ,  $J/\psi\pi^+\pi^-\pi^0$  by the constituent quark model is not found in the literature.

# How to describe $X(3872)$

Uniqueness of  $X(3872)$ , right on the  $D^0\bar{D}^{0*}$  threshold

$$M_{X(3872)} = 3871.69 \pm 0.17 \text{ MeV}, \Gamma_{X(3872)} < 1.2 \text{ MeV}$$

We use the extended Friedrichs scheme.

- Solid mathematical background for describing resonances
- Rigorous solution and non-perturbative
- Almost parameter-free

# Gamow state in rigged Hilbert space

Gamow state, a state with a complex eigenvalue, introduced by Gamow to describe nuclear  $\alpha$  decay. [G.Gamow](#) , *Z.Phys.* 51 (1928) 204-212

However, in the conventional QM, due to Hermitian of the Hamiltonian, the eigenvalues of Hamiltonian are real.

Rigged Hilbert Space (RHS) scheme, developed by Bohm and Gadella, provides a solid mathematical foundation for describing the unstable state.

[A.Bohm, M.Gadella, Dirac kets, Gamow vectors, and Gelfund Triplets, Springer Lectures Notes in Physics Vol.348, Springer, Berlin](#)

One of the properties of Gamow states:

$$\begin{aligned} H|z_R\rangle &= z_R|z_R\rangle, H|z_R^*\rangle = z_R^*|z_R^*\rangle \\ \langle z_R|H &= z_R^*\langle z_R|, \langle z_R^*|H = z_R\langle z_R^*| \\ \langle z_R|z_R\rangle &= 0, \langle z_R^*|z_R^*\rangle = 0 \end{aligned} \quad (3)$$

$|z_R\rangle$  and  $|z_R^*\rangle$  are not in Hilbert space but in RHS.

A plane wave state is also not in the Hilbert space but in RHS.



# The Friedrichs Model

[Friedrichs, Commun. Pure Appl. Math.,1(1948),361]

A free Hamiltonian  $H_0$  with a simple continuous spectrum,  $\mathbb{R}^+ \equiv [0, \infty)$ , plus a discrete eigenvalue  $\omega_0$  ( $\omega_0 > 0$ ). An interaction  $V$  between the continuous and discrete parts is introduced so that the discrete state of  $H_0$  is dissolved in the continuous spectrum and a resonance is produced.

$$H_0|1\rangle = \omega_0|1\rangle, H_0|\omega\rangle = \omega|\omega\rangle. \quad (4)$$

The free Hamiltonian is then

$$H_0 = \omega_0|1\rangle\langle 1| + \int_0^\infty \omega|\omega\rangle\langle\omega|d\omega, \quad (5)$$

and the interaction  $V$  is written as

$$V = \lambda \int_0^\infty [f(\omega)|\omega\rangle\langle 1| + f(\omega)^*|1\rangle\langle\omega|]d\omega. \quad (6)$$

The eigenvalue problem of  $H = H_0 + V$  is exactly solvable.

# The Friedrichs Model

[Friedrichs, Commun. Pure Appl. Math.,1(1948),361] solve the eigenstate  $|\Psi(x)\rangle$  of  $H = H_0 + V$  with eigenvalue  $x$ ,

$$H\Psi(x) = x|\Psi(x)\rangle. \quad (7)$$

Since  $|1\rangle$  and  $|\omega\rangle$  form a complete set, the eigenstate  $|\Psi(x)\rangle$  can be expressed in terms of  $|1\rangle$  and  $|\omega\rangle$ ,

$$|\Psi(x)\rangle = \alpha(x)|1\rangle + \int_0^\infty \psi(x, \omega)|\omega\rangle d\omega. \quad (8)$$

Note that here  $|\Psi\rangle$  is a vector in  $\Phi^\times$ , and it only make sense as an anti-linear functional. So,  $\psi(x, \omega)$  should be treated as a distribution. Substituting (8) into Eq.(7), one can obtain the following relations:

$$\begin{aligned} (\omega_0 - x)\alpha(x) + \lambda \int_0^\infty f(\omega)\psi(x, \omega)d\omega &= 0, \\ (\omega - x)\psi(x, \omega) + \lambda f(\omega)\alpha(x) &= 0. \end{aligned} \quad (9)$$

# The Friedrichs Model

[Friedrichs, Commun. Pure Appl. Math.,1(1948),361]

Then, for real  $x > 0$ , we have

$$\begin{aligned}\psi_{\pm}(x, \omega) &= -\frac{\lambda\alpha(x)f(\omega)}{\omega-x\pm i\epsilon} + \gamma(\omega)\delta(\omega-x), \\ (\omega_0-x)\alpha_{\pm}(x) + \lambda f(x)\gamma(x) - \alpha_{\pm}(x)\lambda^2 \int_0^{\infty} \frac{|f(\omega)|^2}{\omega-x\pm i\epsilon} d\omega &= 0.\end{aligned}\quad (10)$$

one can define the resolvent function

$$\eta^{\pm}(x) = x - \omega_0 - \lambda^2 \int_0^{\infty} \frac{|f(\omega)|^2}{x - \omega \pm i\epsilon} d\omega, \quad (11)$$

and analytically continue  $\eta^{\pm}$  to the complex plane  $\eta(x)$ , and  $\eta^+$  and  $\eta^-$  are the boundary functions of  $\eta(x)$  on the upper rim and lower rim of the cut on the positive axis, respectively.

# Resonance state

If  $\eta(x) = 0$  has a pair of complex conjugate solutions  $z_R \in \mathbb{C}_-$  and  $z_R^* \in \mathbb{C}_+$  on the second sheet, the right eigenstates for eigenvalue  $z_R$  and  $z_R^*$  can be expressed as

$$\begin{aligned} |z_R\rangle &= N_R \left( |1\rangle + \lambda \int_0^\infty d\omega \frac{f(\omega)}{[z_R - \omega]_+} |\omega\rangle \right), \\ |z_R^*\rangle &= N_R^* \left( |1\rangle + \lambda \int_0^\infty d\omega \frac{f(\omega)}{[z_R^* - \omega]_-} |\omega\rangle \right), \end{aligned} \quad (12)$$

which are the Gamow states satisfying  $H|z_R\rangle = z_R|z_R\rangle$  and  $H|z_R^*\rangle = z_R^*|z_R^*\rangle$ .

If  $\eta(x) = 0$  have a solution on the negative real axis on physical Riemann sheet, it represents a bound state. The bound state with eigenvalue  $z_B$  can then be represented as

$$|z_B\rangle = N_B \left( |1\rangle + \lambda \int_0^\infty \frac{f(\omega)}{z_B - \omega} |\omega\rangle d\omega \right) \quad (13)$$

where the normalization factor

$$N_B = (\eta'(z_B))^{-1/2} = \left( 1 + \lambda^2 \int d\omega \frac{|f(\omega)|^2}{(z_B - \omega)^2} \right)^{-1/2}.$$

# Virtual state

If  $\eta(x) = 0$  has a solution on the negative real axis of the second Riemann sheet, it corresponds to a virtual state.

For the simple virtual poles, similar to resonant states, there are two kinds of states,  $|z_v^+\rangle$  by analytical continuation from the upper rim and  $|z_v^-\rangle$  lower rim of the cut to the second sheet

$$|z_v^\pm\rangle = N_v^\pm \left( |1\rangle + \lambda \int_0^\infty \frac{f(\omega)}{[z_v - \omega]_\pm} |\omega\rangle d\omega \right), \quad \langle \tilde{z}_v^\pm | = \langle z_v^\mp |, \quad (14)$$

where  $N_v^- = N_v^{+*} = (\eta'^+(z_v))^{-1/2} = (1 + \lambda^2 \int d\omega \frac{|f(\omega)|^2}{[(z_v - \omega)_+]^2})^{-1/2}$ .

# The Extended Friedrichs Scheme

Z. Xiao and ZYZ, J. Math. Phys. 58, 072102 (2017)

Z. Xiao and ZYZ, J. Math. Phys. 58, 062110 (2017)

For a specific total angular momentum, the interaction between a discrete state  $|0\rangle$  having a bare energy eigenvalue  $m_0$ , and some continuum two-particle states  $|E; n, SL\rangle$ , where  $E$  is the bare energy eigenvalue in the center of mass system (c.m.s) and  $n, S, L$  denote the species, total spin and total orbital angular momentum, respectively, can be expressed in the form of extended Friedrichs scheme

$$H = m_0|0\rangle\langle 0| + \sum_{n,S,L} \int_{E_{th,n}}^{\infty} dE E|E; n, SL\rangle\langle E; n, SL| \\ + \sum_{n,S,L} \int_{E_{th,n}}^{\infty} dE f_{SL}^n(E)|0\rangle\langle E; n, SL| + h.c. \quad (15)$$

The coupling form factor between  $|A\rangle$  and  $|BC\rangle$  in the extended Friedrichs scheme by  $f_{SL}(E)$  could be calculated by theoretical models as the QPC model.

# The Extended Friedrichs Scheme

Scattering amplitudes could be expressed as

$$S_{fi}(E, E') = \delta(E - E') \left( \delta_{fi} - 2\pi i \frac{f_i(E) f_f^*(E)}{\eta^+(E)} \right). \quad (16)$$

where the resolvent  $\eta^\pm(x)$  is defined as

$$\eta^\pm(x) = x - m_0 - \sum_{n,S,L} \int_{E_{n,th}}^{\infty} \frac{|f_{SL}^n(E)|^2}{x - E \pm i0} dE. \quad (17)$$

The bound states, virtual states, or resonance state correspond to the zero points of  $\eta(z)$  on the different Riemann sheets.



# X(3872)

Applying the EFS to study the first radially excited P-wave charmonium state, the related poles could be obtained. [YZZ and Z. Xiao, Phys. Rev. D96, 054031 \(2017\)](#)

Table: Comparison of the experimental masses and the total widths (in MeV).

| $n^{2s+1}L_J$ | $M_{expt}$                         | $\Gamma_{expt}$           | $M_{BW}$ | $\Gamma_{BW}$ | pole                | GI   |
|---------------|------------------------------------|---------------------------|----------|---------------|---------------------|------|
| $2^3P_2$      | $3927.2 \pm 2.6$                   | $24 \pm 6$                | 3920     | 10            | 3920-4i             | 3979 |
| $2^3P_1$      | $3942 \pm 9$<br>$3871.69 \pm 0.17$ | $37^{+27}_{-17}$<br>< 1.2 | 3871     | 0             | 3934-40i<br>3871-0i | 3953 |
| $2^3P_0$      | $3862^{+66}_{-45}$                 | $201^{+242}_{-149}$       | 3878     | 11            | 3878-5i             | 3917 |
| $2^1P_1$      |                                    |                           | 3895     | 37            | 3902-27i            | 3956 |

For X(3872), the “elementariness” and “compositeness”

$$Z_{c\bar{c}} : X_{\bar{D}^0 D^{0*}} : X_{D^+ D^{*-}} : X_{\bar{D}^{*0} D^0} = 1 : (3.1 \sim 9.3) : (0.45 \sim 0.46) : 0.04$$

Only one free parameter  $\gamma = 4.0$ , so does the following calculation.

# X(3872) wave function

Wave function of the X(3872) is expressed explicitly as

$$\begin{aligned} |X(3872)\rangle = & N_B \left( |c\bar{c}\rangle + \int_{M_{00V}}^{\infty} dE \sum_{S,L} \frac{f_{SL}^{00V}(E)}{z_X - E} (|E; D^0 \bar{D}^{0*}, SL\rangle + |E; D^{0*} \bar{D}^0, SL\rangle) \right. \\ & + \int_{M_{+-V}}^{\infty} dE \sum_{S,L} \frac{f_{SL}^{+-V}(E)}{z_X - E} (|E; D^+ D^{-*}, SL\rangle + |E; D^{+*} D^-, SL\rangle) \\ & + \int_{M_{0V0V}}^{\infty} dE \sum_{S,L} \frac{f_{SL}^{0V0V}(E)}{z_X - E} |E; D^{0*} \bar{D}^{0*}, SL\rangle \\ & \left. + \int_{M_{+V-V}}^{\infty} dE \sum_{S,L} \frac{f_{SL}^{+V-V}(E)}{z_X - E} |E; D^{+*} D^{-*}, SL\rangle \right), \end{aligned} \quad (18)$$

where  $N_B = \eta'(z_X)^{-1/2}$  is the normalization factor and  $z_X$  the mass of X(3872).

$$X(3872) \rightarrow \chi_{cJ}\pi^0, J/\psi\pi^+\pi^-, J/\psi\pi^+\pi^-\pi^0$$

$d\Gamma(\alpha \rightarrow \beta) = 2\pi|M_{\beta\alpha}|^2\delta^4(p_{\beta_1} + p_{\beta_2} - p_{\alpha})d^3\vec{p}_{\beta_1}d^3\vec{p}_{\beta_2}$  where  $M_{\beta\alpha}$  is the transition amplitude.

$$\begin{aligned} F_{l's'} &= {}_{l's'}\langle\chi_{cJ}\pi^0|H_I|X(3872)\rangle = N_B\left(\chi_{cJ}\pi^0\langle E'|H_I|c\bar{c}\rangle\right. \\ &+ \int_{M_{00}}^{\infty}dE\sum_{l,s}\frac{f_{ls}^{00}(E)}{z_X-E}({}_{l's'}\langle E'|H_I|E\rangle_{ls}^{D^0\bar{D}^{0*}} + C.C.) \\ &+ \int_{M_{+-}}^{\infty}dE\sum_{l,s}\frac{f_{ls}^{+-}(E)}{z_X-E}({}_{l's'}\langle E'|H_I|E\rangle_{ls}^{D^+D^{-*}} + C.C.) \\ &\left. + \dots\right) \end{aligned} \quad (19)$$

The terms as  $\chi_{cJ}\pi^0\langle E'|H_I|E\rangle_{ls}^{D^0\bar{D}^{0*}}$  could be obtained by using the Barnes-Swanson model.

$$X(3872) \rightarrow \chi_{cJ}\pi^0, J/\psi\pi^+\pi^-, J/\psi\pi^+\pi^-\pi^0$$

## The Barnes-Swanson model

T. Barnes and E. S. Swanson, Phys. Rev. D 46, 131 (1992)

T. Barnes, N. Black, D. J. Dean, and E. S. Swanson, Phys. Rev. C60, 045202 (1999)

T. Barnes, N. Black, and E. S. Swanson, Phys. Rev. C63, 025204 (2001)

The interaction Hamiltonian of the Barnes-Swanson model in the momentum space is

$$T_{fi} = \left\{ \begin{array}{ll} -\frac{8\pi\alpha_s}{3m_1m_2} [\vec{S}_1 \cdot \vec{S}_2] & \text{Spin - spin} \\ \frac{4\pi\alpha_s}{q^2} I & \text{Coulomb} \\ \frac{6\pi b}{q^4} I & \text{Linear} \\ \frac{4i\pi\alpha_s}{q^2} \{ \vec{S}_1 \cdot [\vec{q} \times (\frac{\vec{p}_1}{2m_1^2} - \frac{\vec{p}_2}{m_1m_2})] + \vec{S}_2 \cdot [\vec{q} \times (\frac{\vec{p}_1}{m_1m_2} - \frac{\vec{p}_2}{2m_2^2})] \} & \text{OGE spin - orbit} \\ -\frac{3i\pi b}{q^4} [ \frac{1}{m_1^2} \vec{S}_1 \cdot (\vec{q} \times \vec{p}_1) - \frac{1}{m_2^2} \vec{S}_2 \cdot (\vec{q} \times \vec{p}_2) ] & \text{Linear spin - orbit} \\ \frac{4\pi\alpha_s}{m_1m_2q^2} [\vec{S}_1 \cdot \vec{q} \vec{S}_2 \cdot \vec{q} - \frac{1}{3}q^2 \vec{S}_1 \cdot \vec{S}_2] & \text{OGE tensor} \end{array} \right.$$

$$X(3872) \rightarrow \chi_{cJ}\pi^0, J/\psi\pi^+\pi^-, J/\psi\pi^+\pi^-\pi^0$$

## The Barnes-Swanson Model

Four kinds of diagrams are considered, among which the quark-antiquark interactions are denoted as  $Capture_1$ ,  $Capture_2$ , and the quark-quark(antiquark-antiquark) interactions are denoted as  $Transfer_1$ , and  $Transfer_2$ .

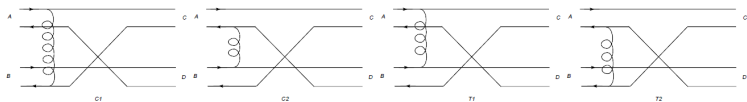


FIG. 2. The four quark rearrangement diagrams of  $AB \rightarrow CD$  meson-meson scatterings. The arrows represent the quark line directions.

To reduce the so-called “prior-poster” ambiguity, the four “poster” diagrams are considered similarly and averaged to obtain the final result.

$$X(3872) \rightarrow \chi_{cJ}\pi^0, J/\psi\pi^+\pi^-, J/\psi\pi^+\pi^-\pi^0$$

$$\begin{aligned}
 F_{l's'} &= l's' \langle \chi_{cJ}\pi^0 | H_I | X(3872) \rangle = N_B \left( \chi_{cJ}\pi^0 \langle E' | H_I | c\bar{c} \rangle \right. \\
 &+ \int_{M_{00}}^{\infty} dE \sum_{l,s} \frac{f_{ls}^{00}(E)}{z_X - E} (\chi_{cJ}\pi^0 \langle E' | H_I | E \rangle)_{ls}^{D^0\bar{D}^{0*}} + C.C.) \\
 &+ \int_{M_{+-}}^{\infty} dE \sum_{l,s} \frac{f_{ls}^{+-}(E)}{z_X - E} (\chi_{cJ}\pi^0 \langle E' | H_I | E \rangle)_{ls}^{D^+D^{-*}} + C.C.) + \dots
 \end{aligned}$$

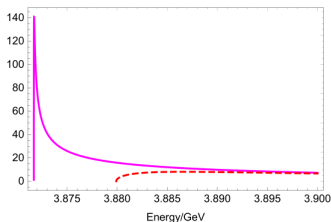


Figure: Comparison of the integrands  $\frac{f_{ls}\mathcal{M}_{l's',ls}}{(z_X-E)}$  for  $D^0\bar{D}^{0*} \rightarrow \chi_{c1}\pi^0$  (solid) and  $D^+\bar{D}^{-*} \rightarrow \chi_{c1}\pi^0$  (dashed).

# $X(3872) \rightarrow J/\psi \rho, \omega$

With the Barnes-Swanson model, one could obtain

$D\bar{D}^* \rightarrow \chi_{c0}\pi^0, \chi_{c1}\pi^0, \chi_{c2}\pi^0, J/\psi\rho, J/\psi\omega$  ( $\rho \rightarrow \pi\pi$  and  $\omega \rightarrow \pi\pi\pi$ ) the branch fraction will be

$$\Gamma_{\chi_{c0}\pi} : \Gamma_{\chi_{c1}\pi} : \Gamma_{\chi_{c2}\pi} : \Gamma_{J/\psi 2\pi} : \Gamma_{J/\psi 3\pi} = 1.5 : 1.3 : 1.0 : 16 : 26. \quad (21)$$

This numerical results of  $\Gamma_{J/\psi 2\pi} : \Gamma_{J/\psi 3\pi}$  are consistent with the measured ratio  $1.0 \pm 0.4 \pm 0.3$  by Belle,  $0.8 \pm 0.3$  by BABAR, and  $1.6_{-0.3}^{+0.4} \pm 0.2$  by BESIII. [hep-ex/0505037](https://arxiv.org/abs/hep-ex/0505037), [PRD82,011101](https://arxiv.org/abs/PRD82,011101), [PRL122, 232002](https://arxiv.org/abs/PRL122,232002)

A rough prediction of  $X(3872)$  decaying to P-wave  $\chi_{cJ}\pi^0$ ,  $J = 0, 1, 2$ , is about one order smaller than its decay to S-wave  $J/\psi\pi\pi$ .

# Comparisons with experiments

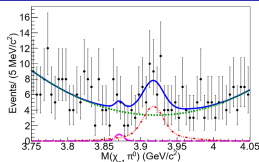
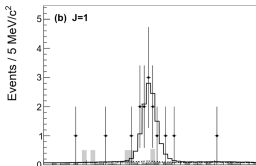


FIG. 2: 1D UML fit to the  $M_{\chi_{c1}\pi^0}$  distribution in the  $-30$  MeV  $< \Delta E < 20$  MeV signal region for the  $B^+ \rightarrow (\chi_{c1}\pi^0)K^+$  decay mode. The curves show the  $B^+ \rightarrow X(3872)(\rightarrow \chi_{c1}\pi^0)K^+$  signal (magenta dashed),  $B^+ \rightarrow X(3915)(\rightarrow \chi_{c1}\pi^0)K^+$  signal (red double dotted-dashed), and the background component (green dotted) for combinatorial as well as the overall fit (blue solid). Points with error bar represent the data.

BES

$$\frac{\mathcal{B}(X(3872) \rightarrow \chi_{c1}\pi^0)}{\mathcal{B}(X(3872) \rightarrow J/\psi\pi^+\pi^-)} = 0.88_{-0.27}^{+0.33} \pm 0.10. \quad (22)$$

Belle

No significant signal of  $X(3872) \rightarrow \chi_{c1}\pi^0$ .

Still an open question.



- The Extended Friedrichs Scheme might shed more light on hadron physics.
- With only one free parameter, the properties of  $X(3872)$  could be understood.
- The isospin breaking effect of  $X(3872)$  is understood easily.
- A rough prediction of  $X(3872)$  decaying to P-wave  $\chi_{cJ}\pi^0$ ,  $J = 0, 1, 2$ , is about one order smaller than its decay to S-wave  $J/\psi\pi\pi$ .

Thanks for your patience!

# Extra slides

# Extra slides