Three body open flavor decays of higher vector charmonium and bottomonium

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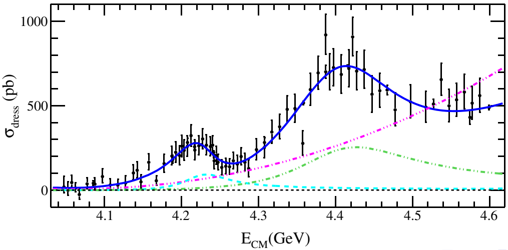
Outline

Introduction

- 2008, Belle, $e^+e^- \to D^0 D^- \pi^+$ PRL 100, 062001 (2008)

$$\frac{\mathcal{B}[\psi(4415)\to D^0 D^- \pi^+_{\text{nonresonant}}]}{\mathcal{B}[\psi(4415)\to D\bar{D}_2^*(2460)\to D^0 D^- \pi^+]} < 0.22 \quad (1)$$

- 2009, Belle, $e^+e^- \to D^0 D^{*-} \pi^+ + \text{c.c.}$: found no evidence of $\psi(4260)$, $\psi(4360)$, $\psi(4415)$, $\psi(4630)$ or $\psi(4660)$ with limited statistics PRD 80, 091101 (2009)
- 2018, BESIII, $e^+e^- \to D^0 D^{*-} \pi^+$: $Y(4220)$, $\psi(4415)$ PRL 122, 102002 (2019)

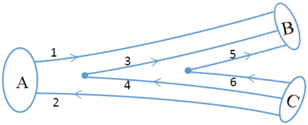


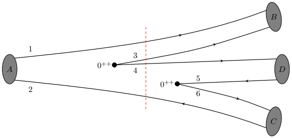
Introduction

- 2010, Belle, $\Upsilon(5S)\to B^{0,+}$ + others: PRD 81, 112003 (2010)

$$f(B\bar{B}\pi) = (0.0 \pm 1.1 \pm 0.3)\%, \tag{2}$$
$$f(B\bar{B}^*\pi + B^*\bar{B}\pi) = (7.3^{+2.3}_{-2.1} \pm 0.8)\%, \tag{3}$$
$$f(B^*\bar{B}^*\pi) = (1.0^{+1.4}_{-1.3} \pm 0.4)\%. \tag{4}$$

- $^3P_0$ model, $X(4660)[c\bar{c}] \to \Lambda_c\bar{\Lambda}_c$: Xiao *et al.* EPJC 78, 605 (2018)
  - $\Gamma_{\psi(4S,5S,6S)} \sim$ MeV
  - $\Gamma_{\psi(3D,4D,5D)} \sim 0.1$ MeV

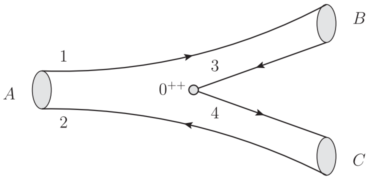




$^3P_0$ Model

- In the $^3P_0$ model, a light $q\bar{q}$ pair is created with the vacuum quantum number $J^{PC} = 0^{++}$, and then rearranged with the quarks within the initial meson to produce two final mesons.

$$J^{PC} = 0^{++} \Rightarrow L = 1\,, S = 1 \Rightarrow {}^{2S+1}L_J = {}^3P_0 \tag{5}$$



L. Micu, Nucl. Phys. B 10, 521 (1969).

Transition operator

- Interaction Hamiltonian PRD 54, 6811 (1996)

$$H_{\text{int}} = g_s \int \mathrm{d}^3\mathbf{x}\bar{\psi}\psi \tag{6}$$

where

$$\gamma = g_s/2m_q \tag{7}$$

- Decay amplitude

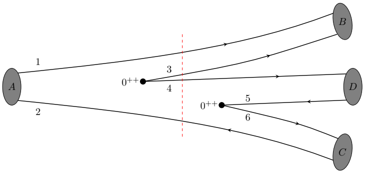
$$\delta^{(3)}\left(\mathbf{P}_f - \mathbf{P}_i\right)\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}} = \langle f \,|\, H_{\text{int}} \,|\, i\rangle \tag{8}$$

- In the nonrelativistic limit, the transition operator reads

$$T = -3\gamma\sum_m \langle 1m; 1-m \,|\, 00\rangle \int \mathrm{d}^3\mathbf{k}_4\mathrm{d}^3\mathbf{k}_5\delta^3(\mathbf{k}_4+\mathbf{k}_5)\mathcal{Y}_1^m\left(\frac{\mathbf{k}_4-\mathbf{k}_5}{2}\right)$$
$$\chi^{45}_{1,-m}\varphi^{45}_0\omega^{45}_0 b^\dagger_{4i}(\mathbf{k}_4)d^\dagger_{5j}(\mathbf{k}_5) \tag{9}$$

Transition operator

- Two pairs of $q\bar{q}$ are created in the three body open flavor decay processes



- At the 2nd order EPJC 78, 605 (2018)

$$\delta^{(3)}\left(\mathbf{P}_f - \mathbf{P}_i\right)\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}} = \sum_k \frac{\langle f \mid H_{\text{int}} \mid k\rangle \langle k \mid H_{\text{int}} \mid i\rangle}{E_k - E_i} \approx \frac{\langle f \mid H_{\text{int}} H_{\text{int}} \mid i\rangle}{2m_q} \quad (10)$$

where we take $E_k - E_i$ as a constant: $\boxed{E_k - E_i \approx 2m_q}$ (closure approximation). PRD 44, 799 (1991); PRL 67, 1066 (1991)

Closure approximation: $E_k - E_i \approx 2m_q$

- (focus on 1^{--} charmonium)
- quark level
  - intermediate state − initial state $\sim q\bar{q}$
- hadron level
  - Intermediate states: $E_k \sim (4.0 - 4.1)$ GeV:

$$D\bar{D}_1, D^*\bar{D}_1, D^*\bar{D}_0, D^*\bar{D}_2, J/\psi f_0(500),$$
$$h_c(1P)\pi, h_c(1P)\eta, \chi_{c0}(1P)\omega, \chi_{c2}(1P)\omega, \ldots$$

  - Higher mass states [$\psi(4660)$, $\psi(4415)$, $\psi(4360)$]: ✓
  - Lower mass states [$\psi(4040)$, $\psi(4160)$]: ×

Transition operator

- Transition operator $T_2 \sim T_1^2/2m_q$

$$T = \frac{9\gamma^2}{2m_q} \sum_{mm'} \langle 1m; 1-m \,|\, 00\rangle \langle 1m'; 1-m' \,|\, 00\rangle \int \mathrm{d}^3\mathbf{k}_3 \mathrm{d}^3\mathbf{k}_4 \mathrm{d}^3\mathbf{k}_5 \mathrm{d}^3\mathbf{k}_6$$
$$\times \delta^3(\mathbf{k}_3+\mathbf{k}_4)\delta^3(\mathbf{k}_5+\mathbf{k}_6)\mathcal{Y}_1^m\left(\frac{\mathbf{k}_3-\mathbf{k}_4}{2}\right)\mathcal{Y}_1^{m'}\left(\frac{\mathbf{k}_5-\mathbf{k}_6}{2}\right)$$
$$\times \chi^{34}_{1,-m}\varphi_0^{34}\omega_0^{34}a^\dagger_{3i}(\mathbf{k}_3)b^\dagger_{4j}(\mathbf{k}_4)\chi^{56}_{1,-m'}\varphi_0^{56}\omega_0^{56}c^\dagger_{5i'}(\mathbf{k}_5)d^\dagger_{6j'}(\mathbf{k}_6)$$

Mock state

- Meson

$$
\begin{aligned}
&\left|B\left(n_B^{2S_B+1}L_B J_B M_{J_B}\right)(\mathbf{P}_B)\right\rangle \\
=&\sqrt{2E_B}\sum_{M_{L_B}M_{S_B}}\langle L_B M_{L_B} S_B M_{S_B} \mid J_B M_{J_B}\rangle \\
&\times\int \mathrm{d}^3\mathbf{k}_a \mathrm{d}^3\mathbf{k}_b \delta^3(\mathbf{k}_a+\mathbf{k}_b-\mathbf{P}_B) \\
&\times\psi_{n_B L_B M_{L_B}}(\mathbf{k}_a,\mathbf{k}_b)\chi^{ab}_{S_B M_{S_B}}\varphi^{ab}_B\omega^{ab}_B\left|q_a(\mathbf{k}_a)\bar{q}_b(\mathbf{k}_b)\right\rangle
\end{aligned}
$$

- Normalization

$$
\langle B(\mathbf{P}_B) \mid B(\mathbf{P}'_B)\rangle = 2E_B\delta^3(\mathbf{P}_B-\mathbf{P}'_B) \tag{11}
$$

- Spatial wave function → simple harmonic oscillator (SHO).

C. Hayne and N. Isgur, Phys. Rev. D 25, 1944 (1982).

Decay width

- Helicity amplitude

$$\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}M_{J_D}}(A \to BCD) = \frac{\gamma^2}{2m_q}\sqrt{16E_AE_BE_CE_D} \times \sum_{mm'}\sum_{M_{L_{A,B,C,D}},M_{S_{A,B,C,D}}} \langle 1m;1-m|00\rangle\langle 1m';1-m'|00\rangle$$
$$\times \langle L_AM_{L_A}S_AM_{S_A}|J_AM_{J_A}\rangle\langle L_BM_{L_B}S_BM_{S_B}|J_BM_{J_B}\rangle\langle L_CM_{L_C}S_CM_{S_C}|J_CM_{J_C}\rangle$$
$$\times \langle L_DM_{L_D}S_DM_{S_D}|J_DM_{J_D}\rangle\langle\chi^{13}_{S_BM_{S_B}}\chi^{26}_{S_CM_{S_C}}\chi^{45}_{S_DM_{S_D}}|\chi^{12}_{S_AM_{S_A}}\chi^{34}_{1,-m}\chi^{56}_{1,-m'}\rangle$$
$$\times \langle\varphi^{13}_B\varphi^{26}_C\varphi^{45}_D|\varphi^{12}_A\varphi^{34}_0\varphi^{56}_0\rangle \times I^{M_{L_A}mm'}_{M_{L_B}M_{L_C}M_{L_D}}(\mathbf{p}),$$

- Decay width

$$\Gamma = \int_0^\infty dE_B dE_C \frac{\pi^3}{M_A}\frac{1}{2J_A+1}\sum_{M_{J_{A,B,C,D}}}\left|\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}M_{J_D}}\right|^2. \quad (12)$$

Parameter

- $\gamma_{c\bar{c}} = 6.95$, $\gamma_{b\bar{b}} = 10.42$ PRD 72, 054026 (2005); PRD 92, 054034 (2015)
- $m_u = m_d = 330$ MeV, $m_s = 419$ MeV,
  $m_c = 1628$ MeV, $m_b = 4977$ MeV PRD 32, 189 (1985)
- SHO strength β PRD 69, 054008 (2004); 72, 054026 (2005); 92, 054034 (2015); 93, 034035 (2016)

$$\int \mathrm{d}^3\mathbf{p} \left|\psi^{\text{SHO}}_{nlm}(\mathbf{p})\right|^2 p^2 = \int \mathrm{d}^3\mathbf{p} \left|\Phi(\mathbf{p})\right|^2 p^2$$

- $\beta_{c\bar{c}} = 500\pm50$ MeV
- $\beta_{b\bar{b}} = 600\pm50$ MeV
- $\alpha_{q\bar{q}} = 400$ MeV

TABLE II. Masses and harmonic oscillator strength β's of final state mesons used in the decays (in units of MeV).

Meson	State	Mass [55]	β [28,51]
π	$^1S_0$	138.0	400
ρ	$^3S_1$	775.3	400
ω	$^3S_1$	782.6	400
η	$^1S_0$	547.9	400
D	$^1S_0$	1867.2	600
D^*	$^3S_1$	2008.6	520
D_s	$^1S_0$	1968.3	650
D_s^*	$^3S_1$	2112.2	560
B	$^1S_0$	5279.5	580
B^*	$^3S_1$	5324.6	540
B_s	$^1S_0$	5366.9	640
B_s^*	$^3S_1$	5415.4	600

Decay width: $\psi(4360)$

TABLE III. The partial decay widths (in MeV) of the vector charmonium with a mass of 4368 MeV.

State	$\psi(4^3S_1)$	$\psi(3^3D_1)$
$\Gamma_{DD\pi}$	0.27	0.14
$\Gamma_{DD^*\pi}$	1.40	1.21
$\Gamma_{D^*D^*\pi}$	0.60	0.25
$\Gamma_{DD\eta}$	0.6 keV	0.3 keV

- Possible assignment: $\psi(3D)$, $\psi(4S)$ PRD 79, 094004 (2009); Int. J. Mod. Phys. E 22, 1330026 (2013).
- S- and D-waves are of same order.
- $DD^*\pi$ mode is dominant.

$$\mathcal{B}[\psi(4^3S_1)\to DD^*\pi] \sim 1.5\%, \quad \mathcal{B}[\psi(3^3D_1)\to DD^*\pi] \sim 1.3\%.$$

Decay width: $\psi(4415)$

TABLE IV. The partial decay widths (in MeV) of the vector charmonium with a mass of 4421 MeV.

State	$\psi(4^3S_1)$	$\psi(5^3S_1)$	$\psi(3^3D_1)$
$\Gamma_{DD\pi}$	0.38	0.11	0.21
$\Gamma_{DD^*\pi}$	2.01	0.96	1.84
$\Gamma_{D^*D^*\pi}$	1.07	0.59	0.52
$\Gamma_{DD\eta}$	5.4 keV	1.7 keV	2.9 keV

- Assignment: $\psi(4S)$ PLB 72, 57 (1977); PRD 72, 054026 (2005);
- $\mathcal{B}[\psi(4S)\to DD^*\pi] \sim 3.2\%$ — PDG upper limit (11%) PRD 98, 030001 (2018)
- $\mathcal{B}[\psi(4S)\to DD\pi] \sim 0.6\%$ — Belle upper limit (2.2%) PRL 100, 062001 (2008)
- Other assignments: $\psi(5S)$, $\psi(3D)$ PRD 79, 094004 (2009); Int. J. Mod. Phys. E 22, 1330026 (2013)

Decay width: $\psi(4660)$

TABLE I: The possible assignments of the $Y(4660)$ with the predicted masses (MeV) from various models.

State	QM [46]	QM [47]	QM [48]	SSE/EA[49]	NR/GI [50]	SP [10]	LP/SP [51]
$\psi(4^3S_1)$	4625	4450	4389	4398/4426	4406/4450	4273	4412/4281
$\psi(5^3S_1)$	...	...	4641	4642/4672	...	4463	4711/4472
$\psi(6^3S_1)$	...	...	...	4804/4828	...	4608	...
$\psi(3^3D_1)$	...	4520	4426	4464/4477	...	4317	4478/4336
$\psi(4^3D_1)$	...	...	4641	4690/4707	...	...	...
$\psi(5^3D_1)$	...	...	...	4840/4855	...	...	...

TABLE V. The partial decay width (MeV) of the vector charmonium with a mass of 4643 MeV.

State	$\psi(4^3S_1)$	$\psi(5^3S_1)$	$\psi(6^3S_1)$	$\psi(3^3D_1)$	$\psi(4^3D_1)$	$\psi(5^3D_1)$
$\Gamma_{DD\pi}$	1.14	0.31	0.09	0.63	0.17	0.05
$\Gamma_{DD^*\pi}$	6.65	2.83	1.10	6.99	2.99	1.16
$\Gamma_{D^*D^*\pi}$	5.97	2.68	1.13	4.12	2.11	0.96
$\Gamma_{DD\rho}$	0.85	0.41	0.16	1.86	0.64	0.22
$\Gamma_{DD\omega}$	0.24	0.12	0.05	0.59	0.20	0.07
$\Gamma_{DD\eta}$	53.2 keV	15.3 keV	4.2 keV	29.1 keV	8.2 keV	2.2 keV
$\Gamma_{DD^*\eta}$	0.25	0.12	0.05	0.20	0.11	0.05
$\Gamma_{D^*D^*\eta}$	58.2 keV	38.7 keV	19.1 keV	8.5 keV	9.9 keV	7.5 keV
$\Gamma_{D_sD_s\eta}$	3.0 keV	0.8 keV	0.2 keV	1.6 keV	0.4 keV	0.1 keV
$\Gamma_{D_sD_s^*\eta}$	1.9 keV	1.3 keV	0.6 keV	12 eV	11 eV	7 eV

- Possible assignment: $\psi(4S, 5S, 6S)$, $\psi(3D, 4D, 5D)$.
- $D^{(*)}D^{(*)}\pi$ modes dominant.
- $DD\rho$, $DD\omega$ and $DD^*\eta$ modes are also important.

Decay width

- The variation of the $\Gamma_{D^+D^-\pi^0}$ with the mass of the charmonium

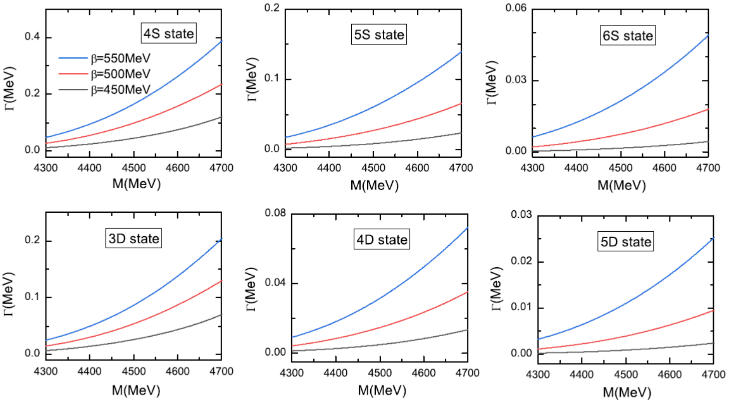


FIG. 2. The variation of the $D^+D^-\pi^0$ partial decay width with the mass of the D-wave vector charmonium. Note that $\Gamma_{D^+D^-\pi^0} = \frac{1}{6}\Gamma_{DD\pi}$ since we have ignored the isospin breaking. The blue, red, and black lines correspond to the predictions with different values of the harmonic oscillator strength $\beta = 450, 500$, and 550 MeV, respectively.

Decay width

- The variation of the $\Gamma_{D^+D^{*-}\pi^0}$ with the mass of the charmonium

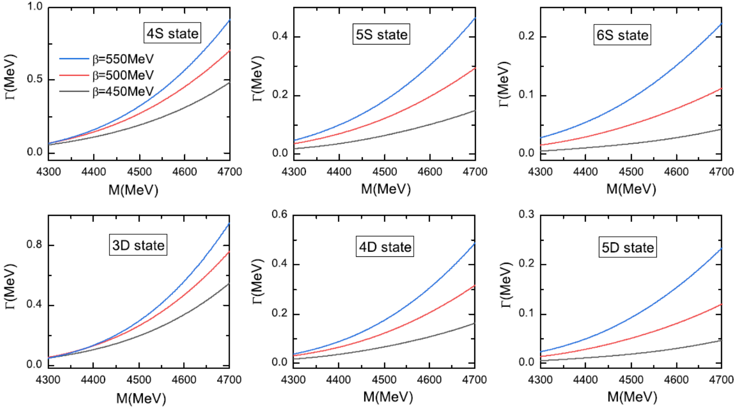


FIG. 3. The variation of the $D^+D^{*-}\pi^0$ partial decay width with the mass of the D-wave vector charmonium. Note that $\Gamma_{D^+D^{*-}\pi^0} = \frac{1}{12}\Gamma_{DD^*\pi}$ since we have ignored the isospin breaking. The blue, red, and black lines correspond to the predictions with different values of the harmonic oscillator strength $\beta = 450, 500,$ and 550 MeV, respectively.

Decay width

- The variation of the $\Gamma_{D^{*+}D^{*-}\pi^0}$ with the mass of the charmonium

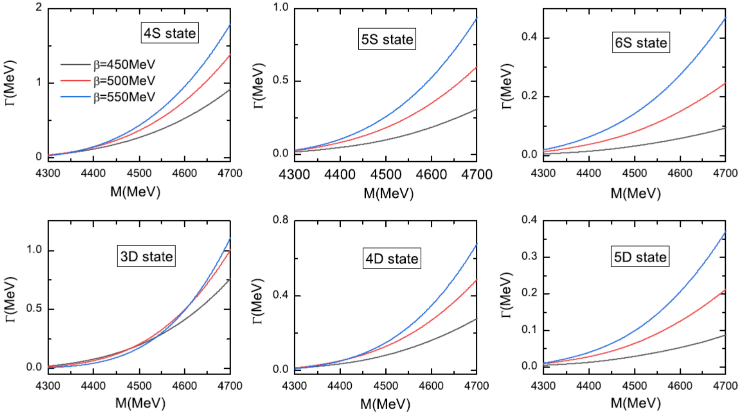


FIG. 4. The variation of the $D^{*+}D^{*-}\pi^0$ partial decay width with the mass of the D-wave vector charmonium. Note that $\Gamma_{D^{*+}D^{*-}\pi^0} = \frac{1}{6}\Gamma_{D^*D^*\pi}$ since we have ignored the isospin breaking. The blue, red, and black lines correspond to the predictions with different values of the harmonic oscillator strength $\beta = 450, 500$, and 550 MeV, respectively.

Decay width

- The partial widths are sensitive to mass.
- The partial widths increased as the mass increase.
- partial widths are not sensitive to β.

$\Upsilon(10860)$, $\Upsilon(11020)$

TABLE VII. The $B^{(*)}\bar{B}^{(*)}\pi$ partial decay widths of the vector bottomonium (in units of MeV). $\mathcal{B}_{\rm exp}$ represents the branching ratio for each corresponding channel.

			$\beta = 550$ MeV		$\beta = 600$ MeV		$\beta = 650$ MeV			
Meson	State	Mode	$\Gamma_{\rm th}$	$\mathcal{B}_i$	$\Gamma_{\rm th}$	$\mathcal{B}_i$	$\Gamma_{\rm th}$	$\mathcal{B}_i$	[36]	$\mathcal{B}_{\rm exp}$ [55]
$\Upsilon(10860)$	5^3S_1	$BB\pi$	0.12	0.2%	0.20	0.4%	0.28	0.5%		$(0.0 \pm 1.2)\%$
		$BB^*\pi$	1.36	2.7%	1.22	2.4%	0.94	1.8%	(23–30) keV	$(7.3 \pm 2.3)\%$
		$B^*B^*\pi$	0.68	1.3%	0.61	1.2%	0.47	0.9%	(5–6.6) keV	$(1.0 \pm 1.4)\%$
$\Upsilon(11020)$	6^3S_1	$BB\pi$	0.17	0.3%	0.34	0.7%	0.55	1.1%		
		$BB^*\pi$	2.50	5.1%	3.17	6.5%	3.41	7.0%		
		$B^*B^*\pi$	2.12	4.3%	2.69	5.5%	2.97	6.1%		

- $\Upsilon(10860)$: $\mathcal{B}_{BB\pi}$ and $\mathcal{B}_{B^*B^*\pi}$ are consistent with the exp. data, while $\mathcal{B}_{BB^*\pi}$ is smaller but very close to the Belle's measurement.
- $\Upsilon(11020)$: $\mathcal{B}_{BB^*\pi}$ and $\mathcal{B}_{B^*B^*\pi}$ are quite large.

PLB 671, 55 (2009)

Summary

- Extended the $^3P_0$ model to study the three body open flavor decays of heavy quarkonium
- Charmonium
  - $\Gamma[\psi(4360)\to DD^*\pi]$ can reach up to 1 MeV.
  - $\mathcal{B}[\psi(4415)\to DD^{(*)}\pi]$ are within the exp. upper limits.
  - $\psi(4660)$ decays dominantly into $D^{(*)}D^{(*)}\pi$.
- Bottomonium
  - $\mathcal{B}[\Upsilon(5S)\to BB\pi/B^*B^*\pi]$ are consistent with the exp.
  - $\mathcal{B}[\Upsilon(5S)\to BB^*\pi]$ is smaller but close to the Belle measurement.
  - $\Gamma[\Upsilon(6S)\to BB^*\pi/B^*B^*\pi]$ can reach up to several MeV.

Thank You!