# Properties of $X(3872)$ beyond effective range expansion

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# Introduction

- A series of exotic hadron candidate XYZ were and are observed, cannot be accommodated by potential model — kinematical effects, molecular, quark-gluon hybrid, et al..
- Typically different scenarios predicts different constituents, in practice, may involve several mechanism
  $\longrightarrow$ compositeness
- Stay close to threshold of meson pairs: only 2-3 MeV above meson pair threshold
  $\longrightarrow$ effective range expansion (ERE)

# Compositeness for resonance

- Weinberg compositeness condition: wave function renormalization constant $Z$=0. In fact, $Z = 1 - X$, where $X = -\gamma^2 \frac{dG(s_R)}{ds_R}$ quantifies the weight of constituents; $\gamma$ is the residue for $t(s)$ in the 1st sheet at the pole, and $G$ is the two-point loop function.
- only applied to bound state — model-independent relation for deuteron [Weinberg 1963; 1965]
- For resonance case, as long as $\sqrt{\text{Re}E_R^2}$ larger than the lightest threshold, $X = |\gamma^2 \frac{dG(s_R)}{s_R}|$, $\gamma$ residue in the 2nd sheet [Guo and Oller, PRD2015]
- Adapted to non-relativistic case, criterion: $M_R > M_{\text{th}}$, applied to $Z_b$ and $Z_c$ states [Kang, Guo and Oller, PRD2016]

# Inclusion of CDD pole and ERE

- Only right-hand cut without crossed-channel effect [Oller and Oset, PRD1999] $t(E) = \left[\sum_i \frac{g_i}{E - M_{\text{CDD},i}} + \beta - ik\right]^{-1}$
- ERE: $t(E) = \left[-1/a + 1/2\, r\, k^2 - ik\right]^{-1}$
- Expansion of Re $t(E)^{-1}$ in powers of $k^2$ is equivalent to ERE, but worry for the small scale $[M_{\text{CDD}} - M_{\text{th}}]$, which restricts the validity range.
- $M_{\text{CDD}}$ far away from $M_{\text{th}}$, then modulu of $r$ is around 1 fm, otherwise $r$ is very large.

$$1/a = \frac{g_i}{M_{\text{CDD}} - M_{\text{th}}} - \beta, \quad r = -\frac{g_i}{\mu(M_{\text{th}} - M_{\text{CDD}})^2}$$

- $M_{\text{CDD}}$ close to $M_{\text{th}}$, then small $X$, i.e., containing also other important components, e.g., compact quark-gluon states; $M_{\text{CDD}}$ far from $M_{\text{th}}$, then the two meson constitute dominates

# Observation of the $X(3872)$

- First observation from Belle, PRL2003, triggering voluminous amount of papers
- PDG determination:

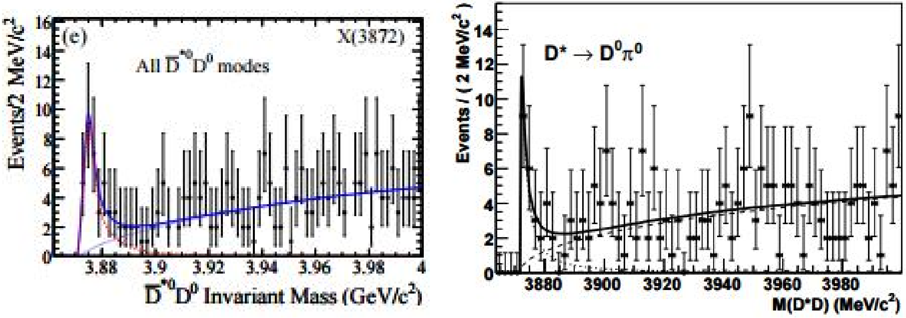
$$
\begin{aligned}
& I^G(J^{PC}) = 0^+(1^{++}), \\
& M = 3871.69 \pm 0.17 \text{ MeV}, \Gamma < 1.2 \text{ MeV}, \ \text{CL} = 90\% \\
& \bar{D}D^* : C = + \text{ combination } (D\bar{D}^* + \bar{D}D^*)/\sqrt{2} \\
& \text{threshold } M_{\text{th}} = 3871.81 \text{ MeV}
\end{aligned}
$$

- From now on, all the energy and $M_{\text{CDD}}$ are measured respective to $M_{\text{th}}$. X(3872) mass: $-0.11 \pm 0.17$ MeV
- Nature: molecular like virtual state (V) and bound state (B), or preexisting state, etc.

# Nature of $X(3872)$

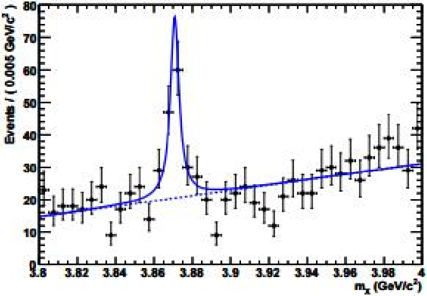
- V or B scenarios are typically based on ERE analysis [Hanhart et al PRD76,034007('07); Braaten et al PRD81, 014019('10)]
- But as mentioned, a nearby CDD pole around threshold could spoil ERE strongly. Recall previous slide!
- all these scenarios can not be excluded
- New points: these are obtained by parameterizing data in more general terms
  - a simultaneous virtual and bound state (V+B);
  - double/triple virtual state $\rightarrow$ higher-order $S$-matrix pole.

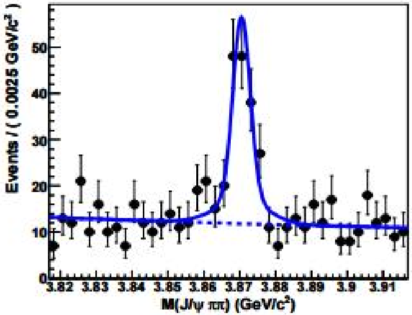
# Experimental situation: $\bar{D}^0 D^{*0}$ channel




- the decay chain: $B \to X(3872)K \to \bar{D}^0 D^{*0} K$
- Left: BaBar2008, Right: Belle2010
- BaBar has total number of $B\bar{B}$ pairs, $N_{B\bar{B}}^{\text{BaBar}} = 3.83 \cdot 10^8$, while $N_{B\bar{B}}^{\text{Belle}} / N_{B\bar{B}}^{\text{BaBar}}$=1.75

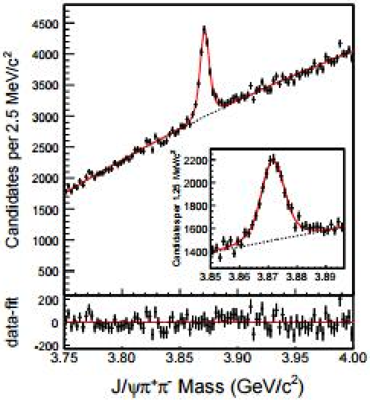
# Experimental situation: $J/\psi\pi\pi$ channel




- the decay chain: $B \to X(3872)K \to J/\psi\pi^+\pi^-K$
- Left: BaBar2008, Right: Belle2008
- Data are compatible with each other.

# Experimental situation: $J/\psi\pi\pi$ channel continued




- the decay chain: $p\bar{p} \to X(3872) +$ all with $X(3872) \to J/\psi\pi^+\pi^-$
- The inset shows an enlargement of the region around the $X(3872)$ peak, with very small bin width of 1.25 MeV.
- *"Precision Measurement of the $X(3872)$ in $J/\psi\pi\pi$ Decays"* from CDF2009.

# Formalism (1)

- Exp summary: Belle $D\bar{D}\pi$ + BaBar $J/\psi\pi\pi$ + Belle $J/\psi\pi\pi$ + CDF $J/\psi\pi\pi$
- As introduced, scattering amplitude

$$t(E) = \left( \frac{\lambda}{E - M_{\text{CDD}}} + \beta - ik(E) \right)^{-1},$$

  more general than ERE

- Removing the extra zeros due to the CDD pole, one ends with the final-state interaction

$$d(E) = \left( 1 + \frac{E - M_{\text{CDD}}}{\lambda} (\beta - ik) \right)^{-1}$$

  [Oller PLB2000, Bugg PLB2003]

- When $M_{\text{CDD}}$ far, $M_{\text{CDD}} \to \infty$ keeping $\lambda / M_{\text{CDD}}$ fixed, one recovers the scattering length approximation

$$t(E) \Longrightarrow f(E) = \frac{1}{-\lambda / M_{\text{CDD}} + \beta - ik} = \frac{1}{-\gamma - ik}$$

# Formalism (2)

- The normalized standard non-relativistic mass distribution for a narrow resonance or bound state ($\Gamma_X \to 0$)

$$\frac{d\hat{M}}{dE} = \frac{\Gamma_X |d(E)|^2}{2\pi|\alpha|^2}$$

- $\alpha$ is a constant, obtained by singling out the pole contribution, in fact, the residue of $d(E)$, $d(E) \sim \frac{\alpha}{E-E_p}$, $E_p$ pole position.
- Normalization integral $\mathcal{N} = \int_{-\infty}^{\infty} dE \frac{d\hat{M}}{dE}$
- For a narrow resonance (including bound state), $\mathcal{N} \approx 1$, but not so when $d(E)$ has a shape strongly departs from a non-relativistic Breit-Wigner, e.g., for a virtual state
- For $f(E)$ (ERE), the integral does not converge, just integrate in the signal region.

# Formalism (3): event distribution for $J/\psi\pi\pi$ channel

- For $B \to KJ/\psi\pi\pi$ channel [simpler]:

$$N_i = 2N_{B\bar{B}}\left[\mathcal{B}_J \int_{E_i-\Delta/2}^{E_i+\Delta/2} dE' \int_{-\infty}^{\infty} dE R(E', E)\frac{d\hat{M}}{dE} + \mathrm{cbg}_J\Delta\right]$$

- For $p\bar{p}$ to $J/\psi\pi\pi$ channel: just replace $2N_{B\bar{B}}$ by $\mathcal{L}\sigma_{p\bar{p}\to X\mathrm{All}}$, with $\mathcal{L}$ luminosity, and total cross section $\sigma$ for $p\bar{p} \to X + \mathrm{All}$.
- $R(E', E)$ is the Gaussian, experimental resolution function
- $\int_{E_i-\Delta/2}^{E_i+\Delta/2} dE'$ indicates the integration in the bin width.

# Formalism (4): event distribution for $\bar{D}^0 D^{*0}$ channel

- For $B \to K \bar{D}^0 D^{*0}$ channel [taking into account the small width of $D^*$, $\Gamma_* \approx 65$ KeV]

$$N_i = 2N_{B\bar{B}} \int_{E_i-\Delta/2}^{E_i+\Delta/2} dE' \int_0^\infty d\mathcal{E}' R(E', \mathcal{E}') \sqrt{\mathcal{E}'}$$
$$\times \left[ \frac{\mathcal{B}_D \Gamma_*}{\sqrt{2}\pi \left( \sqrt{E_X^2 + \Gamma_*^2/4} - E_X \right)^{1/2}} \int_{-\infty}^{\infty} dE \frac{d\hat{M}}{dE} \frac{1}{|\mathcal{E}' - E - i\Gamma_*/2|^2} + \mathrm{cbg}_D \right]$$

- Pole position $E_X - i\Gamma_X/2$, with $E_X$ relative to $\bar{D}^0 D^{*0}$ (reduced mass $\mu \approx 1$GeV) threshold, momentum at pole position $k_X$.

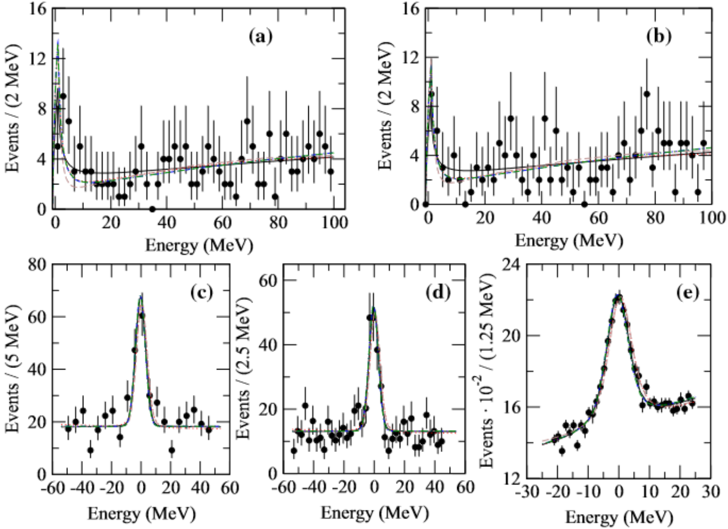
# Fit strategy

- $\mathcal{B}_D$, $\mathcal{B}_J$, $\text{cbg}_D$, $\text{cbg}_J$, overall constants and background are always free parameters
- As mentioned, for bound state, $\mathcal{B}$ can be justified as branching ratio, otherwise not.
- $M_{\text{CDD}}, \lambda, \beta$ characterize the line shape of $d(E)$.
- Braaten et al has also used ERE (only scattering length), but fit separately
- C. Hanhart et al used ERE including the effective range.
- One should use more general $d(E)$, other than ERE!
- Maximize likelihood fit, data errors are asymmetric.

# Different scenarios

- Case i): using ERE, which is a pure bound state, making combined fits to all the existing data
- Case ii): imposing $t(E)$ has a virtual state, one can express $\lambda$ and $\beta$ from $\underline{E_R \text{ and } M_{\mathrm{CDD}}}$.
- Case iii): taking into account the coupled channel effect, $\underline{E_R}$ determines the shape of $t(E)$:
  Quadratic equation: two solutions with case iii). I and case iii).II

# Results


(a)
(b)
(c)
(d)
(e)
Events / (2 MeV)
Events / (5 MeV)
Events / (2.5 MeV)
Events · 10$^{-2}$ / (1.25 MeV)
Energy (MeV)

# Results

| Cases | Pole position [MeV] | $X$ | Residue [GeV$^2$] | $Y_{D1}$ <br> $Y_{D2}$ <br> $Y_J$ <br> $Y_J^{(p)}$ |
|---|---|---|---|---|
| 1 | $-0.19^{+0.01}_{-0.01} - i\,0.0325$ | 1.0 | $14.78^{+0.38}_{-0.14}$ | $7.49^{+0.71}_{-0.41}$ |
| | | | | $6.45^{+0.32}_{-0.47}$ |
| | | | | $79.03^{+5.65}_{-6.11}$ |
| | | | | $5.23^{+0.07}_{-0.11} \times 10^3$ |
| 2.I | $-0.36^{+0.08}_{-0.10} - i\,0.18^{+0.01}_{-0.02}$ | | $-47.48^{+9.75}_{-12.40} - i\,66.06^{+10.87}_{-13.50}$ | $83.13^{+22.42}_{-16.15}$ |
| | $-0.70^{+0.11}_{-0.13} + i\,0.17^{+0.02}_{-0.01}$ | | $82.69^{+14.84}_{-11.88} + i\,66.03^{+13.50}_{-10.87}$ | $40.13^{+11.86}_{-7.25}$ |
| | | | | $8.44^{+3.64}_{-2.59} \times 10^3$ |
| | | | | $5.78^{+2.29}_{-1.65} \times 10^5$ |
| 2.II | $-0.33^{+0.04}_{-0.03} - i\,0.31^{+0.02}_{-0.01}$ | | $-6.24^{+2.80}_{-2.20} - i\,1.41^{+0.14}_{-0.10} \times 10^2$ | $79.75^{+22.46}_{-19.81}$ |
| | $-0.84^{+0.07}_{-0.05} + i\,0.77^{+0.03}_{-0.04}$ | | $(2.32^{+0.16}_{-0.21} - i\,1.77^{+0.11}_{-0.08}) \times 10^2$ | $42.20^{+9.18}_{-8.02}$ |
| | $-1.67^{+0.10}_{-0.08} - i\,0.49^{+0.02}_{-0.02}$ | | $(-3.26^{+0.22}_{-0.16} + i\,3.18^{+0.18}_{-0.25}) \times 10^2$ | $9.23^{+1.60}_{-1.57} \times 10^3$ |
| | | | | $6.23^{+0.71}_{-0.84} \times 10^5$ |
| 3.I | $-0.50^{+0.04}_{-0.03}$ | $0.061^{+0.003}_{-0.002}$ | $1.52^{+0.01}_{-0.01}$ | $25.45^{+4.05}_{-4.15}$ |
| | $-0.68^{+0.05}_{-0.03}$ | | $2.72^{+0.02}_{-0.04}$ | $12.29^{+1.32}_{-1.89}$ |
| | | | | $80.14^{+5.67}_{-5.19}$ |
| | | | | $5.26^{+0.12}_{-0.08} \times 10^3$ |
| 3.II | $-0.51^{+0.03}_{-0.01}$ | $0.158^{+0.001}_{-0.001}$ | $3.96^{+0.03}_{-0.08}$ | $22.90^{+2.94}_{-3.02}$ |
| | $-1.06^{+0.05}_{-0.02}$ | | $7.56^{+0.08}_{-0.20}$ | $11.03^{+1.40}_{-0.77}$ |
| | | | | $80.07^{+5.14}_{-5.36}$ |
| | | | | $5.28^{+0.05}_{-0.17} \times 10^3$ |

# Highlights

- New! Case ii): double/triple virtual-state pole, which degenerates to one in the limit of vanishing $D^*$ width, an example of higher-order $S-$matrix pole. Residues for this case are very large.
- New! Case iii): As mentioned, in this scenario, a simultaneous virtual and bound state, where CDD pole is very close to threshold, resulting a very small compositeness, typically a few percent, which is a preexisting state, also fits with Morgan criterion [Morgan, NPA1992].