REGGE TRAJECTORIES IN LIGHT AND HEAVY MESONS: THE PATTERN OF APPEARANCES AND POSSIBLE DYNAMICAL EXPLANATIONS

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Content

PART I. Light non-strange mesons: linear Regge trajectories, spectral degeneracies and qualitative string explanation

PART II. From light to heavy mesons

PART III.* A novel dynamical picture for natural emergence of linear Regge trajectories

PART I

TOWARDS UNDERSTANDING THE GENERAL PICTURE OF LIGHT NON-STRANGE MESONS

Still unanswered questions in the spectroscopy of light mesons:

1. What spectral regularities and symmetries do we observe?

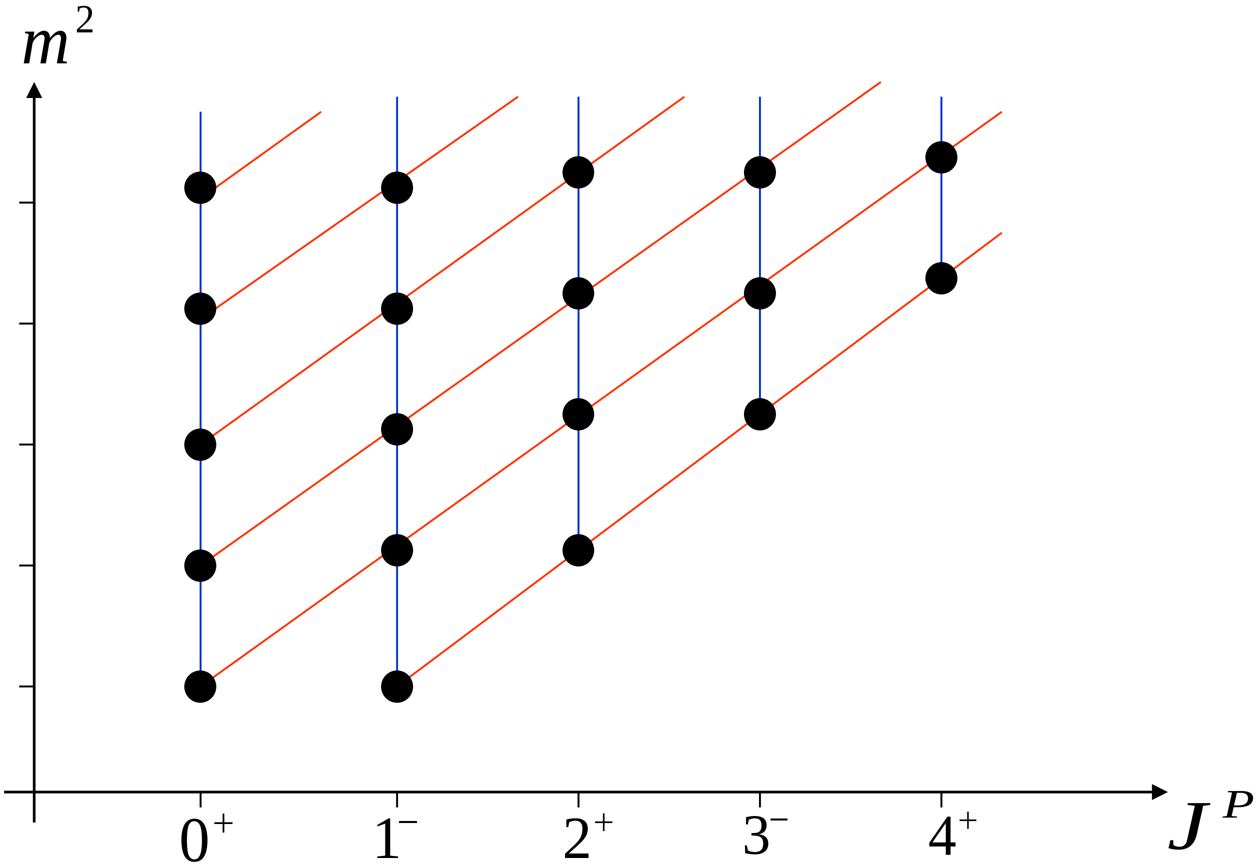
2. How are they related to the fundamental theory (QCD)?

Symmetries

fundamental   dynamical

For example $J \sim M^2$

Regge and radial Regge linear trajectories



$$m^2(J) = m_0^2 + \alpha' J$$ – Regge trajectories

$$m^2(n) = \mu_0^2 + \alpha n$$ – Radial Regge trajectories

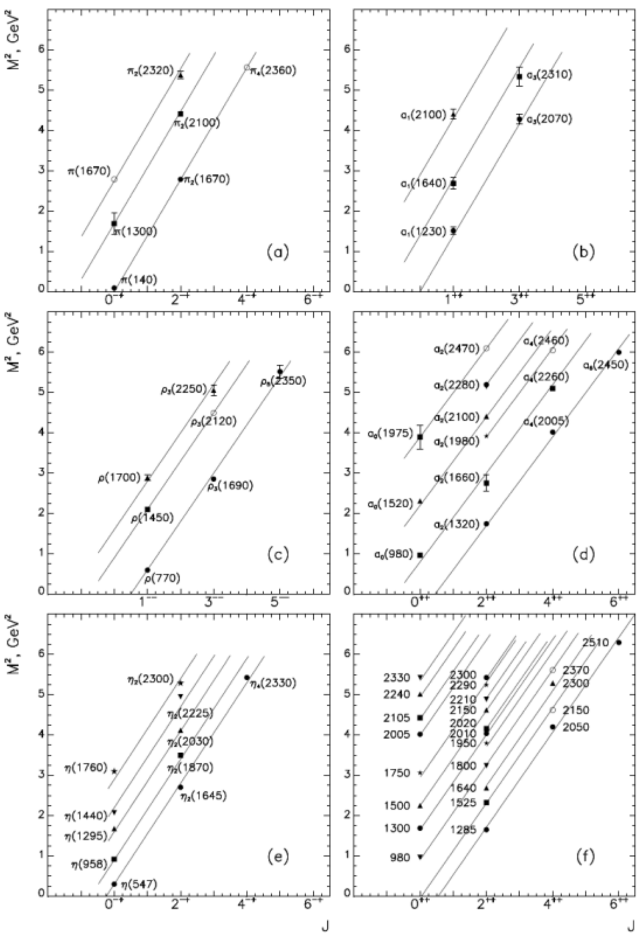
Rich source of spectral data on the light mesons – proton-antiproton annihilation

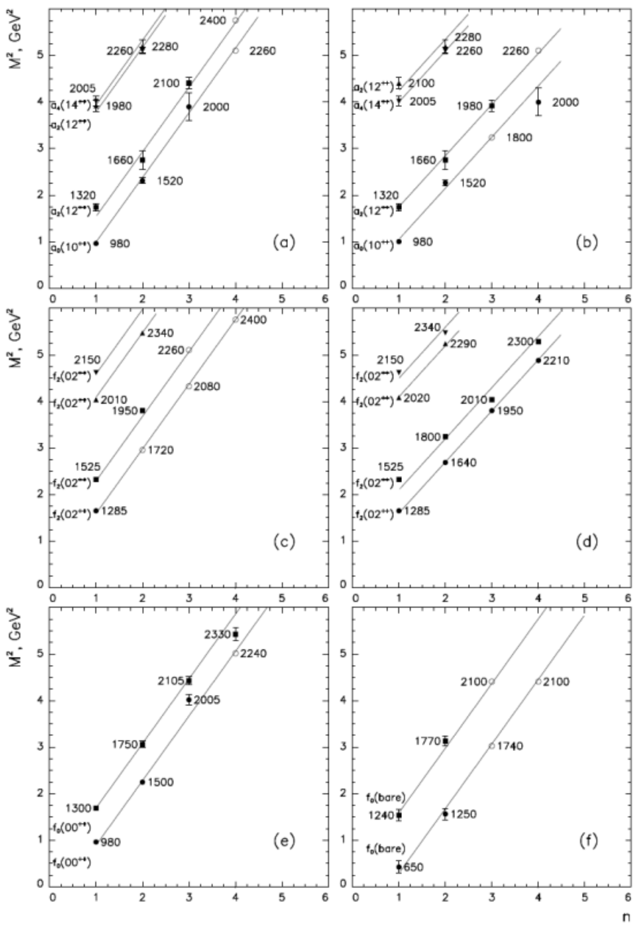
CRYSTAL BARREL

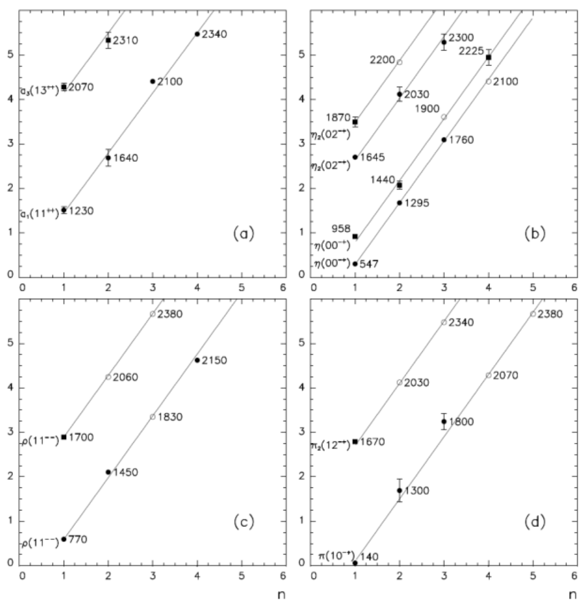
A.V. Anisovich, V.V. Anisovich and A.V. Sarantsev, Phys. Rev. D 62 (2000) 051502

D.V. Bugg, Phys. Rept. 397 (2004) 257

Many new states in 1.9-2.4 GeV range!







Examples

Doubling of some trajectories:

L=0 (*S*-wave): $J = \uparrow\uparrow = ½ + ½ = 1$

$q\bar{q}$

L=2 (*D*-wave): $J = \big|\ \downarrow\downarrow = 2 - ½ - ½ = 1$

$L\ q\bar{q}$

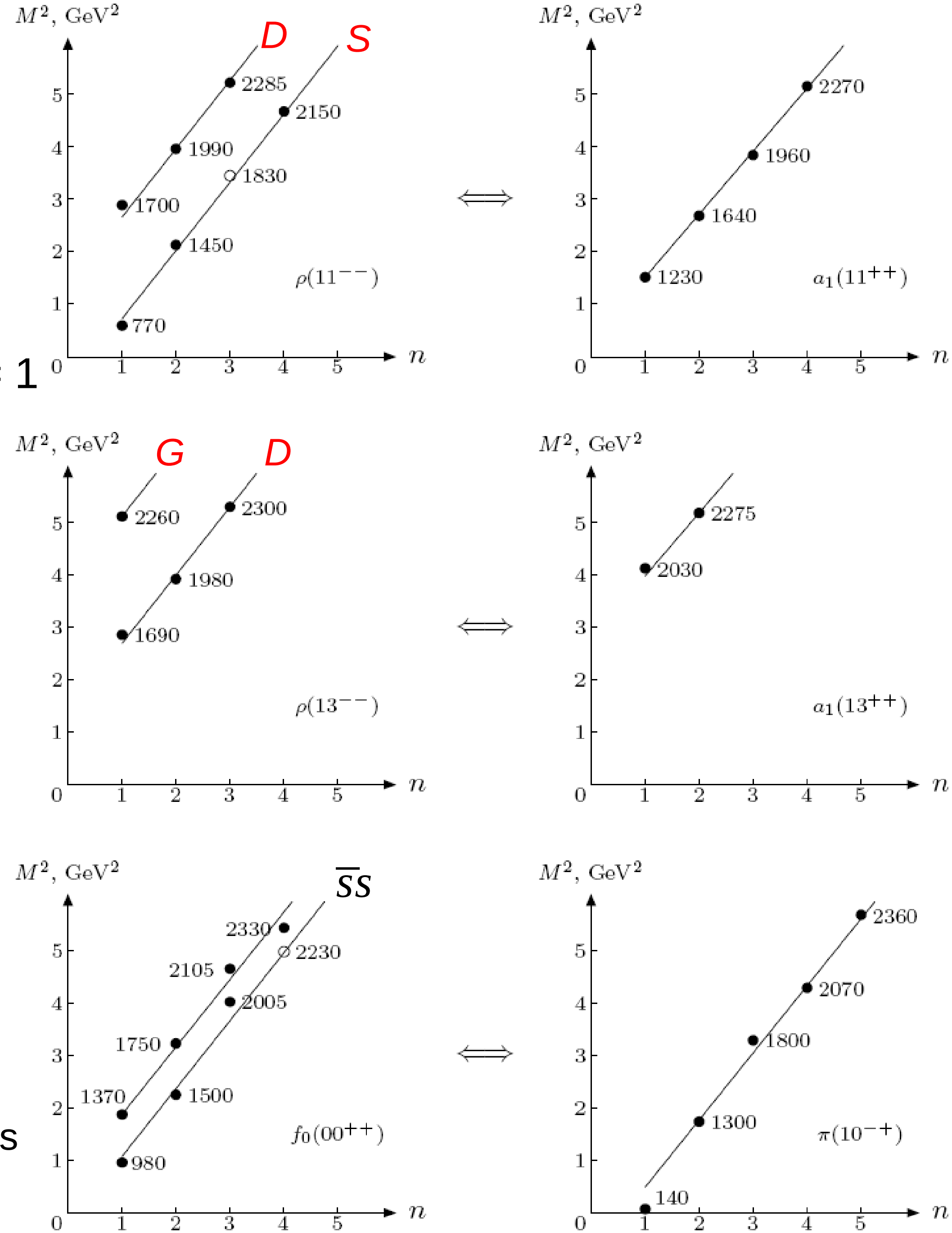
⟹ Two kinds of ρ

Linear trajectories:

$$M_n^2 = a(n+b)$$

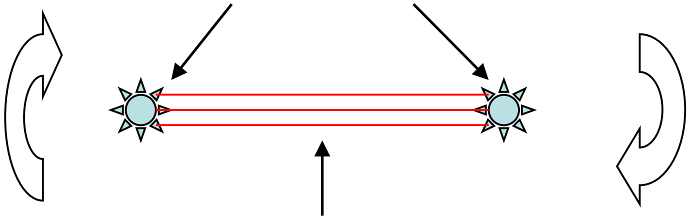
Approximately universal slope

Intercept depends on quantum numbers



A concept of hadron string model

Hadron string picture for mesons:



Rotating string with relativistic massless quarks at the ends $M^2 = 2\pi\sigma L$

σ - string tension, L - angular momentum $(J = L, L \pm 1)$

Bohr-Sommerfeld quantization $\int p(r)dr = 2\pi(n+b)$

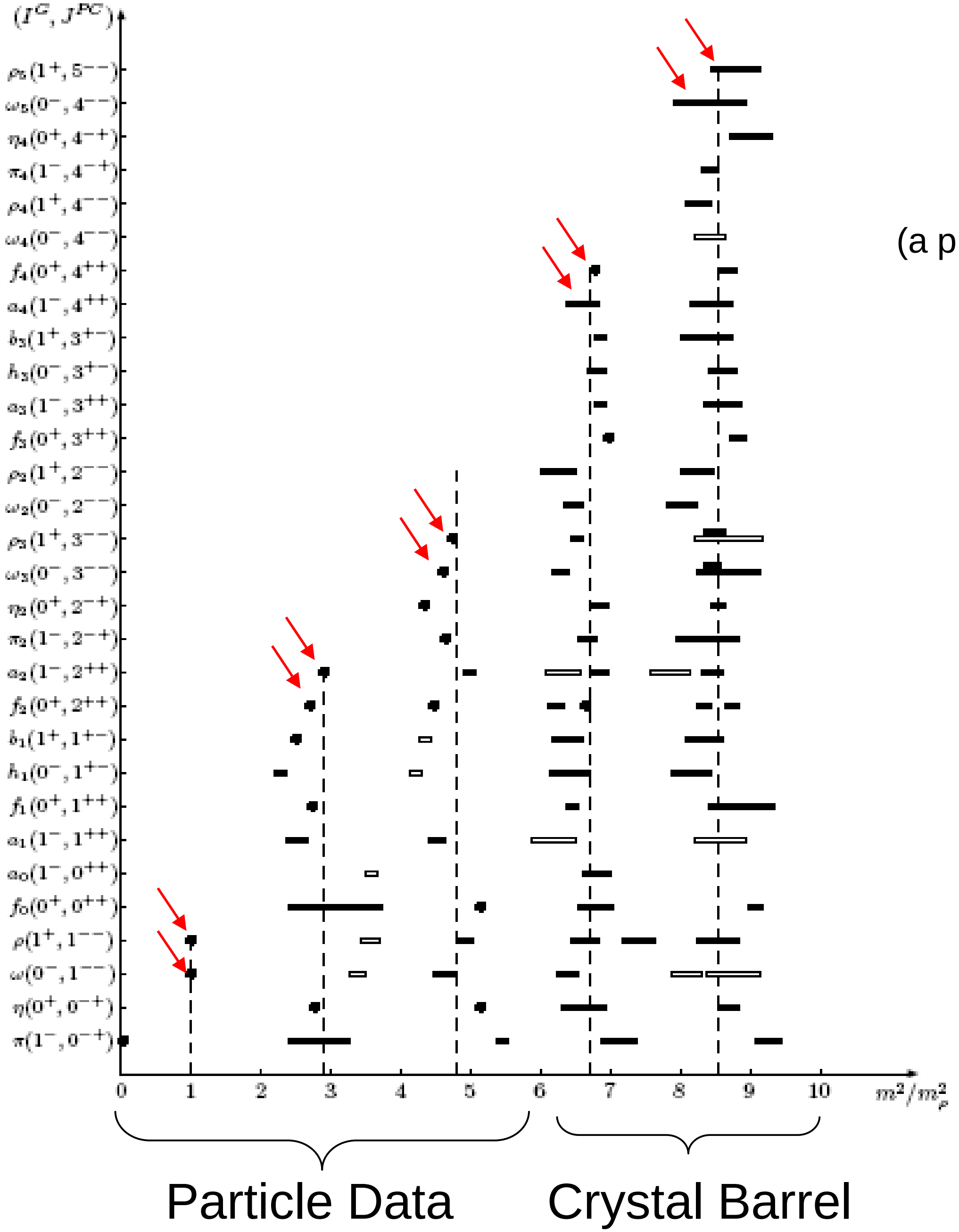
n - radial quantum number, $p(r)$ and r are relative momentum and distance related in the simplest case by $M = 2p + \sigma r$

Taking into account $M = l\sigma$ where l is the string length

the result is $M^2 = 4\pi\sigma(n+b)$

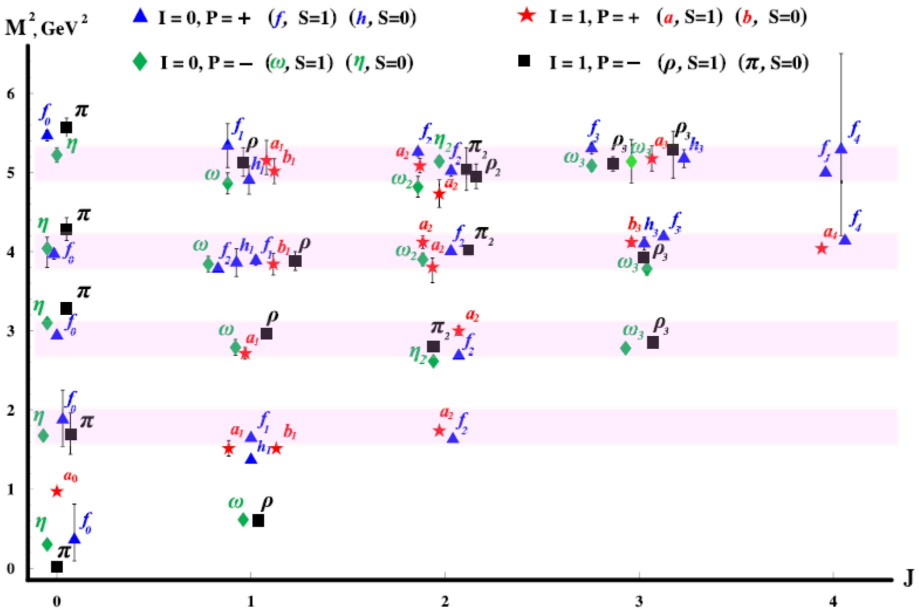
Experimental spectrum of light non-strange mesons

(a plot from S.S. Afonin, Eur. Phys. J. A 29 (2006) 327)

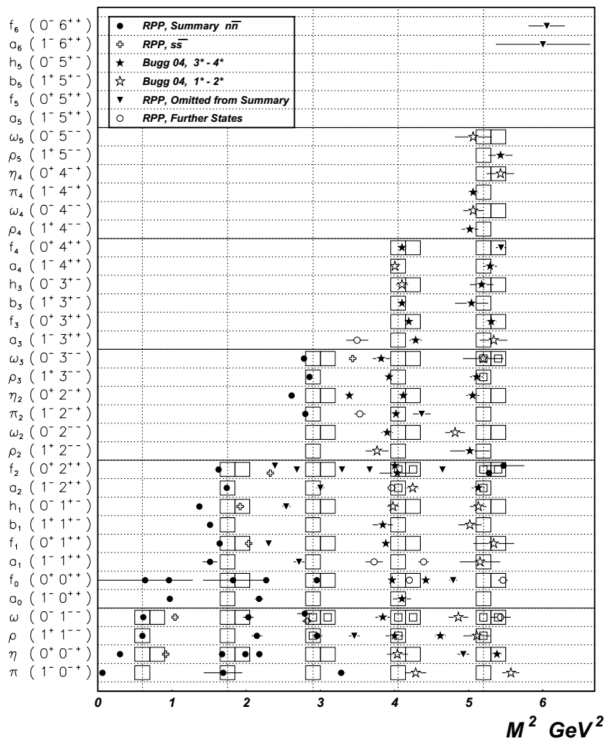


The major feature: Spin-parity clustering

The spectrum of light nonstrange mesons in units of M_ρ^2 . Experimental errors are indicated. Circles stay when errors are negligible. The dashed lines mark the mean mass squared in each cluster of states and the open strips and circles denote the one-star states. The arrows indicate the *J > 0* mesons which have no chiral partners (the hypothetical chiral singlets).



(a plot from M. Shifman and A. Vainshtein, Phys. Rev. D 77 (2008) 034002)



(a plot from E. Klempt and A. Zaitsev, Phys. Rept. 454 (2007) 1)

TABLE I: Classification of light non-strange mesons according to the values of (L, n). The states with the lowest star rating (according to [3]) are marked by the question mark, the states, which presumably have a large admixture of strange quark, are marked by the double question mark.

$L \backslash n$	0	1	2	3	4
0	$\pi(140)$ $\eta(548)(??)$ $\rho(770)$ $\omega(782)$	$\pi(1300)$ $\eta(1295)(??)$ $\rho(1450)$ $\omega(1420)$	$\pi(1800)$ $\eta(1760)$ $\rho(?)$ $\omega(?)$	$\pi(2070)$ $\eta(2010)$ $\rho(1900)$ $\omega(?)$	$\pi(2360)$ $\eta(2285)$ $\rho(2150)$ $\omega(2205)(?)$
1	$f_0(1370)$ $a_0(1450)(??)$ $a_1(1260)$ $f_1(1285)$ $b_1(1230)$ $h_1(1170)$ $a_2(1320)$ $f_2(1275)$	$f_0(1770)$ $a_0(?)$ $a_1(1640)$ $f_1(?)$ $b_1(1620)(?)$ $h_1(1595)(?)$ $a_2(1680)$ $f_2(1640)$	$f_0(2020)$ $a_0(2025)$ $a_1(1930)(?)$ $f_1(1971)$ $b_1(1960)$ $h_1(1965)$ $a_2(1950)(?)$ $f_2(1934)$	$f_0(2337)$ $a_0(?)$ $a_1(2270)(?)$ $f_1(2310)$ $b_1(2240)$ $h_1(2215)$ $a_2(2175)(?)$ $f_2(2240)$	
2	$\rho(1700)$ $\omega(1650)$ $\pi_2(1670)$ $\eta_2(1645)$ $\rho_2(?)$ $\omega_2(?)$ $\rho_3(1690)$ $\omega_3(1670)$	$\rho(2000)$ $\omega(1960)$ $\pi_2(2005)$ $\eta_2(2030)$ $\rho_2(1940)$ $\omega_2(1975)$ $\rho_3(1982)$ $\omega_3(1945)$	$\rho(2265)$ $\omega(2295)(?)$ $\pi_2(2245)$ $\eta_2(2267)$ $\rho_2(2225)$ $\omega_2(2195)$ $\rho_3(2300)(?)$ $\omega_3(2285)$		
3	$f_2(2001)$ $a_2(2030)$ $f_3(2048)$ $a_3(2031)$ $b_3(2032)$ $h_3(2025)$ $f_4(2018)$ $a_4(2005)$	$f_2(2293)$ $a_2(2255)$ $f_3(2303)$ $a_3(2275)$ $b_3(2245)$ $h_3(2275)$ $f_4(2283)$ $a_4(2255)$			
4	$\rho_3(2260)$ $\omega_3(2255)$ $\rho_4(2230)$ $\omega_4(2250)(?)$ $\pi_4(2250)$ $\eta_4(2328)$ $\rho_5(2300)$ $\omega_5(2250)$				

Classification of light nonstrange mesons according to the values of (L,n). The states with the lowest star rating or which are doubtful as non-strange quark-antiquark resonances are marked by the question mark.

(from S.S. Afonin, Phys. Rev. C76 (2007) 015202)

In average (in GeV^2)

$$\bar{M}^2(L,n) \approx 1.1(L+n+0.6)$$

$\Longrightarrow$ The law $\boxed{M^2(L,n) \sim L+n}$ works!

Like in nonrelativistic hydrogen atom:

$$E(L,n) \sim \frac{1}{N^2}, \qquad N = L+n+1$$ - principal quantum number

The symmetry of the spectrum: larger than O(3), it is O(4)
(V.A. Fock, Z. Phys. 98 (1935) 145)

$\Longrightarrow$ Existence of parity (chiral) singlets
follows from the nonrelativistic definition of parity, $P = (-1)^{L+1}$

Mesons on leading Regge trajectories have $n=0$, hence, they are parity singlets

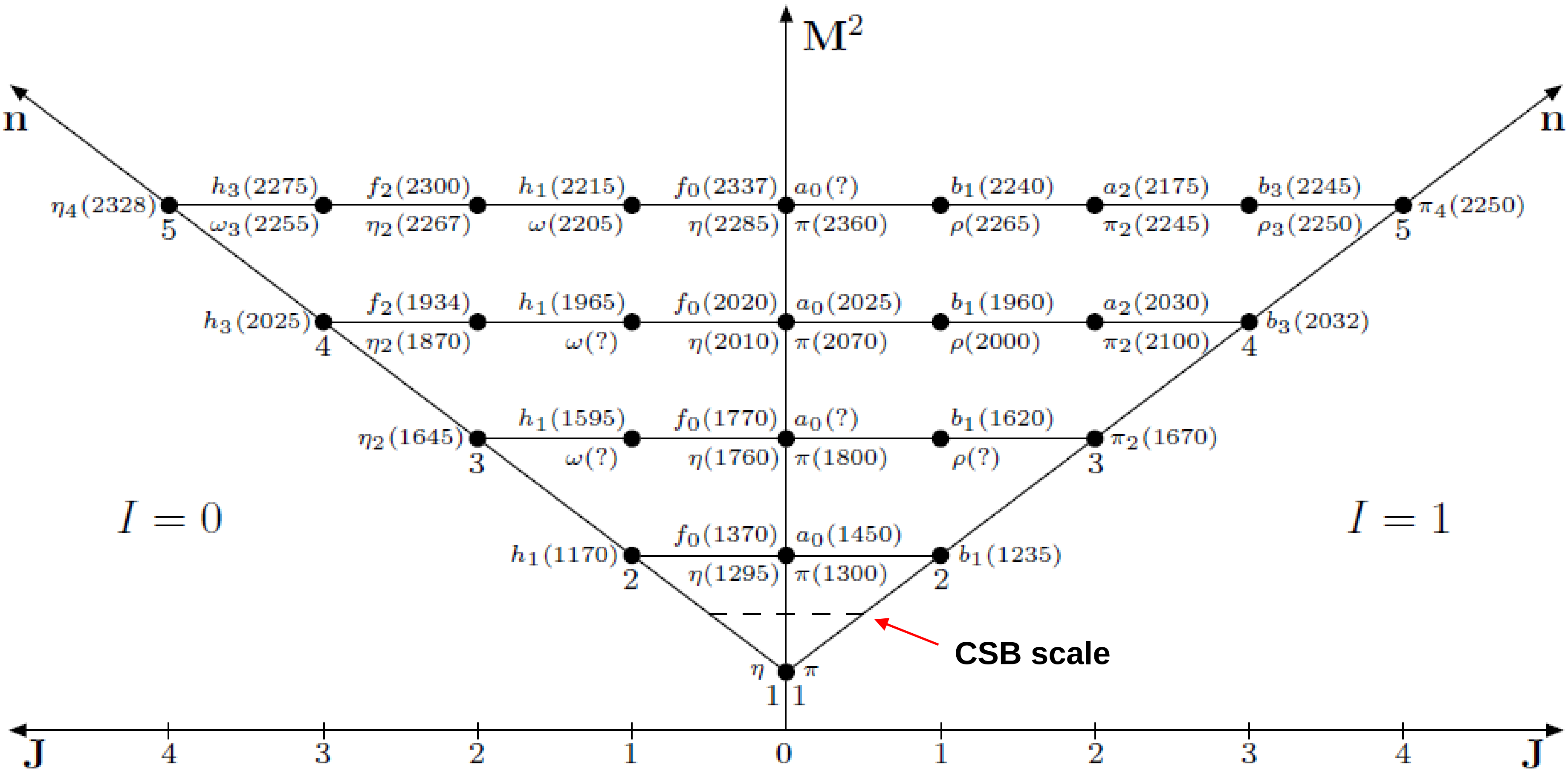
For instance: ρ-meson, (L,n)=(0,0), a_1-meson, (L,n)=(1,0), is partner for ρ´, (0,1)

<u>Potential models cannot explain the existence of “principal” quantum number!</u>

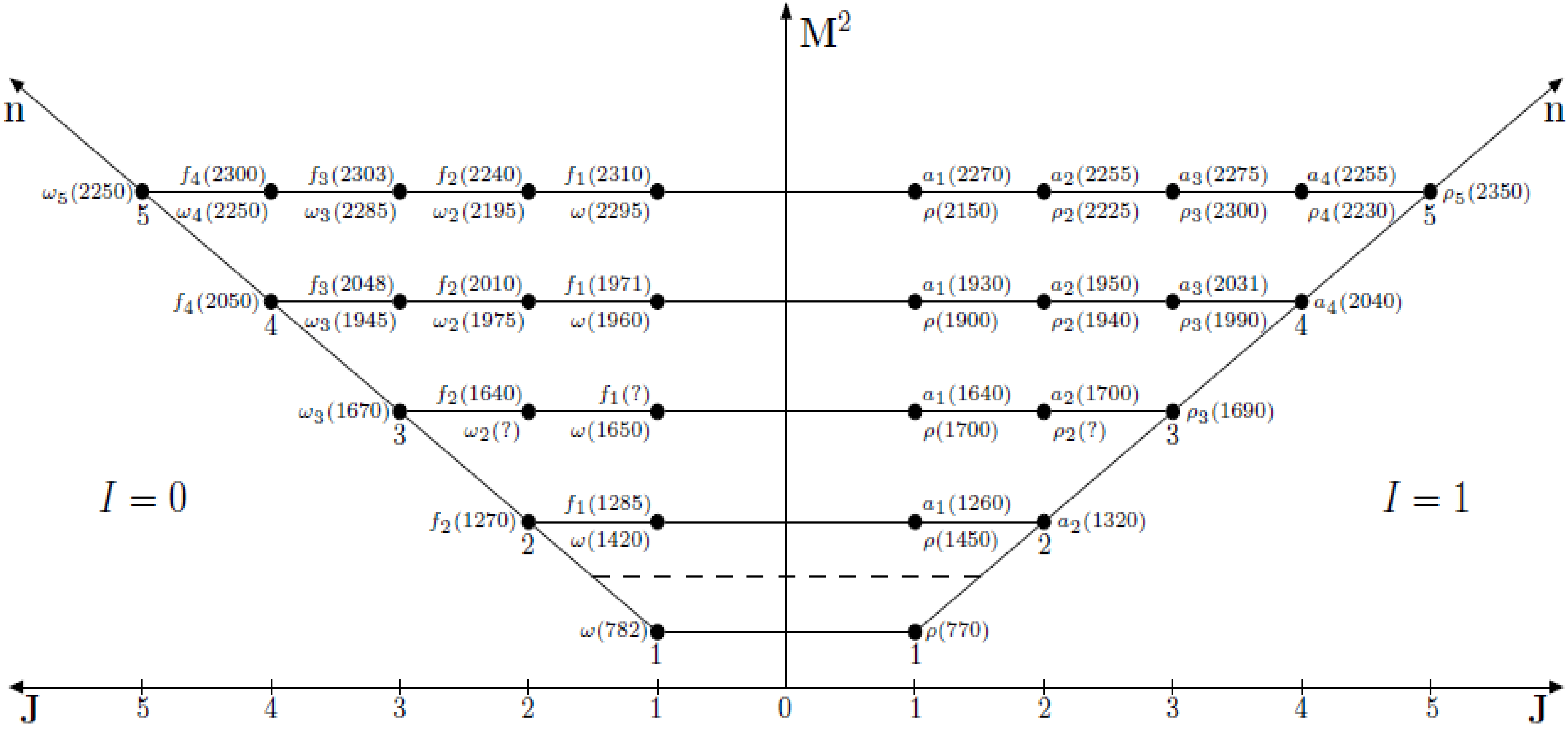
Hydrogen-like classification for light mesons

(S.S. Afonin, Mod. Phys. Lett. A 23 (2008) 4205)

J = L



J = L + 1



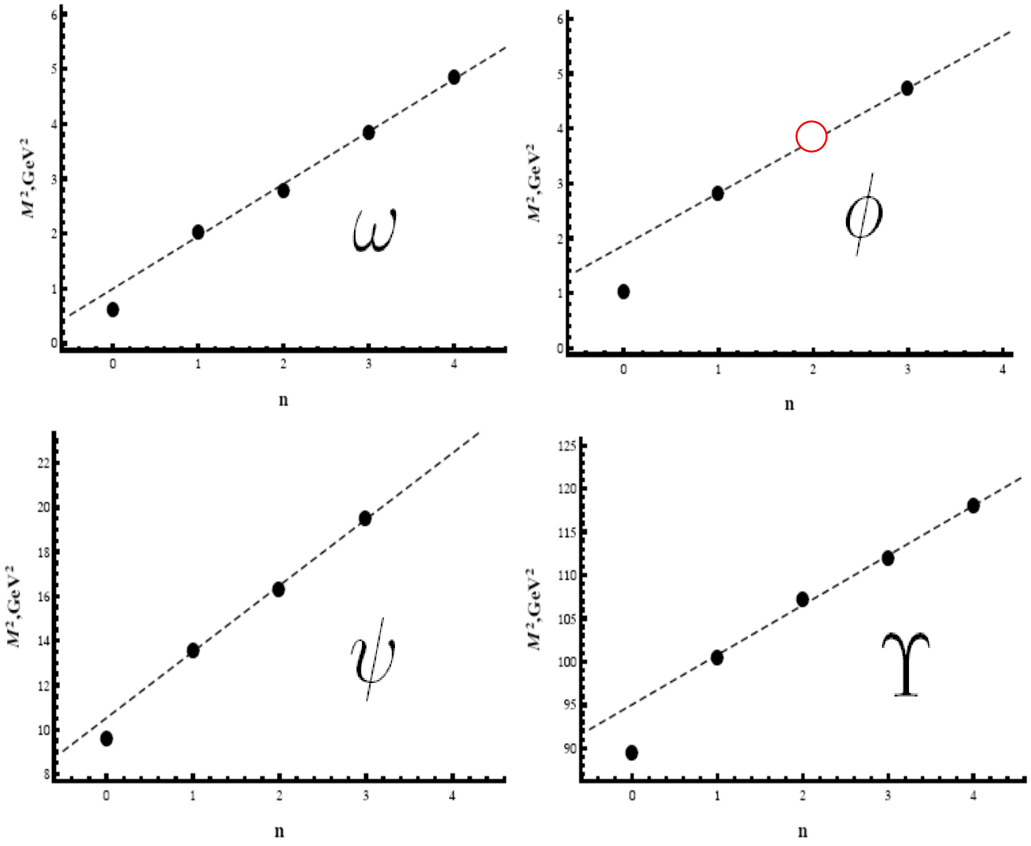
M²
n
n
I = 0
I = 1
J
J
ω₅(2250)
f₄(2300)
ω₄(2250)
f₃(2303)
ω₃(2285)
f₂(2240)
ω₂(2195)
f₁(2310)
ω(2295)
a₁(2270)
ρ(2150)
a₂(2255)
ρ₂(2225)
a₃(2275)
ρ₃(2300)
a₄(2255)
ρ₄(2230)
ρ₅(2350)
f₄(2050)
f₃(2048)
ω₃(1945)
f₂(2010)
ω₂(1975)
f₁(1971)
ω(1960)
a₁(1930)
ρ(1900)
a₂(1950)
ρ₂(1940)
a₃(2031)
ρ₃(1990)
a₄(2040)
ω₃(1670)
f₂(1640)
ω₂(?)
f₁(?)
ω(1650)
a₁(1640)
ρ(1700)
a₂(1700)
ρ₂(?)
ρ₃(1690)
f₂(1270)
f₁(1285)
ω(1420)
a₁(1260)
ρ(1450)
a₂(1320)
ω(782)
ρ(770)
5 4 3 2 1 0 1 2 3 4 5

PART II

FROM LIGHT TO HEAVY

S.S. Afonin and I.V. Pusenkov, Phys. Rev. D **90**, 094020 (2014)

In mesons with heavy quarks – known higher spin states are rare.
Relatively rich spectrum only for radial excitations of vector quarkonia.



The masses of unflavored (presumably S-wave) vector mesons

n	0	1	2	3	4
M_ω	783	1425 ± 25	1670 ± 30	1960 ± 25	2205 ± 30
M_ϕ	1020	1680 ± 20	—	2175 ± 15	—
M_ψ	3097	3686	4039 ± 1	4421 ± 4	—
M_Υ	9460	10,023	10,355	$10{,}579 \pm 1$	$10{,}865 \pm 8$

The radial linear trajectories

M_n^2	Fit (a)	Fit (b)
M_ω^2	$1.03\,(n + 0.74)$	$0.95\,(n + 1.04)$
M_ϕ^2	$1.19\,(n + 1.07)$	$0.95\,(n + 1.96)$
M_ψ^2	$3.26\,(n + 3.03)$	$2.98\,(n + 3.53)$
M_Υ^2	$6.86\,(n + 11.37)$	$5.75\,(n + 16.54)$

The problem: extract the dependence of slope and intercept on quark mass

Heavy quarks?

Usually (potential models, Bethe-Salpeter, hadron strings)

$$p \to \sqrt{p^2 + m^2}$$

The answer

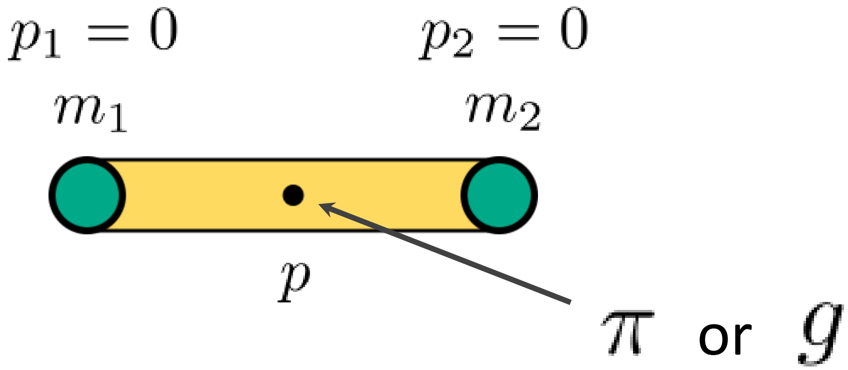
$$M_n \sqrt{M_n^2 - 4m^2} + 4m^2 \ln \frac{M_n - \sqrt{M_n^2 - 4m^2}}{2m} = 4\pi\sigma(n+b)$$

Non-linear in the quark mass *m*!

$$M_n \gg 2m \longrightarrow M_n^2 = 4\pi\sigma(n+b)$$

$$M_n - 2m \ll 2m \longrightarrow M_n \sim n^{\frac{2}{3}}$$

Proposal: An alternative concept for the hadron string picture



(or some collective excitation?)

only this momentum must be quantized!

$$M = m_1 + m_2 + P + \sigma x$$

In the unflavored mesons: $m_1 = m_2 \equiv m$

$$(M_n - 2m)^2 = 2\pi\sigma(n + b)$$

The slope of radial trajectory is automatically equal to the angular one!

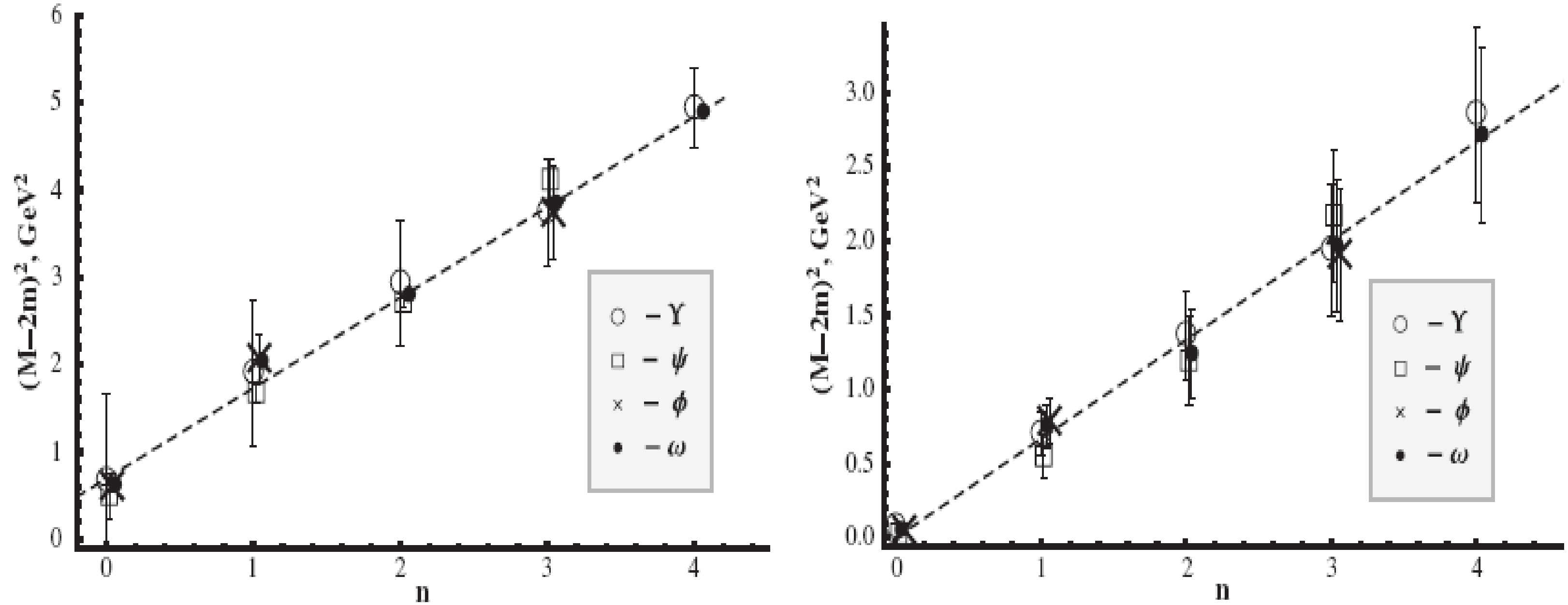
Let us fit $(M_n - 2m)^2 = a(n+b)$ to the observed spectrum

In GeV:

	Fit I	Fit II
$m_{u,d}$	0	0.36
m_s	0.13	0.49
m_c	1.17	1.55
m_b	4.33	4.69
a	1.10	0.49
b	0.57	0.00

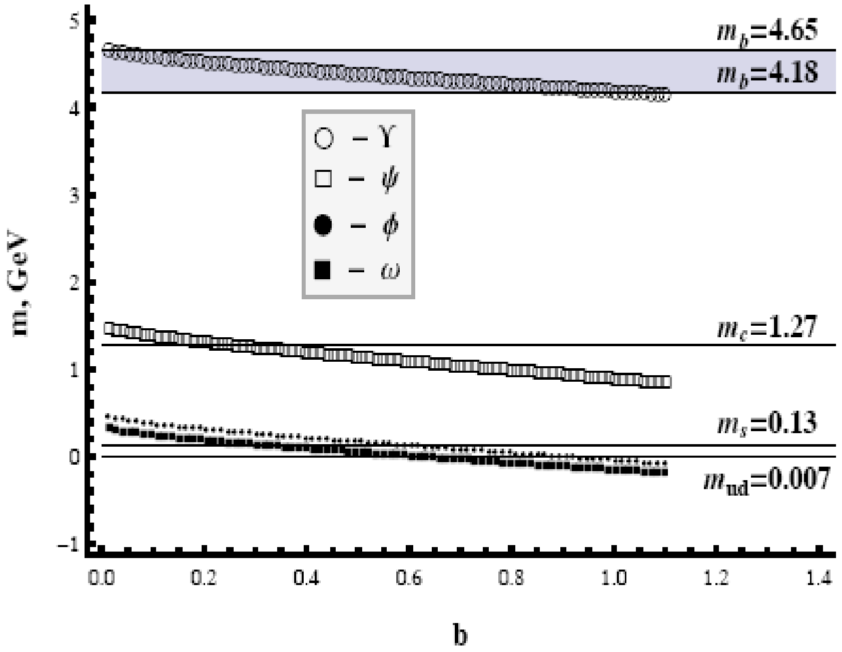
Fit I: The masses of u,d-quarks are set to zero (fixed)

Fit II: The masses of u,d-quarks are unfixed



The dependence of meson mass ***m*** on the intercept parameter ***b***

Fit I



Fit II

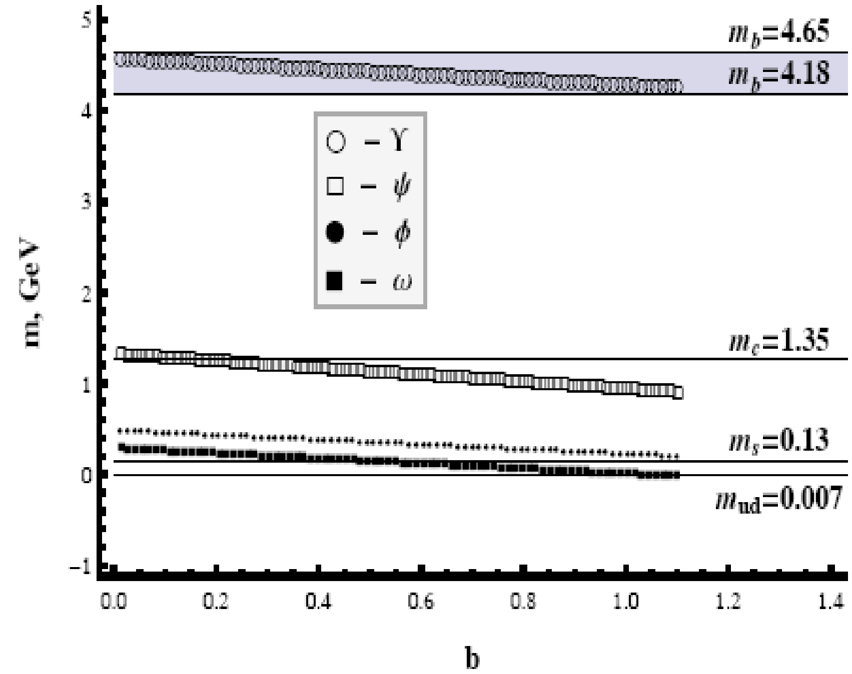


Table 5. The masses of states predicted by the Fits I and II vs. known experimental values[4] (in MeV).

$M_n \setminus n$	0	1	2	3	4	5
M_ω, Fit I	792	1314	1681	1982	2242	2475
M_ω, Fit II	720	1420	1710	1932	2120	2285
M_ω, Exp.	783	1425	1670	1960	2205	—
M_ϕ, Fit I	1052	1574	1941	2242	2502	2735
M_ϕ, Fit II	980	1680	1970	2192	2380	2545
M_ϕ, Exp.	1020	1680	—	2175	—	—
M_ψ, Fit I	3132	3654	4021	4322	4582	4815
M_ψ, Fit II	3100	3800	4090	4312	4500	4665
M_ψ, Exp.	3097	3686	4039	4421	—	—
M_Υ, Fit I	9452	9974	10,341	10,642	10,902	11,135
M_Υ, Fit II	9380	10,080	10,370	10,592	10,780	10,945
M_Υ, Exp.	9460	10,023	10,355	10,579	10,876	11,019

Extension to open flavor: $(M_n - m_1 - m_2)^2 = a(n+b)$

Many predictions!

For instance, the case of heavy-light sector was elaborated in

K. Chen, Y. Dong, X. Liu, Q. F. Lü and T. Matsuki,
``Regge-like relation and a universal description of heavy–light systems,''
Eur. Phys. J. C78 (2018) 20.

PART III

The hadron string picture has serious shortcomings.
For instance:
- predicts too large size for high excitations;
- the tension of chromoelectric field is not gauge-invariant, hence, nonobservable

If not strings, what can give the linear Regge trajectories in a natural way?

A recent proposal – the picture of constituent quark-antiquark pairs
(or constituent gluons?)

S.S. Afonin, Adv. High Energy Phys. 2019 (2019) 1701939

HADRON MASS: THE CENTRAL PROBLEM

Mass of resonance → ***Re*-part of *S*-matrix pole**

→ **gravitational mass**

related with ↓

Energy-momentum tensor in QCD:

$$T^{\mu\nu} = \frac{1}{2}\,\overline{\psi}\, i\overleftrightarrow{D}^{(\mu}\gamma^{\nu)}\psi + \frac{1}{4}\,g^{\mu\nu}F^2 - F^{\mu\alpha}F^{\nu}{}_{\alpha}$$

Its trace:

QCD trace anomaly

anomalous dimension of quark mass

$$T^{\alpha}_{\alpha} = \frac{\beta(g)}{2g}F^{\mu\nu,a}F^{a}_{\mu\nu} + \sum_{q=u,d,s} m_q(1+\gamma_m)\overline{\psi}_q\psi_q$$

$$\beta(g) = -(11 - 2n_f/3)\,g^3/(4\pi)^2 + \ldots$$

$$M = \frac{\langle P|H_{\rm QCD}|P\rangle}{\langle P|P\rangle}|_{\rm rest\ frame} \qquad H_{\rm QCD} = \int d^3\vec{x}\, T^{00}(0,\vec{x})$$

$\langle P|P\rangle = 1$ — non-relativistic normalization

$\langle P|P\rangle = 1/(2E)$ — relativistic normalization

Hadron mass in relativistic case: $2m_h^2 = \langle h|T^\alpha_\alpha|h\rangle$ renorminvariant!

A consequence of the Ward identity $2p_\mu p_\nu = \langle h|T_{\mu\nu}|h\rangle$

Our ansatz:

$$m_h^2 = \Lambda(E_h + 2m_q) = \Lambda E_h + m_\pi^2$$

where $E_h \sim \langle h|G^2_{\mu\nu}|h\rangle \neq 0$ $\qquad m_u = m_d \doteq m_q$

Λ is universal for light hadrons and fixed by GOR relation

$$m_\pi^2 = -\frac{\langle\bar{q}q\rangle}{f_\pi^2}(m_u + m_d) = \Lambda \cdot 2m_q \qquad \Lambda \doteq -\frac{\langle\bar{q}q\rangle}{f_\pi^2}$$

One must specify E_h - interpretations?

$$m_h^2 = \Lambda E_h + m_\pi^2$$

By assumption $E_h \sim \langle G_{\mu\nu}^2 \rangle$

ΛE_h renorminvariant!

Let us fix $\langle \bar{q} q \rangle = -(250\,\mathrm{MeV})^3,\ m_u + m_d = 11\ \mathrm{MeV},\ f_\pi = 92.4\ \mathrm{MeV}$

This yields $\Lambda = 1830\ \mathrm{MeV}$

Consider the rho-meson

$E_\rho \approx 310\ \mathrm{MeV}$ - the energy cost for the given spin flip (looks like a constituent mass!)

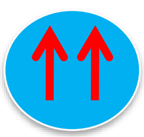
The non-renorminvariant logic does not work! $m_\rho \neq E_\rho + m_\pi$

$$m_h^2 = \Lambda E_h + m_\pi^2$$

Higher spin and radial excitations with correct quantum numbers?

A proposal: Let us assume that gluodynamics leads to formation of some effective degrees of freedom with a certain mass inside hadrons (gluon analogues of positronium? constituent gluons? …)

$J^{PC} = 0^{-+}$ — A_0



$J^{PC} = 1^{--}$ — A_1

notations

$${}^J\pi_{A_0^n A_1^l} \quad (P,C) = \left((-1)^{n+l+1}, (-1)^l\right) \qquad m^2_{\pi_{A_0^n A_1^l}} = \Lambda(nE_0 + lE_1) + m_\pi^2$$

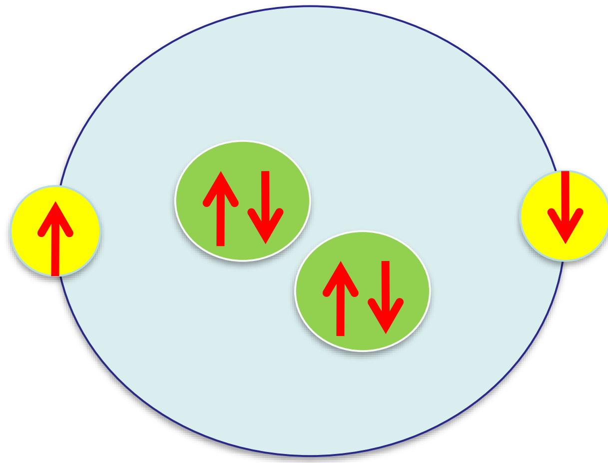
Light non-strange mesons

Regge spectrum!

$${}^J\rho_{A_0^n A_1^l} \quad (P,C) = \left((-1)^{n+l+1}, (-1)^{l+1}\right) \qquad m^2_{\rho_{A_0^n A_1^l}} = \Lambda(E_\rho + nE_0 + lE_1) + m_\pi^2$$

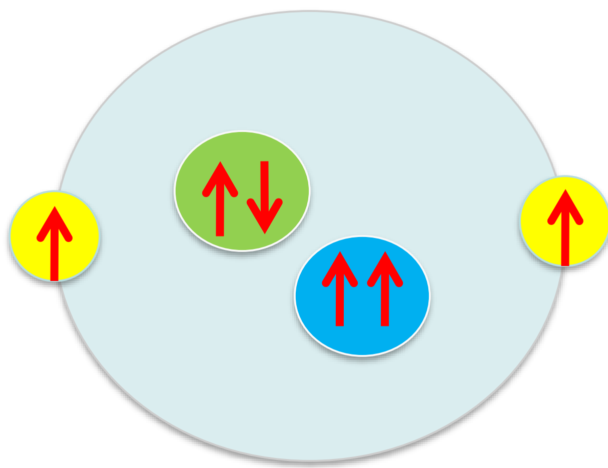
Examples

$${}^{0}\pi_{A_0^2} \Longrightarrow$$



$$\pi(1300)$$

$${}^{J}\rho_{A_0 A_1} \Longrightarrow$$



$$\pi,\ \pi_1 \text{ and } \pi_2$$

Not exotic!

An atomic-like picture emerges!

Phenomenologically: $E_0 \approx 450 \quad E_1 \approx 570 \quad \mathrm{MeV}$

It should be noted that constituent gluons were proposed long ago…

PHYSICAL REVIEW D VOLUME 26, NUMBER 6 15 SEPTEMBER 1982

Dynamical mass generation in continuum quantum chromodynamics

John M. Cornwall
Department of Physics, University of California, Los Angeles, California 90024
(Received 30 April 1982)

We study the formation of a mass gap, or effective gluon mass (and consequent dimensionful parameters such as the string tension, glueball mass, $\langle \mathrm{Tr} G_{\mu\nu}{}^2 \rangle$, correlation lengths) in continuum QCD, using a special set of Schwinger-Dyson equations. These equations are derived from a resummation of the Feynman graphs which represent certain *gauge-invariant* color-singlet Green's functions, and are themselves essentially gauge invariant. This resummation is essential to the multiplicative renormalizability of QCD

Numerical calculations of the mass gap are presented, suggesting an effective gluon mass of 500±200 MeV

Each hadron made of $q\bar{q}$ or qqq will be the lowest member of a series in which the next member is $q\bar{q}g$ or $qqqg$ (g for gluon). The next-lowest members of the series should have opposite parity to the ground state and lie about 600 MeV higher; their angular momentum may or may not

Conclusions

1. We need new experiments for checking the observed pattern of degeneracies in light mesons. The region from 1 to 2.5 GeV must be carefully scanned.

2. Phenomenology seems to suggest an extended universal relation for the meson mass

universal slope

universal intercept

$$(M - m_1 - m_2)^2 = a(L + n + b)$$

radial number

quark masses

quark orbital momentum

3*. Hadron strings are not necessary for a natural explanation of linear Regge trajectories