Catalytic effects of monopoles in QCD on the phase transitions

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HADRON 2019, 16 - 21 August 2019, Guilin Bravo Hotel, Guilin, China

Topics of my talk

1. Catalytic effects of monopoles in QCD at the zero temperature

- This project was started with Prof. A. Di Giacomo (Univ. Pisa, Italy) in 2014.
- Details of computations are in the article, **M. H., arXiv: 1807.04808**.

2. Catalytic effects of monopoles in QCD in the high temperature

- We expand the research project at the zero temperature to the finite temperature.
- We investigate the catalytic effects of monopoles on the phase transitions of the quark confinement and chiral symmetry breaking.

Purpose of this research

- **In condensed matter physics, a research group makes Dirac monopole in a Bose-Einstein condensate** [M. W. Ray, et al., Nature 505 (2014) 657, Science 348 (2015) 544.]**.**
- **In the high energy physics, the experiment to explore the magnetic monopoles at LHC (MoEDAL experiment) has begun** [B. Acharya, et al., JHEP 08 (2016) 067, PRL 118 (2017) 061801.]**.**
- **To give indications to detect monopoles in QCD, we estimate the effects of monopoles on physical quantities by Lattice QCD simulations.**
- We add monopoles in SU(3) quenched configurations by applying a **monopole creation operator** on the QCD vacuum [C. Bonati, et al., PRD 85 (2012) 065001, A. Di Giacomo and M. H. PRD 91 (2015) 054512.]**.**
- We use the **overlap fermions** which preserve the chiral symmetry in the lattice gauge theory [R. G. Edwards, et al., PRD 61 (2000) 074504; **L. Giusti, et al., JHEP 11 (2003) 023**; L. Del Debbio, et al., PRL 94 (2005) 032003; L. Del Debbio, et al., JHEP 02 (2004) 003]**.**

Catalytic effects of monopoles at the zero temperature

We show that the catalytic effects of monopoles in QCD as follows ($V = 18^3 \times 32, \beta = 6.052$) [**arXiv: 1807.04808**]:

(0) The additional monopoles make instantons.

(i) The decay constants of the pseudoscalar increase.

(ii) The values of the chiral condensate decrease.

(iii) The masses of the light quarks and the pseudoscalar increase.

(iv) The decay width of the charged pion becomes wider and the lifetime of the charged pion becomes shorter than experimental results.

These are the catalytic effects of Adriano monopole.

In this presentation, results in the continuum limit are evaluated by interpolations.

The monopole creation operator

- **What are monopoles? G. 't Hooft [NPB 190 (1981) 455]**
- **The Plaquette gauge action is shifted as follows** [C. Bonati, et al., PRD 85 (2012) 065001, A. Di Giacomo and M. H. PRD 91 (2015) 054512.]**:**

$$S + \overline{\Delta S} \equiv \sum_{n,\mu<\nu} \mathrm{Re}(1 - \bar{\Pi}_{\mu\nu}(n))$$

$$\bar{\Pi}_{i0}(t,\vec{n}) = \frac{1}{\mathrm{Tr}[I]} \mathrm{Tr}[U_i(t,\vec{n}) M_i^\dagger(\vec{n}+\hat{i}) U_0(t,\vec{n}+\hat{i}) \times M_i(\vec{n}+\hat{i}) U_i^\dagger(t+1,\vec{n}) U_0^\dagger(t,\vec{n})]$$

$$M_i(\vec{n}) = \exp(+\mathbf{m_c} i A_i^0(\vec{n}-\vec{x}_1))$$

$$M_i^\dagger(\vec{n}) = \exp(-\mathbf{m_c} i A_i^0(\vec{n}-\vec{x}_2))$$

$$\left(g = \sqrt{\frac{6}{\beta}} : \text{Electric charge}\right)$$

A_i^0 : Abelian monopole (Wu - Yang form) in SU(3)

$+\mathbf{m_c}$: Magnetic charges of the monopole

$-\mathbf{m_c}$: Magnetic charges of the anti-monople

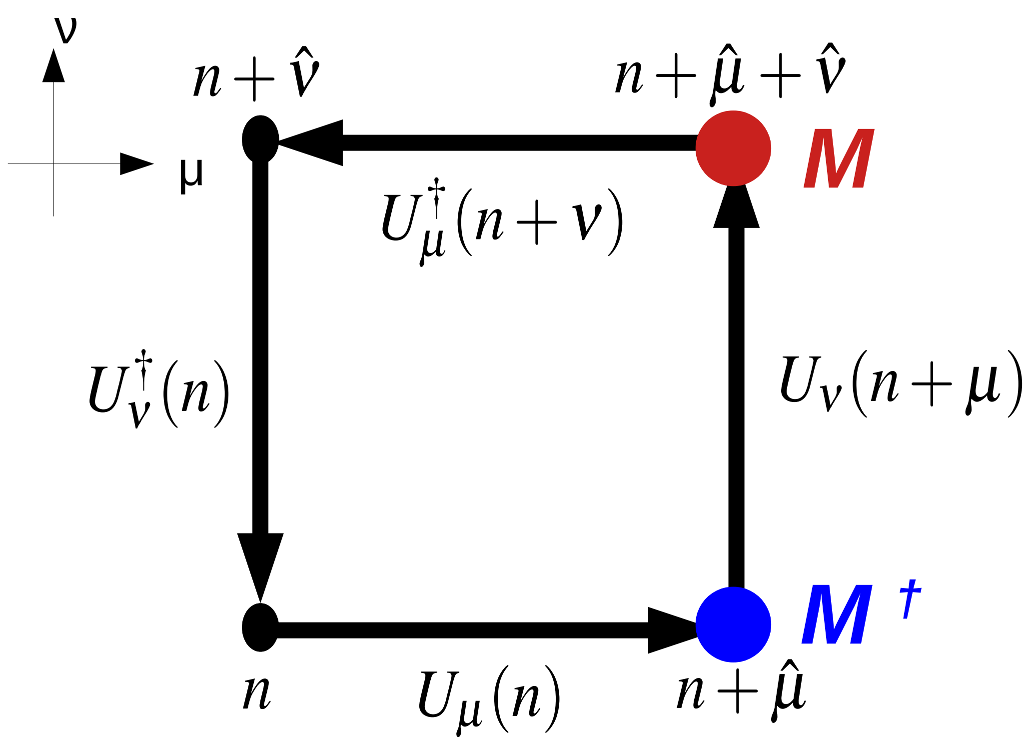
$\mathbf{m_c} = \mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{5}$

[Y. M. Shnir, text book, "Magnetic Monopoles".]

Additional monopoles

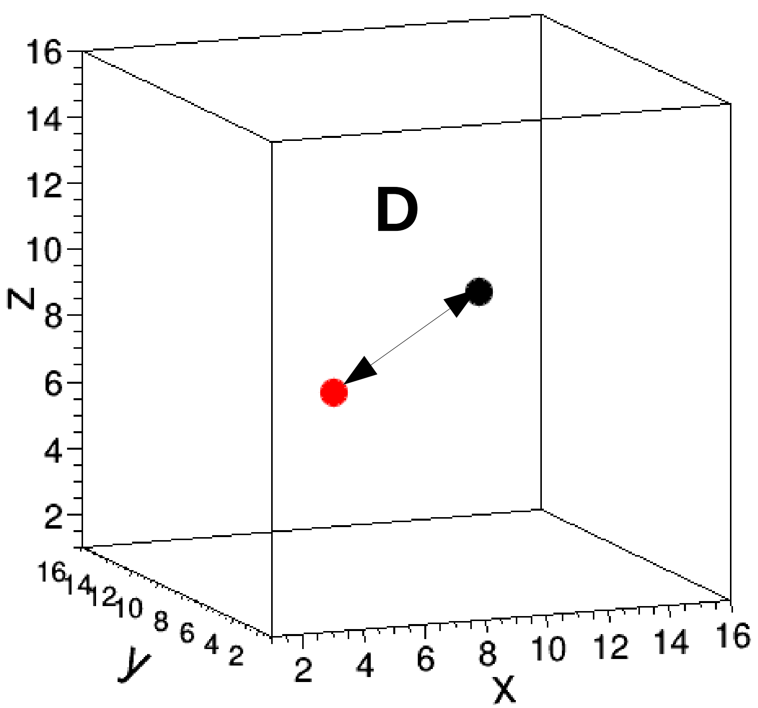
- **Ref. C. Bonati, et al., PRD 85 (2012) 065001.**

Plaquette action



- **The locations of the monopole and the anti-monopole.**

$$\mathbf{V = 16^3 \times 32}$$



$$D = |\vec{x}_1 - \vec{x}_2| \approx 1 \ [\mathrm{fm}]$$

Simulation parameters

β	a/r_0	V	V/r_0^4	N_{conf}
5.778	0.285	$10^3 \times 24$	158	$O(1.0\times10^3)$
5.846	0.248	$12^3 \times 24$	158	$O(1.1\times10^3)$
5.926	0.213	$14^3 \times 28$	158	$O(9\times10^2)$
6.000	0.186	$14^3 \times 28$	93	$O(1.7\times10^3)$
6.000	0.186	$16^3 \times 32$	158	$O(8\times10^2)$
6.052	0.171	$18^3 \times 32$	158	$O(8\times10^2)$
6.137	0.149	$20^3 \times 40$	158	$O(4\times10^2)$

- We add the monopole and anti-monopole with magnetic charges from 0 to 5.
- We use an analytic function from [S. Necco, at al. Nucl. Phys. B622 (2002) 328] and compute the lattice spacing in all of our simulations (r_0 = 0.5 [fm]).
- **We interpolate results at the continuum limit by the fitting a linear function or a constant function.**

Monopole density

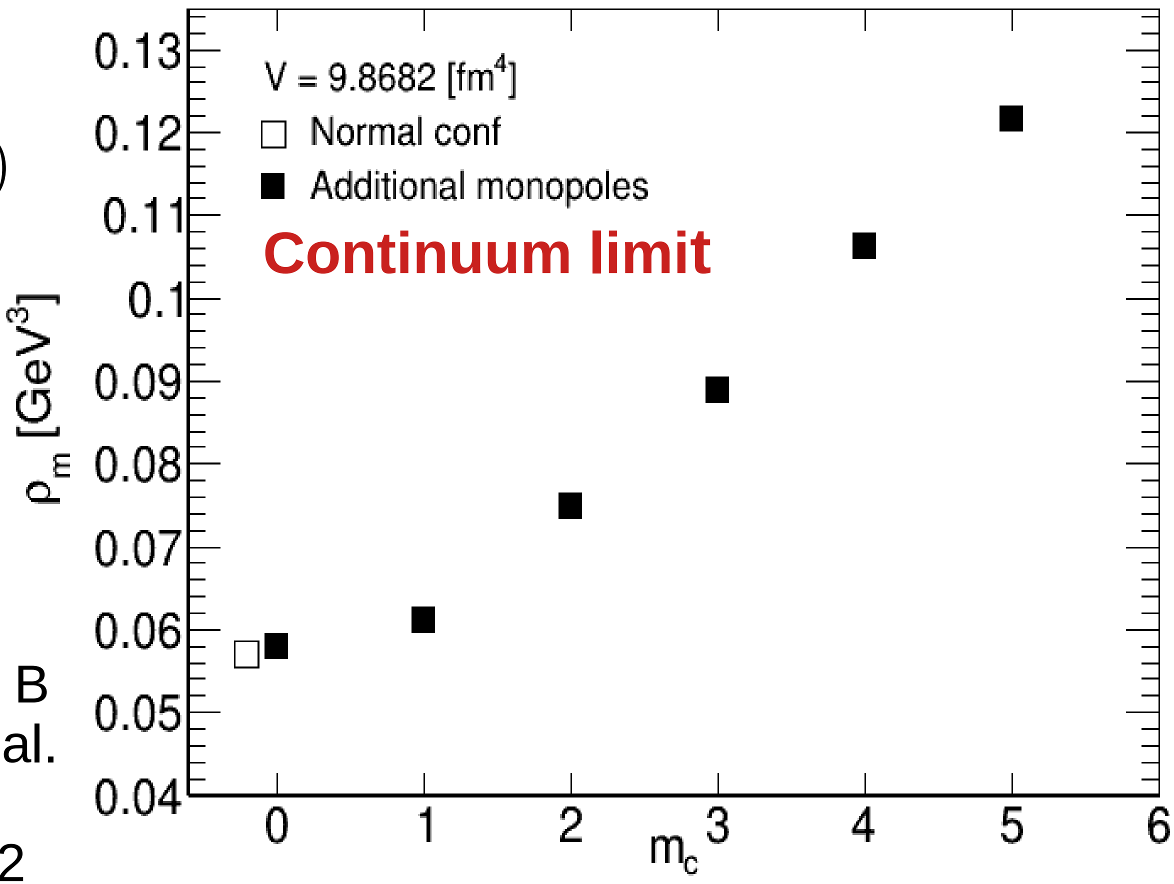
- The monopole currents after the Abelian projection are defined as follows:

$$k_\mu^i(*n) \equiv -\epsilon_{\mu\nu\rho\sigma}\nabla_\nu n_{\rho\sigma}^i(n+\hat{\mu})$$

- The monopole density is defined as follows:

$$\rho_m/a^3 = \frac{1}{12V}\sum_{i,\mu}\sum_{*n}|k_\mu^i(*n)|/a^3$$

[T. DeGrand, et al., PRD 22 (1980) 2478, S. Kitahara, et al. Nucl. Phys. B 533 (1998) 576, M. I. Polikarpov, et al. Phys. Lett. B 316 (1993) 333, F. Brandstaeter, et al. Phys. Lett. B 272 (1991) 319, DIK collaboration, Phys Rev D 70 (2004) 074511]



Overlap fermions

- The overlap fermions preserve the exact chiral symmetry [P. H. Ginsparg and K. G. Wilson PRD 25 (1982) 2649; N. Neuberger, PLB 427 (1998) 353].
- We calculate the overlap Dirac operator ***D(ρ)*** from the gauge links of the configurations (ρ = 1.4) [**L. Giusti, et al., Com. Phys. Comm. 153 (2003) 31, etc**].
- We solve the eigenvalue problem $\mathbf{D}(\rho)|\psi_{\mathbf{i}}\rangle = \lambda_{\mathbf{i}}|\psi_{\mathbf{i}}\rangle$ of the **massless** overlap Dirac operator by using the subroutine ARPACK.

The massless overlap Dirac operator: $D(\rho) = \frac{\rho}{a}\{1 + \gamma_5 \epsilon(H_W(\rho))\}$

- The overlap fermions have the exact zero modes $\mathbf{n}_+, \ \mathbf{n}_-$.
- The topological charge **Q** is defined as follows: $\mathbf{Q} = \mathbf{n}_+ - \mathbf{n}_-$.
- We suppose that the Atiyah–Singer index theorem.

$\mathbf{n}_+$: The number of instantons of the ***positive charge*****.**

$\mathbf{n}_-$: The number of instantons of the ***negative charge*****.**

Instanton density

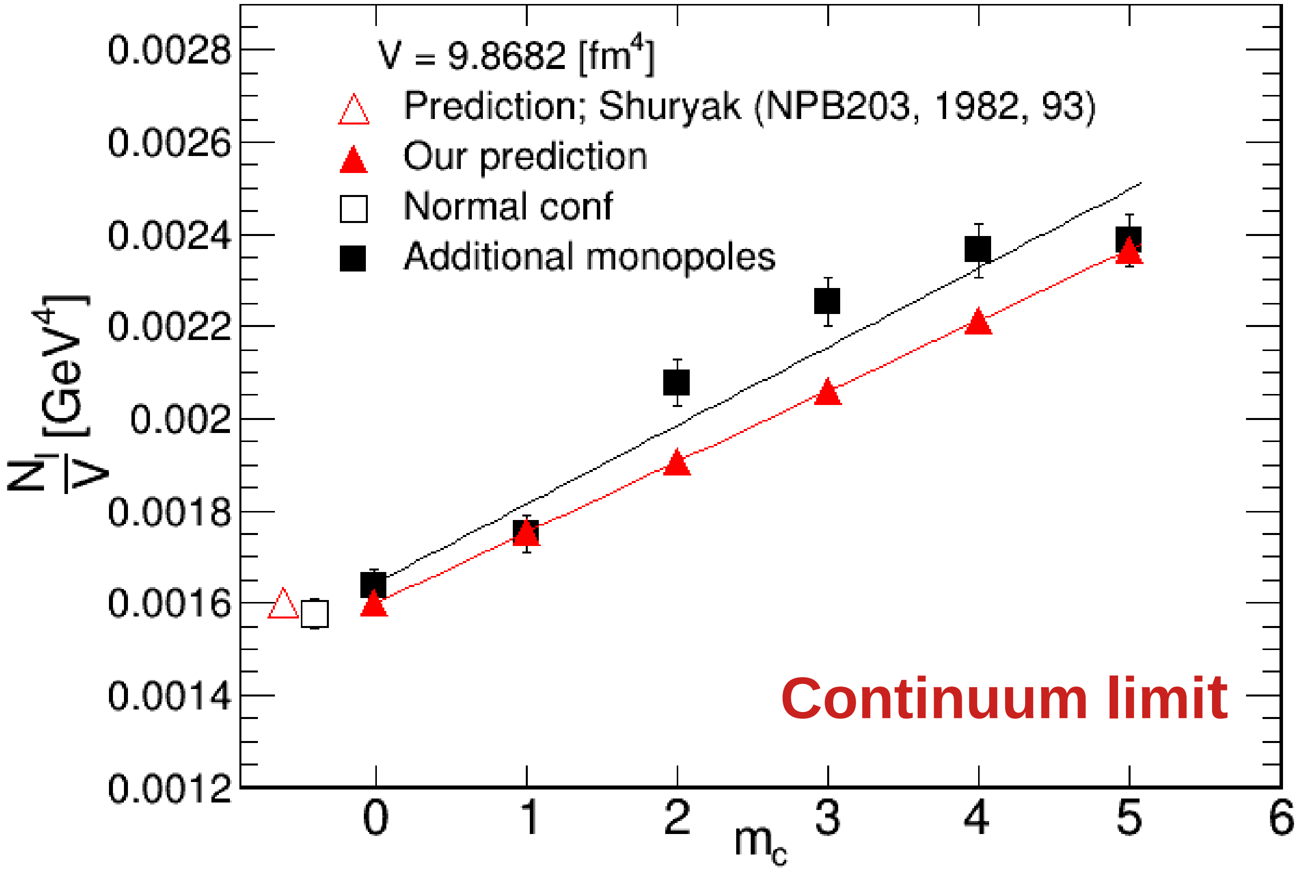
- We never observed the numbers of zero modes of the positive chirality and the negative chirality in the same configuration at the same time.

- **The total number of instantons and anti-instantons N_I is calculated from the average square of the topological charges** [A. Di Giacomo and M. H. PRD 91 (2015) 054512]**:**

$$\mathbf{N_I = \langle Q^2 \rangle}$$

$$A^{Pre} = 1.536 \times 10^{-4}$$
$$B^{Pre} = 1.600 \times 10^{-3}$$

$$A = 1.71(11) \times 10^{-4}$$
$$B = 1.64(3) \times 10^{-3}$$
$$\chi^2/d.o.f. = 13.7/4.0$$



Correlation functions

- **Fermion propagator:** [T. DeGrand, et al. Comp. Phys. Com. 159 (2004) 185.]

$$G(\vec{y}, y^0; \vec{x}, x^0) \equiv \sum_i \frac{\psi_i(\vec{x}, x^0)\psi_i^\dagger(\vec{y}, y^0)}{\lambda_i^{mass}}, \quad \lambda_i^{mass} = \left(1 - \frac{a\bar{m}_q}{2\rho}\right)\lambda_i + \bar{m}_q$$

- **Scalar correlation function:**

$$C_{SS}(\Delta t) = \frac{a^3}{V}\sum_{\vec{x}_1}\sum_{\vec{x}_2,\ t}\langle \mathcal{O}_S^C(\vec{x}_2, t)\mathcal{O}_S(\vec{x}_1, t+\Delta t)\rangle, \quad \mathcal{O}_S = \bar{\psi}_1\left(1 - \frac{a}{2\rho}D\right)\psi_2$$

- **Pseudoscalar correlation function:**

$$C_{PS}(\Delta t) = \frac{a^3}{V}\sum_{\vec{x}_1}\sum_{\vec{x}_2,\ t}\langle \mathcal{O}_{PS}^C(\vec{x}_2, t)\mathcal{O}_{PS}(\vec{x}_1, t+\Delta t)\rangle, \quad \mathcal{O}_{PS} = \bar{\psi}_1\gamma_5\left(1 - \frac{a}{2\rho}D\right)\psi_2$$

- **Correlation function** [T. Blum, et al., PRD 69 (2004) 074502.]**:**

$$C_{PS-SS}(\Delta t) \equiv C_{PS}(\Delta t) - C_{SS}(\Delta t)$$

- **Fitting function** [L. Giusti, et al., PRD 64 (2001) 114508.]**:**

$$C_{PS-SS}(t) = \frac{a^4 G_{PS-SS}}{a m_{PS}} \exp\left(-\frac{m_{PS}}{2}T\right)\cosh\left[m_{PS}\left(\frac{T}{2} - t\right)\right]$$

Decay constant at the chiral limit

- **Decay constant at the chiral limit F_0:**

$$aF_0 \equiv \lim_{a\bar{m}_q \to 0} aF_{PS}$$

$$= \lim_{a\bar{m}_q \to 0} \frac{2a\bar{m}_q \sqrt{a^4 G_{PS-SS}}}{(am_{PS})^2}$$

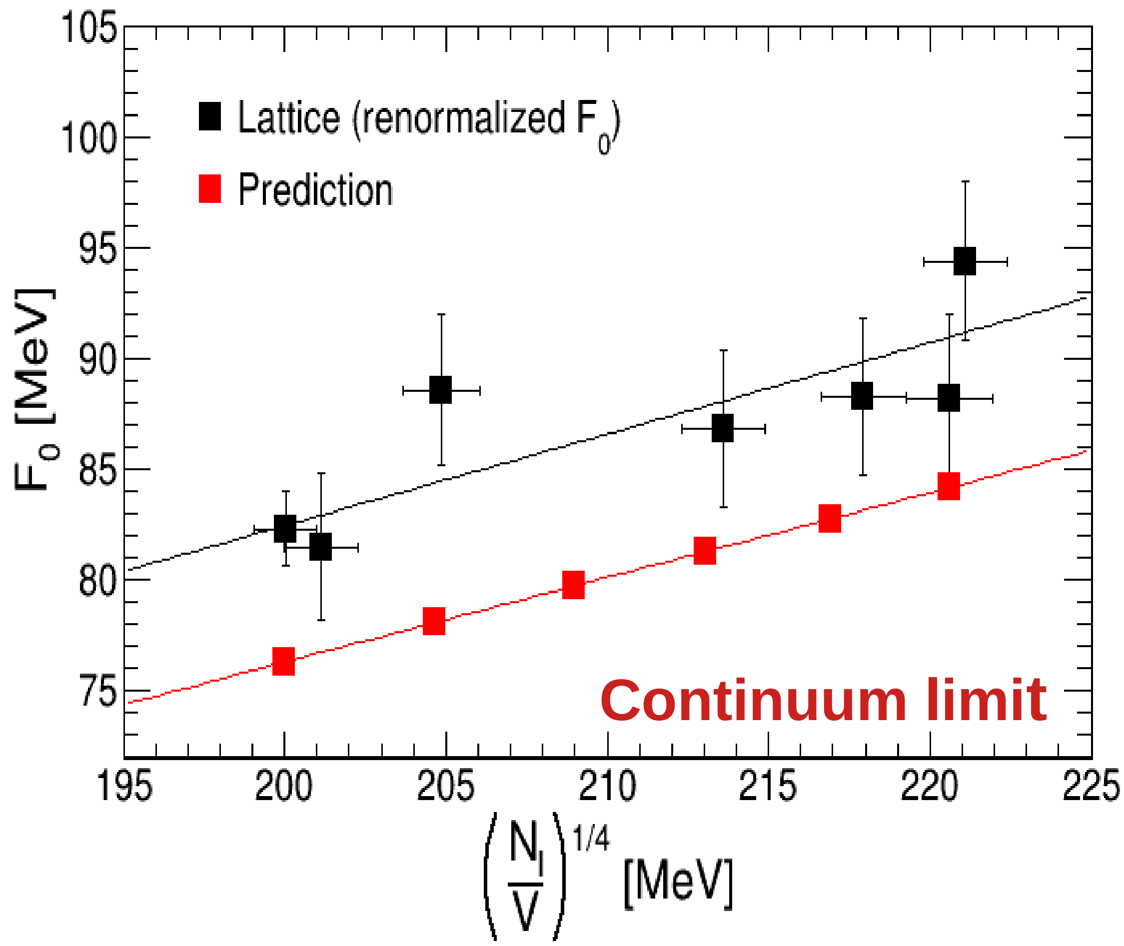
- **Fitting results:**

$$A = 0.412(5),\ \chi^2/d.o.f. = 3.3/6.0$$

- **Prediction:**

$$F_0^{Pre}(m_c) = A^{Pre} \left(\frac{N_I^{Pre}(m_c)}{V} \right)^{\frac{1}{4}}$$

$$A^{Pre} = 0.3816$$



The renormalized F_0 increases in direct proportion to one-fourth root of the instanton density.

Chiral condensate

- **The chiral condensate at the chiral limit:**

$$a^3\langle\bar{\psi}\psi\rangle^{GMOR} = -\frac{aA^{PCAC}}{2}\left(a\hat{F}_0\right)^2$$

- **Fitting results:**

$$A = -0.371(19)\ [\mathrm{GeV}]$$

$$\chi^2/d.o.f. = 1.7/6.0$$

$$\frac{1}{\bar{\rho}} = 5.5(3)\times 10^2\ [\mathrm{MeV}]$$

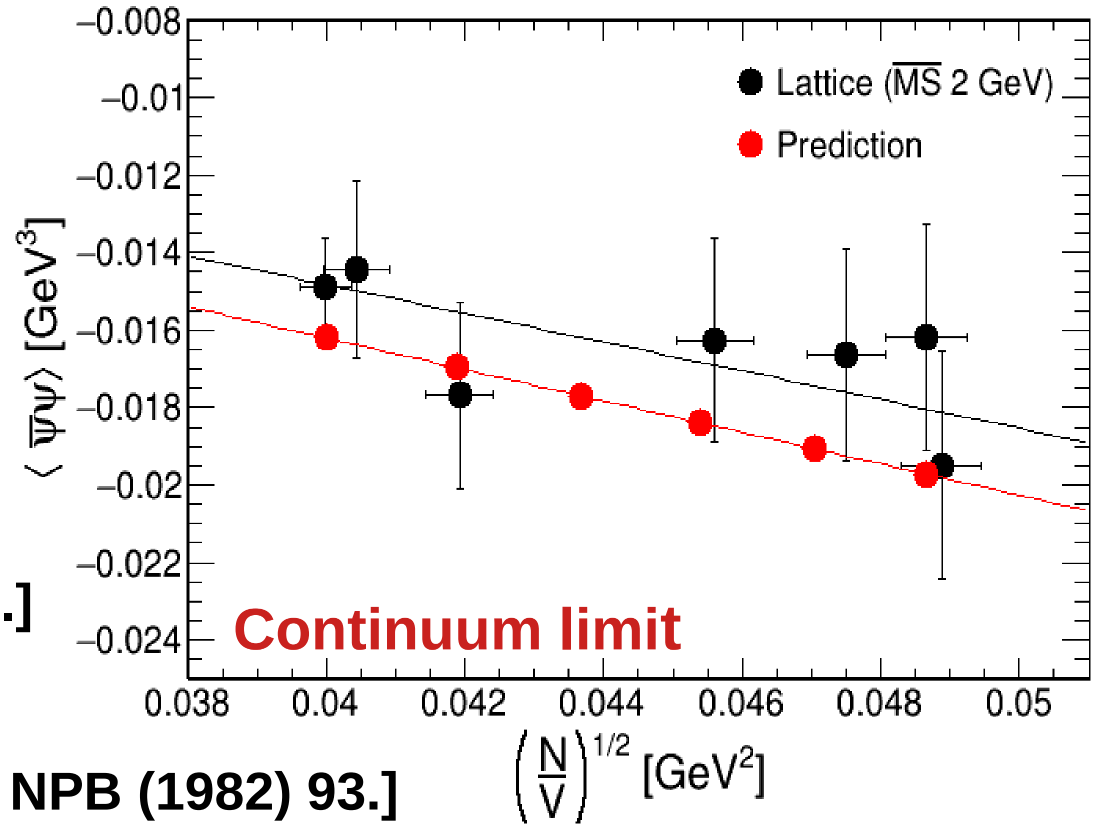
- **Prediction:**

$$\langle\bar{\psi}\psi\rangle^{Pre}(m_c) = A^{Pre}\left(\frac{N_I^{Pre}(m_c)}{V}\right)^{\frac{1}{2}}$$

[T. Schäfer, et al., RMP 70 (1998) 323.]

$$A^{Pre} = -0.405\ [\mathrm{GeV}]$$

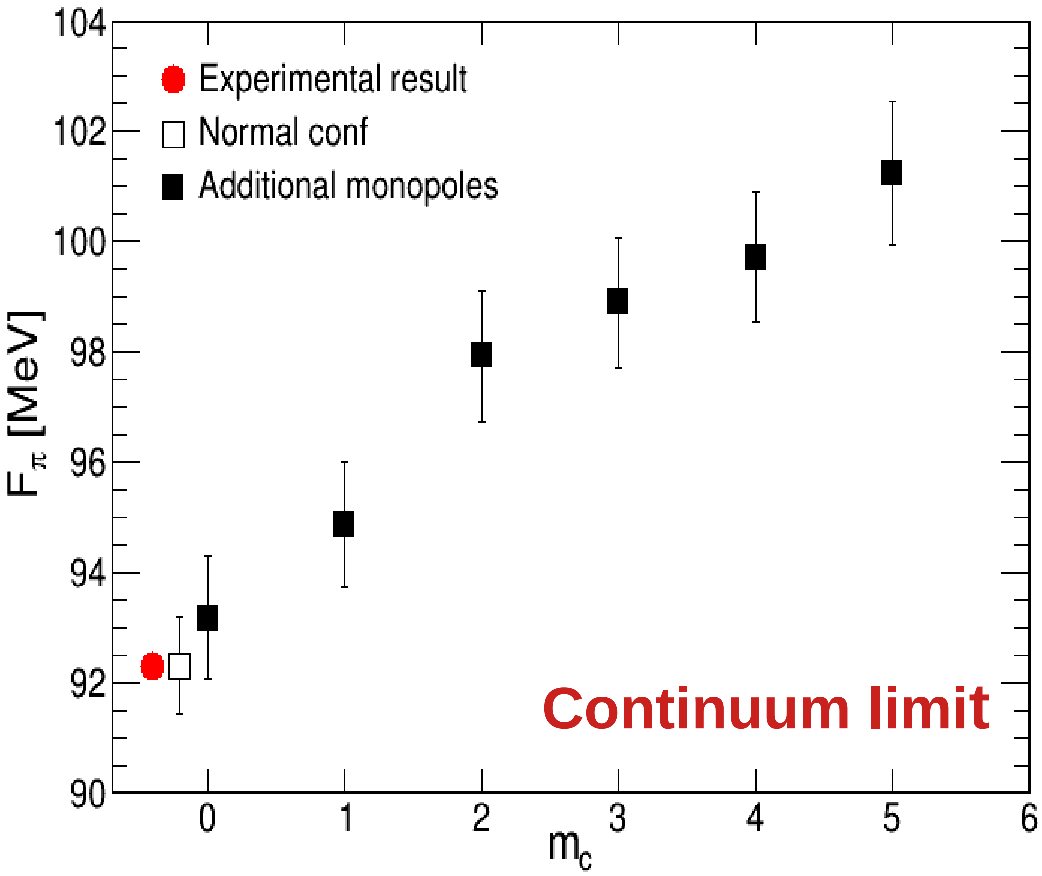
$$\frac{1}{\bar{\rho}} = 6.00\times 10^2\ [\mathrm{MeV}]$$ **[E. V. Shuryak, NPB (1982) 93.]**

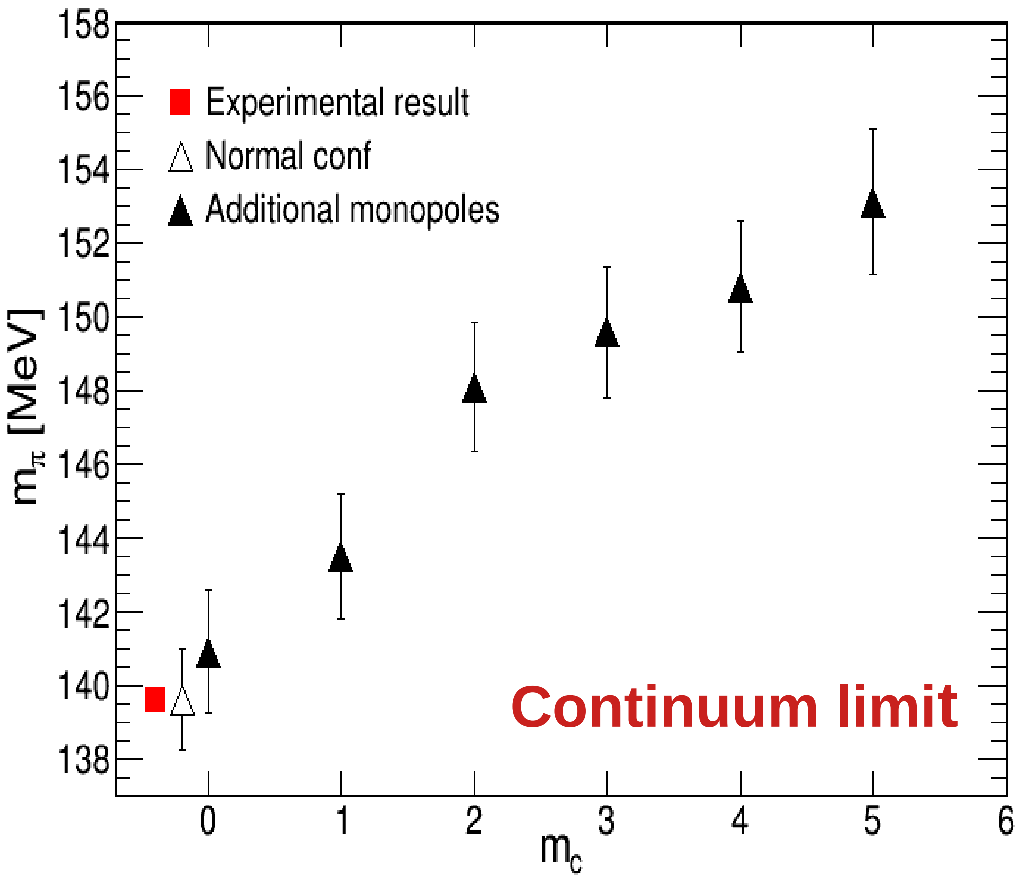


The renormalized chiral condensate decreases in direct proportion to the square root of the instanton density.

Decay constant and mass of pion

- We match the numerical results with the experimental results of pion decay constant and pion mass.



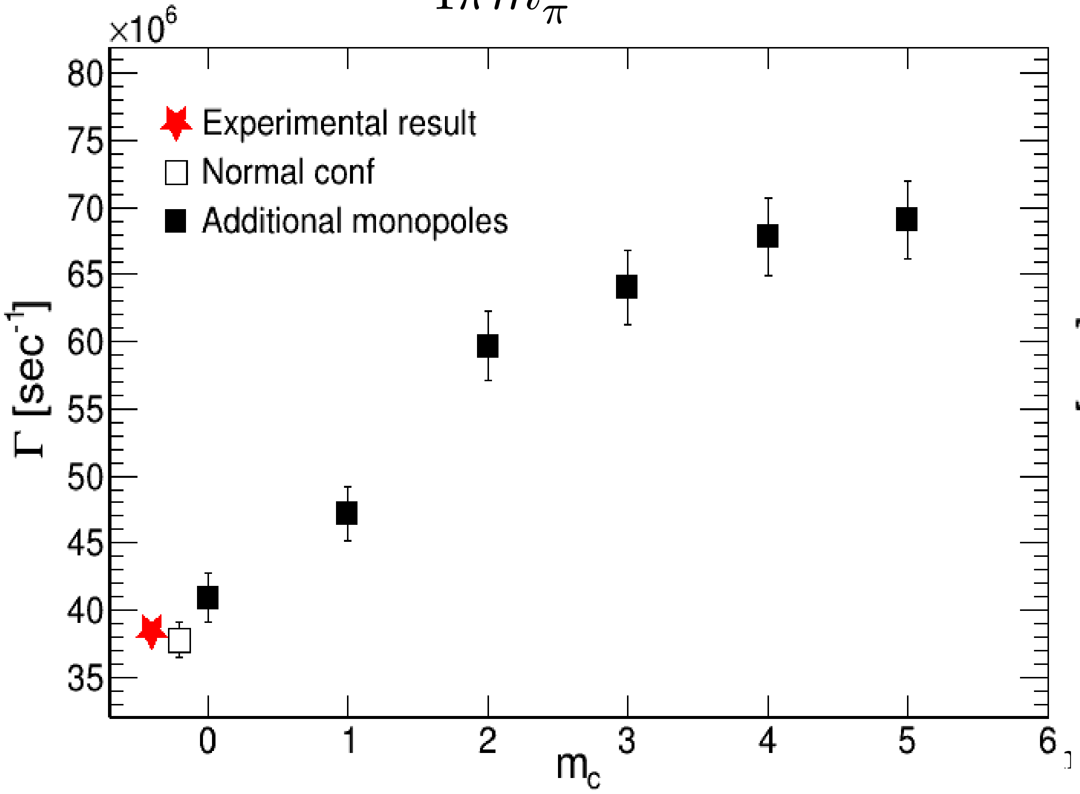


The decay width and lifetime of the charged pion

- **The decay width of the charged pion** [Text book, T. Kugo]**:**

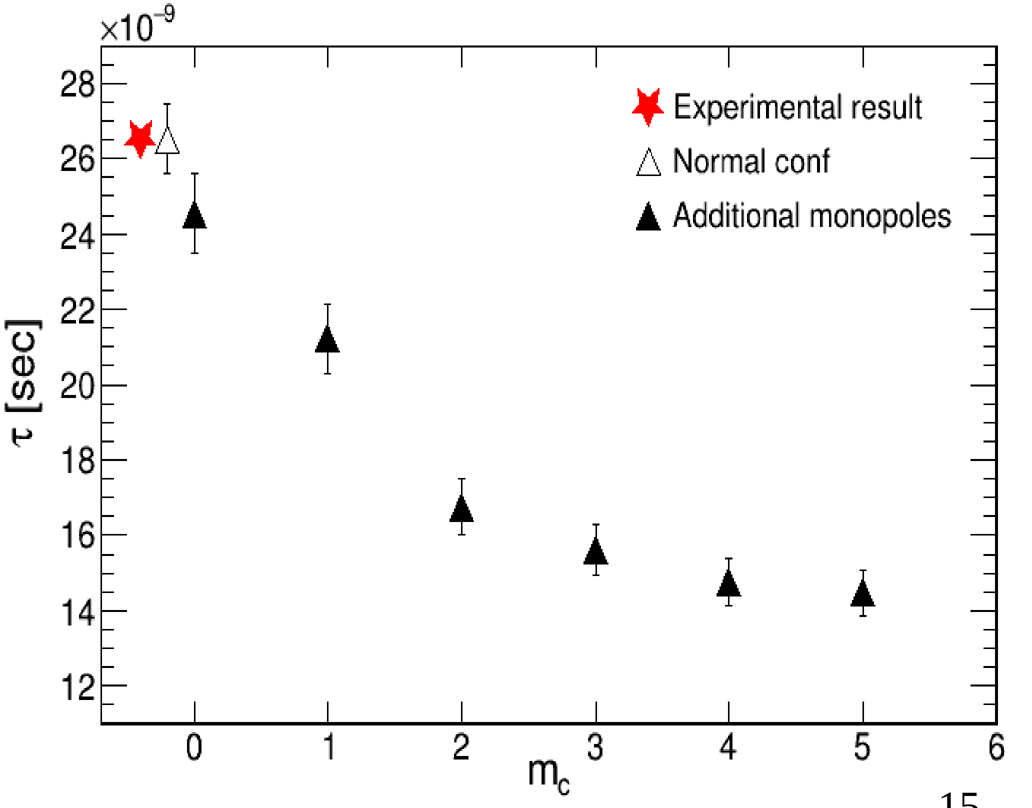
$$\pi^- \to l + \bar{\nu}_l$$

$$\Gamma = \frac{(G_F F_\pi \cos\theta_c)^2}{4\pi m_\pi^3} m_l^2 (m_\pi^2 - m_l^2)^2$$

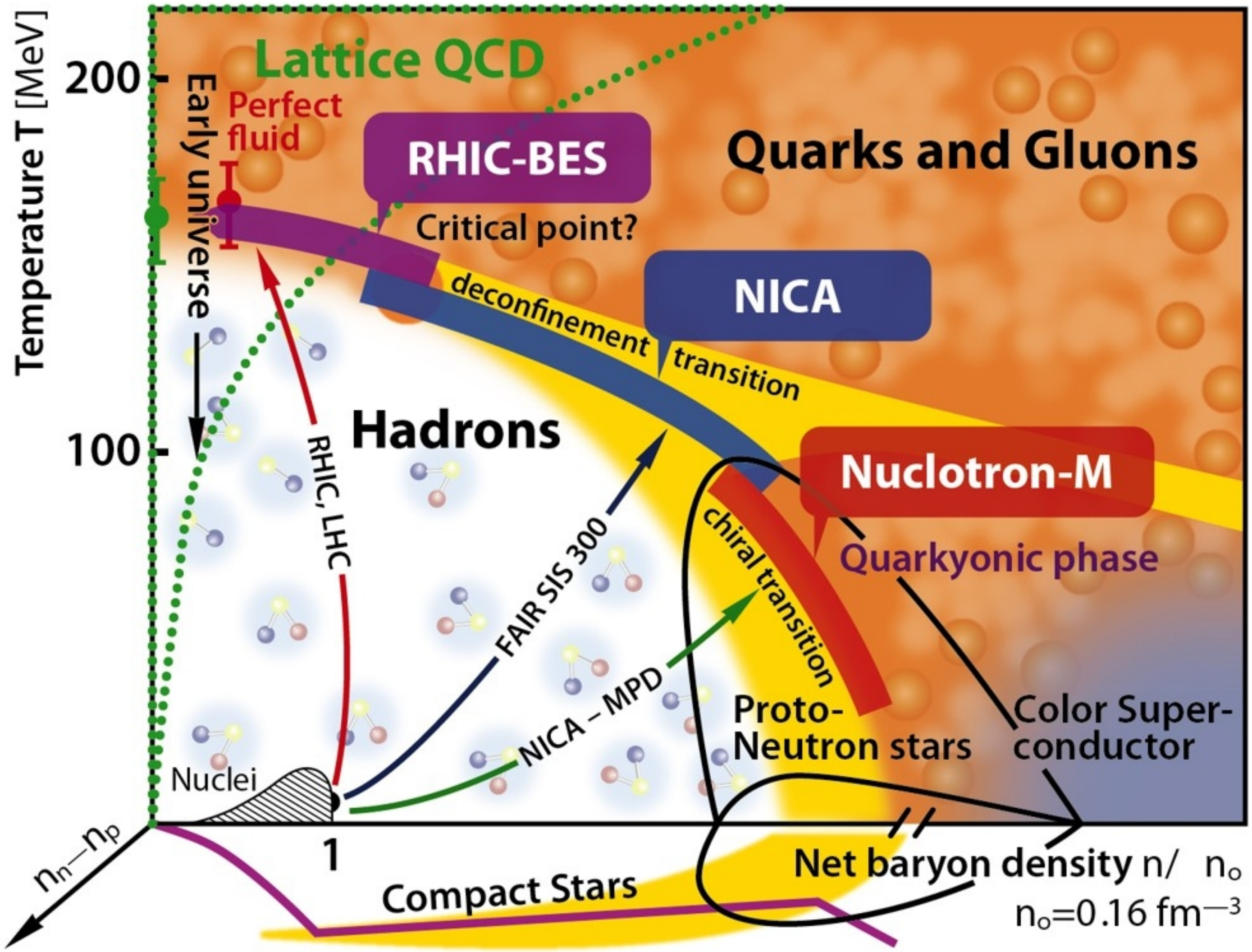


- **The lifetime of the charged pion:**

$$\tau = \frac{1}{\Gamma(\pi^- \to \mu + \bar{\nu}_\mu)}$$



Nuclotron-based Ion Collider fAcility (NICA)



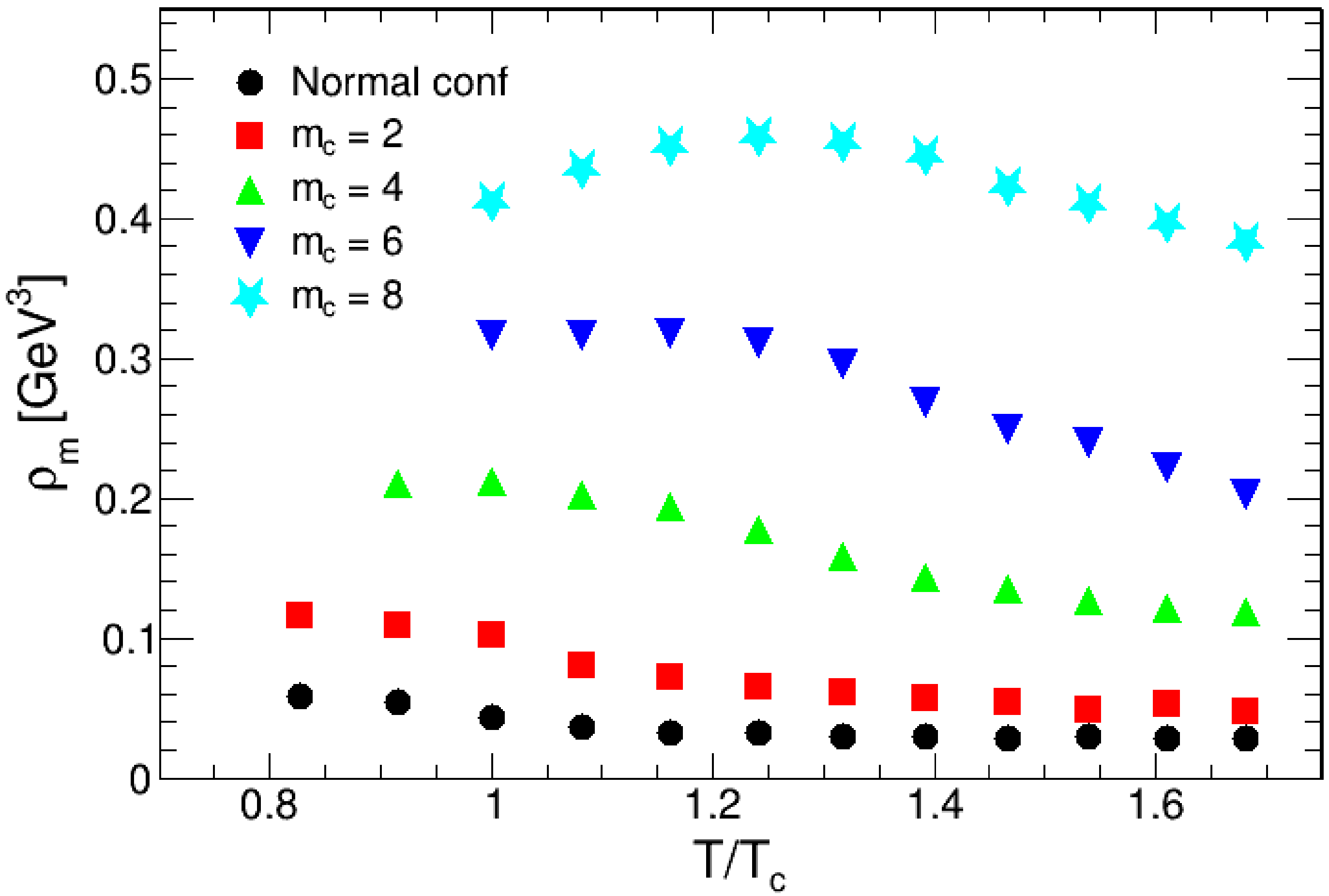
http://nica.jinr.ru/physics.php

Catalytic effects of monopole in the high temperature

- **We investigate the catalytic effects of monopoles on the phase transitions of the quark confinement and deconfinement, and moreover, the chiral symmetry breaking and the restoration.**
- We generate the standard configurations of the finite temperature and the configurations with the additional monopole and anti-monopole.
- The temperatures of the configurations T/T_{c} are from 0.8 to 1.7. The critical temperature is T_{c} = 296 [MeV].
- The length of the temporal direction of the lattice is set to T = 6.
- We vary the spatial volumes of the lattice V_s from 14^3 to 36^3 setting the physical volume to V = 5.4 [fm^4].

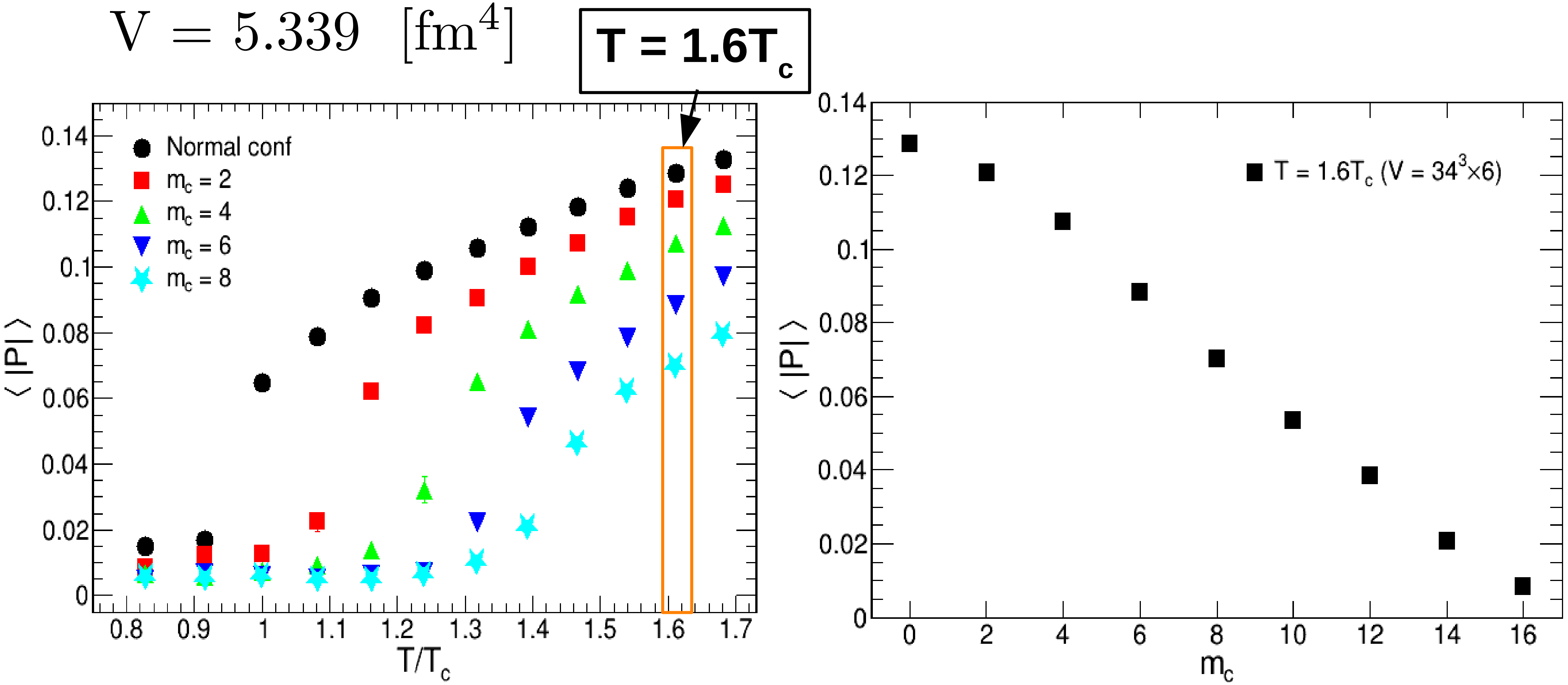
Monopole density

$$V = 5.339 \quad [\mathrm{fm}^4]$$

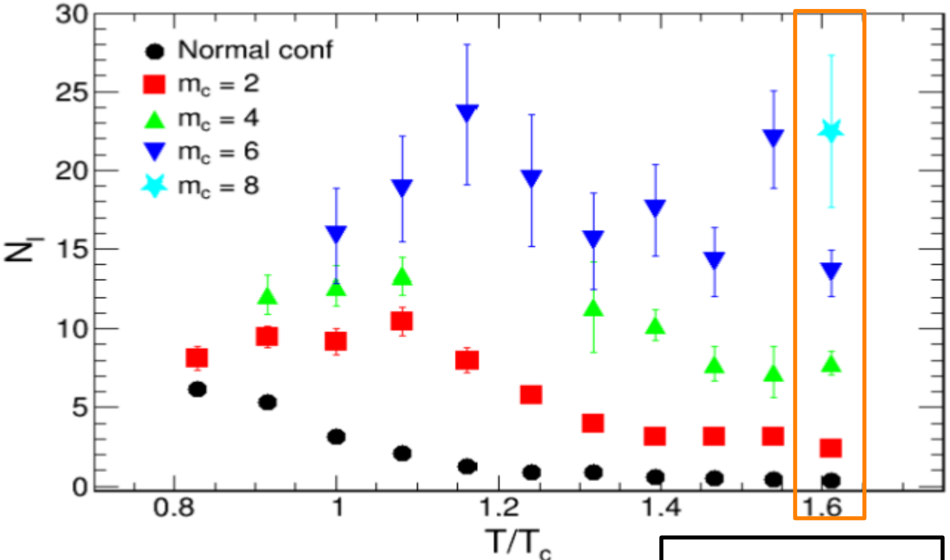


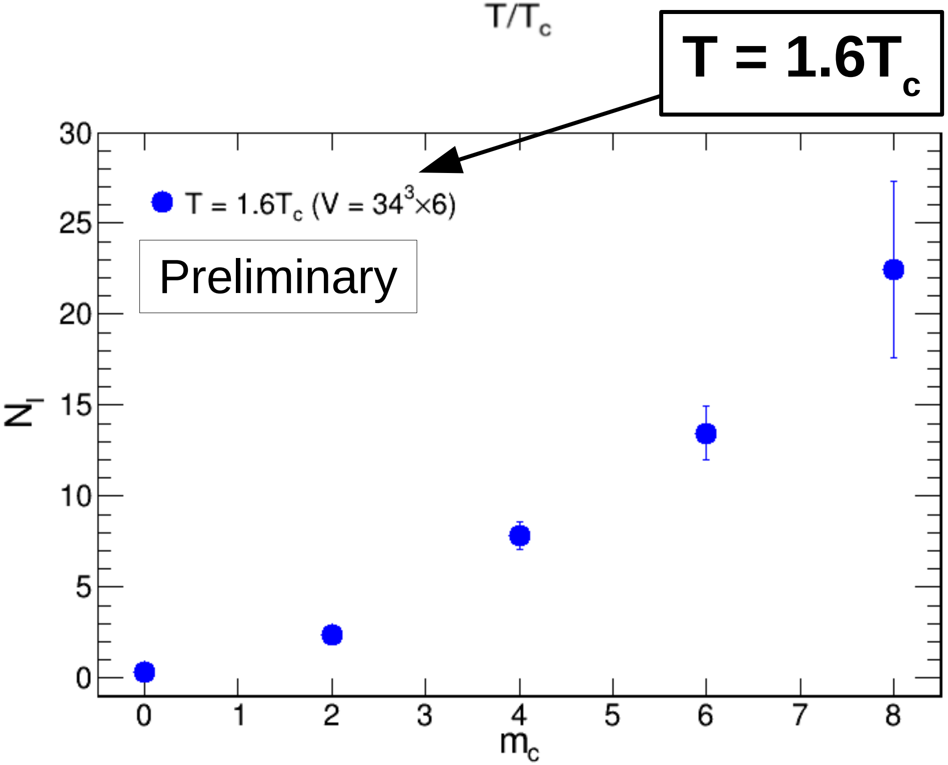
The quark confinement and monopoles

$$V = 5.339 \quad [\mathrm{fm}^4]$$



Instantons and spectrum density

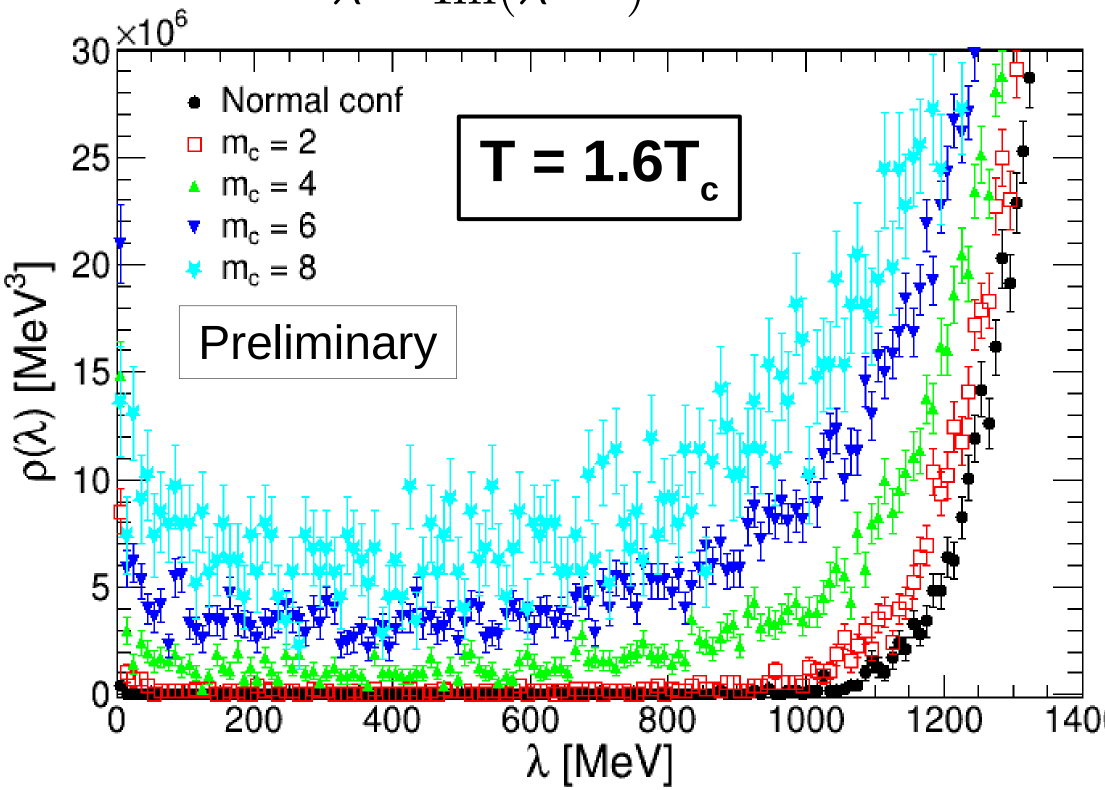




- The spectral density:

$$\rho(\lambda,\ V) = \frac{1}{V}\langle\sum_{\lambda}\delta(\lambda-\bar{\lambda})\rangle$$

$$\bar{\lambda} = \mathrm{Im}(\lambda^{imp})$$



[T. Banks, et al., NPB 169 (1980) 103, R. G. Edwards, et al., PRL 82 (1999) 4188, J. Wennekers, JHEP 09 (2005) 059]

Conclusions

In the study of zero temperature:

- The chiral condensate decreases.
- Pion decay constant and pion mass increase.
- The decay width becomes wider and the lifetime becomes shorter.

In the study of the finite temperature:

- The transition temperature of the quark confinement and deconfinement rises.
- The restoration of the chiral symmetry breaking would not occur.

These are the catalytic effects of the Adriano monopole.

Acknowledgments

We use SX-ACE, PC clusters, OCTOPUS, XC40 at the RCNP and the CMC at Osaka University and at the YITP at Kyoto University. We use the storage elements of JLDG at Osaka University.