Lattice results on dibaryons and baryon-baryon interactions

Sinya AOKI

Center for Gravitational Physics
Yukawa Institute for Theoretical Physics, Kyoto University
Hadron interactions in lattice QCD

Two methods

Finite volume method: successful for meson-meson interactions

Potential method: successful for baryon-baryon interactions

(Lattice) QCD: theory for quarks and gluons

Hadrons

J. Dudek: previous talk

Hadrons to Atomic nuclei

from Lattice QCD
Plan of my talk

I. HAL QCD potential method

II. Dibaryons
   1. at physical pion mass
   2. at heavier pion masses

III. Summary
I. HAL QCD potential method


Our strategy in lattice QCD

**Step 1**  define (Equal-time) Nambu-Bethe-Salpeter (NBS) Wave function

\[
\varphi_k(\mathbf{r}) = \langle 0 | N(\mathbf{x} + \mathbf{r}, 0) N(\mathbf{x}, 0) | N N, W_k \rangle
\]

\[N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x): \text{local operator}\]

\[N N \rightarrow N N\] only elastic scattering

energy

\[W_k = 2 \sqrt{k^2 + m_N^2} < W_{\text{th}} = 2m_N + m_\pi\]

**Asymptotic behavior in the center of mass (CM)**

\[
\varphi_k(\mathbf{r}) \sim \sum_{l,m} C_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} Y_{lm}(\Omega_r)
\]

scattering phase shift (phase of the S-matrix by unitarity) in QCD.
Step 2: define the energy-independent “potential” with derivatives from these NBS wave functions as

$$\left[ \epsilon_k - H_0 \right] \varphi_k(x) = V(x, \nabla) \varphi_k(x)$$

for all \( k \) with \( W_k \leq W_{\text{th}} \)

$$\epsilon_k = \frac{k^2}{2\mu} \quad H_0 = \frac{-\nabla^2}{2\mu}$$

For NN

$$V(x, \nabla) = V_0(r) + V_\sigma(r)(\sigma_1 \cdot \sigma_2) + V_T(r)S_{12} + V_{LS}(r)L \cdot S + O(\nabla^2)$$

LO LO LO NLO NNLO

tensor operator \( S_{12} = \frac{3}{r^2}(\sigma_1 \cdot x)(\sigma_2 \cdot x) - (\sigma_1 \cdot \sigma_2) \)

spins

By construction

potential \( V(x, \nabla) \) is faithful to QCD phase shift \( \delta_l(k) \).

Remark

No non-relativistic approximation in CM

$$-\Box - m^2 = (W_k/2)^2 + \nabla^2 - m^2 = k^2 + \nabla^2$$
Step 3  Determination of local terms order by order

Leading Order potential \( V_0^{LO}(r) := V_0(r) + V_\sigma(r)(\sigma_1 \cdot \sigma_2) + V_T(r)S_{12} \)

One \( \varphi_k(x) \)
\[
V_0^{LO}(r; \varphi_k) = \frac{[\epsilon_k - H_0] \varphi_k(x)}{\varphi_k(x)} \quad \text{LO approximation}
\]

Another \( \varphi_q(x) \)
\[
V_0^{LO}(r; \varphi_q) = \frac{[\epsilon_q - H_0] \varphi_q(x)}{\varphi_q(x)} \quad \text{LO approximation}
\]

If \( V_0^{LO}(r; \varphi_q) \simeq V_0^{LO}(r; \varphi_k) \)  
\( \text{LO approximation is good at } |k| \leq |p| \leq |q| \)

If \( V_0^{LO}(r; \varphi_q) \neq V_0^{LO}(r; \varphi_k) \)  
\( \text{NLO term can be determined from} \)
\[
[\epsilon_k - H_0] \varphi_k = \left[ V_0^{NLO}(r) + V_1^{NLO}(r) L \cdot S \right] \varphi_k
\]
\[
[\epsilon_q - H_0] \varphi_q = \left[ V_0^{NLO}(r) + V_1^{NLO}(r) L \cdot S \right] \varphi_q
\]
Demonstration

Separable potential

\[ U(\vec{x}, \vec{y}) = w v(\vec{x}) v(\vec{y}) \]

\[ v(\vec{x}) = e^{-\mu x}, \quad x := |\vec{x}| \]

highly non-local

L=0 wave function

\[
\psi_k^0(x) = \frac{e^{i\delta(k)}}{kx} \left[ \sin(kx + \delta(k)) - \sin \delta(k)e^{-\mu x} \left( 1 + x \frac{\mu^2 + k^2}{2\mu} \right) \right]
\]

\[
= C \frac{e^{i\delta(k)}}{kx} \sin(kx + \delta_R(k))
\]

phase shift \( \delta_R(k) \) is exactly calculable.

separable potential

\[ U(\vec{x}, \vec{y}) \]

LO potential

\[ V_0^{LO}(r) \quad \text{from } k^2 = 0 \text{ or } k^2 = \mu^2 \]

NLO potential

\[ V_0^{NLO}(r) + V_1^{NLO}(r) \nabla^2 \]
\( \omega/\mu^4 = -0.017, \ m/\mu = 3.30, \ R\mu = 2.5 \)

\[
\begin{align*}
U(\vec{x}, \vec{y}) &= wv(\vec{x})v(\vec{y}) \\
v(\vec{x}) &= e^{-\mu x}, \quad x := |\vec{x}| 
\end{align*}
\]

NLO potential reproduces the exact phase shift rather well.
\[ k \cot(\delta_0(k)) \]

\[ \frac{\omega}{\mu^4} = -0.017, \frac{m}{\mu} = 3.30, R\mu = 2.5 \]

- **LO at** \[ k^2 = 0 \]
- **LO ERE at** \[ k^2 = \mu^2 \]
- **NLO ERE**
- **LO ERE at** \[ k^2 = 0 \]
- **NLO potential is better than NLO ERE.**

\[ \text{ERE = Effective Range Expansion} \]
Improved extraction of potentials

Normalized 4-pt function

\[ R(r, t) \equiv F(r, t)/G_N(t)^2 = \sum_n A_n \varphi^{W_n}(r) e^{-\Delta W_n t} + \cdots \]

\[ \Delta W_n = W_n - 2m_N \]

4-pt function

\[ F(r, t - t_0) = \langle 0| T\{N(x + r, t)N(x, t)\}\mathcal{J}(t_0)|0\rangle \]

NN creation op.

Master equation

\[ \left\{ -H_0 - \frac{\partial}{\partial t} + \frac{1}{4m_N^2} \frac{\partial^2}{\partial t^2} \right\} R(r, t) = V(r, \nabla)R(r, t) + \cdots = V_{0\text{LO}}(r)R(r, t) + \cdots \]

remaining t-dependence of the potential

1. Inelastic contributions

2. Higher order terms in the derivative expansion

Potential

\begin{figure}
\centering
\includegraphics[width=\textwidth]{potential_graph.png}
\end{figure}
II. Dibaryons
Baryon (B=1)

Proton, Neutron, Lambda, Omega,…

Dibaryon (B=2)

Deuteron observed in 1930s
+ $d^*(2380)$ resonance

Dibaryon = two baryon bound state or resonance
SU(3) classification for Dibaryon candidates (B=2)

1) octet-octet system

\[ 8 \otimes 8 = 27 \oplus 8_s \oplus 1 \oplus 10 \oplus 10 \oplus 8_a \]

Jaffe (1977)

H-dibaryon (J=0)

Deuteron (J=1)

2) decuplet-octet system

\[ 10 \otimes 8 = 35 \oplus 8 \oplus 10 \oplus 27 \]

NΩ system and NΔ system (J=2)

Goldman et al (1987)
Dyson, Xuong (1964)

3) decuplet-decuplet system

\[ 10 \otimes 10 = 28 \oplus 27 \oplus 35 \oplus 10 \]

ΩΩ system (J=0)

Zhang et al (1997)

ΔΔ system (J=3)

Dyson, Xuong (1964)
Kamae, Fujita (1977)
Oka, Yazaki (1980)

\[ d^*(2380) \text{ resonance} \]
1. Physical pion mass

Lattice QCD at (almost) physical pion mass

2+1 flavor QCD, $m_\pi \approx 145$ MeV, $a \approx 0.085$ fm, $L \approx 8$ fm
$\Omega^- \Omega^-$

(sssssss)

\( \Omega \Omega (^{1}S_0) \) potential

- \( \delta(\text{deg}) \)
- \( E_{\text{CM}}[\text{MeV}] \)

A similar structure to NN

Strong attraction

repulsive behavior

bound state
Binding energy

\[ H = -\frac{\nabla^2}{m_\Omega} + V^{\text{LQCD}}_\Omega(r) + \frac{\alpha}{r} \]

\[
\begin{aligned}
(B^{(\text{QCD})}_\Omega, B^{(\text{QCD+Coulomb})}_\Omega) &= (1.6(6)\text{MeV}, 0.7(5)\text{MeV}) \\
\text{The most strange (sss sss) dibaryon?} \\
\text{A candidate for the second bound dibaryon.}
\end{aligned}
\]
$N\Omega^-$
$N\Omega$ potential in $^5S_2$ channel

- attractive potential without repulsive core
- long range attraction

qualitatively the same at $m_\pi \approx 875$ MeV

B.E. = 18.9(5.0)(+12.1)(-1.8) MeV
Remark

\[ m_\pi = 146 \text{ MeV} \]

\[ m_\pi = 875 \text{ MeV} \]

\[ L = \infty \quad L = 8.1 \text{ fm} \]
\[ L = \infty \quad L = 1.9 \text{ fm} \]
\[ p_{\text{min}} = 153 \text{ MeV} \]
\[ p_{\text{min}} = 645 \text{ MeV} \]

\[ N\Omega^{(5S_2)} \]
\[ 2676 \]
\[ 2595 \]
\[ \Sigma\Xi^{(3,1D_2)} \]
\[ 2577 \]
\[ 2514 \]
\[ \Lambda\Xi^{(3,1D_2)} \]
\[ 2495 \]

* Single channel analysis only.
* Assume small couplings to D-waves, supported by weak t-dep.
* Coupled channel analysis in the future
Phase shift and binding energy

New dibaryon resonance?
Comparison

\[ \frac{r_{\text{eff}}}{a_0} \text{ VS } r_{\text{eff}} \]

\[ k \cot \delta_0(k) = \frac{1}{a_0} + \frac{r_{\text{eff}}}{2} k^2 + \cdots \]

- \( \Omega\Omega(1S_0) \)
- \( NN(3S_1) \)
- \( N\Omega(5S_2) \)
- \( NN(1S_0) \)

lattice

Unitary region

experiment

universal ?
How can we confirm?
Measurement of two-baryon correlation at RHIC & LHC


**two-baryon interaction ⇔ two-baryon correlation**

$$C_{AB}(Q) = \frac{N_{AB}^{\text{pair}}(Q)}{N_A N_B(Q)} = \begin{cases} 1 & \text{for } Q = \frac{(p_1 - p_2) \cdot P}{p_2^2} \sqrt{\frac{P^2}{2} - \frac{(p_1 - p_2) \cdot P}{p_2^2}} \end{cases}$$

K. Morita et al., PRC94(2016)031901 “$N\Omega$ correlation from HAL pot”
K. Morita et al., NPA967(2017)856 “NXi correlation from HAL pot.”
K. Morita et al., arXiv:1908.05414 “$N\Omega$ & $\Omega\Omega$ correlations from HAL pot.”
\[ N_{AB}(Q) = \int \frac{d^3p_A}{E_A} \frac{d^3p_B}{E_B} N_{AB}(p_A, p_B) \delta(Q - \sqrt{-q^2}) \]

\[ N_{AB}(p_A, p_B) \simeq \int d^4 x d^4 y S_A(x, p_A) S_B(y, p_B) |\Psi(x, y, p_A, p_B)|^2 \]

If the source is approximately known, one can test hadron interactions using the above formula.
Proton-Ω correlation in RHIC

\[ \text{Au} + \text{Au} \]

centrality

40-80% (small)

0-40% (large)

\begin{align*}
V_I : & \text{unbound} \\
V_{II} : & E_B = 6.3 \text{ MeV} \\
V_{III} : & E_B = 26.9 \text{ MeV}
\end{align*}

potential at \( m_\pi = 875 \text{ MeV} \)

Data at \( k^* < 40 \text{ MeV} \) favor \( V_{III} \).
One can also use p+p data (LHC).

Oton Vazquez Doce (ALICE), talk in Session 7 on Aug.18

$p\Omega$ correlations

ALICE Preliminary
$pp \backslash s = 13$ TeV
High-Mult. (0-0.072% INEL)

- $p-\Omega^- \oplus \bar{p}-\Omega^+$
- Coulomb + Sekihara ($^5S_2+^3S_1$)
- Coulomb + HAL-QCD ($^5S_2+^3S_1$)
- Coulomb

- $p-\Omega^- \oplus \bar{p}-\Omega^+$ sideband background

*HAL QCD potential at physical pion seems consistent with data.
*Need more accurate potential/data for a further confirmation.

$\Omega\Omega$ in near future?
2. Heavier pion masses
$\Delta \Delta$ system with $J = 3$

S. Gongyo et al., in preparation.
$d^*(2380)$ observed by WASA@COSY col.

$$p + n(d) \rightarrow d + \pi^0 + \pi^0 (+p_{\text{spectator}})$$

$m \sim 2.38$ GeV, $\Gamma \sim 70$ MeV, $J^\pi = 3^+, I=0$
3-flavor full QCD in the SU(3) limit

$N_f = 2 + 1$ full QCD with $L = 1.93$ fm,

CP-PACS Conf. \hspace{1cm} L = 1.93$ fm

<table>
<thead>
<tr>
<th></th>
<th>[MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{ps}$</td>
<td>1015</td>
</tr>
<tr>
<td>$m_{oct}$</td>
<td>2030</td>
</tr>
<tr>
<td>$m_{dec}$</td>
<td>2220</td>
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</tbody>
</table>

$\Delta$: resonance

$\Delta$: bound state

$\Delta p^+ + \pi^+$

$3045$ MeV

$\Delta$

$2220$ MeV

$\Delta\Delta$

$d^*$: resonance

$d^*$: bound state
We assume that decay to $\text{NN}(^3D_3)$ is neglected

- In short range, there is no repulsive core
- Deep bound state is found

$d^*$ is supported from lattice QCD
H-dibaryon
Baryon potential in the flavor SU(3) limit

\[ m_u = m_d = m_s \]

two octet baryons

\[ 8 \otimes 8 = 27 \oplus 8_s \oplus 1 \oplus 10 \oplus 10 \oplus 8_a \]

Symmetric

Anti-symmetric

6 independent potentials in flavor-basis

\[ V^{(27)}(r), V^{(8_s)}(r), V^{(1)}(r) \]

\[ V^{(10)}(r), V^{(10)}(r), V^{(8_a)}(r) \]
Flavor dependences of BB interactions

$L \simeq 4 \text{ fm}, \quad m_\pi \simeq 470 \text{ MeV}$

same as NN  \quad 8s: strong repulsive core. repulsion only.

1: attractive instead of repulsive core ! attraction only . H-dibaryon.

same as NN  \quad 10: strong repulsive core. weak attraction.

8a: weak repulsive core. strong attraction.

Force for the singlet is attractive at all distances. Bound state ?
Attractive potential in the flavor singlet channel

possibility of a bound state (H-dibaryon)

$\Lambda\Lambda - N\Xi - \Sigma\Sigma$

Inoue et al. (HAL QCD Coll.), PRL106(2011)162002

An H-dibaryon exists in the flavor SU(3) limit.
Binding energy $= 25$-$50$ MeV at this range of quark mass. Real world?
A mild quark mass dependence.
H-dibaryon with the flavor SU(3) breaking

SU(3) limit

\[ \Lambda \Lambda \rightarrow N \Xi \rightarrow \Sigma \Sigma \]

25-50 MeV

Real world

\[ m_u = m_d \neq m_s \]

2386 MeV

129 MeV

2257 MeV

25 MeV

2232 MeV

Bound state energy \( E_0 \) vs. Root-mean-square distance \( \sqrt{\langle r^2 \rangle} \) [fm]

- The figure shows the H-dibaryon with its mass values:
  - \( M_{PS} = 1171 \) [MeV]
  - \( M_{PS} = 1015 \) [MeV]
  - \( M_{PS} = 837 \) [MeV]
  - \( M_{PS} = 672 \) [MeV]
  - \( M_{PS} = 469 \) [MeV]

- The diagram illustrates the energy levels and mass differences in the real world compared to the SU(3) limit.
LG improved gauge action & O(a) improved clover quark action

$\beta = 1.90, \quad a^{-1} = 2.176 \quad [\text{GeV}], \quad 32^3 \times 64$ lattice,

$L = 2.902 \quad [\text{fm}].$

$\kappa_s = 0.13640$ is fixed, $\kappa_{ud} = 0.13700, 0.13727, 0.13754$ are chosen.

Flat wall source is considered to produce S-wave B-B state.

The KEK computer system A resources are used.

$u,d$ quark masses lighter

$\pi$: 701±1, 570±2, 411±2

$K$: 789±1, 713±2, 635±2

$m_\pi / m_K$: 0.89, 0.80, 0.65

$N$: 1585±5, 1411±12, 1215±12

$\Lambda$: 1644±5, 1504±10, 1351±8

$\Sigma$: 1660±4, 1531±11, 1400±10

$\Xi$: 1710±5, 1610±9, 1503±7

In unit of MeV

$\Delta \Delta$: 3288MeV
$N \Xi$: 3295MeV
$\Sigma \Sigma$: 3320MeV

SU(3) breaking effects becomes larger

SU(3) breaking effects becomes larger
ΛΛ and NΞ phase shift

Bound H-dibaryon coupled to NΞ

H as Λ Λ resonance
H as bound NΞ

This suggests that H-dibaryon becomes resonance at physical point.
Below or above NΞ? Need simulation at physical point. (work in progress)

Physically, it is essential that H-dibaryon is a bound state in the flavor SU(3) limit.
III. Summary
• The HAL QCD Potential is a very powerful tool to investigate baryon interactions.

• Dibaryons.

  • Omega-Omega : shallow bound state at physical pion mass

  • N-Omega: dibaryon resonance at physical pion mass ?

  • confirmation by 2-particle correlations in future

  • bound $\Delta \Delta$ at flavor SU(3) limit: support d*(2380)

  • bound H dibaryon at flavor SU(3) limit: physical pion mass ?

• Other applications (rho & sigma resonances, heavy baryons, Tetra quark, Penta quark, 3 body forces)
Thank you for your attention!

HAL QCD Collaboration

Hadrons to Atomic nuclei
from Lattice QCD

* PhD students

YITP, Kyoto: Sinya Aoki, Yutaro Akahoshi*, Daisuke Kawai, Takaya Miyamoto, Koutaro Murakami*, Kenji Sasaki
Riken: Takumi Doi, Takahiro Doi, Sinya Gongyo, Tetsuo Hatsuda, Takumi Iritani
RCNP, Osaka: Yoichi Ikeda, Noriyoshi Ishii, Keiko Murano, Hidekatsu Nemura
Nihon: Takashi Inoue
KEK: Tatsumi Aoyama
Birjand, Iran: Faisal Etminan