hadron spectroscopy from lattice QCD

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CHARTERED 1693

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state of the art in lattice QCD

now routinely done, calculations with

very light quarks (physical pion mass)

QED effects included

breaking of isospin symmetry ($m_u \neq m_d$)

systematic control,
but only for the **simplest observables**

state of the art in lattice QCD

now routinely done, calculations with

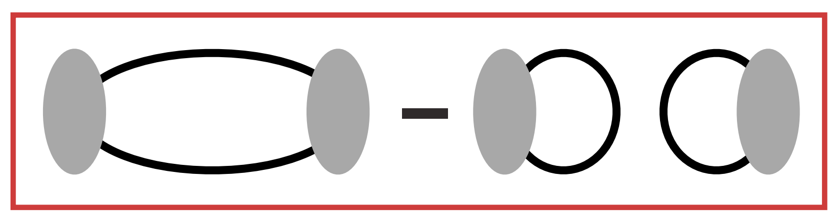
- **very light quarks** (physical pion mass)
- **QED effects included**
- **breaking of isospin symmetry** ($m_u \neq m_d$)

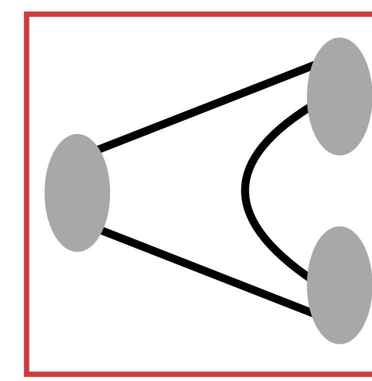
systematic control,
but only for the **simplest observables**

more relevant to excited spectroscopy

large diverse basis of hadron operators (including multi-hadrons)

inclusion of $q\bar{q}$ annihilation effects (access to isoscalar mesons, hadron decays)





'variational' analysis of matrices of correlation functions

(reliable determination of many excited states)

much of this progress follows from

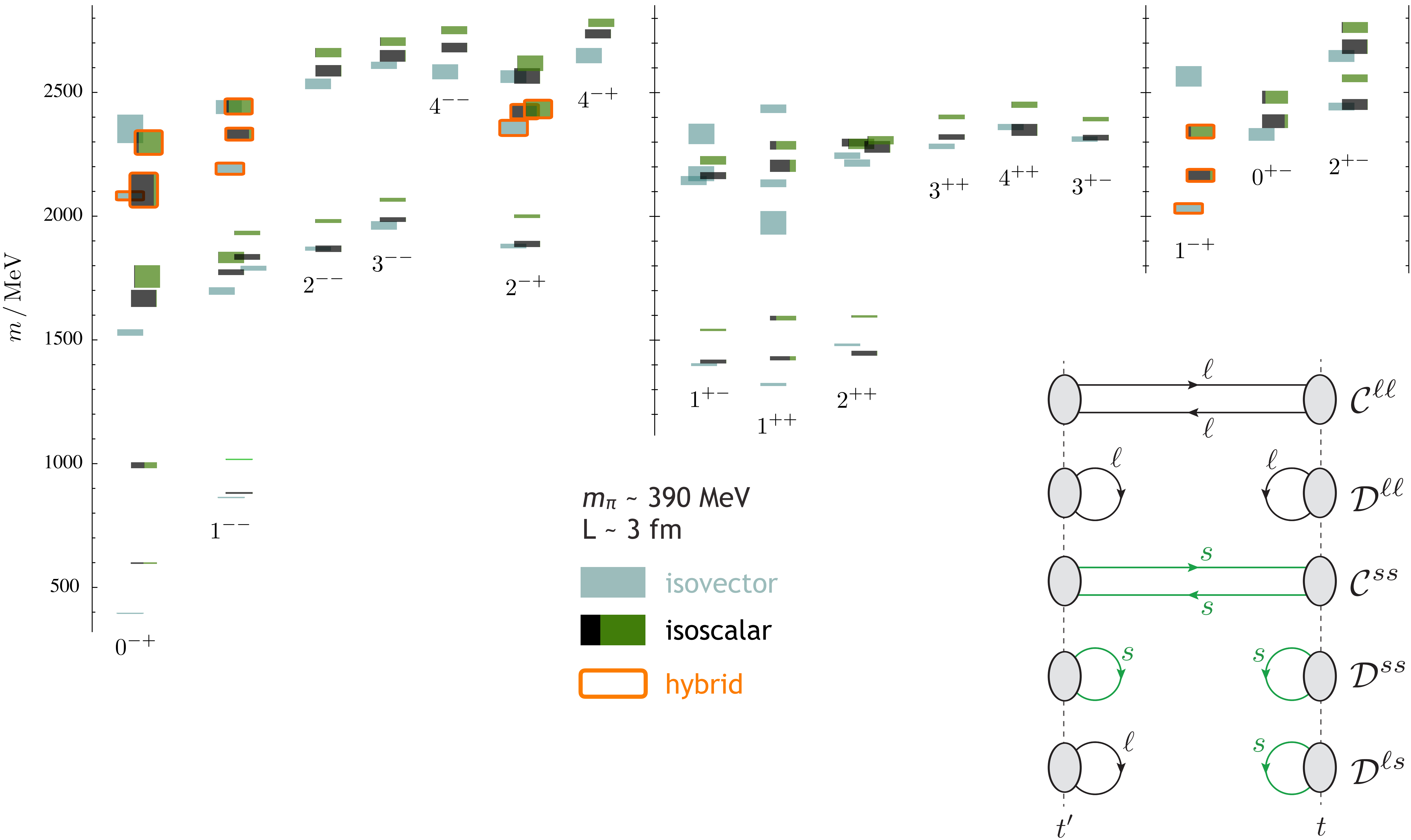
PHYSICAL REVIEW D **80,** 054506 (2009) Peardon et al

Novel quark-field creation operator construction for hadronic physics in lattice QCD

'distilled' and matured for ten years

“stable” meson spectrum

diagonalizing a large basis of operators ~ $\bar{\psi}\Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D}\psi$



tempting to make use of these tools to study contemporary exotic hadrons

e.g. $Z_c(3900)$, a resonance in $J/\psi\,\pi$? — probably $J^P=1^+$

compute correlation functions using a basis of

tetraquark-like $\psi(\mathbf{x})\psi(\mathbf{x})\bar{\psi}(\mathbf{x})\bar{\psi}(\mathbf{x})$ with appropriate spin, color coupling

and

meson-meson-like $\Big(\sum_{\mathbf{x}} e^{i\mathbf{p}_1\cdot\mathbf{x}}\,\bar{\psi}\mathbf{\Gamma}\psi(\mathbf{x})\Big)\Big(\sum_{\mathbf{y}} e^{i\mathbf{p}_2\cdot\mathbf{y}}\,\bar{\psi}\mathbf{\Gamma}\psi(\mathbf{y})\Big)$

operators

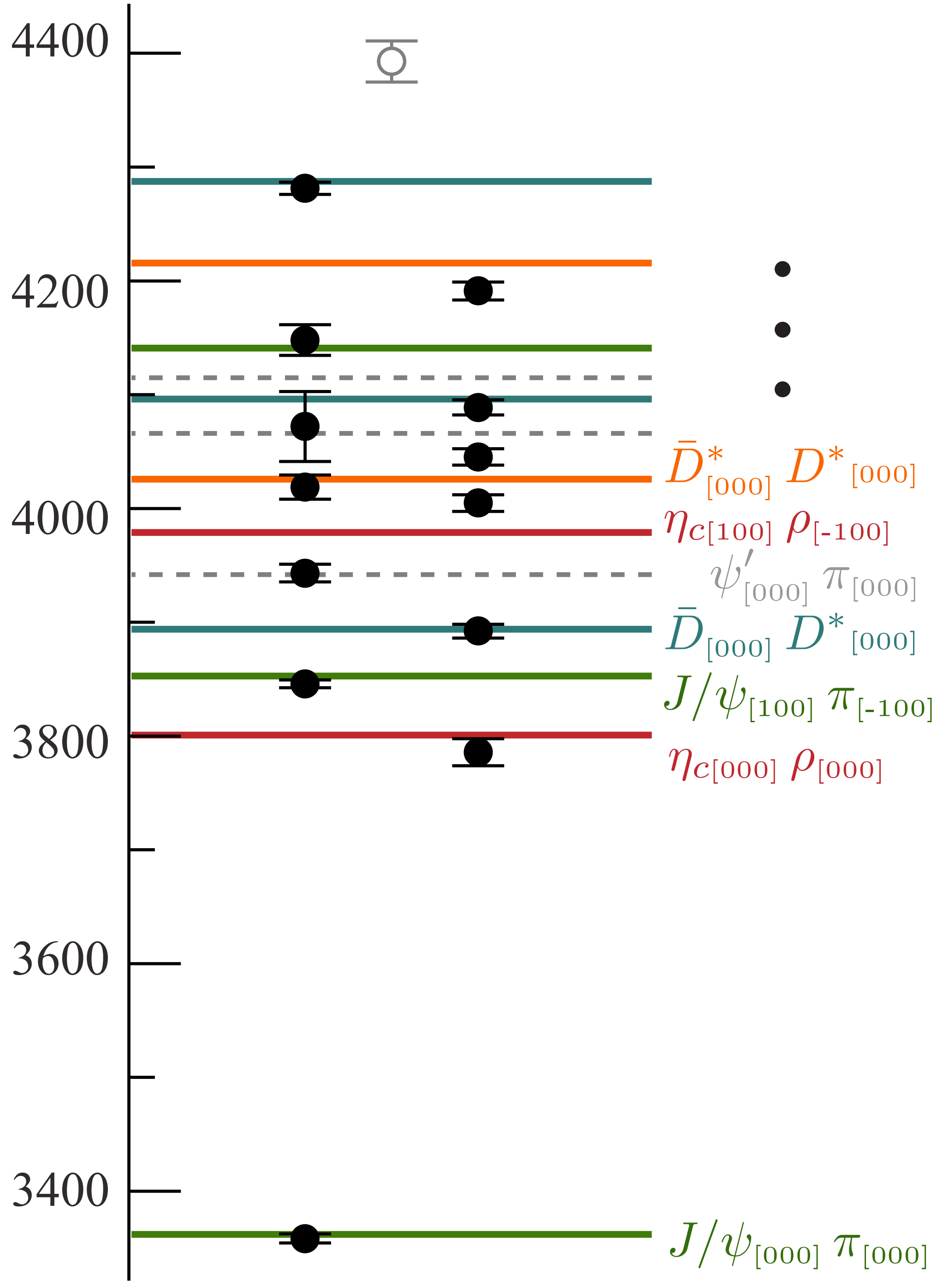
for non-interacting mesons, expect a spectrum

$$\sqrt{m^2+\mathbf{p}_1^2}+\sqrt{m_2^2+\mathbf{p}_2^2}$$

$$\mathbf{p}=\frac{2\pi}{L}(n_x,n_y,n_z)$$

‘Z_c(3900)’ channel — J^P=1$^+$

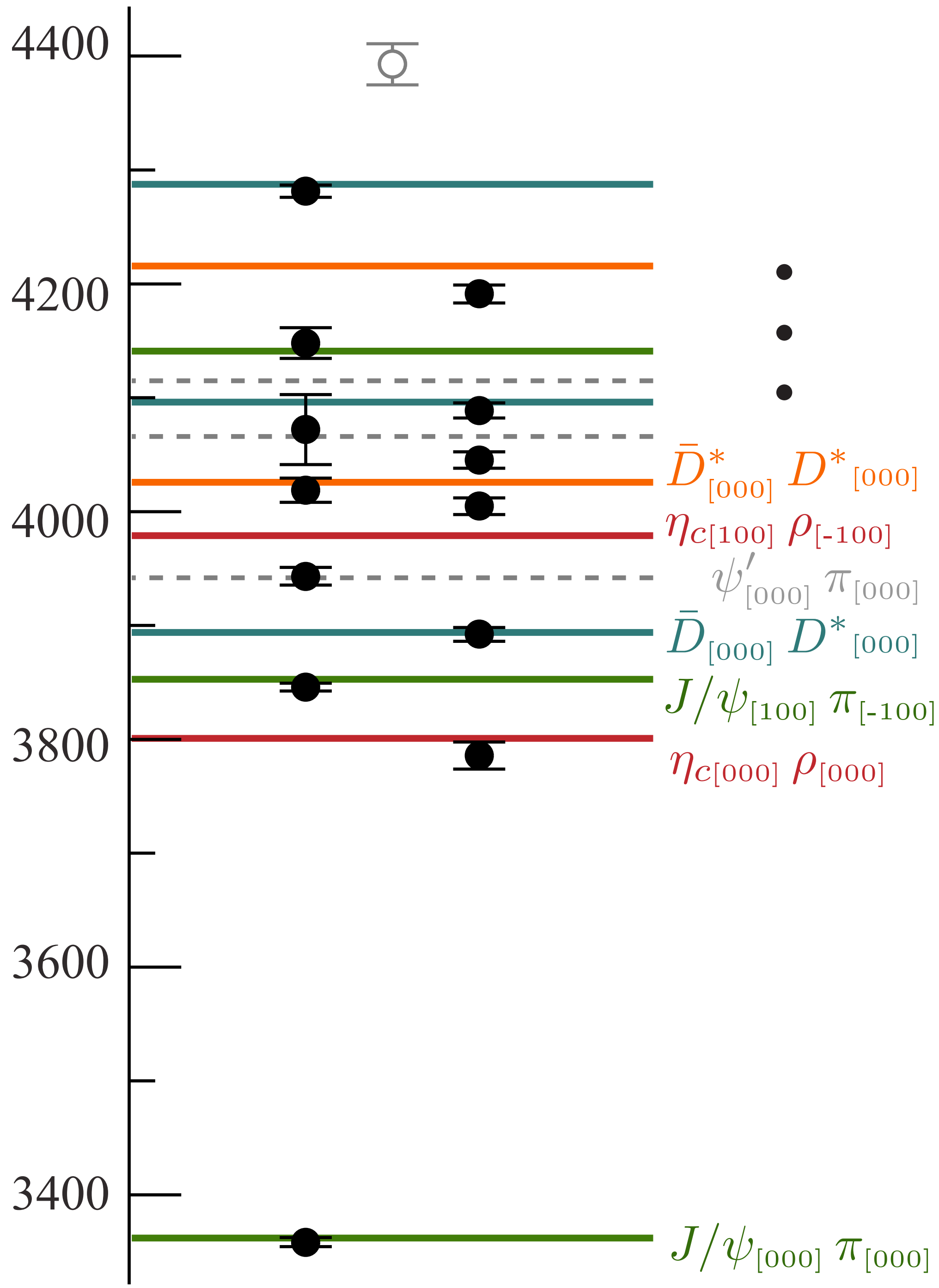
Prelovsek et al (2015)



m_π ~ 270 MeV, L ~ 2 fm

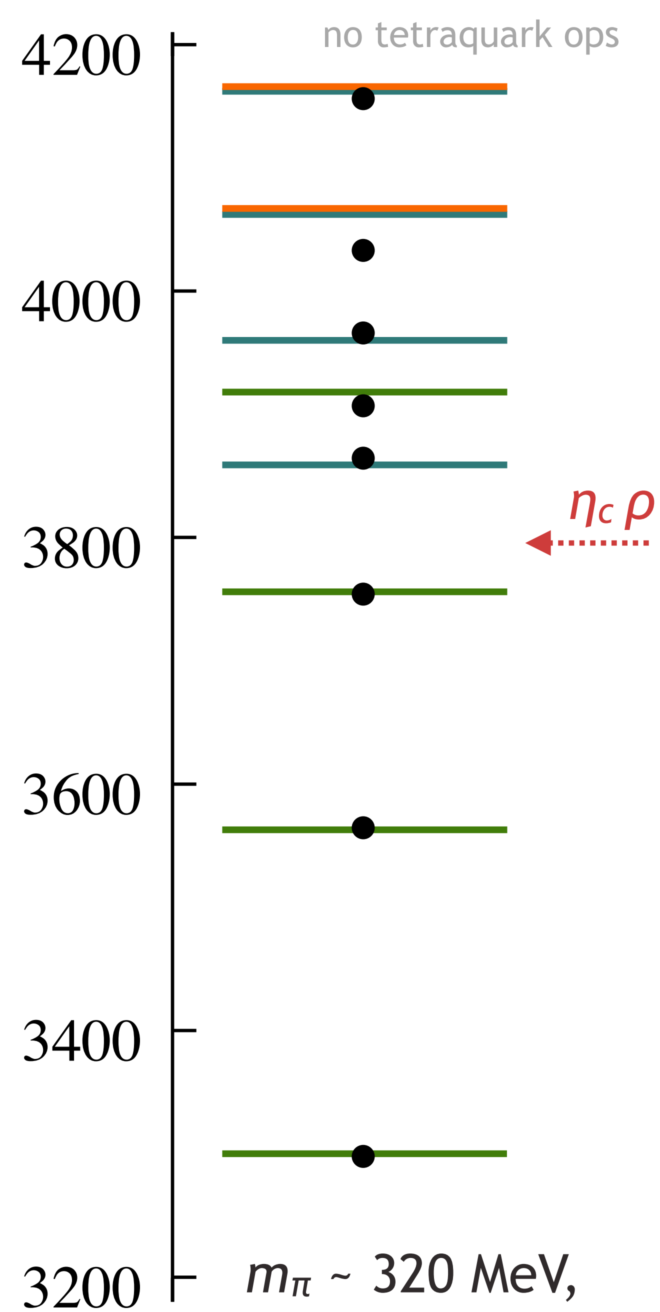
‘$Z_c(3900)$’ channel — $J^P=1^+$

Prelovsek et al (2015)



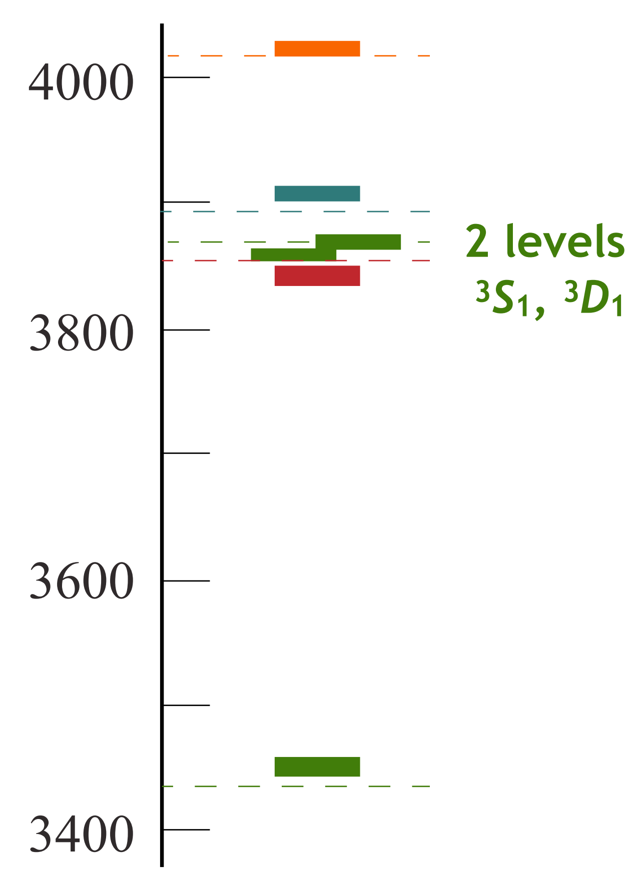
m_π ~ 270 MeV, L ~ 2 fm

Chen et al (2019)



m_π ~ 320 MeV,
L ~ 2.8 fm

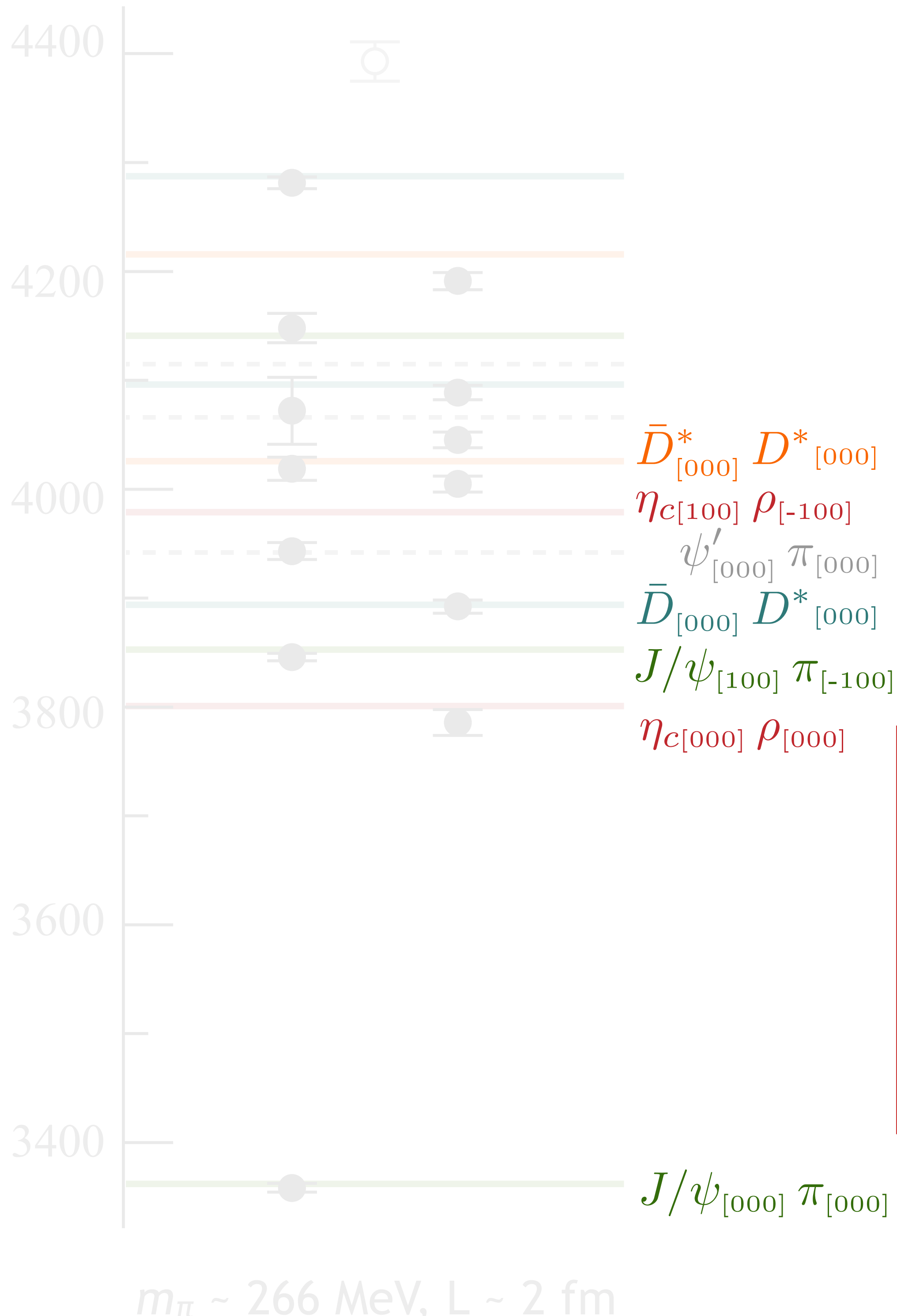
hadspec (2017)

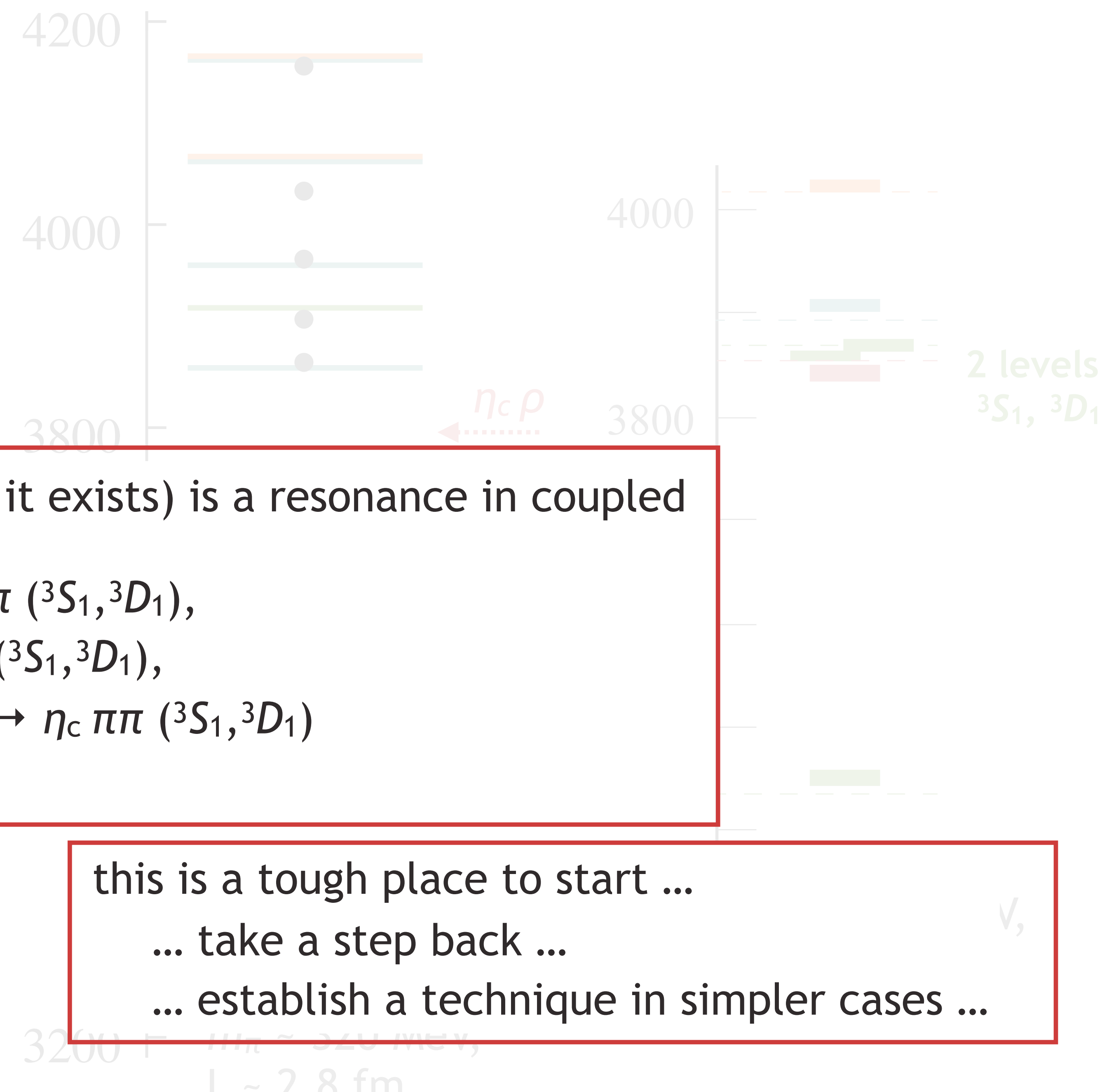


m_π ~ 390 MeV,
L ~ 2 fm

see also HALQCD approach …

‘Z_c(3900)’ as a coupled channel problem





Z_c (if it exists) is a resonance in coupled

$J/\psi\ \pi\ (^3S_1, ^3D_1)$,
$\bar{D}\ D^*\ (^3S_1, ^3D_1)$,
$\eta_c\ \rho \to \eta_c\ \pi\pi\ (^3S_1, ^3D_1)$
...

this is a tough place to start ...
... take a step back ...
... establish a technique in simpler cases ...

‘scattering’ in a finite volume

the important feature of the lattice calculation is the **periodic cubic boundary**

discrete spectrum in a finite-volume ↔ scattering amplitudes in infinite volume *“Lüscher method”*

e.g. in elastic case $E_n(L) \to \delta(E_n)$

recent pedagogic review

REVIEWS OF MODERN PHYSICS, VOLUME 90, APRIL–JUNE 2018

Scattering processes and resonances from lattice QCD

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(published 18 April 2018)

job of the lattice is to compute the discrete spectrum

e.g. $\pi\pi$ scattering in P-wave — expect the ρ resonance ...

operator basis :

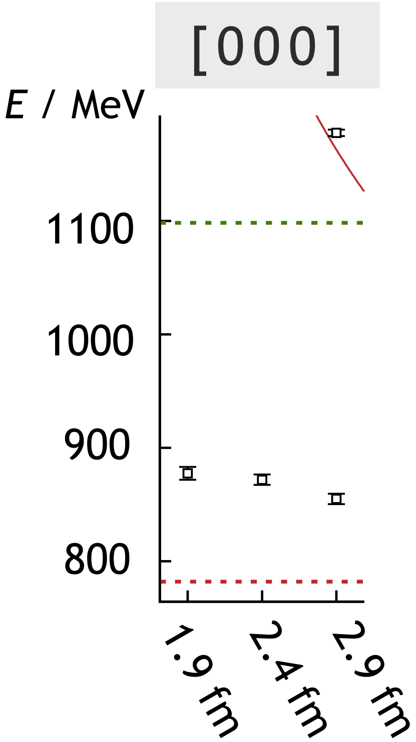
$\bar{\psi}\Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D} \psi$ “$q\bar{q}$-like”

$\pi(\mathbf{p}_1)\pi(\mathbf{p}_2)$

an elastic resonance — the ρ in $\pi\pi$

PRD87 034505 (2013)

m_π ~ 391 MeV



an elastic resonance — the ρ in $\pi\pi$

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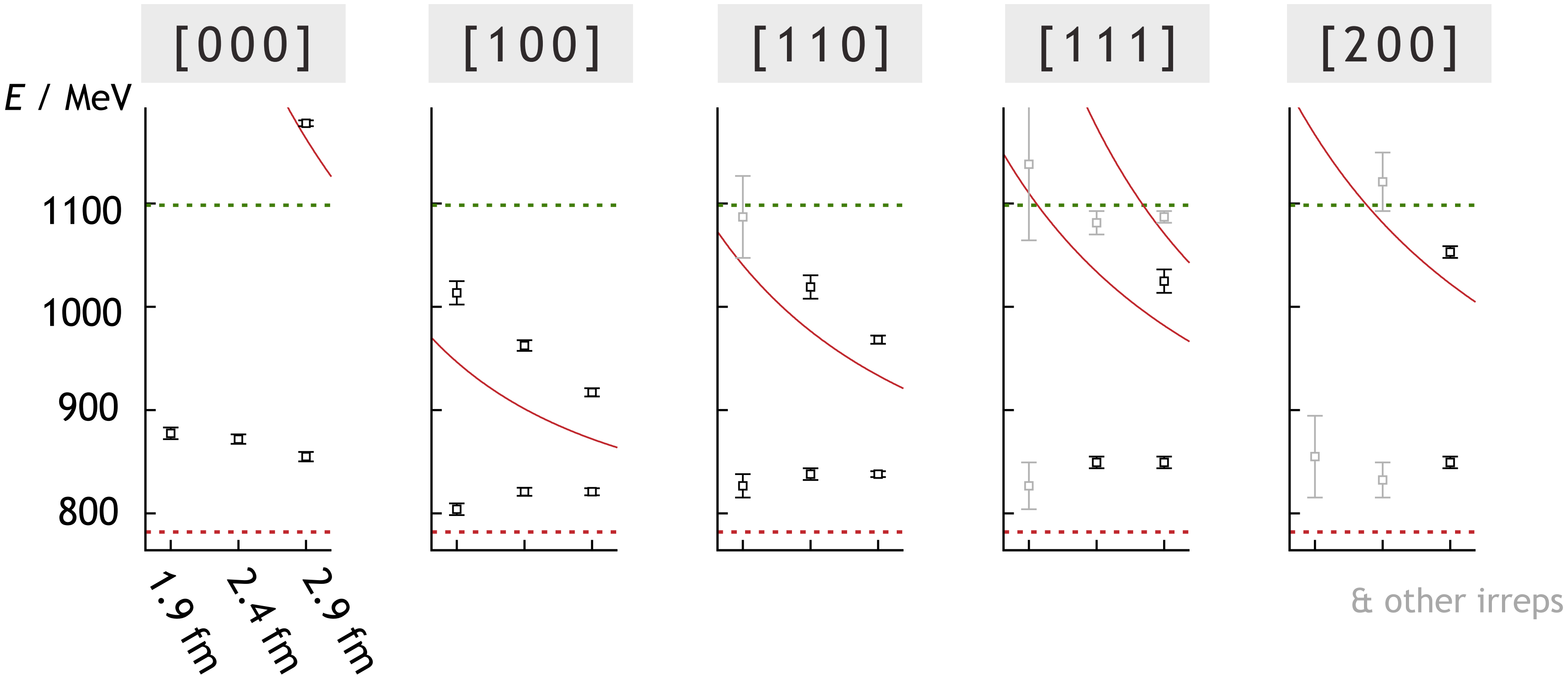

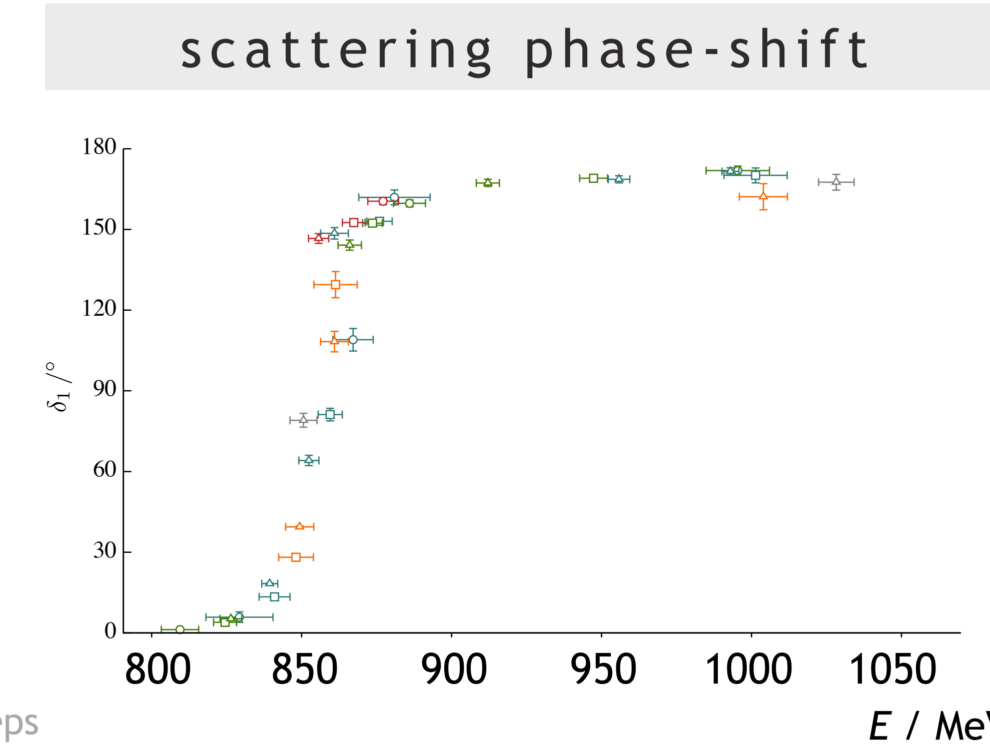

an elastic resonance — the ρ in $\pi\pi$

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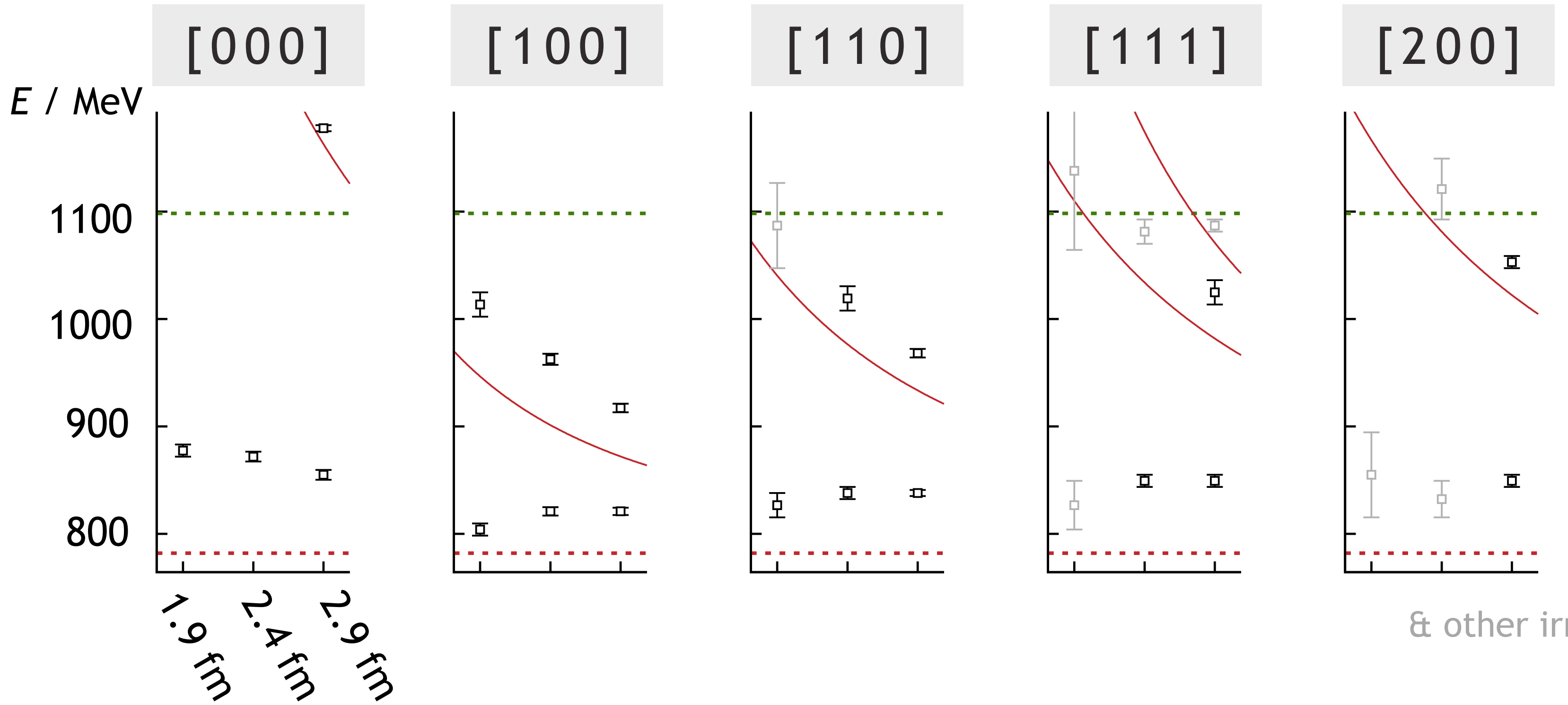
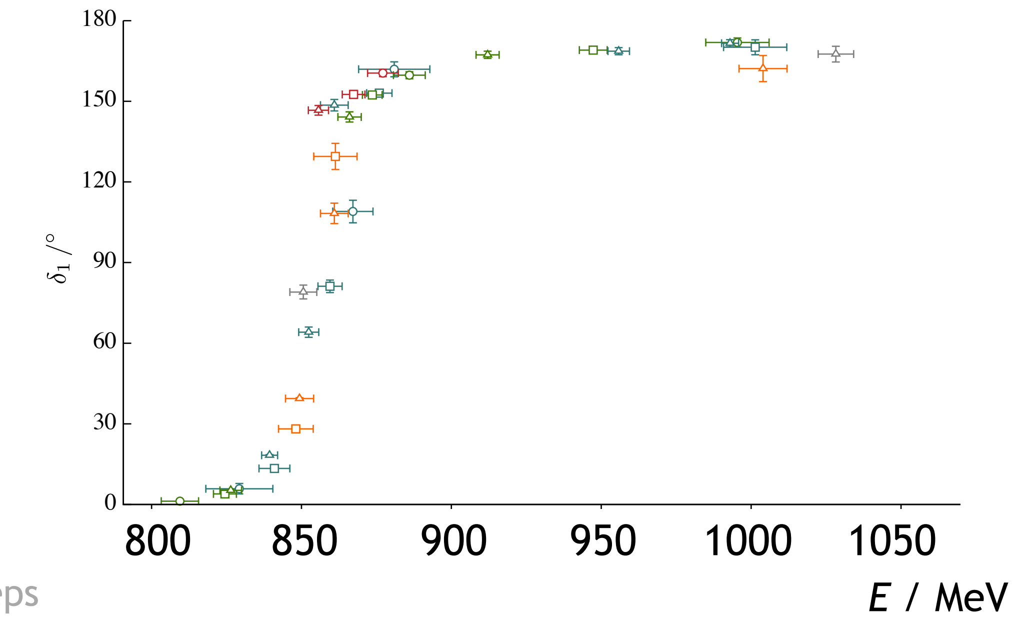


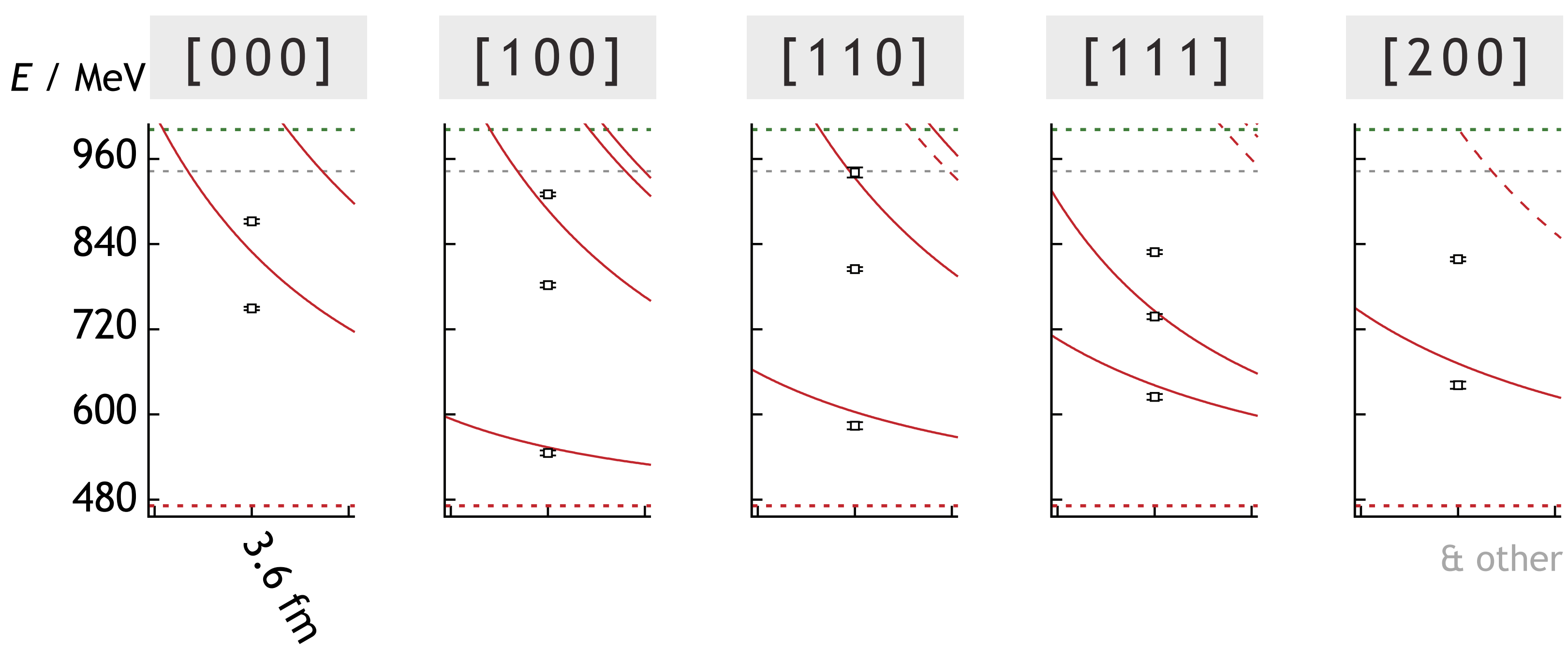
& other irreps

scattering phase-shift



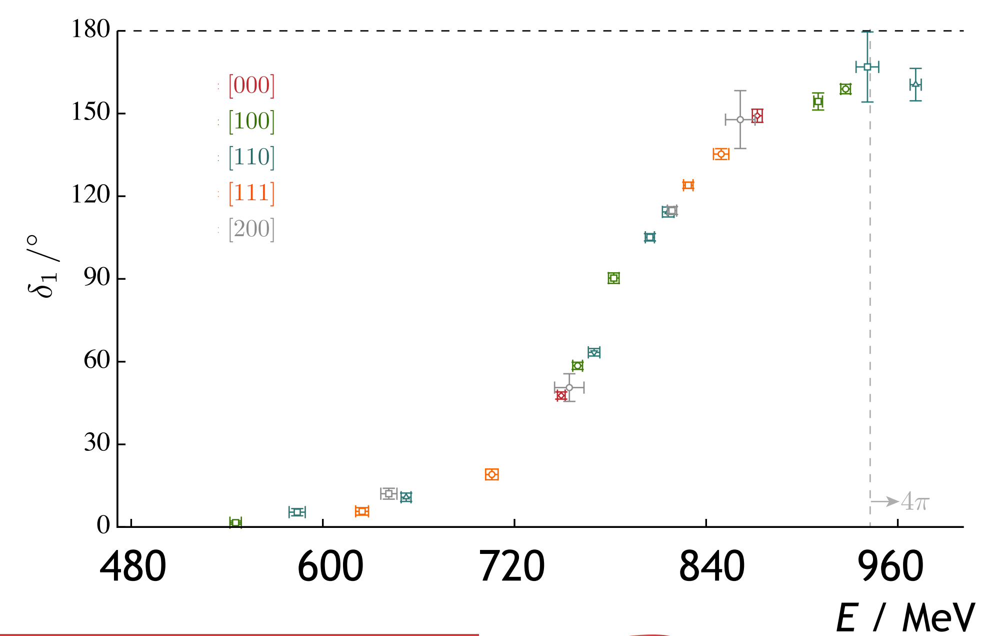
PRD92 094502 (2015)

m_π ~ 236 MeV

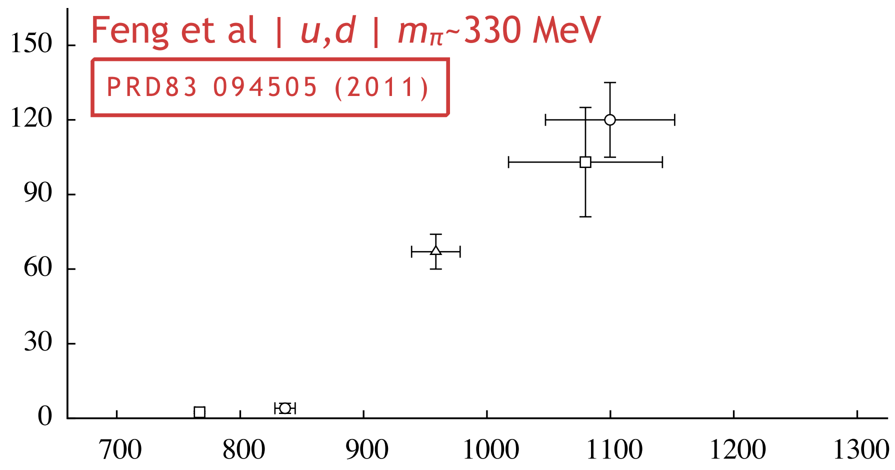


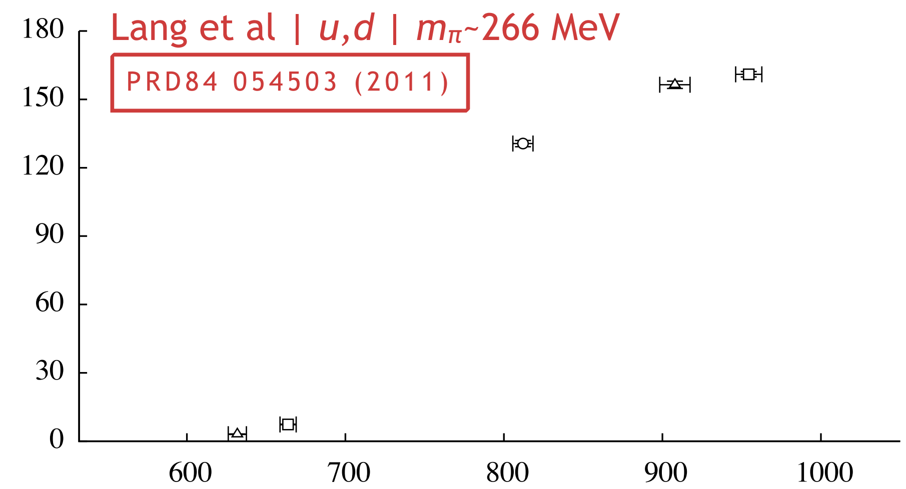
& other irreps

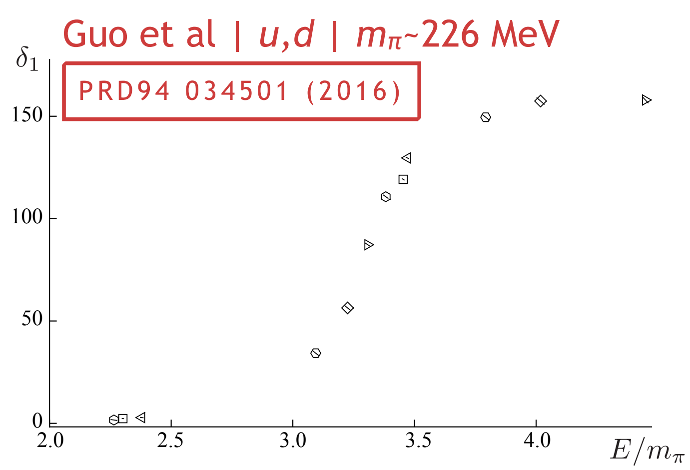
scattering phase-shift

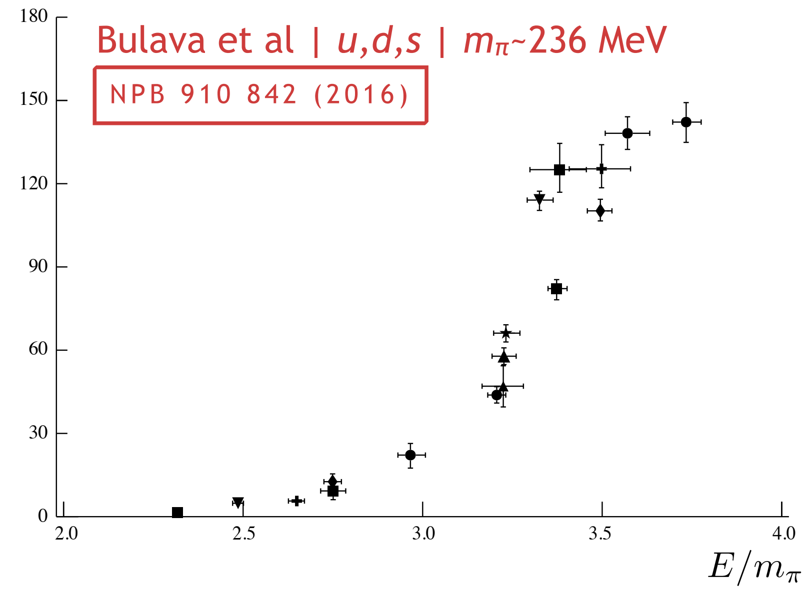


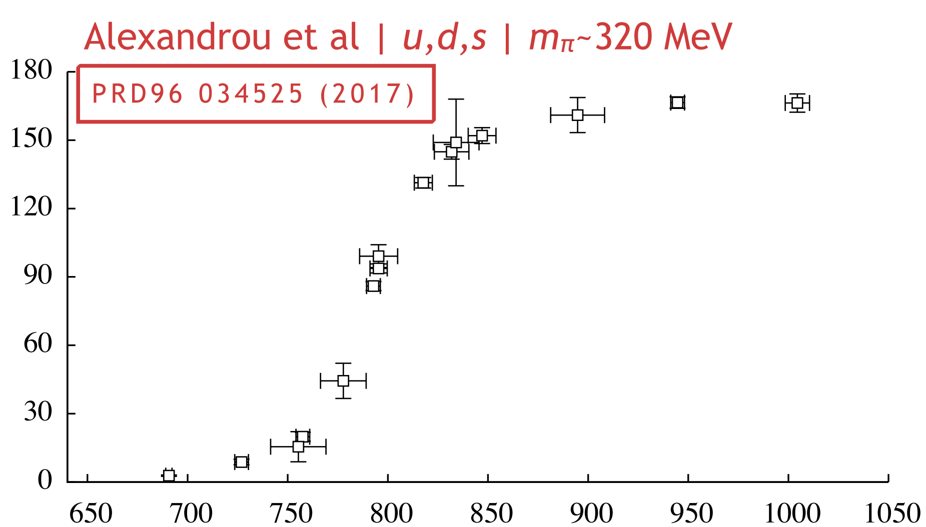
an elastic resonance — the ρ in $\pi\pi$

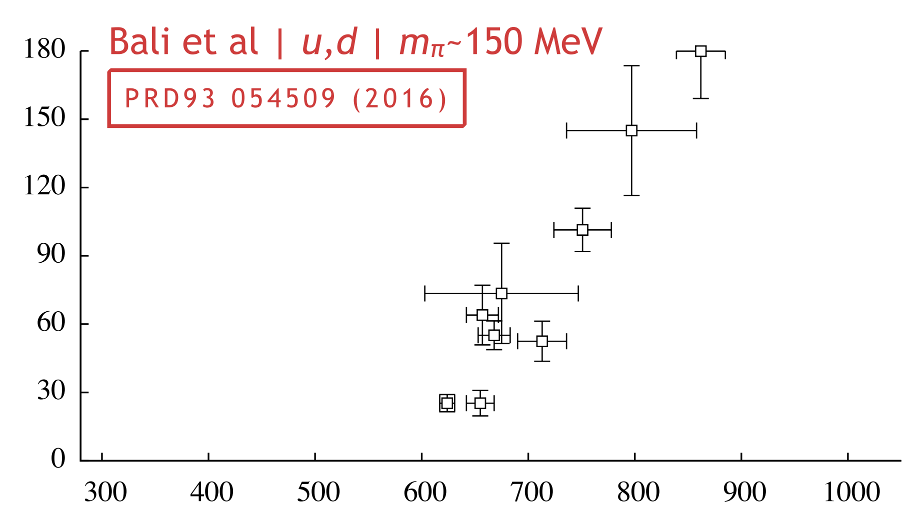












coupled-channel scattering

$$0 = \det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\mathbf{t}(E)\cdot\big(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\big)\right]$$

solved by some discrete $E_n(L)$

phase-space

scattering matrix

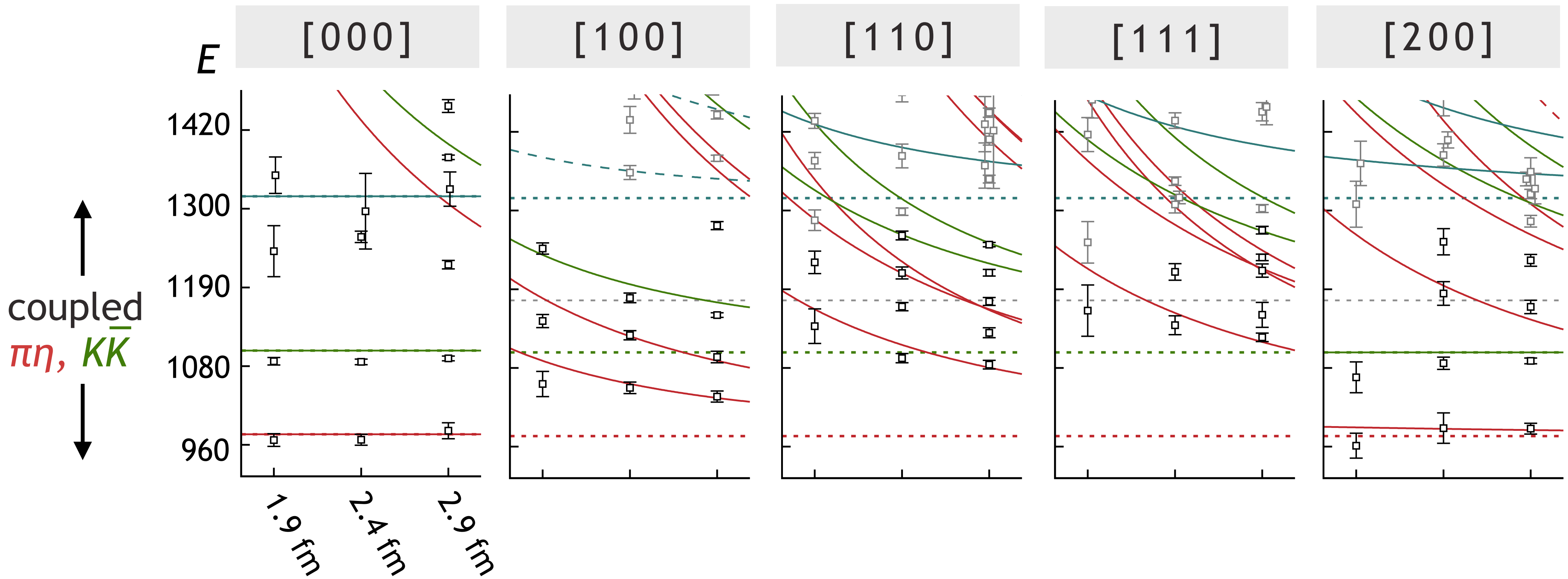
matrix of known finite-volume functions

one approach: parameterize the energy dependence of $\mathbf{t}(E)$

fit parameters by describing lattice spectrum

$\pi\eta$, $K\bar{K}$ S-wave

m_π ~ 391 MeV



$\pi\eta$, $K\bar{K}$ S-wave

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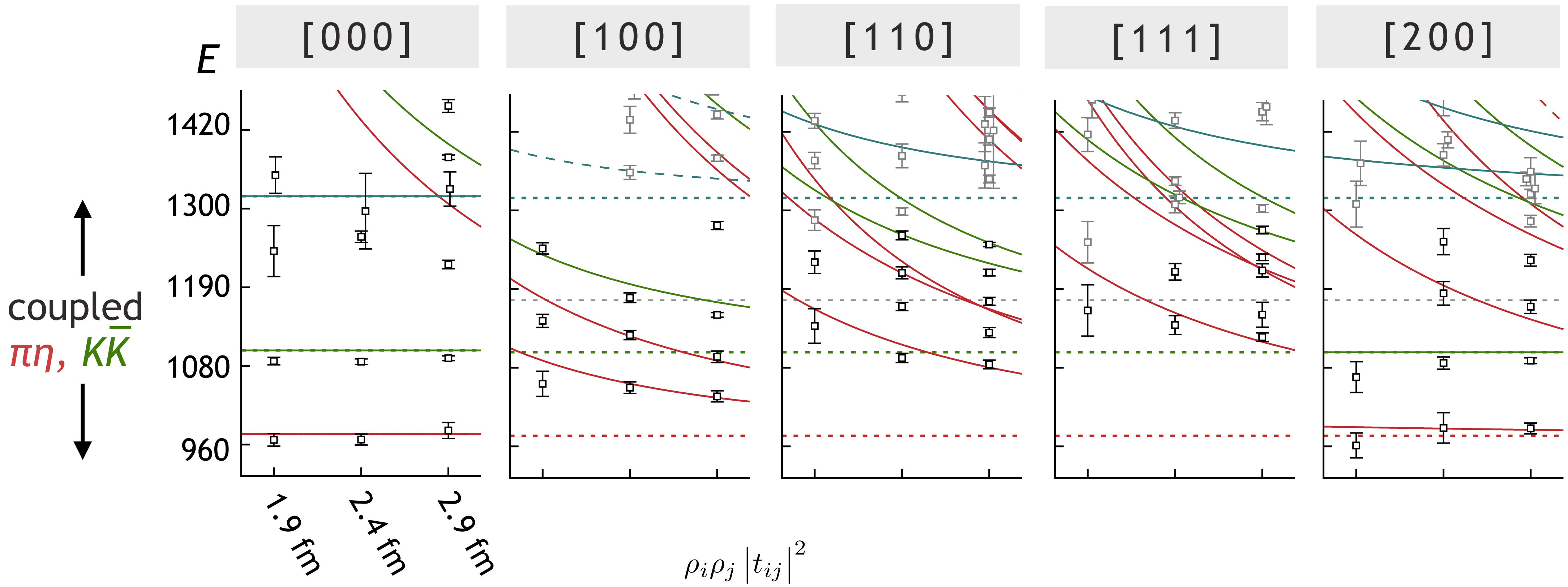
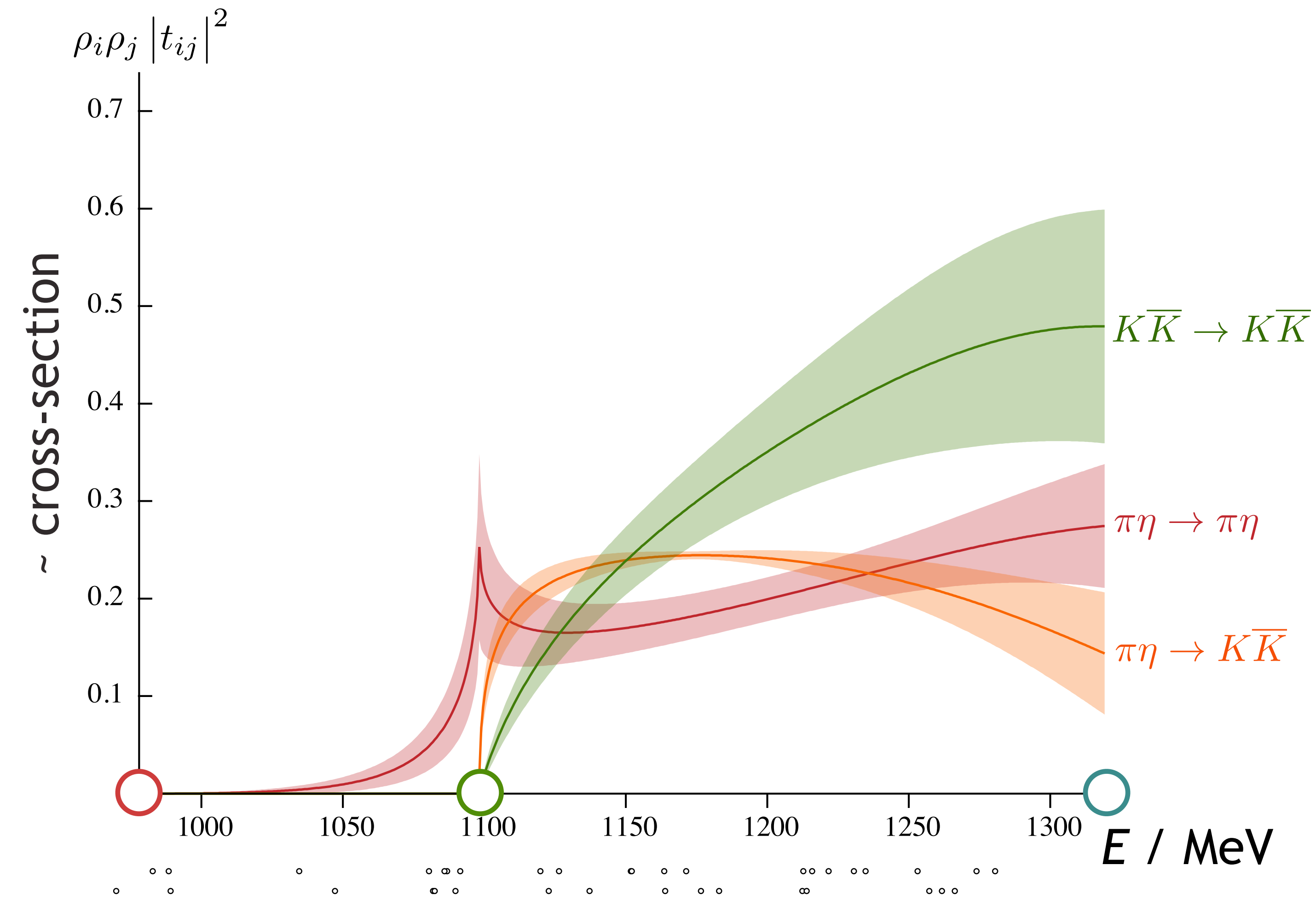


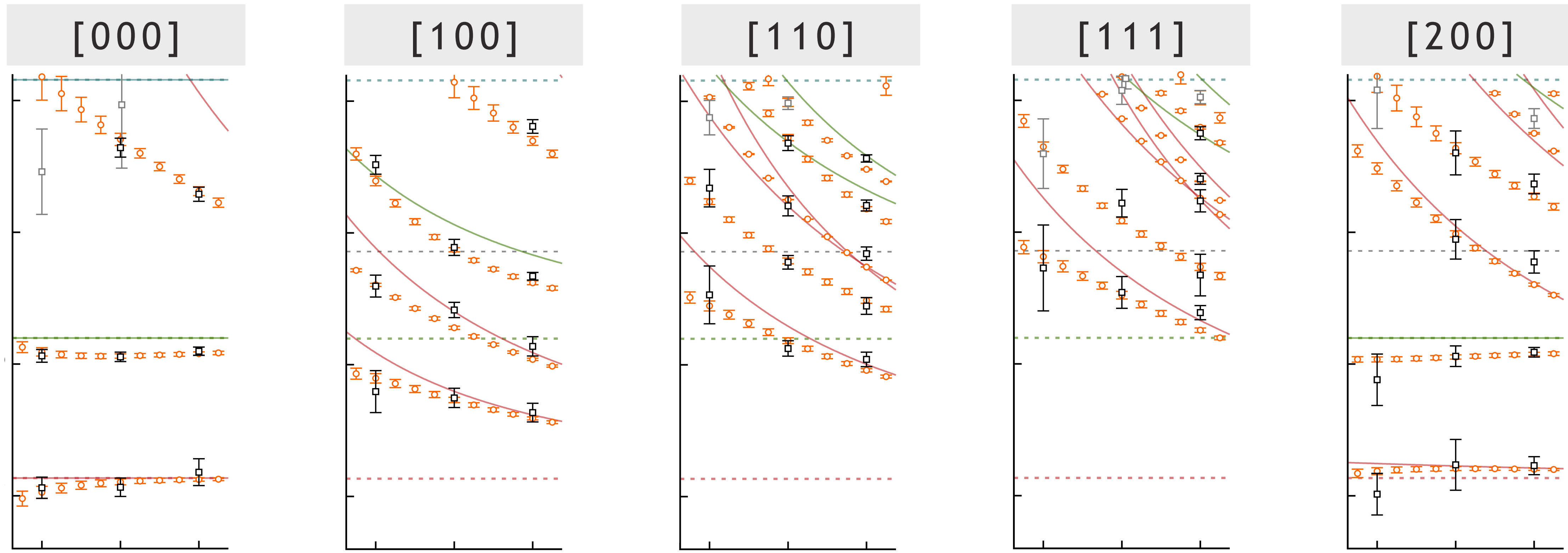


$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I} \qquad \mathrm{Im}\,\mathbf{I} = -\rho$$

$$\mathbf{K} = \frac{1}{m^2 - s}\begin{bmatrix} g_{\pi\eta}^2 & g_{\pi\eta} g_{K\bar{K}} \\ g_{\pi\eta} g_{K\bar{K}} & g_{K\bar{K}}^2 \end{bmatrix} + \begin{bmatrix} \gamma_{\pi\eta,\pi\eta} & \gamma_{\pi\eta,K\bar{K}} \\ \gamma_{\pi\eta,K\bar{K}} & \gamma_{K\bar{K},K\bar{K}} \end{bmatrix}$$

& Chew-Mandelstam phase-space

$\pi\eta$, $K\bar{K}$ S-wave



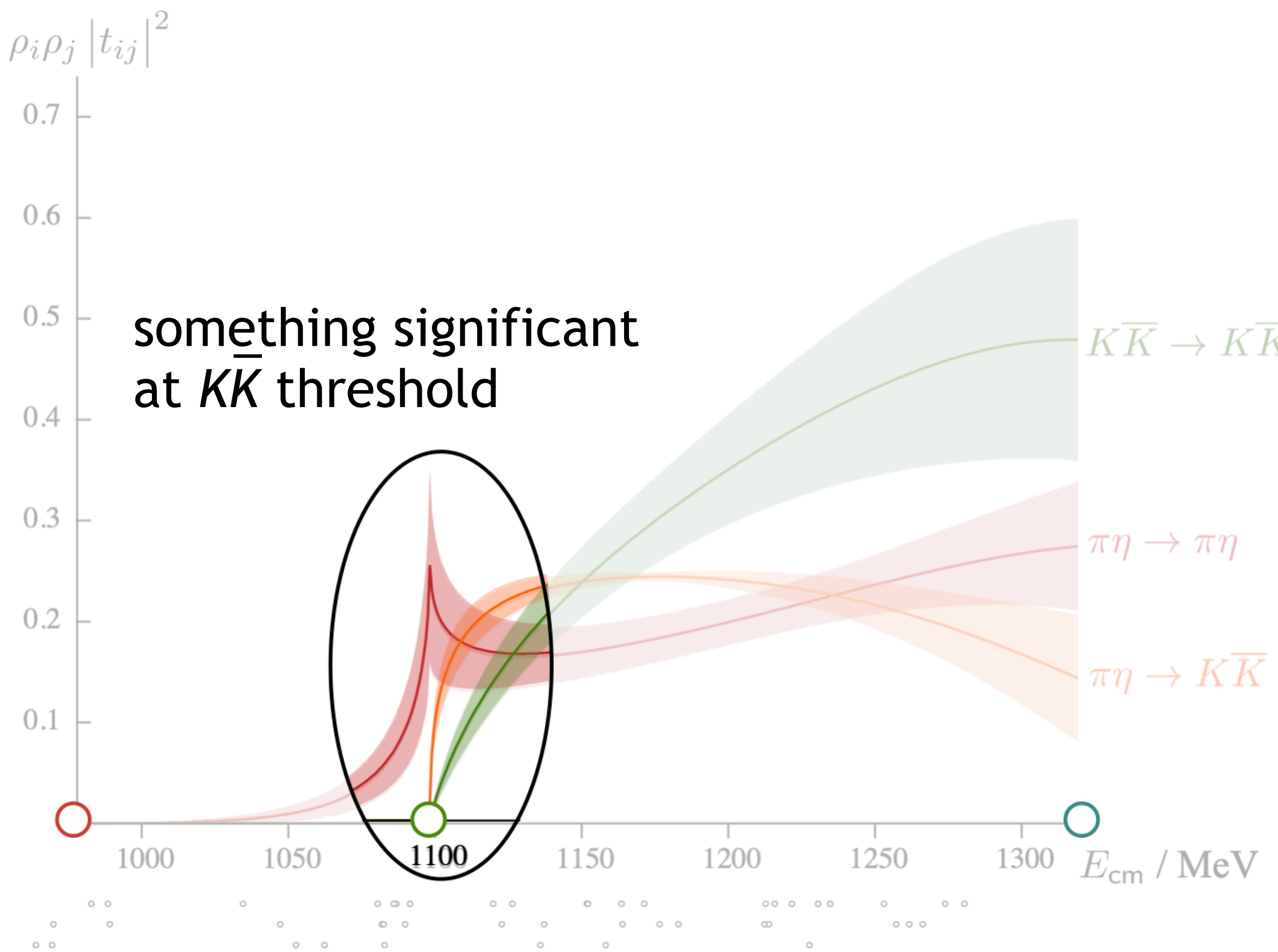
parameterized t-matrix describes the finite volume spectrum well $\chi^2/N_{\mathrm{dof}} = \frac{58.0}{47-6} = 1.41$

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I} \qquad \mathrm{Im}\,\mathbf{I} = -\rho$$

$$\mathbf{K} = \frac{1}{m^2 - s}\begin{bmatrix} g_{\pi\eta}^2 & g_{\pi\eta}g_{K\bar{K}} \\ g_{\pi\eta}g_{K\bar{K}} & g_{K\bar{K}}^2 \end{bmatrix} + \begin{bmatrix} \gamma_{\pi\eta,\pi\eta} & \gamma_{\pi\eta,K\bar{K}} \\ \gamma_{\pi\eta,K\bar{K}} & \gamma_{K\bar{K},K\bar{K}} \end{bmatrix}$$

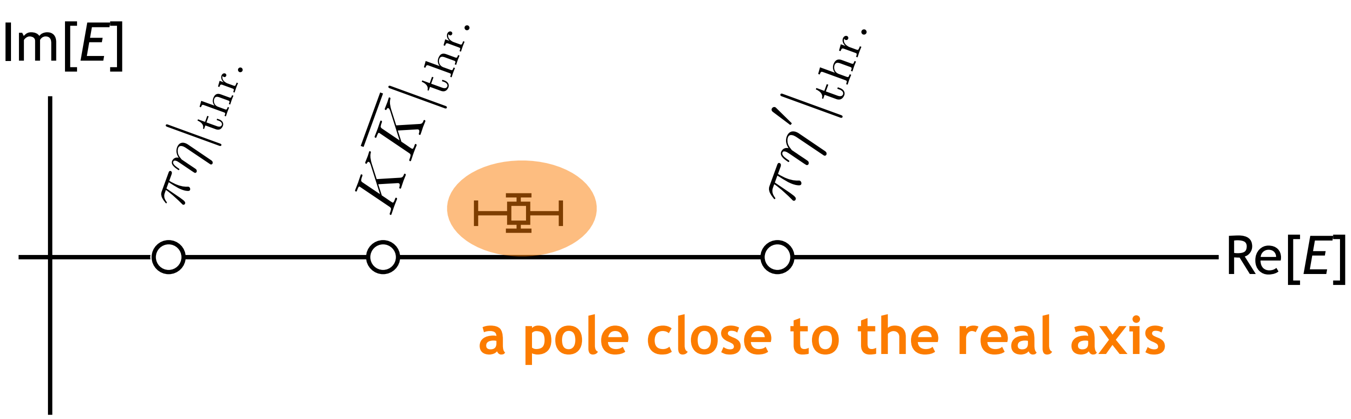
& Chew-Mandelstam phase-space

$\pi\eta$, $K\bar{K}$ S-wave and the a_0 pole



$$t_{ij}(s) \sim \frac{c_i\, c_j}{s_0 - s}$$

$$\sqrt{s_0} = m_R \pm \tfrac{i}{2}\Gamma_R$$



$$m_{a_0} = 1177(27)\ \text{MeV}$$
$$\Gamma_{a_0} = 49(33)\ \text{MeV}$$

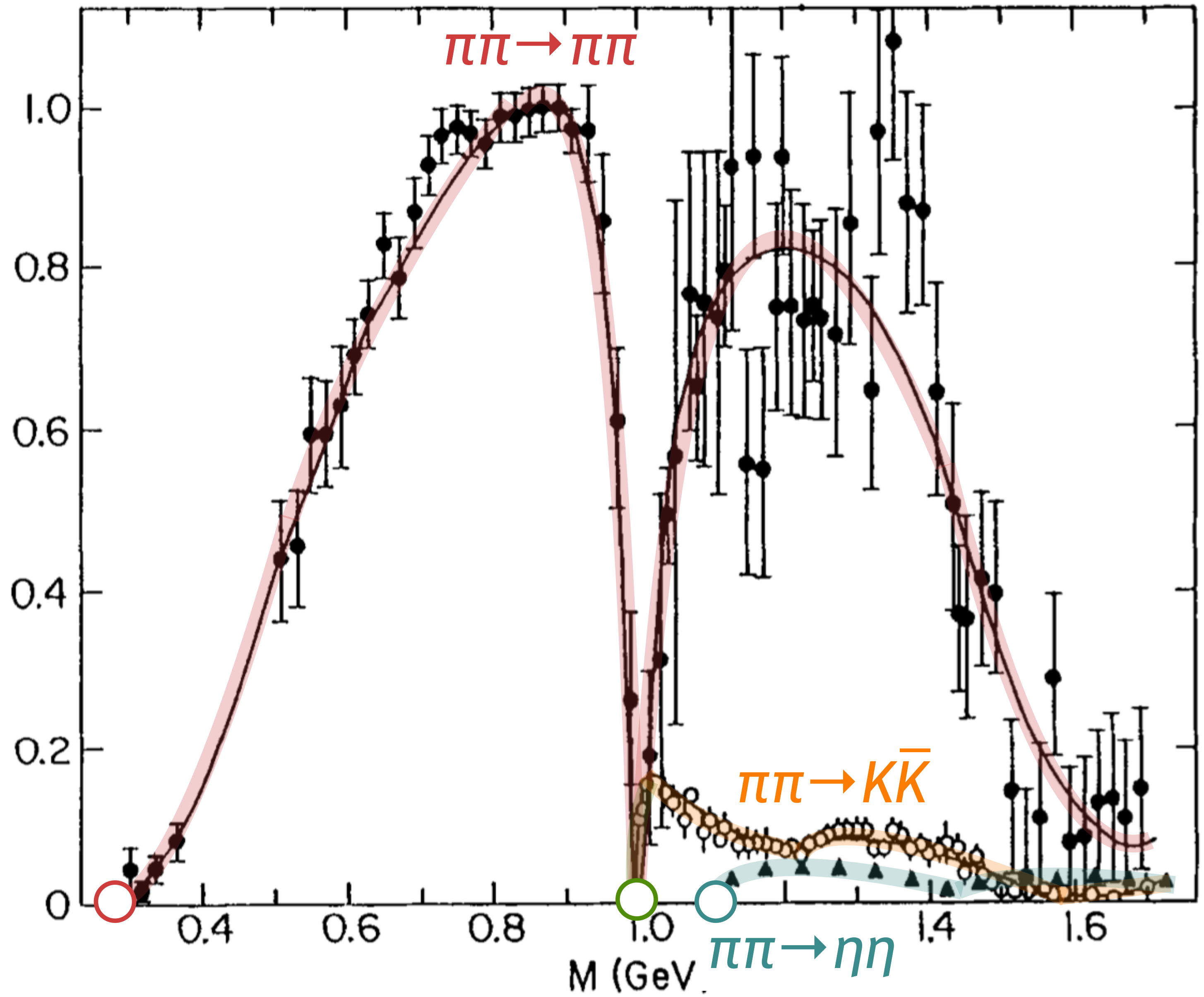
$$\left|\frac{c_{K\bar{K}}}{c_{\pi\eta}}\right| = 1.3(4)$$

m_π ~ 391 MeV

uncertainties include spread over different parameterizations

$\pi\pi$, $K\bar{K}$, $\eta\eta$ S-wave

experimental amplitudes

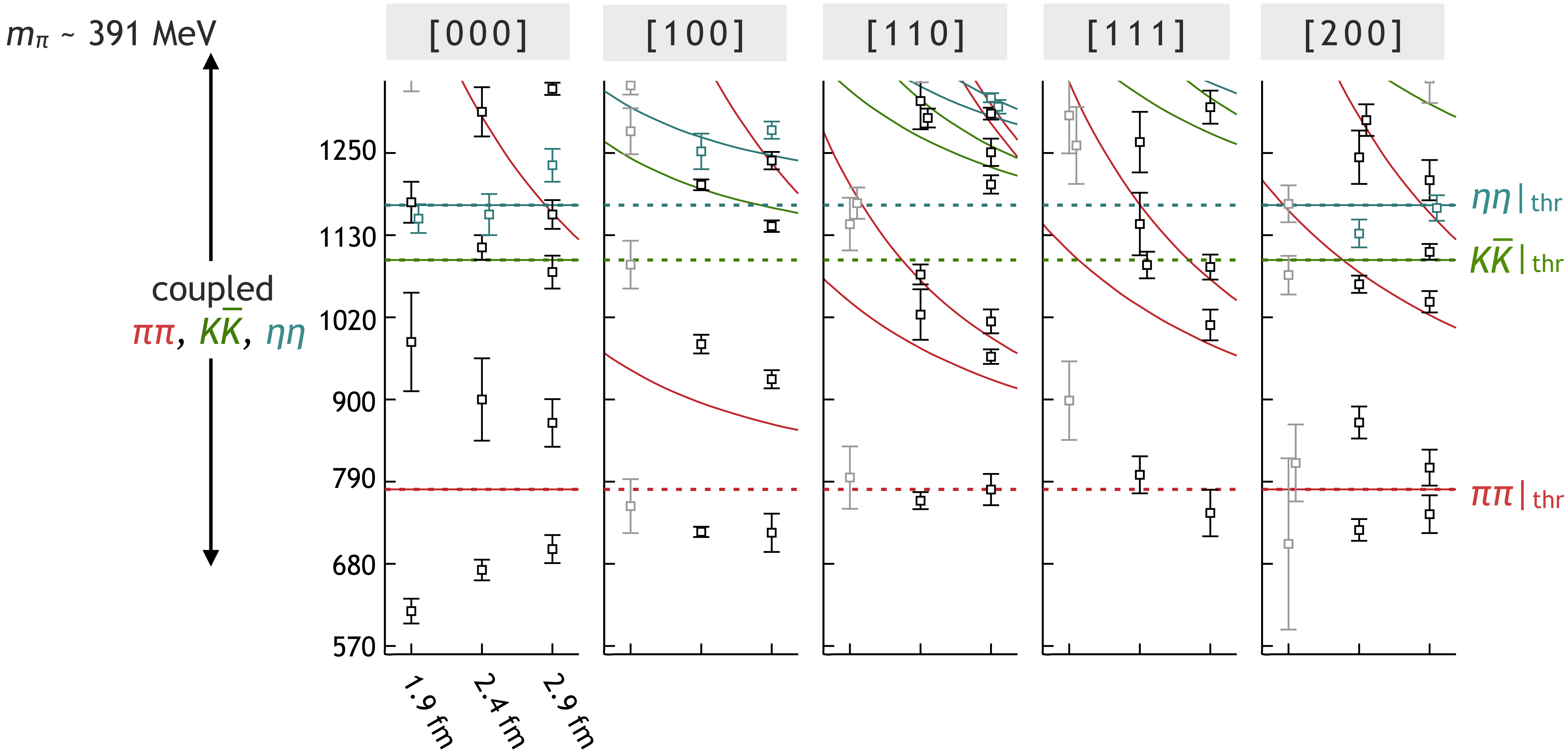


combination of broad σ resonance and narrow $f_0(980)$ at $K\bar{K}$ threshold

two low lying resonances …

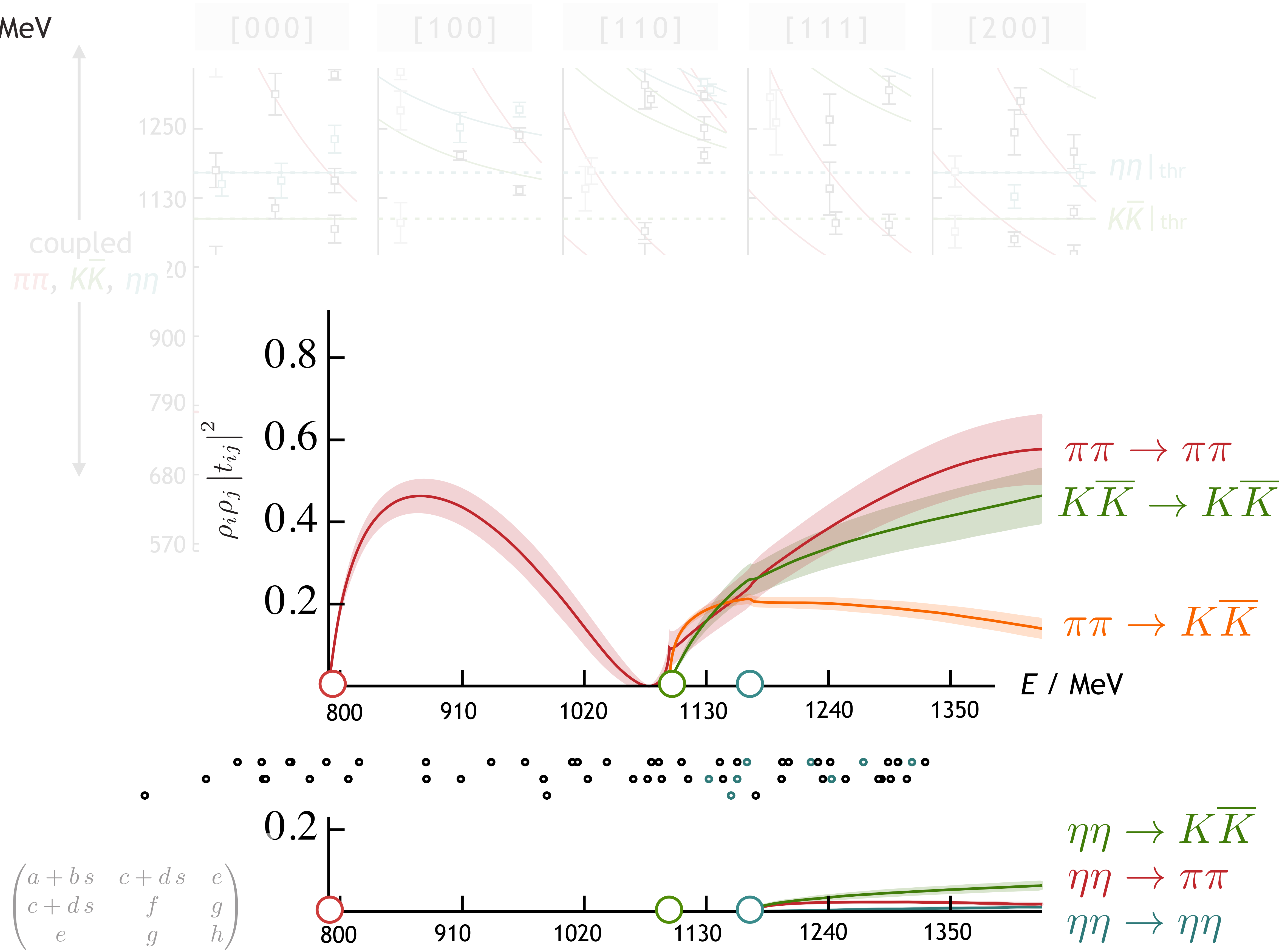
… start at a heavier quark mass …

$\pi\pi$, $K\bar{K}$, $\eta\eta$ S-wave in lattice QCD



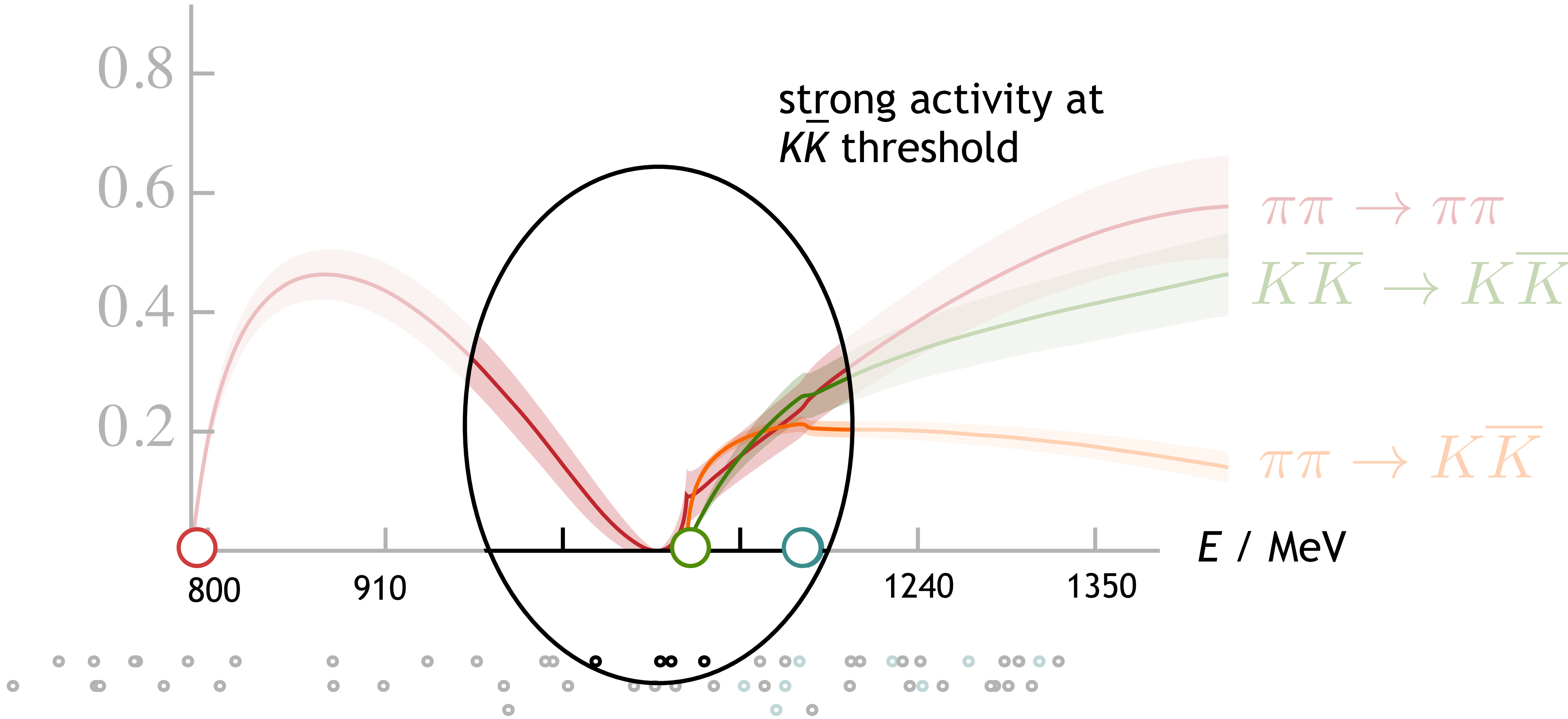
$\pi\pi$, $K\bar{K}$, $\eta\eta$ S-wave in lattice QCD

m_π ~ 391 MeV



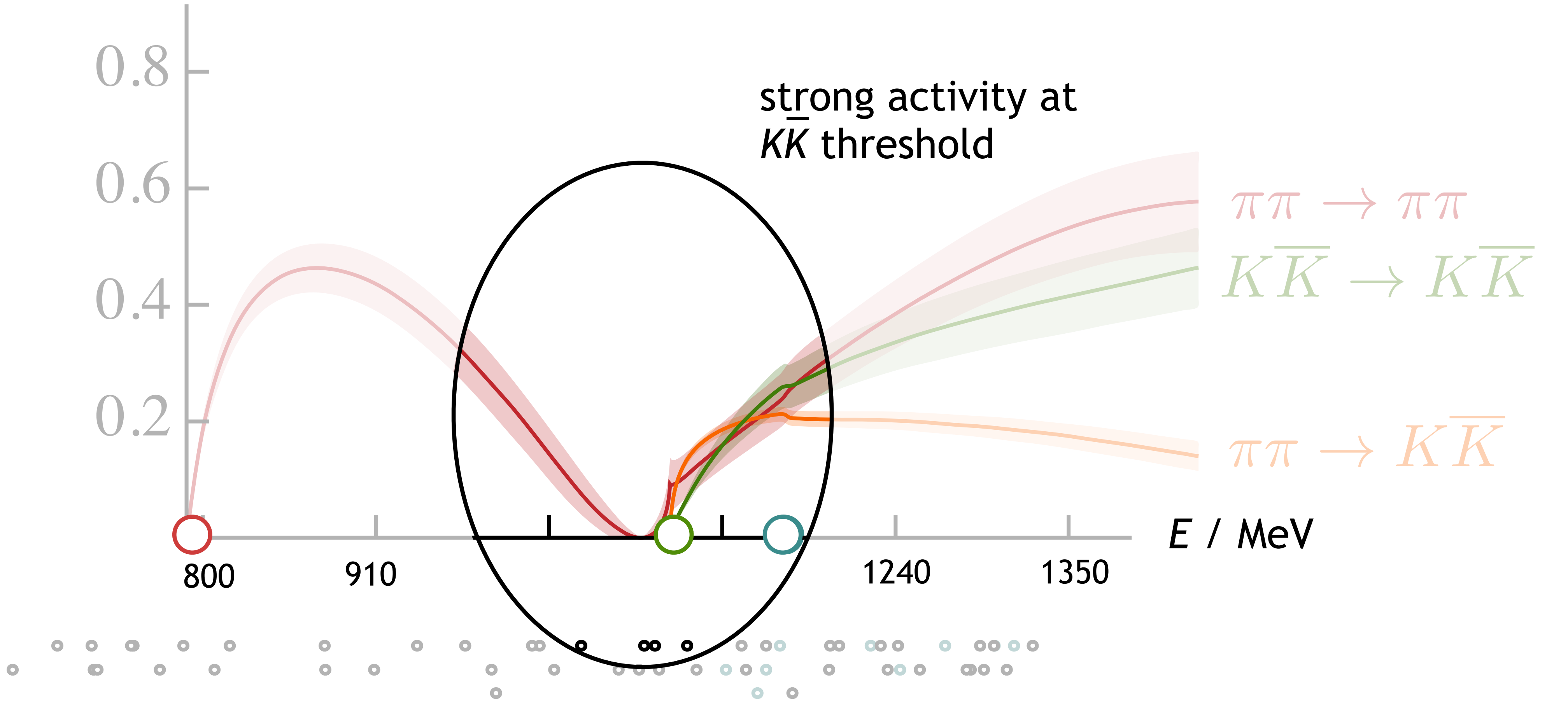
$$\mathbf{K}^{-1}(s) = \begin{pmatrix} a+b\,s & c+d\,s & e \\ c+d\,s & f & g \\ e & g & h \end{pmatrix}$$

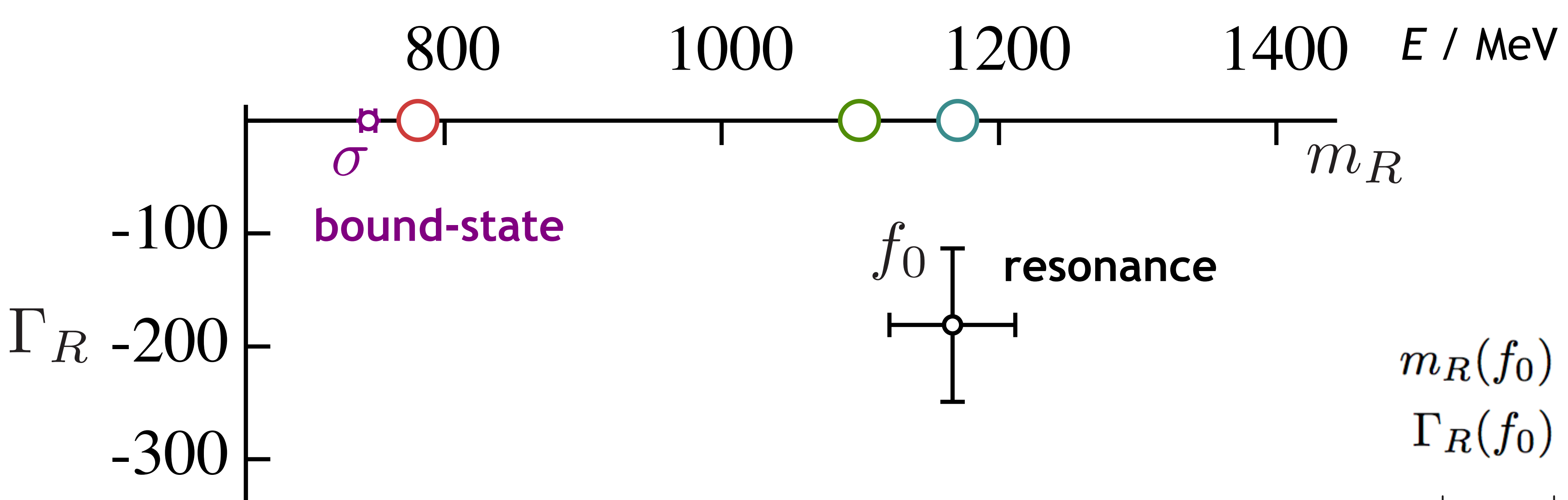
$\pi\pi$, $K\bar{K}$, $\eta\eta$ S-wave in lattice QCD



m_π ~ 391 MeV

$\pi\pi$, $K\bar{K}$, $\eta\eta$ S-wave in lattice QCD





$$m_R(f_0) = 1166(45)\ \text{MeV}$$

$$\Gamma_R(f_0) = 181(68)\ \text{MeV}$$

$$\left|\frac{c_{K\bar{K}}}{c_{\pi\pi}}\right| = 1.2(2)$$

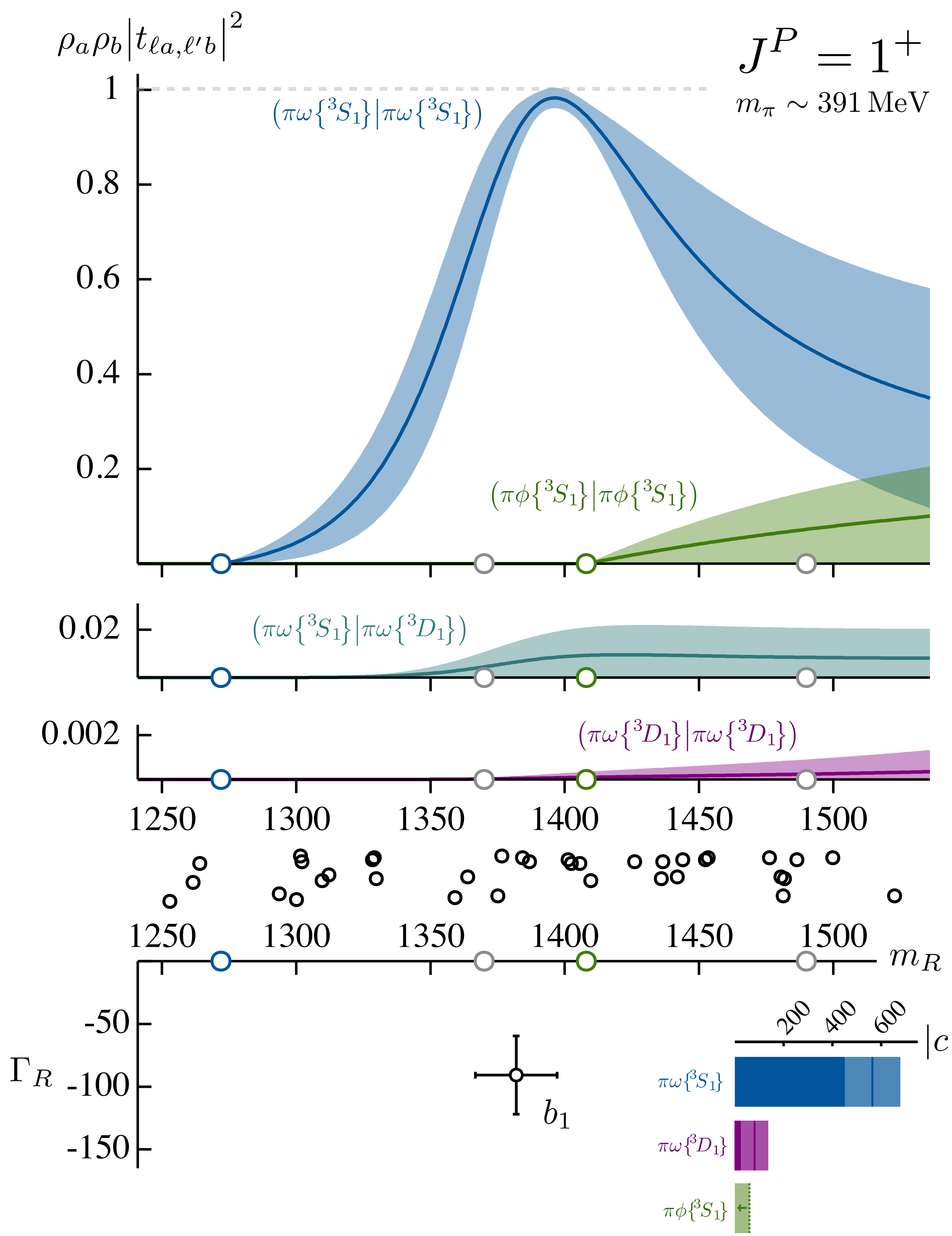
m_π ~ 391 MeV

$\pi\omega$ scattering & the b_1 resonance

m_π ~ 391 MeV

coupled

$\pi\omega$ ($^3S_1$,$^3D_1$), $\pi\varphi$



clear b_1 resonance

— strong $\pi\omega$ S-wave

— weak $\pi\omega$ D-wave

— negligible $\pi\varphi$

so the technology is coming together, will soon be ready to handle complex cases like the $Z_c(3900)$

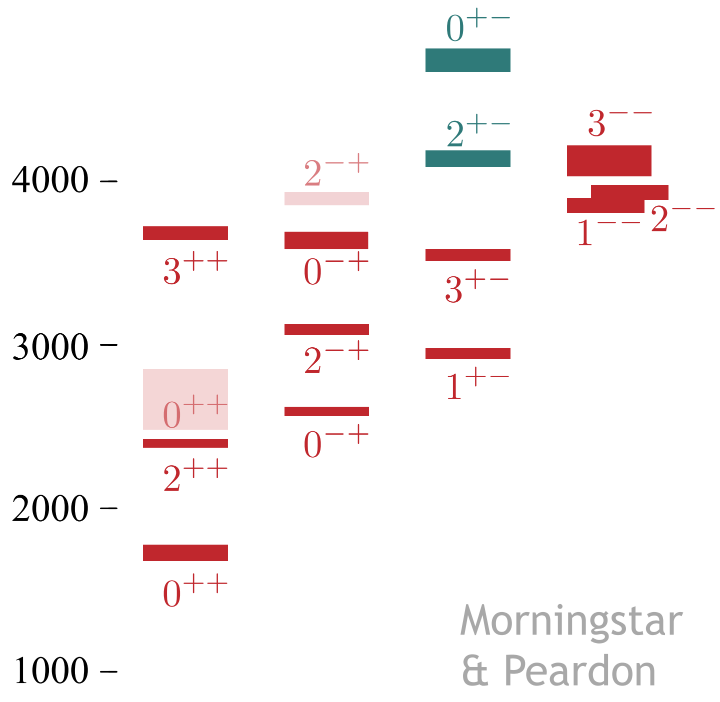
what about ‘exotic hadrons’ in the light sector ?

exotic hadrons — glueballs

long been proposed to study **glueballs** in **J/ψ radiative decay** (“glue-rich”)

+ high quality
new data from BES III
as seen in Beijang’s plenary talk

pure YM glueballs



accessed using
Wilson loop operators

exotic hadrons – glueballs

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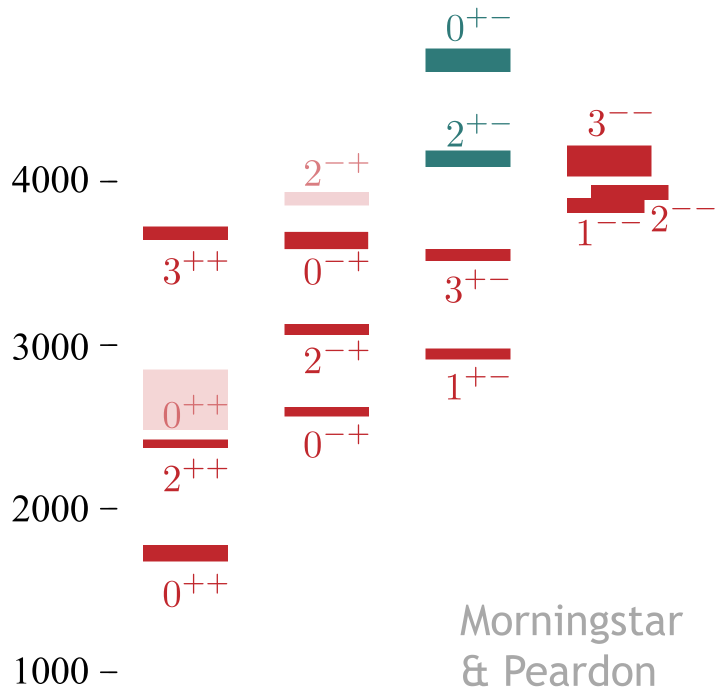
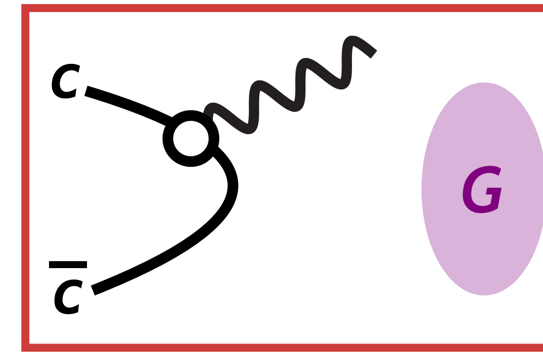
pure YM glueballs

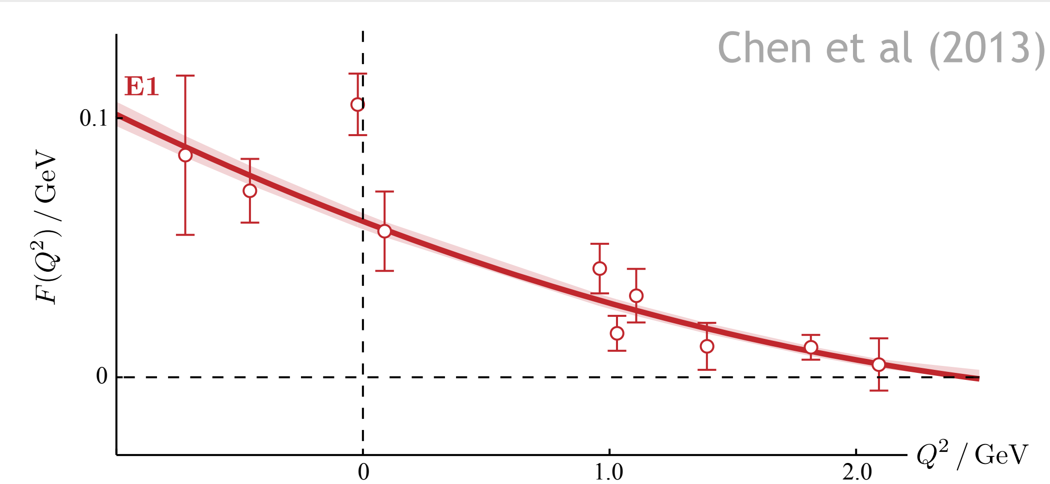


compute three-point correlation functions

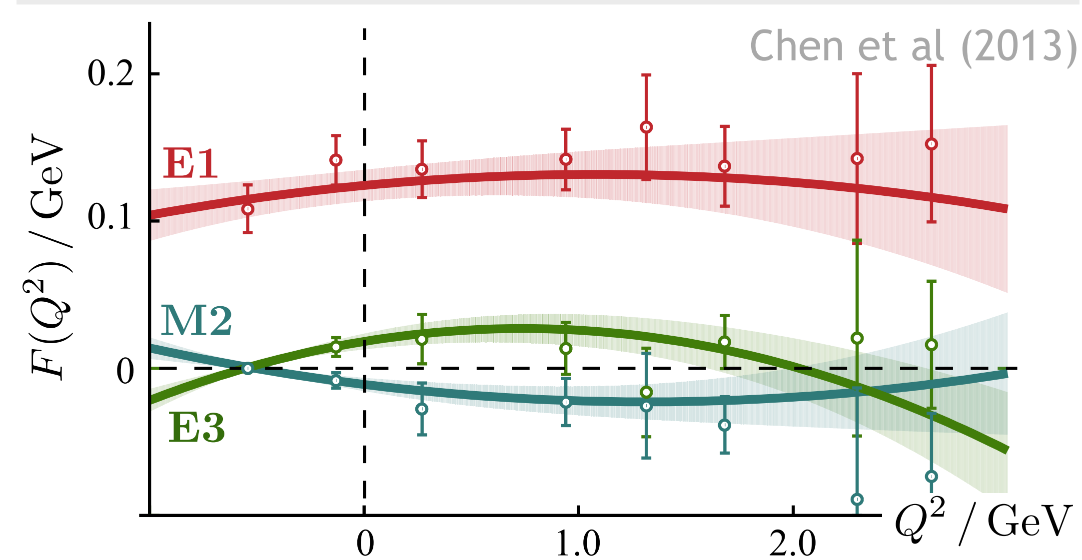
$$\langle 0 \big| \mathcal{O}_{J/\psi}(T)\, j^{\mu}_{\text{em}}(t)\, \mathcal{O}_G(0) \big| 0 \rangle$$



$J/\psi \to \gamma^* \, G[0^{++}]$



$J/\psi \to \gamma^* \, G[2^{++}]$



long been proposed to study **glueballs** in **J/ψ radiative decay** (“glue-rich”)

pure YM glueballs

compute three-point correlation functions

$$\langle 0 | \mathcal{O}_{J/\psi}(T)\, j^{\mu}_{\mathrm{em}}(t)\, \mathcal{O}_G(0) | 0 \rangle$$

$J/\psi \to \gamma^* \, G[0^{++}]$

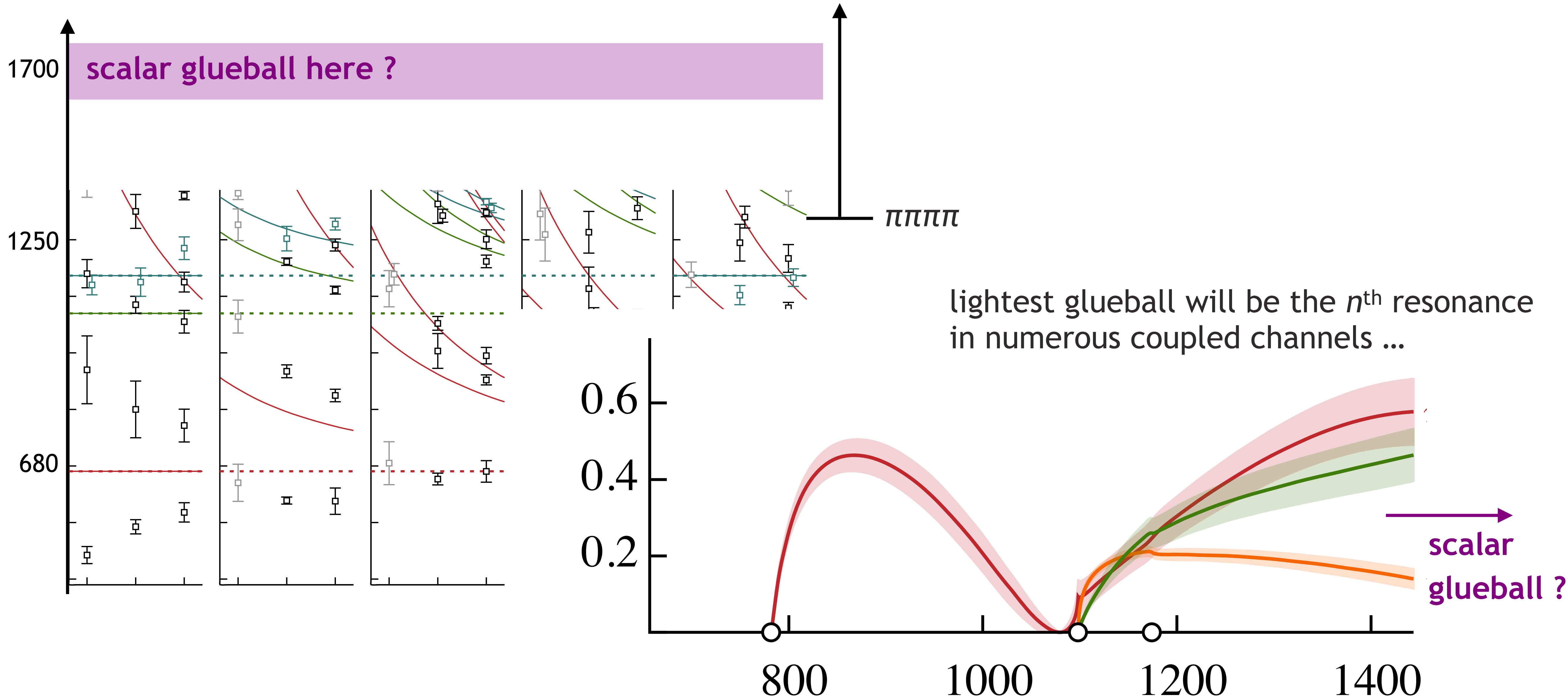
$J/\psi \to \gamma^* \, G[2^{++}]$

helpful to build intuition, but …
this is in pure Yang-Mills — no light quarks !

as seen in Beijang’s plenary talk

glueballs are excited isoscalar meson resonances

glueballs in **QCD** are much more complicated beasts ...



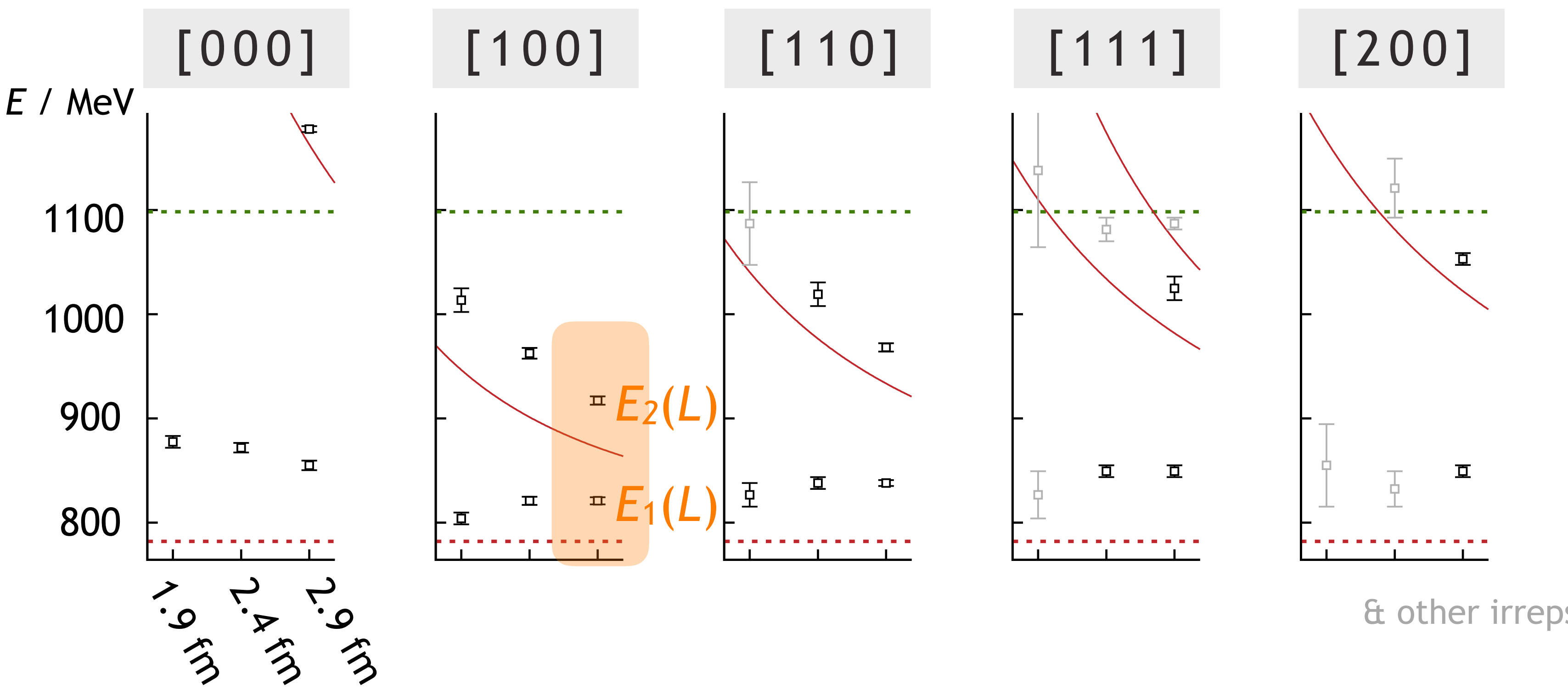
lightest glueball will be the n^{th} resonance in numerous coupled channels ...

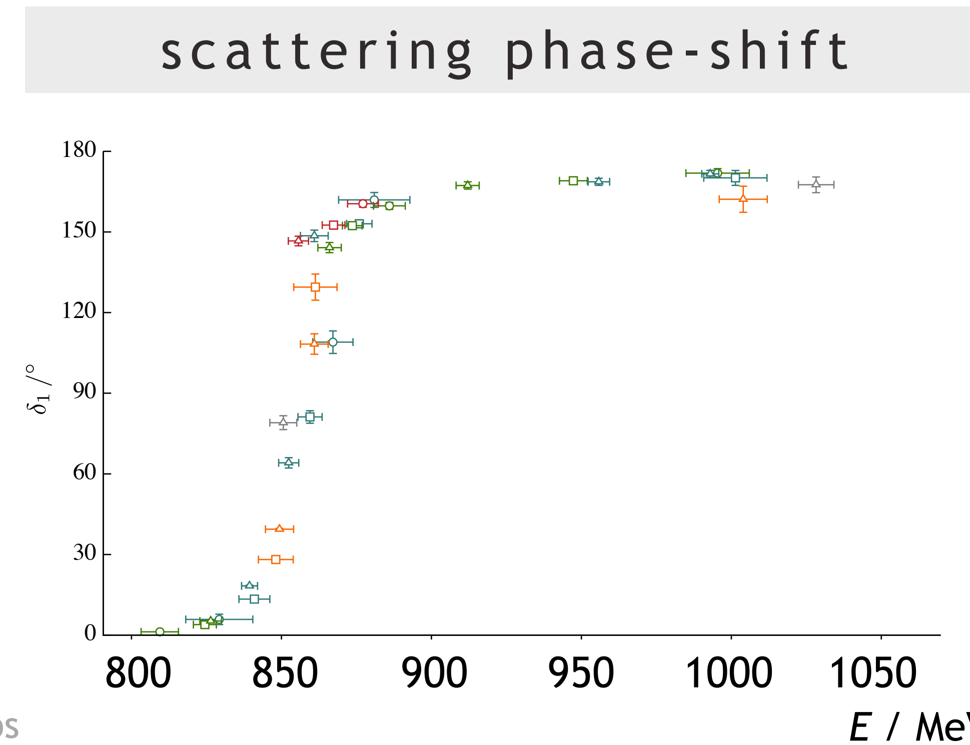
and then we need to couple it to an external current ...

this is too tough to start with ...

... back up and try something simpler ...

compute matrix elements for each finite-volume eigenstate
(no single one of which is the resonance)



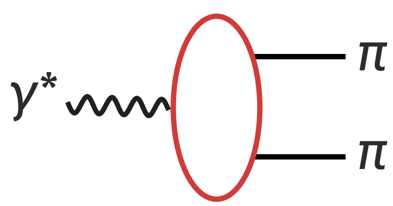


e.g. $\langle 0 | j^\mu_{\rm em} | E_n(L) \rangle$ or $\langle \pi | j^\mu_{\rm em} | E_n(L) \rangle$

and there is a formalism which converts this
to an infinite volume amplitude as a function of $E_{\pi\pi}$ …

transition form-factors of unstable $\rho \to \pi\pi$

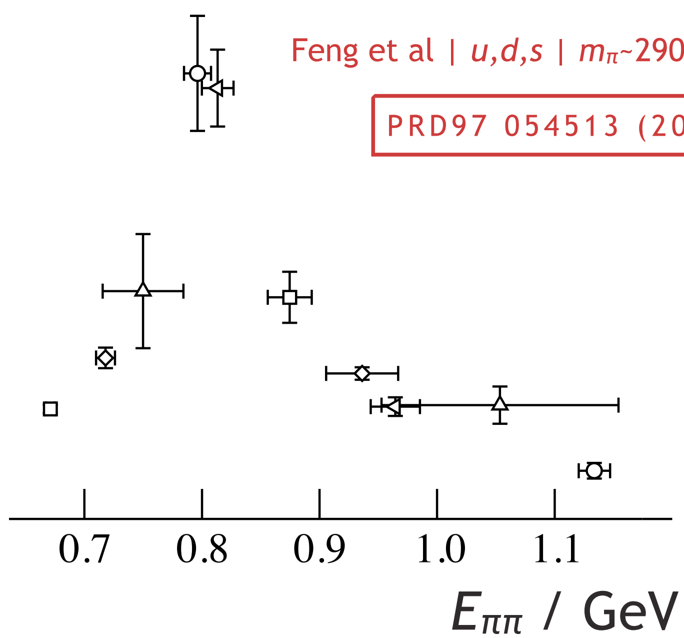
$\gamma^* \to \pi\pi$



$F_\pi(E_{\pi\pi})$

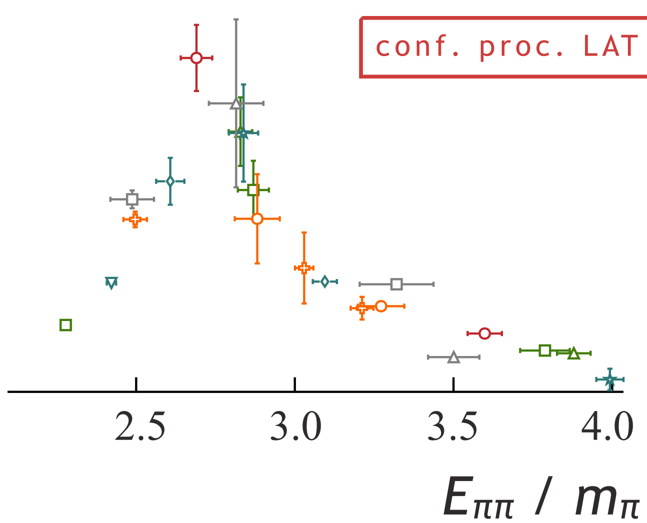
Feng et al | *u,d,s* | m_π~290 MeV

PRD97 054513 (2018)

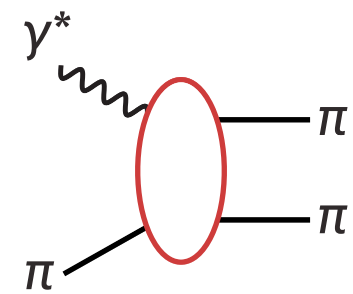


Bulava et al | *u,d,s* | m_π~280 MeV

conf. proc. LAT '15



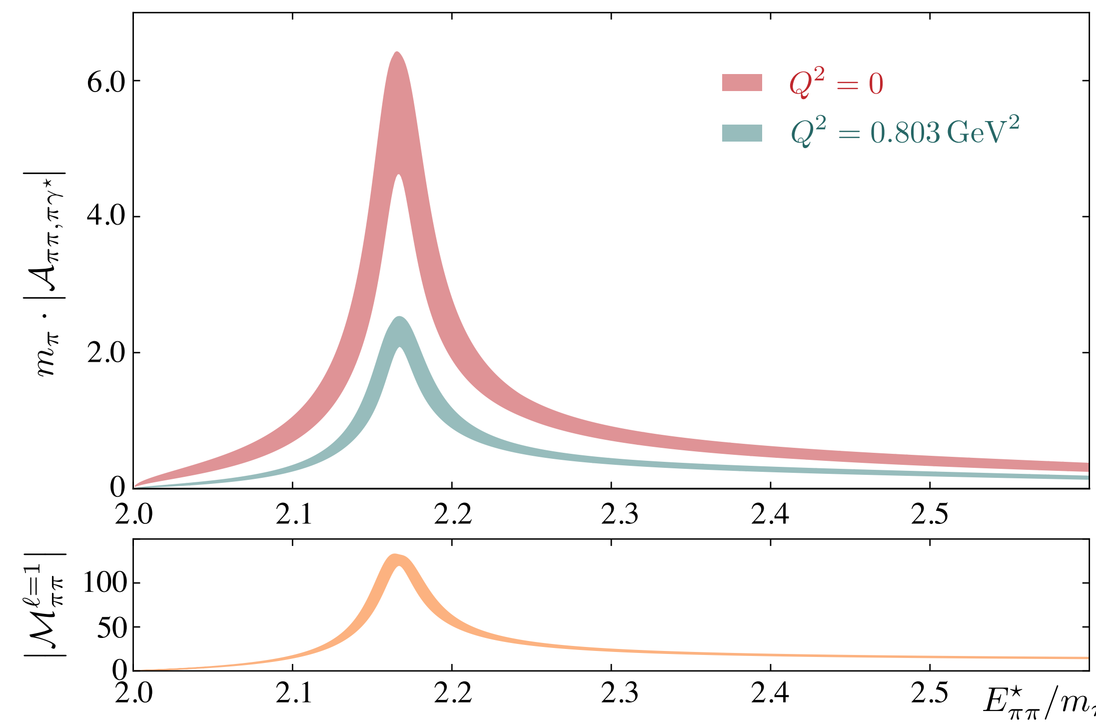
$\gamma^*\pi \to \pi\pi$

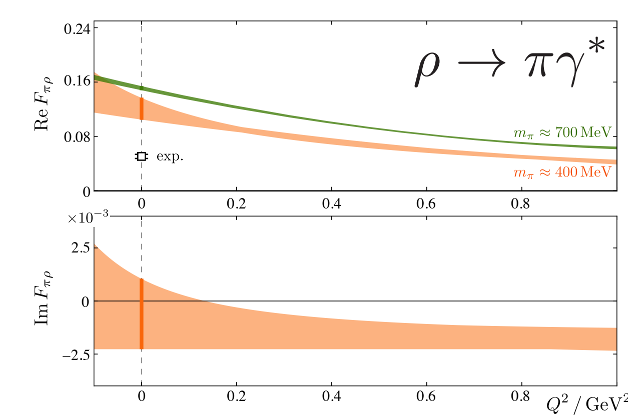


$\mathcal{A}(E_{\pi\pi}, Q^2)$

Briceno et al | *u,d,s* | m_π~391 MeV

PRL115 242001 (2015)





transition form-factors of unstable $\rho \to \pi\pi$

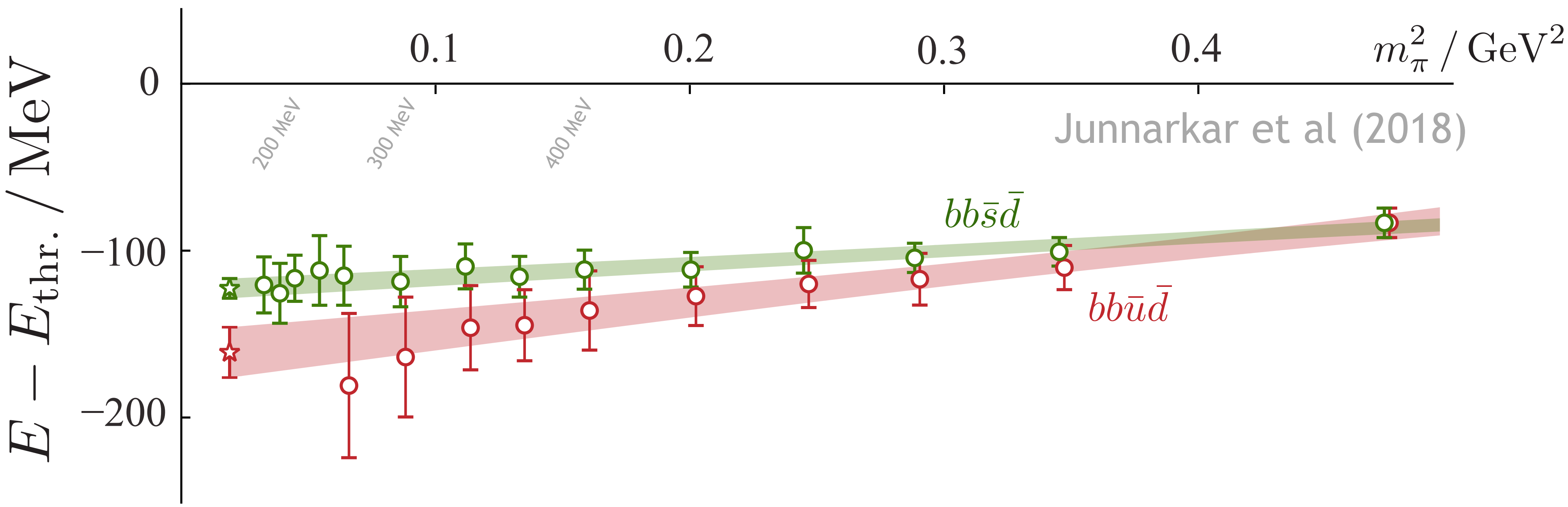
method is practical, next extend to coupled-channel systems …

truly flavor-exotic hadrons

one recent observation in lattice QCD that has a good chance of being robust:

a double-bottom bound-state $bb\bar{u}\bar{d}$ (*I*=0, J^P=1⁺, lying well below *B B** threshold)

and probably a strange partner

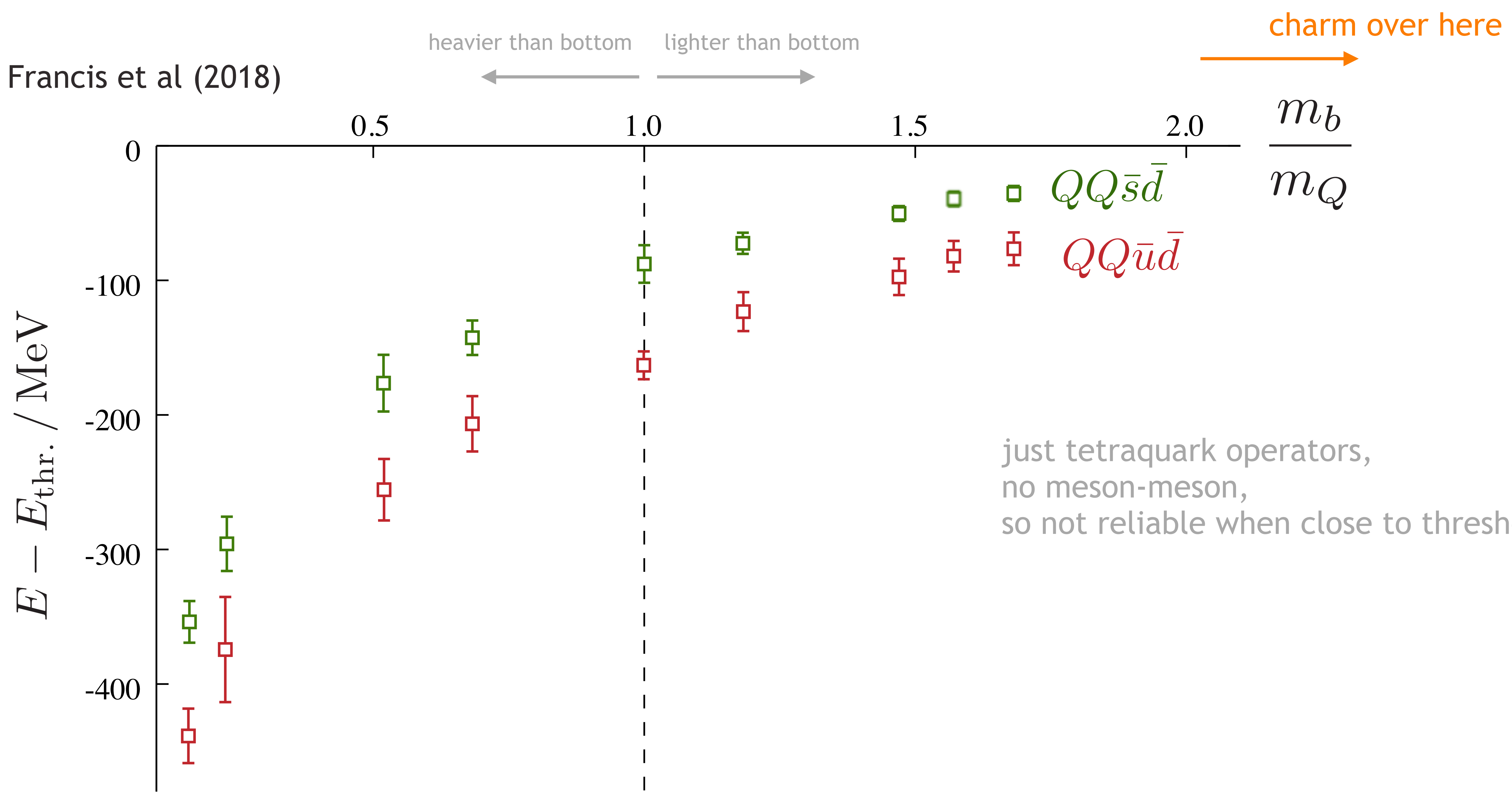


Francis et al (2017)
Junnarkar et al (2018)
Leskovec et al (2018)

$$\left|\Delta E(bb\bar{u}\bar{d})\right| \sim 100 - 200\,\text{MeV}$$
$$\left|\Delta E(bb\bar{s}\bar{d})\right| \sim 90 - 120\,\text{MeV}$$

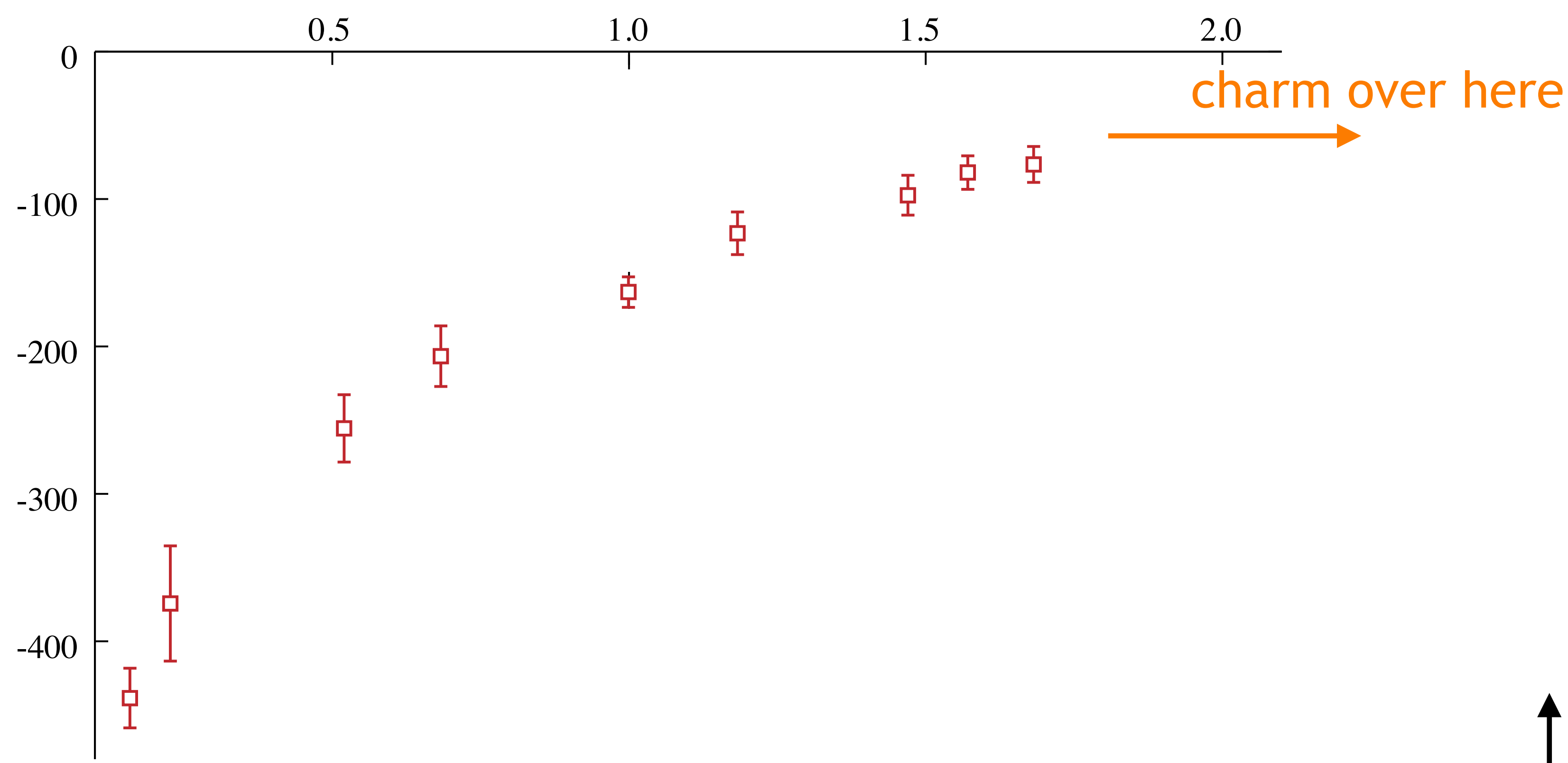
see Eichten & Quigg (2017) for heavy quark symmetry argument

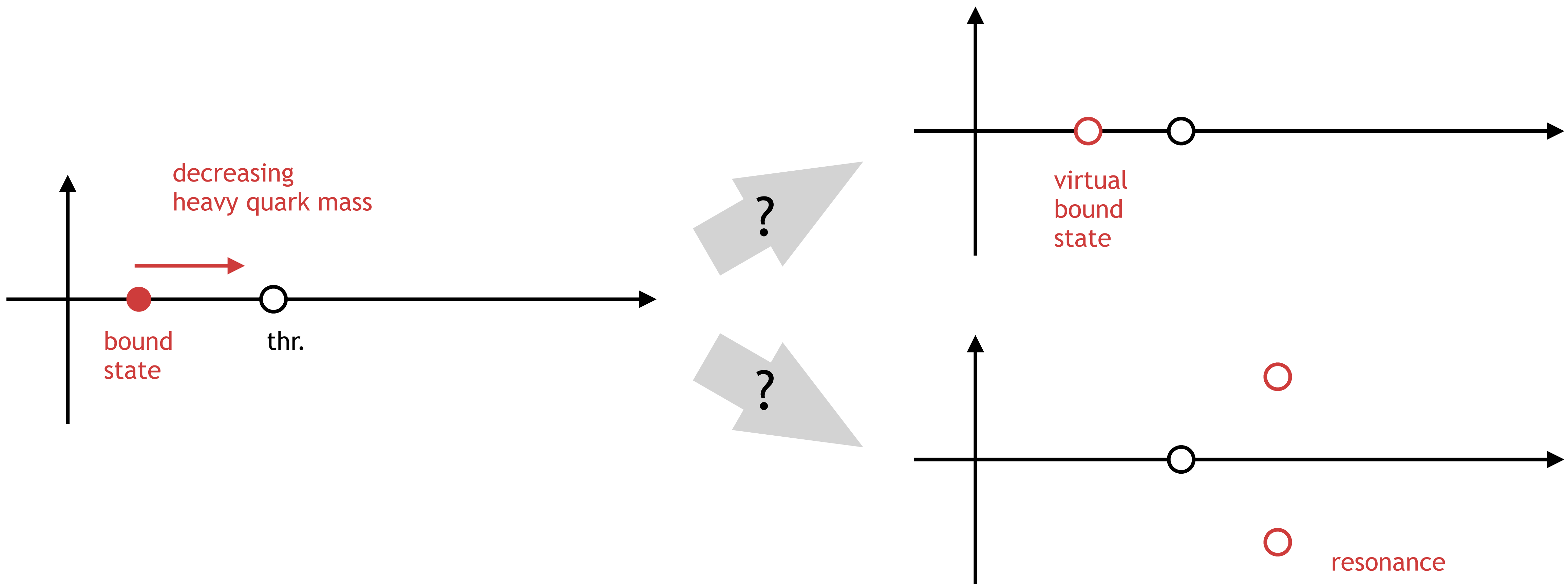
binding energy with changing heavy quark mass



just tetraquark operators,
no meson-meson,
so not reliable when close to threshold

what happens for $cc\bar{u}\bar{d}$?

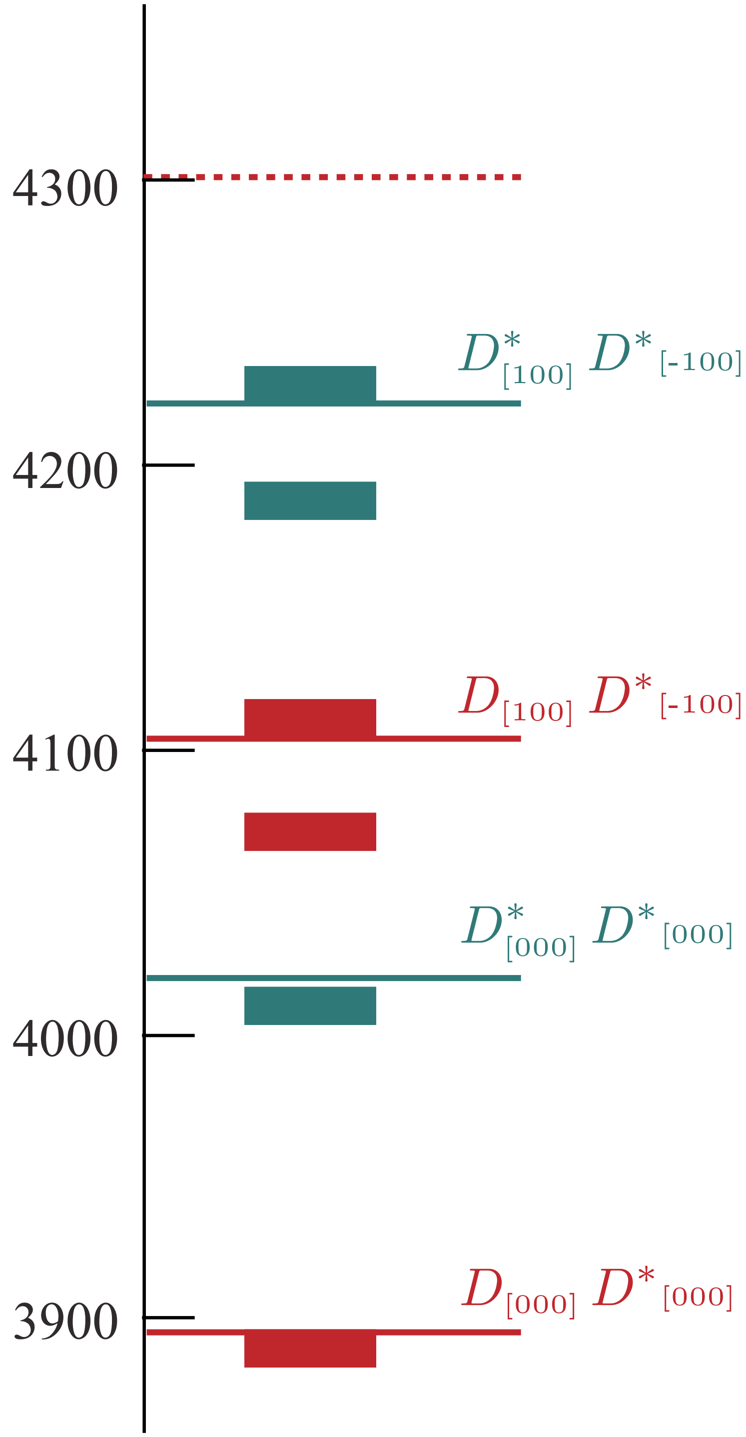




direct calculation of $cc\bar{u}\bar{d}$

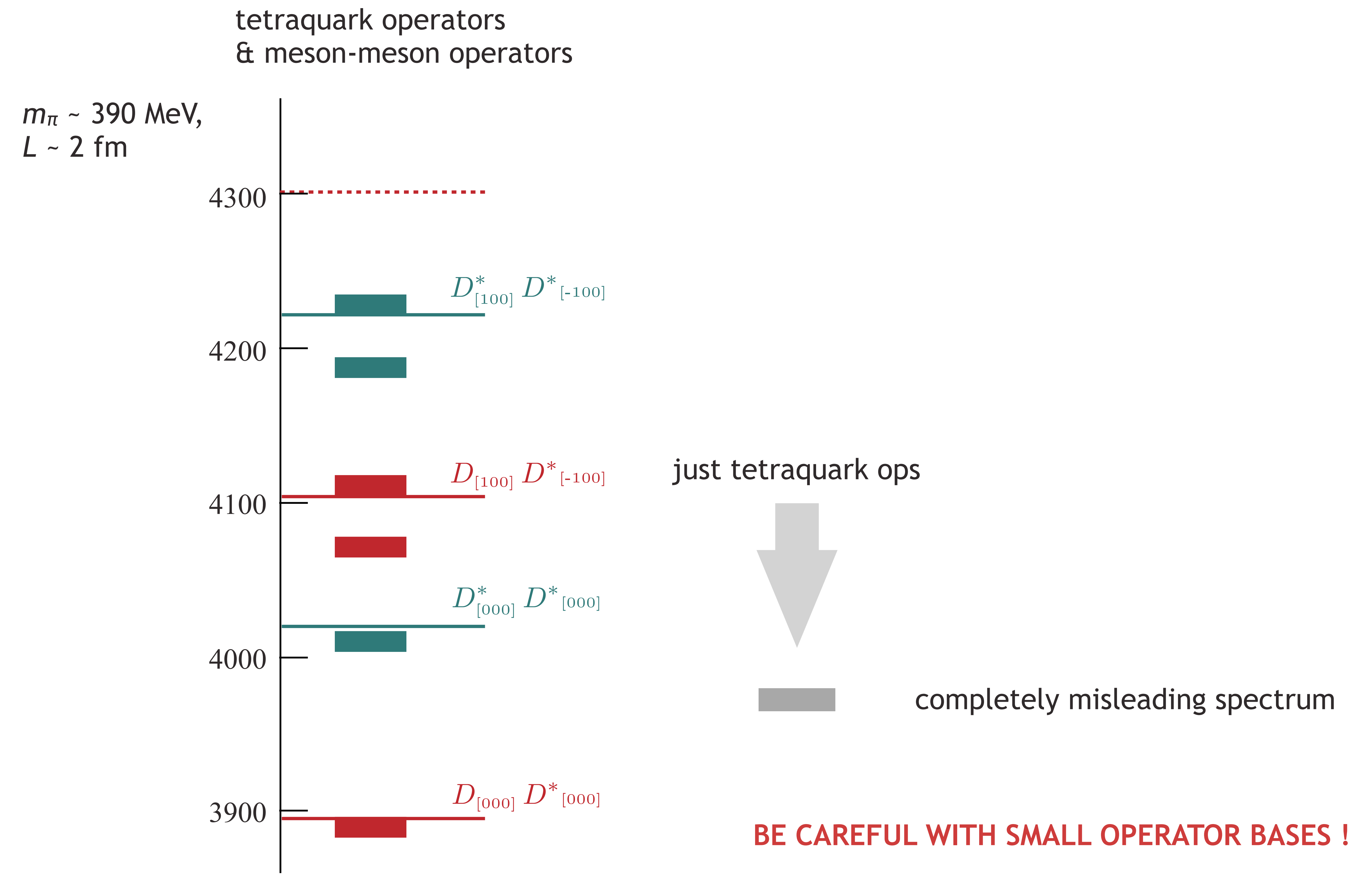
tetraquark operators
& meson-meson operators

m_π ~ 390 MeV,
L ~ 2 fm



no obvious sign of a narrow resonance ...

direct calculation of $cc\bar{u}\bar{d}$



technology exists now in lattice QCD to determine properties of excited states

but it needs to be applied carefully

— avoid jumping to conclusions obtained from incomplete calculations

— a systematic approach starts with the simplest resonances and works up to more exciting cases

elastic scattering now well studied, especially $\rho \to \pi\pi$

a few **coupled-channel cases** have appeared

formalism to **couple resonances to currents** has been applied

ρ, K^*, σ, κ, f_0, a_0, f_2, a_2, b_1

also some D-mesons, ψ'...

initial suggestions that **double-bottom, isospin=0 channel** might house a **QCD-stable bound state**

looks unlikely that the double-charm analogue is bound or resonant, but more calculation needed

limitation:

formalism to handle three-body decays in development

— needed to go to physical pion mass