Excited light baryons from quark-gluon-level calculations

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18th International Conference on Hadron Spectroscopy and Structure
Guilin, China, 16-21 August 2019
Quantum Chromodynamics in its non-perturbative regime

**Emergence**

Low-level rules producing high-level phenomena with enormous apparent complexity

Start from the QCD Lagrangian:

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a + \frac{1}{2\xi} (\partial_\mu A_\mu^a)^2 + \partial_\mu \bar{c}^a \partial_\mu c^a + g f^{abc} (\partial_\mu \bar{c}^a) A_\mu^b c^c + \text{quarks}$$

Lattice, DSEs, ...

And obtain:

- Dynamical generation of fundamental mass scale in pure Yang-Mills (gluon mass).
- Quark constituent masses and chiral symmetry breaking.
- Bound state formation: mesons, baryons, glueballs, hybrids, multiquark systems...
- Signals of confinement.

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Emergent phenomena could be associated with dramatic, dynamically driven changes in the analytic structure of QCD’s Green functions (propagators and vertices).

**Quark propagator:**

\[ -1 = \begin{array}{c}
\text{vertex} \\
\text{loop}
\end{array} -1 + \begin{array}{c}
\text{vertex} \\
\text{loop}
\end{array} \]

**Gluon propagator:**

\[ -1 = \begin{array}{c}
\text{vertex} \\
\text{loop}
\end{array} -1 + \begin{array}{c}
\text{vertex} \\
\text{loop}
\end{array} + \begin{array}{c}
\text{vertex} \\
\text{loop}
\end{array} + \begin{array}{c}
\text{vertex} \\
\text{loop}
\end{array} + \begin{array}{c}
\text{vertex} \\
\text{loop}
\end{array} \]

**Ghost propagator:**

\[ -1 = \begin{array}{c}
\text{vertex} \\
\text{loop}
\end{array} -1 + \begin{array}{c}
\text{vertex} \\
\text{loop}
\end{array} \]

**Ghost-gluon vertex:**

\[ = \begin{array}{c}
\text{vertex} \\
\text{loop}
\end{array} + \begin{array}{c}
\text{vertex} \\
\text{loop}
\end{array} + \begin{array}{c}
\text{vertex} \\
\text{loop}
\end{array} \]

**Quark-gluon vertex:**

\[ = \begin{array}{c}
\text{vertex} \\
\text{loop}
\end{array} + \begin{array}{c}
\text{vertex} \\
\text{loop}
\end{array} + \begin{array}{c}
\text{vertex} \\
\text{loop}
\end{array} + \begin{array}{c}
\text{vertex} \\
\text{loop}
\end{array} + \begin{array}{c}
\text{vertex} \\
\text{loop}
\end{array} \]
Off-shell Green’s (correlation) functions

- Even though they are:
  - Gauge dependent.
  - Renormalization point and scheme dependent.

- However:
  - They capture characteristic features of the underlying dynamics, both perturbative and non-perturbative.
  - When appropriately combined they give rise to physical observables.

Theory tool based on Dyson-Schwinger equations

- Interesting features:
  - Inherently non-perturbative but, at the same time, captures the perturbative behavior → accommodates the full range of physical momenta.
  - Cover smoothly the full quark mass range, from the chiral limit to the heavy-quark domain.

- Main caveats:
  - Truncation of the infinite system of coupled non-linear integral equations that preserves the underlying symmetries of the theory.
  - No expansion parameter → no formal way of estimating the size of the omitted terms ↔ the projection of higher Green’s functions on the lower ones is small.
Dressed-quark propagator in Landau gauge:

\[ S^{-1}(p) = Z_2(i\gamma \cdot p + m) + \Sigma(p) = \left( \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \right)^{-1} \]

- Mass generated from the interaction of quarks with the gluon-medium.
- Light quarks acquire a HUGE constituent mass.
- Responsible of the 98% of proton's mass, the large splitting between parity partners, . . .

Dressed-gluon propagator in Landau gauge:

\[ i\Delta_{\mu\nu} = -iP_{\mu\nu}\Delta(q^2), \quad P_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu/q^2 \]

- An inflexion point at \( p^2 > 0 \).
- Breaks the axiom of reflexion positivity.
- Gluon mass generation ↔ Schwinger mechanism.
Non-perturbative QCD:
Ghost saturation and three-gluon-vertex suppression

Dressed-ghost propagator in Landau gauge:

\[ G^{ab}(q^2) = \delta^{ab} \frac{J(q^2)}{q^2} \]

- No power-like singular behavior at \( q^2 \to 0 \).
- Good indication that \( J(q^2) \) reaches a plateau.
- Saturation of ghost's dressing function.

Three-gluon vertex form factor in Landau gauge:

(\( \propto \) the tree-level tensor structure)

\[ \Gamma_{T,R}^{asym}(q^2) \xrightarrow{q^2 \to 0} F(0) \left[ \frac{\partial}{\partial q^2} \Delta_R^{-1}(q^2) - C_1(r^2) \right] \]

- Appearance of (longitudinally coupled) massless poles.
- Suppression of the form factor in the so-called asymmetric momentum configuration.
- Plausible zero-crossing.
**Non-perturbative QCD:**
Saturation at IR of process-independent effective-charge


Data = running coupling defined from the Bjorken sum-rule.

\[
\int_0^1 dx \left[ g_p(x, k^2) - g_n(x, k^2) \right] = \frac{g_A}{6} \left[ 1 - \frac{1}{\pi} \alpha_{g_1}(k^2) \right]
\]

Curve determined from combined continuum and lattice analysis of QCD’s gauge sector (massless ghost and massive gluon).

The curve is a running coupling that does NOT depend on the choice of observable.

- No parameters.
- No matching condition.
- No extrapolation.

It predicts and unifies an enormous body of empirical data via the matter-sector bound-state equations.

**Perturbative regime:**

\[
\alpha_{g_1}(k^2) = \alpha_{\overline{\text{MS}}}(k^2) \left[ 1 + 1.14\alpha_{\overline{\text{MS}}}(k^2) + \ldots \right]
\]

\[
\hat{\alpha}_{\overline{\text{PI}}}(k^2) = \alpha_{\overline{\text{MS}}}(k^2) \left[ 1 + 1.09\alpha_{\overline{\text{MS}}}(k^2) + \ldots \right]
\]
The bound-state problem in quantum field theory

Extraction of hadron properties from poles in $q\bar{q}$, $qqq$, $qq\bar{q}\bar{q}$... scattering matrices

Use scattering equation (inhomogeneous BSE) to obtain $T$ in the first place: $T = K + KG_0 T$

Homogeneous BSE for BS amplitude:

\[
\begin{align*}
T & = K + KG_0 T \\
\psi & = K \psi
\end{align*}
\]

Baryons. A 3-body bound state problem in quantum field theory:

Faddeev equation in rainbow-ladder truncation

Faddeev equation: Sums all possible quantum field theoretical exchanges and interactions that can take place between the three dressed-quarks that define its valence quark content.
Consequence of solving Poincaré-covariant bound-state equations

**Mesons: \( P = (-1)^{L+1} \)**

<table>
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<tr>
<th>S</th>
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<td>1</td>
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<td>1^{--}</td>
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<tr>
<td>0</td>
<td>1</td>
<td>1^{+-}</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0^{++}</td>
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</table>

\[
\Gamma_{\pi}(P,p) = \gamma_5 \left[ F_1(P,p) + F_2(P,p)iP + F_3(P,p)PPi\not{p} + F_4(P,p)[\not{p},P] \right]
\]

**Baryons: \( P = (-1)^L \)**

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<thead>
<tr>
<th>S</th>
<th>L</th>
<th>( J^P )</th>
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<tr>
<td>1/2</td>
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<td>1/2^{+}</td>
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<tr>
<td>3/2</td>
<td>2</td>
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<table>
<thead>
<tr>
<th>( J^P )</th>
<th>total</th>
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<th>p-wave</th>
<th>d-wave</th>
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<tr>
<td>1/2^{+}</td>
<td>64</td>
<td>8</td>
<td>36</td>
<td>20</td>
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<tr>
<td>3/2^{+}</td>
<td>128</td>
<td>4</td>
<td>36</td>
<td>60</td>
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Diquarks inside baryons

The attractive nature of quark-antiquark correlations in a colour-singlet meson is also attractive for $\bar{3}_c$ quark-quark correlations within a colour-singlet baryon.

Diquark correlations:

- A tractable truncation of the Faddeev equation.
- In $N_c = 2$ QCD: diquarks can form colour singlets and are the baryons of the theory.
- In our approach: Non-pointlike colour-antitriplet and fully interacting.

Diquark-quark approximation:
Owing to properties of charge-conjugation, a diquark with spin-parity $J^P$ may be viewed as a partner to the analogous $J^{-P}$ meson:

$$\Gamma_{q\bar{q}}(p; P) = - \int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q + P) \Gamma_{q\bar{q}}(q; P) S(q) \frac{\lambda^a}{2} \gamma_\nu$$

$$\Gamma_{q\bar{q}}(p; P) C^\dagger = - \frac{1}{2} \int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q + P) \Gamma_{q\bar{q}}(q; P) C^\dagger S(q) \frac{\lambda^a}{2} \gamma_\nu$$

Whilst no pole-mass exists, the following mass-scales express the strength and range of the correlation:

$$m_{[ud]_{0^+}} = 0.7 - 0.8 \text{ GeV}, \quad m_{\{uu\}_{1^+}} = 0.9 - 1.1 \text{ GeV}, \quad m_{\{dd\}_{1^+}} = m_{[ud]_{1^+}} = m_{\{uu\}_{1^+}}$$

Diquark correlations are soft, they possess an electromagnetic size:

$$r_{[ud]_{0^+}} \gtrsim r_\pi, \quad r_{\{uu\}_{1^+}} \gtrsim r_\rho, \quad r_{\{uu\}_{1^+}} > r_{[ud]_{0^+}}$$
Diquark species

Octet and decuplet baryons

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<tr>
<th></th>
<th>[nn]</th>
<th>{nn}</th>
<th>[ns]</th>
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<tr>
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<tr>
<td>Δ</td>
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<tr>
<td>Λ</td>
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<tr>
<td>Σ</td>
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<tr>
<td>Ω</td>
<td></td>
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</tr>
</tbody>
</table>

$\mathbb{I} = 0$, $J^P = 0^+$: Isoscalar-scalar.
$\mathbb{I} = 1$, $J^P = 1^+$: Isovector-pseudovector.
$\mathbb{I} = 0$, $J^P = 0^-$: Isoscalar-pseudoscalar.
$\mathbb{I} = 0$, $J^P = 1^-$: Isoscalar-vector.
$\mathbb{I} = 1$, $J^P = 1^-$: Isovector-vector.

G. Eichmann et al.

Mesons

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$\pi$, $\rho$, $a_0$, $b_1$, $a_1$,

Diquarks

<table>
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<tr>
<th>m [GeV]</th>
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<tbody>
<tr>
<td>1.5</td>
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<td>1.0</td>
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<td>0.5</td>
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<td>0.0</td>
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</table>

$\text{PDG}$, $\text{RL x c}$,

G. Eichmann et al.
The quark+diquark structure of the nucleon

Faddeev equation in the quark-diquark picture

\[ \Psi^a = \Gamma^a \Gamma^b \Psi^b \]

Dominant piece in nucleon’s eight-component Poincaré-covariant Faddeev amplitude: \( s_1(|p|, \cos \theta) \)

- There is strong variation with respect to both arguments in the quark+scalar-diquark relative momentum correlation.

- Support is concentrated in the forward direction, \( \cos \theta > 0 \). Alignment of \( p \) and \( P \) is favoured.

- Amplitude peaks at \( (|p| \sim M_N/6, \cos \theta = 1) \), whereat \( p_q \sim p_d \sim P/2 \) and hence the natural relative momentum is zero.

- In the anti-parallel direction, \( \cos \theta < 0 \), support is concentrated at \( |p| = 0 \), i.e. \( p_q \sim P/3, p_d \sim 2P/3 \).

A baryon can be viewed as a **Borromean bound-state**, the binding within which has two contributions:

- Formation of tight diquark correlations.
- Quark exchange depicted in the shaded area.

The exchange ensures that diquark correlations within the baryon are **fully dynamical**: no quark holds a special place.

The rearrangement of the quarks guarantees that the baryon's wave function complies with **Pauli statistics**.

Modern diquarks are different from the old static, point-like diquarks which featured in early attempts to explain the so-called missing resonance problem.

The number of states in the spectrum of baryons obtained is similar to that found in the three-constituent quark model, just as it is in today's LQCD calculations.

Modern diquarks enforce certain distinct interaction patterns for the singly- and doubly-represented valence-quarks within the baryon.
Spectrum in one to one agreement with experiment.
Correct level ordering (without coupled-channels effects).
Three-body agrees with quark-diquark where applicable.
# Level of progress in 2019

Excited light baryons from quark-gluon-level calculations

<table>
<thead>
<tr>
<th>Level of complexity</th>
<th>I) NJL/contact interaction</th>
<th>II) Quark-diquark model</th>
<th>III) DSE (RL)</th>
<th>IV) DSE (bRL)</th>
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<td>$N, \Delta$ masses</td>
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<td>✓</td>
<td>✓</td>
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<td></td>
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<td>✓</td>
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<td>✓</td>
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<td>$N^<em>, \Delta^</em>$ masses</td>
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<td>$N^<em>, \Delta^</em>$ masses</td>
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- Jorge Segovia (jsegovia@upo.es)

- Excited light baryons from quark-gluon-level calculations

- Cloet, Thomas, Roberts, Segovia, Chen, et al.
- Oettel, Alkofer, Bloch, Roberts, Segovia, Chen, et al.
- Eichmann, Alkofer, Krassnigg, Nicmorus, Sanchis-Alepuz, CF
- Eichmann, Alkofer, Sanchis-Alepuz, CF, Qin, Roberts
- Sanchis-Alepuz, Williams, CF
An example of diquark content

**Λ − Σ mass splitting**

Whilst the Λ and Σ are associated with the same combination of valence-quarks, their spin-flavor wave functions are different.

\[ \downarrow \]

Λ contains more of the (lighter) scalar diquark correlations than Σ

\[ u_\Lambda = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} s[ud]_{0+} \\ d[us]_{0+} - u[ds]_{0+} \\ d\{us\}_{1+} - u\{ds\}_{1+} \end{pmatrix} \leftrightarrow \begin{pmatrix} s^1_{\Lambda} \\ s_{2,3}^\Lambda \\ a^\Lambda_{6,8} \end{pmatrix} ; \quad u_\Sigma = \begin{pmatrix} u[us]_{0+} \\ s\{uu\}_{1+} \\ u\{us\}_{1+} \end{pmatrix} \leftrightarrow \begin{pmatrix} s^2_\Sigma \\ a^\Sigma_4 \\ a^\Sigma_6 \end{pmatrix} \]

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Λ</th>
<th>Σ</th>
<th>Ξ</th>
<th>Δ</th>
<th>Σ*</th>
<th>Ξ*</th>
<th>Ω</th>
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<tr>
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<td>1.37(14)</td>
<td>1.41(14)</td>
<td>1.58(15)</td>
<td>1.35(12)</td>
<td>1.52(14)</td>
<td>1.71(15)</td>
<td>1.93(17)</td>
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<tr>
<td>Exp.</td>
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<td>1.19</td>
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<tr>
<td>The.</td>
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<td>1.88(11)</td>
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<td>1.79(12)</td>
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<td>2.08(12)</td>
<td>2.23(13)</td>
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<tr>
<td>Exp.</td>
<td>1.44(03)</td>
<td>1.51^{+0.10}_{-0.04}</td>
<td>1.66(03)</td>
<td>-</td>
<td>1.57(07)</td>
<td>1.73(03)</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The computed **masses are uniformly larger** than the corresponding empirical values.

The quark-diquark kernel omits all resonant contributions associated with meson-baryon final state interactions, which typically generate a measurable reduction.

The Faddeev equations analyzed to produce the results should be understood as producing the **dressed-quark core** of the bound state, **not** the completely dressed and hence observable object.

---

Angular momenta of the octet and decuplet (I)

<table>
<thead>
<tr>
<th>$L$ content</th>
<th>$N_{n=0}$</th>
<th>$N_{n=1}$</th>
<th>$\Lambda_{n=0}$</th>
<th>$\Lambda_{n=1}$</th>
<th>$\Sigma_{n=0}$</th>
<th>$\Sigma_{n=1}$</th>
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<td>1.85</td>
<td>1.41</td>
<td>1.88</td>
<td>1.58</td>
<td>1.99</td>
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<tr>
<td>$-, P, D$</td>
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<td>1.42</td>
<td>1.84</td>
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<td>1.89</td>
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<td>1.99</td>
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<td>$S, -, -$</td>
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<td>1.84</td>
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<td>1.97</td>
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<table>
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<th>$\Sigma^*_{n=0}$</th>
<th>$\Sigma^*_{n=1}$</th>
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<td>1.79</td>
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<td>1.71</td>
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<td>$-, P, D, F$</td>
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<td>$S, -, D, F$</td>
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<td>$S, P, -, F$</td>
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<td>1.93</td>
<td>2.24</td>
</tr>
<tr>
<td>$S, P, D, -$</td>
<td>1.35</td>
<td>1.79</td>
<td>1.52</td>
<td>1.93</td>
<td>1.71</td>
<td>2.08</td>
<td>1.93</td>
<td>2.23</td>
</tr>
<tr>
<td>$S, -, -, -$</td>
<td>1.35</td>
<td>1.80</td>
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<td>2.08</td>
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<td>2.23</td>
</tr>
</tbody>
</table>

No state is generated by the Faddeev equation unless $S$-wave components are contained in the wave function.

This observation provides support in quantum field theory for the constituent quark model classification of these systems.

The $P$- and $D$-wave components play a measurable role in, respectively, octet and decuplet baryons.
Angular momenta of the octet and decuplet (I)

<table>
<thead>
<tr>
<th>L content</th>
<th>$N_{n=0}$</th>
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<th>$\Lambda_{n=0}$</th>
<th>$\Lambda_{n=1}$</th>
<th>$\Sigma_{n=0}$</th>
<th>$\Sigma_{n=1}$</th>
<th>$\Xi_{n=0}$</th>
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<tbody>
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<td>$S, P, -$</td>
<td>1.24</td>
<td>1.71</td>
<td>1.40</td>
<td>1.83</td>
<td>1.42</td>
<td>1.84</td>
<td>1.59</td>
<td>1.97</td>
</tr>
<tr>
<td>$S, -,-$</td>
<td>1.24</td>
<td>1.71</td>
<td>1.40</td>
<td>1.83</td>
<td>1.42</td>
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<th>$\Sigma^*_{n=1}$</th>
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<th>$\Omega_{n=0}$</th>
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<tbody>
<tr>
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<td>1.79</td>
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<td>1.71</td>
<td>2.08</td>
<td>1.93</td>
<td>2.23</td>
</tr>
<tr>
<td>$-, P, D, F$</td>
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<td>2.23</td>
</tr>
<tr>
<td>$S, -,-, -$</td>
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Angular momenta of the octet and decuplet (II)

(Jorge Segovia (jsegovia@upo.es))

Excited light baryons from quark-gluon-level calculations

20/27
Radial excitations in quantum field theory

\[
S_1 - \frac{1}{3}A_3 + \frac{2}{3}A_5
\]

\[
N(940)
\]

\[
-\frac{1}{3}\Delta_6 + \frac{1}{3}\Delta_8 + \Delta_1
\]

\[
\Delta(1232)
\]

\[
N(1440)
\]

\[
-\frac{1}{3}\Delta_6 + \frac{1}{3}\Delta_8 + \Delta_1
\]

\[
\Delta(1600)
\]

Jorge Segovia (jsegovia@upo.es)

Excited light baryons from quark-gluon-level calculations
Wave function decomposition: $N(1440)$ cf. $\Delta(1600)$

<table>
<thead>
<tr>
<th></th>
<th>$N(940)$</th>
<th>$N(1440)$</th>
<th>$\Delta(1232)$</th>
<th>$\Delta(1600)$</th>
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</thead>
<tbody>
<tr>
<td>scalar</td>
<td>62%</td>
<td>62%</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>pseudovector</td>
<td>29%</td>
<td>29%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>mixed</td>
<td>9%</td>
<td>9%</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>S-wave</td>
<td>0.76</td>
<td>0.85</td>
<td>0.61</td>
<td>0.30</td>
</tr>
<tr>
<td>P-wave</td>
<td>0.23</td>
<td>0.14</td>
<td>0.22</td>
<td>0.15</td>
</tr>
<tr>
<td>D-wave</td>
<td>0.01</td>
<td>0.01</td>
<td>0.17</td>
<td>0.52</td>
</tr>
<tr>
<td>F-wave</td>
<td>—</td>
<td>—</td>
<td>$\sim 0$</td>
<td>0.02</td>
</tr>
</tbody>
</table>

$N(1440)$

- Roper’s diquark content are almost identical to the nucleon’s one.
- It has an orbital angular momentum composition which is very similar to the one observed in the nucleon.

$\Delta(1600)$

- $\Delta(1600)$’s diquark content are almost identical to the $\Delta(1232)$’s one.
- It shows a dominant $\ell = 2$ angular momentum component with its S-wave term being a factor 2 smaller.

The presence of all angular momentum components compatible with the baryon’s total spin and parity is an inescapable consequence of solving a realistic Poincaré-covariant Faddeev equation.
Consequences on, e.g., EM transition form factors

$N(1440)$: Our calculation agrees quantitatively in magnitude and qualitatively in trend with the data on $x \gtrsim 2$.

$N(1440)$: The mismatch between our prediction and the data on $x \lesssim 2$ is due to meson cloud contribution.

$\Delta(1600)$: It is positive defined in the whole range of photon momentum and decreases smoothly with larger $Q^2$-values $\rightarrow D$-wave dominance.

---

**Observations:**

- $N(1440)$: Our calculation agrees quantitatively in magnitude and qualitatively in trend with the data on $x \gtrsim 2$.
- $N(1440)$: The mismatch between our prediction and the data on $x \lesssim 2$ is due to meson cloud contribution.
- $\Delta(1600)$: It is positive defined in the whole range of photon momentum and decreases smoothly with larger $Q^2$-values $\rightarrow D$-wave dominance.

The $N(940)$ and $N(1440)$ are primarily $S$-wave in nature, since they are not supported by the Faddeev equation unless $S$-wave components are contained in the wave function.

The $N(1535)$ and $N(1650)$ are essentially $P$-wave in character, since they are not supported by the Faddeev equation unless $P$-wave components are contained in the wave function.

These observations provide (again) support in quantum field theory for the constituent-quark model classifications of these systems.

<table>
<thead>
<tr>
<th>$L$ content</th>
<th>$N_0^+$</th>
<th>$N_1^+$</th>
<th>$N_0^-$</th>
<th>$N_1^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S, P, D$</td>
<td>1.19</td>
<td>1.73</td>
<td>1.83</td>
<td>1.91</td>
</tr>
<tr>
<td>$-, P, D$</td>
<td>-</td>
<td>-</td>
<td>1.89</td>
<td>1.98</td>
</tr>
<tr>
<td>$S, -, D$</td>
<td>1.24</td>
<td>1.71</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$S, P, -$</td>
<td>1.20</td>
<td>1.74</td>
<td>1.83</td>
<td>1.91</td>
</tr>
<tr>
<td>$S, -, -$</td>
<td>1.24</td>
<td>1.71</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$-, P, -$</td>
<td>-</td>
<td>-</td>
<td>1.90</td>
<td>1.98</td>
</tr>
</tbody>
</table>

Masses of the quark core against values determined for the meson-undressed bare-excitations

<table>
<thead>
<tr>
<th></th>
<th>$m_N$</th>
<th>$m_{N(1440)}^{1/2+}$</th>
<th>$m_{N(1535)}^{1/2-}$</th>
<th>$m_{N(1650)}^{1/2-}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>herein</td>
<td>1.19</td>
<td>1.73</td>
<td>1.83</td>
<td>1.91</td>
</tr>
<tr>
<td>$M_B^0$</td>
<td>1.76</td>
<td>1.80</td>
<td>1.88</td>
<td></td>
</tr>
</tbody>
</table>

Again, the two lightest $1/2^+$ doublets are predominantly $S$-wave in character, whereas the negative parity states are chiefly $P$-wave. In all cases the $D$-wave components are negligible.

$N(940)$ and $N(1440)$ are mostly constituted by scalar and pseudovector diquarks and $g_{DB} < 1$ has little impact on the nucleon and Roper, so we do not draw $g_{DB} = 1$ results.

$g_{DB} < 1$ has a significant effect on the structure of the negative parity baryons, it increases both the effective energy-cost (mass) of positive parity diquarks and the fraction of pseudoscalar- and vector-diquarks they contain.

Epilogue

Take home messages:

- Baryon spectrum: fair agreement with experiment!
- Results for up/down, strange and heavy quarks.
- Three-body vs diquark-quark: good agreement.

QFT combined with quark-diquark picture:

- The running of the strong coupling constant which is expressed in e.g. the momentum dependence of the dressed-quark mass produces DCSB.
- DCSB and its correct implementation produces pions as well as nonpointlike and fully-dynamical diquark correlations inside baryons.
- The Faddeev kernel ensures that every valence-quark participates actively in all diquark correlations to the fullest extent allowed by kinematics and symmetries.
- Poincaré covariance demands the presence of dressed-quark orbital angular momentum in the baryon.