Spectrum of the fully-heavy tetraquark state $QQQ'\bar{Q}'$
Background

• Since the discovery of X(3872) in 2003, numerous exotic structures “XYZ” have been observed in experiments.

> C. Z. Yuan, Int.J.Mod.Phys. A33,1830018

• The theoretical interpretations of “XYZ” include the loosely bound molecular states, the compact tetraquark states, and the hybrids, etc.

> H.X. Chen et al., Phys. Rept. 639, 1
> F. K. Guo et al., Rev. Mod. Phys., 015004
> Y. R. Liu et al., Prog.Part.Nucl.Phys. 107 237-320
Motivation

- PhD thesis result using CMS data:
  - Best mass: $18.4 \pm 0.1$ (stat.) $\pm 0.2$ (syst.) GeV
  - $M(bb\bar{b}\bar{b}) < 2M(\Upsilon(1S))$
  - Global significance was $3.6\sigma$

- LHCb:
  - No significant excess is found for $X_{bb\bar{b}\bar{b}}$ in the mass range (17.5-20.0) GeV.

JHEP 1705, 013 (2017)
S. Durgut (CMS), Search for Exotic Mesons at CMS (2018),
http://meetings.aps.org/Meeting/APR18/Session/U09.6

JHEP 1810, 086 (2018)
Motivation

• Theoretical works: existence of the stable fully-heavy tetraquark state

✓ Positive: $b\bar{b}b\bar{b} \sim 18 - 20$ GeV, $c\bar{c}c\bar{c} \sim 5 - 7$ GeV:
  Phys. Lett. B 718, 545, Phys. Rev. D 70, 014009 ...  

✓ Negative: no stable $QQ\bar{Q}\bar{Q}$ states exist.

• A fully-heavy tetra-quark state:

✓ Two color configurations: $\bar{3}_c \otimes 3_c = 1_c$ and $6_c \otimes \bar{6}_c = 1_c$.
✓ Short-range one-gluon-exchange (OGE) potential dominates.
✓ A good candidate for compact tetraquark state.
✓ Nonrelativistic quark model.
Quark model

• Model I

\[ V_{ij}(r_{ij}) = \frac{\lambda_i \lambda_j}{2} \left( V_{\text{coul}} + V_{\text{conf}} + V_{\text{hyp}} + V_{\text{cons}} \right) \]

\[ = \frac{\lambda_i \lambda_j}{2} \left( \frac{\alpha_s}{r_{ij}} - \frac{3b}{4} r_{ij} - \frac{8\pi \alpha_s}{3m_i m_j} \mathbf{s}_i \cdot \mathbf{s}_j e^{-\frac{r^2}{\tau^2}} \frac{\tau^3}{\pi^{3/2}} + V_{\text{cons}} \right) \]

The running coupling constant

\[ \alpha_s(Q^2) = \frac{12\pi}{(33 - 2N_f) \ln(A + Q^2/B^2)} \]

• Model II

\[ V_{ij}(r_{ij}) = -\frac{3}{16} \sum_{i<j} \lambda_i \lambda_j \left( -\frac{\kappa(1 - \exp(-r_{ij}/r_c))}{r_{ij}} + \lambda r_{ij}^p \right) \]

\[ -\Lambda + \frac{8\pi}{3m_i m_j} \kappa' (1 - \exp(-r_{ij}/r_c)) \frac{\exp(-r_{ij}^2/r_0^2)}{\pi^{3/2} r_0^3} \mathbf{s}_i \cdot \mathbf{s}_j \]

C. Y. Wong et al., Phys. Rev. C 65, 014903

B. Silvestre-Brac, Few Body Syst. 20, 1

All the mass information
Quark model

• The parameters Mass spectra of the mesons

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td></td>
<td>1.776</td>
<td>5.102</td>
<td>0.18</td>
<td>0.897</td>
<td>0.62</td>
<td>10</td>
<td>0.31</td>
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</table>

<table>
<thead>
<tr>
<th>Model II</th>
<th>$p$</th>
<th>$r_c$</th>
<th>$m_c$ [GeV]</th>
<th>$m_b$ [GeV]</th>
<th>$\kappa$</th>
<th>$\kappa'$</th>
<th>$\lambda$ [GeV$^2$]</th>
<th>$\Lambda$ [GeV]</th>
<th>$A$ [GeV$^{B-1}$]</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>1.836</td>
<td>5.227</td>
<td>0.5069</td>
<td>1.8609</td>
<td>0.1653</td>
<td>0.8321</td>
<td>1.6553</td>
<td>0.2204</td>
</tr>
</tbody>
</table>

TABLE I. The values of parameters in quark model I and model II.

TABLE II. The mass spectra of the heavy quarkonia in units of MeV. The $M_{ex}$, $M_{th}^I$, and $M_{th}^{II}$ refer to the mass spectra of mesons from PDG, in model I, and in model II, respectively.

<table>
<thead>
<tr>
<th></th>
<th>$M_{ex}$</th>
<th>$M_{th}^I$</th>
<th>$M_{th}^{II}$</th>
<th>$M_{ex}$</th>
<th>$M_{th}^I$</th>
<th>$M_{th}^{II}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_c$</td>
<td>6274.9</td>
<td>6319.4</td>
<td>6293.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta_c$</td>
<td>2983.9</td>
<td>3056.5</td>
<td>3006.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta_c(2S)$</td>
<td>3637.6</td>
<td>3637.6</td>
<td>3621.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J/\psi$</td>
<td>3096.9</td>
<td>3085.1</td>
<td>3102.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\psi(2S)$</td>
<td>3686.1</td>
<td>3652.4</td>
<td>3657.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta_b$</td>
<td></td>
<td></td>
<td></td>
<td>9399.0</td>
<td>9497.8</td>
<td>9427.9</td>
</tr>
<tr>
<td>$\Upsilon(1S)$</td>
<td></td>
<td></td>
<td></td>
<td>9460.30</td>
<td>9503.6</td>
<td>9470.4</td>
</tr>
<tr>
<td>$\Upsilon(2S)$</td>
<td></td>
<td></td>
<td></td>
<td>10023.26</td>
<td>9949.7</td>
<td>10017.8</td>
</tr>
<tr>
<td>$\Upsilon(3S)$</td>
<td></td>
<td></td>
<td></td>
<td>10355.2</td>
<td>10389.8</td>
<td>10440.6</td>
</tr>
</tbody>
</table>

Phys.Rev.D 98, 030001
Wave function

- Wave function of tetraquark state: No. of basis $N^3 = 2^3$.

$$
\psi_{J J z} = \sum [\varphi_{n_a J_a}(r_{12}, \beta_a) \otimes \varphi_{n_b J_b}(r_{34}, \beta_b) \otimes \phi_{N L_{ab}}(r, \beta)]_{J J z},
$$

- Basic wave function of each Jacobi coordinate

$$
\varphi_{n_a J_a M_a} = [\phi_{n_a l_a}(r_{12}, \beta_a) \chi_{s_a} J_a M_a \chi_f \chi_c] J_a \chi_s \chi_f \chi_c
$$

$\chi_{s,f,c}$: the wave function in the spin, flavor, and color space.

- Gaussian function:

$$
\phi_{n_a l_a m_a}(r_{12}, \beta_a) = i^{l_a} r_{12}^{l_a} \sqrt{\frac{4\pi}{(2l_a + 1)!!}} \frac{n_a \beta_a^2}{\pi}^{3/4} \times (2n_a \beta_a^2)^{l_a/2} e^{-r^2 \beta_a^2 n_a/2} Y_{l_a m_a}(\Omega_{12})
$$

S-wave tetraquark state

• S-wave tetraquark states: $J = S$

✓ The ground S-wave state: $l_a = l_b = L_{ab} = 0$
✓ The couple with higher orbital excitations is neglected.

• Color-flavor-spin configuration of $QQ\bar{Q}'\bar{Q}'$:  Fermi statistics

\[ J^{PC} = 0^{++} \]

\[
\chi_1 = \left[ [QQ]_3^1 [\bar{Q}\bar{Q}]_3^1 \right]_{1_0}^0,
\]
\[
\chi_2 = \left[ [QQ]_6^0 [\bar{Q}\bar{Q}]_6^0 \right]_{1_0}^0.
\]

\[ J^{PC} = 1^{+-} \]

\[
\chi_1 = \left[ [QQ]_3^1 [\bar{Q}\bar{Q}]_3^1 \right]_{1_1}^1.
\]

\[ J^{PC} = 2^{++} \]

\[
\chi_1 = \left[ [QQ]_3^1 [\bar{Q}\bar{Q}]_3^1 \right]_{1_2}^2.
\]
Hamiltonian

\[ H = \sum_{i=1}^{4} \frac{p_i^2}{2m_i} + \sum_i m_i + \sum_{i<j} V_{ij} = \frac{p^2}{2u} + V_I + h_{12} + h_{34}, \]

With

\[ V_I = V_{13} + V_{14} + V_{23} + V_{24}, \]

\[ h_{ij} = \frac{p_{ij}^2}{2u_{ij}} + V_{ij} + m_i + m_j, \]

- \( h_{12}/h_{34} \): diagonal in the color-flavor-spin space.
- \( V_I \): the mixing between different color-spin-flavor configurations.
- Solving the Schrödinger equation by variational method.
\[ J^{PC} = 0^{++} \]

- **0^{++} state:** an admixture of $\bar{3}_c \otimes 3_c$ and $6_c \otimes \bar{6}_c$ configurations.

- The two quark models lead to similar mass spectra up to tens of MeV.

The left (right) half: without (with) mixing between $\bar{3}_c \otimes 3_c$ and $6_c \otimes \bar{6}_c$
$J^{PC} = 0^{++}$

- $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$: $M(\bar{3}_c \otimes 3_c) > M(6_c \otimes \bar{6}_c)$ in two quark models.

- $bb\bar{c}\bar{c}$: $M(\bar{3}_c \otimes 3_c) > M(6_c \otimes \bar{6}_c)$ in model I;
  $M(\bar{3}_c \otimes 3_c) < M(6_c \otimes \bar{6}_c)$ in model II;

$\left\langle \frac{\lambda_i \lambda_j}{2} \right\rangle = -\frac{5}{6}$, attractive

$\left\langle \frac{\lambda_i \lambda_j}{2} \right\rangle = \frac{1}{3}$, repulsive

$\left\langle \frac{\lambda_i \lambda_j}{2} \right\rangle = -\frac{2}{3}$, attractive

Compete
$J^{PC} = 0^{++}$

- The mixture:

<table>
<thead>
<tr>
<th>$J^{PC} = 0^{++}$</th>
<th>Model I</th>
<th>Model II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_a = \beta_b = 0.4, \beta = 0.6$</td>
<td>$6.377$</td>
<td>$\beta_a = \beta_b = 0.5, \beta = 0.7$</td>
</tr>
<tr>
<td>$\gamma_a = \gamma_b = 0.4, \gamma = 0.7$</td>
<td>$6.425$</td>
<td>$\gamma_a = \gamma_b = 0.5, \gamma = 0.8$</td>
</tr>
</tbody>
</table>

- $6_c \otimes \bar{6}_c$ is important even dominant in the ground state.

- The proportions in the two models are quite different. The mixing is more stronger in model II.

- S-wave tetraquark state $Q_1 Q_2 \bar{Q} \bar{Q}$:
  - Orthogonality of $\chi_S$: Color interactions do not contribute to the mixing.
  - Only the hyperfine interaction contributes to the couple-channel effects.
Mixture

• A tetraquark state is an admixture of different color configurations.

• For a $Q_1 Q_2 \bar{Q}_3 \bar{Q}_4$ ($Q_1 \neq Q_2$ & $Q_3 \neq Q_4$):

$$\left( \sum_{n}^{4} \lambda_n \right)^2 |\chi_{i,j} \rangle = 0$$

$$\langle \chi_i |(\lambda_1 + \lambda_2)^2 |\chi_j \rangle = 0$$

$$\langle \chi_i |(\lambda_3 + \lambda_4)^2 |\chi_j \rangle = 0$$

$$\langle \chi_i |(\lambda_1 + \lambda_2)(\lambda_3 + \lambda_4) |\chi_j \rangle = 0$$

<table>
<thead>
<tr>
<th></th>
<th>Model I</th>
<th>Model II</th>
</tr>
</thead>
<tbody>
<tr>
<td>OGE Coulomb</td>
<td>$\alpha_s(m_i, m_j)$ ✓</td>
<td>constant $\kappa$ ✗</td>
</tr>
<tr>
<td>Linear confinement</td>
<td>constant $b$ ✗</td>
<td>constant $\lambda$ ✗</td>
</tr>
<tr>
<td>Hyperfine</td>
<td>$\alpha_s(m_i, m_j)$ ✓</td>
<td>constant $\kappa'$ ✓</td>
</tr>
</tbody>
</table>
Scattering state vs tetraquark state

\[ J^{PC} = 0^{++} \]

<table>
<thead>
<tr>
<th>( N^3 = 2^3 )</th>
<th>Model I</th>
</tr>
</thead>
<tbody>
<tr>
<td>After mixing</td>
<td></td>
</tr>
<tr>
<td>( 3_c \otimes 3_c )</td>
<td>( \sqrt{\langle r_{12}^2 \rangle} ) fm</td>
</tr>
<tr>
<td>( ccc\bar{c} )</td>
<td>6.377</td>
</tr>
<tr>
<td>( bbbb )</td>
<td>19.215</td>
</tr>
<tr>
<td>( bb\bar{c}\bar{c} )</td>
<td>12.847</td>
</tr>
</tbody>
</table>

**Graphical Representation:**

- Considerable \( 8_c \otimes 8_c \)
- Small \( r_{rms} \)
- Stable spatial extension

**Compact tetraquark states**
\[ J^{PC} = 1^{+-} \text{ and } 2^{++} \]

- Only one color configuration: \( \bar{3}_c \otimes 3_c \)

- The mass spectra of \( cc\bar{c}\bar{c}, bb\bar{b}\bar{b} \) and \( bb\bar{c}\bar{c} \) states

<table>
<thead>
<tr>
<th></th>
<th>Model I</th>
<th>( nS )</th>
<th>( J^{PC} = 1^{+-} )</th>
<th>( J^{PC} = 2^{++} )</th>
<th>Model II</th>
<th>( nS )</th>
<th>( J^{PC} = 1^{+-} )</th>
<th>( J^{PC} = 2^{++} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( cc\bar{c}\bar{c} )</td>
<td>( \beta_a = 0.4 )</td>
<td>1S</td>
<td>6.425</td>
<td>6.432</td>
<td>( \beta_a = 0.5 )</td>
<td>1S</td>
<td>6.450</td>
<td>6.479</td>
</tr>
<tr>
<td></td>
<td>( \beta_b = 0.4 )</td>
<td>2S</td>
<td>6.856</td>
<td>6.864</td>
<td>( \beta_b = 0.5 )</td>
<td>2S</td>
<td>6.894</td>
<td>6.919</td>
</tr>
<tr>
<td></td>
<td>( \beta = 0.6 )</td>
<td>3S</td>
<td>6.915</td>
<td>6.919</td>
<td>( \beta = 0.6 )</td>
<td>3S</td>
<td>7.036</td>
<td>7.058</td>
</tr>
<tr>
<td>( bb\bar{b}\bar{b} )</td>
<td>( \beta_a = 0.7 )</td>
<td>1S</td>
<td>19.247</td>
<td>19.249</td>
<td>( \beta_a = 1.0 )</td>
<td>1S</td>
<td>19.311</td>
<td>19.325</td>
</tr>
<tr>
<td></td>
<td>( \beta_b = 0.7 )</td>
<td>2S</td>
<td>19.594</td>
<td>19.596</td>
<td>( \beta_b = 1.0 )</td>
<td>2S</td>
<td>19.813</td>
<td>19.823</td>
</tr>
<tr>
<td></td>
<td>( \beta = 0.9 )</td>
<td>3S</td>
<td>19.681</td>
<td>19.682</td>
<td>( \beta = 1.1 )</td>
<td>3S</td>
<td>20.065</td>
<td>20.077</td>
</tr>
<tr>
<td>( bb\bar{c}\bar{c} )</td>
<td>( \beta_a = 0.7 )</td>
<td>1S</td>
<td>12.864</td>
<td>12.868</td>
<td>( \beta_a = 0.7 )</td>
<td>1S</td>
<td>12.924</td>
<td>12.940</td>
</tr>
<tr>
<td></td>
<td>( \beta_b = 0.5 )</td>
<td>2S</td>
<td>13.259</td>
<td>13.262</td>
<td>( \beta_b = 0.5 )</td>
<td>2S</td>
<td>13.321</td>
<td>13.334</td>
</tr>
<tr>
<td></td>
<td>( \beta = 0.7 )</td>
<td>3S</td>
<td>13.297</td>
<td>13.299</td>
<td>( \beta = 0.7 )</td>
<td>3S</td>
<td>13.364</td>
<td>13.375</td>
</tr>
</tbody>
</table>

- Results in two quark models are similar.

- The mass difference of two states arises from the hyperfine potential.
Numerical results

- The lowest $0^{++}$ states are located about 300 ~ 450 MeV above the lowest scattering state.
- No bound states exist in the two quark models.
\(m_q\)-dependence

- The constituent quark mass dependence of the tetraquark spectra

\[0^{++} QQ\bar{Q}\bar{Q}\]

\(m_Q\) [GeV]

Energy [GeV]

(a) The mass spectra of the tetraquark states \(QQ\bar{Q}\bar{Q}\) with \(J^{PC} = 0^{++}\).

\[0^{++} QQ\bar{Q}\bar{Q}\]

\(M_{QQ\bar{Q}\bar{Q}} - M_{\eta_Q\bar{\eta}_Q}\) [GeV]

(b) The mass difference between the tetraquark states and the mass threshold of \(\eta_Q\bar{\eta}_Q\).

- \(M(QQ\bar{Q}\bar{Q}) > M(\eta_Q\bar{\eta}_Q)\): no bound tetraquark states exist.
Summary and Outlook

➢ The mass spectra of tetraquark states $QQ\bar{Q}'\bar{Q}'$ in two quark models,

• $6_c \otimes \bar{6}_c$ is important even dominant in the ground state.

• Only the hyperfine potential contributes to the mixing between different color configurations.

• No bound $cc\bar{c}\bar{c}$, $bb\bar{b}\bar{b}$, and $bb\bar{c}\bar{c}$ (or $cc\bar{b}\bar{b}$) states exist in the two quark models.

➢ The extension to the $Q_1 Q_2 \bar{Q}_3 \bar{Q}_4$ state.

➢ The confinement mechanism for multi-quark system need more investigation.

➢ The existence of the tetraquark resonances with narrow decay width.
Thank you for your attention!
Backup slides
Wave function

- The Jacobi coordinates transfer as

\[
    r_{jk} = r_j - r_k = r + c_{jk}^a r_{12} + c_{jk}^b r_{34},
\]

\[
    r = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} - \frac{m_3 r_3 + m_4 r_4}{m_3 + m_4},
\]

\[
    r' = \frac{(m_1 m_3 - m_2 m_4) r + M_T u_{12} r_{12} - M_T u_{34} r_{34}}{(m_1 + m_4)(m_2 + m_3)},
\]

\[
    r'' = \frac{(m_1 m_4 - m_2 m_3) r + M_T u_{12} r_{12} - M_T u_{34} r_{34}}{(m_1 + m_3)(m_2 + m_4)},
\]

- Use the first coordinate configuration.

FIG. 1. The Jacobi coordinates in the tetraquark state.
Numerical results

- The dependence of the mass spectra on the number of the expanding base.

\[ N^3 = 2^3 \]

**FIG. 2.** The dependence of the mass spectrum on the number of Gaussian basis \( N^3 \). The line and dashed line represent the numerical results in model I and model II, respectively.
TABLE III. The coefficient $c_{ij}$.

<table>
<thead>
<tr>
<th>$c_{14}^a$</th>
<th>$c_{13}^a$</th>
<th>$c_{23}^a$</th>
<th>$c_{24}^a$</th>
<th>$c_{14}^b$</th>
<th>$c_{13}^b$</th>
<th>$c_{23}^b$</th>
<th>$c_{24}^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{m_2}{m_1+m_2}$</td>
<td>$\frac{m_2}{m_1+m_2}$</td>
<td>$-\frac{m_1}{m_1+m_2}$</td>
<td>$-\frac{m_1}{m_1+m_2}$</td>
<td>$\frac{m_3}{m_3+m_4}$</td>
<td>$-\frac{m_4}{m_3+m_4}$</td>
<td>$-\frac{m_4}{m_3+m_4}$</td>
<td>$\frac{m_3}{m_3+m_4}$</td>
</tr>
</tbody>
</table>

TABLE IV. The configurations of the diquark (antiquark) constrained by Pauli principle. “S” and “A” represent symmetry and antisymmetry.

<table>
<thead>
<tr>
<th>$J^P = 1^+$</th>
<th>$QQ$</th>
<th>$J^P = 0^+$</th>
<th>$QQ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-wave(L=0)</td>
<td>S</td>
<td>S-wave(L=0)</td>
<td>S</td>
</tr>
<tr>
<td>Flavor</td>
<td>S</td>
<td>Flavor</td>
<td>S</td>
</tr>
<tr>
<td>Spin(S=1)</td>
<td>S</td>
<td>Spin(S=0)</td>
<td>A</td>
</tr>
<tr>
<td>Color($\bar{3}_c$)</td>
<td>A</td>
<td>Color($6_c$)</td>
<td>S</td>
</tr>
</tbody>
</table>
FIG. 1: The dependence of the root mean square radius $\sqrt{\langle r_{12} \rangle}$ ($\sqrt{\langle r_{34} \rangle}$) and $\sqrt{\langle r \rangle}$ on the extension of the wave function.

$$
\rho(r) = \int |\psi(r_{12}, r_{34}, r)|^2 d\vec{r}_{12} d\vec{r}_{34} d\vec{r}
$$

$$
\rho(r_{12}) = \int |\psi(r_{12}, r_{34}, r)|^2 d\vec{r} d\vec{r}_{34} d\vec{r}_{12}
$$
TABLE VIII. The comparison of the mass spectra of $0^{++}$ $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ from Ref. [48] and our results using the same quark model. In the right table, we remove the constrains on the wave functions used in Ref. [48].

<table>
<thead>
<tr>
<th>$J^{PC} = 0^{++}$</th>
<th>$w = 0.325$</th>
<th>M [GeV]</th>
<th>$3_c \otimes 3_c$</th>
<th>$6_c \otimes \bar{6}_c$</th>
<th>M [GeV]</th>
<th>$3_c \otimes 3_c$</th>
<th>$6_c \otimes \bar{6}_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$cc\bar{c}\bar{c}$</td>
<td>$\beta_a = \beta_b = 0.49$, $\beta = 0.69$</td>
<td>6470</td>
<td>66%</td>
<td>34%</td>
<td>$\beta_a = \beta_b = 0.4$, $\beta = 0.6$</td>
<td>6417</td>
<td>33%</td>
</tr>
<tr>
<td></td>
<td>$\gamma_a = \gamma_b = 0.49$, $\gamma = 0.69$</td>
<td>6559</td>
<td>34%</td>
<td>66%</td>
<td>$\gamma_a = \gamma_b = 0.4$, $\gamma = 0.7$</td>
<td>6509</td>
<td>67%</td>
</tr>
<tr>
<td>$bb\bar{b}\bar{b}$</td>
<td>$\beta_a = \beta_b = 0.88$, $\beta = 1.24$</td>
<td>19268</td>
<td>66%</td>
<td>34%</td>
<td>$\beta_a = \beta_b = 0.7$, $\beta = 0.9$</td>
<td>19226</td>
<td>18%</td>
</tr>
<tr>
<td></td>
<td>$\gamma_a = \gamma_b = 0.88$, $\gamma = 1.24$</td>
<td>19306</td>
<td>34%</td>
<td>66%</td>
<td>$\gamma_a = \gamma_b = 0.7$, $\gamma = 0.9$</td>
<td>19268</td>
<td>82%</td>
</tr>
</tbody>
</table>