Tri-hadron bound state with heavy flavor

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1 Introduction

2 The delocalized $\pi$ bond in the $DD^*K$ and $D\bar{D}^*K$

3 Numerical results for the Double heavy tri-meson bound states

4 Tri-meson bound state $BBB^*$

5 Application to the $NNN$ system

6 Numerical results for the tri-meson bound state $BBB^*$
The delocalized $\pi$ bond in the $DD^*K$

Figure: Diagrams (a), (b) and (c) are the leading OPE diagrams for the transitions among the relevant three-body channels, i.e. $DD^*K$, $DDK^*$ and $D^*DK$ channels. The TPE diagrams, i.e. (d) and (e), are the next-to-leading order contributions.

Monopole form factor $\mathcal{F}(q) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - q^2}$, with $q$ the four-momentum of the pion and $\Lambda$ the cutoff parameter. $\Lambda_{D^*K}, \Lambda_{DD^*}$. 
Fix \( \Lambda_{D^*K} \) by reproducing the mass of \( D_{s1}(2460) \)

- \( SU(2) \) flavor symmetry. To the order \( \mathcal{O}\left(\frac{p_K}{m_K}\right) \). S-D wave mixing, and the coupled channels \( D^*K \) and \( DK^* \). With \( \Lambda_{D^*K} = 803.2 \text{ MeV} \), we find a \( D^*K \) bound state with mass at \( D_{s1}(2460) \).
- G-parity rule, i.e., \( V_{A\bar{B}} = (-1)^I G V_{AB} \). \( V_{D^*K} = V_{\bar{D}^*K} \).
- \( B^*\bar{K} \) with the mass \( 5772 \text{ MeV} \). \( \Lambda_{B^*\bar{K}} = 1451.0 \text{ MeV} \).

\[ \text{(b)} \]

Born-Oppenheimer (BO) approximation

- Firstly, we keep the two heavy mesons, i.e. $D$ and $D^*$, at a given fixed location $R$ and study the dynamical behavior of the light kaon.
- Then, we solve Schrödinger equation of the $DD^*$ system with the effective BO potential created from the interaction with the kaon.
- The BO approximation is based on the factorized wave function

$$ |\Psi_T(\vec{R}, \vec{r})\rangle = |\Phi(\vec{R})\psi(\vec{r}_1, \vec{r}_2)\rangle , $$

with the two charmed mesons and the light kaon located at $\pm \vec{R}/2$ and $\vec{r}$. Here, $\vec{r}_1 = \vec{r} + \vec{R}/2$ and $\vec{r}_2 = \vec{r} - \vec{R}/2$ are the coordinates of the kaon relative to the first and second interacting $D^*$. 

The delocalized $\pi$ bond in the $DD^* K$ and $D\bar{D}^* K$
Numerical results for the Double heavy tri-meson bound states

When the distance $R$ is larger than a certain value, the kaon energy of the three-body system equal to the binding energy of the isosinglet $D^*K$ or $B^*\bar{K}$ two-body system. The two-body binding energies are from the calculations $^3$.

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<th>$BB^*\bar{K}$</th>
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<th>$BB^*\bar{K}$</th>
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<td>$4317.92^{+6.13}_{-6.55}$</td>
<td>$11013.65^{+8.68}_{-9.02}$</td>
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</table>

Numerical results for the Double heavy tri-meson bound states

\[ \pi \quad (*) \quad (*) \quad (*) \]

\[ u_{DD}^* S \quad u_{DD}^* S \]

\[
\begin{align*}
\text{(a)} & \quad 7.92 \text{ MeV} \\
\text{(b)} & \quad 1.98 \text{ MeV}
\end{align*}
\]

The red point is the critical point which indicates the lower limit of the required binding energy of the isosinglet \( D^* K \) or \( B^* \bar{K} \) to form a three-body bound state. The vertical dashed lines and bands are the central values and uncertainties of the binding energies of the two-body subsystems from the analysis \(^4\).

Aiming at the $\chi(3872)$, LHCb, Belle and BABAR have collected quite numerous data for $B$ decays in the $J/\psi \pi \pi K$ channel. The existence of the $D\bar{D}^* K$ bound state could be checked from the experimental side by analyzing the current world data on the channels $J/\psi \pi^+ K^0$, $J/\psi \pi^0 K^+$, $J/\psi \pi^0 K^0$, and $J/\psi \pi^- K^+$. 

"Numerical results for the Double heavy tri-meson bound states"
Tri-meson bound state $BBB^*$

The OPEP indicates that there is only one virtual pion exchanged by any two constituents as shown in the following.

Figure: Dynamical illustration of the $BBB^*$ system with a circle describing the delocalized $\pi$ bond inside. Since the three constituents have the same probabilities to be the $B$ and $B^*$, one can rewrite the system as $B_a^{(*)} B_b^{(*)} B_c^{(*)}$. 
The OPE interaction for the $BBB^*$

Under SU(2) chiral symmetry, the OPE interaction is of order $O(p^0)$ for the three-body system.

Figure: The leading order OPE diagrams for the transitions among the relevant three-body channels, i.e. $B_a^*B_bB_c$, $B_aB_b^*B_c$, $B_aB_bB_c^*$, $B_a^*B_b^*B_c$, $B_a^*B_bB_c^*$ and $B_aB_b^*B_c^*$. 
Considering that the particle $b$ and $c$ are static with the separation $r_{bc}$, one can separate the degree of freedom of $a$ from the three-body system.

We assume the distance $r_{bc}$ is a parameter. The mesons $b$ and $c$ are static, and have one-pion interactions with meson $a$, which can be viewed as two static sources.

We explore the dynamics for the meson $a$ in the limit $r_{bc} \to \infty$, and subtract the binding energy for the break-up state which is trivial for the three-body bound state.
Interpolating wave function of meson $a$

\[
\frac{1}{\sqrt{2}} \psi(\vec{r}_{ab}) |B_a^* B_b B_c\rangle + \frac{1}{\sqrt{2}} \psi(\vec{r}_{ab}) |B_a B_b^* B_c\rangle + \psi'(\vec{r}_{ab}) |B_a^* B_b^* B_c\rangle
\]

Pion exchanged between $a$ and $b$.

\[
\frac{1}{\sqrt{2}} \psi(\vec{r}_{ac}) |B_a^* B_b B_c\rangle + \frac{1}{\sqrt{2}} \psi(\vec{r}_{ac}) |B_a B_b B_c^*\rangle + \psi'(\vec{r}_{ac}) |B_a B_b B_c^*\rangle
\]

Pion exchanged between $a$ and $c$.

The final wave function for the meson $a$ could be the superposition of these two components

\[
\psi(\vec{r}_{ab}, \vec{r}_{ac}) = C \left\{ \left[ \frac{1}{\sqrt{2}} \psi(\vec{r}_{ab}) + \frac{1}{\sqrt{2}} \psi(\vec{r}_{ac}) \right] |B_a^* B_b B_c\rangle + \frac{1}{\sqrt{2}} \psi(\vec{r}_{ab}) |B_a B_b^* B_c\rangle \right. \\
+ \frac{1}{\sqrt{2}} \psi(\vec{r}_{ac}) |B_a B_b B_c^*\rangle + \psi'(\vec{r}_{ab}) |B_a^* B_b^* B_c\rangle + \psi'(\vec{r}_{ac}) |B_a^* B_b B_c^*\rangle \right\}
\]

Accordingly, one can obtain the energy eigenvalue of the meson $a$

\[
E_a(\Lambda, \vec{r}_{bc}) = \langle \psi(\vec{r}_{ab}, \vec{r}_{ac}) | H_a | \psi(\vec{r}_{ab}, \vec{r}_{ac}) \rangle
\]
BO potential and Its physical meaning (Intensity of "glue")

We define the BO potential as

\[ V_{BO}(\Lambda, \vec{r}_{bc}) = E_a(\Lambda, \vec{r}_{bc}) - E_2(\Lambda). \]

The BO potential can describe the contribution for the one meson on the dynamics of the two remaining mesons. The meson \( a \) here works like a kind of "glue".

Figure: Here we chose the parameter \( \Lambda = 1440 \) MeV. \( E^{BB*}_{l=1} = -5.08 \) MeV.

\[ \Psi_T = \alpha \Phi(\vec{r}_{bc}) \psi(\vec{r}_{ab}, \vec{r}_{ac}). \]
The configurations of the three-body systems

Figure: Every meson can be considered to be a lighter one and separated from the three-body system. Each of them can generate the "glue" for the remaining mesons.

Figure: (a), (b) and (c) correspond to the wave functions $\psi_a = \Phi(\vec{r}_{bc}) \psi(\vec{r}_{ab}, \vec{r}_{ac})$, $\psi_b = \Phi(\vec{r}_{ac}) \psi(\vec{r}_{ab}, \vec{r}_{bc})$ and $\psi_c = \Phi(\vec{r}_{ab}) \psi(\vec{r}_{bc}, \vec{r}_{ac})$, respectively.
Interpolating wave functions

The basis constitute a configuration space \( \{ \psi_\hat{a}, \psi_\hat{b}, \psi_\hat{c} \} \).

\[
\Psi_T = \alpha \Phi(\vec{r}_{bc})\psi(\vec{r}_{ab}, \vec{r}_{ac}) + \beta \Phi(\vec{r}_{ac})\psi(\vec{r}_{ab}, \vec{r}_{bc}) + \gamma \Phi(\vec{r}_{ab})\psi(\vec{r}_{bc}, \vec{r}_{ac})
\]

\[
= \alpha \psi_\hat{a} + \beta \psi_\hat{b} + \gamma \psi_\hat{c} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix},
\]

Expand \( \Phi(\vec{r}_{bc}) \), \( \Phi(\vec{r}_{ac}) \) and \( \Phi(\vec{r}_{ab}) \) as a set of Laguerre polynomials

\[
\chi_{nl}(r) = \sqrt{\frac{(2\lambda)^{2l+3}n!}{\Gamma(2l+3+n)}}r^l e^{-\lambda r} L_n^{2l+2}(2\lambda r), \quad n = 1, 2, 3...
\]

\[
\psi_\hat{a} = \sum_i \phi_i(\vec{r}_{bc})\psi(\vec{r}_{ab}, \vec{r}_{ac}), \quad \psi_\hat{b} = \sum_i \phi_i(\vec{r}_{ac})\psi(\vec{r}_{ab}, \vec{r}_{bc}), \quad \psi_\hat{c} = \sum_i \phi_i(\vec{r}_{ab})\psi(\vec{r}_{bc}, \vec{r}_{ac}).
\]

Here the subscript \( i \) is the order of \( \hat{i} \) Laguerre polynomials. We define the \( i^{th} \) order of the configuration functions as \( \psi^i_\hat{a} = \phi_i(\vec{r}_{bc})\psi(\vec{r}_{ab}, \vec{r}_{ac}) \), \( \psi^i_\hat{b} = \phi_i(\vec{r}_{ac})\psi(\vec{r}_{ab}, \vec{r}_{bc}) \) and \( \psi^i_\hat{c} = \phi_i(\vec{r}_{ab})\psi(\vec{r}_{bc}, \vec{r}_{ac}) \).
Orthonormalization

We orthonormalize the \( \{ \psi_{\hat{a}}, \psi_{\hat{b}}, \psi_{\hat{c}} \} \) into a new basis \( \{ \tilde{\psi}_{\hat{a}}, \tilde{\psi}_{\hat{b}}, \tilde{\psi}_{\hat{c}} \} \).

\[
\begin{align*}
\tilde{\psi}_{\hat{a}}^i &= \frac{1}{N_i} \left[ (\psi_{\hat{a}}^i + \psi_{\hat{b}}^i + \psi_{\hat{c}}^i) - \sum_i x_{ij} \psi_{\hat{j}}^i \right], \\
\tilde{\psi}_{\hat{b}}^i &= \frac{1}{N_i} \left[ (\psi_{\hat{a}}^i + \psi_{\hat{b}}^i + \psi_{\hat{c}}^i) - \sum_i x_{ij} \psi_{\hat{j}}^i \right], \\
\tilde{\psi}_{\hat{c}}^i &= \frac{1}{N_i} \left[ (\psi_{\hat{a}}^i + \psi_{\hat{b}}^i + \psi_{\hat{c}}^i) - \sum_i x_{ij} \psi_{\hat{j}}^i \right],
\end{align*}
\]

where the \( x_{ij} \) is a parameter matrix which will be determined later. The \( N_i \) are normalization coefficients.

Then the eigenvector for the three-body system \( B_{a}^{(*)} B_{b}^{(*)} B_{c}^{(*)} \) can be written as a vector in the configuration space \( \{ \tilde{\psi}_{\hat{a}}, \tilde{\psi}_{\hat{b}}, \tilde{\psi}_{\hat{c}} \} \). Therefore, we have

\[
\Psi_T = \sum_i \tilde{\alpha}_i \tilde{\psi}_{\hat{a}}^i + \sum_j \tilde{\beta}_j \tilde{\psi}_{\hat{b}}^j + \sum_k \tilde{\gamma}_k \tilde{\psi}_{\hat{c}}^k,
\]
Application to the $N^{NN}$ system (Triton or Helium-3 nucleus)

Figure: Dependence of the reduced three-body binding energy on the binding energy of its two-body subsystem (the deuteron). The result is comparable with the empirical binding energies of the triton (8.48 MeV) and helium-3 (7.80 MeV) nuclei.
Numerical results for the $NNN$ system (Triton or Helium-3 nucleus)

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<th>$\Lambda$(MeV)</th>
<th>$E_2$(MeV)</th>
<th>$E_3$(MeV)</th>
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<th>$V_{BO}(0)$(MeV)</th>
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Table: Bound state solutions for the $NNN$ system with isospin $I_3 = 1/2$. $E_2$ is the energy eigenvalue of its subsystem. $E_3$ is the reduced three-body energy eigenvalue relative to the break-up state of the $NNN$ system. $E_T$ is the total three-body energy eigenvalue relative to the $NNN$ threshold.
Numerical results for the tri-meson bound state $BBB^*$

**Table:** Bound state solutions of the $BBB^*$ with the isospin $I_3 = 3/2$. $\alpha$ and $\beta$ are the probabilities for the components $BBB^*$ and $BB^*B^*$, respectively.

<table>
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<tr>
<th>$E_2$(MeV)</th>
<th>$E_3$(MeV)</th>
<th>$E_T$(MeV)</th>
<th>$V_{BO}(0)$(MeV)</th>
<th>S wave(%)</th>
<th>D wave(%)</th>
<th>$r_{rms}$(fm)</th>
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Numerical results for the tri-meson bound state $BBB^*$

Figure: Here we chose the parameter $\Lambda = 1440$ MeV in (a) and $\Lambda = 1107.7$ MeV in (b) for a better comparison of all the cases, since they have the same two-body binding energy of 5.08 MeV.
Summary

- Based on the attractive force of the isosinglet $D^*K$ and $B^*\bar{K}$ systems and the BOA method, we predict four double heavy tri-meson bound states, i.e. $DD^*K$, $D\bar{D}^*K$, $BB^*\bar{K}$ and $B\bar{B}^*\bar{K}$ bound states.

- Hopefully the future analysis on the B meson decay data and NN collisions may unveil the existence of the tri-meson structures.

- We also predict that a triple heavy tri-meson molecular state for the $BBB^*$ system is probably existent as long as the molecular states of its two-body subsystem $BB^*$ exist.

- In our calculations, we use the Born-Oppenheimer potential method to construct our interpolating wave functions, which can be regarded as a version of the variational principle which always gives an upper limit of the energy of a system.

Thank you very much!