STRONG INTERACTION STABLE HEAVY TETRAQUARKS FROM THE LATTICE

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with

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HADRON 2019, 桂 林, Aug. 16-21, 2019
OUTLINE

- Basic features of doubly heavy tetraquark channels
- Brief review of doubly bottom results
- Variable b mass test of underlying binding picture
- Bottom-charm results
- $I(J^P) = 0(0^+), 0(1^+) \text{ bottom-strange channel results}$
- Conclusions and future work
BASIC FEATURES OF DOUBLY HEAVY TETRAQUARK CHANNELS

- Interactions in localized $qq' \bar{Q}\bar{Q}'$ absent for separated heavy meson pairs
  - $\bar{Q}\bar{Q}'$ 3$_c$ color Coulomb attraction
    - binding proportional to $\bar{Q}\bar{Q}'$ reduced mass $\mu_h$
    - dominant as $\mu_h \to \infty$ ($\Rightarrow$ bound tetraquarks for $Q=Q'$, $m_Q \to \infty$)
    - for s-wave $\bar{Q}\bar{Q}'$: $J_h = 1$ ($Q=Q'$); $J_h = 0, 1$ ($Q \neq Q'$)

- Attractive “good” light $qq'$ diquark interaction in field of heavy 3$_c$ source
  - “good” ($J = 0, F = \frac{3}{3}, C = \frac{3}{3}$) vs "bad" ($J = 1, F = 6, C = \frac{3}{3}$) $qq'$ configuration
  - phenomenological constraints from heavy baryon splittings
    - $\Sigma_b - \Lambda_b = 194$ MeV
    - $\Xi_b' - \Xi_b = 142$ MeV
    - $\Sigma_c - \Lambda_c = 167$ MeV
    - $\Xi_c' - \Xi_c = 109$ MeV
Implications/observations

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- Good ud, & s diquark attraction relative to corresponding spin averages: $\sim 145, 105$ MeV
  $\Rightarrow$ increased attraction with decreased $m_q$

- Coulomb BE $\propto \mu_h$

- $h=c$ vs. $h=b$: residual ($\propto 1/m_h$) light-heavy interactions eat into good diquark attraction with decreasing $m_h$

- $\Rightarrow$ Binding most likely for $J^p=1^+ u\bar{d}b\bar{b}, s\bar{s}b\bar{b}$
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- $\Rightarrow$ Binding most likely for $J^P=1^+ \ u\bar{d} b\bar{b}$, $\ell s\bar{b}\bar{b}$
Lattice studies/results for doubly bottom channels

- Earlier work with Born-Oppenheimer, static b quark potential
  - recent e.g.: Bicudo, Scheunert, Wagner [1612.02758]

- Studies with non-static (NRQCD) b
  - Francis, Hudspith, Lewis, KM: $qq' = ud, \ell s$  
    $n_f = 2+1$, PACS-CS Wilson-clover; Iwasaki gauge  
    $a = 0.091$ fm, $m_{\pi} = 164 \rightarrow 415$ MeV  
    [1607.05214] (FHLFM16)

  - Junnarkar, Mathur, Padmanath: $qq' = ud, \ell s$  
    overlap on $n_f = 2+1+1$ MILC HISQ; one-loop, tadpole-improved Symanzik gauge  
    $a = 0.058, 0.089, 0.121$ fm, $m_{\pi} = 257$ (ud), 189 ($\ell s$) $\rightarrow 688$ MeV (PQ)  
    [1810.12285] (JMP18)

  - Leskovec, Meinel, Pflaumer, Wagner: $qq' = ud$  
    $n_f = 2+1$ RBC/UKQCD DWF; Iwasaki gauge  
    $a = 0.083, 0.111, 0.114$ fm, $m_{\pi} = 139 \rightarrow 431$ MeV  
    [1904.04197] (LMPW19)
LMPW19
Summary of current status for $u\bar{d}b\bar{b}$, $s\bar{s}b\bar{b}$

- $u\bar{d}b\bar{b}$: FHLM16, JMP18, LMPW19 all see sub-\(BB^*\)-threshold $I(J^P) = 0(1^+)$ state
  - Binding in all cases below EM decay threshold $\Rightarrow$ weak decays only
  - All see increased binding with decreasing $m_q$, as per good-light-diquark expectation
  - LMPW19 Lüscher analysis confirms bound state interpretation
  - FHLM16 updates (FHLM+Colquhoun) in progress: larger volumes, more light $m_q$,
    *extended sinks for improved plateaus* (preliminary results: no volume dependence)

- $s\bar{s}b\bar{b}$: FHLM16, JMP18 both see bound $J^P = 1^+$ isodoublet
  - Also below EM decay threshold, weak decays only
  - Less bound than $I(J^P) = 0(1^+)$, as per expected light-quark mass dependence of good
diquark attraction
  - FHML16 updates as for $u\bar{d}b\bar{b}$ $I(J^P) = 0(1^+)$ in progress
Test of color-Coulomb + good-light-diquark binding picture

- FHLM18 [1810.10550] study, $m_\pi = 299$ MeV ensemble
- $qq'b\bar{b}$, $qq'b\bar{b}$ with $m_{b'}$ in range $0.6m_b \to 6.3m_b$ still amenable to use of NRQCD
- Fit to model with expected color Coulomb + good light diquark $m_q$, $m_Q$ dependence
- Suggests $ud\bar{c}\bar{b}$ as next best channel for study
The \( ud\bar{c}b \) (and \( ud\bar{c}\bar{c} \)) channels

- Detecting doubly bottom state exclusive decay modes experimentally challenging: low production rates, low weak exclusive mode BFs, low daughter BFs and experimental efficiencies in sequential decay chains

- Alternate Gershon, Poluetkov [1810.06657] suggestion (inclusive search at LHCb using \( B_c \)'s coming from displaced vertices) still challenging

- Much higher production rates expected for states with one or both \( \bar{b}' \)'s replaced by a \( \bar{c} \), IF such states exist

- Though NRQCD breakdown prevents variable \( b' \) mass study from reaching \( b'=c \), results suggest \( ud\bar{c}\bar{b} \) most likely among bottom-charm, charm-charm channels to support a bound tetraquark state
udcb studies

- **FHL18** [1810.10550] + preliminary updates (CFHLM19); Mathur et al. in progress
- **FHL18**
  - $n_f = 2+1$ PACS-CS ($32^3 \times 64$, $m_\pi = 164, 299, 415$ MeV) as for FHL16 ud$\bar{b}b$, $\ell s\bar{b}b$
  - Charm: Tsukuba RHQ action [Namekawa et al. 1104.4600 tuning]; bottom: NRQCD
  - Also as in FHL16: gauge-fixed wall sources, local sinks, local discrete “meson-meson” (“DB*”, “D*B”), “diquark-antidiquark” operators
    - $I(J^P) = 0(1^+)$ only
- **CFHLM19** (B. Colquhoun + FHL)
  - Expanded local operator set
  - Extended (box) sinks
  - Supplement PACS-CS with new Wilson-clover, Iwasaki gauge ensembles
    - $I(J^P) = 0(0^+)$ in addition to 0(1+)
FHLM18: $I(J^P) = 0(1^+)$ $udc\bar{b}$ results

- No binding for $m_\pi = 415$ MeV
- Evidence for binding $\sim 15 \rightarrow 61$ MeV from $m_\pi = 299, 164$ MeV ensembles

- E.g. $m_\pi=164$ MeV GEVP results for ground, first excited states
- Reduced binding c.f. $ud\bar{b}\bar{b}$, as per expectations
- Improved ground-excited state separation, better ground state plateau, FV study desirable
**CFHLM19 (1):** $udc\bar{b}$, $I(J^P) = 0(1^+)$: old-style WL, larger $V$, $m_{\text{eff}}$ results

- E.g.: with 94 new $48^3 \times 64$, $\kappa = 0.13781$ configs (c.f. FHLM18: 195 PACS-CS $32^3 \times 64$, $\kappa = 0.13781$)

- Wall source, local sink (as in FHLM18): both “meson-meson” and “diquark-antidiquark” operators show good overlap with ground state

- Effective mass plateaus late (also as in FHLM18)
CFHLM19 (2): \(u\bar{d}\bar{c}\bar{b}\), \(I(J^P) = 0(1^+)\) update 1: WL with expanded basis

- \(48^3 \times 64\), \(\kappa = 0.13781\) results (currently: \(~1/2\) FHLM18 statistics, more to come)

- Wall-local setup, as in FHLM18

- Increased local operator basis

- \(4 \times 4\) GEVP: ground state effective mass
c.f. FHLM18 range

* Improvements:
  - Increased operator basis
  - Wall-box set-up

\[
D_{\rho,\nu} = (u^c \gamma c_d) \left[ (\bar{c} \gamma \bar{b} c_b^\gamma) - (\bar{c} \gamma \bar{b} c_b) \right] \\
D_{\lambda,\sigma} = (u^c \gamma c_d) \left[ (\bar{c} \gamma \bar{b} c_b^\gamma) - (\bar{c} \gamma \bar{b} c_b) \right] \\
M_{\rho,\nu} = (\bar{c}_a \gamma \bar{c}_d) (\bar{b}_a \gamma b_d) - (\bar{c}_a \gamma \bar{c}_d) (\bar{b}_a \gamma b_d), \\
M_{\lambda,\sigma} = (\bar{c}_a \gamma \bar{c}_d) (\bar{b}_a \gamma \bar{c}_d) - (\bar{c}_a \gamma \bar{c}_d) (\bar{b}_a \gamma b_d).
\]

\[u\bar{d}\bar{c}\bar{b}, \ I(J^P) = 0(1^+)\]
CFHLM19 (3): $ud\bar{c}\bar{b}$, $I(J^P)=0(1^+)$ update 2: extended sinks

- $48^3 \times 64$, $\kappa=0.13781$ results (currently: ~1/2 FHLM18 statistics, more to come)

- Local sink $\rightarrow$ extended (~0.5 fm) “box” sink for improved ground state overlap

- Increased local operator basis

- $4 \times 4$ GEVP: ground state effective mass c.f. FHLM18 range

- $47(3)$ MeV (stat only) below DB*
CFHLM19 (4): preliminary results \( ud\bar{c}\bar{b} \ I(J^P)=0(0^+) \)

- \( 48^3 \times 64, \kappa=0.13781 \) (more statistics, more \( m_\pi < 200 \) MeV to come)

- 4x4 GEVP, wall-box setup for improved ground state plateau, as for \( I(J^P) = 0(1^+) \)

- 37(3) MeV (stat error only) below DB

- 33(1) MeV below \( I(J^P) = 0(1^+) \)

\( \Rightarrow 0(1^+) \) decay to \( 0(0^+) + \gamma \)
For completeness: the $I(J^P) = 0(1^+) \, ud\bar{c}\bar{c}$ channel

- **HadSpec [1709.01417]**
  - $n_f=2+1$, anisotropic clover + improved Symanzik gauge, $m_\pi = 391$ MeV
  - Large "meson-meson" + tetraquark basis
  - No evidence for $ud\bar{c}\bar{c}$ or $\bar{c}s\bar{c}c$ bound tetraquarks

- **JMP18 [1810.12285]**
  - MILC $n_f=2+1+1$, $m_\pi = 257 \rightarrow 688$ MeV (PQ), 3 lattice spacings
  - Results extrapolated to continuum, physical $m_\pi$: $ud\bar{c}\bar{c}$ bound by 23(11) MeV, no sign of binding for $\bar{c}s\bar{c}c$

**For future investigation**
- Differing HadSpec, JMP18 $ud\bar{c}\bar{c}$ conclusions due to larger HadSpec $m_\pi$ (reduced good light diquark attraction)?
- FV effects on small JMP18 $ud\bar{c}\bar{c}$ binding?
The singly bottom $I(J^P) = 0(1^+), 0(0^+) \; ud\bar{s}\bar{b}$ channels

- Further exploration of light-quark configurations accessible in the field of a heavy color source (good light $ud$ diquark for localized $ud\bar{s}\bar{b}$, but impact of $\bar{s}$?)

- Light-quark configuration not present in ordinary mesons, baryons, hence (unlike doubly heavy channels) no phenomenological constraints from the ordinary hadron spectrum

- Initial $X(5568)$ motivation, binding wrt $B^*K$, $BK$ in some phenomenological models, e.g., Chen & Ping [1806.10505], Huang & Ping [1902.05778], but some model light-quark interactions necessarily unconstrained by fits to ordinary meson and baryon spectra

- Situation unlike that of the doubly heavy sector, where phenomenology constrains the dominant (color Coulomb and light-quark spin-spin) interactions ⇒ success of models in predicting bound doubly heavy states no guarantee of utility for other exotic channels
\( ud\bar{s}b \) results: no binding for either \( I(J^P) = 0(0^+) \) or \( 0(1^+) \)

- Preliminary 3x3 GEVP results with 94 new \( 48^3 \times 64 \), \( \kappa_1 = 0.13781 \) configurations
Agreement on $\bar{3}_F$ of doubly bottom $J^P = 1^+$ tetraquarks bound wrt strong and EM decay thresholds (but challenging experimentally)

Previous evidence for $I(J^P) = 0(1^+) \; ud\bar{c}\bar{b}$ bound below DB* threshold (hence also strong-interaction-stable) significantly strengthened

New results show good evidence for second $ud\bar{c}\bar{b}$ tetraquark, with $I(J^P) = 0(0^+)$
  - bound wrt both DB and $0(1^+)$ tetraquark partner
  - $\Rightarrow$ expect $0(1^+)$ tetraquark decay to $0(0^+)+\gamma$, $0(0^*)$ by weak decay only

Further work needed on doubly charmed states
A COUPLE OF QUALITATIVE OBSERVATIONS

- Absence of binding in doubly heavy analogue $I(J^P) = 0(0^+), 0(1^+) \, ud\bar{s}s\bar{b}$ channels, contrary to some model expectations ⇒ caution required when using models in channels where previously untested and/or unconstrained aspects of the models become relevant.

- Similar couplings of nominally “meson-meson”, “diquark-antidiquark” local operators to tetraquark ground states: “meson-meson” coupling does NOT mean these states are meson-meson molecules: caution re interpreting nature of exotic states based solely on the discrete structure of local operators to which they couple (a problem in some sum rule literature).
LMPW19 overlap factors

- $O_1$: local “BB*” operator
- $O_2$: local “B*B*” operator
- $O_3$: local diquark-antidiquark operator
- $O_4$: non-local “BB*” operator
- $O_5$: non-local “B*B*” operator

Normalized to largest overlap

FIG. 5. The normalized overlap factors $|Z|^2$ as determined on ensemble C005, indicating the relative contributions of the energy eigenstates $|n\rangle$ to the trial state $O_\Omega|\Omega\rangle$. The upper row corresponds to a two-exponential fit with $11 \leq t/a \leq 24$, while the lower row corresponds to a three-exponential fit with $10 \leq t/a \leq 24$. 

20/22
FUTURE WORK

- **Forthcoming/desirable**
  - Upcoming (2019) Mathur et al. $I(J^P) = 0(0^+), 0(1^+)$ $u\bar{d}c\bar{b}$ analysis
  - FV-dependence, additional near-physical $m_\pi$ for $u\bar{d}c\bar{c}$ to clarify relation of JMP18 to HadSpec results

- **CFHLM near-term/in progress**
  - Updated $u\bar{d}b\bar{b}$, $u\bar{d}c\bar{b}$ results with $a=0.09$ fm, $\kappa_1 = 0.13777, 0.13779, 0.13781$, $32^3x64$ and $48^3x64$ ensembles
    - $48^3$ vs. $32^3$ for volume dependence studies
    - $\kappa_1 \leftrightarrow m_\pi \lesssim 200$ MeV ⇒ improved physical point extrapolation
    - Wall-box setup (improved ground state plateaus)
    - c.f. FHL16, FHLM18: PACS-CS, $a=0.09$ fm, $\kappa_1 = 0.13754, 0.13770, 0.13781$, $32^3x64$ ensembles, $m_\pi = 164, 299, 415$ MeV
    - c.f. CFHLM19 results: so far, 94 configs, $a=0.09$ fm, $\kappa_1 = 0.13781$, $48^3x64$ only (c.f. 195 for near-physical-point $a=0.09$ fm, $\kappa_1 = 0.13781$, $32^3x64$ PACS-CS ensemble used in FHL16, FHLM18)
Current status CFHLM $32^3 \times 64$ and $48^3 \times 64$ configurations

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Stay tuned for more
谢谢

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BACKUP SLIDES
FIG. 4. Results for the lowest two $\bar{bb}ud$ energy levels relative to the $BB^*$ threshold, $\Delta E_n = E_n - E_B - E_{B^*}$, as determined on ensemble C005 from several different fits. The five bars below each column indicate the interpolators used, as explained in the main text. Above each column, we give the number of exponentials, the fit range, and the value of $\chi^2/\text{d.o.f.}$. The shaded horizontal bands correspond to our final estimates of $\Delta E_0$ and $\Delta E_1$, obtained from a bootstrap average of the subset of fits that are shown with filled symbols.