Nature of the \( Y(4260) \): a light quark perspective

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The nature of $Y(4260)$ has remained controversial since its discovery in 2005.

- **hybrid state**: $c\bar{c}g$
  

- **excited charmonium**
  
  [F. J. Llanes-Estrada, PRD’2005; B.-Q. Li, K.-T. Chao, PRD’2009]

- **hadrocharmonium**
  
  [S. Dubynskiy, M. B. Voloshin, PLB’2008; X. Li, M. B. Voloshin, MPLA’2014]

- **tetraquark state**
  

- **hadronic molecule of $\bar{D}D_1(2420)$ or $\omega\chi_{c0}$**
  

- **interference effect**
  
If the $Y(4260)$ contains no light quarks (as hybrid state or charmonium), the light-quark source provided by the $Y(4260)$ has to be an SU(3) singlet state.

- The $\pi\pi$ invariant mass distribution in the $e^+e^- \rightarrow Y(4260) \rightarrow J/\psi\pi^+\pi^-$ process, at $\sqrt{s} = 4.23$ GeV (left) and $\sqrt{s} = 4.26$ GeV (right). BESIII [PRL’2017]

- The ratio of $\sigma(e^+e^- \rightarrow J/\psi K^+K^-)/\sigma(e^+e^- \rightarrow J/\psi\pi^+\pi^-)$. BESIII [PRD’2018]
Our strategy:

- dispersion theory, based on unitarity, analyticity and crossing symmetry ⇒ account for the $\pi\pi$ rescattering and the $K\bar{K}$ coupled channel in the $S$-wave in a model-independent way

- consider the effects of the $Z_c(3900)$ and the triangle diagrams, which provide the left-hand-cut contribution

- the subtraction constants are obtained by matching the dispersive amplitudes to the heavy quark chiral effective theory
\[ |Y(4260)\rangle = a |V_1\rangle + b |V_8\rangle \]

**SU(3) singlet:** \[ |V_1\rangle \equiv V_1^{\text{light}} \otimes V_8^{\text{heavy}} = \frac{1}{\sqrt{3}} (u \bar{u} + d \bar{d} + s \bar{s}) \otimes V_8^{\text{heavy}} \]

**SU(3) octet:** \[ |V_8\rangle \equiv V_8^{\text{light}} \otimes V_8^{\text{heavy}} = \frac{1}{\sqrt{6}} (u \bar{u} + d \bar{d} - 2s \bar{s}) \otimes V_8^{\text{heavy}} \]

\[ \mathcal{L}_{Y\psi \Phi \Phi} = g_1 \langle V_1^\alpha J_\alpha^\dagger \rangle \langle u_\mu u^\mu \rangle + h_1 \langle V_1^\alpha J_\alpha^\dagger \rangle \langle u_\mu u_\nu \rangle v^\mu v^\nu + g_8 \langle J_\alpha^\dagger \rangle \langle V_8^\alpha u_\mu u^\mu \rangle + h_8 \langle J_\alpha^\dagger \rangle \langle V_8^\alpha u_\mu u_\nu \rangle v^\mu v^\nu + \text{h.c.} \] (1)
No strange partner of the $Z_c$ states, thus the SU(3) singlet and octet components of the $Y(4260)$ are not distinguishable in the $Z_c Y(4260)\pi$ interaction.

\begin{align*}
\mathcal{L}_{Z_c Y\pi} &= C_{Z_c Y\pi} Y^i \langle Z_c^i u_{\mu} \rangle v^\mu + \text{h.c.}, \\
\mathcal{L}_{Z_c \psi\pi} &= C_{Z_c \psi\pi} \psi^i \langle Z_c^i u_{\mu} \rangle v^\mu + \text{h.c.}, \\
\mathcal{L}_{Y D_1 D} &= \frac{y}{\sqrt{2}} Y^i \left( \bar{D}_a^i D_{1a}^i - \bar{D}_{1a}^i D_a^i \right) + \text{h.c.}, \\
\mathcal{L}_{D_1 D^* P} &= \frac{i h'}{F} \left[ 3 D_{1a}^i (\partial^i \Phi_{ab}) D_{b}^* j^j - D_{1a}^i (\partial^j \Phi_{ab}) D_{b}^* i^i \right] + \text{h.c.}, \\
\mathcal{L}_{\psi D^* D P} &= \frac{g_{\psi P}}{2} \langle \psi \bar{H}_a^i H_b^j \rangle u_{ab}^0, 
\end{align*}
• elastic unitarity (single channel, no left-hand cut)

\[
\frac{1}{2i} \text{disc } F_l(s) = \text{Im } F_l(s) = F_l(s) \sin \delta^I_l(s) e^{-i\delta^I_l(s)}.
\]  

(7)

Watson’s theorem: phase of \( F_l(s) \) is just \( \delta^I_l(s) \), the elastic \( \pi\pi \) phase shift

• traditional solution to this homogeneous integral equation [Omnès, Nuovo Cim’1958]

\[
F_l(s) = P_n(s) \Omega^I_l(s), \quad \Omega^I_l(s) = \exp \left\{ \frac{s}{\pi} \int_{4m^2_\pi}^{\infty} \frac{dx}{x} \frac{\delta^I_l(x)}{x - s} \right\}.
\]  

(8)

\( P_n(s) \): polynomial; \( \Omega^I_l(s) \): Omnès function
Dispersion theory

- modified Omnès solution (right-hand and left-hand cuts are separated)

\[ \text{Im} \, M_l(s) = \left[ M_l(s) + \hat{M}_l(s) \right] \sin \delta^I_l(s) e^{-i\delta^I_l(s)}. \] (9)

- \( M_l(s) \): right-hand-cut contributions
- \( \hat{M}_l(s) \): left-hand-cut contributions, approximated by \( Z_c \)-exchange + triangle diagrams

\[ M_l(s) = \Omega^I_l(s) \left\{ P^{n-1}_l(s) + \frac{s^n}{\pi} \int_{4m^2/\pi}^{\infty} \frac{dx \hat{M}_l(x) \sin \delta^I_l(x)}{x^n |\Omega^I_l(x)|(x - s)} \right\}, \] (10)

- \( P^{n-1}_l(s) \): subtraction polynomial determined by matching to heavy quark chiral effective theory
Dispersion theory

- **two-channel unitarity conditions**

\[ \text{Im } M_0(s) = 2iT_0^0*(s)\Sigma(s) \left[ M_0(s) + \hat{M}_0(s) \right], \quad (11) \]

\[ M_0(s) = \left( \begin{array}{c} M_0^\pi(s) \\ \frac{2}{\sqrt{3}} M_0^K(s) \end{array} \right), \quad \hat{M}_0(s) = \left( \begin{array}{c} \hat{M}_0^\pi(s) \\ \frac{2}{\sqrt{3}} \hat{M}_0^K(s) \end{array} \right). \quad (12) \]

\[ T_0^0(s) = \left( \begin{array}{cc} \eta_0^0(s)e^{2i\delta_0^0(s)-1} & |g_0^0(s)|e^{i\psi_0^0(s)} \\ |g_0^0(s)|e^{i\psi_0^0(s)} & \eta_0^0(s)e^{2i(\psi_0^0(s)-\delta_0^0(s))-1} \end{array} \right) \quad (13) \]

\[ \Sigma(s) = \text{diag}(\sigma_\pi(s)\theta(s - 4m_\pi^2), \sigma_K(s)\theta(s - 4m_K^2)). \quad (14) \]

- \( \delta_0^0(s) \): \( \pi\pi \) S-wave isoscalar phase shift
- \( |g_0^0(s)|, \psi_0^0(s) \): modulus and phase of \( \pi\pi \to K\bar{K} \) S-wave amplitude
- \( \eta_0^0(s) \): inelasticity, \( = \sqrt{1 - 4\sigma_\pi(s)\sigma_K(s)|g_0^0(s)|^2\theta(s - 4m_K^2)} \)

Two different \( T_0^0(s) \) matrices will be used: the Dai–Pennington (DP) and the Bern/Orsay (BO) parametrizations.

\[ M_0(s) = \Omega(s) \left\{ M_0^\chi(s) + \frac{s^3}{\pi} \int_{4m_\pi^2}^\infty dx \frac{\Omega^{-1}(x)T(x)\Sigma(x)\hat{M}_0(x)}{x^3} \right\}, \quad (15) \]
• \( Y(4260) \rightarrow J/\psi \pi^+ \pi^- \): the crossed-channel exchanged \( Z_c \) and \( DD^* \) can be on-shell, the left-hand cut intersects and overlaps with the right-hand cut.

![Diagram](image)

• **Solution:** using the spectral representation of the resonance propagator, and application of \( q^2 \rightarrow q^2 + i\epsilon \).

\[
\widetilde{BW}_R(x) = \frac{1}{\pi} \int_{x^{thr}}^{\infty} dx' \frac{\text{Im}[BW_R(x')]}{x' - x},
\]

(16)

where \( BW_R(x') = (M_R^2 - x' - iM_R\Gamma_R(x'))^{-1} \).
• Shapes of the $\pi\pi$ mass spectra contributed from SU(3) singlet (left) and octet (right) contact terms using DP (top) or BO (bottom) parametrizations in $e^+e^- \to Y(4260) \to J/\psi \pi^+\pi^-$. $h_i/g_i \ (i = 1, 8)$ fixed at 0.1, 0.3, 1, 3, and 10, respectively.

**Singlet:** a bump below 1 GeV; around 1 GeV a dip for $h_1/g_1 \lesssim 1$

**Octet:** little contribution below 0.9 GeV; a sharp peak around 1 GeV.
Phenomenological discussion

- **Fitting to the BESIII data**

Fit results of the $\pi\pi$ mass spectra in $e^+e^- \rightarrow J/\psi\pi^+\pi^-$. 

$E = 4.23$ GeV:
- **Fit Ia (top left):** Only SU(3) singlet
- **Fit Ib (top right):** SU(3) singlet + octet

$E = 4.26$ GeV:
- **Fit IIa (bottom left):** Only SU(3) singlet
- **Fit IIb (bottom right):** SU(3) singlet + octet

<table>
<thead>
<tr>
<th>$\frac{\sigma(J/\psi K^+ K^-)}{\sigma(J/\psi \pi^+ \pi^-)} \times 10^2$, $E = 4.23$ GeV</th>
<th>Experiment</th>
<th>Fit Ia, DP</th>
<th>Fit Ib, DP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$6.44 \pm 1.15$</td>
<td>$7.82 \pm 0.83$</td>
<td>$7.75 \pm 1.10$</td>
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<th>$\frac{\sigma(J/\psi K^+ K^-)}{\sigma(J/\psi \pi^+ \pi^-)} \times 10^2$, $E = 4.26$ GeV</th>
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<tr>
<td></td>
<td>$4.99 \pm 1.10$</td>
<td>$4.46 \pm 0.82$</td>
<td>$4.67 \pm 0.98$</td>
</tr>
</tbody>
</table>
Table 1: Fit parameters using the DP \( T \)-matrix parametrization.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fit Ia, DP</th>
<th>Fit Ib, DP</th>
<th>Fit IIa, DP</th>
<th>Fit IIb, DP</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_1 ) ( [GeV^{-1}] )</td>
<td>(-0.29 \pm 0.04)</td>
<td>(1.87 \pm 0.13)</td>
<td>(0.21 \pm 0.04)</td>
<td>(-0.99 \pm 0.11)</td>
</tr>
<tr>
<td>( h_1 ) ( [GeV^{-1}] )</td>
<td>(-0.29 \pm 0.02)</td>
<td>(-0.31 \pm 0.06)</td>
<td>(-0.32 \pm 0.02)</td>
<td>(0.03 \pm 0.04)</td>
</tr>
<tr>
<td>( g_8 ) ( [GeV^{-1}] )</td>
<td>(0 ) (fixed)</td>
<td>(1.25 \pm 0.11)</td>
<td>(0 ) (fixed)</td>
<td>(-1.18 \pm 0.03)</td>
</tr>
<tr>
<td>( h_8 ) ( [GeV^{-1}] )</td>
<td>(0 ) (fixed)</td>
<td>(-1.96 \pm 0.10)</td>
<td>(0 ) (fixed)</td>
<td>(1.70 \pm 0.18)</td>
</tr>
<tr>
<td>( C_{Zc}^\Psi \times 10^2 )</td>
<td>(0.7 \pm 0.6)</td>
<td>(2.0 \pm 0.8)</td>
<td>(4.6 \pm 0.3)</td>
<td>(6.9 \pm 0.3)</td>
</tr>
<tr>
<td>( C_{Y\Psi}^{loop} ) ( [GeV^{-3}] )</td>
<td>(4.5 \pm 1.0)</td>
<td>(38.8 \pm 2.5)</td>
<td>(12.5 \pm 0.8)</td>
<td>(-19.4 \pm 2.1)</td>
</tr>
</tbody>
</table>

\[ \chi^2/d.o.f. \]

\[
\begin{array}{c}
\frac{405.1}{44-4} = 10.13 \\
\frac{102.1}{44-6} = 2.69 \\
\frac{182.7}{46-4} = 4.35 \\
\frac{63.9}{46-6} = 1.60
\end{array}
\]

- Ratio of the SU(3) octet component relative to the SU(3) singlet component:
  - In the \( \bar{D}D_1 \) hadronic molecule scenario of \( Y(4260) \): \( 1/\sqrt{2} \)
    since \( |Y(4260)\rangle = \frac{1}{2} \left[ |D_1^0\bar{D}^0\rangle + |D_1^+D^-\rangle \right] + c.c. \), from which the light-quark component \( |u\bar{u} + d\bar{d}|/\sqrt{2} = (\sqrt{2}V_{1}^{\text{light}} + V_{8}^{\text{light}})/\sqrt{3} \).
  - Our results in Fit IIb, DP: \( g_8/g_1 = 1.2 \pm 0.2 \) and \( h_8/h_1 = 57 \pm 76 \)
Assuming the strengths of the light-quark components from the $\bar{D}D_1$ hadronic molecule and the other SU(3) singlet source, e.g., from $|c\bar{c}\rangle$ or a hybrid, are $\alpha$ and $\beta$, respectively,

$$\frac{\alpha}{\sqrt{3}}\left(\sqrt{2}V_{1}^{\text{light}} + V_{8}^{\text{light}}\right) + \beta V_{1}^{\text{light}},$$

we can estimate the ratio of $\beta/\alpha = -0.30 \pm 0.05$ based on our results of $g_8/g_1$.

The $\bar{D}D_1$ component of the $Y(4260)$ may not be completely dominant.

$$M_{Y(4260)} - M_D - M_{D_1} \simeq (4220 - 1870 - 2420) \text{ MeV} \simeq -70 \text{ MeV}$$
• Moduli of the $S$- (left) and $D$-wave (right) amplitudes for $e^+e^- \rightarrow J/\psi \pi^+\pi^-$. 

$D$-wave contribution is comparable to the $S$-wave contribution

– $Y(4260)$ cannot be a conventional charmonium state, for which the $\pi\pi$ $S$-wave should be dominant
– In the $\bar{D}D_1$ hadronic molecule interpretation, the $\pi\pi$ $D$-wave emerges naturally since the $D_1$ decays dominantly into $D$-wave $D^*\pi$

Conclusions

• Dispersion theory can consider pion-pion final-state interaction in a model-independent way.

• $Y(4260)$ contains a large light-quark component, thus it is in all likelihood neither a hybrid nor a conventional charmonium state.

• Our findings are consistent with the $Y(4260)$ having a sizeable $\bar{D}D_1$ component which, however, is not completely dominant.
Thanks for your patience