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# Determination of resonance properties from lattice energy levels using chiral EFT


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# Outline:

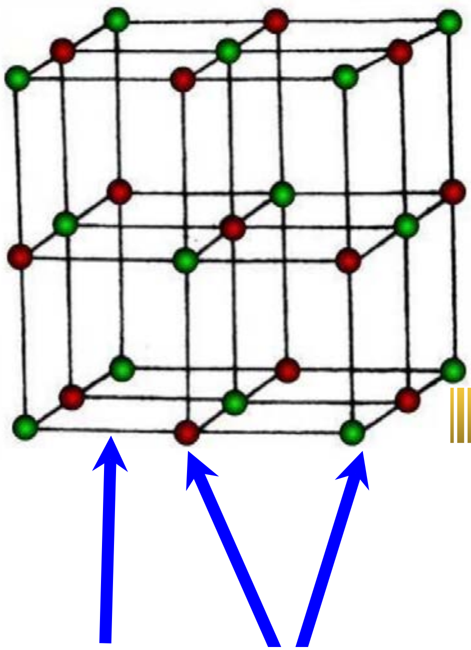
1. Background & Introduction

2. ChPT amplitudes and Finite-volume effects

3. Results and Discussions

4. Summary

# Background & Introduction

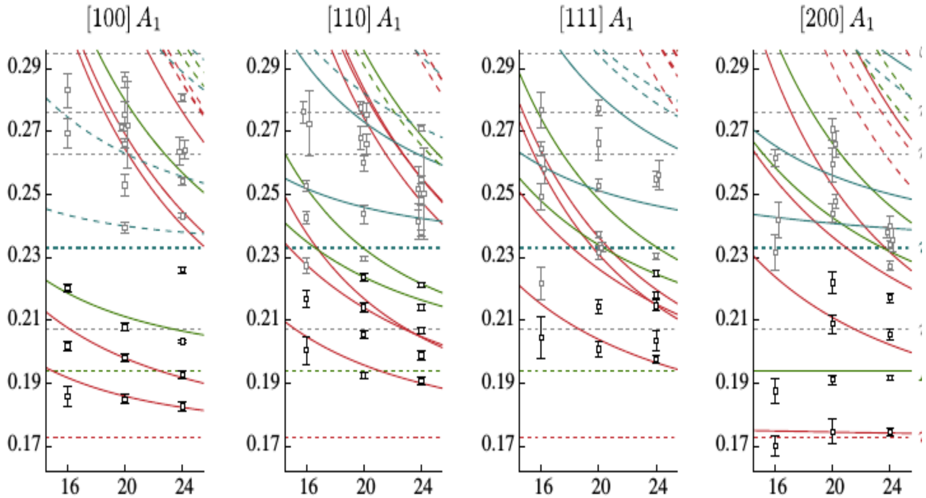


Gluon Quark

Significant progress on meson-meson scattering in lattice simulation

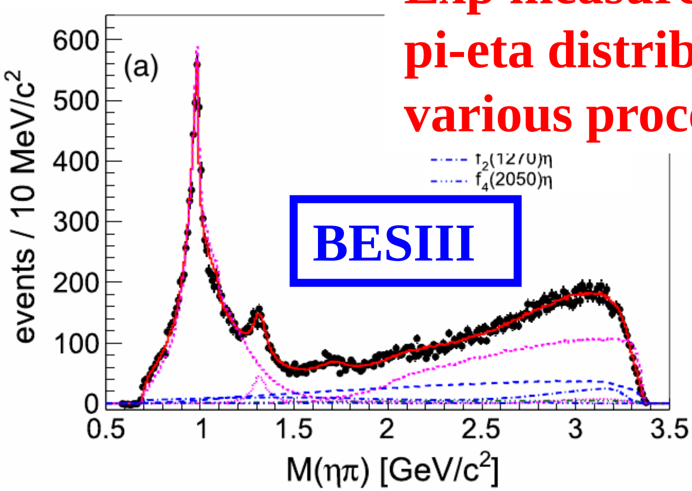
E.g. Pi-eta, KKbar, Pi-eta' scattering

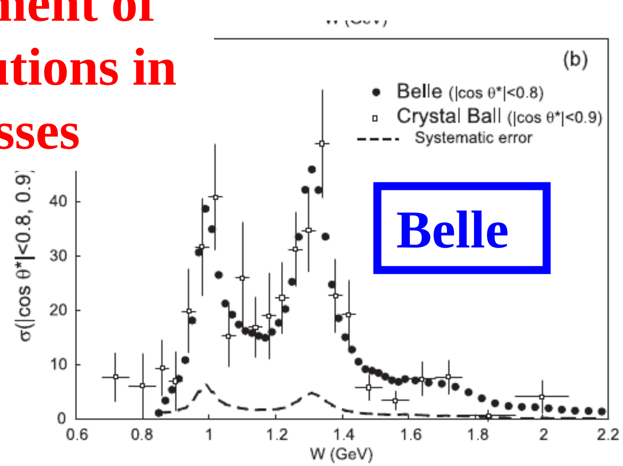
[Dudek,Edwards,Wilson, PRD'16]




Data "measured" by lattice: finite-volume energy levels

Exp measurement of pi-eta distributions in various processes

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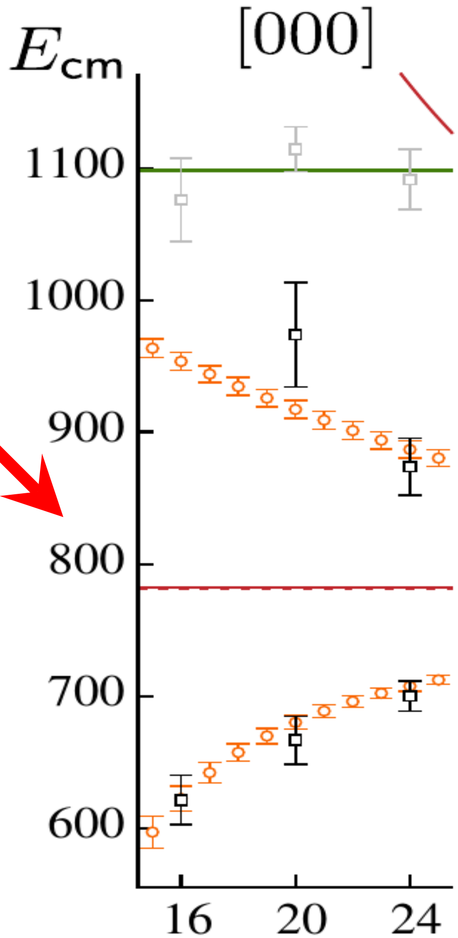
# Lattice simulation data:

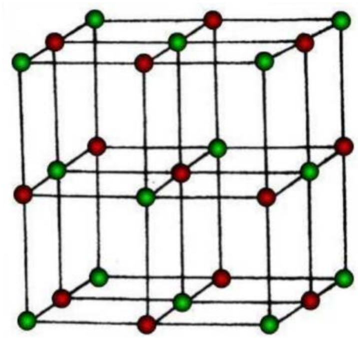
# finite-volume spectra in meson-meson scattering

**Eigenenergies in finite box**




**An example:**
**Isoscalar S-wave pi-pi scattering**

[Briceno,Dudek,Edwards,Wilson, PRL'17]



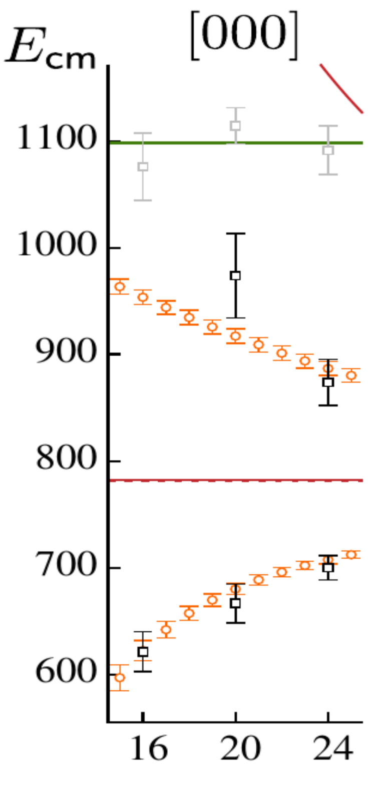
**Length of finite box**

**Luscher's Approach:**

**connect the discrete spectra in finite box to the scattering amplitudes in the infinite volume**

[Luscher, NPB '91]

**Elastic scattering case:** $p \cot \delta(p) = \frac{2\pi}{L} \pi^{-3/2} \mathcal{Z}_{00}(1, \hat{p}^2)$




**Phase shifts**

$\hat{p} = \frac{pL}{2\pi}$

**Luscher's Zeta function**
**( function of *L*, parameter free)**

- **For the elastic case, one has the one-to-one correspondance between the phase shifts and energy levels.**
- **The one-to-one correspondance will be lost in the inelastic case.**

[He,Feng,Liu,JHEP'05] [Wilson,Briceno,Dudek,Edwards,Thomas, PRD '15]
[Lang,Leskovec,Mohler,Prelovsek,Woloshyn, PRD'14] [Fu,PRD'12]
[Gockeler,Horsley,Lage,Meissner,Rakow,Rusetksy,Schierholz,Zanotti,PRD'12] ......

A widely used approach in the inelastic scattering case:

K matrix + Luscher function

$$\det\left[K^{-1}(E) + \mathcal{M}(E,\vec{p},L)\right] = 0$$

| K matrix: polynomial + possible pole terms | Include Luscher's functions (complex objects) |
|---|---|

- **Free parameters in K matrix are determined by the finite-volume spectra. Then one can determine amplitudes in infinite volume.**
- **K matrix does not automatically respect the QCD symmetries, such as the chiral symmetry.**

# Our approach:

**Step 1:** **Put chiral perturbation theory (ChPT) in finite volume.**

**Step 2:** **The free parameters in ChPT, which are indepdent of quark masses and volumes, are fitted to the finite-volume energy levels obtained at (un)physical quark masses.**

**Step 3:** **Perform the chiral extrapolation and give the predictions in infinite volume with physical quark masses, including phase shifts, inelasticities, resonance poles, etc.**

# Two topics focused in this talk:

- **pi-eta, K-Kbar, pi-eta' coupled-channel scattering : a0(980)**
- **D-pi, D-eta, Ds-Kbar coupled-channel scattering : D*0(2400)**

  **D-K, Ds-eta scattering: D*s0(2317)**

# Unitarized ChPT
# and its finite-volume effects

# Three relevant coupled channels: πη, K-Kbar, πη'

**In this case, it is essential to generalize from SU(3) to U(3) ChPT**

$$\begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta_8+\frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta_8+\frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \bar{K}^0 & \frac{-2}{\sqrt{6}}\eta_8+\frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}$$

## Leading order:

$$\mathcal{L}_2 = \frac{F^2}{4}\langle u_\mu u^\mu \rangle + \frac{F^2}{4}\langle \chi_+ \rangle + \boxed{\frac{F^2}{3} M_0^2 \ln^2 \det u}$$

**Leads to a massive $\eta_0$**

$$\begin{aligned} \eta_8 &= c_\theta \overline{\eta} + s_\theta \overline{\eta}' \,, \\ \eta_0 &= -s_\theta \overline{\eta} + c_\theta \overline{\eta}' \,, \end{aligned}$$

$$\sin\theta = -\left( \sqrt{1 + \frac{\left(3M_0^2 - 2\Delta^2 + \sqrt{9M_0^4 - 12M_0^2\Delta^2 + 36\Delta^4}\right)^2}{32\Delta^4}} \right)^{-1} \qquad \Delta^2 = \overline{m}_K^2 - \overline{m}_\pi^2$$

**Instead of using higher order local counterterms, resonance saturations are assumed in our study.**

$$\mathcal{L}_S = c_d \langle\, S_8 u_\mu u^\mu \,\rangle + c_m \langle\, S_8 \chi_+ \,\rangle + \widetilde{c}_d S_1 \langle\, u_\mu u^\mu \,\rangle + \widetilde{c}_m S_1 \langle\, \chi_+ \,\rangle$$

$$\mathcal{L}_V = \frac{iG_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle \,,$$

[Ecker, Gasser, Pich, de Rafael, NPB'89]

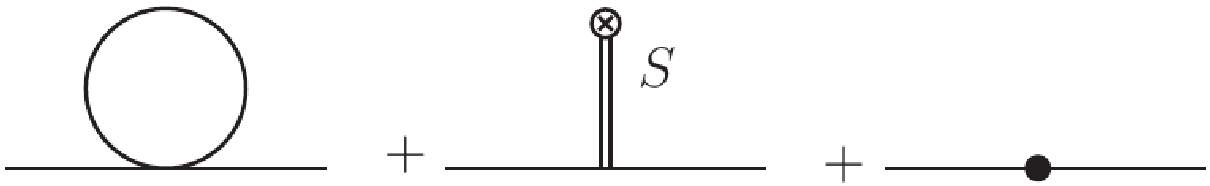
**One important local U(3) operator is also considered:**

$$-\Lambda_2 \frac{F^2}{12} \langle\, U^+ \chi - \chi^+ U \,\rangle \ln \det u^2$$

# Meson-meson scattering: πη → πη, πη → KKbar, πη → πη' ......

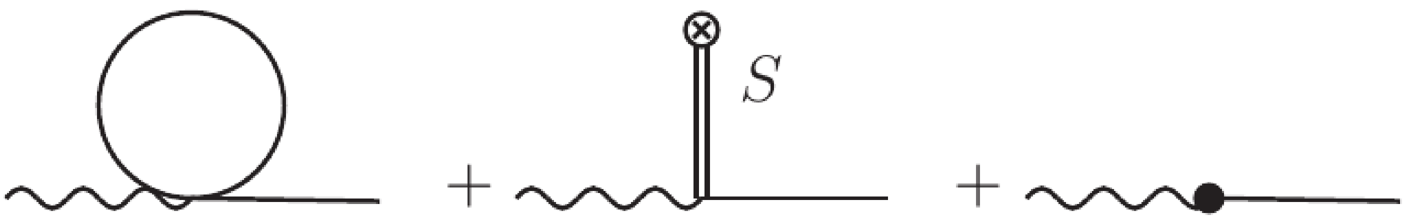
[Guo,Oller,PRD '11] [Guo,Oller,Ruiz de Elvira, PLB '12, PRD'12]

*Self energy :*



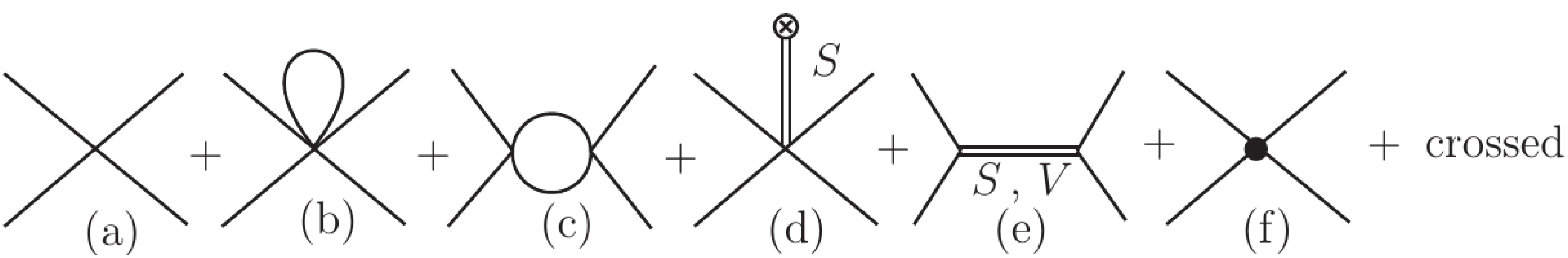

$m_\pi$, $Z_\pi$, ...

*Goldstone decay constant :*




$F_\pi$

*Scattering amplitude :*



# Heavy-light meson ChPT

## LO ChPT with heavy-light mesons

$$\mathcal{L}^{(1)}_{\mathcal{P}\phi} = \mathcal{D}_\mu \mathcal{P} \mathcal{D}^\mu \mathcal{P}^\dagger - \overline{M}_D^2 \mathcal{P}\mathcal{P}^\dagger$$

## NLO ChPT with heavy-light mesons [F.K.Guo,et al.,PLB'08]

$$\mathcal{L}^{(2)}_{\mathcal{P}\phi} = \mathcal{P}\Big( -h_0\langle\chi_+\rangle - h_1\chi_+ + h_2\langle u_\mu u^\mu\rangle - h_3 u_\mu u^\mu \Big)\mathcal{P}^\dagger$$
$$+\mathcal{D}_\mu\mathcal{P}\Big( h_4\langle u_\mu u^\nu\rangle - h_5\{u^\mu, u^\nu\}\Big)\mathcal{D}_\nu\mathcal{P}^\dagger$$

**with** $u_\mu = i\left(u^\dagger\partial_\mu u - u\,\partial_\mu u^\dagger\right)$ $\quad u = \exp\left(\frac{i\phi}{\sqrt{2}F_0}\right)$ $\quad \chi_\pm = u^\dagger\chi u^\dagger \pm u\chi u$

$$\phi = \begin{pmatrix} \frac{\sqrt{3}\pi^0+\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\sqrt{3}\pi^0+\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & \frac{-2\eta_8}{\sqrt{6}} \end{pmatrix}$$

# $D_{(s)}$ and light pseudoscalar meson scattering amplitudes

$$V^{(S,I)}_{D_1\phi_1\to D_2\phi_2}(s,t,u) =$$

$$\frac{1}{F_\pi^2}\Big[\frac{C_{\rm LO}}{4}(s-u) - 4C_0 h_0 + 2C_1 h_1$$

$$-2C_{24}H_{24}(s,t,u) + 2C_{35}H_{35}(s,t,u)\Big]$$

| $(S,I)$ | Channels | $C_{\rm LO}$ | $C_0$ | $C_1$ | $C_{24}$ | $C_{35}$ |
|---|---|---|---|---|---|---|
| $(-1,0)$ | $D\bar{K}\to D\bar{K}$ | $-1$ | $m_K^2$ | $m_K^2$ | 1 | $-1$ |
| $(-1,1)$ | $D\bar{K}\to D\bar{K}$ | 1 | $m_K^2$ | $-m_K^2$ | 1 | 1 |
| $(2,\frac{1}{2})$ | $D_sK\to D_sK$ | 1 | $m_K^2$ | $-m_K^2$ | 1 | 1 |
| $(0,\frac{3}{2})$ | $D\pi\to D\pi$ | 1 | $m_\pi^2$ | $-m_\pi^2$ | 1 | 1 |
| $(1,1)$ | $D_s\pi\to D_s\pi$ | 0 | $m_\pi^2$ | 0 | 1 | 0 |
| | $DK\to DK$ | 0 | $m_K^2$ | 0 | 1 | 0 |
| | $DK\to D_s\pi$ | 1 | 0 | $-(m_K^2+m_\pi^2)/2$ | 0 | 1 |
| $(1,0)$ | $DK\to DK$ | $-2$ | $m_K^2$ | $-2m_K^2$ | 1 | 2 |
| | $DK\to D_s\eta$ | $-\sqrt{3}$ | 0 | $\frac{-5m_K^2+3m_\pi^2}{2\sqrt{3}}$ | 0 | $\frac{1}{\sqrt{3}}$ |
| | $D_s\eta\to D_s\eta$ | 0 | $\frac{4m_K^2-m_\pi^2}{3}$ | $\frac{4(m_\pi^2-2m_K^2)}{3}$ | 1 | $\frac{4}{3}$ |
| $(0,\frac{1}{2})$ | $D\pi\to D\pi$ | $-2$ | $m_\pi^2$ | $-m_\pi^2$ | 1 | 1 |
| | $D\eta\to D\eta$ | 0 | $\frac{4m_K^2-m_\pi^2}{3}$ | $\frac{-m_\pi^2}{3}$ | 1 | $\frac{1}{3}$ |
| | $D_s\bar{K}\to D_s\bar{K}$ | $-1$ | $m_K^2$ | $-m_K^2$ | 1 | 1 |
| | $D\eta\to D\pi$ | 0 | 0 | $-m_\pi^2$ | 0 | 1 |
| | $D_s\bar{K}\to D\pi$ | $-\frac{\sqrt{6}}{2}$ | 0 | $\frac{-\sqrt{6}(m_K^2+m_\pi^2)}{4}$ | 0 | $\frac{\sqrt{6}}{2}$ |
| | $D_s\bar{K}\to D\eta$ | $-\frac{\sqrt{6}}{2}$ | 0 | $\frac{5m_K^2-3m_\pi^2}{2\sqrt{6}}$ | 0 | $\frac{-1}{\sqrt{6}}$ |

# Unitarization: Algebraic approximation of N/D (a variant version of K-matrix) [Oller,Oset, PRD '99]

$$T_J(s) = \frac{N(s)}{1 + G(s)\ N(s)}$$

- **The s-channel unitariy is exact. The crossed-channel dyanmics is included in a perturbative manner.**

- **Unitarity condition:** $\mathrm{Im}G(s) = -\rho(s)$

$$G(s) = \boxed{a^{SL}(s_0)} - \frac{s - s_0}{\pi} \int_{4m^2}^{\infty} \frac{\rho(s')}{(s'-s)(s'-s_0)} ds'$$

- ***N*(*s*): given by the partial wave chiral amplitudes**

$$\mathcal{V}^{(S,I)}_{J,D_1\phi_1\to D_2\phi_2}(s) = \frac{1}{2}\int_{-1}^{+1} \mathrm{d}\cos\varphi P_J(\cos\varphi) V^{(S,I)}_{D_1\phi_1\to D_2\phi_2}(s, t(s,\cos\varphi))$$

# Finite-volume effects

**Two types of finite volume dependence of scattering amplitudes:**

- **Exponentially suppressed type ∝ *exp*(-$m_p$L) : *s, t, u* channels**
- **Power suppressed type ∝ $1/L^3$ : only *s* channel**

We ignore the exponentially suppressed terms, indicating that finite-volume effects only enter through s channel.

$$T_J(s) = \frac{N(s)}{1 + G(s)\, N(s)}$$

**I.e. We only consider the finite-volume corrections for G(s).**

$$G(s) = i\int \frac{\mathrm{d}^4 q}{(2\pi)^4}\frac{1}{(q^2-m_1^2+i\epsilon)[(P-q)^2-m_2^2+i\epsilon]}\,, \qquad s\equiv P^2$$

## Sharp momentum cutoff to regularize G(s)

$$G(s)^{\rm cutoff} = \int^{|\vec{q}|<q_{\rm max}} \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}\, I(|\vec{q}|)\,, \quad \begin{aligned} I(|\vec{q}|) &= \frac{w_1+w_2}{2w_1w_2\,[E^2-(w_1+w_2)^2]}\,, \\ w_i &= \sqrt{|\vec{q}|^2+m_i^2}\,, \quad s=E^2 \end{aligned}$$

## G(s) in a finite box of length L with periodic boundary condition

$$\widetilde{G} = \frac{1}{L^3}\sum_{\vec{n}}^{|\vec{q}|<q_{\rm max}} I(|\vec{q}|)\,, \qquad \vec{q} = \frac{2\pi}{L}\vec{n}, \qquad \vec{n}\in\mathbb{Z}^3$$

## Finite-volume correction ΔG

[Doring, Meissner, Oset, Rusetsky, EPJA11]

$$\Delta G = \widetilde{G} - G^{\rm cutoff}$$

$$= \left\{ \frac{1}{L^3}\sum_{\boldsymbol{q}}^{|\boldsymbol{q}|<q_{\rm max}} - \int^{|\boldsymbol{q}|<q_{\rm max}} \frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3} \right\} \frac{1}{2\omega_1(\boldsymbol{q})\,\omega_2(\boldsymbol{q})}\, \frac{\omega_1(\boldsymbol{q})+\omega_2(\boldsymbol{q})}{E^2-(\omega_1(\boldsymbol{q})+\omega_2(\boldsymbol{q}))^2}$$

# Finite-volume effects in the moving frames

**Lorentz invariance is lost in finite box. One needs to work out the explicit form of the loops when boosting from one frame to another.**

transforming $\vec{q}_{i=1,2}$ to $\vec{q}^{\,*}_{i=1,2}$ → **CM quantities**

$$\vec{q}_i^{\,*} = \vec{q}_i + \left[\left(\frac{P^0}{E} - 1\right)\frac{\vec{q}_i \cdot \vec{P}}{|\vec{P}|^2} - \frac{q_i^0}{E}\right]\vec{P}$$

moving frame with total four-momentum $P^\mu = (P^0, \vec{P})$ $\quad s = E^2 = (P^0)^2 - |\vec{P}|^2$

**Impose on-shell condition** $\quad q_i^{*\,0} = \sqrt{|\vec{q}_i^{\,*}|^2 + m_i^2}$

$$q_i^0 = \frac{q_i^{*\,0} E + \vec{q}_i \cdot \vec{P}}{P^0} \longrightarrow q_i^0 = \sqrt{|\vec{q}_i|^2 + m_i^2}$$

**G function in the moving frame**

$$\int^{|\vec{q}_1|^* < q_{\max}} \frac{d^3\vec{q}_1^{\,*}}{(2\pi)^3} I(|\vec{q}_1^{\,*}|) \Longrightarrow \widetilde{G}^{MV} = \frac{E}{P^0 L^3} \sum_{\vec{q}_1}^{|\vec{q}_1^{\,*}| < q_{\max}} I(|\vec{q}_1^{\,*}(\vec{q}_1)|)$$

$$\vec{q}_1 = \frac{2\pi}{L}\vec{n}, \quad \vec{n} \in \mathbb{Z}^3, \qquad \vec{P} = \frac{2\pi}{L}\vec{N}, \quad \vec{N} \in \mathbb{Z}^3$$

**Finite-volume correction $\Delta G^{MV}$:** $\quad \Delta G^{MV} = \widetilde{G}^{MV} - G^{cutoff}$

[Doring, Meissner, Oset, Rusetsky, EPJA12]

# Mixing of different partial waves in finite volume

**The mixing between different partial waves is absent in the infinite volume:**

$$\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta Y_{\ell m}(\theta,\phi) Y^*_{\ell' m'}(\theta,\phi) = \delta_{\ell\ell'}\delta_{mm'}$$

**The mixing appears in finite-volume case, due to the absence of the general orthogonal conditions of spherical harmonic functions.**

**The mixing patterns vary in different irreducible representations and different moving frames.**

$$\det\left[K^{-1}(E) + \mathcal{M}(E,\vec{p},L)\right] = 0$$

[He,Feng,Liu,JHEP'05] [Wilson,Briceno,Dudek,Edwards,Thomas, PRD '15] [Lang,Leskovec,Mohler,Prelovsek,Woloshyn, PRD'14] [Fu,PRD'12]
[Gockeler,Horsley,Lage,Meissner,Rakow,Rusetksy,Schierholz,Zanotti,PRD'12] ......

**We adapt the following approach to proceed the study of unitarized ChpT in finite volume.**

[Gockeler,Horsley,Lage,Meissner,Rakow,Rusetksy,Schierholz,Zanotti,PRD'12]

**Finite-volume correction to G function:**

$$\Delta G^{\rm MV}_{\ell m} = \widetilde{G}^{\rm MV}_{\ell m} - G^{\rm cutoff}\delta_{\ell\,0}\delta_{m\,0}$$

$$\widetilde{G}^{\rm MV}_{\ell m} = \sqrt{\frac{4\pi}{2\ell+1}}\frac{1}{L^3}\frac{E}{P^0}\sum_{\vec{n}}^{|\vec{q}^*|<q_{\rm max}}\left(\frac{|\vec{q}^*|}{|\vec{q}^{\rm on*}|}\right)^{\ell} Y_{\ell m}(\hat{q}^*)\, I(|\vec{q}^*|)$$

**It is equavelent to the $w_{lm}$ function, up to exponentially suppressed terms** [Gockeler,Horsley,Lage,et al.,PRD'12]

$$w_{lm} = \frac{1}{\pi^{3/2}\sqrt{2l+1}}\gamma^{-1}q^{-l-1}Z^{\Delta}_{lm}(1, q^2)$$

$$Z^{\Delta}_{js}(\delta, q^2) = \sum_{z\in P_\Delta}\frac{\mathcal{Y}_{js}(z)}{(z^2-q^2)^\delta}$$

$$\mathcal{Y}_{lm}(\boldsymbol{r}) = |\boldsymbol{r}|^l Y_{lm}(\hat{\boldsymbol{r}}), \qquad \hat{\boldsymbol{r}} = \frac{\boldsymbol{r}}{|\boldsymbol{r}|}$$

$$\widetilde{G}^{\rm MV}_{\ell m} = -\frac{|\vec{q}^{\,\rm on*}|}{8\pi E}w_{\ell m}$$

**Final expression for the G function:**

$$\widetilde{G}^{\rm MV}_{\ell m} = G^{V\infty}\delta_{\ell 0}\delta_{m0} + \Delta G^{\rm MV}_{\ell m}$$

## To determine the energy levels in different frames with only S and P waves:

**$A_1^+$ (0,0,0) :** $$\det[1 + N_0(s).\widetilde{G}_{00}] = 0$$

**$T_1^-$ (0,0,0):** $$\det[1 + N_1(s).\widetilde{G}_{00}] = 0$$

**$A_1$ (0,0,1):** $$\det[I + N_{0,1} \cdot \mathcal{M}_{0,1}^{A_1}] = 0\,,$$

$$N_{0,1} = \begin{pmatrix} N_0 & 0 \\ 0 & N_1 \end{pmatrix}, \qquad \mathcal{M}_{0,1}^{A_1} = \begin{pmatrix} \widetilde{G}_{00} & i\sqrt{3}\widetilde{G}_{10} \\ -i\sqrt{3}\widetilde{G}_{10} & \widetilde{G}_{00} + 2\widetilde{G}_{20} \end{pmatrix}$$

$$N_{0,1} \cdot \mathcal{M}_{0,1}^{A_1} = \begin{pmatrix} N_{0,11}\widetilde{G}_{00,1} & N_{0,12}\widetilde{G}_{00,2} & N_{0,13}\widetilde{G}_{00,3} & i\sqrt{3}N_{0,11}\widetilde{G}_{10,1} \\ N_{0,21}\widetilde{G}_{00,1} & N_{0,22}\widetilde{G}_{00,2} & N_{0,23}\widetilde{G}_{00,3} & i\sqrt{3}N_{0,21}\widetilde{G}_{10,1} \\ N_{0,31}\widetilde{G}_{00,1} & N_{0,32}\widetilde{G}_{00,2} & N_{0,33}\widetilde{G}_{00,3} & i\sqrt{3}N_{0,31}\widetilde{G}_{10,1} \\ -i\sqrt{3}N_1\widetilde{G}_{00,1} & 0 & 0 & N_1(\widetilde{G}_{00,1} + 2\widetilde{G}_{20,1}) \end{pmatrix}$$

[ZHG,Liu,Meissner,Oller,Rusetsky, EPJC'19]

## Results and Discussions

## for pi-eta, KKbar, pi-eta' scattering

# Fits to lattice finite-volume energy levels

$m_\pi = 391.3\pm0.7$ MeV , $m_K = 549.5\pm0.5$ MeV , $m_\eta = 587.2\pm1.1$ MeV , $m_{\eta'} = 929.8\pm5.7$ MeV

[Dudek,Edwards,Wilson, PRD16]

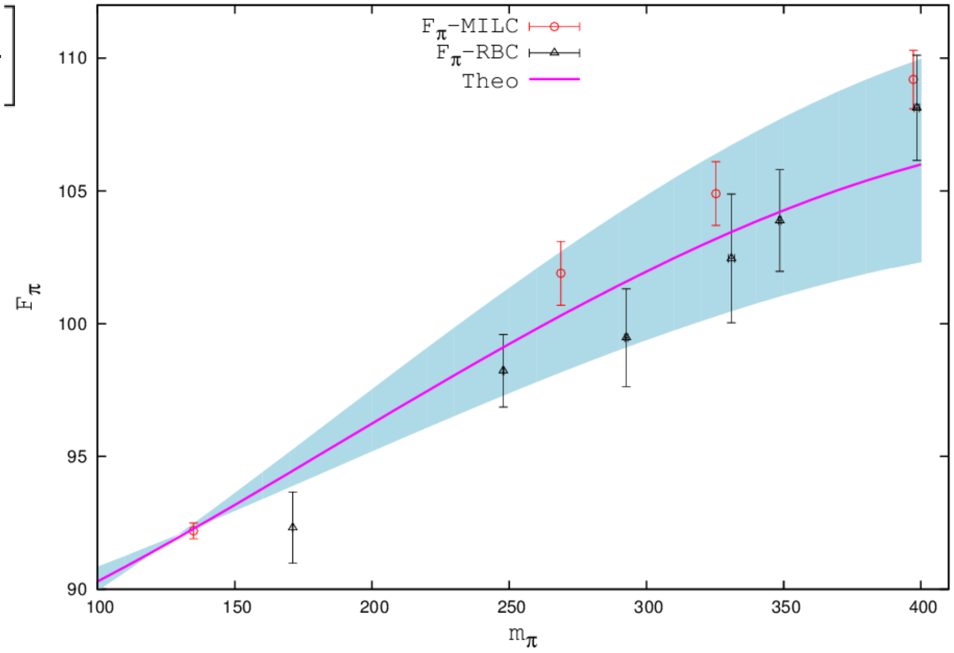
**Our estimate of the leading order η-η' mixing angle at unphysical masses**

$$\theta = (-10.0 \pm 0.1)^\circ \quad ( \theta^{\rm phys} = -16.2^\circ \ )$$

**We also need to estimate $F_\pi$ at the unphysical meson masses.**

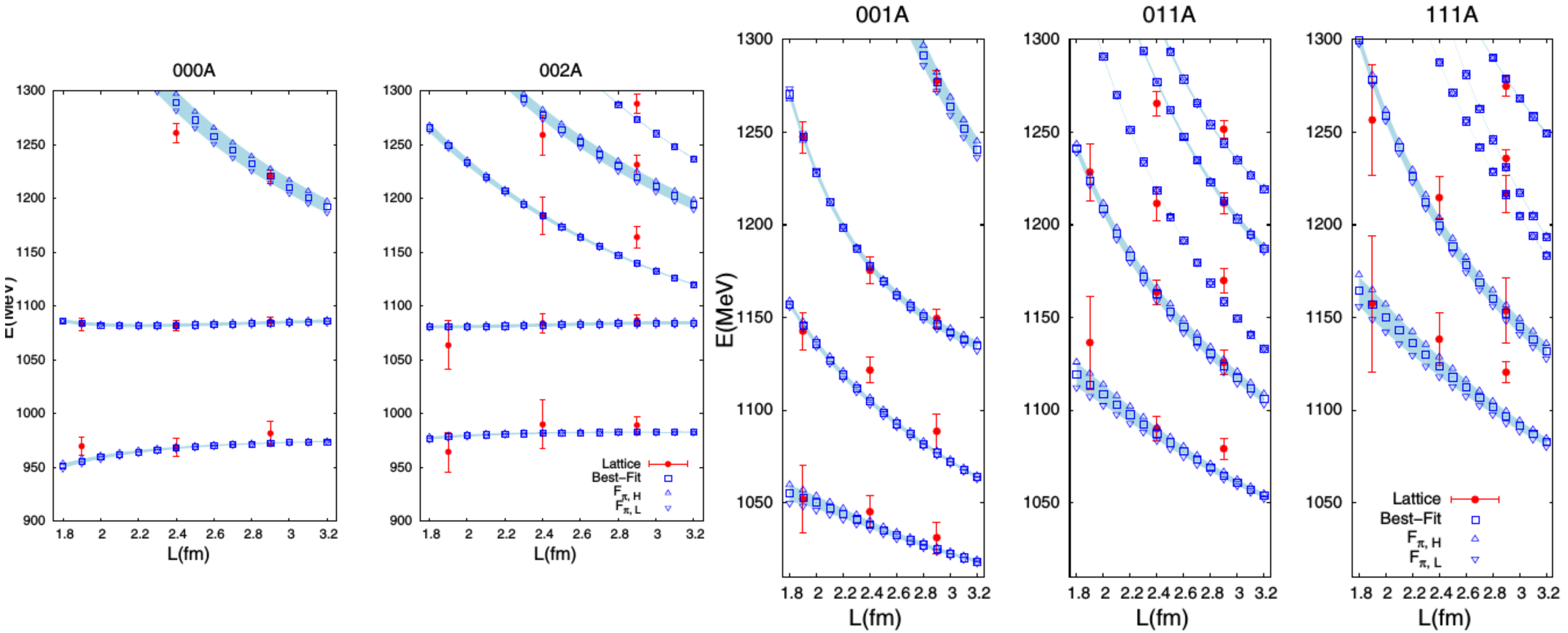
$$F_\pi = F\Big\{1 - \frac{1}{16\pi^2 F^2}\Big[m_\pi^2 \ln\frac{m_\pi^2}{\mu^2} + \frac{m_K^2}{2}\ln\frac{m_K^2}{\mu^2}\Big] + \Big[\frac{4\widetilde{c}_d\,\widetilde{c}_m(m_\pi^2+2m_K^2)}{F^2 M_{S_1}^2} - \frac{8c_d\,c_m\,(m_K^2-m_\pi^2)}{3F^2 M_{S_8}^2}\Big]\Big\}$$

[Guo, Oller, PRD'11]



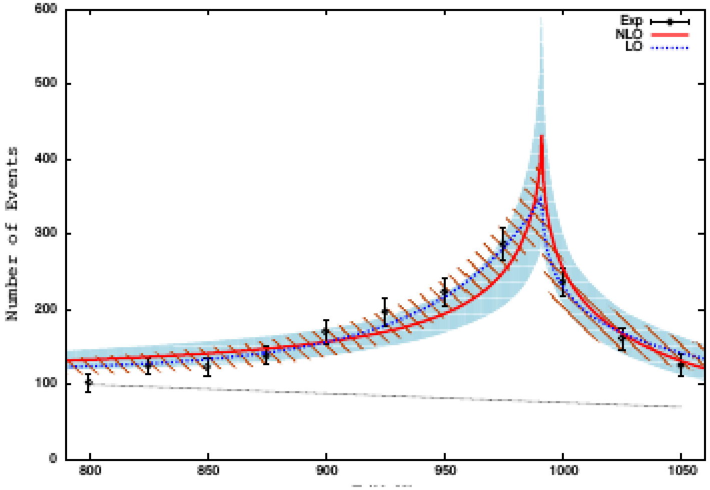
# Leading order Fit (only LO amplitudes are included in the N(s) function.)

[ZHG,Liu,Meissner,Oller,Rusetsky, PRD'17]

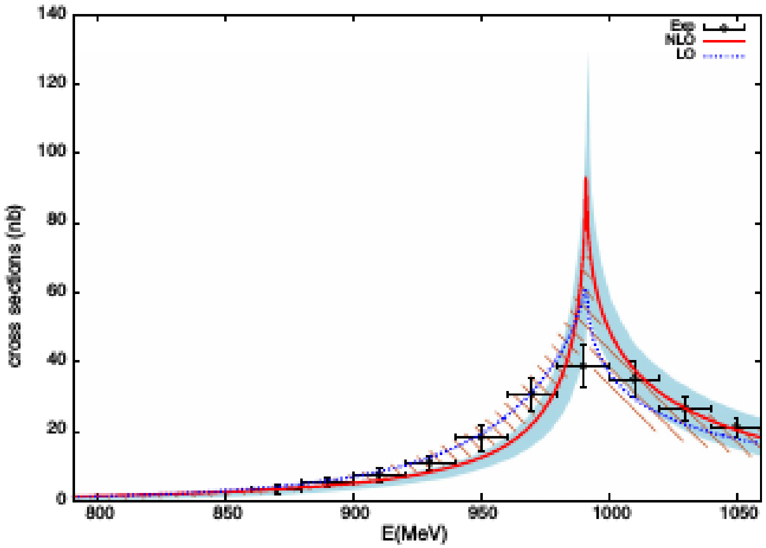


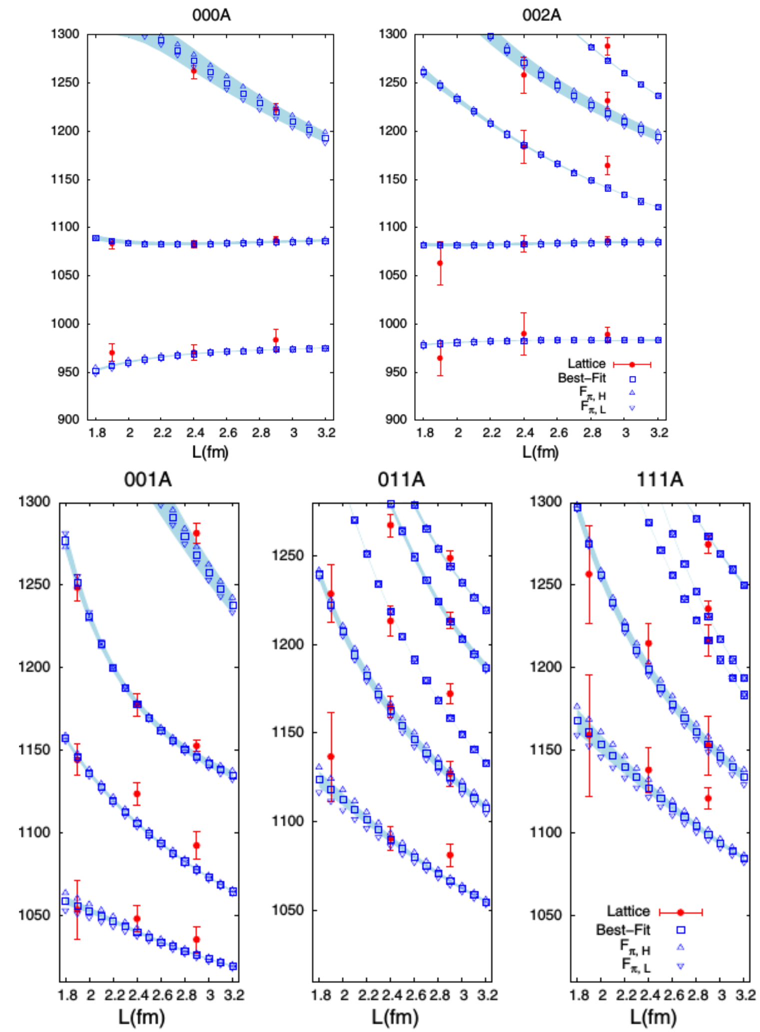

**Remark: there is only one free parameter in the fits, i.e. the common subtraction constant !**

$$\frac{dN_{\pi\eta}}{dE_{\pi\eta}} = q_{\pi\eta}|c_1 D^{-1}(s)_{\pi\eta\to\pi\eta} + c_2 D^{-1}(s)_{\pi\eta\to K\bar{K}}|^2$$




$$\sigma(s) = \frac{\alpha^2 q_{\pi\eta}}{2s^{3/2}}|c_1' D^{-1}(s)_{\pi\eta\to\pi\eta} + c_2' D^{-1}(s)_{\pi\eta\to K\bar{K}}|^2$$



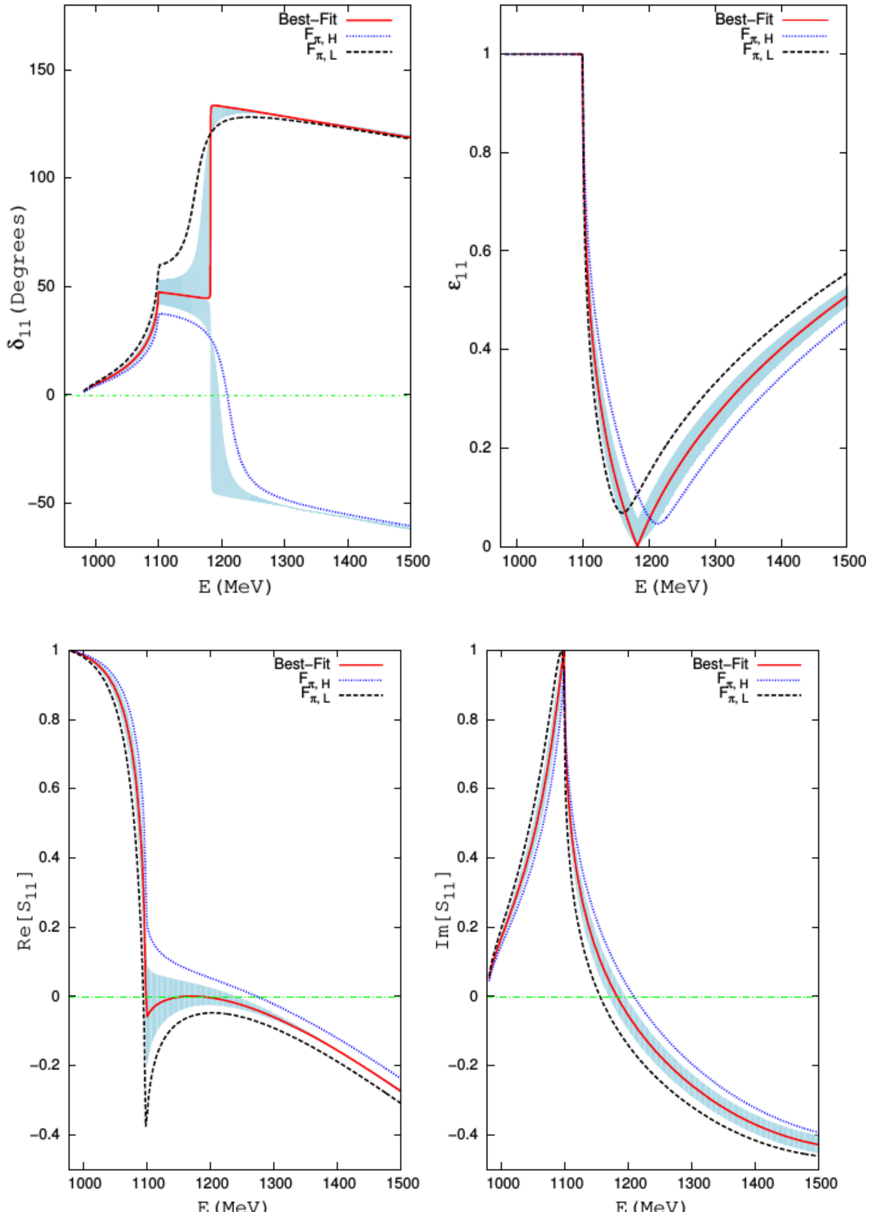



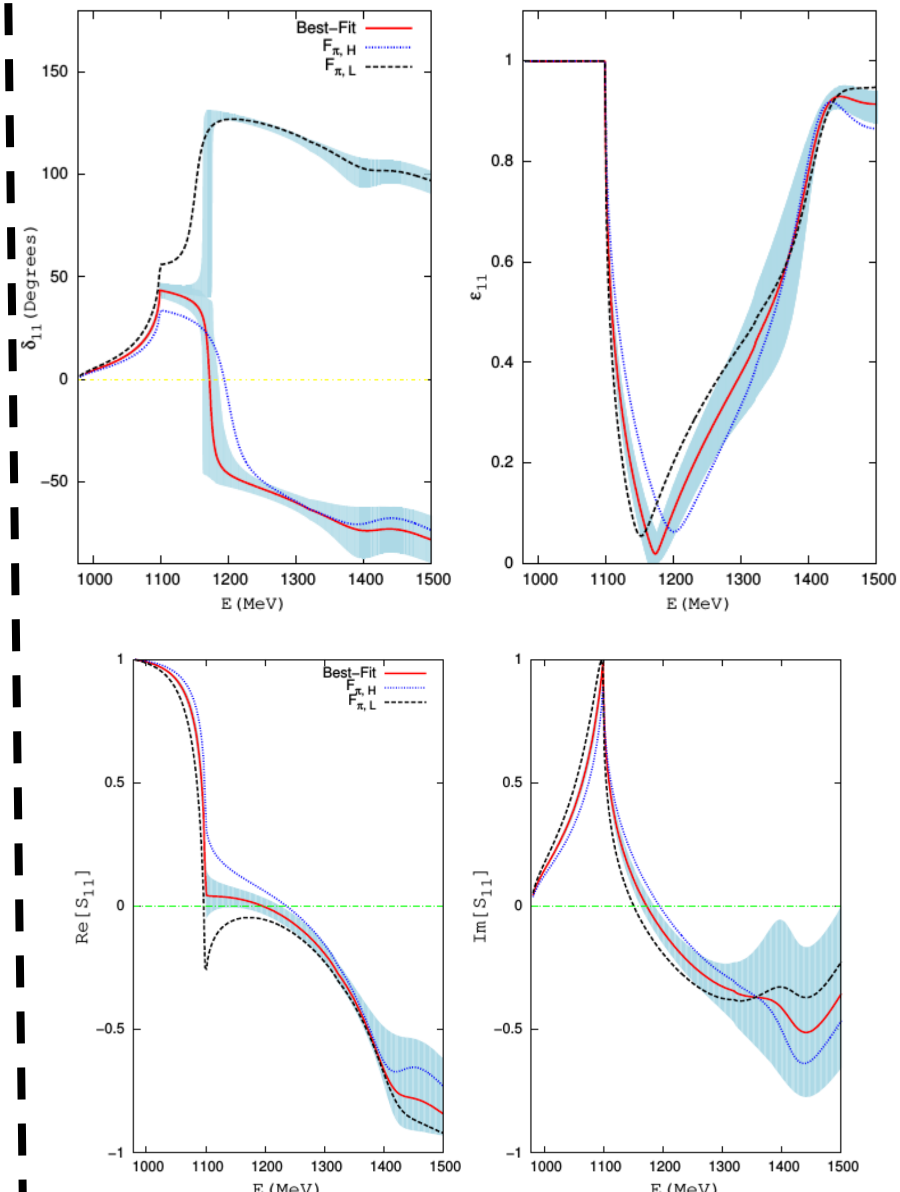

**Next-to-Leading order Fit ( Both loops and resonance exchanges are included in the N(s) function.) Similar fit quality from the two cases.**

# Phase shifts and inelasticities at unphysical meson masses

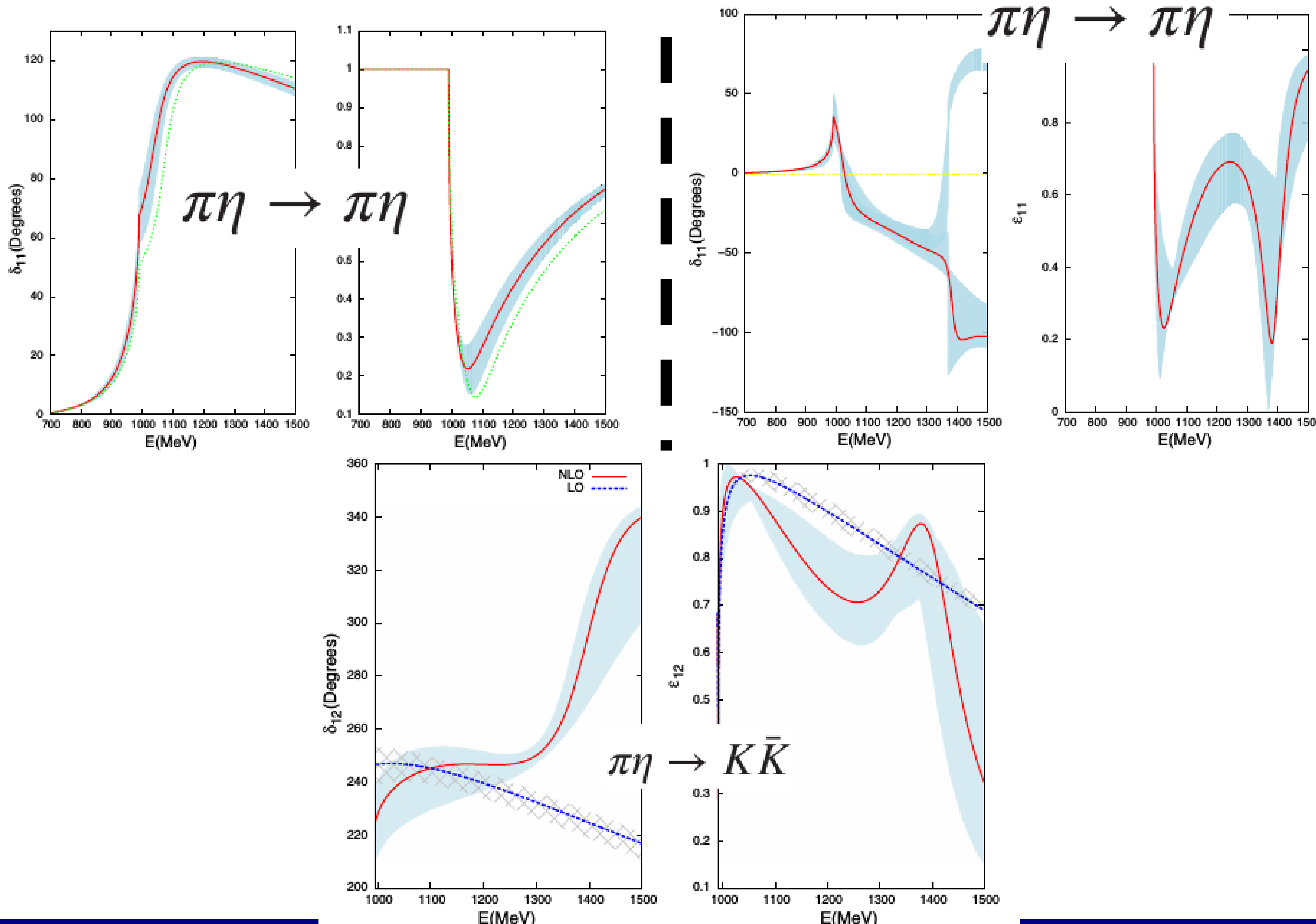
$$S = 1 + 2i\sqrt{\rho(s)} \cdot \mathcal{T}(s) \cdot \sqrt{\rho(s)}$$

$$S_{kk} = \varepsilon_{kk} e^{2i\delta_{kk}}, \qquad S_{kl} = i\varepsilon_{kl} e^{i\delta_{kl}}$$



# Phase shifts and inelasticities at physical meson masses



# Pole positions and residues at physical meson masses

| Resonance | RS | Mass (MeV) | Width/2 (MeV) | $\lvert\text{Residue}\rvert^{1/2}_{\pi\eta}$ (GeV) | Ratios | |
|---|---|---|---|---|---|---|
| LO | | | | | | |
| $a_0(980)$ | II | $1037^{+17}_{-14}$ | $44^{+6}_{-9}$ | $3.8^{+0.3}_{-0.2}$ | $1.43^{+0.03}_{-0.03}$ $(K\bar{K}/\pi\eta)$ | $0.05^{+0.01}_{-0.01}$ $(\pi\eta'/\pi\eta)$ |
| NLO | | | | | | |
| $a_0(980)$ | IV | $1019^{+22}_{-8}$ | $24^{+57}_{-17}$ | $2.8^{+1.4}_{-0.6}$ | $1.8^{+0.1}_{-0.3}$ $(K\bar{K}/\pi\eta)$ | $0.01^{+0.06}_{-0.01}$ $(\pi\eta'/\pi\eta)$ |
| $a_0(1450)$ | V | $1397^{+40}_{-27}$ | $62^{+79}_{-8}$ | $1.7^{+0.3}_{-0.4}$ | $1.4^{+2.4}_{-0.6}$ $(K\bar{K}/\pi\eta)$ | $0.9^{+0.8}_{-0.2}$ $(\pi\eta'/\pi\eta)$ |

[ZHG,Liu,Meissner,Oller,Rusetsky, PRD'17]

# Results for the D-pi, D-eta, Ds-Kbar scattering

# Fits

**Fpi, # data = 56** | **F, # data = 56** | **Fpi, # data = 65** | **F, # data = 65** | **[L.Liu,Orginos, F.K.Guo,Meissner, PRD'13]**

| | Fit-1A | Fit-1B | Fit-2A | Fit-2B | Table V [24] |
|---|---|---|---|---|---|
| $h_{24}$ | $-0.50^{+0.12}_{-0.11}$ | $-0.64^{+0.17}_{-0.11}$ | $-0.42^{+0.18}_{-0.15}$ | $-0.14^{+0.10}_{-0.14}$ | $-0.10^{+0.05}_{-0.06}$ |
| $h'_4$ | $-1.45^{+0.68}_{-0.61}$ | $-1.30^{+0.50}_{-0.68}$ | $-0.49^{+0.23}_{-0.23}$ | $-0.02^{+0.34}_{-0.36}$ | $-0.32^{+0.35}_{-0.34}$ |
| $h_{35}$ | $0.83^{+0.13}_{-0.19}$ | $0.77^{+0.14}_{-0.21}$ | $0.76^{+0.16}_{-0.22}$ | $0.05^{+0.16}_{-0.12}$ | $0.25^{+0.13}_{-0.13}$ |
| $h'_5$ | $0.74^{+0.78}_{-0.68}$ | $0.68^{+0.56}_{-0.51}$ | $-0.49^{+0.17}_{-0.17}$ | $-0.81^{+0.33}_{-0.32}$ | $-1.88^{+0.63}_{-0.61}$ |
| $a^{0,1/2}_{D\pi}$ | $-1.73^{+0.21}_{-0.19}$ | $-1.45^{+0.19}_{-0.14}$ | $-2.00^{+0.13}_{-0.12}$ | $-1.52^{+0.07}_{-0.06}$ | $-1.88^{+0.07}_{-0.09}{}^{*}$ |
| $a^{0,1/2}_{D\eta}$ | $-2.68^{+0.21}_{-0.19}$ | $-2.53^{+0.24}_{-0.25}$ | $-2.43^{+0.21}_{-0.24}$ | $-2.02^{+0.08}_{-0.10}$ | $-1.88^{+0.07}_{-0.09}{}^{*}$ |
| $a^{1,0}_{DK}$ | $-1.58^{+0.17}_{-0.22}$ | $-1.62^{+0.16}_{-0.18}$ | $-1.86^{+0.18}_{-0.27}$ | $-1.60^{+0.11}_{-0.17}$ | $-1.88^{+0.07}_{-0.09}{}^{*}$ |
| $a_{EC}$ | $-2.72^{+0.20}_{-0.21}$ | $-2.69^{+0.18}_{-0.20}$ | $-2.45^{+0.23}_{-0.19}$ | $-1.91^{+0.18}_{-0.25}$ | $-1.88^{+0.07}_{-0.09}$ |
| $\chi^2$/d.o.f | 116.7/(56 − 8) | 124.1/(56 − 8) | 221.8/(65 − 8) | 215.5/(65 − 8) | 1.06 |

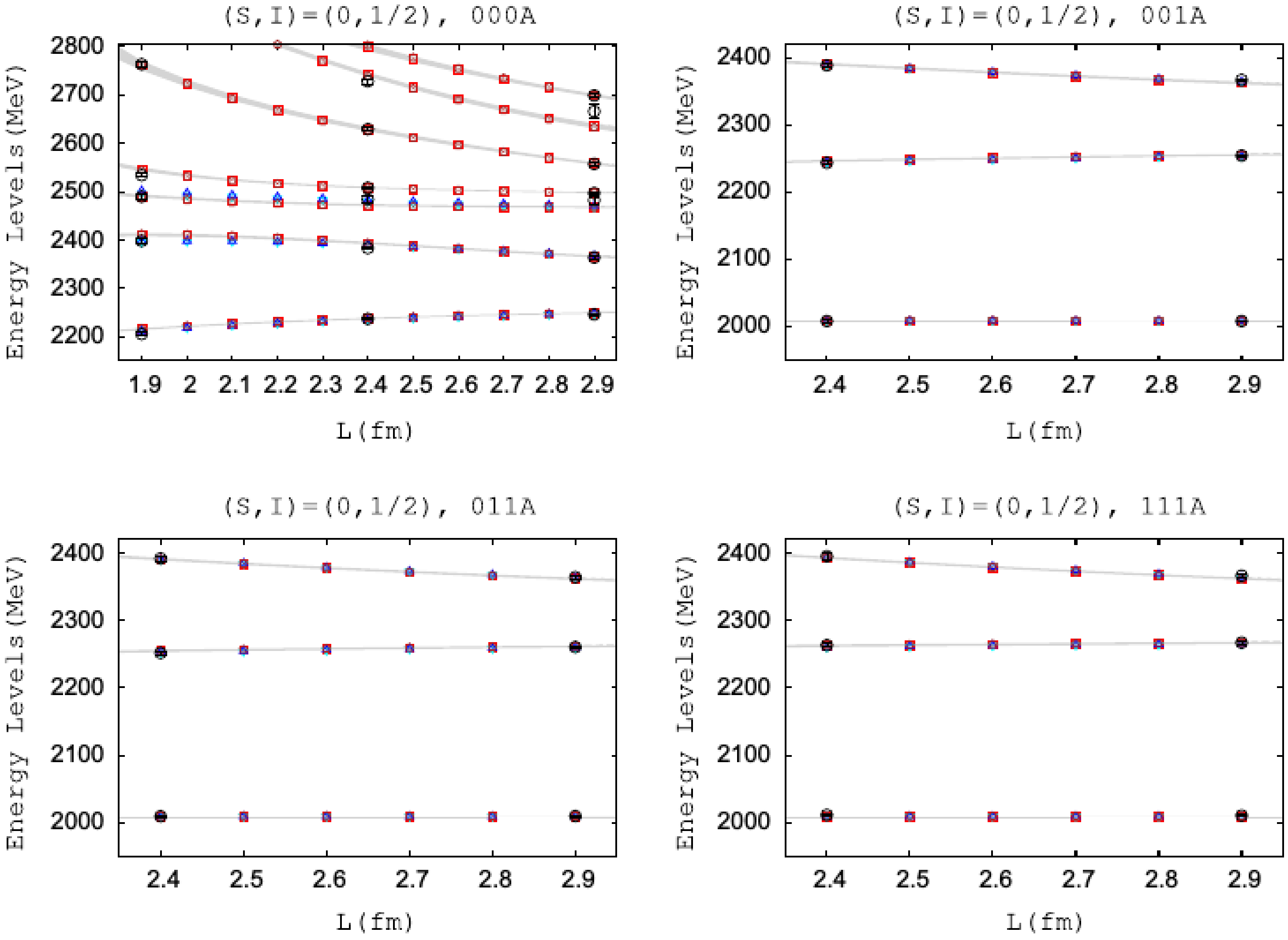
**Data: (1) D-pi lattice energy levels from HSC, 38/47**

**(2) DK lattice energy levels and scattering length from Lang et al., 3**

**(3) Single-channel scattering lengths from Liu et al., 15**

# Reproduction of the finite-volume energy levels

[Moir,Peardon,Ryan,Thomas, Wilson, JHEP'16]

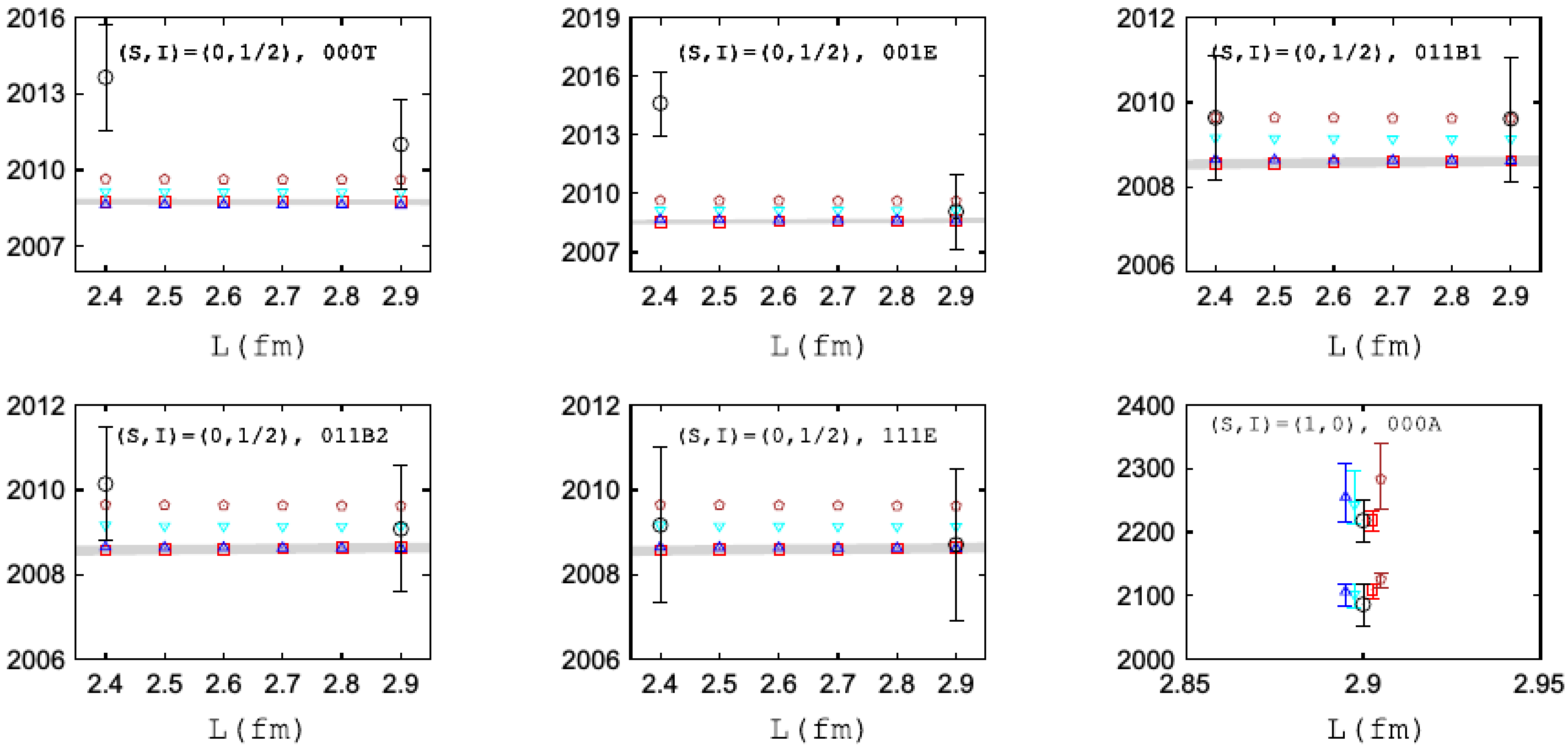



[ZHG,Liu,Meissner,Oller,Rusetsky, EPJC'19]

# Reproduction of the finite-volume energy levels

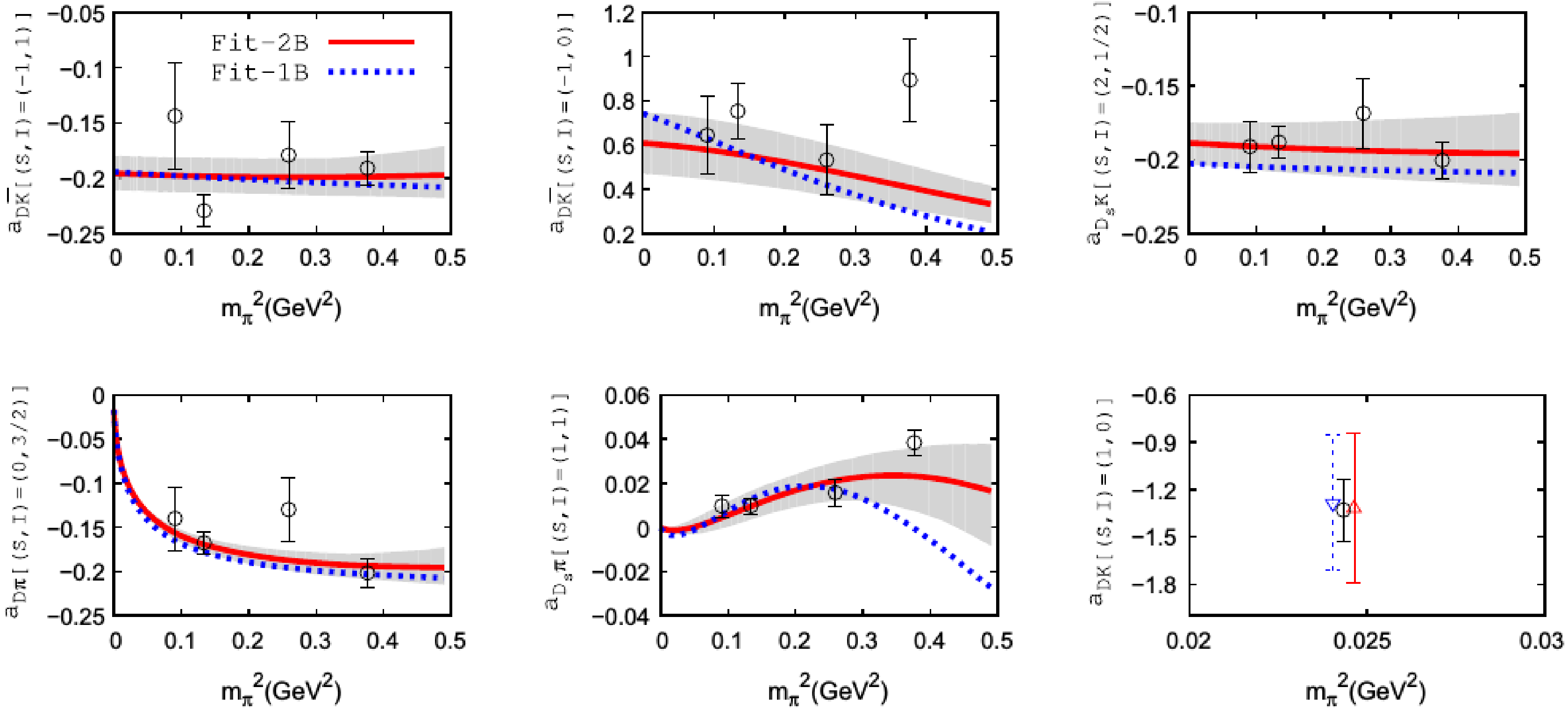
[Moir,Peardon,Ryan,Thomas, Wilson, JHEP'16]

[Lang,Leskovec,Mohler,Prelovsek,Woloshyn,PRD'14]



# Reproduction of scattering lengths

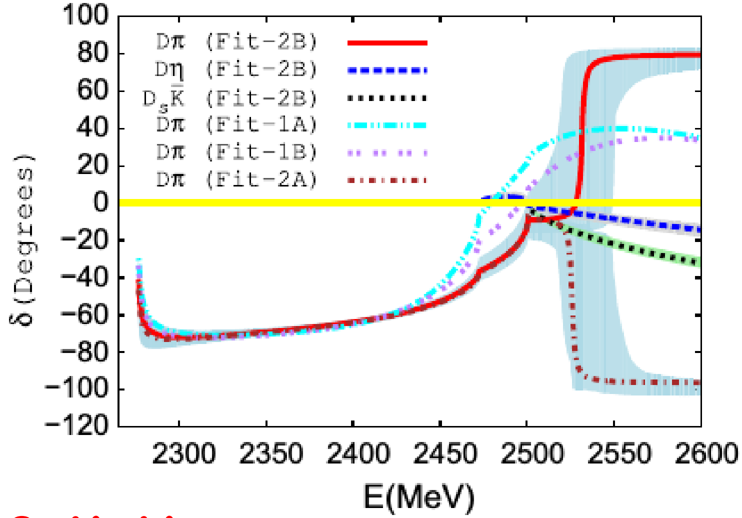
**[L.Liu,Orginos,F.K.Guo,Meissner, PRD'13]**
**[Lang,Leskovec,Mohler,Prelovsek,Woloshyn,PRD'14]**

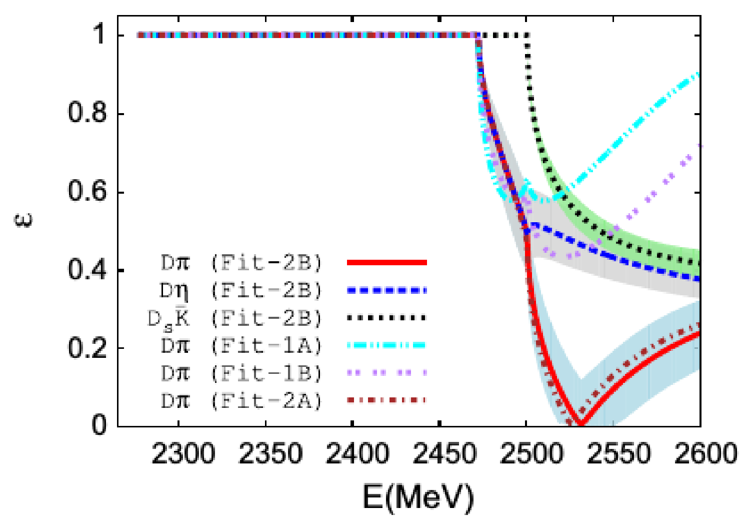


# Prediction of the D-pi phase shifts and inelasticities

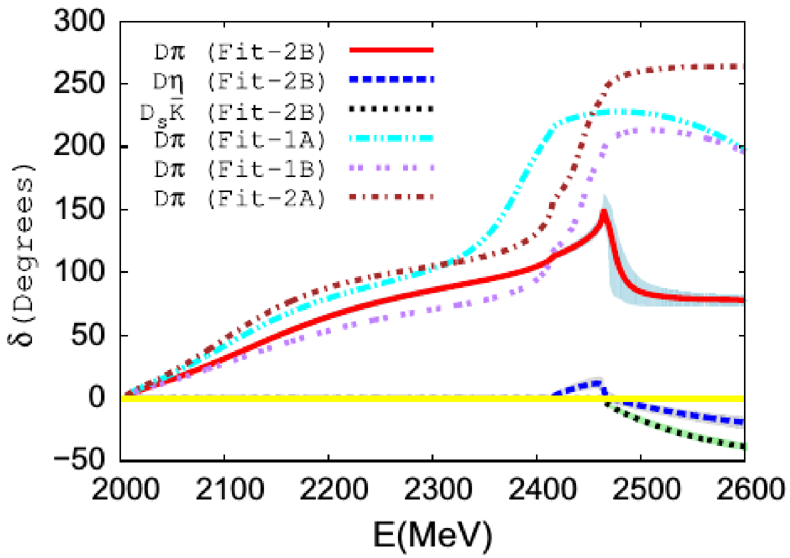
## mpi=391 MeV

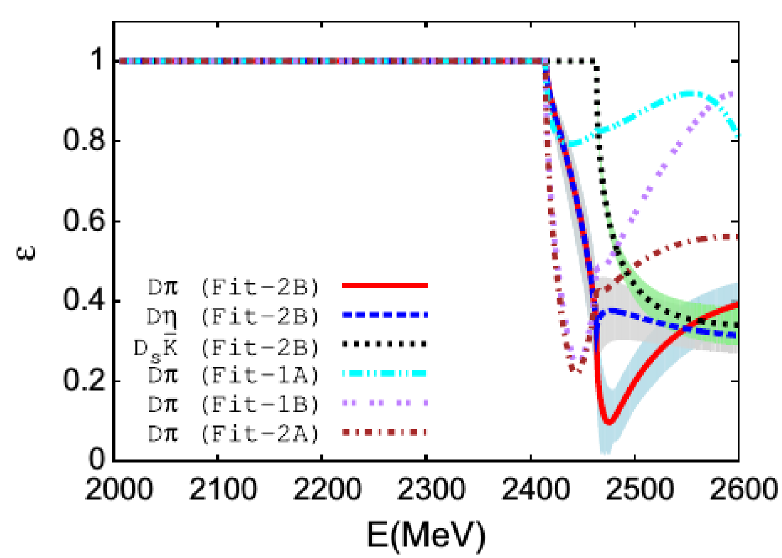
[ZHG,Liu,Meissner,Oller,Rusetsky, EPJC'19]

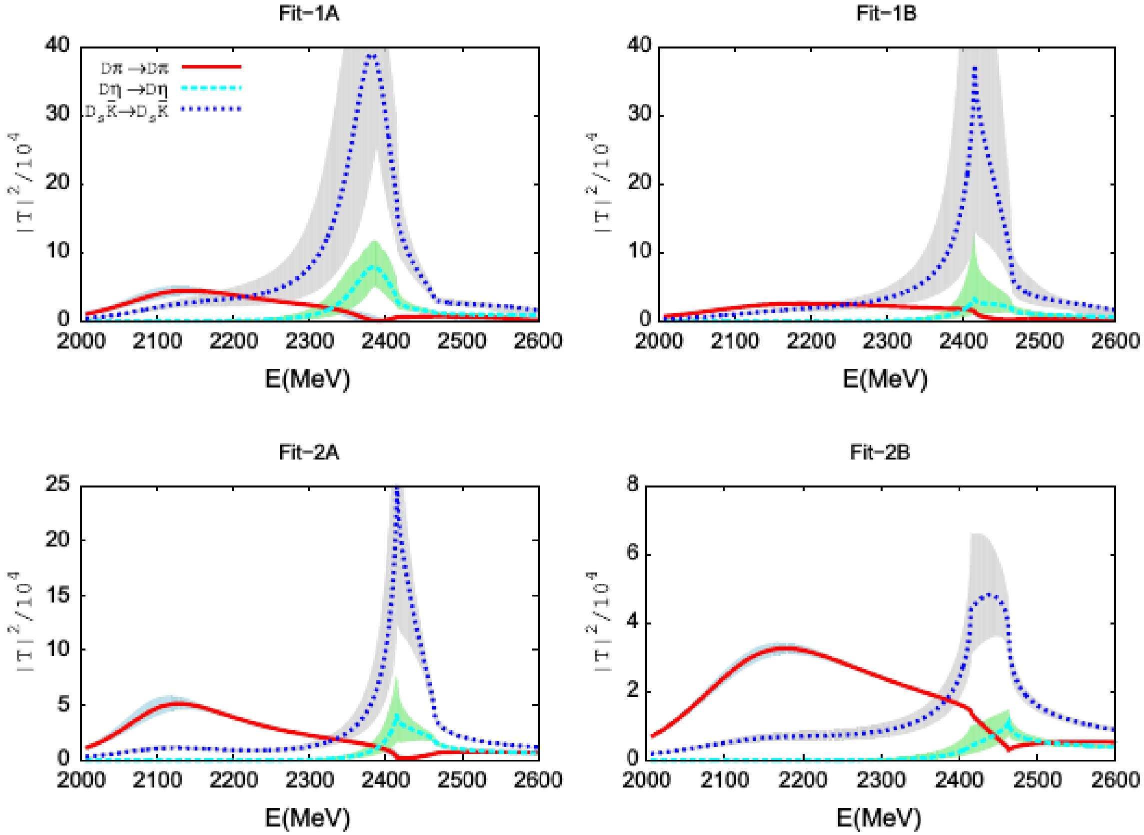


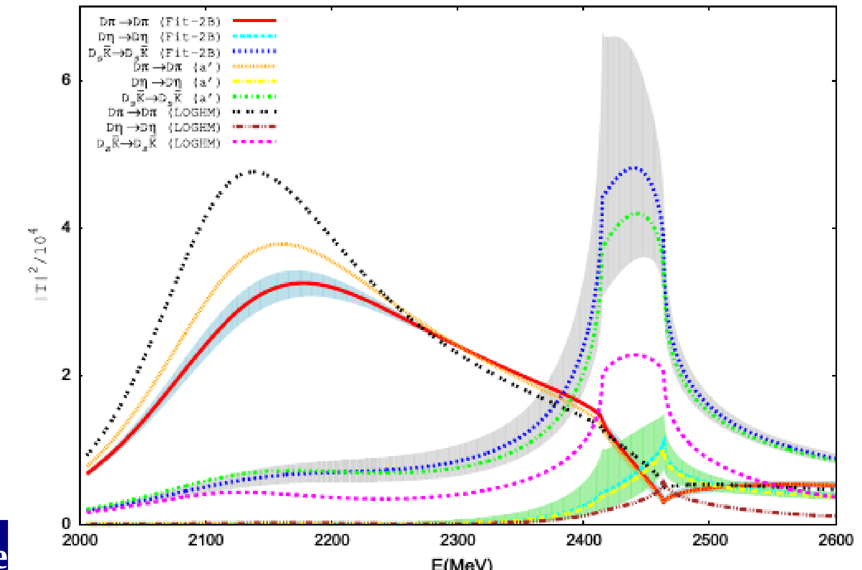




## mpi=138 MeV

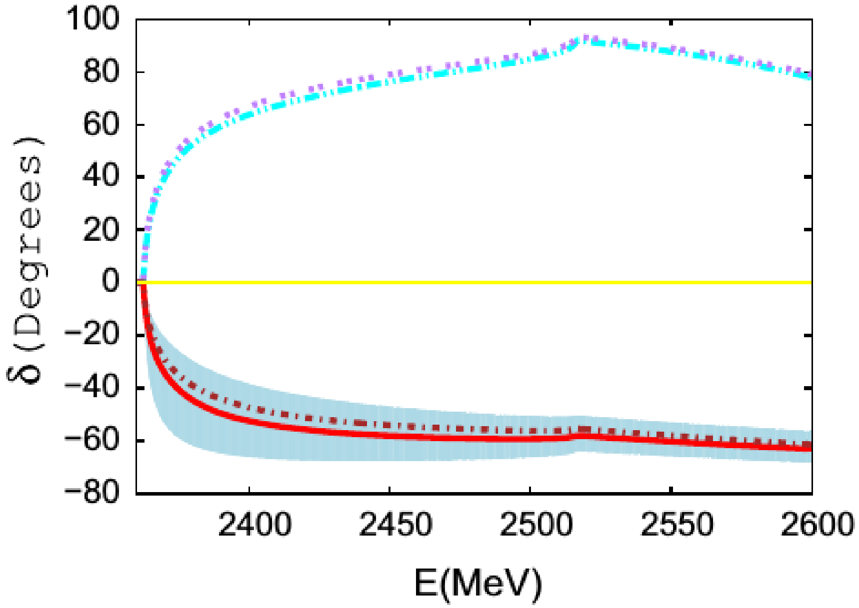


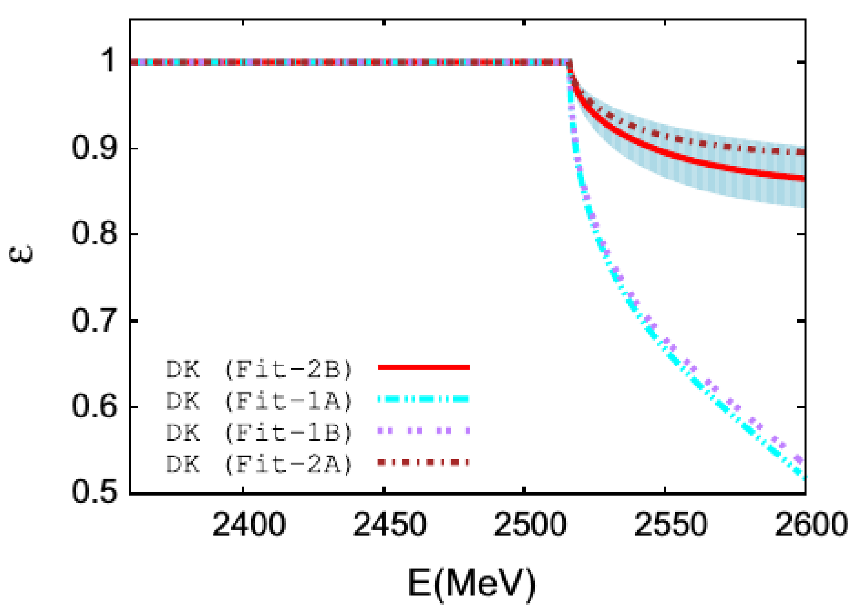




Fit−1A
Fit−1B
Fit−2A
Fit−2B
Dπ →Dπ
Dη →Dη
D_sK̄→D_sK̄
|T|²/10⁴
E(MeV)


Dπ →Dπ (Fit-2B)
Dη →Dη (Fit-2B)
D_sK̄→D_sK̄ (Fit-2B)
Dπ →Dπ (a')
Dη →Dη (a')
D_sK̄→D_sK̄ (a')
Dπ →Dπ (LOGEM)
Dη →Dη (LOGEM)
D_sK̄→D_sK̄ (LOGEM)
|T|²/10⁴
E(MeV)

# Prediction of the D-K with I=0 phase shifts and inelasticities



# Poles of D*0(2400)

| Fit | RS | M | $\Gamma/2$ (MeV) | $\|\gamma_1\|$ (GeV) | $\|\gamma_2/\gamma_1\|$ | $\|\gamma_3/\gamma_1\|$ |
|---|---|---|---|---|---|---|
| Fit-1A | II | $2097.7^{+6.8}_{-6.1}$ | $112.2^{+16.5}_{-14.2}$ | $9.6^{+0.3}_{-0.3}$ | $0.10^{+0.05}_{-0.04}$ | $0.78^{+0.08}_{-0.08}$ |
| Fit-1A | II | $2384.4^{+26.4}_{-23.6}$ | $36.0^{+9.9}_{-10.0}$ | $4.8^{+0.5}_{-0.6}$ | $1.51^{+0.15}_{-0.16}$ | $2.09^{+0.18}_{-0.18}$ |
| Fit-1B | II | $2106.4^{+5.1}_{-5.0}$ | $170.6^{+12.5}_{-13.0}$ | $10.1^{+0.3}_{-0.2}$ | $0.11^{+0.07}_{-0.07}$ | $0.79^{+0.07}_{-0.07}$ |
| Fit-1B | III | $2409.0^{+22.7}_{-24.5}$ | $78.6^{+20.5}_{-15.2}$ | $6.1^{+0.7}_{-0.6}$ | $1.22^{+0.19}_{-0.19}$ | $2.72^{+0.48}_{-0.49}$ |
| Fit-2A | II | $2095.7^{+5.2}_{-6.8}$ | $97.1^{+10.3}_{-10.7}$ | $9.4^{+0.2}_{-0.2}$ | $0.10^{+0.02}_{-0.02}$ | $0.63^{+0.03}_{-0.03}$ |
| Fit-2A | III | $2401.3^{+20.4}_{-19.6}$ | $55.0^{+14.5}_{-10.8}$ | $5.1^{+0.5}_{-0.5}$ | $1.31^{+0.19}_{-0.15}$ | $2.50^{+0.31}_{-0.28}$ |
| Fit-2B | II | $2117.7^{+3.8}_{-3.4}$ | $145.0^{+8.0}_{-6.8}$ | $10.2^{+0.2}_{-0.1}$ | $0.09^{+0.03}_{-0.03}$ | $0.58^{+0.04}_{-0.03}$ |
| Fit-2B | III | $2470.5^{+25.1}_{-24.9}$ | $104.1^{+16.0}_{-12.5}$ | $6.7^{+0.7}_{-0.6}$ | $1.14^{+0.12}_{-0.12}$ | $2.06^{+0.16}_{-0.16}$ |

# Poles of D*s0(2317)

| Fit | RS | M (MeV) | $\Gamma/2$ (MeV) | $\|\gamma_1\|$ (GeV) | $\|\gamma_2/\gamma_1\|$ |
|---|---|---|---|---|---|
| Fit-1A | I | 2356.7–2362.8 | 0 | 1.3–6.9 | 1.03–1.20 |
| Fit-1A | II | 2316.7–2362.8 | 0 | 0.4–10.1 | 1.14–1.50 |
| Fit-1B | I | 2357.1–2362.8 | 0 | 0.5–6.7 | 1.05–1.22 |
| Fit-1B | II | 2316.0–2362.8 | 0 | 0.6–10.3 | 1.12–1.56 |
| Fit-2A | I | $2345.1^{+14.7}_{-41.5}$ | 0 | $8.3^{+2.3}_{-2.6}$ | $0.96^{+0.06}_{-0.08}$ |
| Fit-2B | I | $2350.7^{+9.0}_{-25.7}$ | 0 | $7.7^{+2.1}_{-2.0}$ | $0.83^{+0.08}_{-0.06}$ |

# Summary

• The chiral approach illustrated in this talk provides an efficient way to study the finite-volume energy levels.

• It can build a bridge to connect the lattice eigenenergies in finite box obtained at unphysical masses with the physical observables, such as phase shifts, inelasticities, at physical meson masses.

• We have successfully applied this approach to the pi-eta, K-Kbar , pi-eta' and D-pi, D-eta, Ds-Kbar coupled-channel scattering.

• Similar study in other systems can be straightforwardly extended.

**Thanks for your attention!**