Baryon–baryon scattering in manifestly Lorentz–invariant formulation of ChPT

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Introduction

Theoretical Framework

Results and discussion

Summary
Residual quark-gluon strong interaction

Understood from Quantum Chromo-Dynamics

At low-energy region

- Running coupling constant $\alpha_s > 1$
- Nonperturbative QCD -- unsolvable

Studies of BB interactions

- Phenomenological models (since 1935~)
- Lattice QCD simulation (since 2010~)
- Chiral effective field theory (since 1990~)

S. Bethke, PPNP(2013)
Residual quark-gluon strong interaction

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S.Bethke, PPNP(2013)
Weinberg's proposal

- NN potential calculated in chiral perturbation theory order by order

\[ V(p', p) = V_{\text{LO}} + V_{\text{NLO}} + V_{\text{NNLO}} + \cdots \]

Weinberg's Power Counting: \( \mathcal{O}(Q^0) \), \( \mathcal{O}(Q^2) \), \( \mathcal{O}(Q^3) \)  

- Scattering amplitude obtained by solving the Schrödinger or Lippmann-Schwinger equations

\[ T(p', p) = V(p', p) + \int_0^\infty \frac{k^2 dk}{(2\pi)^3} \frac{m}{p^2 - k^2 + i\epsilon} V(p', k) \frac{m}{p^2 - k^2 + i\epsilon} T(k, p). \]

- BUT, e.g., a series of ladder diagrams

\[ \longrightarrow k \to \infty \text{ Ultraviolet Divergence!!!} \]

cannot be absorbed by contact terms

WPC is inconsistent with renormalization, even at leading order (LO)!

\[ \mathcal{O}(Q^0) \mathcal{O}(Q^2) \mathcal{O}(Q^3) \]
Possible solutions **(still controversial...)**

- Keep cutoff lower than hard scale: $\Lambda < \Lambda_{\chi PT} \sim 1 \text{ GeV}

  ✓ WPC is consistent

  ✓ Achieve great successes

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- R. Machleidt, D. R. Entem, Phys. Rept. 503 (2011) 1

Short-range loop contrib. still missing.
Possible solutions (still controversial...)  

- Keep cutoff lower than hard scale: \( \Lambda < \Lambda_{\chi PT} \sim 1 \text{ GeV} \)
  
  ✓ WPC is consistent  \( G.P. \) Lepage, nucl-th/9706029. E.Epelbaum, J.Gegelia, Ulf-G. Meißner, NPB925(2017)161
  
  R. Machleidt, D. R. Entem, Phys. Rept. 503 (2011) 1

- Kaplan, Savage, and Wise (KSW) power counting
  
  
  
  ✓ Recently, some improvements of KSW proposed by Kaplan  D.B. Kaplan, arXiv: 1905.07485
Possible solutions *(still controversial...)*

- Keep cutoff lower than hard scale: $\Lambda < \Lambda_{\chi PT} \sim 1$ GeV

- Kaplan, Savage, and Wise (KSW) power counting
  - ✓ Recently, some improvements of KSW proposed by Kaplan [D. B. Kaplan, *arXiv: 1905.07485*]

- Modified WPC with renormalization group invariance (RGI)
Possible solutions (still controversial…)

- Keep cutoff lower than hard scale: \( \Lambda < \Lambda_{\chi PT} \approx 1 \text{ GeV} \)
  - √ WPC is consistent
    - G.P. Lepage, nucl-th/9706029.
    - E. Epelbaum, J. Gegelia, Ulf-G. Meißner, NPB925(2017)161
  - √ Achieve great successes
    - R. Machleidt, D. R. Entem, Phys. Rept. 503 (2011) 1

- Kaplan, Savage, and Wise (KSW) power counting
  - √ Treat the exchange of pions perturbatively
  - √ Fail to converge in certain spin-triplet channels
  - √ Recently, some improvements of KSW proposed by Kaplan
    - D.B. Kaplan, arXiv: 1905.07485

- Modified WPC with renormalization group invariance (RGI)
  - √ Rearrange the higher order contact terms to the lower chiral order
    - A. Nogga, et al., PRC72(2005)054006
    - M. C. Birse, PRC74(2006)014003
    - M. Pavon Valderrama, PRC72(2005) 054002
    - B. Long and C.-J. Yang, PRC84(2011)057001 …

- Lorentz invariant framework to reformulate chiral force
  - √ The fundamental symmetry of our nature
Relativistic studies of chiral force

Talk by Prof. Li-Sheng Geng @ Session4, 8/20

- Relativistic chiral force (covariant form)
  \[ u = u_0 + u_1 + u_2 + \cdots \]
  \[ V(p', p) = \bar{u}_1 \bar{u}_2 \mathcal{A} u_1 u_2 \text{ with } u = u_0 + u_1 + u_2 + \cdots \]

Here, we focus on the **renormalization issue** of chiral force

- Modified Weinberg approach

  - Based on the Lorentz invariant chiral Lagrangians
  - Adopt **WPC** to expand the NN potential and the relativistic corrections are perturbatively included

  \[ T(p', p) = V(p', p) + \int \frac{k^2 dk}{(2\pi)^3} \frac{m^2}{2\sqrt{k^2 + m^2}} \frac{1}{\sqrt{p^2 + m^2} - \sqrt{k^2 + m^2} + i\epsilon} T(k, p) . \]

  - Use the Kadyshevsky equation to calculate the scattering T-matrix

  - Milder ultraviolet behavior than in LS equation

  - Result in a renormalizable LO potential!

  - All divergences absorbed in parameters of the LO potential
In this work

- Based on the idea of modified Weinberg approach, we proposed a systematic framework within the old-fashioned (time-ordered) perturbation theory using the Lorentz invariant chiral Lagrangians
  
  - Derive the rules of time-ordered diagrams, especially for the rules with spin-1/2 fermion (as far as we know, there was no such rules in the literatures)
  
  - Extend the framework from the nucleon-nucleon scattering to the baryon-baryon sector with different strangeness
    ✓ Calculate the NN and YN scatterings up to leading order
    ✓ Discuss the renormalization issue of our obtained potentials


XLR, E.Epelbaum, J.Gegelia, Hyperon-nucleon scattering, in preparation
Time-ordered perturbation theory (TOPT)

**Definition**

- Re-express the Feynman integral in a form that makes the connection with on-shell state explicit. This form is called **TOPT or old-fashioned PT**.
- (In short) Instead the propagators for internal lines as the energy denominators for intermediate states.

**Advantages**

- Explicitly show the unitarity
- One-to-one relation between internal lines and intermediate states
- Easily to tell the contributions of a particular diagram

**Derive the rules for time-ordered diagrams**

- Perform Feynman integrations over the zeroth components of the loop momenta
- Decompose Feynman diagram into sums of time-ordered diagrams
- Match to the rules of time-ordered diagrams
Diagram rules in TOPT

- **External lines**
  - Incoming (outgoing) baryon lines: $u(p) \ [\bar{u}(p')]$ Dirac spinors

- **Internal lines**
  - Pseudo-scalar meson lines: $\frac{1}{2\omega(q, M)}$
  - Baryon lines: $\frac{m_i}{\omega(p_i, m)} \sum u(p_i)\bar{u}(p_i)$
  - Anti-baryon lines: $\frac{m_i}{\omega(p_i, m)} \sum u(p_i)\bar{u}(p_i) - \gamma_0$
  \[ \omega(q, M) = \sqrt{q^2 + M^2} \]

- **Interaction vertices**
  - Follow the standard Feynman rules
  - Take care of zeroth components of momenta $p^0$
    ✓ Replaced as $\omega(p, m)$ for particle
    ✓ Replaced as $-\omega(p, m)$ for antiparticle

- **Intermediate state:** a set of lines between any two vertices
  \[ [E - \sum_{i} \omega(p_i, m_i) + i\epsilon]^{-1} \]

  $E$ is the total energy of the system
Scattering amplitude $T$

\[
\begin{array}{c c c}
\frac{E}{2} + p & \frac{E}{2} + p' & \frac{E}{2} + k \\
\hline
\hline
\frac{E}{2} - p & \frac{E}{2} - p' & \hline
\end{array}
\]

- **Potential $V$:** sum up the two-particle irreducible time-ordered diagrams

  ✓ Employ the Weinberg power counting to perturbatively calculate potential

- **Two-body Green functions $G$:**

\[
G_{ij}(E) = \frac{1}{\omega(k, m_i) \omega(k, m_j)} \frac{m_i m_j}{E - \omega(k, m_i) - \omega(k, m_j) + i\epsilon}
\]

✓ This is the generalized Kadyshhevsky propagator of NN scattering

✓ **SELF-CONSIDERENTLY obtained** in TOPT!
Leading order contributions in TOPT

- Time-ordered diagrams at LO

- Lorentz-invariant effective Lagrangians

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_\phi + \mathcal{L}_{\phi B} + \mathcal{L}_{BB} + \cdots \]

- **Mesonic Lagrangian**
  - \[ \mathcal{L}^{(2)}_\phi = \frac{F_0^2}{4} \text{Tr} \left\{ u_\mu u^\mu + \chi^+ \right\} \]
  - \text{J. Gasser and H. Leutwyler, Ann. Phys. 158, 142 (1984)}

- **Meson-baryon Lagrangian**
  - \[ \mathcal{L}^{(1)}_{\phi B} = \text{Tr} \left\{ \bar{B} \left( i \gamma_\mu D^\mu - m \right) B \right\} + \frac{D/F}{2} \text{Tr} \left\{ \bar{B} \gamma_\mu \gamma_5 [u^\mu, B] \right\} \]

- **Baryon-baryon Lagrangian**
  - \[ \mathcal{L}^{(0)}_{BB} = C_1^1 \text{Tr} \left\{ \bar{B}_\alpha B_\beta (\Gamma_i B)_\alpha (\Gamma_i B)_\beta \right\} + C_2^2 \text{Tr} \left\{ \bar{B}_\alpha (\Gamma_i B)_\alpha \bar{B}_\beta (\Gamma_i B)_\beta \right\} \]
  - \[ + C_3^3 \text{Tr} \left\{ \bar{B}_\alpha (\Gamma_i B)_\alpha \right\} \text{Tr} \left\{ \bar{B}_\beta (\Gamma_i B)_\beta \right\} \]
  - \text{H. Polinder, J. Haidenbauer, Ulf-G. Mei\ss{}ner, NPA 779 (2006) 244-266}

\[ \Gamma_1 = 1, \quad \Gamma_2 = \gamma^\mu, \quad \Gamma_3 = \sigma^{\mu\nu}, \quad \Gamma_4 = \gamma^\mu \gamma_5, \quad \Gamma_5 = \gamma_5. \]
Leading order potentials

Contact baryon-baryon interaction

- **S = 0, NN single channel**
  \[ V_{0,C}^{IJ,KL} = C_S(\bar{u}_K u_I)(\bar{u}_L u_J) + C_A(\bar{u}_K \gamma_5 u_I)(\bar{u}_L \gamma_5 u_J) + C_V(\bar{u}_K \gamma_{\mu} u_I)(\bar{u}_L \gamma^\mu u_J) \]
  \[ + C_{AV}(\bar{u}_K \gamma_{\mu} \gamma_5 u_I)(\bar{u}_L \gamma^\mu \gamma_5 u_J) + C_T(\bar{u}_K \sigma_{\mu\nu} u_I)(\bar{u}_L \sigma^{\mu\nu} u_J) \]
  ✓ Contains higher order contributions according to WPC
  ✓ Perform the expansion for the baryon energies
  \[ \sqrt{\omega(p,m) + m} = \sqrt{2m + O(p^2)} \]
  \[ V_{LO,C}^{IJ,KL} = (C_S + C_V) - (C_{AV} - 2C_T)\vec{\sigma}_1 \cdot \vec{\sigma}_2 \]
  Same as the non-relativistic potential
  S. Weinberg, PLB251(1990)288-292

- **S = -1, ΛN-ΣN two coupled channels**
  ✓ Expressions are the same as the non-relativistic potential
  H.Polinder, J. Haidenbauer, Ulf-G. Meißner, NPA779(2006)244-266

- **S = -2, ΛΛ, ΩN, ΣΣ, ΣΛ four coupled channels**
  ✓ Expressions are the same as the non-relativistic potential
Leading order potentials

One-meson-exchange contribution

\[
V_{0,MP}^{IJ,KL} = \frac{f_{IKP} f_{JLP} I_{IJ,KL}}{2 \omega(q, M_P)} \left[ \frac{(\bar{u}_I \gamma_\mu \gamma_5 q^\mu u_K) (\bar{u}_J \gamma_\nu \gamma_5 q^\nu u_L)}{\omega(q, M_P) + \omega(p_K, m_K) + \omega(p_J, m_J) - E - i \epsilon} \right.
\]

\[
+ \frac{(\bar{u}_I \gamma_\mu \gamma_5 q^\mu u_K) (\bar{u}_J \gamma_\nu \gamma_5 q^\nu u_L)}{\omega(q, M_P) + \omega(p_L, m_L) + \omega(p_I, m_I) - E - i \epsilon} \right]
\]

- Contains higher order contributions according to WPC
- Perform the expansion for the baryon energies

\[
\sqrt{\omega(p, m) + m} = \sqrt{2m} + \mathcal{O}(p^2)
\]

✓ Keep the baryon energies in denominator (consistent with Kadysevsky eq.)

\[
V_{LO,MP}^{IJ,KL} = -\frac{f_{IKP} f_{JLP} I_{IJ,KL}}{2 \omega(q, M_P)} \left[ \frac{1}{\omega(q, M_P) + \omega(p_K, m_K) + \omega(p_J, m_J) - E - i \epsilon} \right.
\]

\[
+ \frac{1}{\omega(q, M_P) + \omega(p_L, m_L) + \omega(p_I, m_I) - E - i \epsilon} \right]
\]

\[
\times \frac{(m_I + m_K) (m_J + m_L)}{\sqrt{m_I m_J m_K m_L}} \frac{(m_K \bar{\sigma}_1 \cdot \vec{p}_I - m_I \bar{\sigma}_1 \cdot \vec{p}_K) (m_L \bar{\sigma}_2 \cdot \vec{p}_J - m_J \bar{\sigma}_2 \cdot \vec{p}_L)}{\sqrt{\omega(p_I, m_I) + m_I} \sqrt{\omega(p_J, m_J) + m_J} \sqrt{\omega(p_K, m_K) + m_K} \sqrt{\omega(p_L, m_L) + m_L}}.
\]

It has a milder ultraviolet behaviour than the non-relativistic OMEP
Behavior of long-range potential

One-loop integral $V_{MGV_M}$

$$I_{VGV} = \sum_{Q,R} \int \frac{d^3k}{(2\pi)^3} V^{IJ,QR}_{LO,M_P} G^{QR}(E) V^{QR,KL}_{LO,M_P}$$

$k \equiv \mid \vec{k} \mid \to \infty$ & $\text{Our: } I_{VGV}^{Our} \to \int dk^3 \frac{1}{k^5}$ & Ultraviolet convergent!

$\text{NR: } I_{VGV}^{NR} \to \int dk^3 \frac{1}{k^2}$ & Ultraviolet divergent!

Iteration of our OMEP

$k \to \infty$ \quad \text{Finite diagram!}

- Scattering amplitude from OMEP is cutoff independent

$$T_M = V_M + V_M G T_M$$

Renormalizable!
Phase shifts: cutoff-independent

- **NN single channel**

  \( ^1S_0 \)

  ![Graph](image)

- **NN couple channels**

  \( ^3S_1 \)

  ![Graph](image)

- **ΛN-ΣN single channel**

  \( ^3P_0 \)

  ![Graph](image)

- **ΛN-ΣN couple channels**

  \( ^3S_1 \)

  ![Graph](image)
Phase shifts of NN scattering

- **Leading order chiral NN potential**

\[
V_C = C_S - C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2
\]

\[
V_\pi (\vec{p}', \vec{p}) = -\frac{g_A^2}{4 F_0^2} \frac{\tau_1 \cdot \tau_2}{\sqrt{(\vec{p} - \vec{p}')^2 + M_\pi^2}} \frac{4m_N^2}{(m_N + \sqrt{p'^2 + m_N^2})(m_N + \sqrt{p^2 + m_N^2})} \times \frac{[\vec{\sigma}_1 \cdot (\vec{p} - \vec{p}')][\vec{\sigma}_2 \cdot (\vec{p} - \vec{p}')}}{\sqrt{(\vec{p} - \vec{p}')^2 + M_\pi^2 + \sqrt{p'^2 + m_N^2} + \sqrt{p^2 + m_N^2} - E - i\epsilon}},
\]

- **Phase shifts at LO with cutoff \( \rightarrow \infty \)**
Leading order chiral NN potential

\[ V_C = C_S - C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2 \]

\[ V_\pi (\vec{p}', \vec{p}) = -\frac{g_A^2}{4F_0^2} \frac{\vec{\tau}_1 \cdot \vec{\tau}_2}{(\vec{p} - \vec{p}')^2 + M_\pi^2} \frac{4m_N^2}{(m_N + \sqrt{\vec{p}'^2 + m_N^2})(m_N + \sqrt{\vec{p}^2 + m_N^2})} \times \frac{[\vec{\sigma}_1 \cdot (\vec{p} - \vec{p}')][\vec{\sigma}_2 \cdot (\vec{p} - \vec{p}')]}{\sqrt{(\vec{p} - \vec{p}')^2 + M_\pi^2 + \sqrt{\vec{p}'^2 + m_N^2} + \sqrt{\vec{p}^2 + m_N^2} - E - i\epsilon}}, \]

Phase shifts at LO with cutoff → ∞

\(^1S_0 \text{ and } ^3P_0: \]

- Large differences
  - At least a part of the subleading corrections must be treated non-perturbatively
  - For simplicity, we choose to promote the NLO contact terms up to LO.
Role of NLO contact terms in renormalization

- Take $^3P_0$ partial wave for example
  - Promote the NLO contact term to the lowest order
  - Potential: $V_{3P_0}(p', p) = C p' p + V_\pi$
  - Amplitude: $T_{3P_0} = T_\pi + \left[ (1 + T_\pi G)p' \right] \left[ p (1 + G T_\pi) \right] \frac{C^{-1} - p G p' - p G T_\pi G p'}{C^{-1} - p G p' - p G T_\pi G p'}$

$V_\pi T_\pi$

$T_\pi = V_\pi + V_\pi G T_\pi : \quad k \to \infty \quad \text{Finite term}$

$(1 + T_\pi G)p' : \quad \text{Finite term}$

$p G p' : \quad \text{Logarithmic divergence}$

$p G T_\pi G p' : \quad \text{Logarithmic divergence}$

It is cutoff dependent, not renormalizable!
From the point view of Renormalization Group Invariance

- The $^3P_0$ potential is not singular, and therefore does not require a contact term to achieve RG-invariance.
- Promoting a contact term to LO in the non-perturbative treatment will destroy the RG, unless extra care is taken to further subtract the divergences.
Use subtractive renormalization to subtract all the divergences in loop diagrams

\[ (pGp')^R = \frac{m^2 p^2}{4\pi^2 E} \left[ 2p \left( \sinh^{-1} \frac{p}{m} - i\pi \right) - \pi m \right] + \frac{m^2 p^2}{8\pi^2 \sqrt{m^2 - \mu^2}} \left( 2\mu \left( \sin^{-1} \frac{\mu}{m} - \pi \right) + \pi m \right) \].

Renormalized amplitude:

\[ T_{3P0}^R = T_\pi + \frac{[(1 + T_\pi G)p'][p(1 + G T_\pi)]}{(C^R)^{-1} - (pGp')^R - pGT_\pi Gp'} \]

Cutoff independent!

\[ \delta \text{ [deg]} \]

\[ E_{\text{lab}} \text{ [MeV]} \]

Scale dependence is weak!

\[ \delta \text{ [deg]} \]

\[ \text{Lab. Energy [MeV]} \]
\( \Lambda N - \Sigma N \) scattering

- **\( ^3P_0 \) phase shifts up to LO**

![Graph showing phase shifts](image)

- **In order to improve the description of \( ^3P_0 \) phases**
  - NLO contact terms are promoted to LO
  
  \[
  V_{LO}^{^3P_0} (p'_1, p'_2; p_1, p_2) = V_C(p'_1, p'_2; p_1, p_2) + V_{LO,MP}^{IJ,KL}
  \]

  \[
  V_C = \xi(p'_1, p'_2) C \xi(p_1, p_2), \quad C = \begin{pmatrix}
  C_{\Lambda N, \Lambda N} & C_{\Lambda N, \Sigma N} \\
  C_{\Sigma N, \Lambda N} & C_{\Sigma N, \Sigma N}
  \end{pmatrix}, \quad \xi(p_1, p_2) = \begin{pmatrix} p_1 & 0 \\
  0 & p_2 \end{pmatrix}
  \]

  - Use the **subtractive renormalization** to achieve a renormalizable potential

- H. Polinder, et al., *NPA* 779(2006)244-266
- J. Haidenbauer, et al., 1906.11681
$\Lambda N - \Sigma N$ scattering

- $^3P_0$ phase shifts

![Graph showing $^3P_0$ phase shifts with various models compared.](image)

XLR, E. Epelbaum, J. Gegelia, Hyperon-nucleon scattering, in preparation
We proposed a systematic framework to formulate the baryon-baryon interactions based on the 
**time-ordered perturbation theory** using the Lorentz invariant effective Lagrangian

- Obtained the rules of time-ordered diagrams with spin-1/2 fermions
- Derived the generalized Kadyshevsky equation **self-consistently**
- Calculated the baryon-baryon interactions up to leading order, which is **renormalizable**
- Achieved a rather good description of $^3P_0$ phases by promoting the NLO contact terms in a renormalizable way

Higher order studies are in progress

- Perturbatively/Non-perturbatively include **NLO/NNLO contributions**
- Keep the momentum cutoff $\Lambda < \Lambda_{\chi PT}$ or $\Lambda \sim \infty$
THANK YOU FOR YOUR ATTENTION!
Backup slides
Whether $1/r^2$ potential has a unique solution depends on the strengths

\[ V(r) = \frac{\hbar^2}{m} \frac{c}{r^2} \quad \text{with} \quad r \equiv |r| \quad c \equiv -\frac{1}{4} - v^2 \]

- For the couplings larger than critical value (-1/4) equations do not have unique solutions.
- For the couplings smaller than critical value (-1/4) equations have unique solutions.

For potentials more singular than $1/r^2$, the equations do not have unique solutions.
Subtractive renormalization of the considered problem corresponds to the inclusion of contributions of an infinite number of counter terms generated by bare parameters of the effective Lagrangian.