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Radiative corrections for Dalitz decays of π^0 , $\eta^{(\prime)}$ and Σ^0

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Guilin

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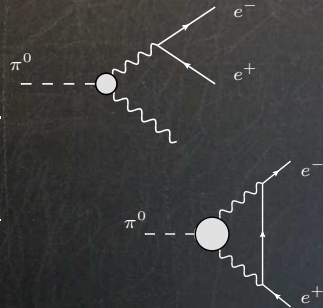
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Decay modes of the neutral pion:

Process	Branching ratio
$\pi^0 \rightarrow \gamma\gamma$	$(98.823 \pm 0.034) \%$
$\pi^0 \rightarrow e^+e^-\gamma$	$(1.174 \pm 0.035) \%$
$\pi^0 \rightarrow e^+e^+e^-e^-$	$(3.34 \pm 0.16) \times 10^{-5}$
$\pi^0 \rightarrow e^+e^-$	$(6.46 \pm 0.33) \times 10^{-8}$



Rare decay $\pi^0 \rightarrow e^+e^-$

- suppressed compared to $\pi^0 \rightarrow \gamma\gamma$ by factor of $2(\alpha m_e/M_\pi)^2$
 - \hookrightarrow one-loop structure + helicity suppression
 - \hookrightarrow may be sensitive to possible effects of new physics

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KTeV-E799-II experiment at Fermilab (*Abouzaid et al., PRD 75 (2007)*)
 \hookrightarrow **precise** measurements of branching ratio $\pi^0 \rightarrow e^+e^-$ (794 candidates)

$$\frac{\Gamma(\pi^0 \rightarrow e^+e^-(\gamma), x > 0.95)}{\Gamma(\pi^0 \rightarrow e^+e^-\gamma, x > 0.232)} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}$$

Extrapolate the Dalitz decay branching ratio to full range of x

$$B_{\text{KTeV}}(\pi^0 \rightarrow e^+e^-(\gamma), x > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}$$

\hookrightarrow PDG average value $(6.46 \pm 0.33) \times 10^{-8}$ mainly based on this result
 \hookrightarrow **final result** for lowest order (no final-state radiation)

$$B_{\text{KTeV}}^{\text{no-rad}}(\pi^0 \rightarrow e^+e^-) = (7.48 \pm 0.29 \pm 0.25) \times 10^{-8}$$

Comparison with SM prediction (*Dorokhov and Ivanov, PRD 75 (2007)*)

$$B_{\text{SM}}^{\text{no-rad}}(\pi^0 \rightarrow e^+e^-) = (6.23 \pm 0.09) \times 10^{-8}$$

\hookrightarrow interpreted as **3.3 σ discrepancy** between theory and experiment

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Size of the radiative corrections

- ↪ *Vaško and Novotný, JHEP 1110 (2011)*
- ↪ *TH, Kampf and Novotný, EPJC 74 (2014)*

$$\delta(0.95) \equiv \delta^{\text{virt.}} + \delta^{\text{BS}}(0.95) = (-5.5 \pm 0.2) \%$$

- ↪ differs significantly from previous approximate calculations

Bergström, Z.Ph.C 20 (1983): $\delta(0.95) = -13.8 \%$
Dorokhov et al., EPJC 55 (2008): $\delta(0.95) = -13.3 \%$

- ↪ original KTeV vs. SM discrepancy reduced to the 2σ level (or less)

New measurement by NA62 under preparation

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Quantity **really** measured by KTeV

$$\left. \frac{\Gamma(\pi^0 \rightarrow e^+e^-\gamma), x > 0.95)}{\Gamma(\pi^0 \rightarrow e^+e^-\gamma), x > 0.2319)} \right|_{\text{KTeV}} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}$$

↔ Dalitz decay comes into play

- **second** most important decay channel of the neutral pion
↔ branching ratio $(1.174 \pm 0.035) \%$
- first studied by **Richard H. Dalitz, PPSA 64 (1951)**, whose name it carries
- experimental data of this process provide the information about **singly-virtual pion transition form factor** $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0, q^2)$
↔ in particular about its **slope** parameter a_π

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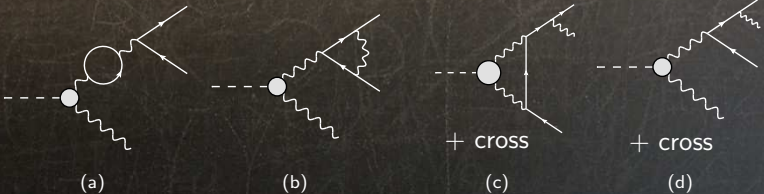
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- radiative corrections to the **total** decay rate of the Dalitz decay
 \hookrightarrow first addressed (numerically) by *Joseph, NC 16 (1960)*
- pioneering study of corrections to the **differential** decay rate
 \hookrightarrow *Lautrup and Smith, PRD 3 (1971)*
 \hookrightarrow soft-photon approximation
- extended by *Mikaelian and Smith, PRD 5 (1972)*
 \hookrightarrow hard-photon corrections
 \hookrightarrow **whole** range of bremsstrahlung photon energy
 \hookrightarrow table of values

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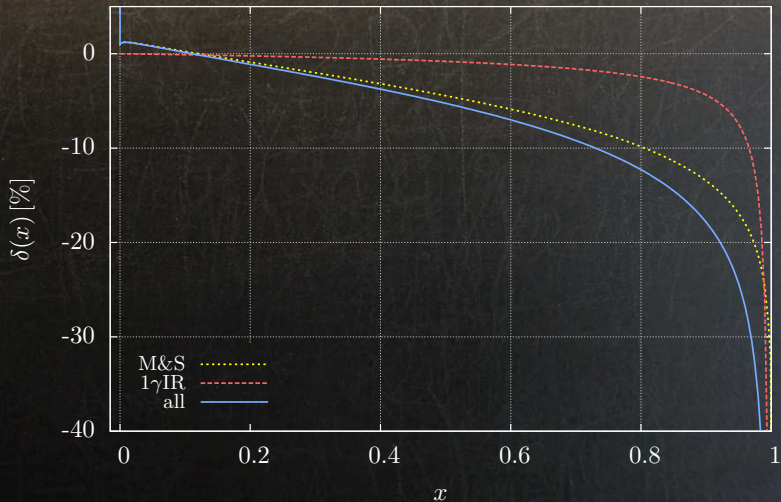
- new calculations motivated by needs of NA48/NA62 experiments at CERN
- ↪ measure the slope of $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0, q^2)$: *Lazzeroni et al., PLB 768 (2017)*

$$a_{\pi}^{\text{NA62}} = 3.68(57) \%$$

- unlike before **no approximation** was used
- ↪ can be used also for related decays $\eta \rightarrow \ell^+\ell^-\gamma$ etc.
- C++ code returns the correction for any given x and y
- ↪ propagated into **MC generator** of NA62 experiment
- *TH, Kampf and Novotný, PRD 92 (2015)*

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Precise and reliable determination of $R \equiv \frac{\Gamma(\pi^0 \rightarrow e^+e^-\gamma)}{\Gamma(\pi^0 \rightarrow \gamma\gamma)}$

\hookrightarrow for small slope and up to NLO radiative corrections

$$R \simeq \frac{\alpha}{\pi} \iint (1 + a_\pi x)^2 (1 + \delta(x, y)) \frac{(1-x)^3}{4x} \left[1 + y^2 + \frac{4m_e^2}{M_\pi^2 x} \right] dx dy$$

Conservative estimate for uncertainty (a_π , NNLO): $R = 1.1978(5)(3) \%$

\hookrightarrow chosen $a_\pi^{\text{univ}} = 3.55(70) \%$, covers

$$a_\pi^{\text{VMD}} = 3.0 \%, \quad a_\pi^{\text{PDG}} = 3.35(31) \%, \quad a_\pi^{\text{NA62}} = 3.68(57) \%$$

Constraint: $1 \simeq \mathcal{B}(\pi^0 \rightarrow \gamma\gamma) + \mathcal{B}(\pi^0 \rightarrow e^+e^-\gamma(\gamma)) + \mathcal{B}(\pi^0 \rightarrow e^+e^-e^+e^-)$

$$\mathcal{B}(\pi^0 \rightarrow \gamma\gamma) = 98.8131(6) \%, \quad \mathcal{B}(\pi^0 \rightarrow e^+e^-\gamma(\gamma)) = 1.1836(6) \%$$

TH, Goudzovski and Kampf, PRL 122 (2018)

PDG

$$R = 1.188(35) \%, \quad \mathcal{B}(\pi^0 \rightarrow \gamma\gamma) = 98.823(34) \%, \quad \mathcal{B}(\pi^0 \rightarrow e^+e^-\gamma) = 1.174(35) \%$$

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TFF normalization $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,0)$ drops out in R
 \hookrightarrow TFF dependence solely represented by its **shape**

for π^0 , transferred momentum significantly (kinematically) limited
 \hookrightarrow **linear** expansion very good approximation
 \hookrightarrow slope a_π constitutes the **only** parameter of the low-energy QCD sector

allowing for 20 % uncertainty on a_π^{univ} due to:

- smallness of the slope
- strong suppression of the region $x \simeq 1$ where the $a_\pi x$ term matters

$$R \simeq \frac{\alpha}{\pi} \iint (1 + a_\pi x)^2 (1 + \delta(x, y)) \frac{(1-x)^3}{4x} \left[1 + y^2 + \frac{4m_e^2}{M_\pi^2 x} \right] dx dy$$

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Abouzaid et al., PRD 100 (2019)

↔ based on 1999 data and *E. Abouzaid*, Ph.D. thesis (2007)

$$\frac{\Gamma(\pi^0 \rightarrow e^+ e^- \gamma)}{\Gamma(\pi^0 \rightarrow \gamma\gamma)} = (1.1559 \pm 0.0046 \pm 0.0106) \%$$

TH, Goudzovski and Kampf, PRL 122 (2018)

Conservative estimate for uncertainty (a_π , NNLO): $R = 1.1978(5)(3) \%$

↔ chosen $a_\pi^{\text{univ}} = 3.55(70) \%$

⇒ 3.6σ discrepancy between theory and experiment

PDG average: $R = 1.188(35) \%$

↔ most recent (archived ALEPH data) *Beddall and Beddall*, EPJC 54 (2008)

Need for new measurements: e.g. NA62

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$\eta^{(\prime)}$ Dalitz decays

- small branching ratios
 \hookrightarrow hadronic decay modes are open
- access to electromagnetic transition form factors
 $\hookrightarrow \eta^{(\prime)}$ -meson structure
 \hookrightarrow valuable input for other quantities and e.g. $g - 2$ of a muon
 \hookrightarrow radiative corrections crucial to **extract** relevant information from data

naive rad. corrections for $\eta \rightarrow e^+e^-\gamma$: *Mikaelian and Smith, PRD 5 2890 (1972)*

- numerical values correspond to simple change $M_{\pi^0} \rightarrow M_\eta$
 $\hookrightarrow \pi^0$ case: *Mikaelian and Smith, PRD 5 1763 (1972)*

inclusive radiative corrections

\hookrightarrow **no** momentum or angular cuts on the bremsstrahlung photon applied

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The $\eta^{(\prime)}$ case compared to π^0

- larger rest mass

↪ M_η above muon-pair threshold: $M_\eta > 2m_\mu$

↪ $M_{\eta'}$ above lowest-lying resonances: $M_{\eta'} > M_\rho, M_\omega$

↪ sensitive to the **widths** of resonances

↪ ω narrow, ρ **broad** resonance in $\pi\pi$ scattering

- **strange-flavor content**

↪ quark-flavor basis

Feldmann et al., PLB 449 (1999), *Escribano et al.*, JHEP 06 (2005)

$$j^\ell \equiv \frac{i}{2} [\bar{u}\gamma_5 u + \bar{d}\gamma_5 d], \quad j^s \equiv \frac{i}{\sqrt{2}} [\bar{s}\gamma_5 s]$$

- $\eta\text{-}\eta'$ **mixing**: $\langle 0 | j^A | \eta^B \rangle = B_0 F_\pi f_A \delta^{AB}$, $\langle \eta^A | \eta^B \rangle = \delta^{AB}$, $A, B \in \{\ell, s\}$

$$|\eta\rangle = \cos \phi |\eta^\ell\rangle - \sin \phi |\eta^s\rangle$$

$$|\eta'\rangle = \sin \phi |\eta^\ell\rangle + \cos \phi |\eta^s\rangle$$

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Full set of NLO QED radiative corrections:

TH, Kampf, Leupold and Novotný, PRD 97 (2018)

- compared to previous approach:
 - ↔ muon loops + **hadronic** VP
 - ↔ **1 γ IR** at one-loop level
 - ↔ **form-factor** effects (also in BS)
 - ↔ higher orders in the final-state-lepton mass **not** neglected
- general framework: **three** additional processes
 - ↔ also muon decay modes

η case: **most** of the ingredients in *TH, Kampf and Novotný, PRD 92 (2015)*

η' case: real challenge

↔ resulting framework also **applicable** to the π^0 case (numerically compatible)

↔ overkill (correction to the correction of order 1%)

Radiative corrections for $\eta' \rightarrow e^+e^-\gamma$ decays

The overall NLO correction $\delta(x, 0)$ in comparison to its constituents

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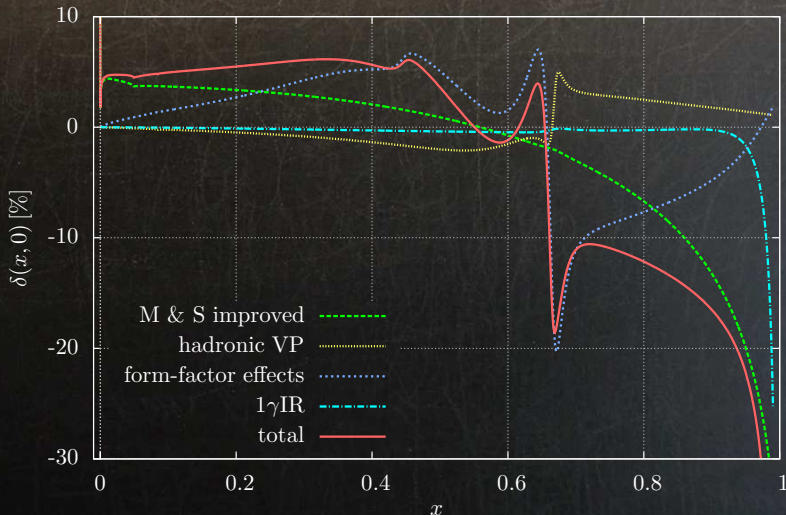
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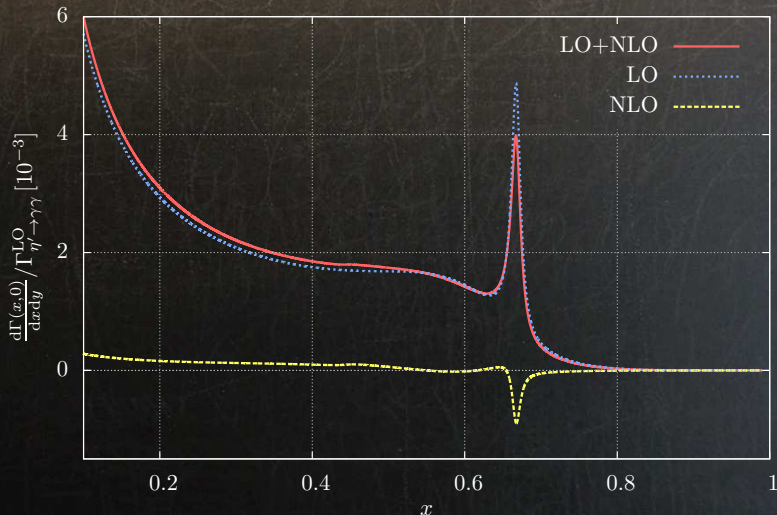


Radiative corrections for $\eta' \rightarrow e^+e^-\gamma$ decays

The two-fold differential decay width $d\Gamma(x,0)$ at NLO

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How are the building blocks **distributed** inside of the composite objects?

↪ electron-nucleon scattering

↪ electromagnetic form factors

↪ known LE quantities: el. charge, mag. moment, electric and magnetic **radii**

Replacement of down quarks (of a nucleon/ Δ) by **strange** quark(s)?

↪ revealing quarks as building blocks of nucleons and hadrons in general

↪ intimate relation among the intrinsic structures of hyperons and nucleons

↪ hyperon electromagnetic and transition form factors contain

complementary information to the nucleon (and Δ) ones



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Experimental knowledge of hyperons rather **limited**

↔ **unstable** (electron scattering difficult)

↔ magnetic moments (and electric charges) known

Form factors at **high** energies

↔ e^+e^- scattering to hyperon + antihyperon

↔ direct and transition form factors accessible

At **low** energies

↔ **Dalitz decay** $Y \rightarrow Y'e^+e^-$

↔ possibly high statistics in **future** at FAIR (PANDA: $p\bar{p}$, HADES: pp)

Dalitz decays in baryon-octet sector?

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Dalitz decay $\Sigma^0 \rightarrow \Lambda e^+ e^-$

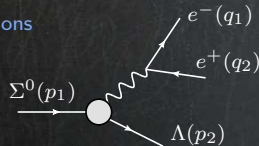
↔ $e^+ e^-$ invariant mass only up to $M_\Sigma - M_\Lambda \simeq 77 \text{ MeV}$

↔ electric and magnetic transition form factors of $\Sigma^0 \rightarrow \Lambda$ transition

↔ extracting transition radii challenging

↔ **high-precision** measurement required

↔ competing with **QED radiative corrections**



Predictions of electric and magnetic radii

↔ *Granados, Leupold and Perotti, EPJA 53 (2017)*

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$\Sigma^0 \Lambda \gamma$ vertex: $\langle 0 | j^\mu | \Sigma^0 \bar{\Lambda} \rangle = e \bar{v}_\Lambda(\vec{p}_2) G^\mu(p_1 + p_2) u_\Sigma(\vec{p}_1)$, with

$$G^\mu(q) \equiv \left[\gamma^\mu - (M_\Sigma - M_\Lambda) \frac{q^\mu}{q^2} \right] G_1(q^2) - \frac{i \sigma^{\mu\nu} q_\nu}{M_\Sigma + M_\Lambda} G_2(q^2)$$

Define magnetic and electric form factors

$$G_M(q^2) \equiv G_1(q^2) + G_2(q^2) = \kappa \left(1 + \frac{1}{6} \langle r_M^2 \rangle q^2 + \mathcal{O}(q^4) \right)$$

$$G_E(q^2) \equiv G_1(q^2) + \frac{q^2}{(M_\Sigma + M_\Lambda)^2} G_2(q^2) = \frac{1}{6} \langle r_E^2 \rangle q^2 + \mathcal{O}(q^4)$$

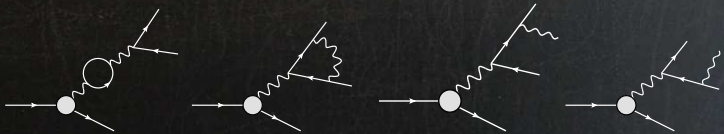
Matrix element squared dominated by the magnetic part

$$\overline{|\mathcal{M}^{\text{LO}}(x, y)|^2} \simeq 2e^4 |G_M(\Delta_M^2 x)|^2 \frac{(1-x)}{x} \left(1 + y^2 + \frac{\nu^2}{x} \right)$$

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Radiative corrections to the differential decay width in **soft-photon** approximation
 \hookrightarrow *Sidhu and Smith, PRD 4 3344 (1971)*

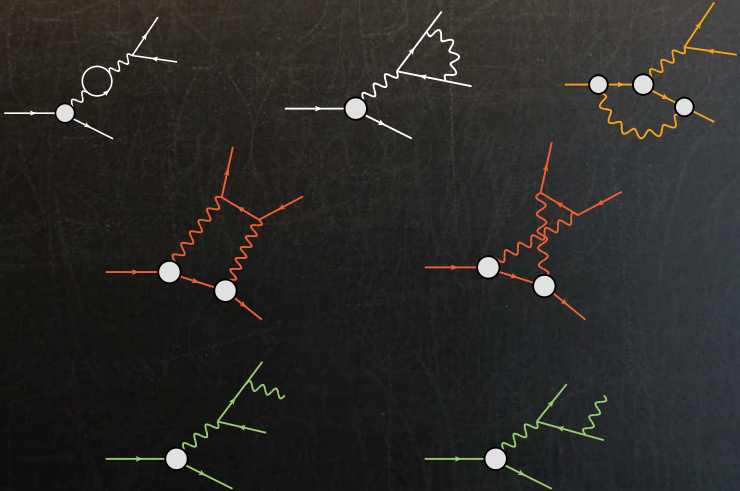


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TH and Leupold, in preparation

↔ inclusive radiative corrections **beyond** the soft-photon approximation



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Low-energy expansion of the form factors:

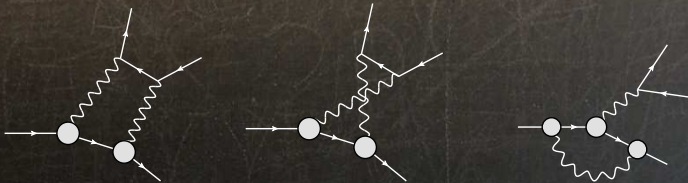
$$G_M((k + q_1 + q_2)^2) \simeq G_M((q_1 + q_2)^2) \left\{ 1 + \frac{1}{6} \langle r_M^2 \rangle [2k \cdot (q_1 + q_2)] \right\},$$

$$G_E((k + q_1 + q_2)^2) \simeq G_E((q_1 + q_2)^2) \left\{ 1 + \frac{2k \cdot (q_1 + q_2)}{(q_1 + q_2)^2} \right\}.$$

Subsequently, integrate over the energy and emission angle of bremsstr. photon
 \hookrightarrow radiative corrections for **inclusive** process

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By loop-momenta-power counting, FFs required to regulate the UV region

1γ IR: UV-convergence already achieved in the simplest case with **constant** FFs
 $\hookrightarrow G_E(q^2) = G_E(0) = 0$ and $G_M(q^2) = G_M(0) = \kappa$

$$G_1(q^2) = \kappa \frac{q^2}{q^2 - M_V^2}, \quad G_2(q^2) = -\kappa \frac{M_V^2}{q^2 - M_V^2}$$

Ansatz satisfying high-energy constraints

$$G_1(q^2) = \kappa \left(3 - \frac{M_V^2 \langle r_M^2 \rangle}{6} \right) \frac{q^2 M_V^4}{(q^2 - M_V^2)^3}, \quad G_2(q^2) = -\kappa \frac{M_V^6}{(q^2 - M_V^2)^3}$$

These contributions to the NLO decay width are found to be **negligible**

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x \ y	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.99	
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π^0 rare decay	0.02	2.67	2.61	2.49	2.34	2.19	2.00	1.75	1.40	0.84	-0.26	-5.84
Introduction	0.03	2.71	2.66	2.55	2.41	2.24	2.04	1.78	1.41	0.83	-0.33	-5.75
Radiative corrections	0.04	2.71	2.67	2.56	2.42	2.26	2.06	1.79	1.41	0.80	-0.40	-5.84
Introduction	0.05	2.69	2.65	2.55	2.42	2.26	2.05	1.78	1.39	0.77	-0.47	-5.98
Radiative corrections	0.06	2.66	2.62	2.53	2.40	2.24	2.04	1.76	1.37	0.73	-0.53	-6.13
π^0 Dalitz dec.	0.07	2.61	2.58	2.49	2.37	2.21	2.01	1.74	1.34	0.69	-0.60	-6.29
Introduction	0.08	2.56	2.53	2.45	2.34	2.18	1.98	1.71	1.30	0.65	-0.66	-6.44
Radiative corrections	0.09	2.51	2.48	2.41	2.29	2.15	1.95	1.68	1.27	0.60	-0.73	-6.60
Introduction	0.10	2.45	2.43	2.36	2.25	2.11	1.91	1.64	1.23	0.56	-0.79	-6.75
Results	0.15	2.14	2.12	2.07	1.99	1.87	1.69	1.42	1.01	0.31	-1.12	-7.47
$\eta^{(\prime)}$ Dalitz ds.	0.20	1.79	1.78	1.75	1.69	1.59	1.43	1.17	0.75	0.04	-1.46	-8.14
Introduction	0.25	1.43	1.42	1.40	1.36	1.28	1.14	0.89	0.48	-0.26	-1.81	-8.78
Radiative corrections	0.30	1.05	1.05	1.04	1.01	0.95	0.82	0.59	0.17	-0.57	-2.18	-9.40
Introduction	0.35	0.65	0.65	0.65	0.64	0.59	0.48	0.26	-0.15	-0.91	-2.57	-10.0
Radiative corrections	0.40	0.23	0.23	0.24	0.24	0.21	0.11	-0.10	-0.51	-1.28	-2.99	-10.6
Results	0.45	-0.22	-0.22	-0.20	-0.18	-0.20	-0.29	-0.49	-0.89	-1.68	-3.43	-11.3
Σ^0 Dalitz dec.	0.50	-0.71	-0.70	-0.67	-0.64	-0.65	-0.72	-0.91	-1.31	-2.11	-3.91	-11.9
Introduction	0.55	-1.23	-1.22	-1.19	-1.15	-1.14	-1.20	-1.38	-1.78	-2.59	-4.43	-12.6
Radiative corrections	0.60	-1.81	-1.79	-1.75	-1.70	-1.68	-1.73	-1.90	-2.30	-3.12	-5.01	-13.3
Introduction	0.65	-2.46	-2.44	-2.38	-2.32	-2.29	-2.32	-2.49	-2.89	-3.72	-5.65	-14.1
Radiative corrections	0.70	-3.19	-3.16	-3.11	-3.03	-2.99	-3.01	-3.17	-3.56	-4.41	-6.38	-14.9
Results	0.75	-4.04	-4.01	-3.94	-3.86	-3.80	-3.81	-3.96	-4.36	-5.22	-7.23	-15.9
Introduction	0.80	-5.06	-5.03	-4.96	-4.86	-4.79	-4.79	-4.93	-5.33	-6.21	-8.26	-17.0
Radiative corrections	0.85	-6.36	-6.33	-6.24	-6.14	-6.05	-6.04	-6.18	-6.58	-7.47	-9.56	-18.4
Introduction	0.90	-8.16	-8.12	-8.03	-7.91	-7.81	-7.79	-7.92	-8.32	-9.24	-11.4	-20.3
Radiative corrections	0.95	-11.2	-11.1	-11.0	-10.9	-10.8	-10.8	-10.9	-11.3	-12.2	-14.4	-23.4
Results	0.99	-18.0	-18.0	-17.9	-17.7	-17.6	-17.6	-17.7	-18.1	-19.0	-21.2	-30.3

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x \ y	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.99	
Introduction	0.01	2.50	2.44	2.31	2.18	2.03	1.87	1.65	1.33	0.83	-0.20	-8.26
π^0 rare decay	0.02	2.67	2.61	2.49	2.34	2.19	2.00	1.75	1.40	0.84	-0.26	-5.84
Introduction	0.03	2.71	2.66	2.55	2.41	2.24	2.04	1.78	1.41	0.83	-0.33	-5.75
Radiative corrections	0.04	2.71	2.67	2.56	2.42	2.26	2.06	1.79	1.41	0.80	-0.40	-5.84
Introduction	0.05	2.69	2.65	2.55	2.42	2.26	2.05	1.78	1.39	0.77	-0.47	-5.98
Radiative corrections	0.06	2.66	2.62	2.53	2.40	2.24	2.04	1.76	1.37	0.73	-0.53	-6.13
π^0 Dalitz dec.	0.07	2.61	2.58	2.49	2.37	2.21	2.01	1.74	1.33	0.69	-0.60	-6.29
Introduction	0.08	2.56	2.53	2.45	2.34	2.18	1.98	1.71	1.30	0.55	-0.66	-6.44
Radiative corrections	0.09	2.51	2.48	2.41	2.29	2.15	1.95	1.68	1.27	0.60	-0.73	-6.60
Introduction	0.10	2.45	2.43	2.36	2.25	2.11	1.91	1.64	1.23	0.56	-0.79	-6.75
Results	0.15	2.14	2.12	2.07	1.99	1.87	1.70	1.44	1.01	0.31	-1.12	-7.47
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Results	0.40	0.23	0.23	0.24	0.24	0.21	0.11	-0.10	-0.51	-1.28	-2.99	-10.6
Introduction	0.45	-0.22	-0.22	-0.20	-0.18	-0.20	-0.29	-0.49	-0.89	-1.68	-3.43	-11.3
Σ^0 Dalitz dec.	0.50	-0.71	-0.70	-0.65	-0.65	-0.65	-0.72	-0.91	-1.31	-2.11	-3.91	-11.9
Introduction	0.55	-1.23	-1.23	-1.19	-1.15	-1.14	-1.20	-1.38	-1.78	-2.59	-4.43	-12.6
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Integrate over the Dalitz plot

$$\frac{\Gamma(\Sigma^0 \rightarrow \Lambda e^+ e^-)}{\Gamma(\Sigma^0 \rightarrow \Lambda \gamma)} = 5.545 \times 10^{-3}$$

↔ consistent with the NLO result in *Sidhu and Smith, PRD 4 3344 (1971)*

$$\frac{\Gamma(\Sigma^0 \rightarrow \Lambda e^+ e^-)}{\Gamma(\Sigma^0 \rightarrow \Lambda \gamma)} = (5.532 + 0.627a) \times 10^{-3} \simeq 5.544 \times 10^{-3}$$

LO result:

$$\frac{\Gamma^{\text{LO}}(\Sigma^0 \rightarrow \Lambda e^+ e^-)}{\Gamma(\Sigma^0 \rightarrow \Lambda \gamma)} = 5.495 \times 10^{-3}$$

⇒ QED NLO correction to the $\Sigma^0 \rightarrow \Lambda e^+ e^-$ decay rate: $\delta = 0.90\%$

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All NLO QED radiative corrections for discussed processes are now available
 \hookrightarrow can be taken into account in **future** experimental analyses

- $\pi^0 \rightarrow e^+e^-$

Vaško and Novotný, JHEP 1110 (2011)

TH, Kampf and Novotný, EPJC 74 (2014)

\hookrightarrow THS model: *TH and S. Leupold, EPJC 75 (2015)*

- $\pi^0 \rightarrow e^+e^-\gamma$

TH, Kampf and Novotný, PRD 92 (2015)

\hookrightarrow **precise** determination of R : *TH, Goudzovski and Kampf, PRL 122 (2018)*

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TH, Kampf, Leupold and Novotný, PRD 97 (2018)

Baryon sector

- $\Sigma^0 \rightarrow \Lambda e^+e^-$

TH and Leupold, in preparation

Ancillary files available together with the papers

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Thank you for listening!