$\pi\pi$ and $K\pi$ scattering amplitudes from lattice QCD

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Hadron 2019
Hadron decays, production and interactions

Guilin Bravo Hotel
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Outline

The Team

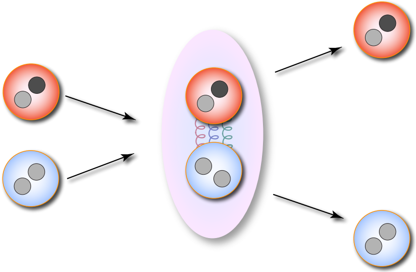
LHPC - Cyprus - Bonn connection

- Stefan Meinel, Gumaro Rendon* (U Arizona)
- John W. Negele, Andrew Pochinsky (MIT)
- Luka Leskovec (JLab)
- Sergey Syritsyn (RIKEN BNL & Stony Brook U)
- Constantia Alexandrou, Srijit Paul (U Cyprus & CyI)
- Giorgio Silvi, Stefan Krieg (FZJ, U Wuppertal)
- M. P. (U Bonn)

* Lattice 2019

Outline

Motivation — $\pi\pi$ and $K\pi$ scattering



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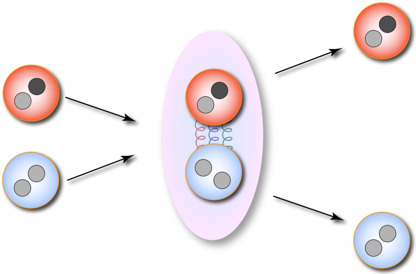

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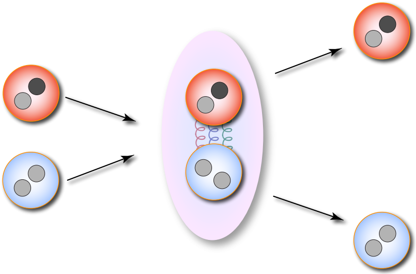

- $\pi\pi$ P-wave scattering with $I = 1$, featuring ρ
- $K\pi$ S- and P-wave scattering with $I = 1/2$, featuring K^*

Motivation — resonant transition amplitudes for QCD-unstable states

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  constraints on hadronic light-by-light scattering contribution to muon $g-2$ (dispersive approach)

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  $|V_{cd}|$, $|V_{cs}|$ known to high precision $\Rightarrow$ laboratories for lattice calculation

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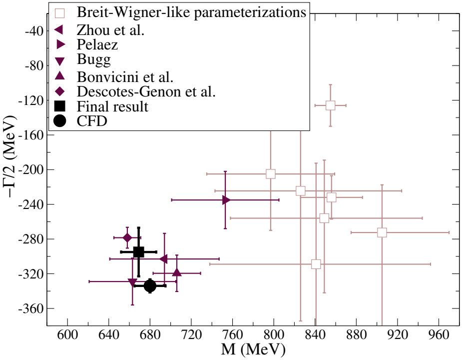
Scattering amplitudes to convert finite-volume $\to$ infinite-volume transition amplitudes (Lellouch-Lüscher factor [Commun.Math.Phys. 219 (2001) 31-44])

Motivation — $K\pi$ scattering with $I = 1/2$

- $K_0^*(700)$ $I\,(J^P) = 1/2\,(0^+)$ (or κ, $K_0^*(800)$) needs confirmation
- $K\pi$ scattering excludes above 825 MeV [Cherry, Pennington, Nucl.Phys. A688 (2001) 823-841]
- reported in production channels $J/\psi \to K^+\pi^-K^-\pi^+$ and $D \to K\pi\pi$ from BES and E791 [Bugg, Eur.Phys.J. A25 (2005) 107-114, Phys.Lett. B632 (2006) 471-474]



[Rodas,Peláez, Ruiz de Elvira, PoS Hadron2017 (2018) 043]

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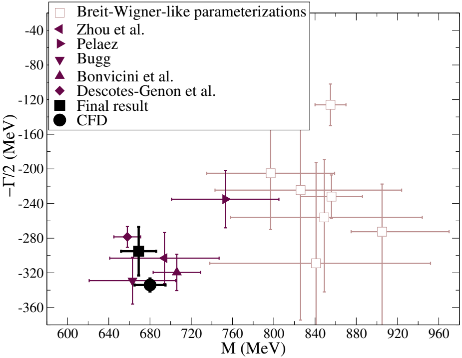

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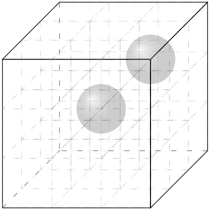
LQCD methods now enable study of quark mass dependence towards physical pion mass [Wilson et al.,Phys.Rev.Lett. 123 (2019) no.4, 042002; Brett et al., Nucl. Phys. B932 (2018), p. 29-51]

Motivation

- Discretized, finite-volume Euclidean space, reduced rotational symmetry



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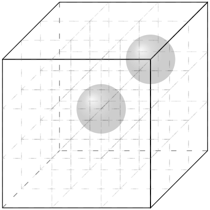

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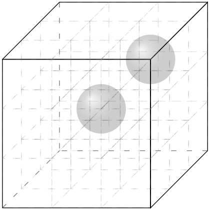

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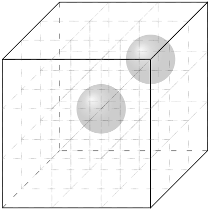

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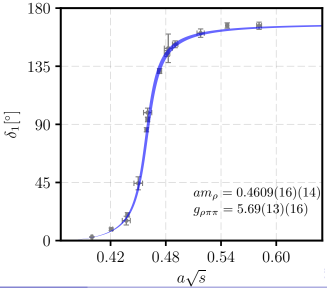
- QCD-unstable states (e.g. ρ, K^* ...)
- Lüscher Quantization condition: lattice spectrum $\rightarrow$ scattering amplitudes

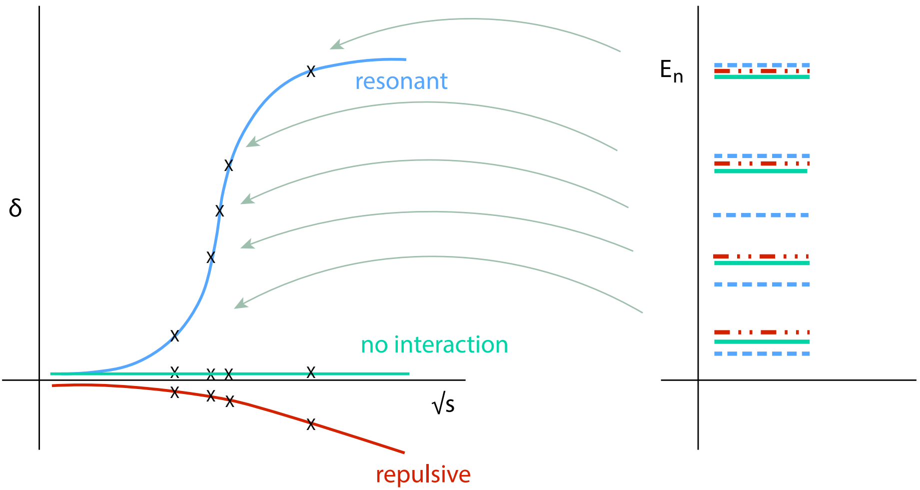
$$\det\left[\mathcal{M}^{-1}\left(E^*, \vec{P}, L\right) + T\left(E^*\right)\right] = 0$$



Outline

Lüscher formalism

Infinite-volume scattering amplitudes $\Longleftarrow$ finite-volume lattice spectrum



Lattice irreducible operator basis for variational analysis ρ, $\pi\pi$ isospin-1

Correlation matrix of 2-point functions

$$C_{ij}(t,\,\vec{P}) = \langle O_i(t,\vec{P})\; O_j(0,\vec{P})^\dagger \rangle$$

$$O_{\bar{q}q} = \sum_{\vec{x}} e^{i\vec{P}\cdot\vec{x}}\,\bar{u}(\vec{x})\Gamma d(\vec{x}), \quad \Gamma = \gamma_i, \gamma_0\gamma_i$$

$$O_{\pi\pi} = \frac{1}{\sqrt{2}}(\pi^+(\vec{p}_1)\pi^0(\vec{p}_2) - \pi^+(\vec{p}_2)\pi^0(\vec{p}_1))$$

$$O_{K\bar{K}} = K^+(\vec{p}_1)\bar{K}^0(\vec{p}_2)\,, \quad \vec{p}_1 + \vec{p}_2 = \vec{P}$$

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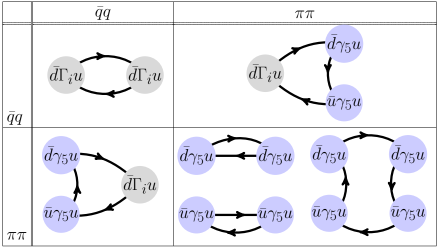

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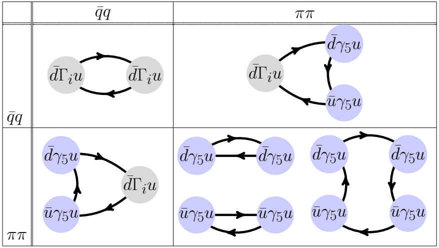

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Quark propagators from numerically solving the lattice Dirac equation

$$D\,S = \eta\,, \qquad \text{huge sparse linear system}$$

Solve the Generalized Eigenvalue Problem

$$\sum_j C_{ij}(t)\,u_j^n = \lambda_n(t, t_0)\sum_j C_{ij}(t_0)\,u_j^n$$

$$\lambda_n(t, t_0) \propto e^{-E_n t} \qquad \text{for large } t$$

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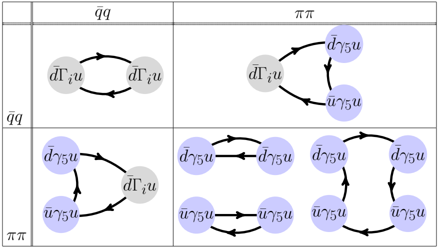

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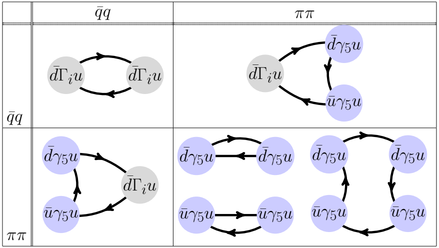

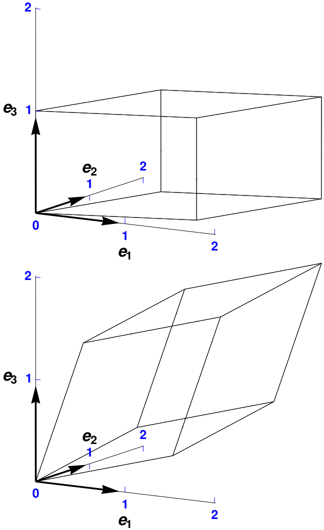

[Göckeler et al. 2012]

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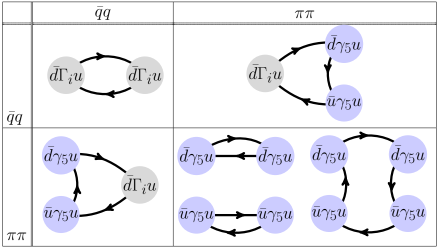

$\vec{P}$ $[\frac{2\pi}{L}]$	$LG(\vec{P})$	Irrep Λ	ℓ
$(0,0,0)$	O_h	T_1^-	$1^-, 3^-, \dots$
$(0,0,1)$	C_{4v}	A_2^-	$1^-, 3^-, \dots$
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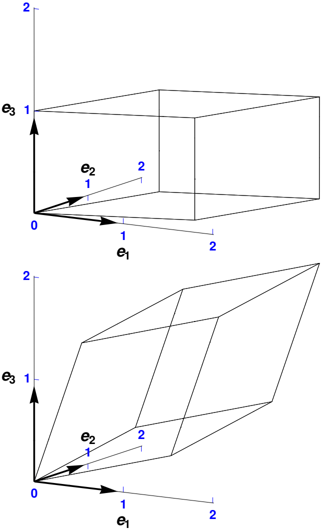

[Göckeler et al. 2012]

$\pi\pi$ $I = 1$ elastic P-wave phase shift [Phys.Rev. D96 (2017) no.3, 034525]

$N_f = 2 + 1$ isotropic, clover-improved Wilson [K. Orginos et al.]

Label	$a/\,\mathrm{fm}$	$L/\,\mathrm{fm}$	$m_\pi/\,\mathrm{MeV}$	$m_K/\,\mathrm{MeV}$	$m_\pi\, L$
C13	0.11403 (77)	3.649 (25)	≈ 317	≈ 530	5.865 (32)

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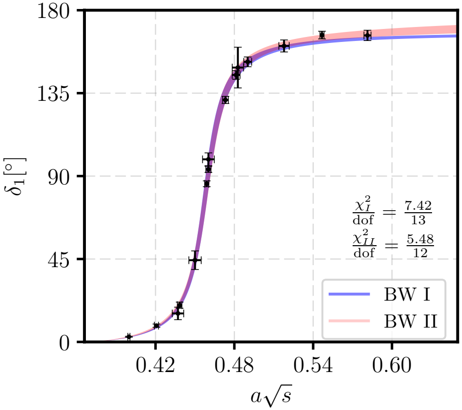

BWI $\Gamma_I(s) = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{k^3}{s}$

BWII $\Gamma_{II}(s) = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{k^3}{s}\frac{1+(k_R r_0)^2}{1+(k r_0)^2}$

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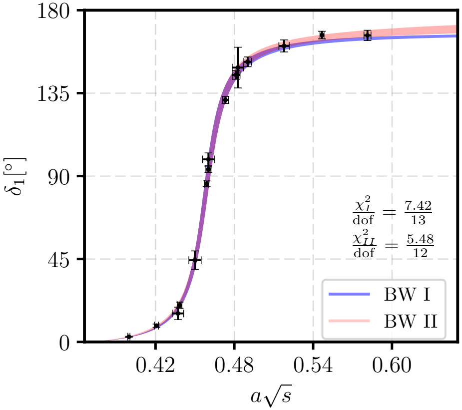

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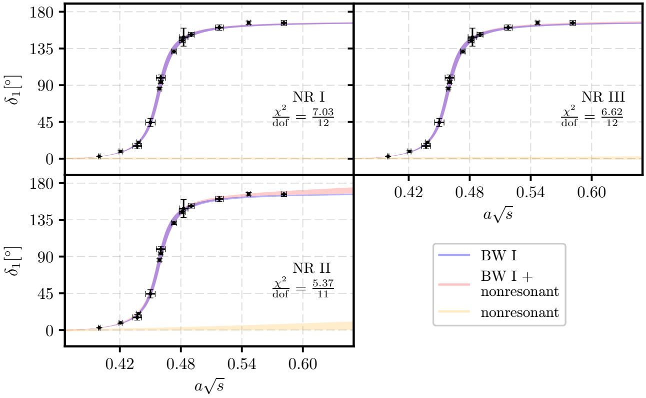
	BWI	BWII
am_ρ	0.4599 (19) (13)	0.4600 (18) (13)
$g_{\rho\pi\pi}$	5.76 (16) (12)	5.79 (16) (12)
$(ar_0)^2$	— — —	8.6 (8.0) (1.2)

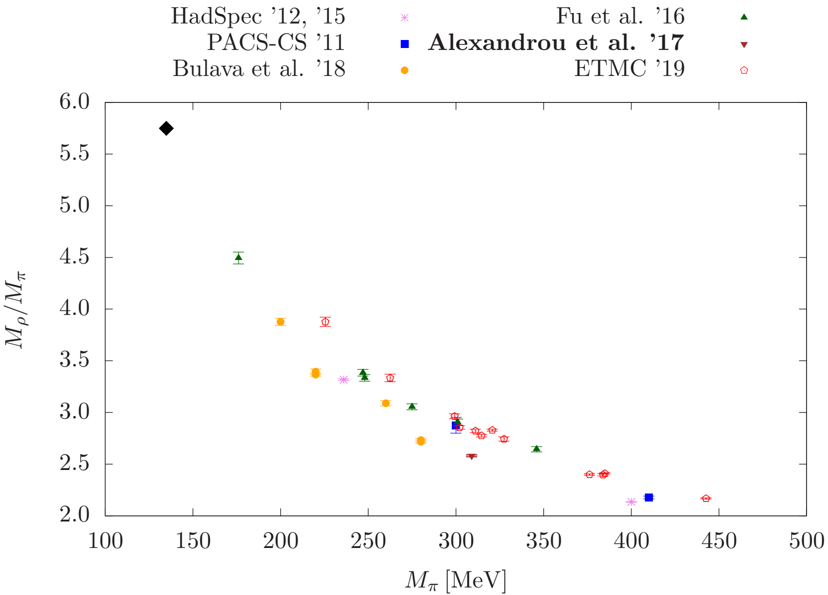
Probing non-resonant background contributions



Model	$\frac{\chi^2}{\text{dof}}$	am_ρ	$g_{\rho\pi\pi}$		
NR I	0.586	0.4600 (19) (13)	5.74 (17) (14)	$A = 0.16\,(31)\,(18)^\circ$	
NR II	0.488	0.4602 (19) (13)	5.84 (21) (20)	$A = -2.9\,(2.7)\,(3.4)^\circ$	$a^{-2}B = 19.2\,(16.6)\,(20.1)^\circ$
NR III	0.552	0.4601 (19) (13)	5.74 (16) (13)	$aa_1^{-1} = -19.8\,(27.4)\,(98.1)$	

const (NR I), linear (NR II) and $\delta_1^{\mathrm{NR}}(s) = \mathrm{arccot}\left(2a_1^{-1}/\sqrt{s - s_{\mathrm{thresh}}}\right)$ (NR III)

Comparison of ρ resonance parameters for $N_f = 2+1,\ 2+1+1$ quark flavors



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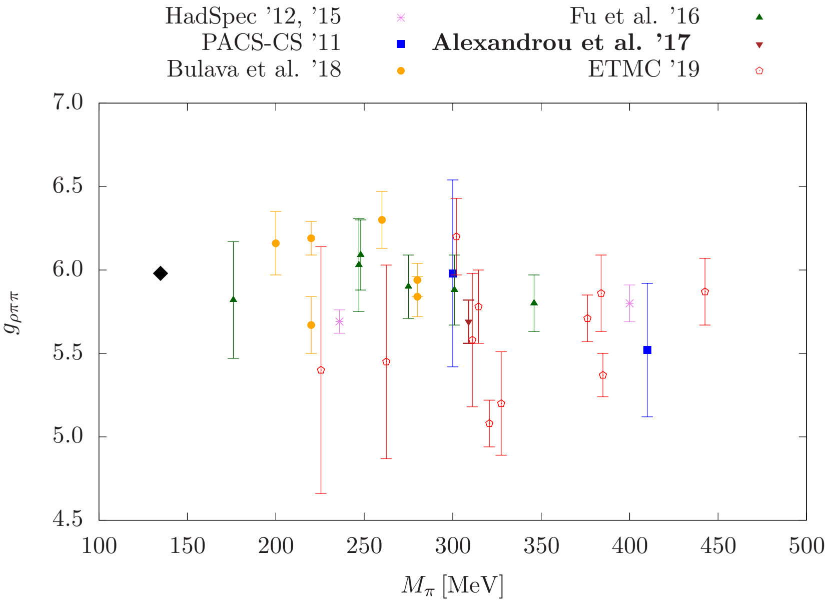

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$K\pi$ interpolators and diagrams

$$K_i^{*+}(p) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \bar{s}(x)\gamma_i u(x)\,, \quad K_{ti}^{*+}(p) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \bar{s}(x)\gamma_t\,\gamma_i\,u(x)$$

$$K_0^{+}(p) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \bar{s}(x)\,u(x)$$

$$O_{K\pi}\,(\vec{p}_1, \vec{p}_2) = \sqrt{\frac{2}{3}}\,\pi^+(\vec{p}_1)\,K^0(\vec{p}_2) - \sqrt{\frac{1}{3}}\,\pi^0(\vec{p}_2)\,K^+(\vec{p}_1)$$

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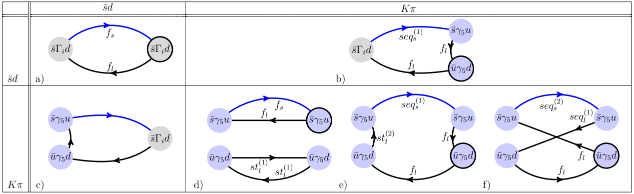

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$K\pi$ system lattice irreducible representations

$\vec{P}\,[\frac{2\pi}{L}]$	Little Group ($LG^{\vec{P}}$)	irrep ($\Lambda^{\vec{P},r}$)	spin content	dimension
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[Leskovec, Prelovsek, Phys. Rev. D85 (2012), p. 114507]

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(0,1,1)	C_{2v}	B_1	J=1,2,...	1
(0,1,1)	C_{2v}	B_2	J=1,2,...	1
(1,1,1)	C_{3v}	$A_1(A_2)$	J=0,1,...	1
(1,1,1)	C_{3v}	E	J=1,2,...	2

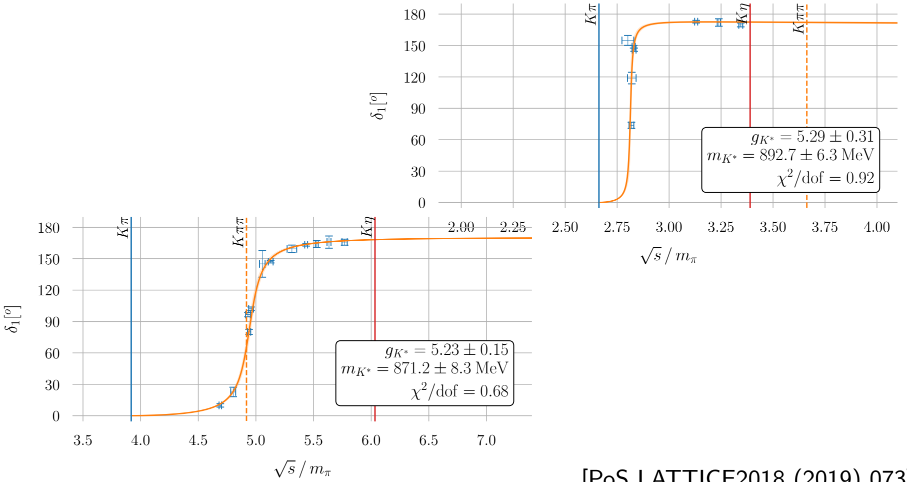
[Leskovec, Prelovsek, Phys. Rev. D85 (2012), p. 114507]

$$\vec{P}_{K\pi} = \frac{2\pi}{L}\,(0,0,1), \quad \Lambda = A_1:$$

$$\left(\left(\mathbf{K}_{00}^{\ell=0}\right)^{-1} - w_{0,0}^{\vec{P}_{K\pi}}\right)\left(\left(\mathbf{K}_{0,0}^{\ell=1}\right)^{-1} - w_{0,0}^{\vec{P}_{K\pi}} - 2w_{2,0}^{\vec{P}_{K\pi}}\right) - 3(w_{1,0}^{\vec{P}_{K\pi}})^2 = 0$$

$K\pi$ leading P-wave only

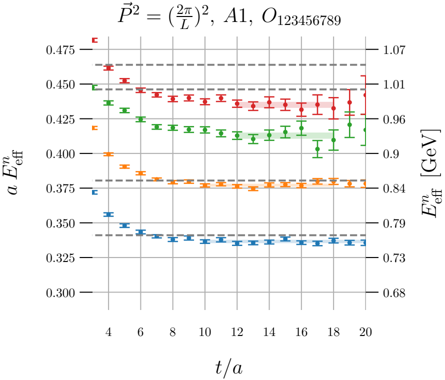
Label	$N_L^3 \times N_t$	a (fm)	L (fm)	m_π (MeV)	m_K (MeV)	N_{config}
C13	$32^3 \times 96$	0.11403(77)	3.65	317(2)	527(4)	896
D6	$48^3 \times 96$	0.08766(79)	4.21	178(2)	514(5)	328



[PoS LATTICE2018 (2019) 073]

Combined S- and P-wave analysis

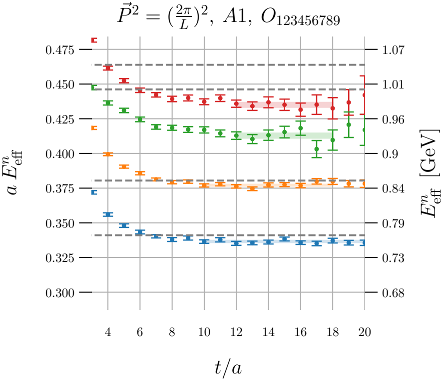
"Inverse Lüscher" approach:



Combined S- and P-wave analysis

"Inverse Lüscher" approach:

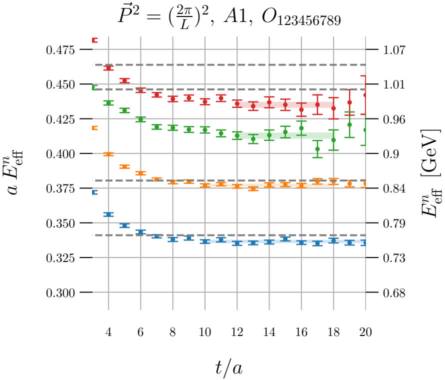
- choose a parametrization for the K matrix



Combined S- and P-wave analysis

"Inverse Lüscher" approach:

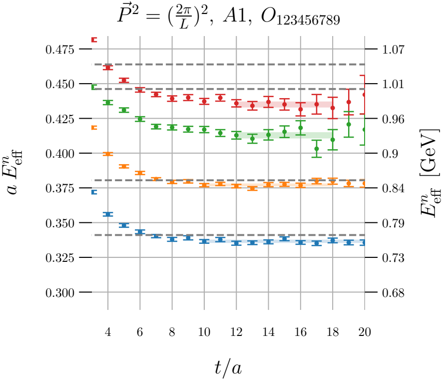
- choose a parametrization for the K matrix
- given K-matrix parameter set + Quantization condition $\Longrightarrow$ predict the finite-volume lattice spectrum



Combined S- and P-wave analysis

"Inverse Lüscher" approach:

- choose a parametrization for the K matrix
- given K-matrix parameter set + Quantization condition $\Longrightarrow$ predict the finite-volume lattice spectrum
- fit K-matrix model parameters to lattice spectrum



Combined *S*- and *P*-wave analysis

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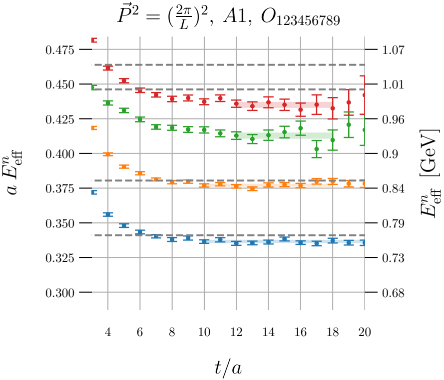

- recover phase shifts, investigate T-matrix poles

Combined S- and P-wave analysis

"Inverse Lüscher" approach:

- choose a parametrization for the K matrix
- given K-matrix parameter set + Quantization condition $\Longrightarrow$ predict the finite-volume lattice spectrum
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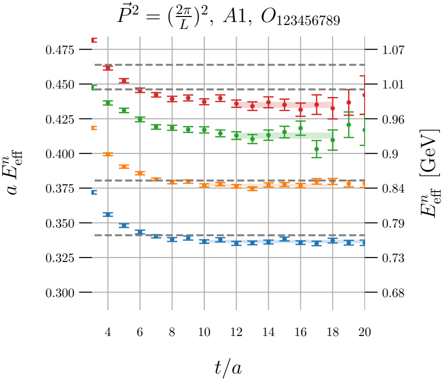


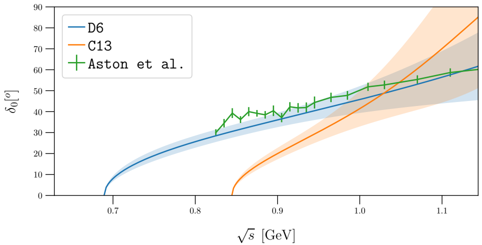
- recover phase shifts, investigate T-matrix poles
- probe systematic uncertainty from dependence on chosen parametrization

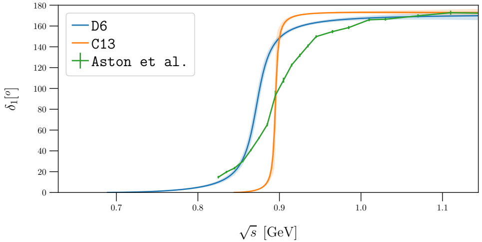
Combined S- and P-wave analysis — (I) Chung's parametrization

Single K-matrix pole:
[Chung et al., Annalen Phys. 4 (1995), p. 404-430]

$$K_{00} = \frac{g^2(s)}{m_R^2 - s}$$

$$g(s) = g^0\, B^\ell(q, q_R)\, \sqrt{\rho_{00}}$$





[Aston et al., Nucl. Phys. B296 (1988), p. 493-526]

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$$g(s) = g^0\, B^\ell(q, q_R)\, \sqrt{\rho_{00}}$$

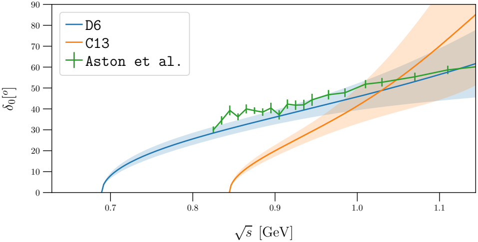
S-wave poles:

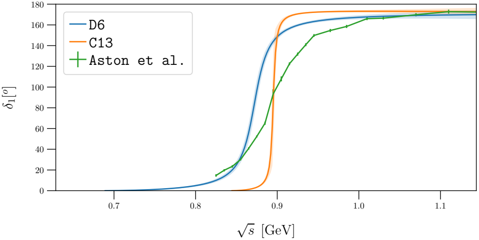
$m_\pi = 317\,\mathrm{MeV}$:

$(1.11 \pm 0.03)\,\mathrm{GeV}\,,\ (-0.14 \pm 0.04)\,\mathrm{GeV}$

$m_\pi = 178\,\mathrm{MeV}$:

$(1.37 \pm 0.27)\,\mathrm{GeV}\,,\ (-0.39 \pm 0.19)\,\mathrm{GeV}$





[Aston et al., Nucl. Phys. B296 (1988), p. 493-526]

Combined S- and P-wave analysis — (II) parametrizations with Adler zero

Bugg's parametrization:

$$\hat{K}_{00}^{(\ell=0)} = \frac{g^2(s)}{m_R^2 - s}$$

$$g(s) = g^0 \sqrt{(s - s_A)} \sqrt{\rho_{00}}\, k^\ell\,, \quad s_A \approx m_K^2 - m_\pi^2/2$$

[Bugg, Phys. Rev. D81 (2010), p. 014002]

Combined S- and P-wave analysis — (II) parametrizations with Adler zero

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[Bugg, Phys. Rev. D81 (2010), p. 014002]

Conformal mapping:

$$(K_{00}^{(\ell)})^{-1} = \cot \delta_\ell(s) = \frac{\sqrt{s}}{2k^{2\ell+1}} F(s) \sum_n B_n \omega^n(s)$$

$$F(s) = 1/(s - s_A) \quad \text{for } S \text{ wave}$$

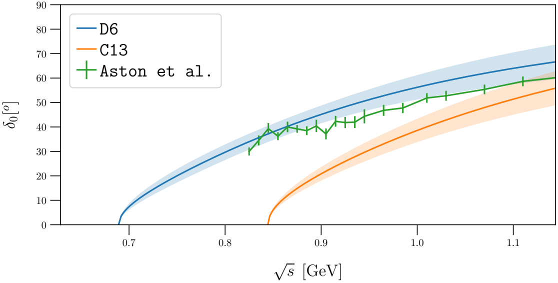
$$\omega(y) = \frac{\sqrt{y} - \alpha\sqrt{y_0 - y}}{\sqrt{y} + \alpha\sqrt{y_0 - y}}, \quad y(s) = \left(\frac{s - \Delta_{K\pi}}{s + \Delta_{K\pi}}\right)^2$$

$$\Delta_{K\pi} = m_K^2 - m_\pi^2$$

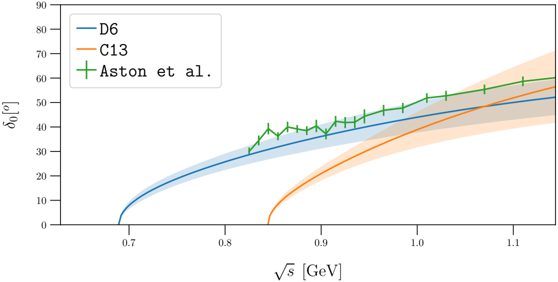
[Peláez, Rodas, Phys. Rev. D93.7 (2016), p. 074025]

Combined S- and P-wave analysis — (II) parametrizations with Adler zero

Bugg's parametrization (S-wave)



Conformal mapping (S-wave)



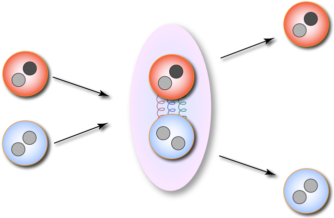
Combined S- and P-wave analysis — (II) parametrizations with Adler zero

S- and P-wave T-matrix poles

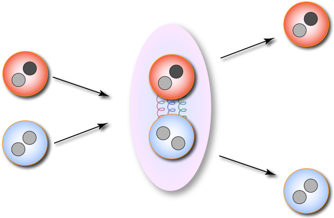
m_π [MeV]	Model	T-matrix Poles [GeV]	χ^2/dof
317	Conformal Mapping	S-wave: $(0.841, -0.329) \pm (0.149, 0.081)$ P-wave: $(0.895, -0.00250) \pm (0.006, 0.00021)$	0.758
	Bugg's parametrization	S-wave: $(0.840, -0.342) \pm (0.077, 0.044)$ P-wave: $(0.895, -0.00250) \pm (0.006, 0.00021)$	0.757
178	Conformal mapping	S-wave: $(0.712, -0.440) \pm (0.040, 0.081)$ P-wave: $(0.872, -0.013) \pm (0.008, 0.001)$	1.01
	Bugg's parametrization	S-wave: $(0.711, -0.297) \pm (0.074, 0.029)$ P-wave: $(0.872, -0.013) \pm (0.008, 0.001)$	1.39

Outline

Outlook

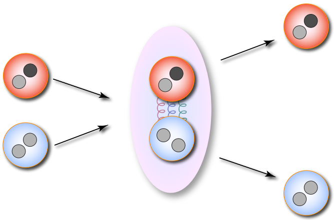


Outlook



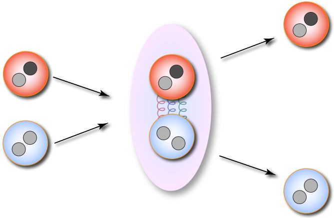
- $\pi\pi$ @ $m_\pi = 178\,\mathrm{MeV}$

Outlook



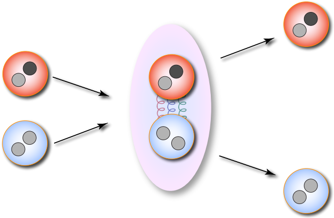
- $\pi\pi$ @ $m_\pi = 178\,\mathrm{MeV}$
- coupled channel analysis $\pi\pi - K\bar{K}$ and $K\pi - K\eta$

Outlook



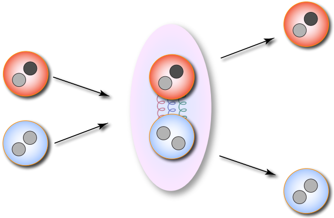
- $\pi\pi$ @ $m_\pi = 178\,\mathrm{MeV}$
- coupled channel analysis $\pi\pi - K\bar{K}$ and $K\pi - K\eta$
- resonant transition amplitudes ("$1 \to 2$")
- resonance form factors

Outlook



- $\pi\pi$ @ $m_\pi = 178\,\mathrm{MeV}$
- coupled channel analysis $\pi\pi - K\bar{K}$ and $K\pi - K\eta$
- resonant transition amplitudes (“1 $\to$ 2”)
- resonance form factors
- reduction of statistical uncertainty

Outlook

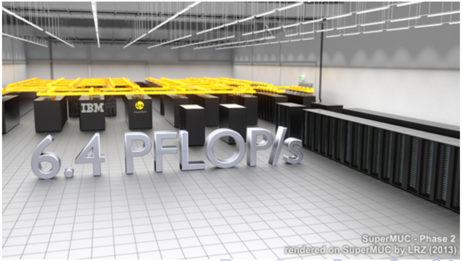


- $\pi\pi$ @ $m_\pi = 178\,\mathrm{MeV}$
- coupled channel analysis $\pi\pi - K\bar{K}$ and $K\pi - K\eta$
- resonant transition amplitudes ("$1 \to 2$")
- resonance form factors
- reduction of statistical uncertainty
- $K\pi\pi$ interpolators, 3-particle quantization

Acknowledgements — The Machines

National Energy Research Scientific Computing Center, **Cori** @ NERSC

Gauss-Center for Supercomputing, **SuperMUC** @ LRZ Munich



Thank you very much for your attention.

Lüscher formalism — 2-pt correlation functions [Rev.Mod.Phys. 90 (2018) no.2, 025001]

$$C_L\left(x_4 - y_4, \vec{P}\right) = \int\limits_L d^3x \int\limits_L d^3y \, \mathrm{e}^{i\vec{P}(\vec{x}-\vec{y})} \left[\langle 0 \,|\, \mathrm{T}\, \mathcal{A}(x)\, \mathcal{B}^{\dagger}(y) \,|\, 0 \rangle\right]_L$$

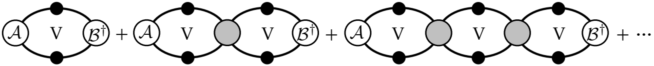
Lüscher formalism — 2-pt correlation functions [Rev.Mod.Phys. 90 (2018) no.2, 025001]

$$C_L\left(x_4 - y_4, \vec{P}\right) = \int\limits_L d^3x \int\limits_L d^3y \, \mathrm{e}^{i\vec{P}(\vec{x}-\vec{y})} \left[\langle 0 \,|\, \mathrm{T}\,\mathcal{A}(x)\,\mathcal{B}^{\dagger}(y) \,|\, 0\rangle\right]_L$$

$$= L^6 \sum_n \langle 0 \,|\, \mathcal{A}(0) \,|\, E_n, \vec{P}, L\rangle \, \langle E_n, \vec{P}, L \,|\, \mathcal{B}^{\dagger}(0) \,|\, 0\rangle \, \mathrm{e}^{-E_n\,(x_4 - y_4)}$$

Lüscher formalism — 2-pt correlation functions [Rev.Mod.Phys. 90 (2018) no.2, 025001]

$C_L(P) =$



$$C_L\left(x_4 - y_4, \vec{P}\right) = \int_L d^3x \int_L d^3y \, \mathrm{e}^{i\vec{P}(\vec{x}-\vec{y})} \left[\langle 0 \,|\, \mathrm{T}\, \mathcal{A}(x)\, \mathcal{B}^\dagger(y) \,|\, 0 \rangle\right]_L$$

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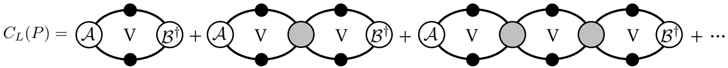
$$= L^3 \int \frac{dP_4}{2\pi} \, \mathrm{e}^{iP_4(x_4-y_4)} \left[C_\infty(P) + A^*(P) \left[\mathcal{M}^{-1}(P,L) + T(P)\right]^{-1} B^*(P) \right]$$

Lüscher formalism — 2-pt correlation functions [Rev.Mod.Phys. 90 (2018) no.2, 025001]



$$C_L\left(x_4 - y_4, \vec{P}\right) = \int_L d^3x \int_L d^3y \, \mathrm{e}^{i\vec{P}(\vec{x}-\vec{y})} \left[\langle 0 \,|\, \mathrm{T}\, \mathcal{A}(x)\, \mathcal{B}^\dagger(y) \,|\, 0 \rangle\right]_L$$

$$= L^6 \sum_n \langle 0 \,|\, \mathcal{A}(0) \,|\, E_n, \vec{P}, L \rangle \langle E_n, \vec{P}, L \,|\, \mathcal{B}^\dagger(0) \,|\, 0 \rangle \, \mathrm{e}^{-E_n\,(x_4 - y_4)}$$

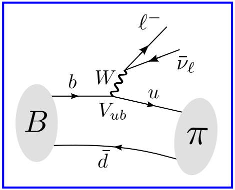
$$= L^3 \int \frac{dP_4}{2\pi} \mathrm{e}^{iP_4(x_4-y_4)} \left[C_\infty(P) + A^*(P) \left[\mathcal{M}^{-1}(P,L) + T(P)\right]^{-1} B^*(P)\right]$$

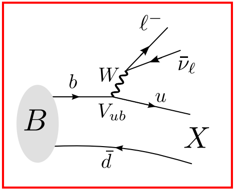
$$\mathcal{M}_{\ell m;\ell' m'}(P,L) = \left[\frac{1}{L^3}\sum_{\vec{k}} - \int \frac{d^3k}{(2\pi)^3}\right] \frac{4\pi\, Y_{\ell m}(\hat{k}^*)\, Y^*_{\ell' m'}(\hat{k}^*)}{2\omega_k\, 2\omega_{P-k}\, \left(E - \omega_k - \omega_{P-k} + i\epsilon\right)} \left(\frac{k^*}{q^*}\right)^{\ell+\ell'}$$

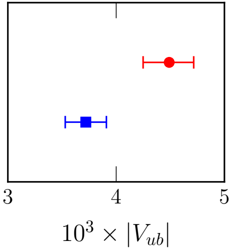
$$\mathbf{0} = \mathbf{det}\left[\mathcal{M}^{-1}\left(E_n, \vec{P}; L\right) + T(E_n)\right]$$

Lüscher Quantization Condition

Motivation - CKM matrix element V_{ub}

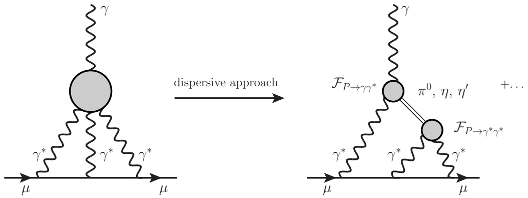


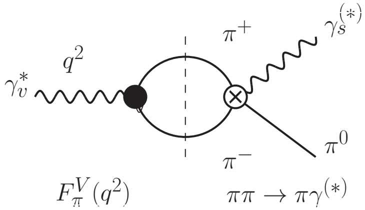




- precision of V_{ub} limited by tension between
  - exclusive B-decay, $B \to \pi\, l\, \bar{\nu}$ and
  - inclusive B-decays, $B \to X_u\, l\, \bar{\nu}$
- $B \to (\rho \to \pi\pi)\, l\, \bar{\nu}$ sensitive probe for transition $b \to u\, l\, \bar{\nu}$
- experiment: Babar and Belle Collaborations
- needs matrix element $\langle \pi\pi \,|\, \bar{u}\, \Gamma\, b \,|\, B \rangle$ from lattice QCD

Motivation - $\pi\gamma \to \pi\pi$ and muon $g-2$ @ HLbL





$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(M_\pi^2, q_1^2, q_2^2) = \mathcal{F}_{vs}(q_1^2, q_2^2) + (q_1 \leftrightarrow q_2)$$

$$\mathcal{F}_{vs}(s, 0) = f_{\pi^0\gamma}(s) = f_{\pi^0\gamma}(0) + \frac{s}{12\pi^2} \int_{(2M_\pi)^2}^{\infty} ds' \, \frac{q_\pi^3(s') \, \left(F_\pi^V(s')\right)^* \, f_{\ell=1}(s')}{s'^{3/2}\,(s'-s)}$$

$$\mathcal{F}_{\gamma\pi\to\pi\pi} = \sum_{\text{odd } \ell} f_\ell \, P'_\ell$$

[Colangelo et al. 2014, Hoferichter et al. 20012, 2014]