The first observation of narrow peak and isospin-violating Lambda(1405) production

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Outline

- $\Lambda_c^+ \to \pi^+ K^* N$ (exp data)
- triangle singularities
- The dynamical origin of the $\Lambda(1405)$
- The $\Lambda_c \to \pi^+ \pi^0 \pi^0 \Sigma^0$ and isospin forbidden $\Lambda(1405)$ production


Anomalous enhancement of the isospin-violating $\Lambda(1405)$ production by a triangle singularity in $\Lambda_c \to \pi^+ \pi^0 \pi^0 \Sigma^0$
\[ \Lambda_c^+ \rightarrow \pi^+ \bar{K}^* N \text{ weak decay} \]

decay mechanism at quark level

At the quark level, the **Cabibbo-allowed vertex** is formed through an **external emission** of a \( W \) boson which is also color-favored, producing a \( u \bar{d} \) pair that forms the \( \pi^+ \), with the remaining \( sud \) quarks hadronizing ...
Hadronization

through $\bar{q}q$ creation with vacuum quantum numbers

$\Lambda^+_{c} \rightarrow W^+_{uu} + \bar{d}u + \bar{s}s + \pi^+$

$ud(I=0)$ spectators $\implies$ also have $I=0$ in the final state

This, added to the $s$ quark that has no isospin, gives us $I=0$ for the final baryon before the hadronization, and this continues to be the case after the hadronization which is based on strong interaction.

$H = \sum_{i=1}^{3} s\bar{q}_i q_i \frac{1}{\sqrt{2}} (ud - du) = \sum_{i=1}^{3} M_{3i} q_i \frac{1}{\sqrt{2}} (ud - du)$  \hspace{1cm} (1)
\[
M = \begin{pmatrix}
  u \bar{u} & u \bar{d} & u \bar{s} \\
  d \bar{u} & d \bar{d} & d \bar{s} \\
  s \bar{u} & s \bar{d} & s \bar{s}
\end{pmatrix}
\]

\[
M \rightarrow V = \begin{pmatrix}
  \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\
  \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\
  K^{*0} & K^{*-} & \phi
\end{pmatrix}
\]

Wave functions of mesons

\[
\rho^0 = \frac{1}{\sqrt{2}}(u \bar{u} - d \bar{d}), \quad \rho^+ = u \bar{d}, \quad \rho^- = d \bar{u},
\]

\[
\omega = \frac{1}{\sqrt{2}}(u \bar{u} + d \bar{d}), \quad \phi = s \bar{s},
\]

\[
K^{*0} = d \bar{s}, \quad K^{*-} = s \bar{u}, \quad K^{*+} = u \bar{s}, \quad \bar{K}^{*0} = s \bar{d}.
\]
\[ H = K^{*-} u \frac{1}{\sqrt{2}} (ud - du) + \bar{K}^{*0} d \frac{1}{\sqrt{2}} (ud - du) + \phi s \frac{1}{\sqrt{2}} (ud - du) \quad (2) \]

Wave functions of baryons

\[ p = \frac{1}{\sqrt{2}} u(ud - du), \quad n = \frac{1}{\sqrt{2}} d(ud - du), \]
\[ \Lambda = \frac{1}{2 \sqrt{3}} [u(ds - sd) + d(su - us) - 2s(ud - du)], \]
\[ \Sigma^0 = \frac{1}{2} [u(ds - sd) - d(su - us)]. \]

After the hadronization

\[ H = K^{*-} p + \bar{K}^{*0} n - \sqrt{\frac{2}{3}} \phi \Lambda \quad (3) \]

- **neglect the \( \phi \Lambda \) component** ⇐ not contribute to triangle singularity mechanism
- **\( s \frac{1}{\sqrt{2}} (ud - du) \)** has zero overlap with \( \Sigma^0 \) ⇒ \( \phi \Sigma^0 \) component not appear \((I = 1)\)
Isospin forbidden 1) cancellation of diagrams if equal masses; 2) The different masses of the kaons make the cancellation partial and we can see the $\Lambda(1405)$ [PRD97,116004]

Triangle Singularity if three intermediate particles are on shell and $K^*$ and $\pi^0$ are parallel $\Rightarrow$ the mechanism generates a singularity in the amplitude for zero width of the $K^*$, or a peak if the width is considered [L. D. Landau, Nucl. Phys. 13 (1959) 181]
Triangle Singularity (TS)


Triangle singularity at the physical boundary can be obtained by solving the equation

\[ q_{on+} = q_{a-}, \quad \text{with} \quad q_{on+} = \frac{1}{2M} \sqrt{\lambda (M^2, m_1^2, m_2^2)}, \quad q_{a-} = \gamma (\beta E_2^* - p_2^*) , \]  

where \( E_2^* = \frac{(m_{23}^2 + m_2^2 - m_3^2)}{(2m_{23})} \) and \( p_2^* = \sqrt{\lambda (m_{23}^2, m_2^2, m_3^2)}/(2m_{23}) \)

see 1) E. Oset’s talk at session 5 (8:55 am, 20/8);
2) H. X. Chen’s talk at session 5 (9:55 am, 20/8);
3) also P. Pavao’s talk at session 5 (11:30 am, 20/8)

Triangle Singularity

TS in simulating a resonance requires very special kinematics → process dependent !!!

In explaining successfully the COMPASS “a_1(1420)” peak

[2] Aceti, Dai & Oset, “a_1(1420) peak as the \(\pi f_0(980)\) decay mode of the \(a_1(1260)\) ”, PRD94(2016)096015

In some particular modes, the production rate is enhanced by the presence of a TS in the reaction mechanism.
**$ar{K}N$ Interaction and $\Lambda(1405)$ resonance**


\[ t_3 \equiv t_{\bar{K}N \to \pi^0 \Sigma^0} , \]
\[ T = [1 - VG]^{-1} V . \]

where $V_{ij}$ are obtained from the chiral Lagrangians [NPA635(1999)99]

$G$ is the meson-baryon loop function for the intermediate states

Very good reproduction is obtained of scattering data and the threshold parameters

Two $\Lambda(1405)$ are generated from this interaction
$\Lambda_c^+ \rightarrow \pi^+ \bar{K}^* N$

since the $\Lambda_c^+ \rightarrow \pi^+ K^* p$ process can proceed via $s$-wave, the amplitude

$$t_{\Lambda_c^+ \rightarrow \pi^+ K^* p} = A \vec{\sigma} \cdot \vec{\epsilon},$$

where a scalar function is made between the spin and the $\bar{K}^*$ polarization.

The $K^* p$ invariant mass distribution

$$\frac{d\Gamma_{\Lambda_c^+ \rightarrow \pi^+ K^* p}}{dM_{\text{inv}}(K^* p)} = \frac{1}{(2\pi)^3} \frac{2M_{\Lambda_c^+} 2M_p}{4M_{\Lambda_c^+}^2} p_{\pi^+} \tilde{p}_{K^*} \sum \sum |t_{\Lambda_c^+ \rightarrow \pi^+ K^* p}|^2,$$

where $p_{\pi^+}$ is the momentum of $\pi^+$ in the $\Lambda_c^+$ rest frame, and $\tilde{p}_{K^*}$ is the momentum of $K^*$ in the $K^* p$ rest frame.

By calculating the width of this decay, using the experimental branching ratio of this decay $Br(\Lambda_c^+ \rightarrow \pi^+ K^* p) = (1.5 \pm 0.5) \times 10^{-2}$ [PRD98(2018)030001], we can determine the value of the constant $|A|$. [PRD97, 116004]
For the **first** diagram

\[
\Lambda_c^+ \rightarrow \pi^+ \pi^0 \Sigma^0 = -A \frac{1}{\sqrt{2}} g \vec{\sigma} \cdot \vec{k} \quad t_{K^- p \rightarrow \pi^0 \Sigma^0} \quad t_T,
\]

where \( t_T \equiv t_T(M_{K^*}, M_p, M_{K^-}) \) for the triangle loop function for the decay

\[
t_T = i \int \frac{d^4 q}{(2\pi)^4} \frac{2M_p}{q^2 - M_p^2 + i\epsilon} \frac{(2 + \frac{\vec{q} \cdot \vec{k}}{k^2})}{(P - q)^2 - m_{K^*}^2 + i\epsilon (P - q - k)^2 - m_{K^-}^2 + i\epsilon}.
\]
The final differential distributions

\[
\frac{1}{\Gamma_{\Lambda_c^+}} \frac{d^2\Gamma}{dM_{\text{inv}}(\pi^0\Lambda(1405))dM_{\text{inv}}(\pi^0\Sigma^0)} = \frac{1}{(2\pi)^5} \frac{M_{\Sigma^0}}{M_{\Lambda_c^+}} \tilde{p}_\pi + \tilde{q}_{\Sigma^0} \frac{1}{2} g^2 \frac{A^2}{\Gamma_{\Lambda_c^+}} |\vec{k}|^3 \\
\times \left| t_T(m_{K^*}, M_p, m_{K^-})t_{K^-p \rightarrow \pi^0\Sigma^0} - t_T(m_{\bar{K}^*0}, M_n, m_{\bar{K}^0})t_{\bar{K}^0n \rightarrow \pi^0\Sigma^0} \right|^2.
\]

PRD97, 116004
The final results

$\bar{K}N$ Interaction (real & imaginary parts)

\[
\begin{align*}
\{ & t_{14} & t_{K^-p \rightarrow \pi^0\Sigma^0} \\
 & t_{24} & t_{\bar{K}^0n \rightarrow \pi^0\Sigma^0} 
\end{align*}
\]

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<tbody>
<tr>
<td>$K^-p$</td>
<td>$\bar{K}^0n$</td>
<td>$\pi^0\Lambda$</td>
<td>$\pi^0\Sigma^0$</td>
<td>$\eta\Lambda$</td>
<td>$\eta\Sigma^0$</td>
<td>$\pi^+\Sigma^-$</td>
<td>$\pi^-\Sigma^+$</td>
<td>$K^+\Xi^-$</td>
<td>$K^0\Xi^0$</td>
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Amplitudes [MeV$^{-1}$]

- $\text{Re}(t_{14})$
- $\text{Re}(t_{24})$
- $\text{Im}(t_{14})$
- $\text{Im}(t_{24})$

$M_{\text{inv}}(\bar{K}N)$ [MeV]
**Triangle amplitude**

The peak of $t_T(m_{K^*-}, M_p, m_{K^-})$ is related to the $K^*-p$ threshold.

- **Real part** $\text{Re}(t_T) \sim 1838\text{ MeV}$, related to the $K^*-p$ threshold.
- **Imaginary part** $\text{Im}(t_T) \sim 1908\text{ MeV}$, dominating for the larger invariant masses for $\pi^0 R$ due to the triangle singularity.
- **Absolute value** $|t_T| \sim 1868\text{ MeV}$

$M_{\text{inv}}(R) \equiv M_{\text{inv}}(\pi^0 \Sigma^0)$ fixed at 1420 MeV.
The remarkable observation of a peak tied to the $\Lambda(1405)$ state close to the $\bar{K}N$ threshold of 1432 MeV.

It is also remarkably narrow and is tied to the difference of masses, mostly from the $K^-$ and $\bar{K}^0$ mass difference (see next page)

FIRST TIME!!!
7 MeV!!!
Unusual narrow width
Discussion on separate effects

Double differential width of $\Lambda^+ \rightarrow \pi^+ \pi^0 \pi^0 \Sigma^0$

Fixed at $M_{\text{inv}}(\pi^0R) = 1890\text{MeV}$

(a) scattering amplitude $t_{K^-p \rightarrow \pi^0\Sigma^0}$, $t_{K^0n \rightarrow \pi^0\Sigma^0}$ with the physical masses (isospin violation)

but Triangle loop $m_p = m_n$, $m_{\bar{K}^0} = m_{K^-}$ (isospin symmetric);

(b) $t_{K^-p \rightarrow \pi^0\Sigma^0}$, $t_{K^0n \rightarrow \pi^0\Sigma^0}$ isospin symmetric (equal) and physical masses for $p$, $n$, $K^-$, $\bar{K}^0$;

(c) scattering amplitude isospin symmetric, $m_{\bar{K}^0} = m_{K^-}$, but $m_p$ and $m_n$ with their physical values

(d) scattering amplitude isospin symmetric, $m_p = m_n$, but $m_{\bar{K}^0}$ and $m_{K^-}$ with their physical values

It can be seen that the peak comes mostly from the $K^-$ and $\bar{K}^0$ mass difference

[PRD97,116004]
The appearance of a **narrow resonance** in the isospin forbidden reactions due to **different kaon masses** also appears in the $f_0(980)$ or $a_0(980)$ isospin forbidden production.


**“Isospin violation in $J/\Psi \rightarrow \phi\pi^0\eta$ decay and the $f_0 - a_0$ mixing”**, L. Roca, Phys. Rev. D 88 (2013) 014045

**“Isospin breaking and $f_0(980)-a_0(980)$ mixing in the $\eta(1405) \rightarrow \pi^0 f_0(980)$ reaction”**, F. Aceti, W. H. Liang, E. Oset, J. J. Wu and B. S. Zou, Phys. Rev. D 86 (2012) 114007
Differential distribution and branching ratio

\[
\frac{1}{\Gamma_{\Lambda_c^+}} \frac{d\Gamma}{dM_{\text{inv}}(\pi^0 R)} [10^{-9} \text{ MeV}^{-1}]
\]

\[M_{\text{inv}}(\pi^0 R) \ [\text{MeV}]\]

\[
\Gamma_{\Lambda^+ c} \frac{d\Gamma}{dM_{\text{inv}}(\pi^0 R)} \times 10^{-9} \text{ MeV}^{-1}
\]

\[
Br(\Lambda_c^+ \rightarrow \pi^+\pi^0\Lambda(1405); \Lambda(1405) \rightarrow \pi^0\Sigma^0) = (4.17 \pm 1.39) \times 10^{-6}
\]

\[\implies \text{this number is within a measurable range}!!!\]

The errors come from the experimental errors in the branching ratio of

\[
Br(\Lambda_c^+ \rightarrow \pi^+K^*-p)
\]
Conclusions

Triangle singularities show a great potential to enhance suppressed processes.

In the present case we showed how the $\Lambda(1405)$ could be produced in an isospin forbidden mode.

It stresses the nature of this resonance as dynamically generated from the meson-baryon interaction, resulting from cancellation of diagrams involving the $\bar{K}N \rightarrow \pi\Sigma$ amplitudes.

One signal of this is the narrow shape of the resonance, which would not be justified if the resonance was a genuine state.

Triangle singularity also enhances the production of resonances that appear around the singular point.

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THE ENDING!!!