

$Z_c(4430)$, $Z_c(4200)$, $Z_1(4050)$, and $Z_2(4250)$ as triangle singularities

arXiv:1901.07385 (to appear in PRD, Rapid Comm.)

PRD 100 011504(R) (2019)

Satoshi Nakamura

University of Science and Technology of China

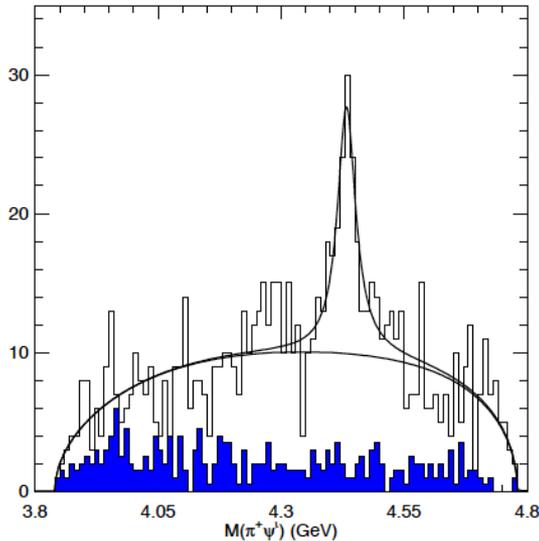
Collaborator : Kazuo Tsushima (Univ. Cruzeiro do Sul, Brazil)

Introduction

Discoveries of Z_c^\pm and Z_b^\pm

$Z_c^+(4430)$

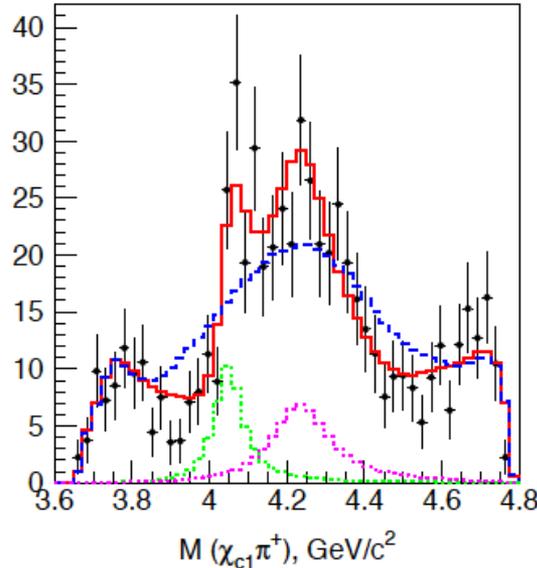
Belle (2008)



$$\bar{B}^0 \rightarrow \psi(2S)\pi^+K^-$$

$Z_1^+(4050), Z_2^+(4250)$

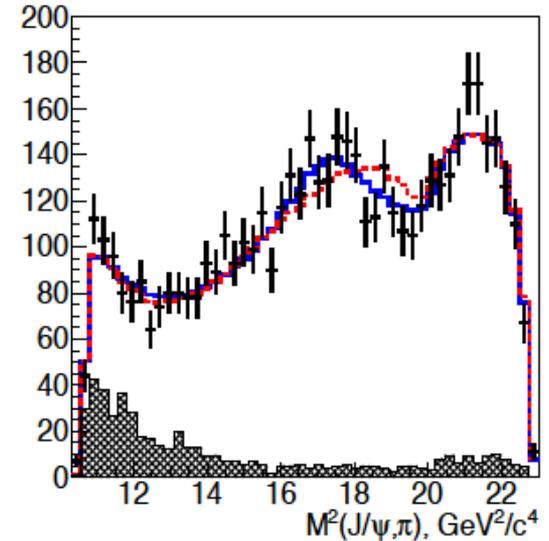
Belle (2008)



$$\bar{B}^0 \rightarrow \chi_{c1}\pi^+K^-$$

$Z_c^+(4200)$

Belle (2014)



$$\bar{B}^0 \rightarrow J/\psi\pi^+K^-$$

Beyond the conventional quark model ?

If they are charged quarkonium-like states $Z_c^+ : c\bar{c}u\bar{d}$ $Z_b^+ : b\bar{b}u\bar{d}$

Minimally 4-quark states and not $q\bar{q}$ → clear signature of exotics

$Z_c(4430)$ has been outstanding exotic candidate

[ABOUT](#)[NEWS](#)[SCIENCE](#)[RESOURCES](#)[SEARCH](#) | [EN](#) ▾

Voir en [français](#)

LHCb confirms existence of exotic hadrons

The LHCb collaboration today published an unambiguous observation of an exotic particle that cannot be classified within the traditional quark model

9 APRIL, 2014 | By [Cian O'Luanaigh](#)

The [Large Hadron Collider beauty](#) (LHCb) collaboration today announced results that confirm the existence of exotic hadrons – a type of matter that cannot be classified within the traditional quark model.

Hadrons are subatomic particles that can take part in the strong interaction – the force that binds protons inside the nuclei of atoms. Physicists have theorized since the 1960s, and ample experimental evidence since has confirmed, that hadrons are made up of quarks and antiquarks that determine their properties. A subset of hadrons, called mesons, is formed from quark-antiquark pairs, while the rest – baryons – are made up of three quarks.

But since it was first proposed physicists have found several particles that do not fit into this model of hadron structure. Now the LHCb collaboration has published an unambiguous observation of an exotic particle – the [\$Z_c\(4430\)\$](#) – that does not fit the quark model.

Related Articles



Incomplete list of previous interpretations of Z_c

$Z_c(4430)$

diquark-antidiquark (tetraquark) : Ebert et al. EPJC 58 (2008); Maiani et al. PRD 89 (2014);
Deng et al. PRD 92 (2015)

hadron molecule : Liu et al, PRD 77 (2008); Ding et al, PRD 79 (2009); Lee et al,
PLB 661 (2008); Zhang et al. PRD 80 (2009); Ma et al, PRD 90 (2014)

kinematical cusp : Rosner, PRD 76 (2007); Bugg, JPG 35 (2008) ← **already ruled out**

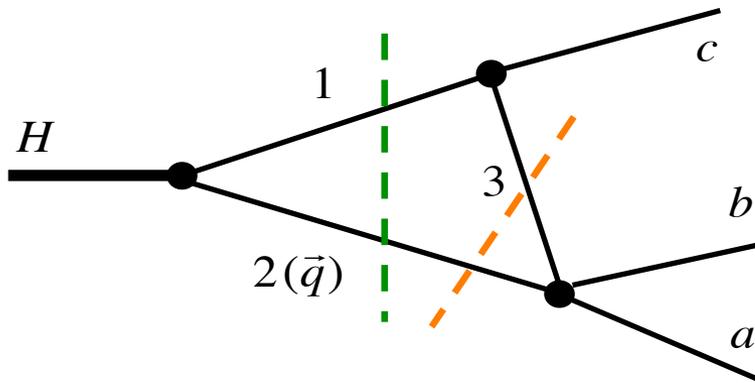
$Z_c(4050)$

tetraquark : Patel et al. EPJA 50 (2014); Deng et al, PRD 92 (2015)

hadron molecule : disfavored by meson exchange models of Liu et al. EPJC 61 (2009);
Liu et al, PRC 80 (2009); Ding et al, PRD 79 (2009)

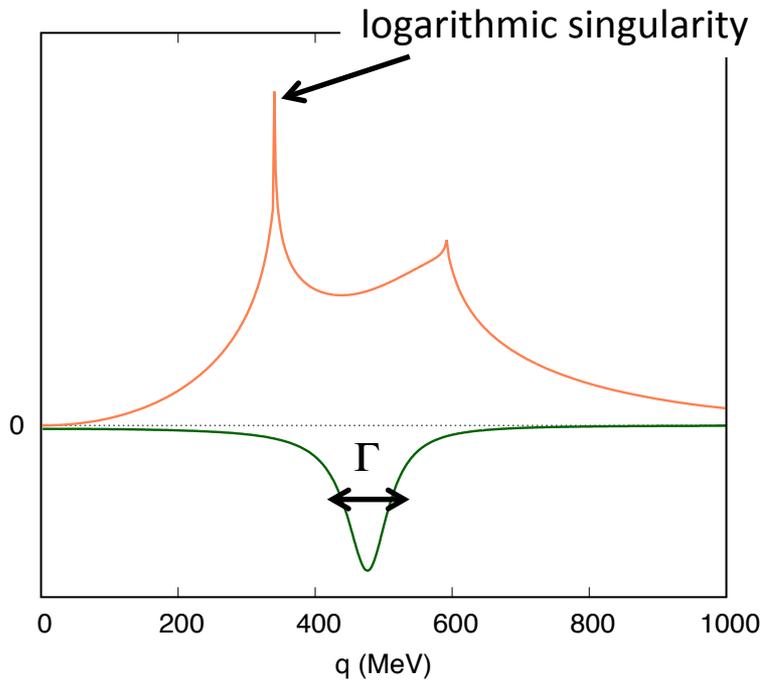
All surviving theoretical works interpret Z_c as tetraquark (including hadron molecule)

Alternative interpretation: Triangle Singularity

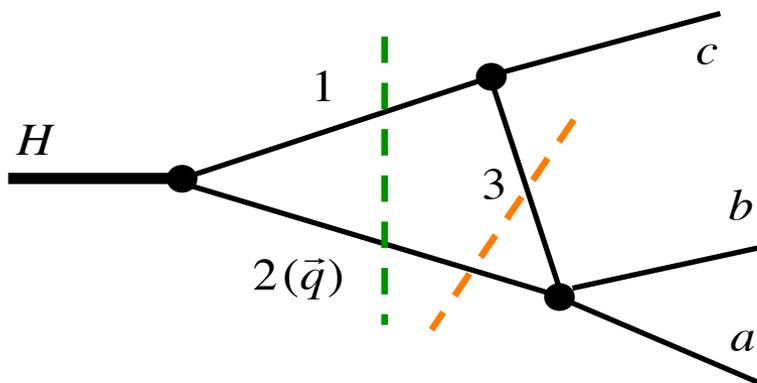


$$A \sim \int dq \frac{1}{E - E_1 - E_2 + i\frac{\Gamma}{2}}$$

$$\times \int d\Omega_q \frac{q^2}{E - E_2 - E_3 - E_c + i\epsilon}$$

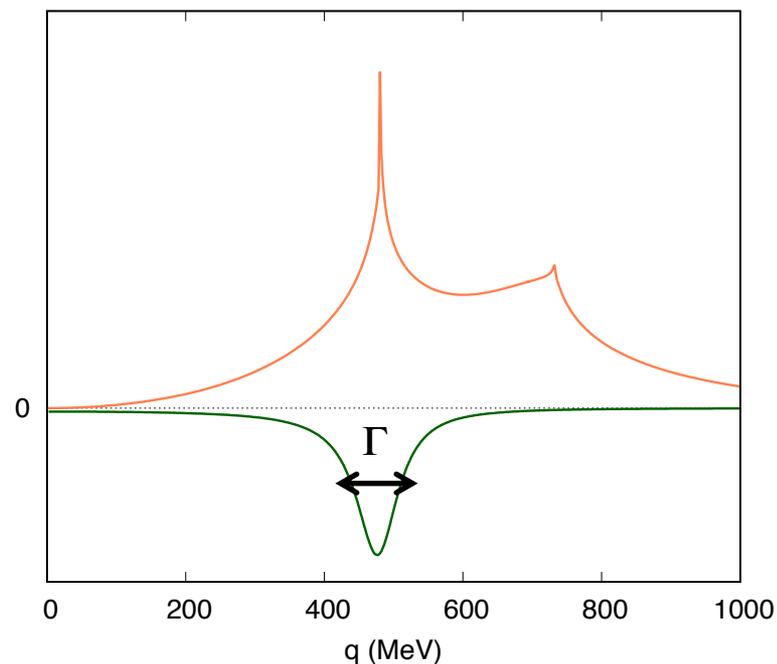
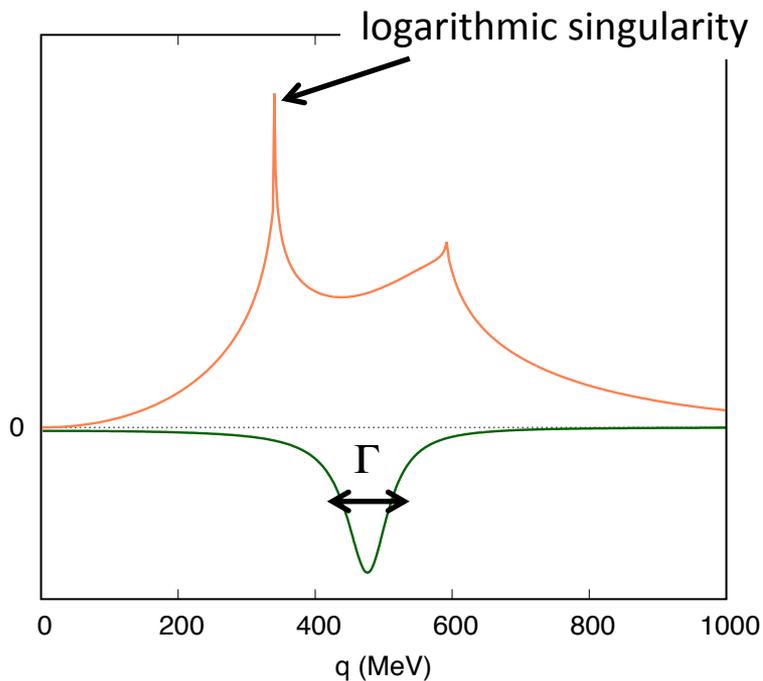


Alternative interpretation: Triangle Singularity

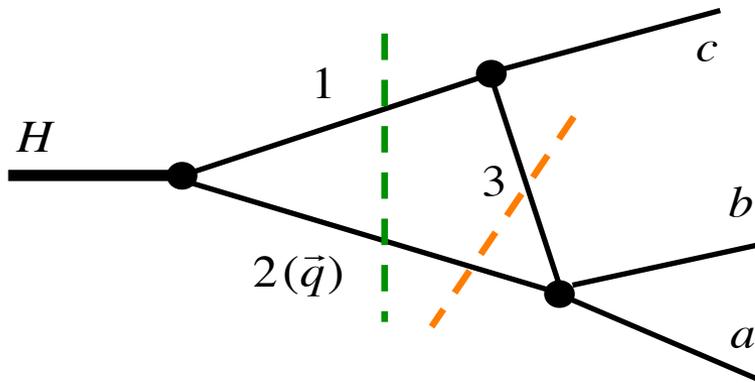


$$A \sim \int dq \frac{1}{E - E_1 - E_2 + i\frac{\Gamma}{2}}$$

$$\times \int d\Omega_q \frac{q^2}{E - E_2 - E_3 - E_c + i\epsilon}$$

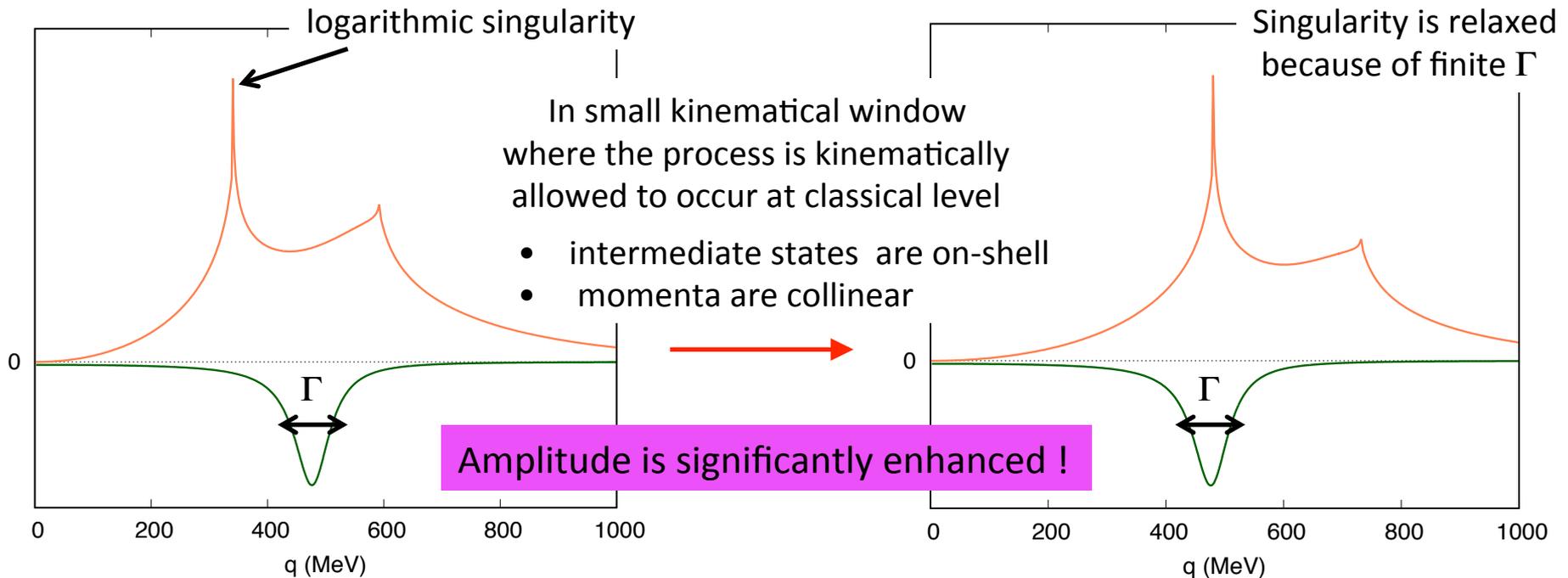


Alternative interpretation: Triangle Singularity

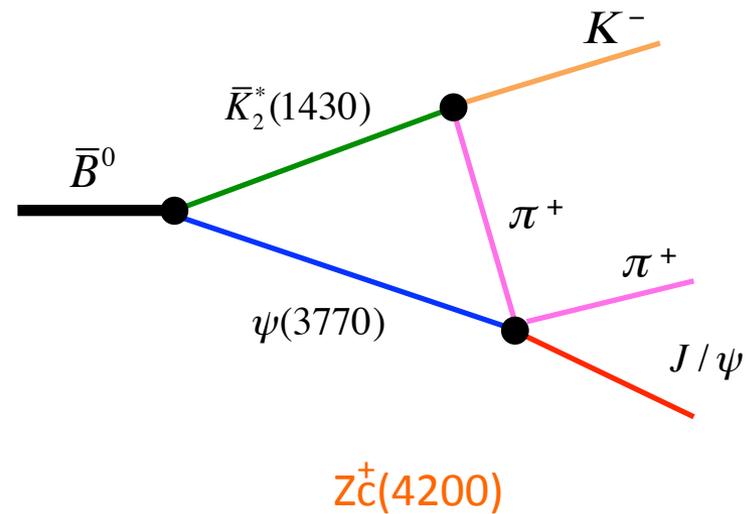
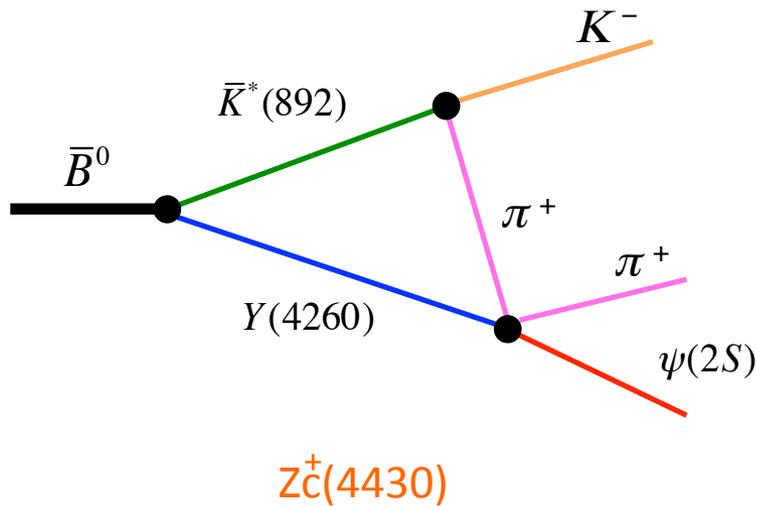


$$A \sim \int dq \frac{1}{E - E_1 - E_2 + i\frac{\Gamma}{2}}$$

$$\times \int d\Omega_q \frac{q^2}{E - E_2 - E_3 - E_c + i\varepsilon}$$



Triangle singularities for $Z_c(4430)$, $Z_c(4200)$



At zero-width limit, diagrams exactly hit triangle singularities at (using PDG averaged masses)

$$m_{\psi(2S)\pi^+} = 4420 \text{ MeV}$$

$$m_{J/\psi\pi^+} = 4187 \text{ MeV}$$

For finite (realistic) widths, triangle singularities are somewhat relaxed

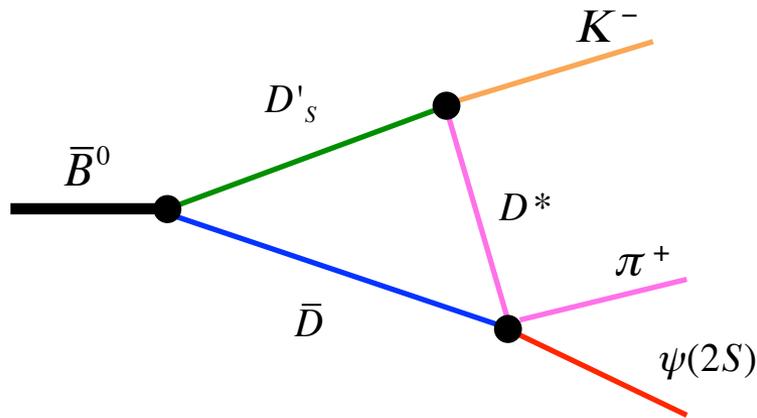
→ Spectrum peak position associated with TS can be a bit different from above

→ will be examined by numerical calculation

Comment on Pakhlov et al.'s model

Phys. Lett. B 702, 139 (2011)

Phys. Lett. B 748, 183 (2015)



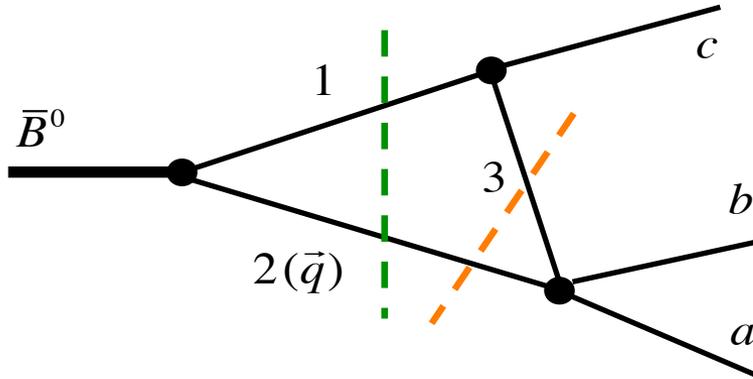
D'_s : hypothetical charmed-strange hadron

CLAIM: The above triangle diagram generates Zc(4430)-like bump

- Comments**
- The process is kinematically forbidden at the classical level
→ no triangle singularity (Coleman-Norton theorem)
 - Argand plot is clockwise
→ ruled out by LHCb data (counter-clockwise Argand plot)
 - Our calculation does not find Zc(4430)-like bump from above triangle diagram
→ expected from Coleman-Norton theorem

Model

Triangle amplitudes for Zc(4430), Zc(4200)



$$T_{\bar{B}^0 \rightarrow abc} = \int d^3q v_{23 \rightarrow ab}$$

$$\frac{1}{E - E_2 - E_3 - E_c + i\epsilon}$$

$$\times \Gamma_{1 \rightarrow 3c} \frac{1}{E - E_1 - E_2 + i\frac{\Gamma}{2}} \Gamma_{\bar{B}^0 \rightarrow 12}$$

$v_{23 \rightarrow ab}(\vec{p}_a, \vec{p}_b; \vec{p}_2, \vec{p}_3) = f(p_{ab})f(p_{23}) \vec{\epsilon}_a \cdot \vec{\epsilon}_2$: s-wave interaction; consistent with $J^P=1^+$ of Zc(4430), Zc(4200)

$$\Gamma_{R \rightarrow ij}(\vec{p}_R; \vec{p}_i, \vec{p}_j) = \sum_{LS} f(p_{ij}) (s_i s_i^z s_j s_j^z | SS^z)(LMSS^z | S_R S_R^z) Y_{LM}(\hat{p}_{ij})$$

$f(p_{ij})$: dipole form factor with cutoff 1 GeV (data not available to fix form factors)

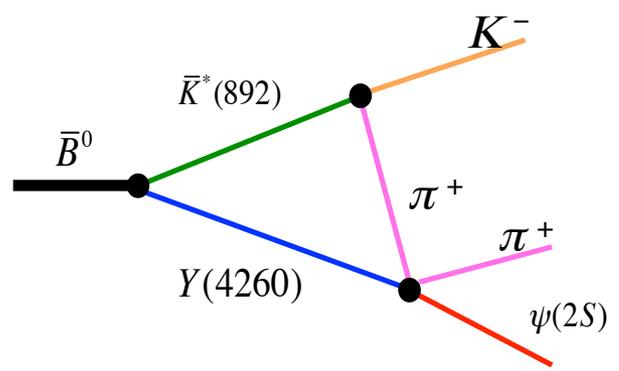
Spectrum shape is mostly determined by kinematical effect → insensitive to cutoff
 → to be checked

All particle masses and widths are taken from PDG average

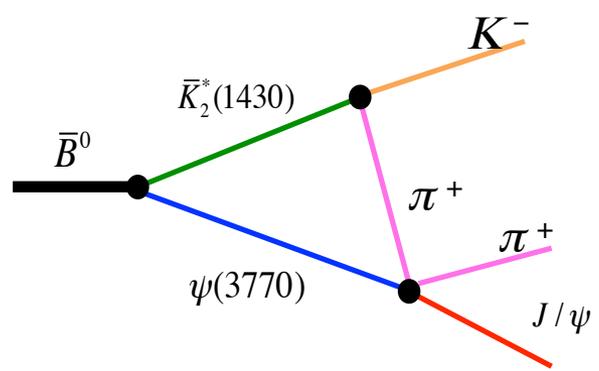
Results for
 $Z_c(4430)$ and $Z_c(4200)$

Invariant mass spectrum

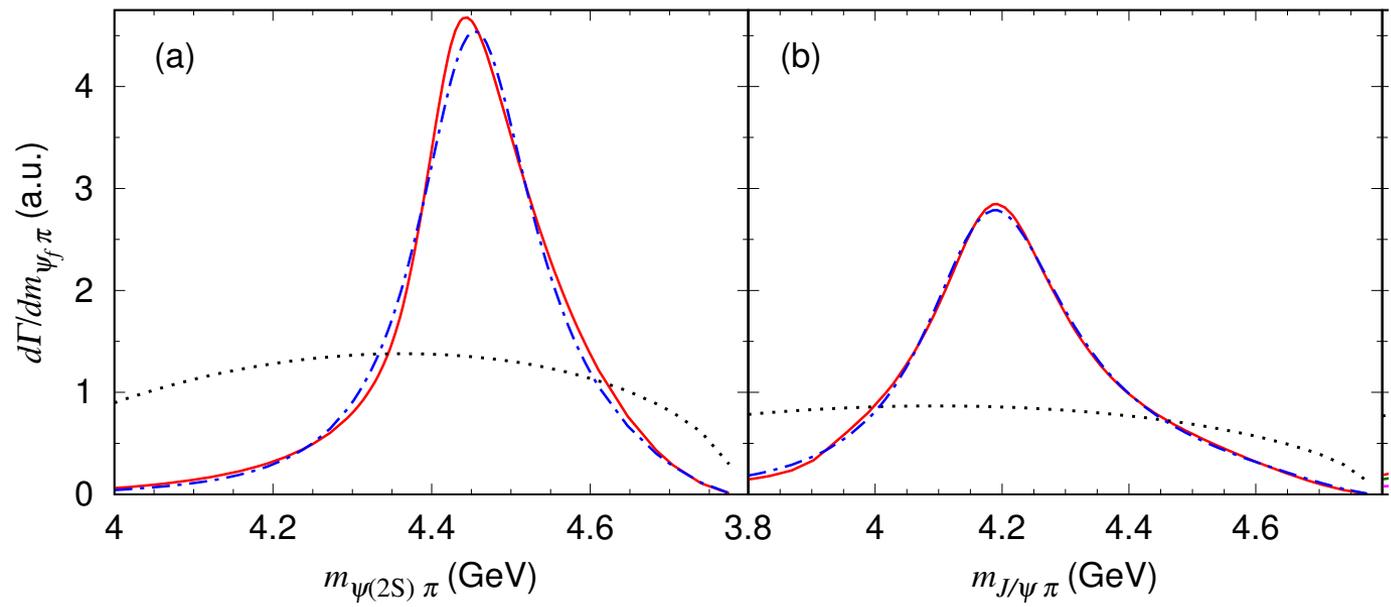
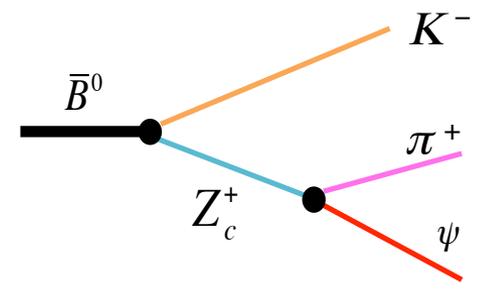
$Z_c^+(4430)$



$Z_c^+(4200)$



Breit-Wigner model



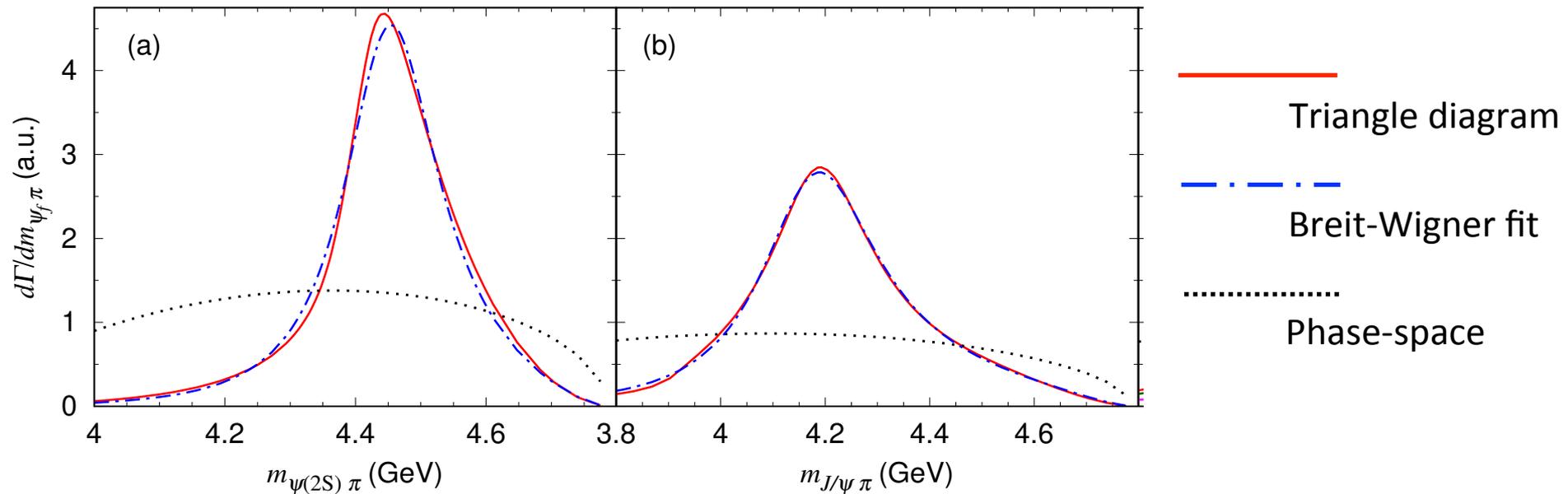
- Triangle diagram
 - · - · Breit-Wigner fit
 - Phase-space
- Spectra are normalized to give unity when integrated

Clear resonance-like peaks are generated by triangle diagrams
 Absolute magnitude is unknown → experimental inputs needed

Invariant mass spectrum

$Z_c^+(4430)$

$Z_c^+(4200)$



Breit-Wigner parameters

$Z_c^+(4430)$

$Z_c^+(4200)$

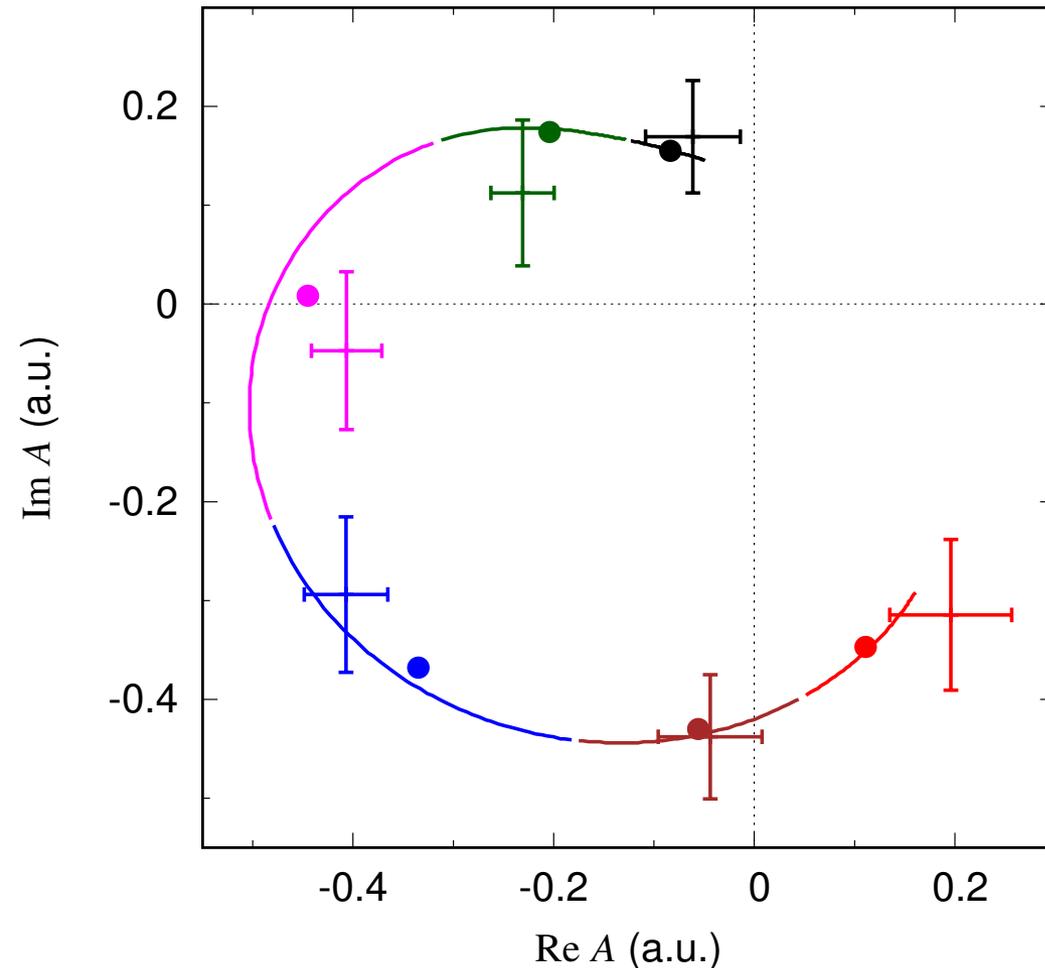
	(a)	Belle (2013)	LHCb (2014)	(b)	Belle (2014)
Mass (MeV)	4463 ± 13	$4485 \pm 22^{+28}_{-11}$	$4475 \pm 7^{+15}_{-25}$	4233 ± 48	4196^{+31+17}_{-29-13}
Width (MeV)	195 ± 16	200^{+41+26}_{-46-35}	$172 \pm 13^{+37}_{-34}$	292 ± 56	$370 \pm 70^{+70}_{-132}$

Remarkable agreement with data

Ranges of the parameters from the model are cutoff-dependence ($\Lambda=0.5-2$ GeV)

Angle-independent part of amplitude + constant background

$$A(m_{\psi(2S)\pi^+}^2) = c_{\text{bg}} + c_{\text{norm}} \int d\Omega_{K^-} Y_1(\hat{p}_{K^-}) M_{\bar{B}^0 \rightarrow \psi(2S)\pi^+ K^-} \quad c_{\text{bg}}, c_{\text{norm}} : \text{fitted to data}$$



Resonance-like counter-clockwise motion is reproduced by triangle diagram, not a resonance

Curved segment and point of same color belong to same $m_{\psi(2S)\pi^+}^2$ bin

Data: LHCb, PRL 112 222002 (2014)

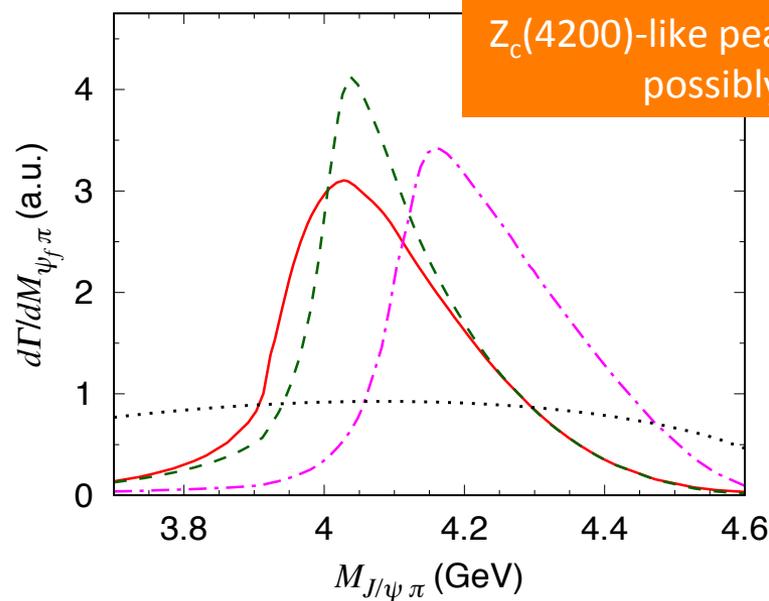
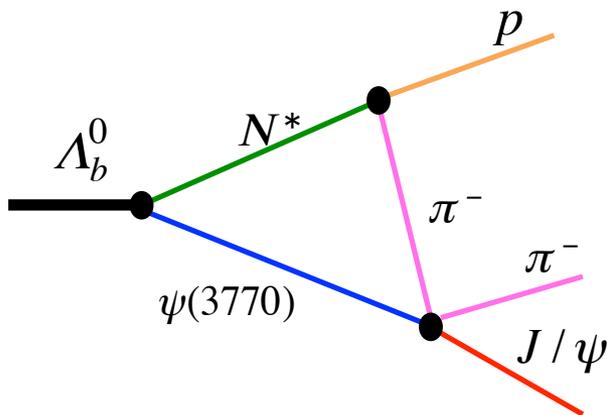
$Z_c(4430)$ and $Z_c(4200)$ in $\Lambda_b^0 \rightarrow J/\psi \pi^- p$

LHCb PRL 117 082003 (2016) :

- $Z_c(4200)$ contribution significantly improves the description of data
- $Z_c(4430)$ contribution hardly improves

$Z_c(4430)$ and $Z_c(4200)$ as triangle singularities give a consistent explanation

No other explanation yet



— $N^* = N(1440) 1/2^+$

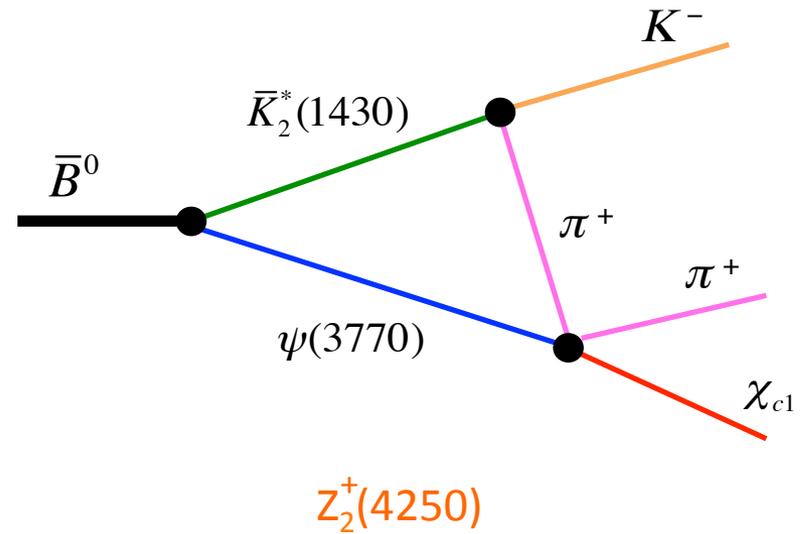
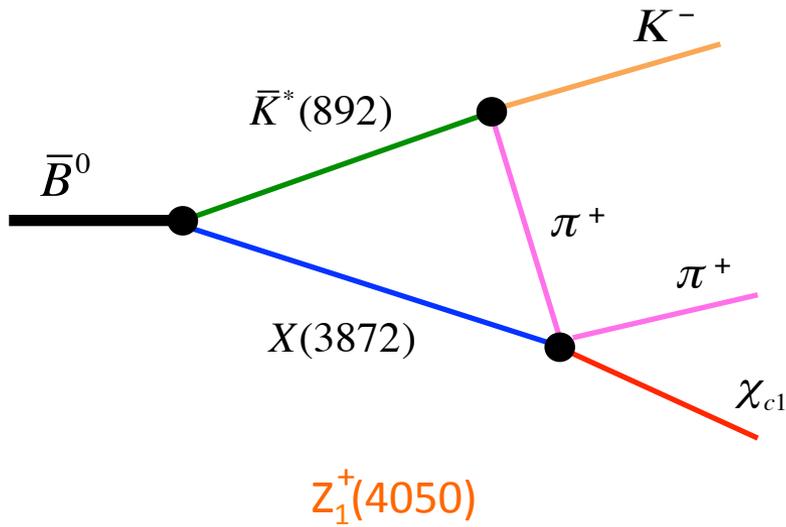
- - - $N^* = N(1520) 3/2^-$

- · - · $N^* = N(1680) 5/2^+$

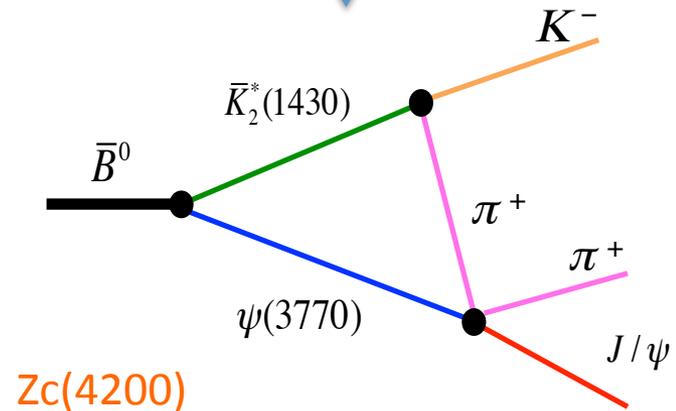
No triangle diagram is available to create $Z_c(4430)$ -like peak

Results for
 $Z_1(4050)$ and $Z_2(4250)$

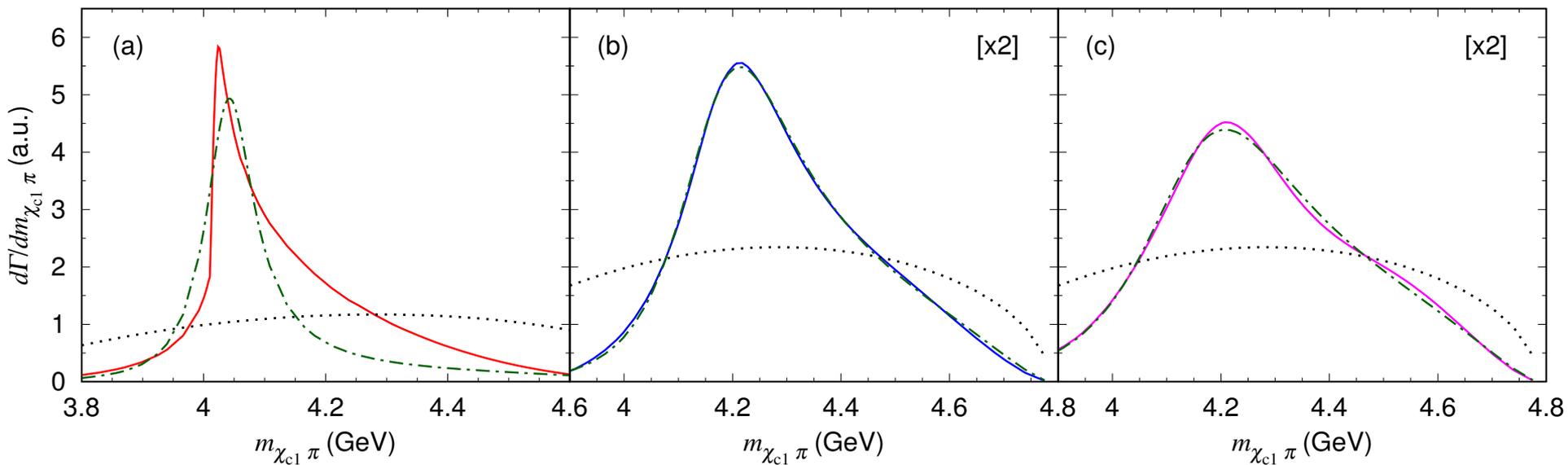
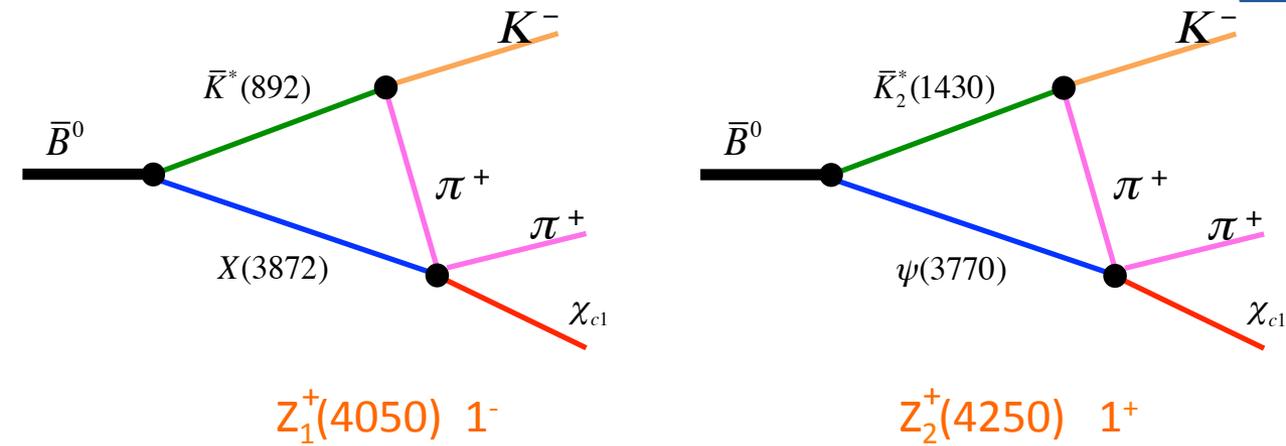
Triangle singularities for $Z_1(4050)$, $Z_2(4250)$



similar



Invariant mass spectrum

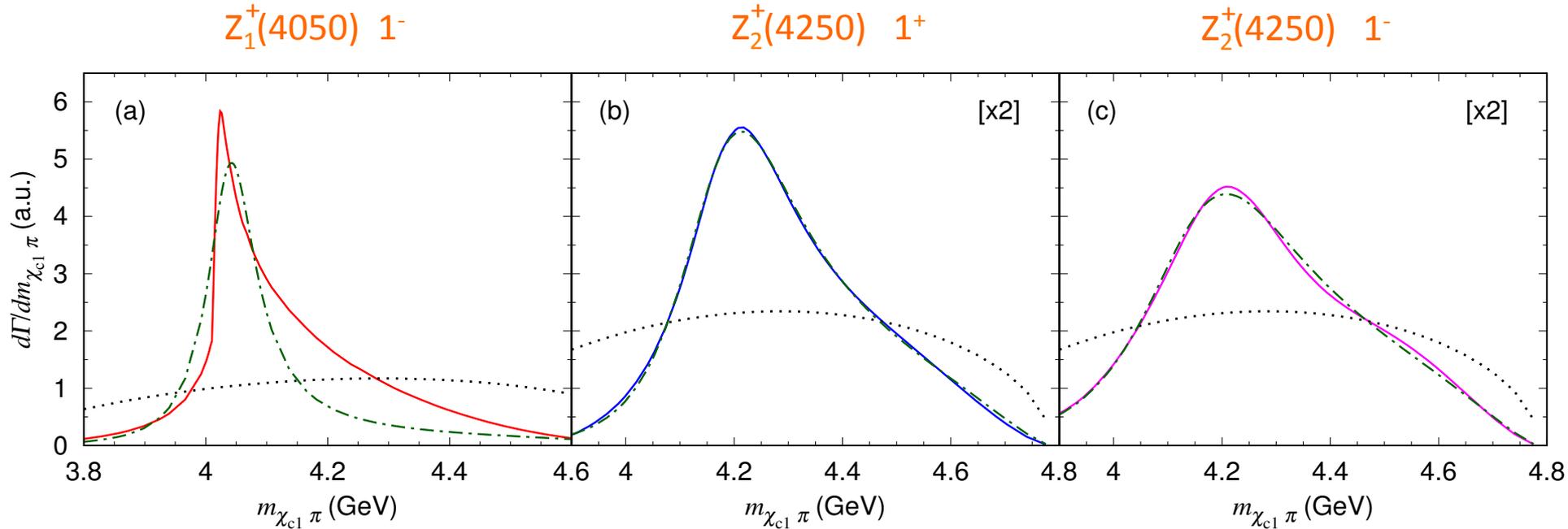


Clear resonance-like peaks are generated by triangle diagrams

Absolute magnitude is unknown \rightarrow experimental inputs needed

- Triangle diagram
- · - · - Breit-Wigner fit
- Phase-space

Invariant mass spectrum



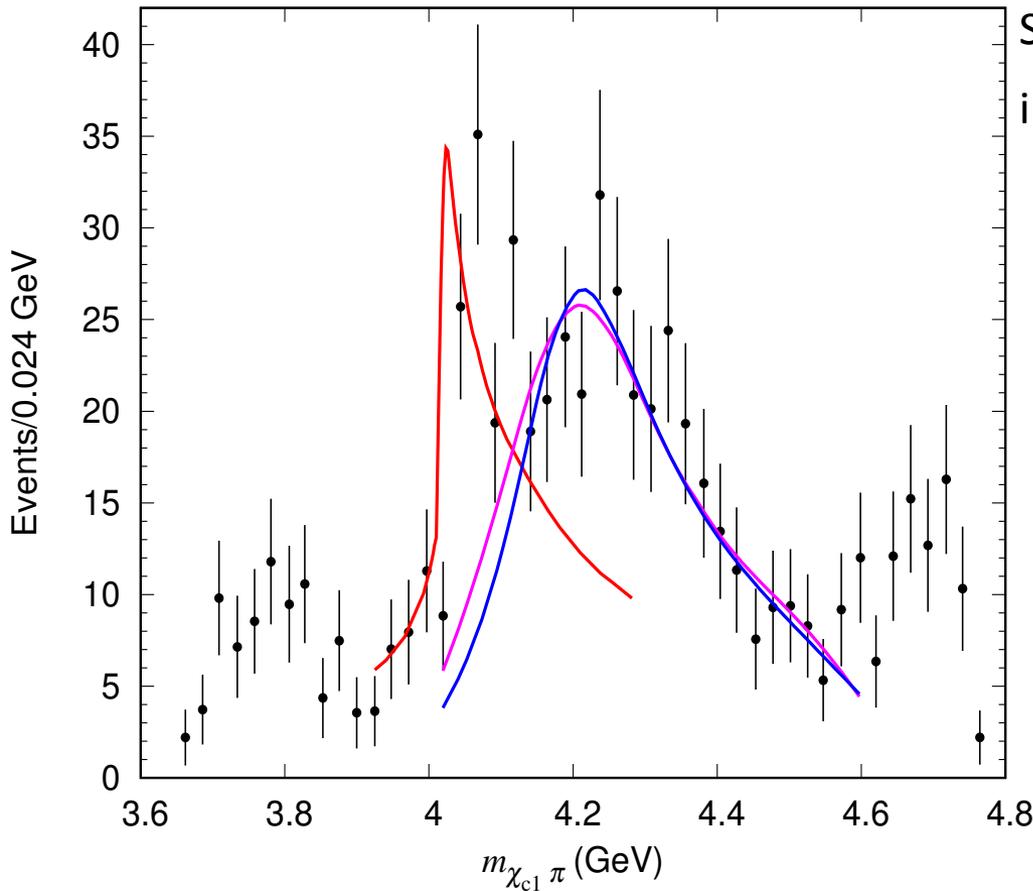
Very good agreement with data

(range \leftarrow cutoff dependence: $\Lambda=1-2$ GeV)

Breit-Wigner parameters

	(a)	Belle (2008)	(b)	(c)	Belle (2008)
J^P	1^-	$??$	1^+	1^-	$??$
Mass (MeV)	4041 ± 1	$4051 \pm 14_{-41}^{+20}$	4247 ± 53	4309 ± 116	$4248_{-29-35}^{+44+180}$
Width (MeV)	115 ± 17	82_{-17-22}^{+21+47}	345 ± 67	468 ± 90	$177_{-39-61}^{+54+316}$

Comparison with spectrum from Belle



Spectra from triangle diagrams are scaled and incoherent background is added to fit data

Triangle diagrams similar to



 $Z_1^+(4050) 1^-$

 $Z_2^+(4250) 1^+$

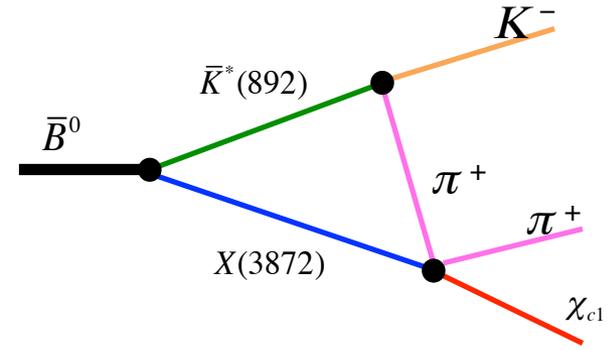
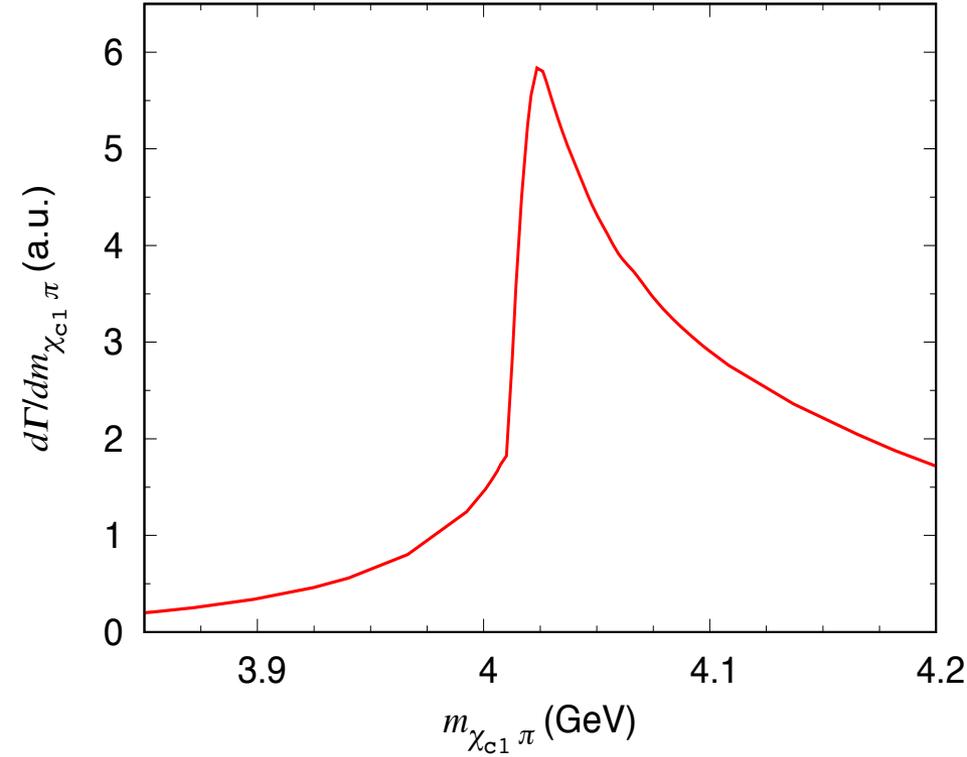
 $Z_2^+(4250) 1^-$

Data: Belle, PRD 78 072004 (2008)

- Spectra from triangle diagrams capture the Belle data feature
- highly asymmetric $Z_1(4050)$ peak is well reproduced

Note : Qualitative comparison; no interference with other mechanisms considered

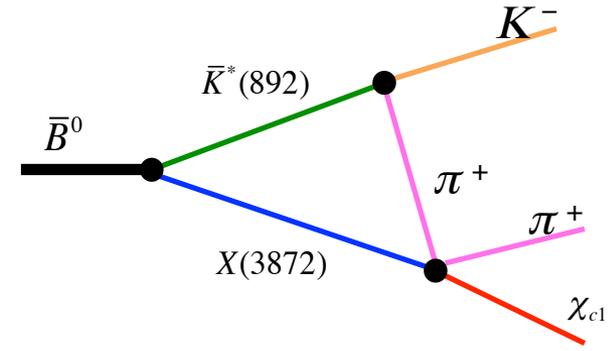
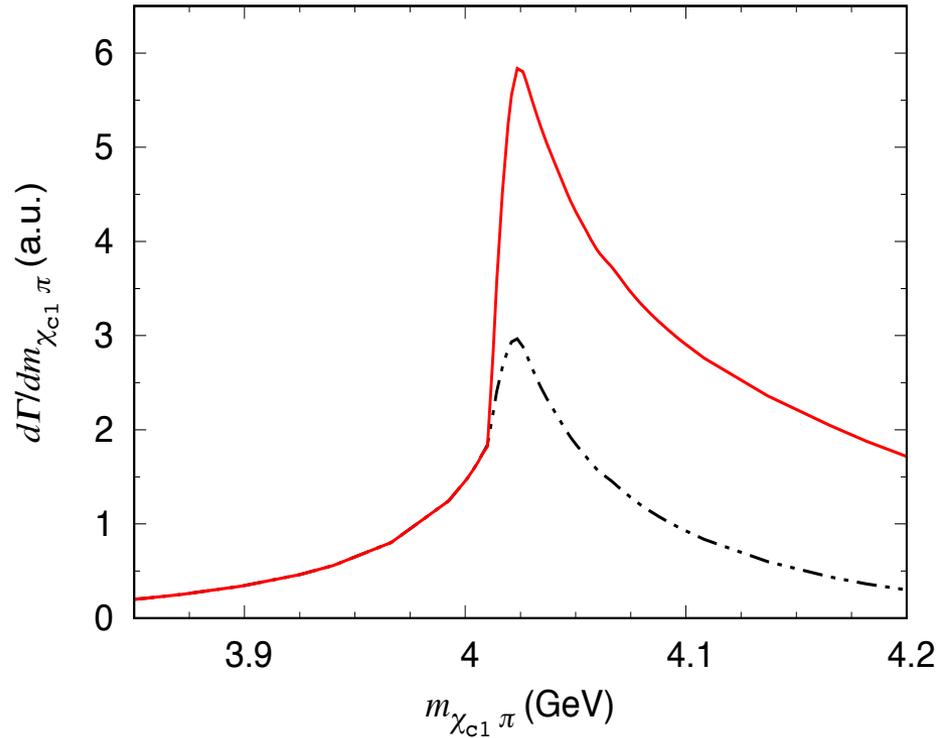
Origin of asymmetric shape of $Z_1(4050)$ peak



Spectrum has abrupt bent at $m_{\chi_{c1}\pi^+} \sim 4.01$ GeV
 where $X(3872)\pi^+$ channel opens

— : original triangle diagram

Origin of asymmetric shape of $Z_1(4050)$ peak

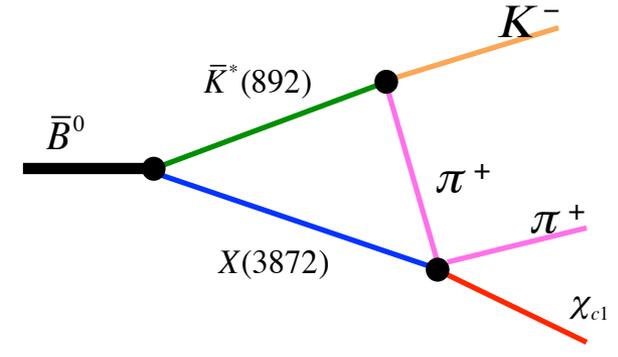
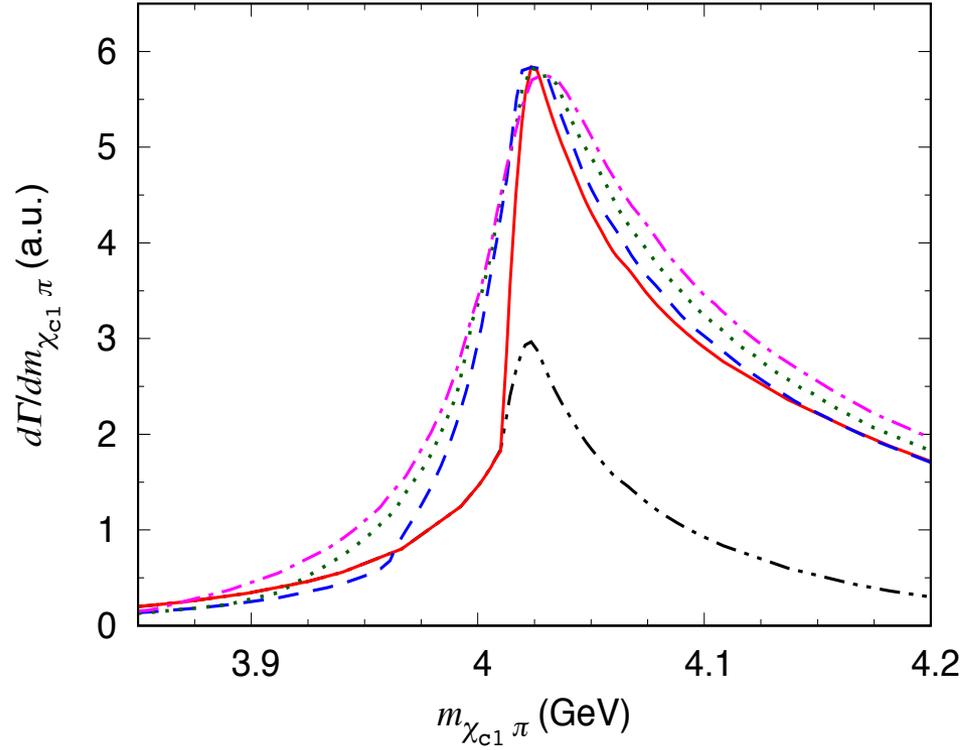


— : original triangle diagram

- · · - · · : on-shell $X(3872)\pi^+$ contribution turned off

- on-shell $X(3872)\pi^+$ makes the spectrum significantly asymmetric
- $X(3872)\pi^+$ threshold energy is close to the peak position \rightarrow effect of proximity ?

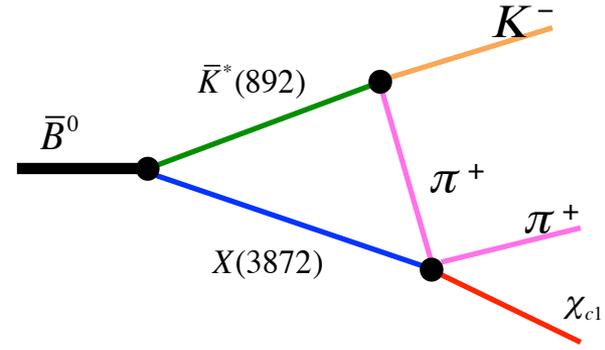
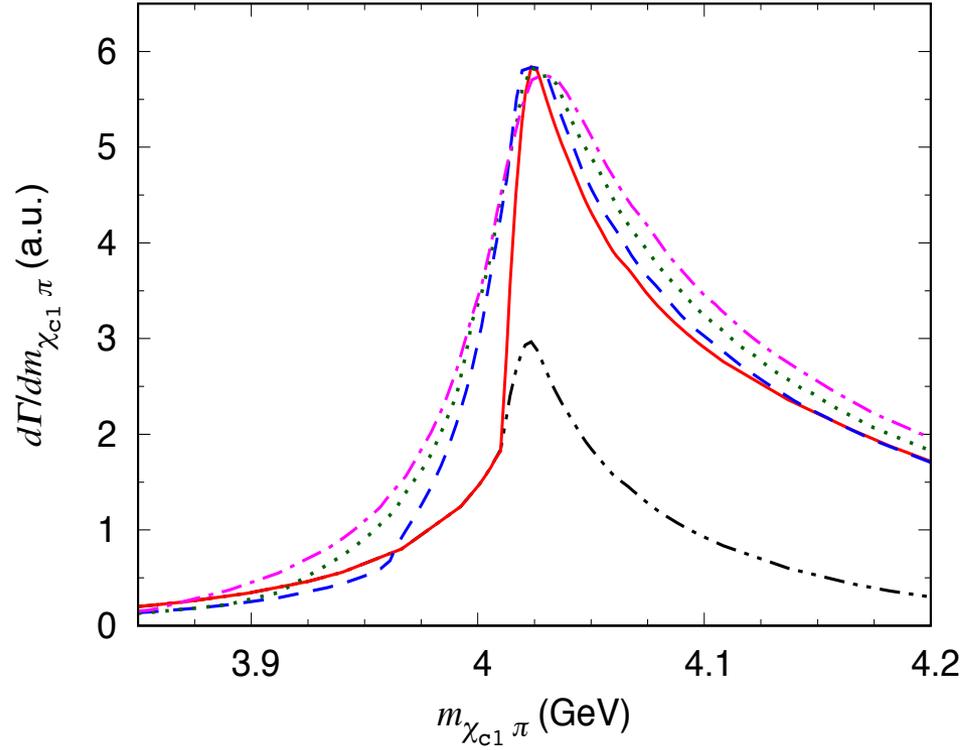
Origin of asymmetric shape of $Z_1(4050)$ peak



- : original triangle diagram
- · - · : on-shell $X(3872)\pi^+$ contribution turned off
- - - : $X(3872)\pi^+$ threshold energy lowered by 50 MeV
- · · : 100 MeV
- · - · : 150 MeV

↓
less asymmetric

Origin of asymmetric shape of $Z_1(4050)$ peak



Origin of the asymmetric peak shape

- on-shell $X(3872)\pi^+$ contribution
- Proximity of $X(3872)\pi^+$ threshold

The triangle diagram includes both

No other explanation yet

— : original triangle diagram

- · - · - : on-shell $X(3872)\pi^+$ contribution turned off

- - - : $X(3872)\pi^+$ threshold energy lowered by 50 MeV

· · · : 100 MeV

- · - · - : 150 MeV

less asymmetric



Conclusion

Conclusion

- Identified triangle diagrams (singularities) generating spectrum bumps similar to $Z_c(4430)$, $Z_c(4200)$, $Z_1(4050)$, and $Z_2(4250)$
- Experimentally determined properties (spin, parity, mass, width, Argand plot) are all explained well by the triangle diagrams
- Cutoff dependence is small \rightarrow kinematical effect dominates
- Appearance [absence] of $Z_c(4200)$ [$Z_c(4430)$] in $\Lambda_b^0 \rightarrow J/\psi \pi^- p$ is consistently understood
- Origin of asymmetric spectrum shape of $Z_1(4050)$ is understood

Backup

Current trend in hadron spectroscopy

Establish existence of **exotic hadrons** (beyond conventional quark model)
driven largely by remarkable experimental developments

- Tetraquark, Pentaquark
- Hadronic molecule
- Hybrid ... etc.

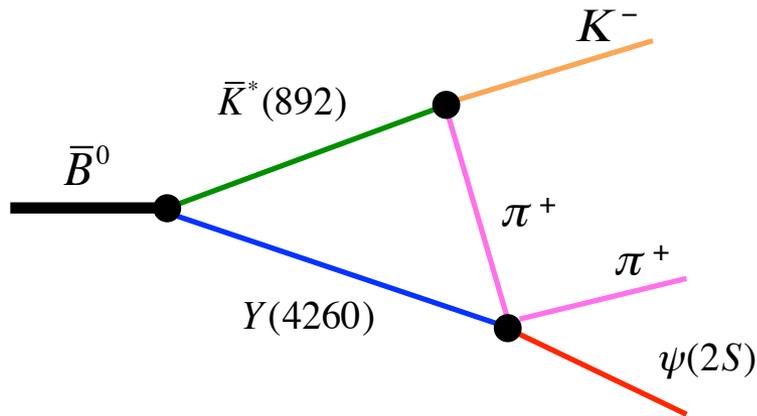
How can we distinguish exotic hadrons from ordinary ones ?

- Mass not predicted by quark model
- High gluon contents predicted by LQCD
- Peculiar decay patterns ... etc.

... seems model-dependent criteria

More unambiguous signature ?

Triangle diagram for $Z_c(4430)$

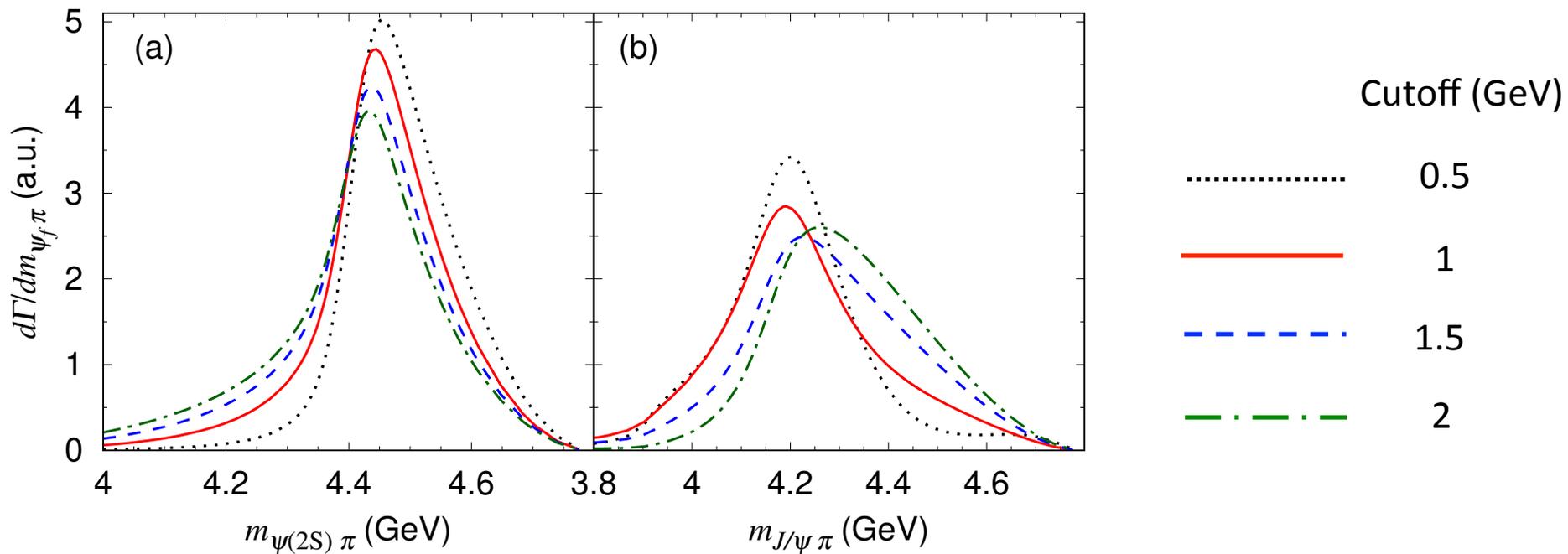
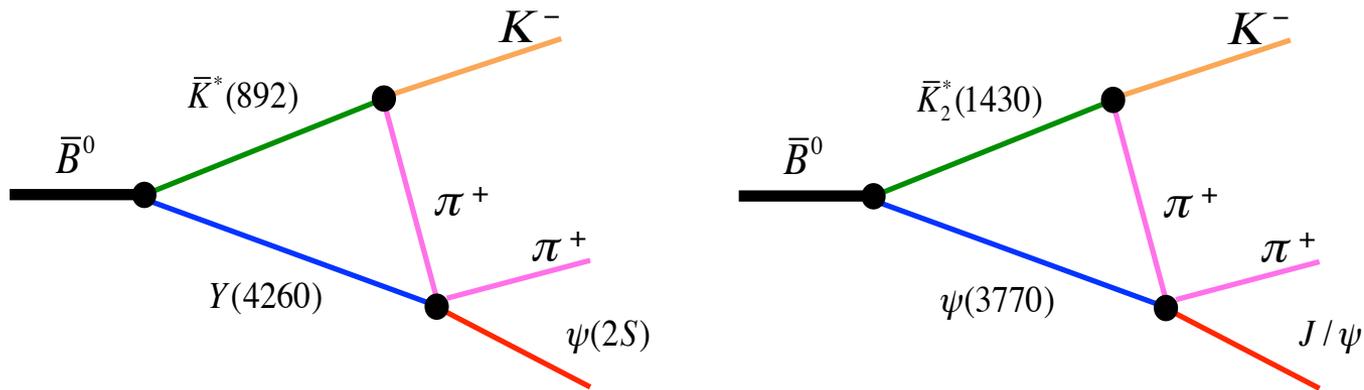


Reasonability of $\bar{B}^0 \rightarrow \bar{K}^*(892) Y(4260)$

- Belle found excess of $B \rightarrow Y(4260)K$ events above the background
PRD 99, 071102 (2019)
- D0 data can be consistently interpreted that some b-flavored hadrons weakly decay into states including $Y(4260)$ PRD 100, 012005 (2019)

$Z_c^+(4430)$

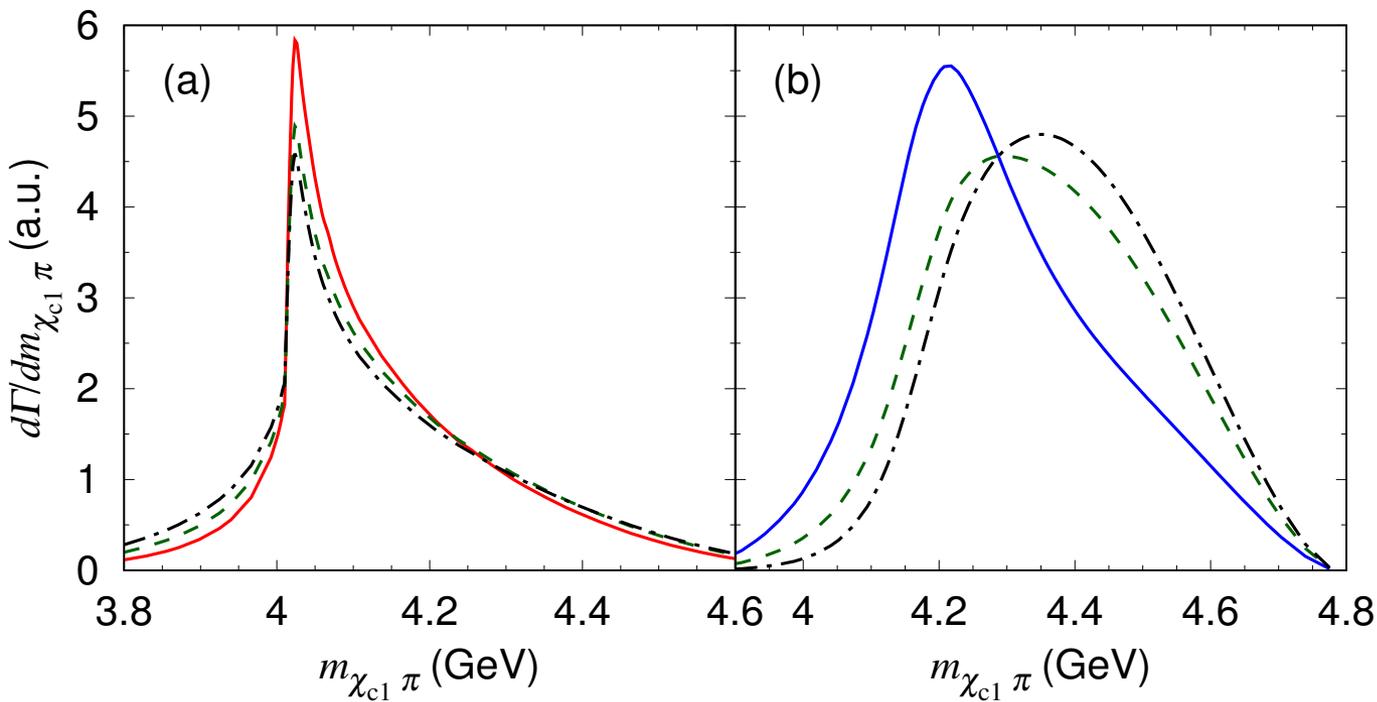
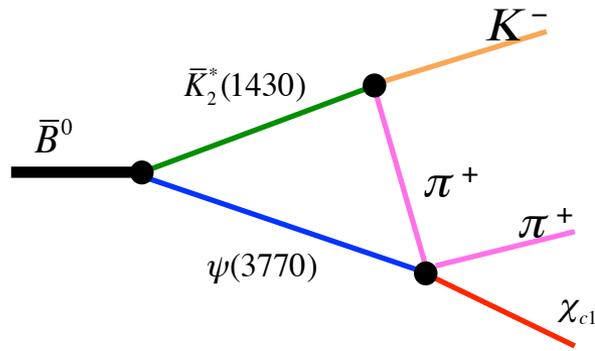
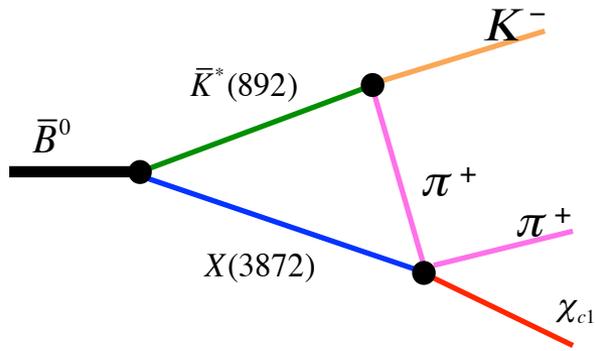
$Z_c^+(4200)$



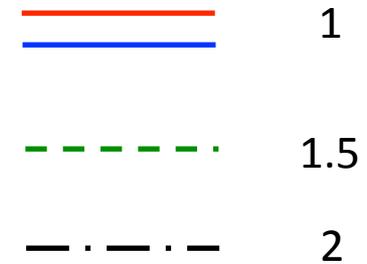
Clear peaks are not largely changed by cutoff values ← triangle singularities dominate

$Z_1^+(4050)$

$Z_2^+(4250)$



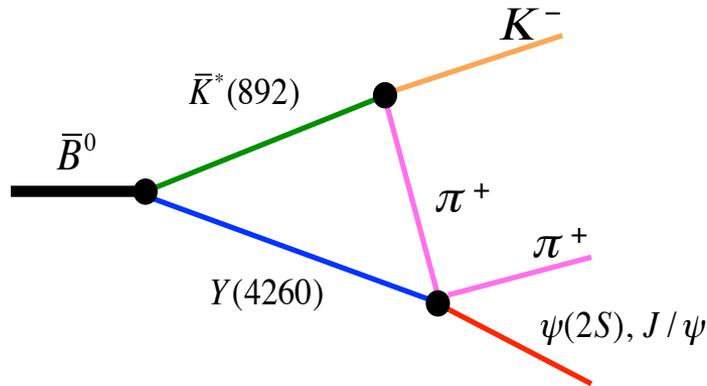
Cutoff (GeV)



Clear peaks are not largely changed by cutoff values ← triangle singularities dominate

$$R_{Z_c^+(4430)}^{\text{exp}} = \frac{\text{Br}(Z_c^+(4430) \rightarrow \psi(2S)\pi^+)}{\text{Br}(Z_c^+(4430) \rightarrow J/\psi\pi^+)} \approx 11$$

Qualitative understanding with triangle diagram and data for $Y(4260)$ decays



$$R_{Y(4260)}^{\text{exp}} = \frac{\text{Br}(Y(4260) \rightarrow \psi(2S)\pi^+\pi^-)}{\text{Br}(Y(4260) \rightarrow J/\psi\pi^+\pi^-)} \approx (0.11 \pm 0.03 \pm 0.03) - (0.55 \pm 0.18 \pm 0.19)$$

Zhang and Yuan, EPJC 77, 727 (2017)

$R_{Y(4260)}^{\text{exp}}$ can fix ratio of coupling strengths : $c_{\psi\pi}^R \equiv \frac{C [Y(4260)\pi^+ \rightarrow \psi(2S)\pi^+]}{C [Y(4260)\pi^+ \rightarrow J/\psi\pi^+]}$

With $|c_{\psi\pi}^R| \sim 1.8$, $R_{Y(4260)}^{\text{model}} = 0.17 \times |c_{\psi\pi}^R|^2 \sim 0.54$ and $R_{Z_c^+(4430)}^{\text{model}} \approx 11$

Assumption: $c_{\psi\pi}^R$ is same for $Y(4260)$ decays and B^0 decays; rather different in energy