Freed-Isobar Analysis of Light Mesons at COMPASS

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for the COMPASS collaboration

Max Planck Institut für Physik

XVIII Conference on Hadron Spectroscopy and Structure

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Guilin, China

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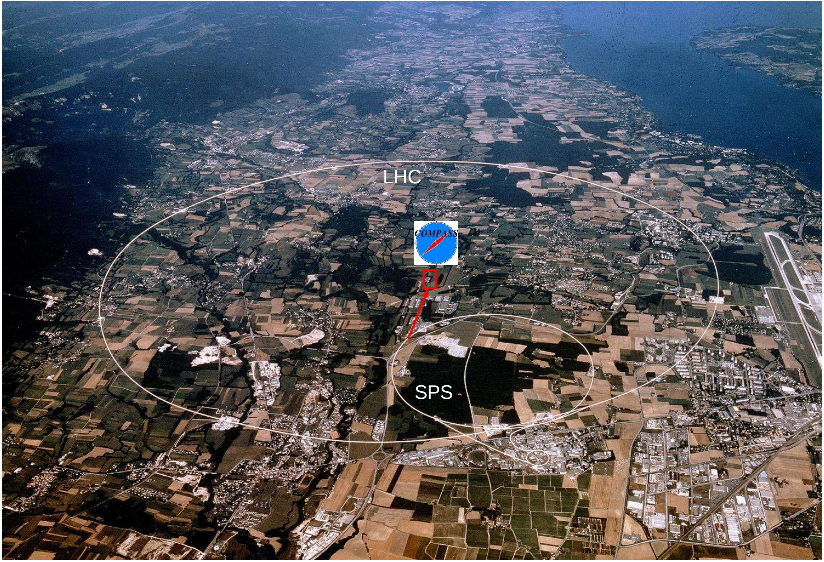
The COMPASS experiment
Common **M**uon **P**roton **A**pparatus for **S**tructure and **S**pectroscopy



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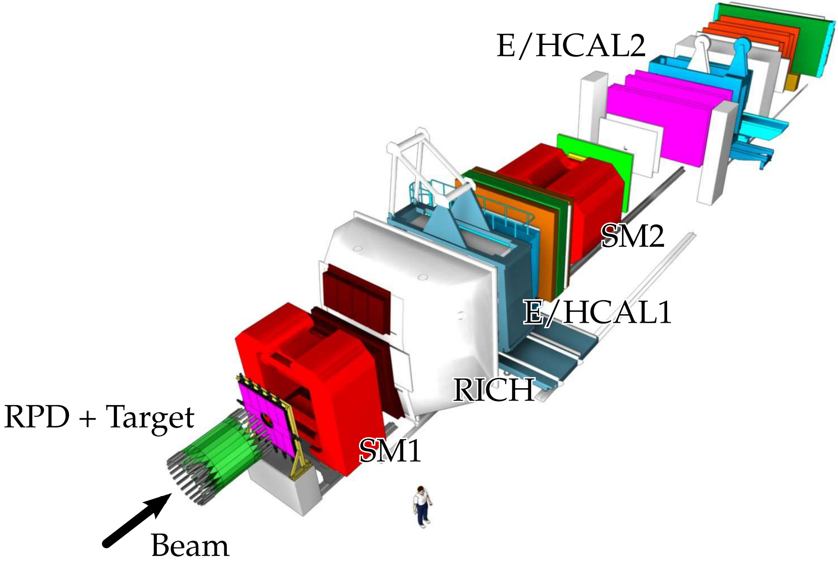

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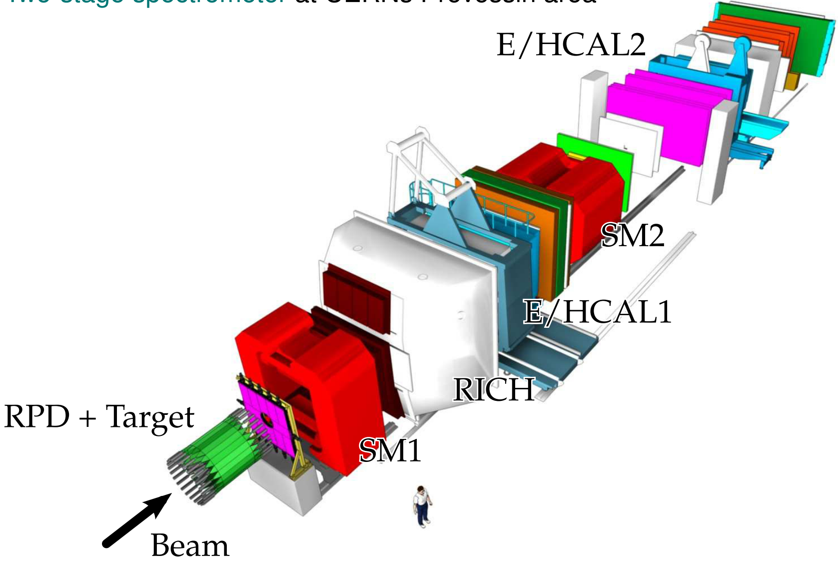
Two-stage spectrometer at CERNs Prévessin area



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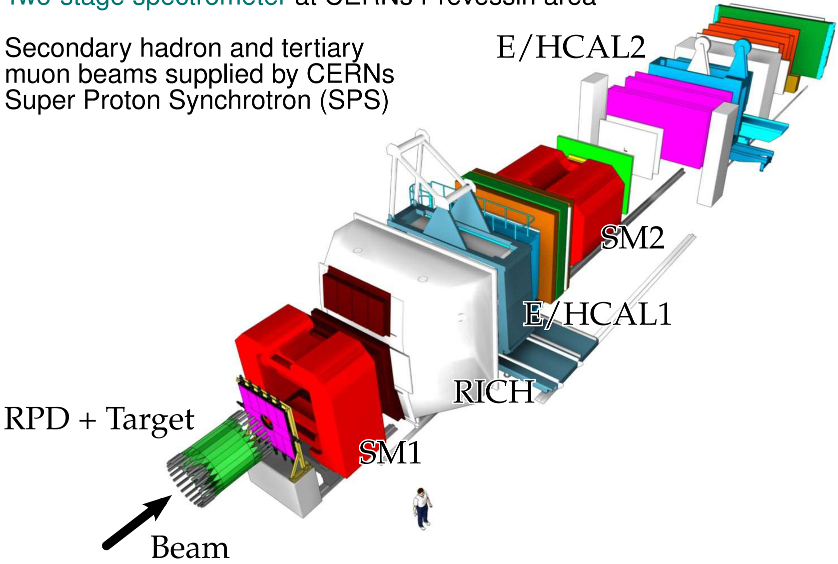

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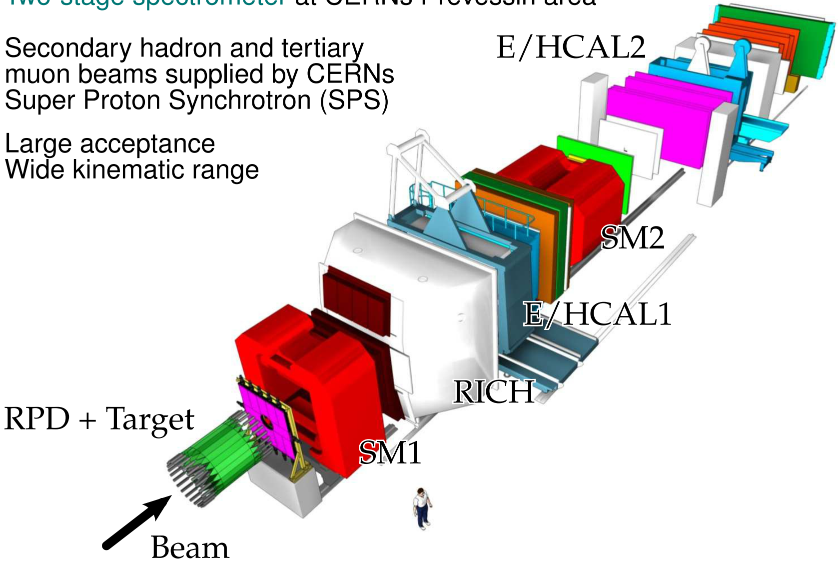

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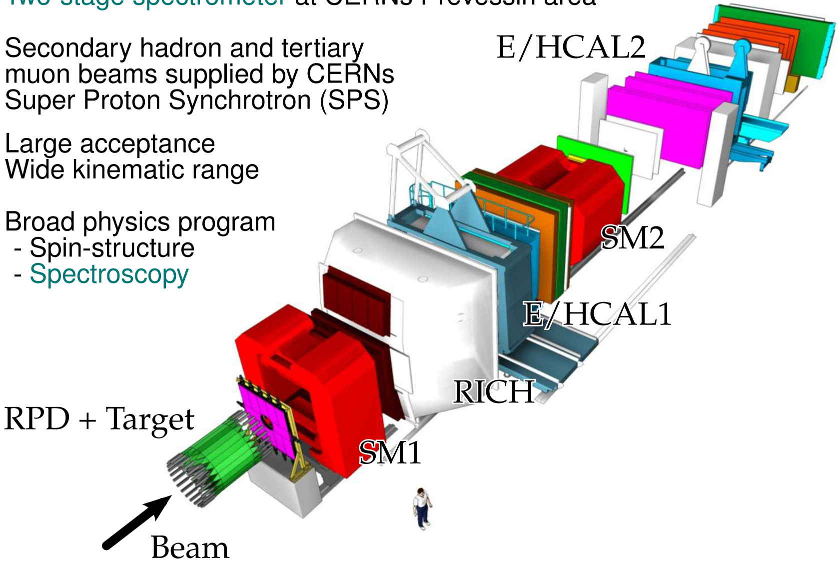

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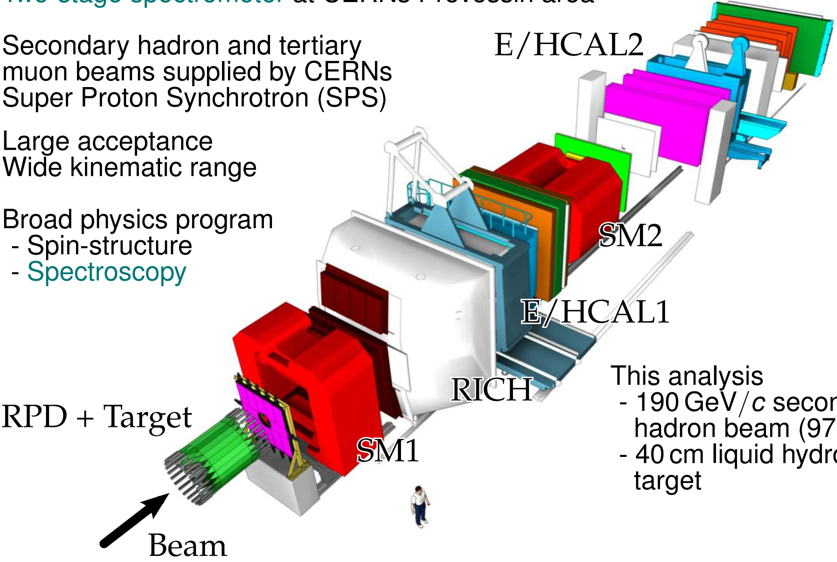

This analysis
- 190 GeV/c secondary hadron beam (97% π^-)
- 40 cm liquid hydrogen target

Diffractive 3π production

- COMPASS: Large data set for the diffractive process
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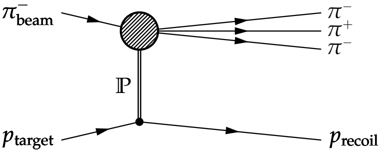

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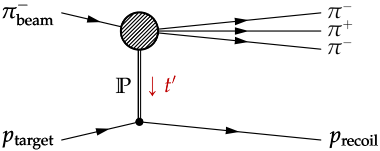

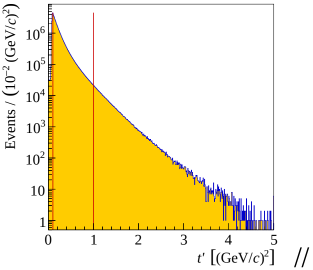

COMPASS collaboration, PR **D95** (2017) 032004

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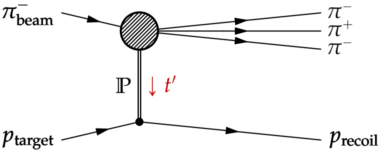

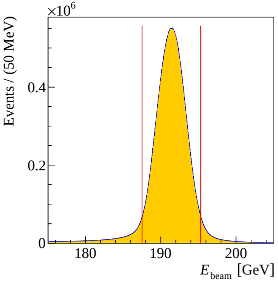

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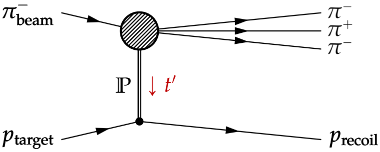

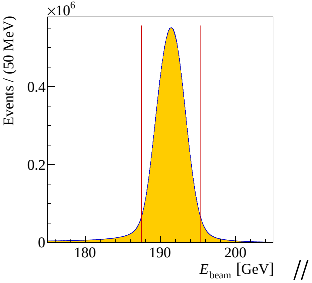

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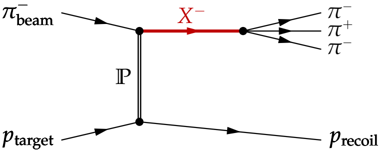

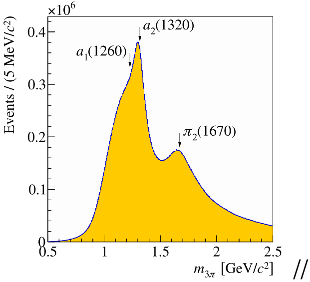

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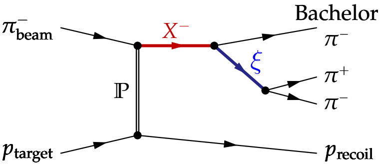
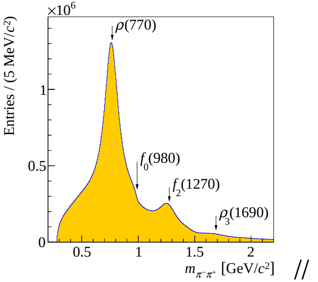
- Also structure in $\pi^+\pi^-$ subsystem: Intermediate states ξ (isobar)





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Modelling resonances

- Intermediate states: Dynamic amplitudes $\Delta(m)$: (line shape)
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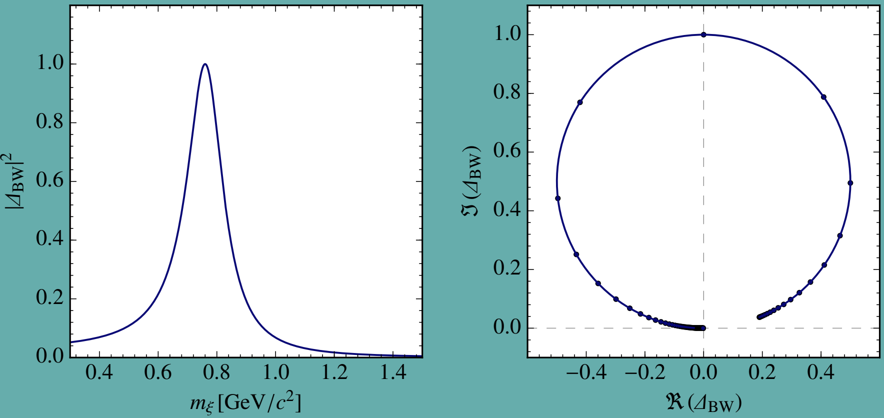
Modelling resonances

Example: dynamic isobar amplitude for $\rho(770)$



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  - Parameterizations neglect e.g. final-state interactions

Partial-Wave Analysis (PWA)

For $m_{3\pi} = const.$ (and $t' = const.$)

- Expand intensity distribution $\mathcal{I}(\vec{\tau})$ over phase-space variables $\vec{\tau}$ as sum over partial waves:

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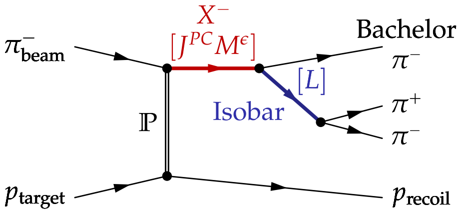
  - Not given by first principles
  - Have to be known beforehand

Wave naming scheme

$$\mathcal{I}(\vec{\tau}) = \left| \sum_i \mathcal{T}_i \psi_i(\vec{\tau}) \Delta_i(m_{\pi^-\pi^+}) \right|^2$$

Waves defined by:

$$i = J_{X^-}^{PC} M^{\varepsilon} \xi \pi L$$



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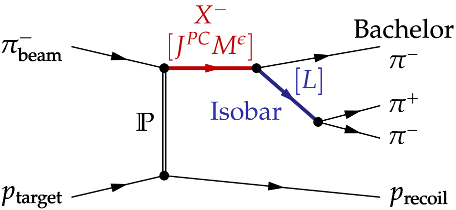

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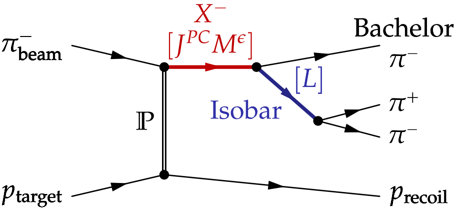

- $J^{PC}_{X^-}$: Spin and eigenvalues under parity and charge conjugation of X^-
- M^{ε}: Spin projection on the beam and naturality of the exchange particle

  Natural: $P = (-1)^J$

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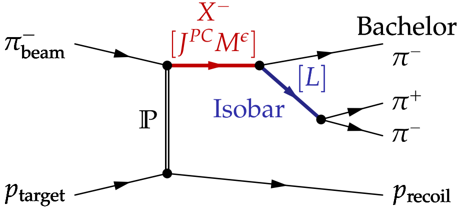

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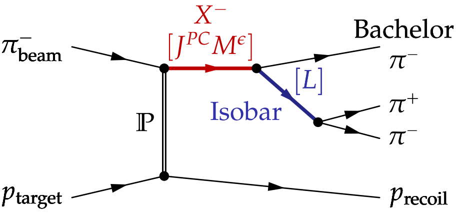

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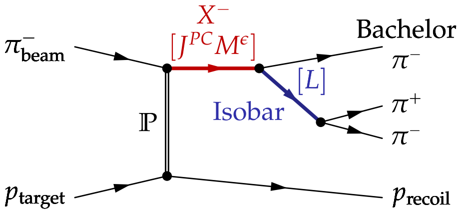

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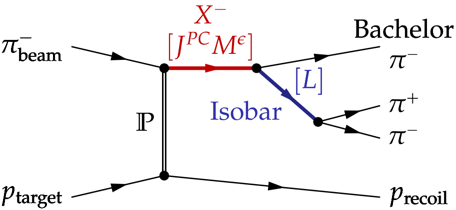

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Various coherent possibilities for quantum-number combinations

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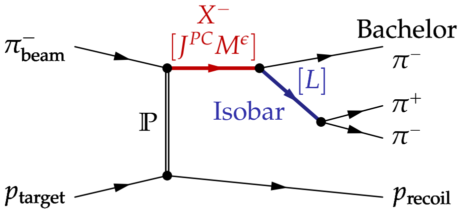

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Fit parameters: Transition amplitudes $\mathcal{T}_i$

Fixed: Angular amplitudes $\psi_i(\vec{\tau})$, dynamic isobar amplitudes $\Delta_i(m_{\pi^-\pi^+})$

Freed-isobar method
Step-like isobar amplitudes

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- Fixed isobar amplitudes $\rightarrow$ Sets of $m_{\pi^-\pi^+}$ bins:

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Freed-isobar method
Step-like isobar amplitudes

- Total intensity in one $(m_{3\pi}, t')$ bin as function of phase-space variables $\vec{\tau}$:

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Freed-isobar method

Step-like isobar amplitudes

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Fit paramet…

Fixed: Angu… $\Delta_i\left(m_{\pi^-\pi^+}\right)$

- Fixed isoba…

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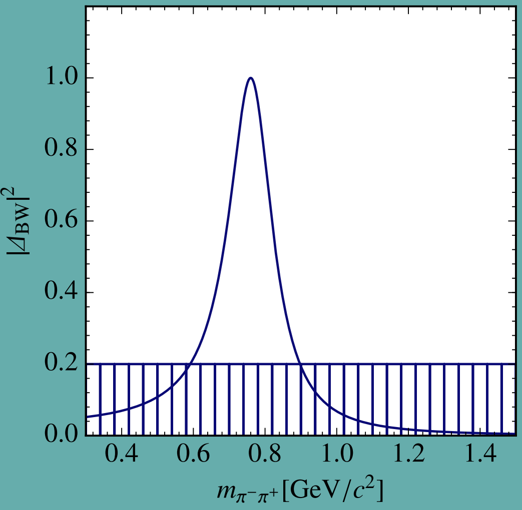
$\mathcal{I}(\vec{\tau}$ … m.$\Big]\Big|^2$

Illustration



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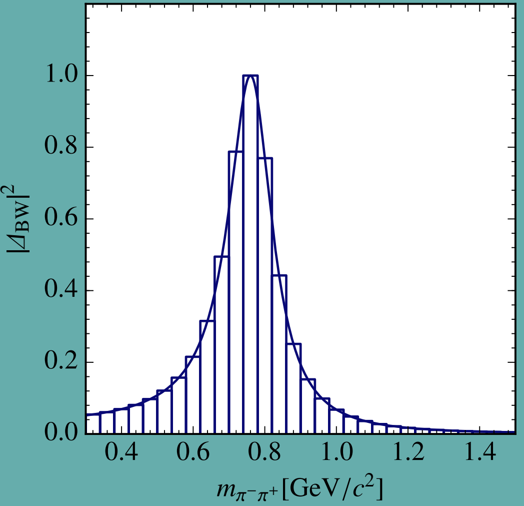

Fitting model

Freed-isobar analysis

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$1^{++}0^{+}[\pi\pi]_{0^{++}}\pi$P	$2^{-+}0^{+}[\pi\pi]_{1^{--}}\pi$F
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- Depending on $m_{3\pi}$ and wave, up to 62 $m_{\pi^-\pi^+}$ bins per freed wave

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Ambiguity in this fit model!

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- **"Zero mode"**: Shift of dynamic isobar amplitude by arbitrary $\mathcal{C}$

$$\Delta^{\mathrm{meas}}(m_\xi) = \Delta^{\mathrm{phys}}(m_\xi) + \mathcal{C}\Delta^0(m_\xi)$$

  leaves $\mathcal{A}_{1^{-+}}^{\mathrm{symm}}$ and therefore intensity and likelihood invariant
  $\Rightarrow$ **Ambiguous solutions**

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Zero mode in the spin-exotic wave

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- In the case of the $1^{-+}1^{+}[\pi\pi]_{1^{--}}\pi \text{P}$ wave:
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  - limit fit range to $m_{\pi^-\pi^+} < 1.12\,\text{GeV}$ to minimize effects from possible excited ρ' states

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- Note: Resolving the ambiguity fixes only a **single** complex-valued degree of freedom, $\mathcal{C}$. $\mathbf{n_{bins} - 1}$ complex-valued degrees of freedom remain free.
  FK, D. Greenwald, D. Ryabchikov, B. Grube, S. Paul, PRD **97** (2018) 114008

- Superfluo … fit
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- In the cas…
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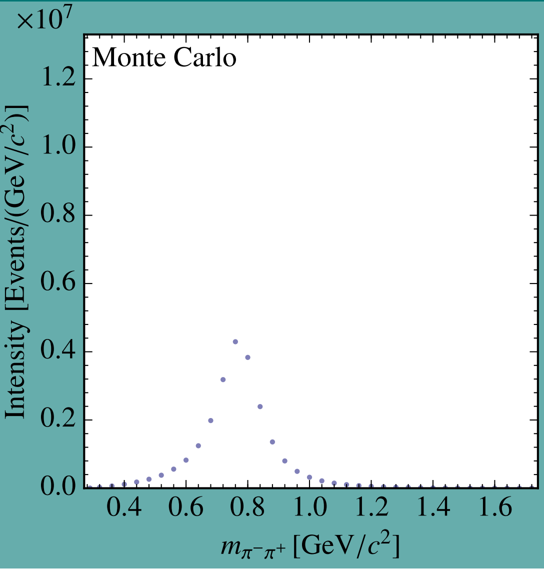
FK, D. Gre… 2018) 114008

Verification of the method



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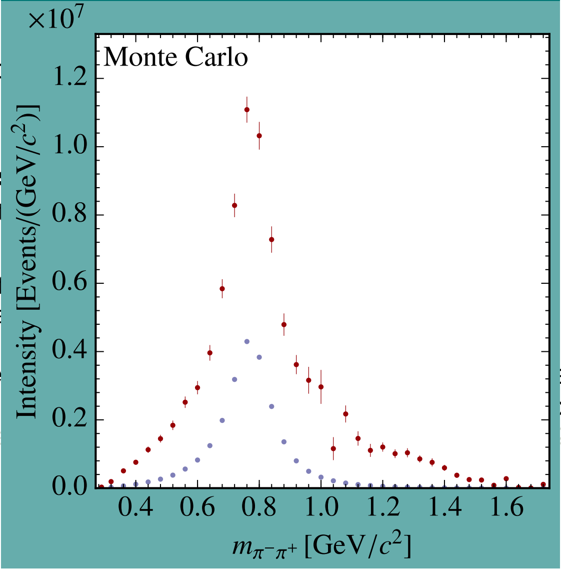

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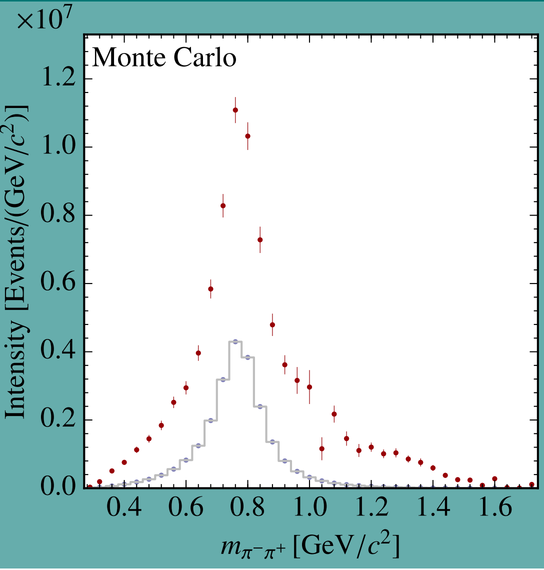

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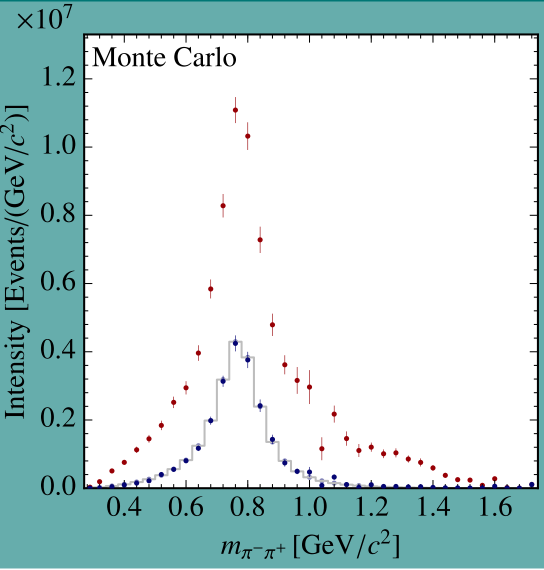

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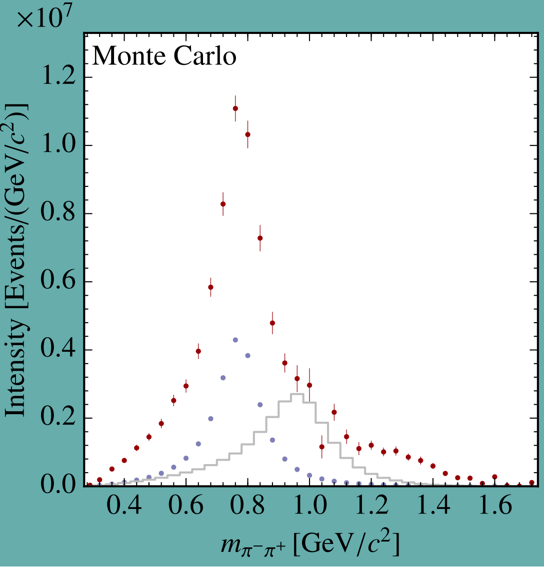

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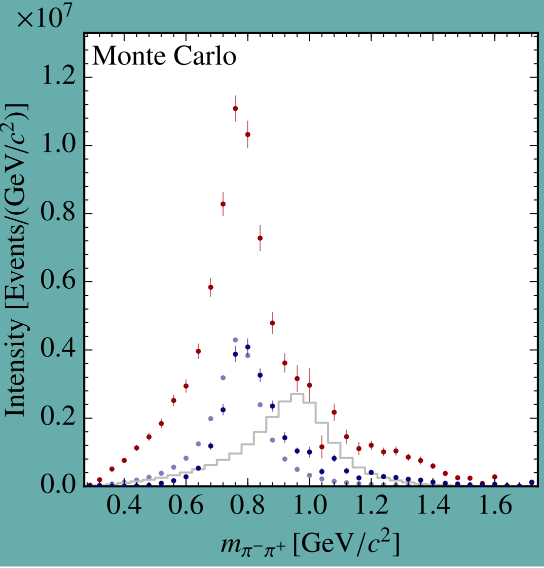
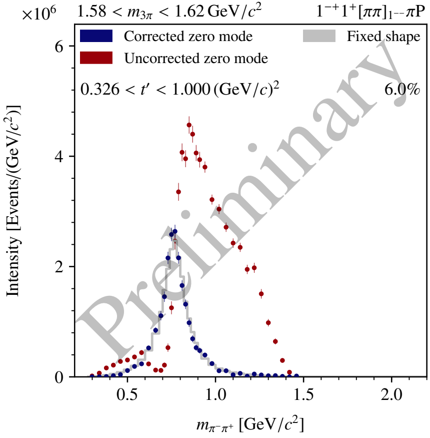
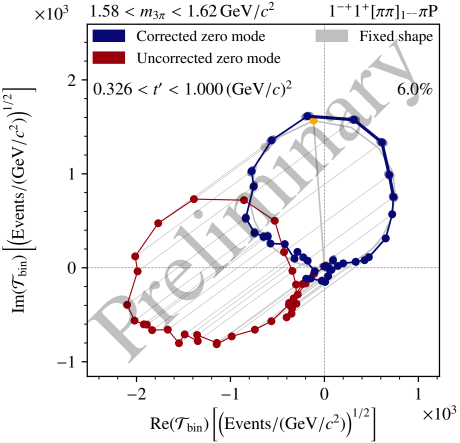
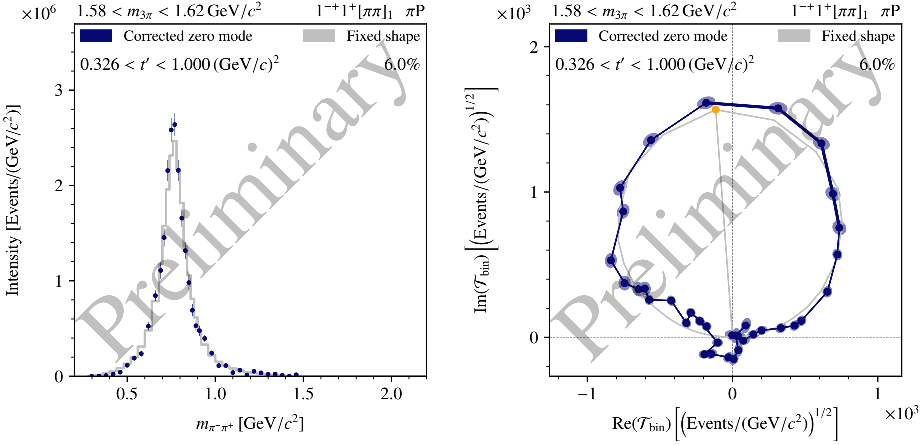


Freed-isobar result

Results for the spin-exotic wave $1^{-+}1^{+}[\pi\pi]_{1^{--}}\pi P$







$\times 10^6$
$1.58 < m_{3\pi} < 1.62\,\mathrm{GeV}/c^2$
$1^{-+}1^{+}[\pi\pi]_{1--}\pi P$
Corrected zero mode
Fixed shape
$0.326 < t' < 1.000\,(\mathrm{GeV}/c)^2$
6.0%
Intensity [Events/(GeV/c^2)]
$m_{\pi^-\pi^+}$ [GeV/c^2]
0
1
2
3
0.5
1.0
1.5
2.0
Preliminary
$\times 10^3$
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$\mathrm{Im}(\mathcal{T}_{\mathrm{bin}})\left[\left(\mathrm{Events}/(\mathrm{GeV}/c^2)\right)^{1/2}\right]$
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$\times 10^3$
−1
0
1
2
Preliminary

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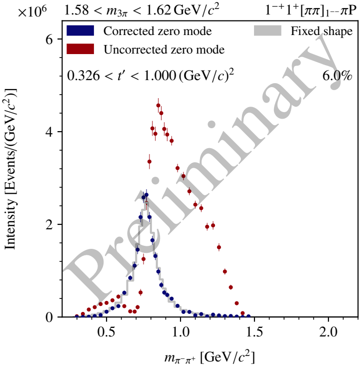

Ambiguity resolved in every $m_{3\pi}$ bin separately

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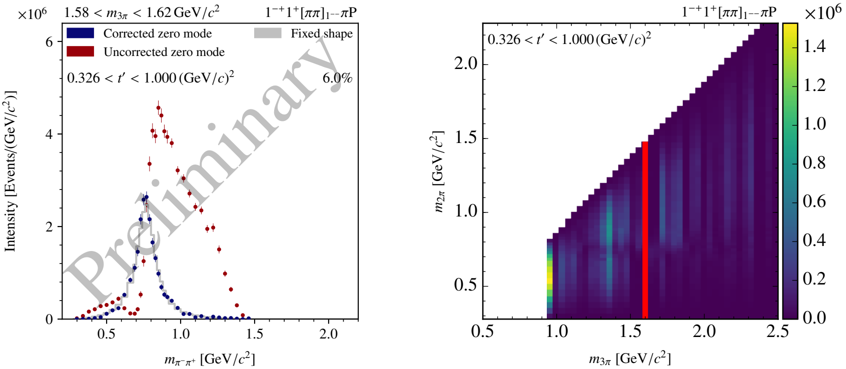

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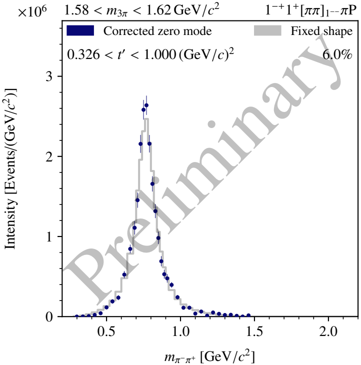

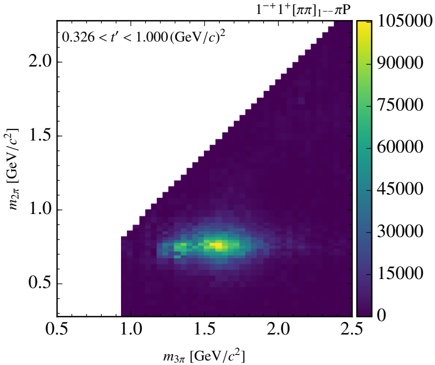

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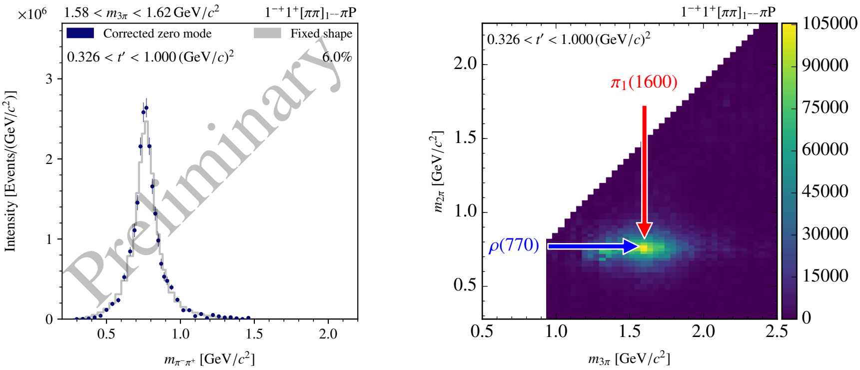

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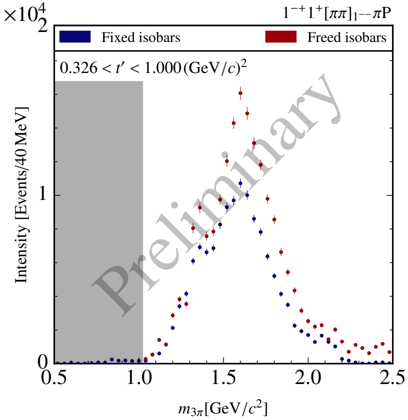
Correlation of $\pi_1(1600)$ with $\rho(770)$ confirmed

- Coherently sum up all $m_{\pi^-\pi^+}$ bins to obtain $m_{3\pi}$ spectrum



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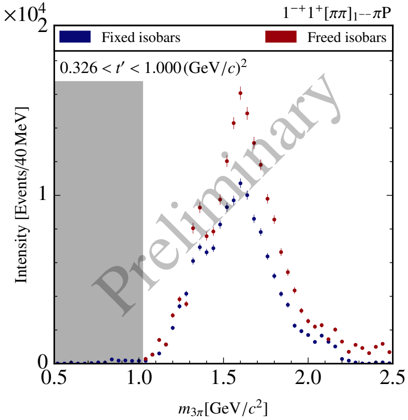
- Zero mode exactly cancels



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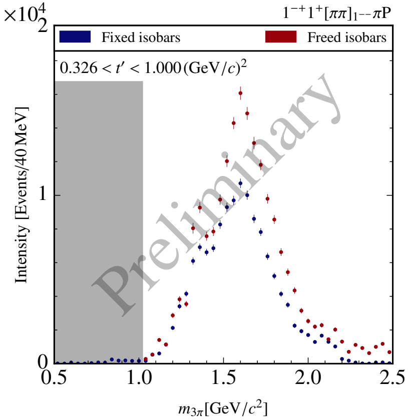

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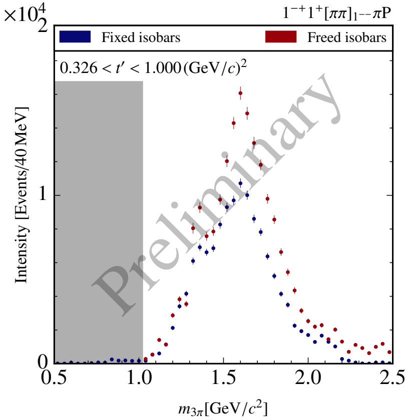

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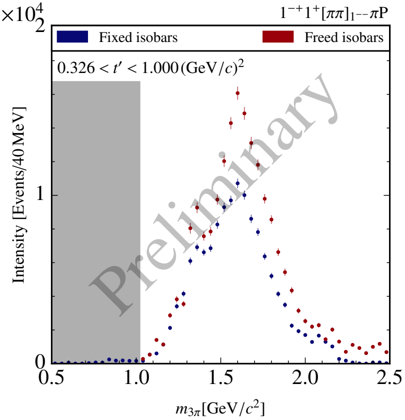
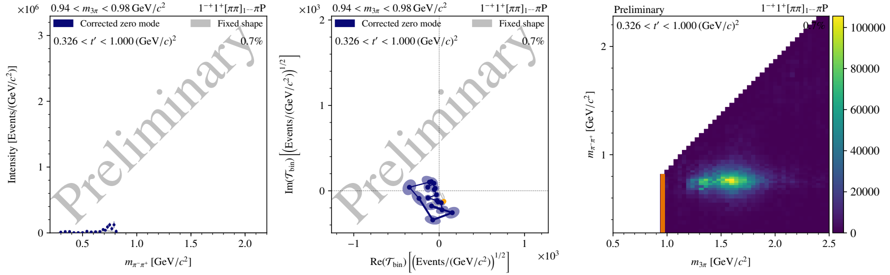
- Similar to fixed-isobar result for π_1 (1600)
- Isobar model: Valid assumption
- Observed deviations hint to:
  - Excited isobar resonances
  - Final state interactions
  - Non-resonant contributions



$$0.326 < t' < 1.000 (\text{GeV}/c)^2$$



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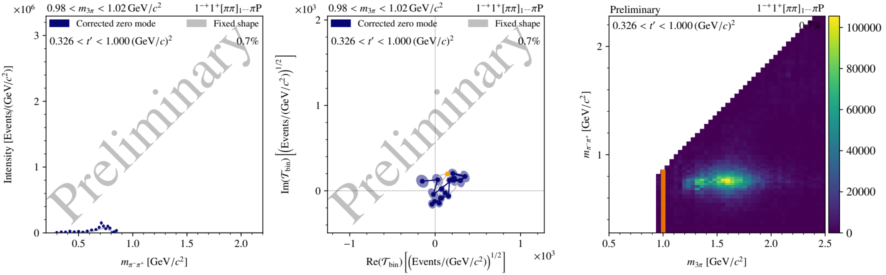

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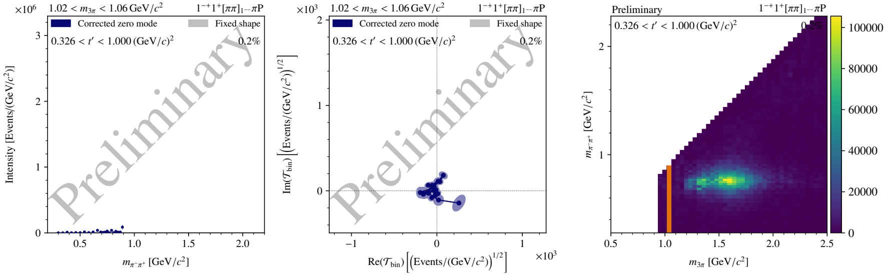

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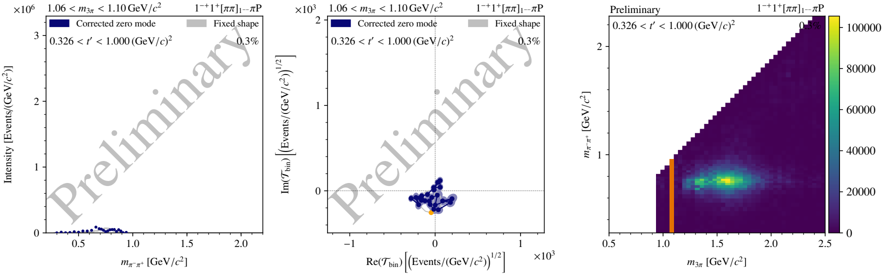

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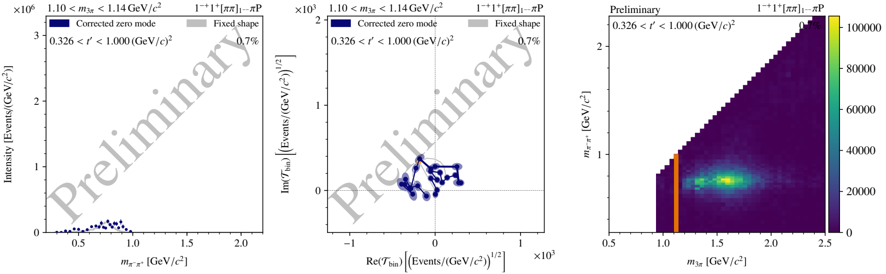

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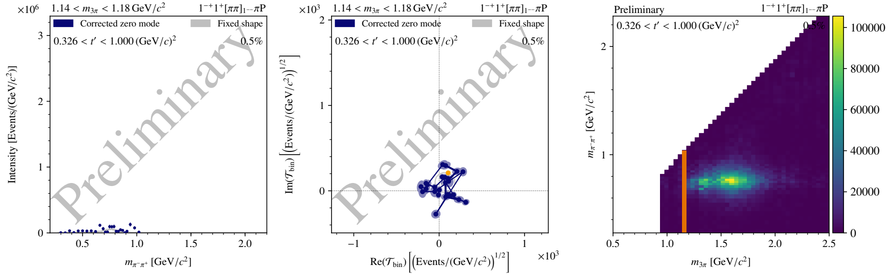

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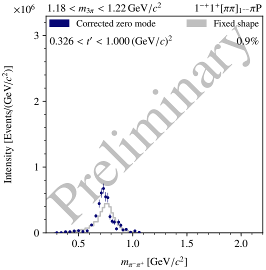
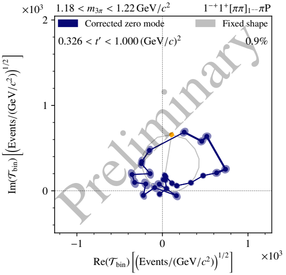
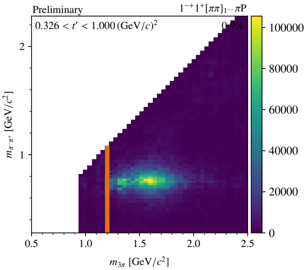


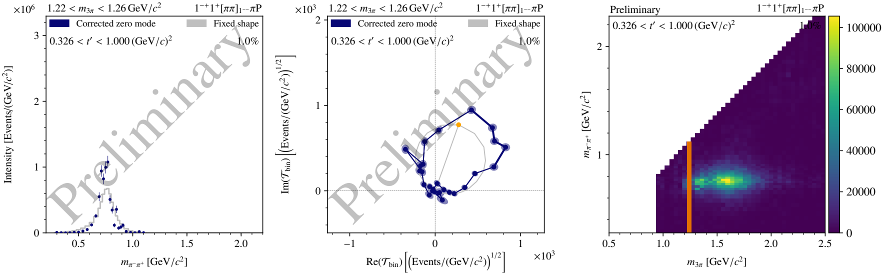




Freed-isobar result

Different mass slices

$$0.326 < t' < 1.000 (\mathrm{GeV}/c)^2$$



Freed-isobar result
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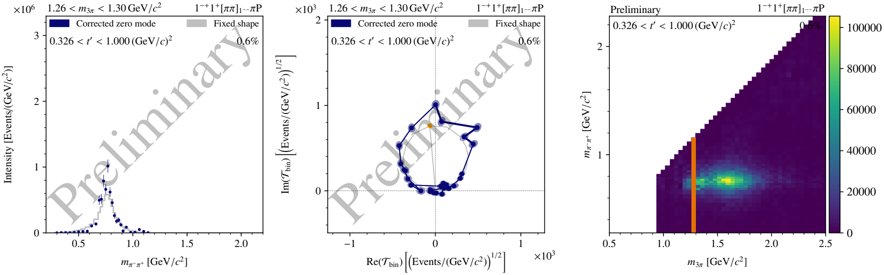

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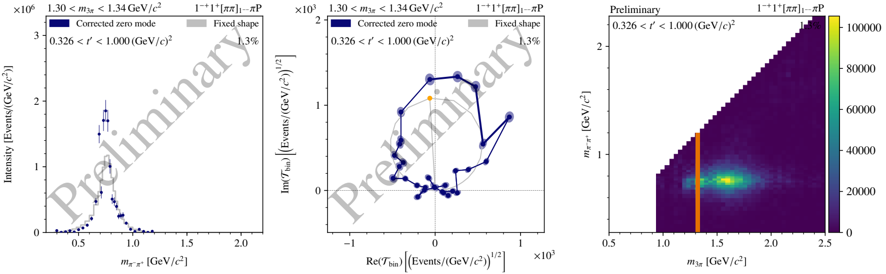

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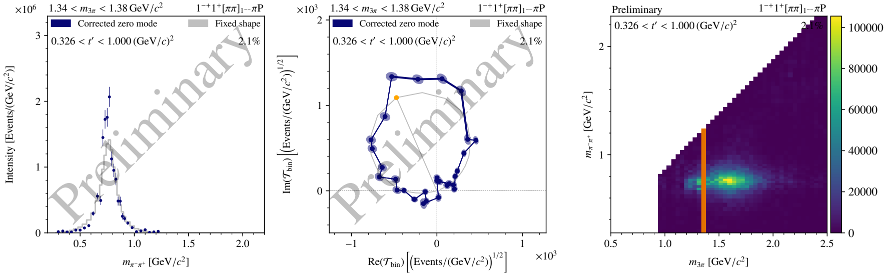

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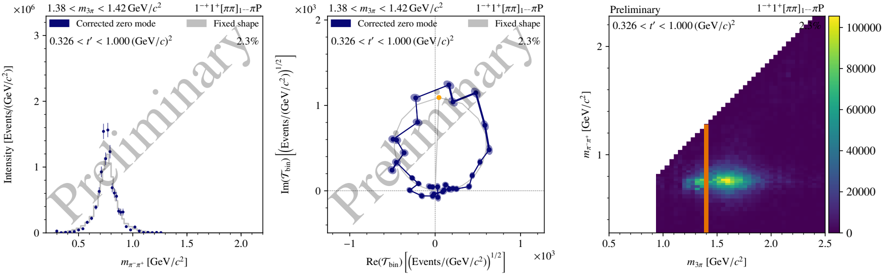

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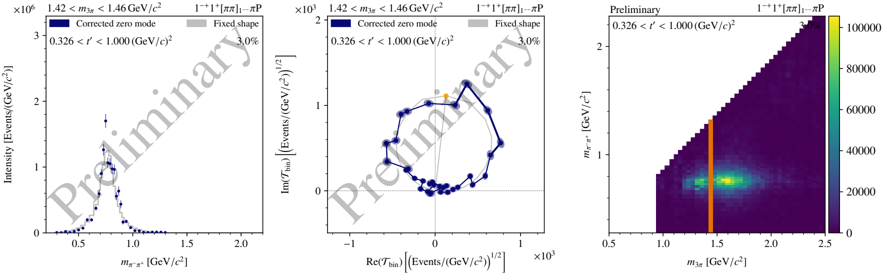

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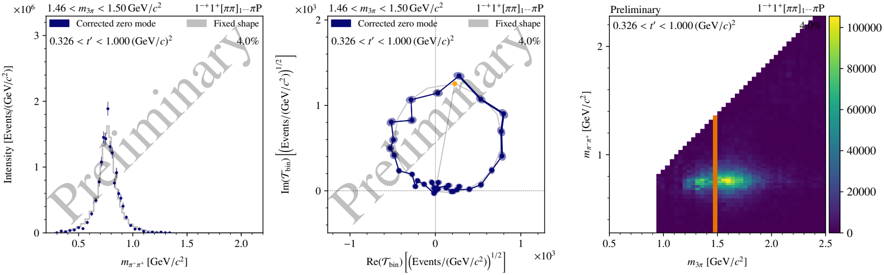

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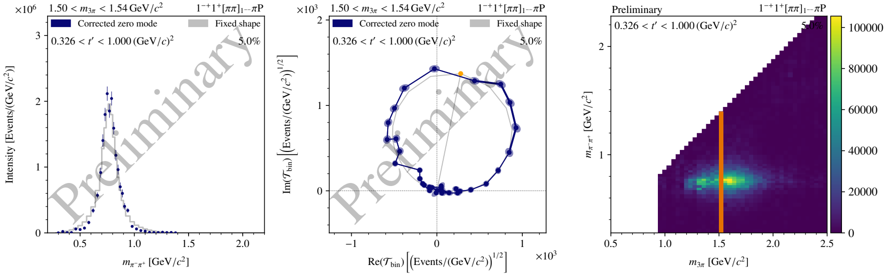

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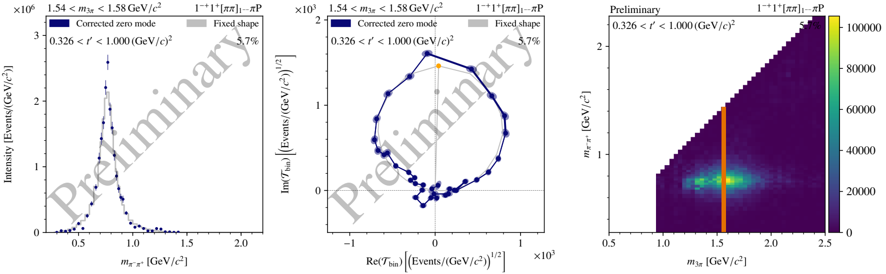

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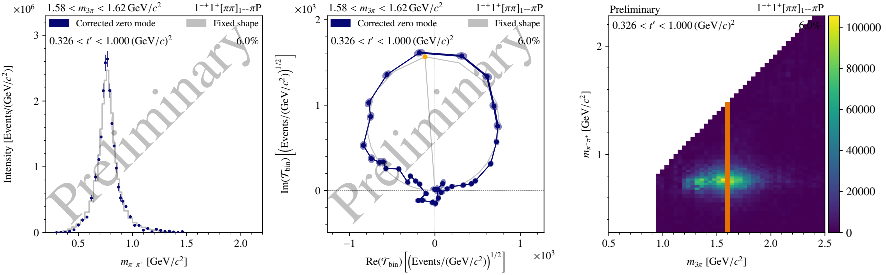

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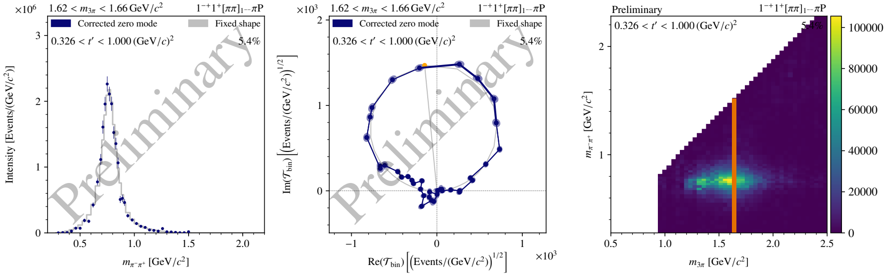

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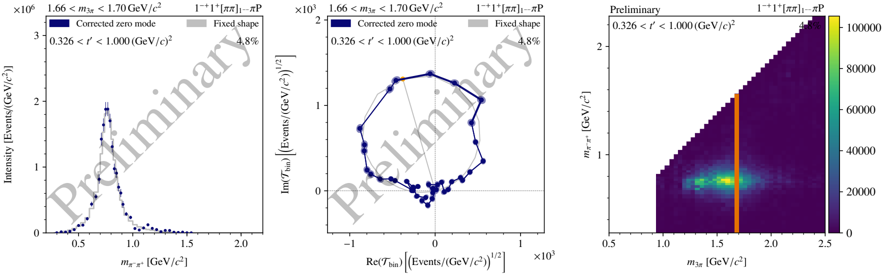

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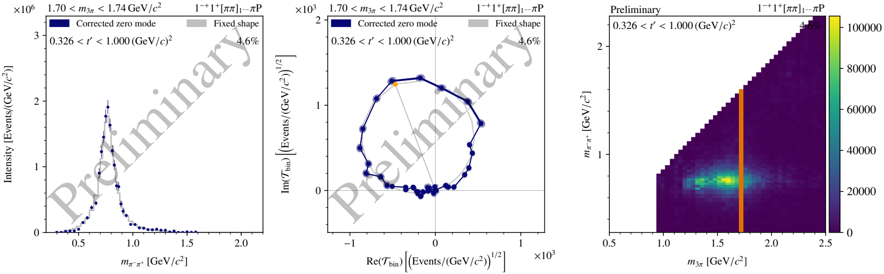

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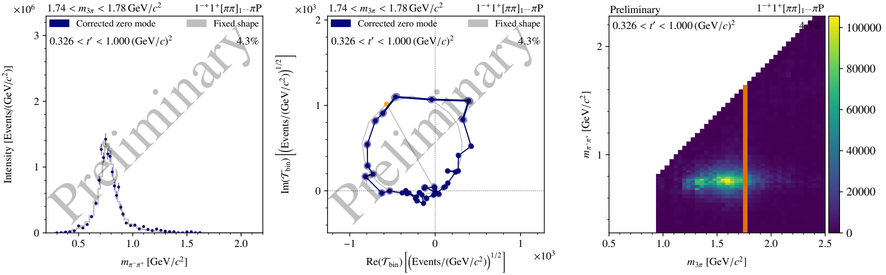

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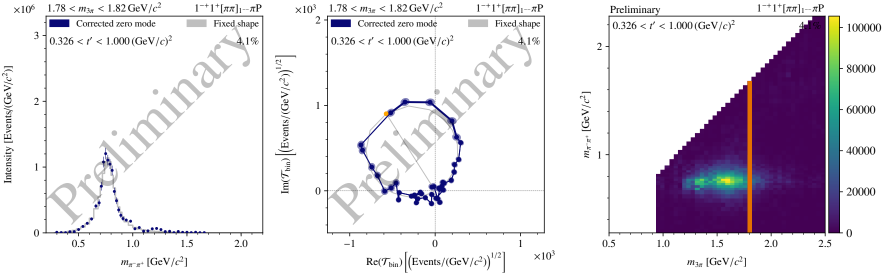

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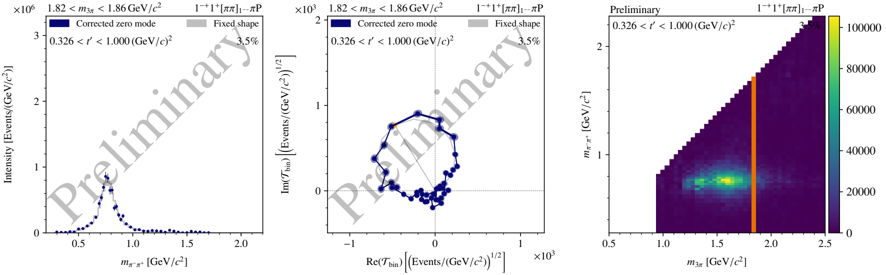

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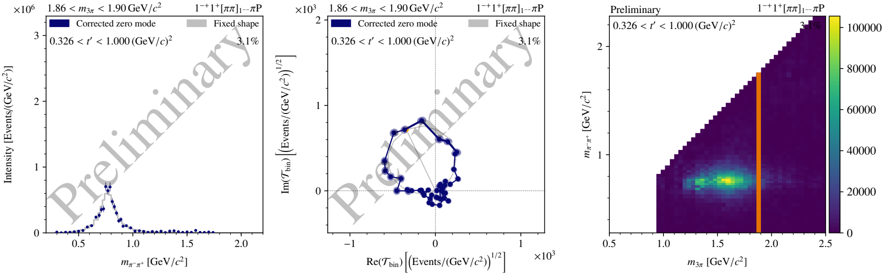

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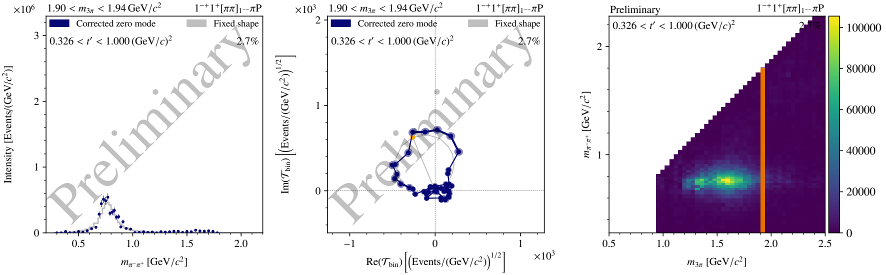

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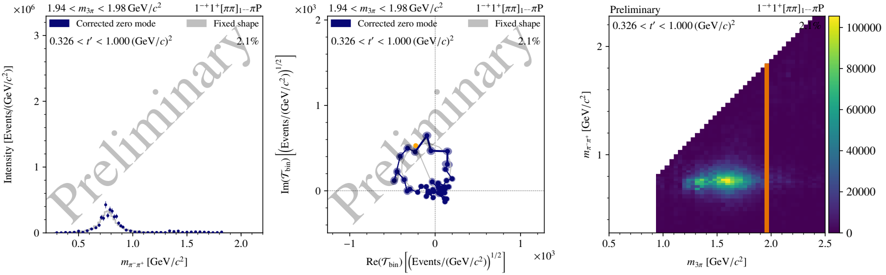

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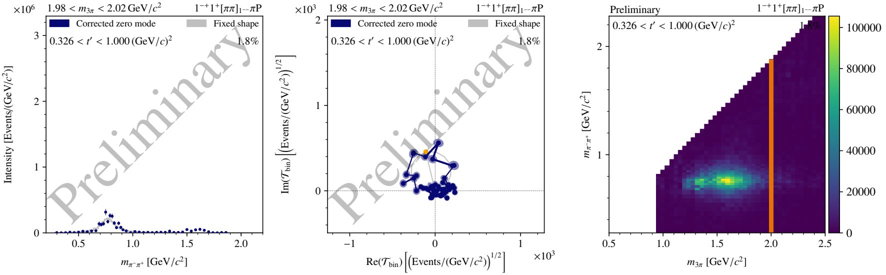

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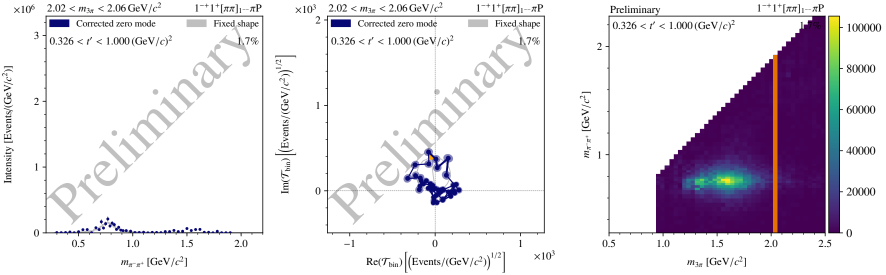

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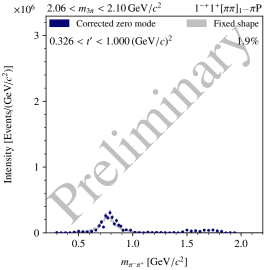

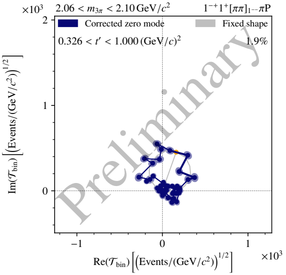

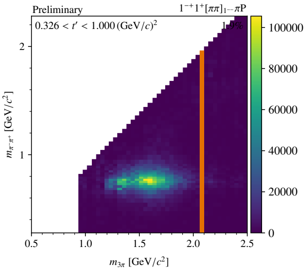

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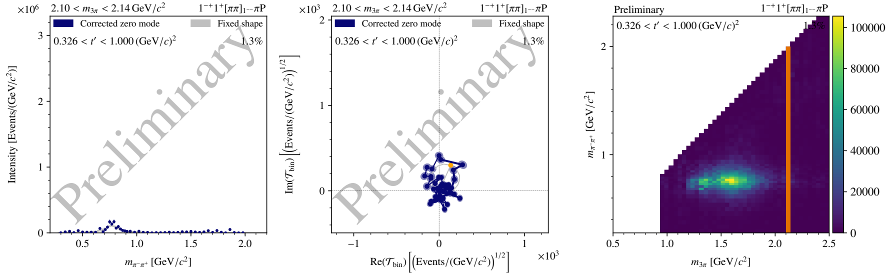

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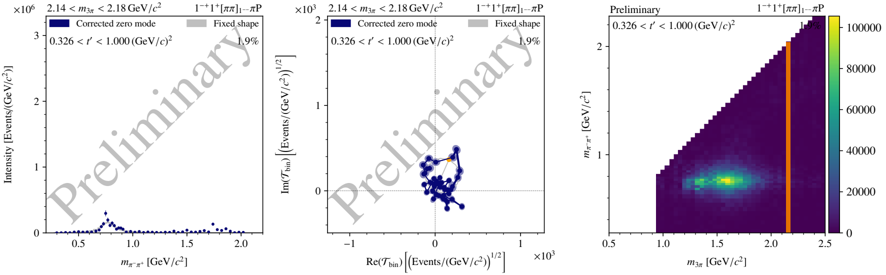

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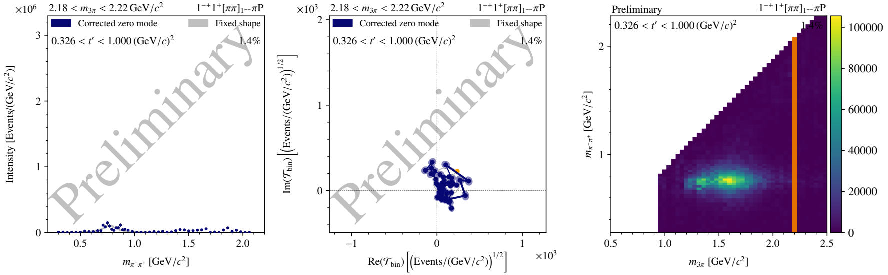

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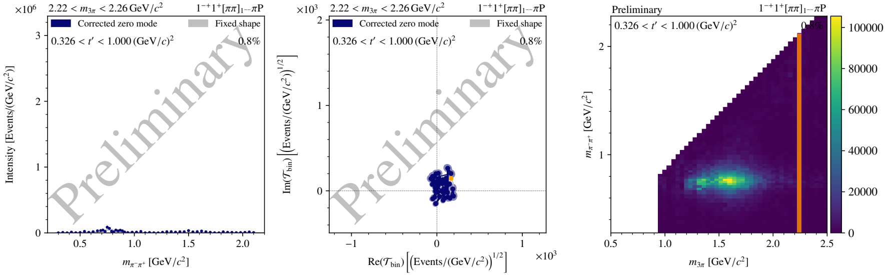

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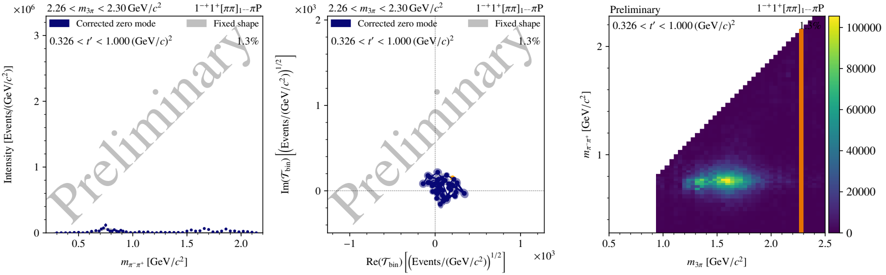

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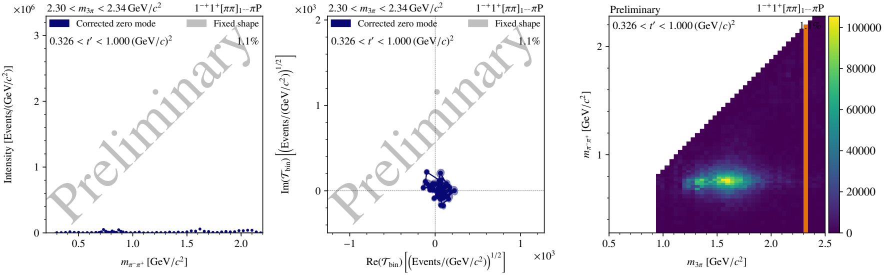

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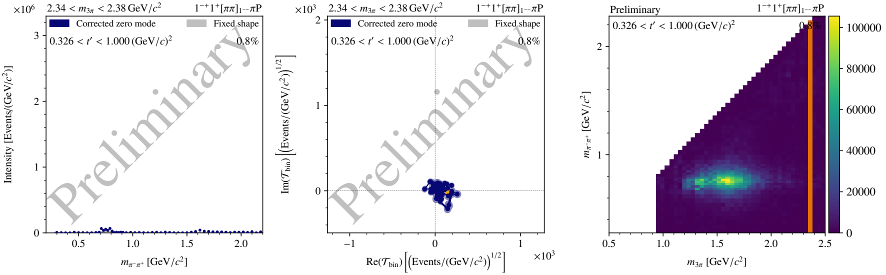

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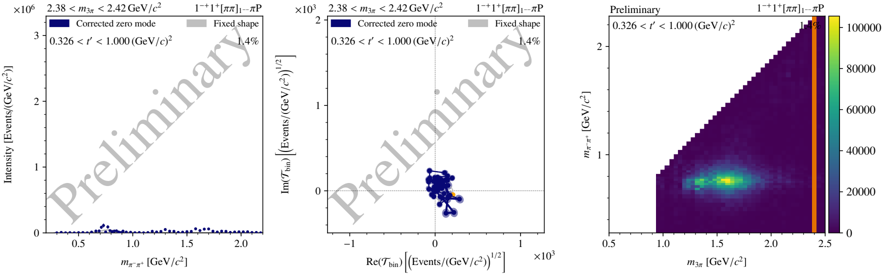

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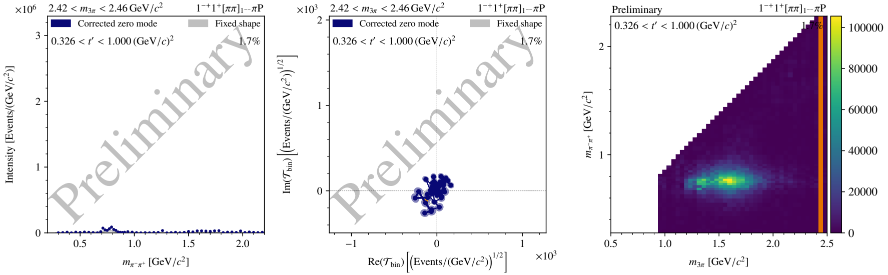

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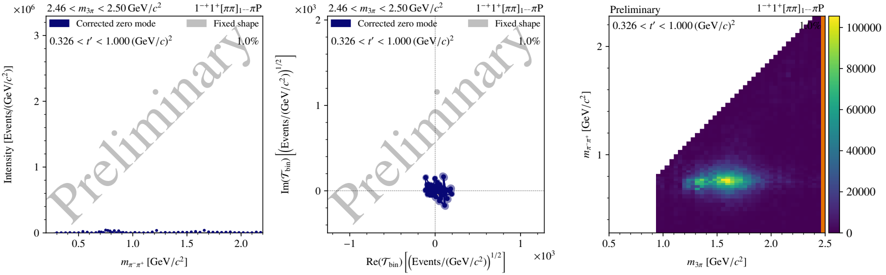

Freed isobars at COMPASS: Conclusions

Conclusion: Extended freed-isobar analysis with 12 out of 88 freed waves

- In total 200 independent fits in $m_{3\pi}$ and t' bins
- Independent dynamic isobar amplitudes obtained in every fit
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Next step: Analyze extracted dynamic isobar amplitudes

- Pin down resonance parameters of ρ'
- Study 2-body resonances in 3-body environment
  - Study 3-body re-scattering effects
  - e.g. Khuri-Treiman amplitudes (Bonn group)