valence quark distribution inside pion from lattice QCD

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the enigma of pion mass

- contribution of gluon trace-anomaly to pion mass:

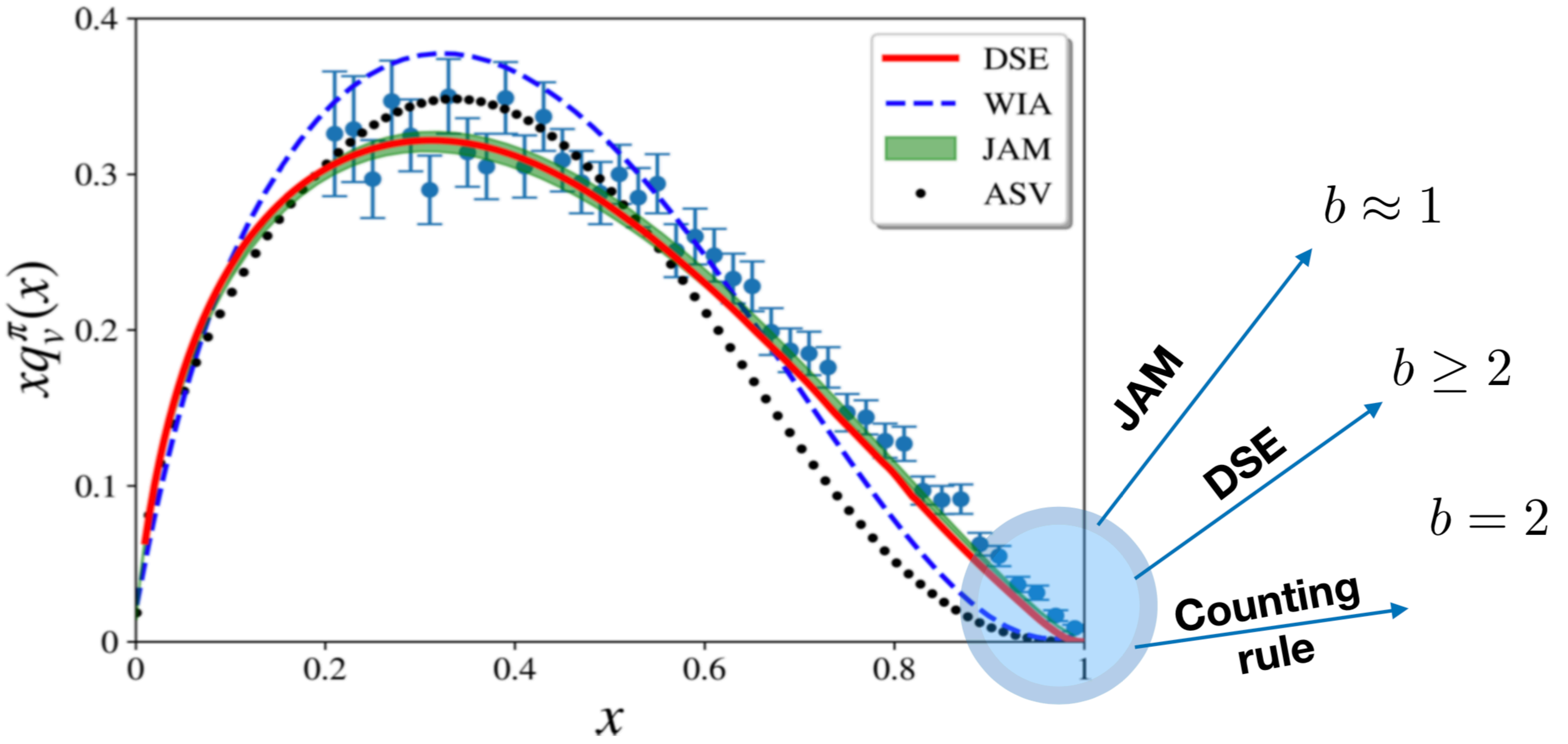
$$m_q \to 0 : \ \langle \pi(p) | \Theta^\mu_\mu | \pi(p) \rangle = -\, p^\mu p_\mu = m_\pi^2 \to 0$$

- but must be large contributions of gluons to proton mass even in the chiral limit

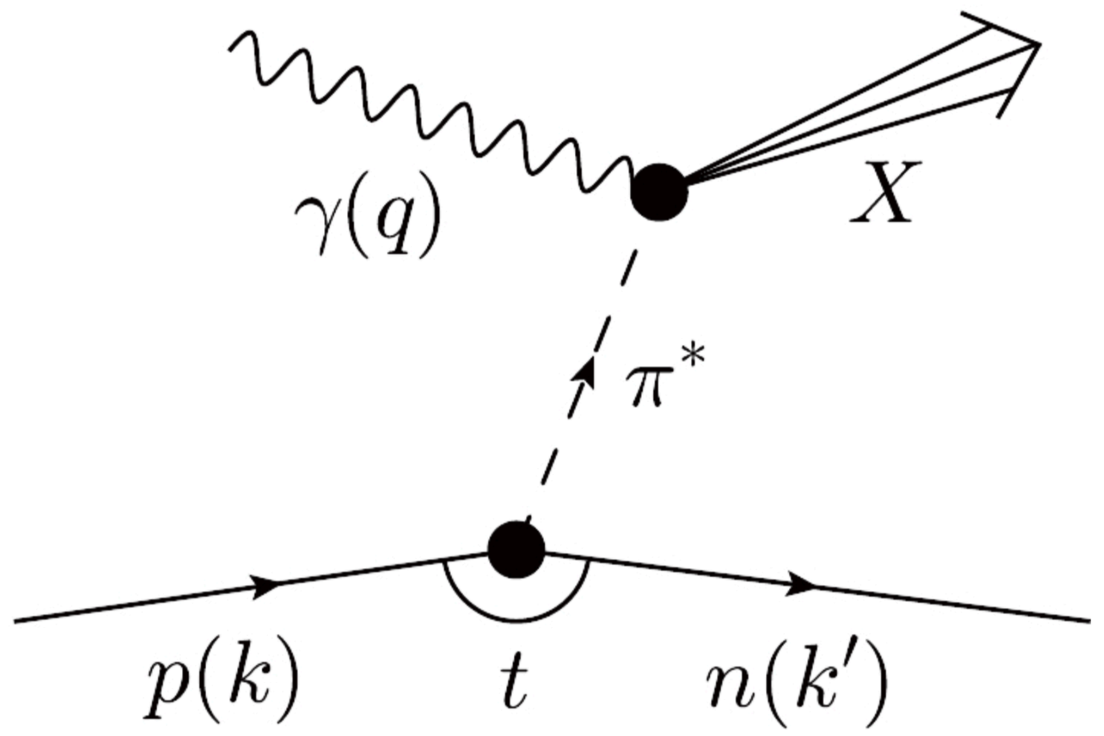
a puzzling dichotomy !!

understanding of proton mass will remain incomplete without simultaneously explaining the *absence* of pions mass *large-x behavior of pion valance PDF*

$$f_v^\pi(x) = Ax^a(1-x)^b$$



EIC: Sullivan process with off-shell pion



… and Parton structure of pion from QCD

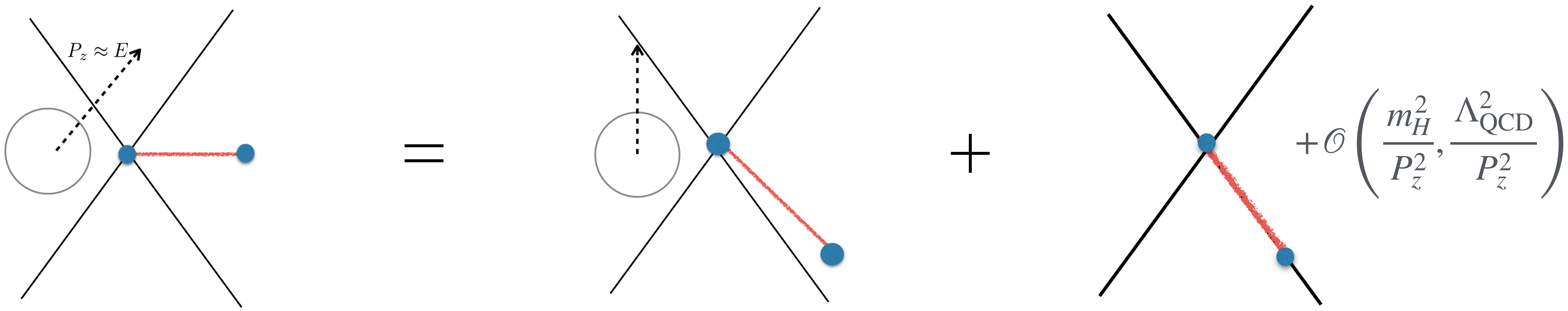
whitepaper: arXiv:1907.08218

Pion and Kaon Structure at the Electron-Ion Collider

abstract

1 GeV mass-scale that characterizes atomic nuclei appear; why does it have the observed value; and, enigmatically, why are the composite Nambu-Goldstone (NG) bosons in quantum chromodynamics (QCD) abnormally light in comparison? In this perspective, we provide an analysis of the mass budget of the pion and proton in QCD; discuss the special role of the kaon, which lies near the boundary between dominance of strong and Higgs mass-generation mechanisms; and explain the need for a coherent effort in QCD phenomenology and continuum calculations, in exa-scale computing as provided by lattice QCD, and in experiments to make progress in understanding the origins of hadron masses and the distribution of that mass within them. We compare the unique capabilities foreseen at the electron-ion collider (EIC) with those at the hadron-electron ring accelerator (HERA), the

X. Ji: Phys. Rev. Lett. 110, 262002 (2013)



rest frame of operator

rest frame of hadron

perturbative matching

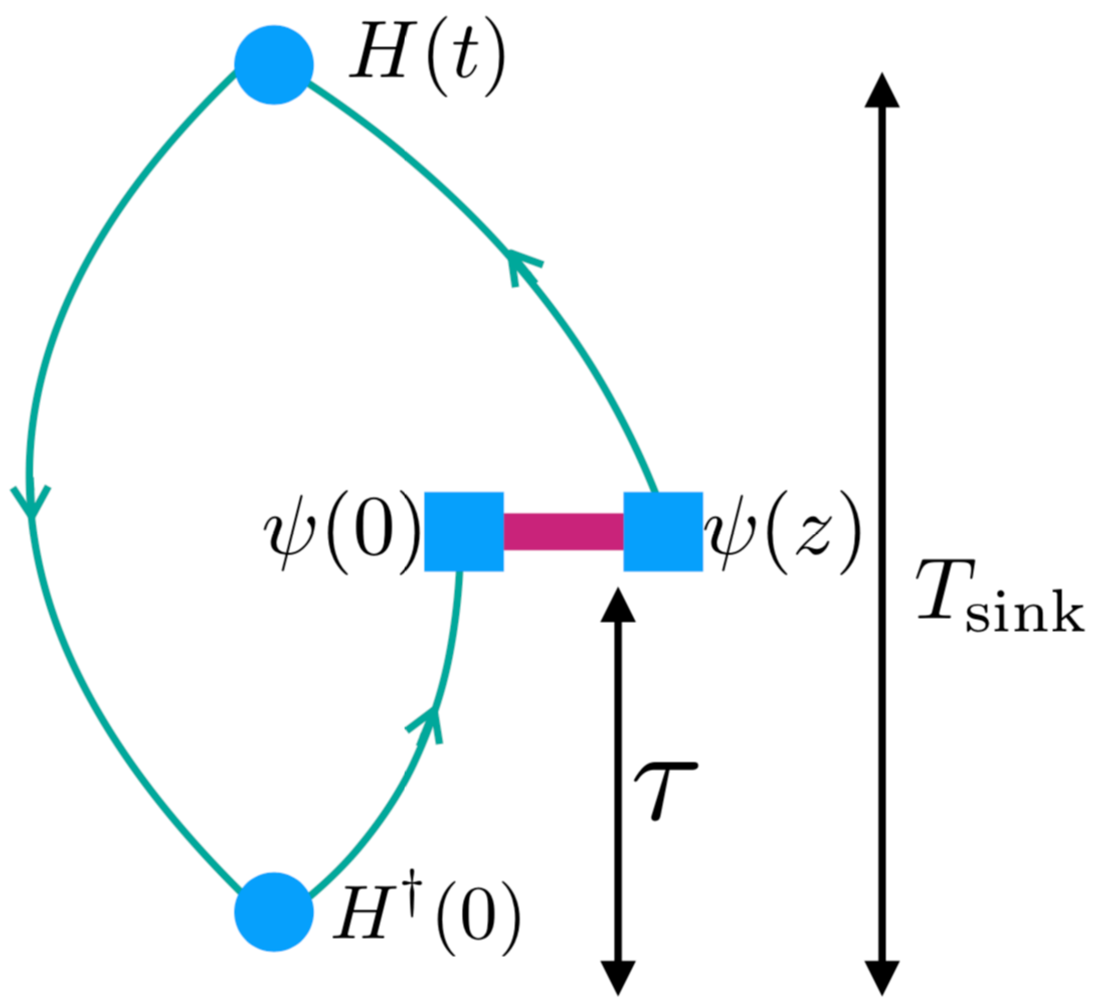
target-mass & higher-twist corrections

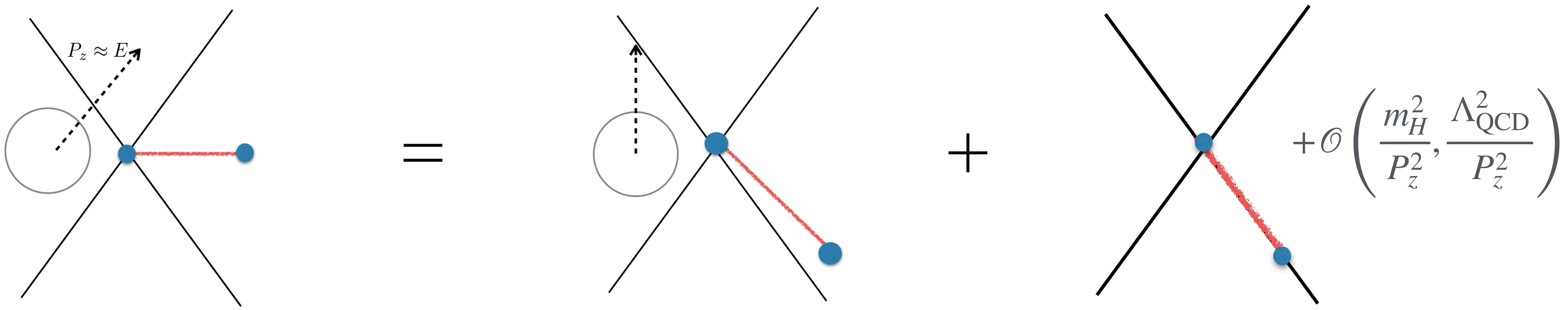
- bare matrix element, extract ground-state aptitude

$$Q_b(z, p_z) = \langle H(p_z, 0) \,|\, \overline{\psi}(0, \tau)\gamma_0 W(0, z)\psi(z, \tau) \,|\, H(p_z, T_{\rm sink})\rangle$$

equal-time, non-local operator within boosted hadron

$W(0,z)$: Wilson line from *0* to *z*





rest frame of operator

rest frame of hadron

perturbative matching

target-mass & higher-twist corrections

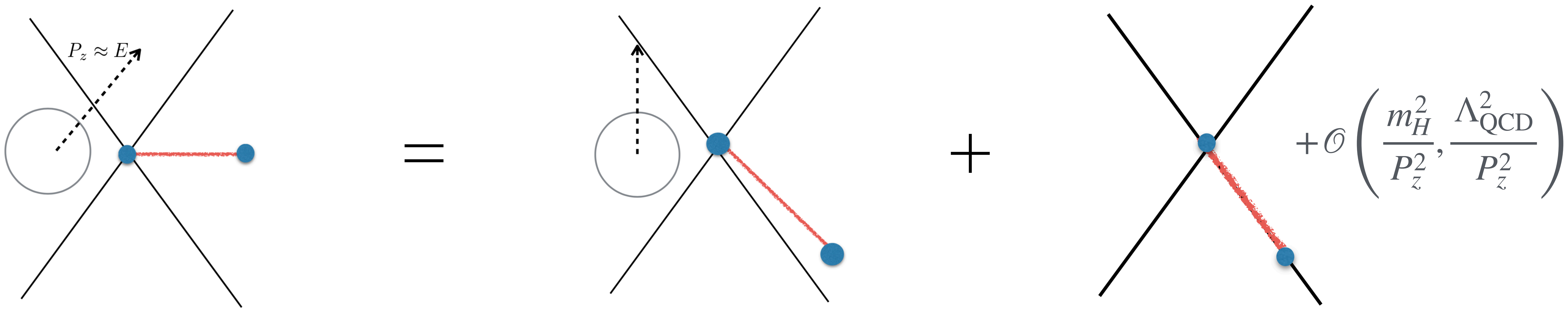
- renormalized operator:

renormalization condition: $Z(z, p^R, a) Q_b(z, p = p^R, a) \equiv e^{izp_z^R}$

by computing the same matrix element for Landau-gauge fixed off-shell quarks with $p^2 > 0$

$$Q(z, p_z, p^R) = Z(z, p^R) Q_b(z, p_z)$$

non-perturbative RI-MOM



rest frame of operator

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perturbative matching

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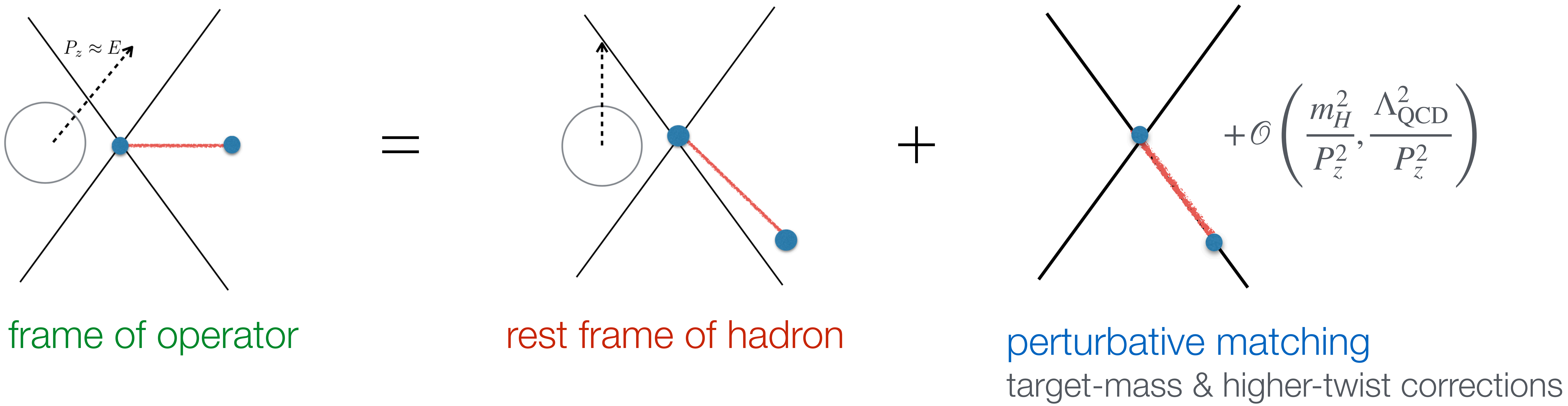
RI-MOM renormalized quasi-PDF (qPDF):

$$q(x, p_z, p^R) = \frac{1}{4\pi}\int_0^\infty dz e^{-izp_z x} Q(z, p_z, p^R)$$

Fourier transform w.r.t z

fixed p_z , small x → large $|z|$ → contaminations of Λ_{QCD}, m_H



RI-MOM qPDF to $\overline{\rm MS}$ -PDF:

$$q(x, P_z, P^R) = \int \frac{dy}{|y|} f(x,\mu)\, C\left(\frac{x}{y}, \frac{\mu}{yP_z}, \frac{P_\perp^R}{P_z^R}, \frac{yP_z}{P_z^R}\right) + \mathcal{O}\left(\frac{m_h^2}{P_z^2}, \frac{\Lambda_{QCD}^2}{P_z^2}\right)$$

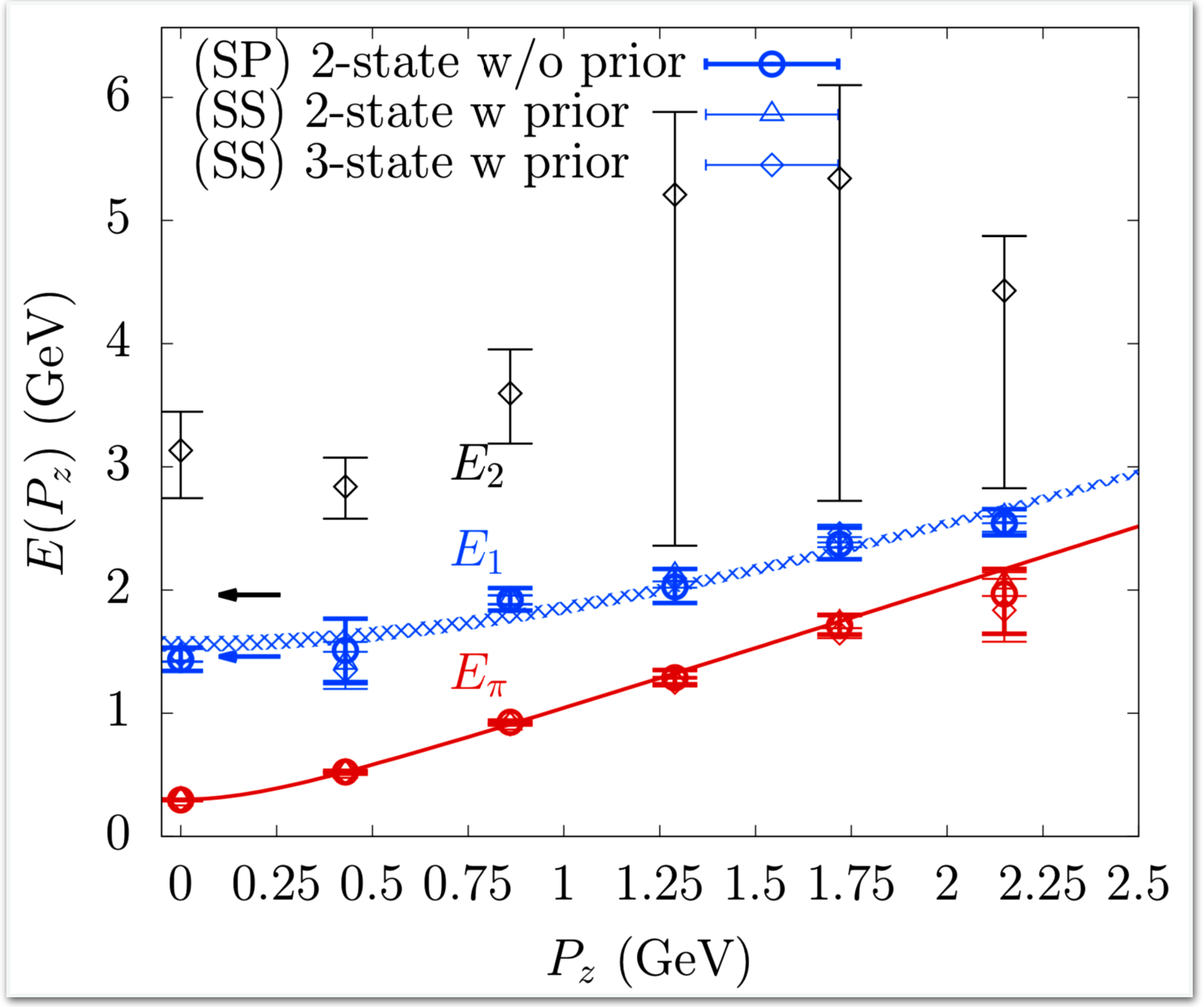
first continuum limit, then $p_z \to 0$

universal for all hadrons,
presently only 1-loop

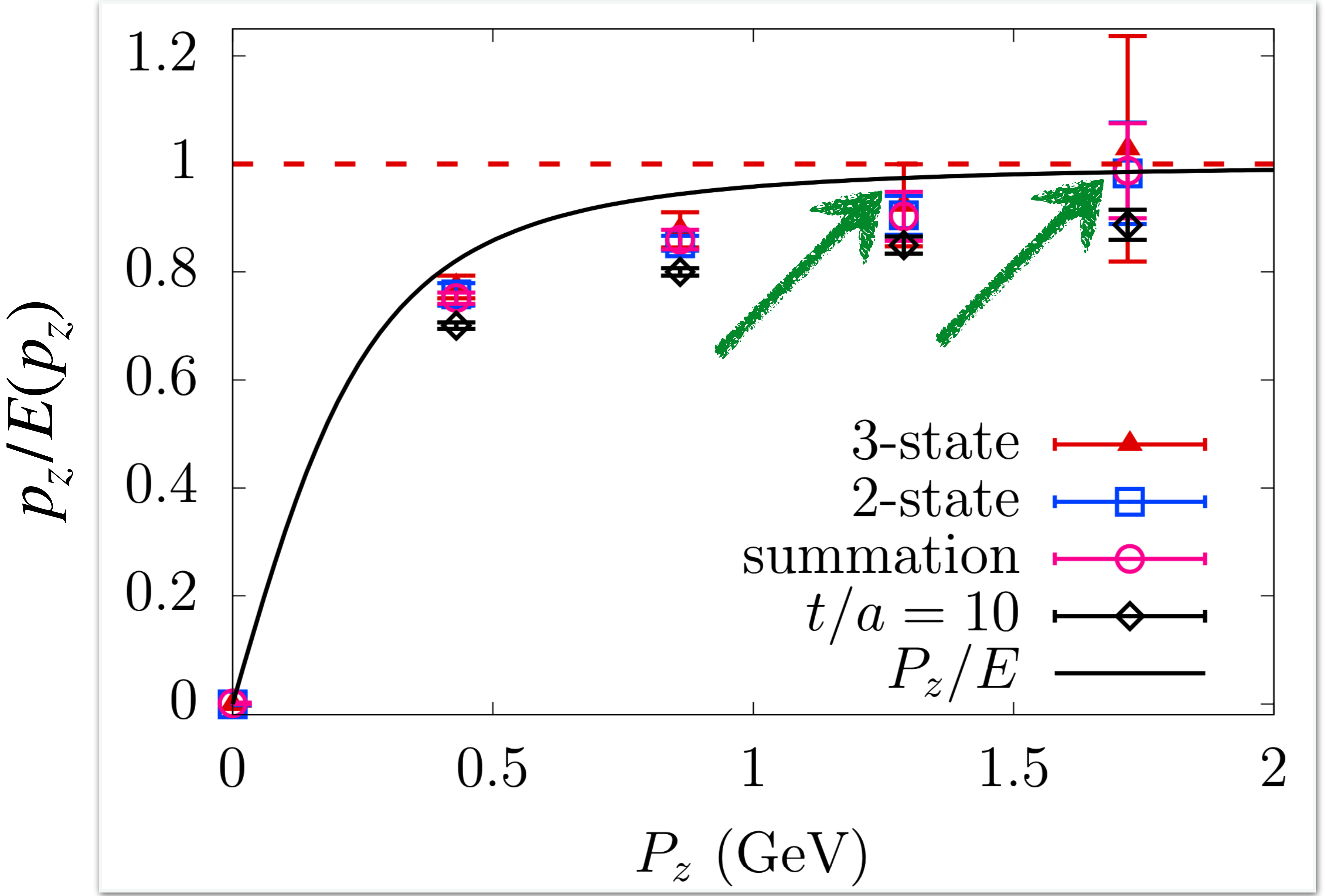
Stewart, Zhao: Phys. Rev. D 97, 054512 (2018) from QCD to $f_v^{\pi}(x)$ …

Nikhil Karthik *et. al.* (BNL-SBU): Phys. Rev. D (in press); arXiv:1905.06349

energy levels of boosted pion

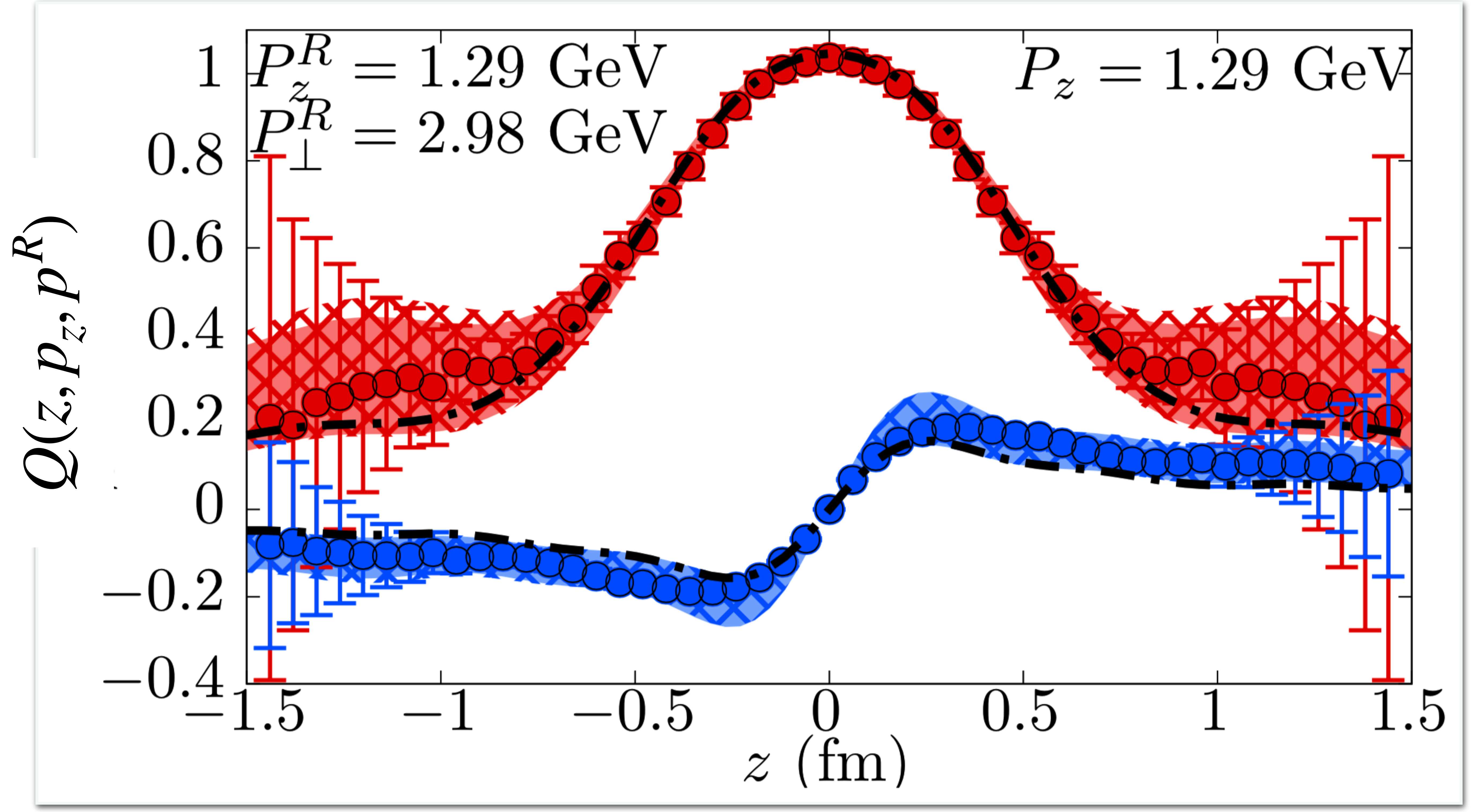


pion almost on the light-cone



- Wilson-Clover valance quarks: $m_\pi^{val} = 300$ MeV $p_z = 1.29,\ 1.72$ GeV
- latice spacing: $a = 0.06$ fm
- lattice size: 3.84×2.88^3 fm^4 (64×48^3) $m_\pi^{val} L = 4.4$
- 2+1 flavor HISQ HotQCD gauge configurations (1-HYP smeared): $m_\pi^{sea} \simeq 160$ MeV, $m_K^{sea} \simeq 500$ MeV

renormalized 3-pt operator in position space



red: real part; blue: imaginary part

dashed black lines: reconstructions from JAM global fits by reversing the procedure

Barry *et. al.* (JAM collaboration): Phys. Rev. Lett. 121, 152001 (2018) pion valance PDF

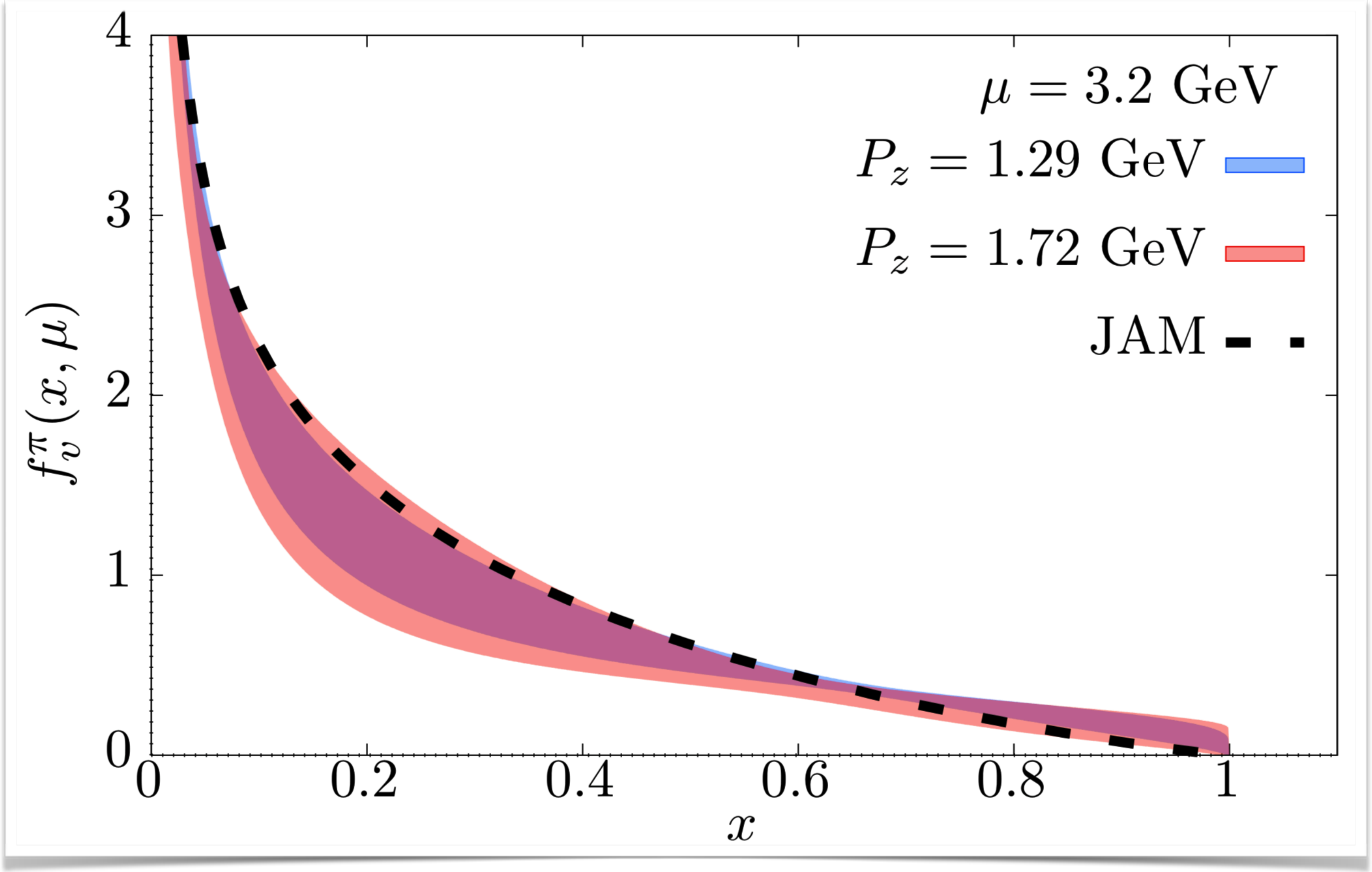
BNL-SBU LQCD:

$2\langle x \rangle = 0.43(6)$

BNL-SBU: Phys. Rev. D (in press); arXiv:1905.06349

JAM:

$2\langle x \rangle = 0.437$



$f_v^\pi(x) = f_u^\pi(x) - f_d^\pi(x) \;\ldots\; 0 < x < 1$

more is better … alternative representation

- pseudo Ioffe-time distribution: Lorentz invariant

Radyushkin: Phys.Rev. D96, 034025 (2017)

$$M_b(z^2,\nu) = \langle H(p_z,0)|\overline{\psi}(0,\tau)\gamma_0 W(0,z)\psi(z,\tau)|H(p_z,T_{\rm sink})\rangle \qquad \nu \equiv zp_z$$

- approach to light-cone: $z^2 \to 0,\ \nu = \text{fixed}$

$$\lim_{z^2\to 0} M(z^2,\nu) \to M(x,\mu) = \int_{-1}^{1} C(\mu,z) f(x,\mu) e^{ix\mu} d\nu$$

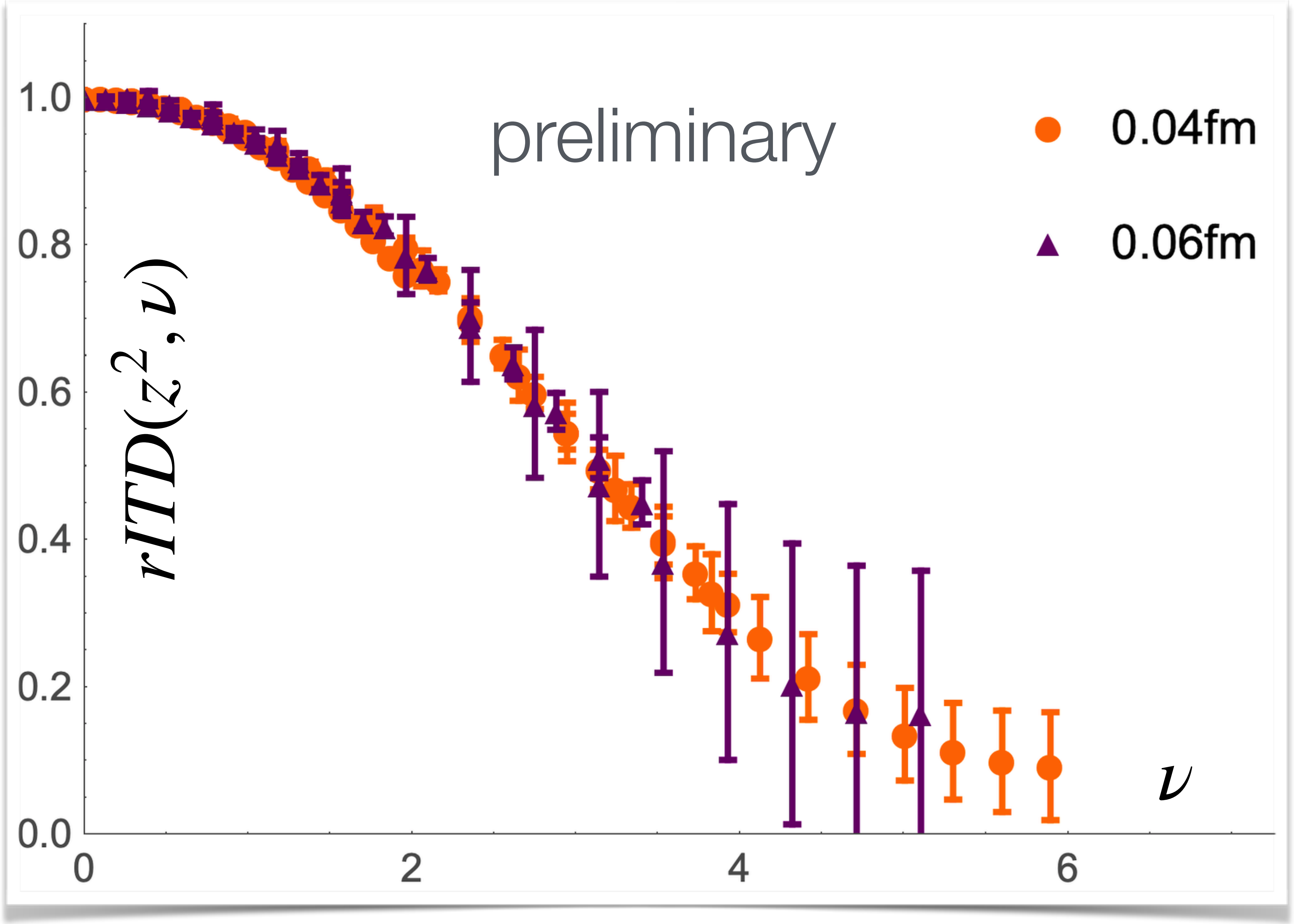
- reduced Ioffe-time distribution: ⟷ moments of $\overline{\rm MS}$-PDF

$$rITD(z^2,\nu) = \frac{M_b(z^2,\nu)}{M_b(z^2,0)} = \sum_{n=0}^{\infty} \frac{(-i\nu)^n}{n!} \frac{C_n(\mu^2 z^2)}{C_0(\mu^2 z^2)} \langle x^n\rangle(\mu) \quad + \mathcal{O}\left(z^2 m_H^2, z^2\Lambda_{\rm QCD}\right)$$

renormalized — perturbative, 1-loop — target-mass, higher-twist

Izubuchi *et. al.*: Phys. Rev. D98, 056004 (2018)

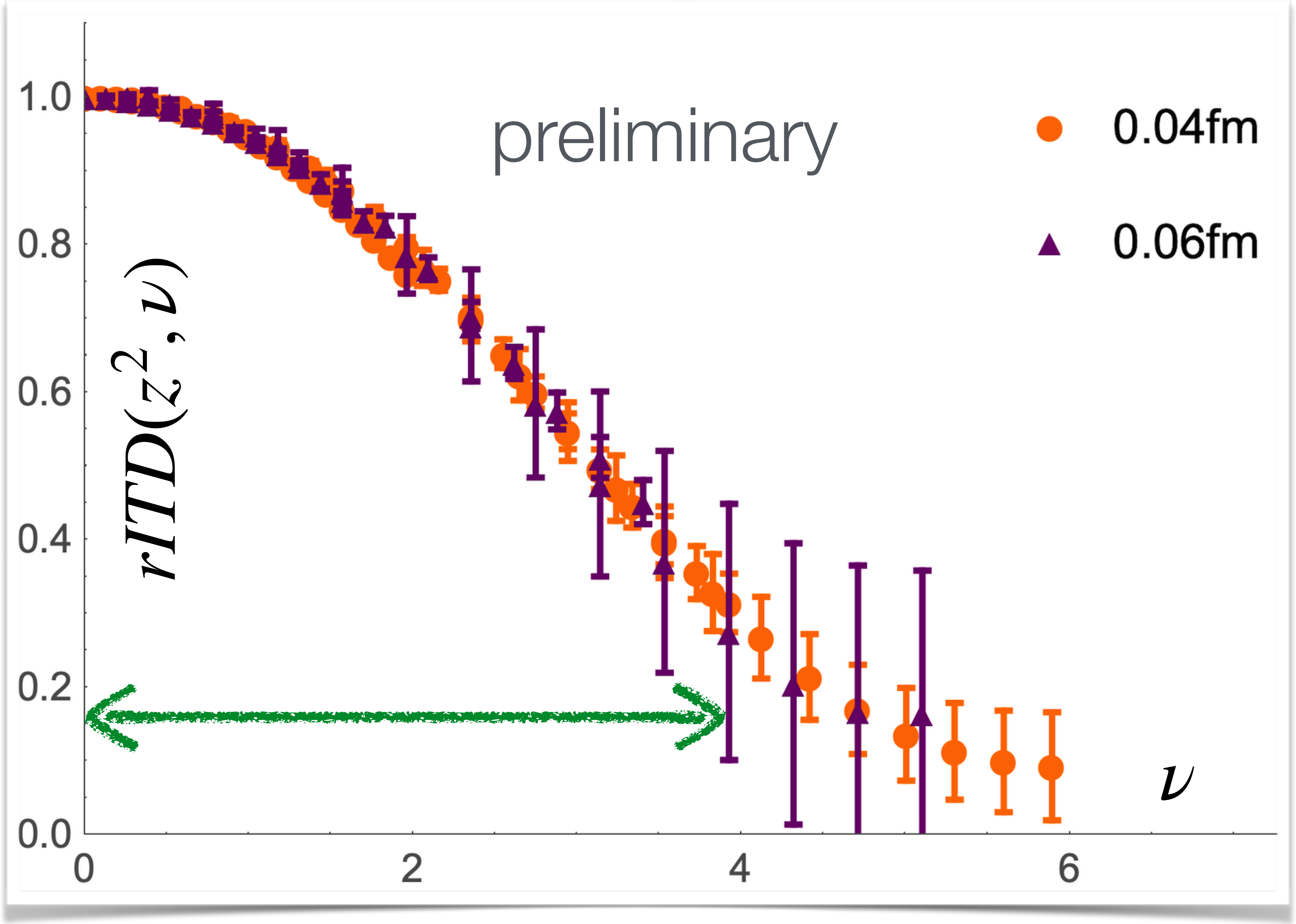
reduced Ioffe-time distribution



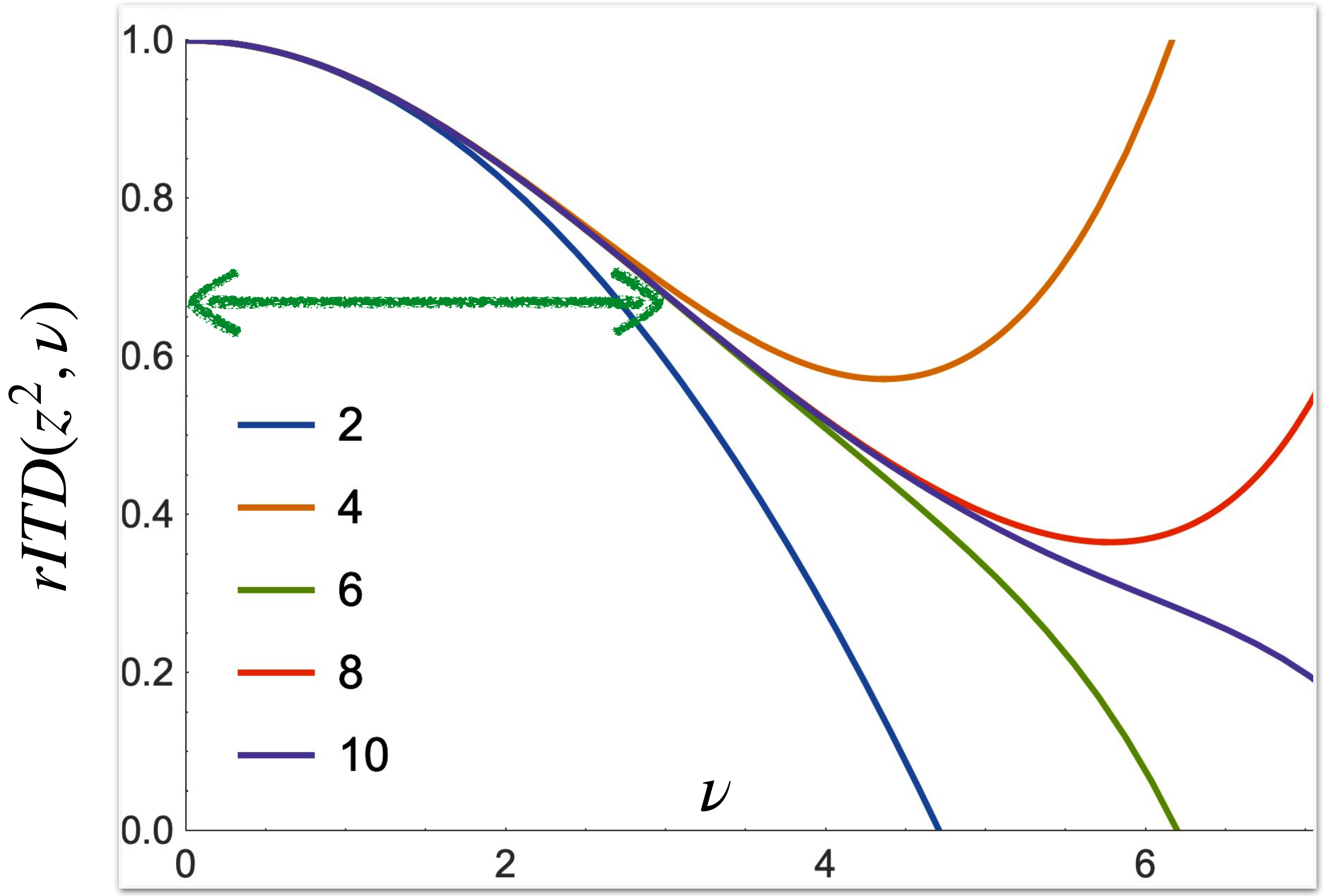
Xiang Gao *et. al.*: BNL-SBU-Tsinghua, on-going

$$a = 0.04 \text{ fm}, \ p_z = 0 - 1.48 \text{ GeV}, \ z = 0 - 0.8 \text{ fm}, \ m_\pi^{val} = 300 \text{ MeV}, \ 64 \times 64^3$$

$$a = 0.06 \text{ fm}, \ p_z = 0 - 1.29 \text{ GeV}, \ z = 0 - 0.8 \text{ fm}, \ m_\pi^{val} = 300 \text{ MeV}, \ 64 \times 48^3$$



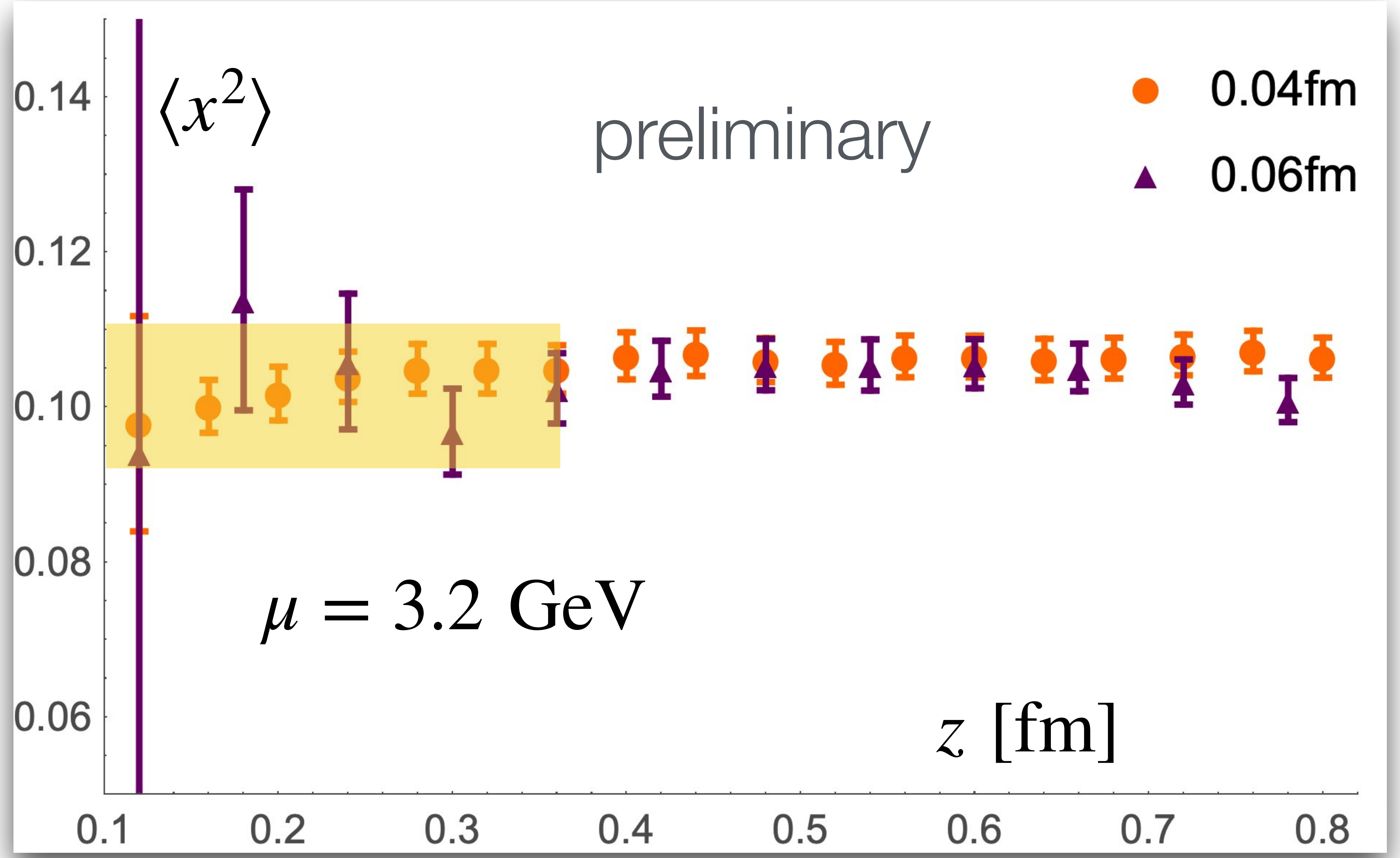
rITD reconstructed form JAM results using n=2-10 moments



joint fit data for all z, p_z for $\nu = zP_z = 0 - 4$ sensitive only to $\langle x^2 \rangle$, $\langle x^4 \rangle$

$$rITD(zp_z, z^2) = \sum_{n=0}^{\infty} \frac{(-izP^z)^n}{n!} \frac{C_n(\mu^2 z^2)}{C_0(\mu^2 z^2)} \langle x^n \rangle(\mu)$$

2nd moment of $f_v^\pi(x)$



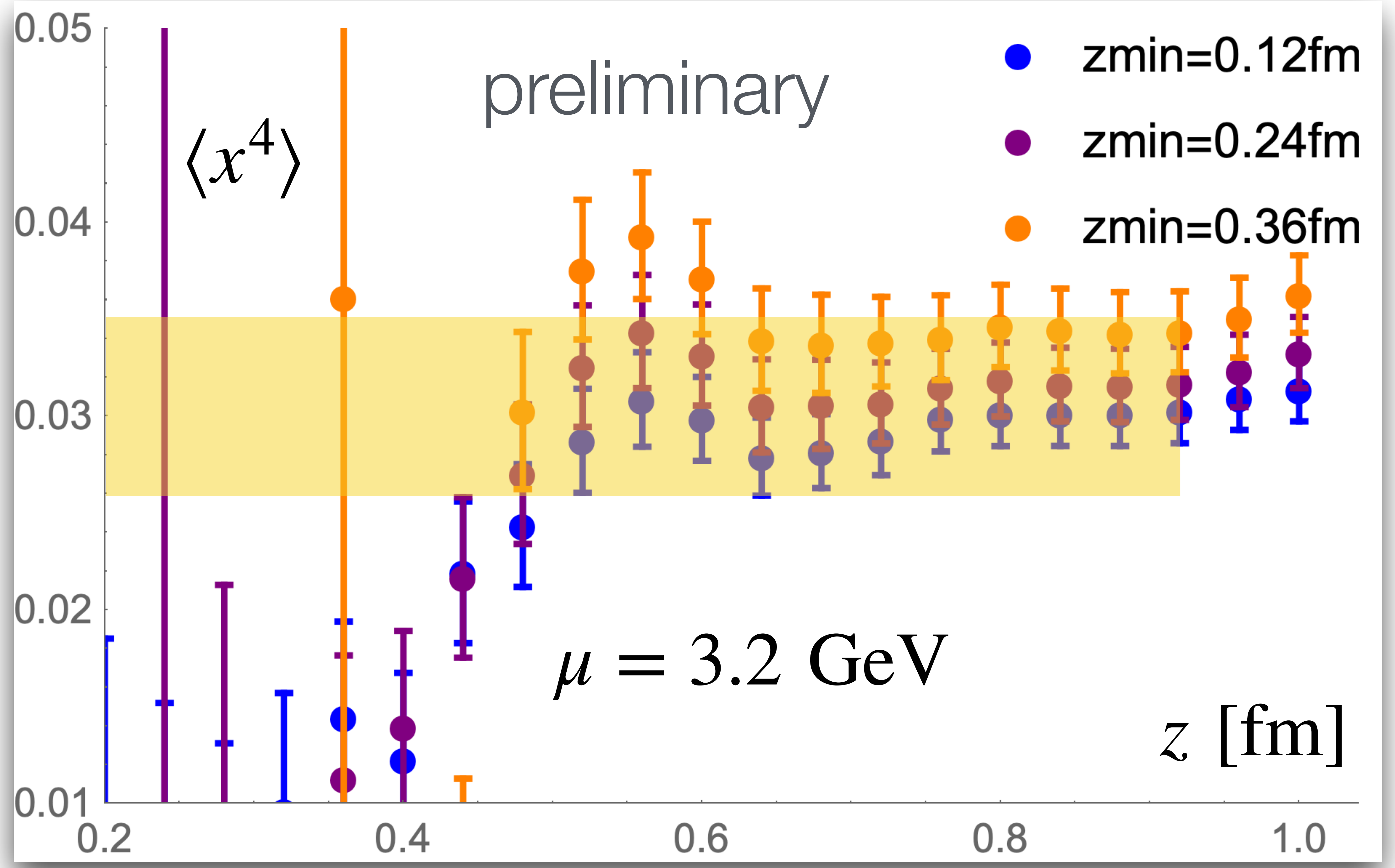
BNL-SBU-Tsinghua, on-going

BNL-SBU-Tsinghua LQCD: $\langle x^2 \rangle \simeq 0.10(1)$

$\mu = 3.2$ GeV

JAM: $\langle x^2 \rangle = 0.095$

4th moment of $f_v^{\pi}(x)$



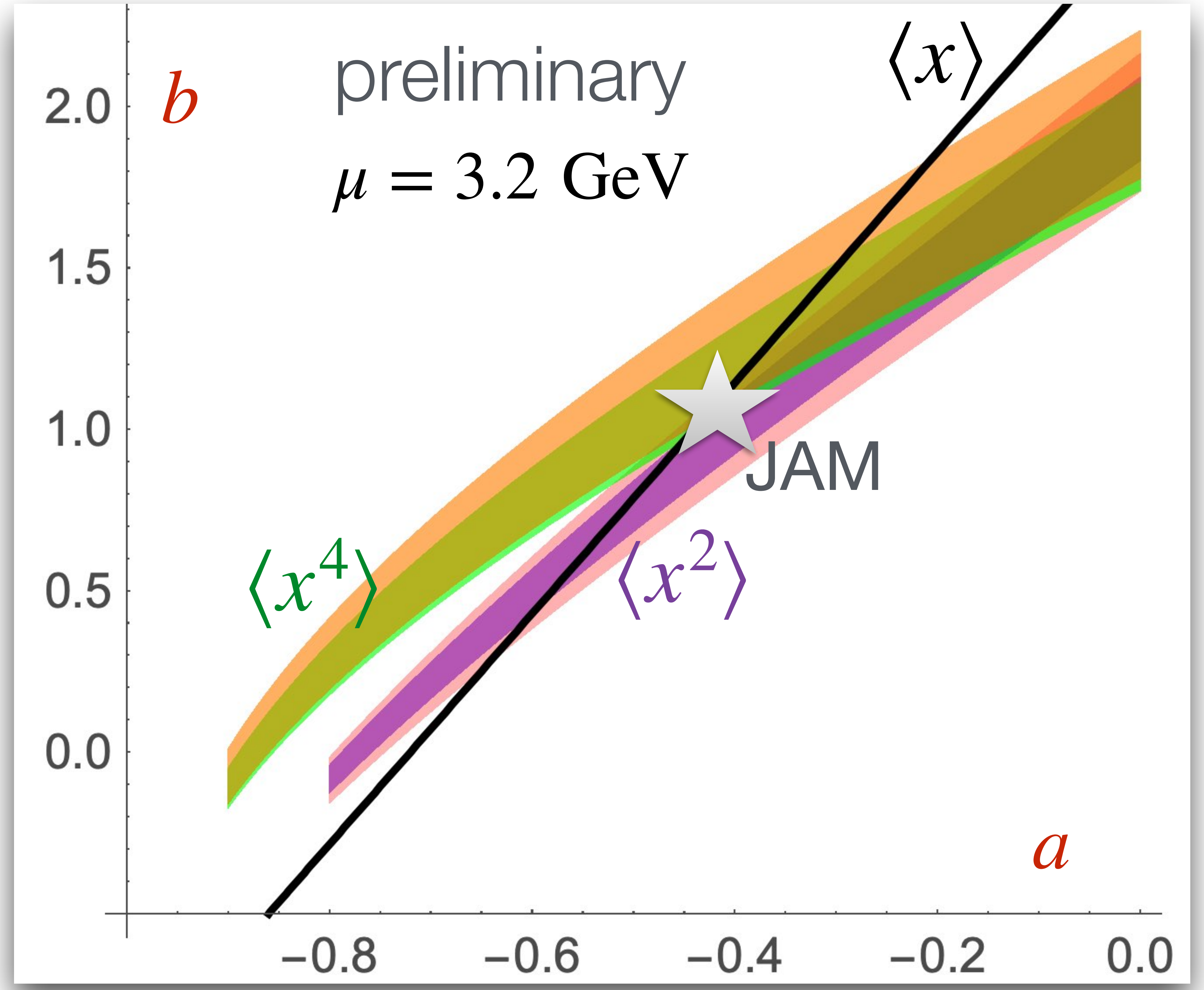
BNL-SBU-Tsinghua, on-going

BNL-SBU-Tsinghua LQCD: $\langle x^4 \rangle \simeq 0.030(5)$

$\mu = 3.2$ GeV

JAM: $\langle x^4 \rangle = 0.032$

$$f_v^\pi(x) = Ax^a(1-x)^b$$



BNL-SBU-Tsinghua, on-going