Dense nuclear matter based on a chiral model with parity doublet structure

Masayasu Harada (Nagoya University)

@ The 18th International Conference on Hadron Spectroscopy and Structure (HADRON2019) (August 18, 2019)

Based on

Introduction
Origin of Mass

One of the interesting problems of QCD
spontaneous chiral symmetry breaking

\[ \langle q\bar{q} \rangle \neq 0 \text{ (chiral condensate)} \]

\[ \langle q\bar{q} \rangle = 0 \]

- The spontaneous chiral symmetry breaking is expected to generate a part of hadron masses.
- It causes mass difference between chiral partners.

- How much mass of nucleon is from the spontaneous chiral symmetry breaking?
- What is the chiral partner of the nucleon?
Parity Doublet models for nucleons

- How much mass of nucleon is from the spontaneous chiral symmetry breaking?
- What is the chiral partner of nucleon?

- A Parity doublet model for light baryons
  - In [C.DeTar, T.Kunihiro, PRD39, 2805 (1989)], N*(1535) is regarded as the chiral partner to the N(939) having the chiral invariant mass.

\[ m_N = m_0 + m_{\langle \bar{q}q \rangle} \]

- This model can be extended to include different excited nucleons.
• We constructed an extended parity doublet model including four light nucleons N(939), N(1440), N(1535) and N(1650).
• We showed that the chiral invariant masses are constrained by the saturation properties of nuclear matter and neutron star properties.

Outline

1. Introduction
2. An Extended Parity Doublet Model for Nucleons: Constraints to chiral invariant masses at vacuum
3. Constraints from Nuclear Matter and Neutron Star Properties
4. Summary
2. An Extended Parity Doublet Model for Nucleons: Constraints to chiral invariant masses at vacuum

chiral representation of baryons

• representation of quark under $SU(2)_R \times SU(2)_L$

\[ q \sim q_r + q_l \sim (2,1) \oplus (1,2) \]

• representation of baryon under $SU(2)_R \times SU(2)_L$

\[ \psi \sim q \otimes q \otimes q \sim \left[(2,1) \oplus (1,2)\right]^3 \]

\[ = 5 \left[(2,1) \oplus (1,2)\right] \oplus 3 \left[(3,2) \oplus (2,3)\right] \oplus \left[(4,1) \oplus (1,4)\right] \]

Chiral symmetry is broken and the isospin symmetry remains

\[ I = \frac{1}{2} \] baryons

\[ I = \frac{3}{2} \] baryons
Parity Doublet models with $[(2,1)\oplus(1,2)]$ nucleons

S. Gallas, F. Giacosa, D. Rischke, PRD82, 014004 (2010)

• An excited nucleon with negative parity such as $N(1535)$ is regarded as **the chiral partner** to the $N(939)$.

• $N(939)$ and $N(1535)$ have a chiral invariant mass:
  $m_0[\bar{\psi}_1\gamma_5\psi_2 - \bar{\psi}_2\gamma_5\psi_1]$

• Spontaneous chiral symmetry breaking generates the mass difference between chiral partners.
  $-g_1[\bar{\psi}_{1l}M\psi_{1r} + \bar{\psi}_{1r}M^\dagger\psi_{1l}] - g_2[\bar{\psi}_{2r}M\psi_{2l} + \bar{\psi}_{2l}M^\dagger\psi_{2r}]$

  - $M = \sigma + i \hat{\tau} \cdot \hat{\pi}$ transforms $M \rightarrow g_L M g_R^\dagger$
  - $\langle M \rangle = \bar{\sigma} \neq 0$ causes the spontaneous chiral symmetry breaking.

$m_{\pm} = \frac{1}{2} \left[ \sqrt{(g_1 + g_2)^2 \bar{\sigma}^2 + 4m_0^2} \mp (g_2 - g_1) \bar{\sigma} \right]$

$m_+ = m(N(939))$
$m_- = m(N(1535))$
A model with \([1,2) \oplus (2,1)] \text{ and } [(2,3) \oplus (3,2)] \text{ representations}


- We include two representations, \( \psi \in [(1,2) \oplus (2,1)] \) and \( \eta \in [(2,3) \oplus (3,2)] \) to study N(939), N(1440), N(1535), N(1650).

- There are 2 chiral invariant masses.
  \[ -m_0^{(1)} [\bar{\psi}_1 \gamma_5 \psi_2 - \bar{\psi}_2 \gamma_5 \psi_1] - m_0^{(2)} [\bar{\eta}_1 \gamma_5 \eta_2 - \bar{\eta}_2 \gamma_5 \eta_1] \]

- 6 Yukawa Interactions
  \[ -g_1 [\bar{\psi}_{1l} M \psi_{1r} + \bar{\psi}_{1r} M^\dagger \psi_{1l}] - g_2 [\bar{\psi}_{2r} M \psi_{2l} + \bar{\psi}_{2l} M^\dagger \psi_{2r}] \]
  etc.

- We also have 4 terms with one derivative.
  \[ a_1 [\bar{\psi}_{1l} \gamma^\mu \partial_\mu M \psi_{2l} - \bar{\psi}_{1r} \gamma^\mu \partial_\mu M^\dagger \psi_{2r}] \text{ etc} \]
Physical inputs

We first fix the values of the chiral invariant masses $m_0^{(1)}$ and $m_0^{(2)}$ to some constants, and use the following 10 physical inputs to determine 10 parameters (10 couplings).

<table>
<thead>
<tr>
<th>Masses</th>
<th>$m_{N(939)} = 939\text{MeV}$</th>
<th>$m_{N(1440)} = 1430\text{MeV}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$m_{N(1535)} = 1535\text{MeV}$</td>
<td>$m_{N(1650)} = 1650\text{MeV}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decay widths</th>
<th>$\Gamma(N(1440) \to N(939) + \pi) = 228\text{MeV}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Gamma(N(1535) \to N(939) + \pi) = 68\text{MeV}$</td>
</tr>
<tr>
<td></td>
<td>$\Gamma(N(1650) \to N(939) + \pi) = 84\text{MeV}$</td>
</tr>
<tr>
<td></td>
<td>$\Gamma(N(1650) \to N(1440) + \pi) = 22\text{MeV}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Axial charges</th>
<th>$g_A(N(939)) = 1.27$</th>
</tr>
</thead>
</table>
|              | $g_A(N(1650)) = 0.55$ | (Lattice analysis [T.T.Takahashi, T.Kunihiro, PRD78 (2008)] )

Constraint $-0.25 \leq g_A(N(1535)) \leq 0.25$

(Lattice analysis [T.T.Takahashi, T.Kunihiro, PRD78 (2008)] shows $g_A \sim O(0.1)$.
5 Groups of solutions

Group 4, 5
N(939) is dominated by \([(1,2) \oplus (2,1)]\)
Chiral partner to N(939)
= a mixture of N(1535) & N(1650)

Group 3
Small \(m_0^{(2)}\) : N(939) is dominated by (1,2)\(\oplus\)(2,1).
Large \(m_0^{(2)}\) : N(939) is dominated by (2,3)\(\oplus\)(3,2).
Chiral Partner \(\approx\) N(1535)

Group 2
N(939) is dominated by (2,3)+(3,2)
Chiral partner to N(939)
= a mixture of 3 nucleons

Group 1
N(939) is dominated by (2,3)+(3,2) representation.
Chiral partner to n(939) = N(1440)
Prediction - Axial charges -

• In our model the following relation is satisfied:
  \[- \sum_{i=1}^{4} g_{A}(N_{i}) = 0\]

Constraint
\[-0.25 \leq g_{A}(N(1535)) \leq 0.25\]

Input
\[g_{A}(N(939)) = 1.27\]
\[g_{A}(N(1650)) = 0.55\]

\[-2.07 \leq g_{A}(N(1440)) \leq -1.57\]
3. Constraints from Nuclear Matter and Neutron Star Properties

Construction of Nuclear Matter

- We include the omega and rho mesons into our model using the hidden local symmetry.
- We calculate the thermodynamic potential in the nuclear medium in our model, using the mean field approximation.
- Then, we adjust model parameters to reproduce the following physical inputs for given values of the chiral invariant masses $m_0^{(1)}$ and $m_0^{(2)}$.
  - Nuclear saturation density
    $$\rho(\mu_B^* = 923\text{MeV}) = \rho_0 = 0.16\text{fm}^{-3}$$
  - Binding energy at normal nuclear density
    $$\left[\frac{E}{A} - m(939)\right]_{\rho_0} = \left[\frac{\varepsilon}{\rho_B} - m(939)\right]_{\rho_0} = -16\text{MeV}$$
  - Incompressibility
    $$K = 9\rho_0^2 \left. \frac{\partial^2 (E/A)}{\partial \rho^2} \right|_{\rho_0} = 9\rho_0 \left. \frac{\partial \mu_B}{\partial \rho} \right|_{\rho_0} = 240\text{MeV}$$
  - Symmetry energy
    $$E_{\text{sym}}(\rho_0) = 31\text{ MeV}$$
• We checked whether the saturation properties are satisfied for the parameter choices indicated by • marks.

• We found that, for the parameter choices indicated by • marks, the saturation properties are NOT satisfied.

• So, these parameter choices are excluded.

• In particular, please note that the parameter choices in Group 1 are all excluded.
Constraint from Neutron Star Properties

- We obtained constraint to the chiral invariant masses from the tidal deformability of Neutron Stars.
  - Tidal deformability: $\tilde{\Lambda} \leq 800$ with $M_{\text{chirp}} = 1.188 \, M_\odot$

- ● (yellow dots) are excluded, and ● (red dots) are allowed
The reason why smaller chiral invariant masses are excluded.

- The attractive force mediated by sigma contribution is larger for smaller chiral invariant masses. 
  \[ m_{\pm} = \frac{1}{2} \left[ \sqrt{(g_1 + g_2)^2\bar{\sigma}^2 + 4m_0^2 \mp (g_2 - g_1)\bar{\sigma}} \right] \]

- The repulsive force mediated by omega contribution is then larger for larger sigma contribution to satisfy the saturation properties of normal nuclear matter.

- The attractive force by the sigma contribution becomes smaller for larger density, while the repulsive force by the omega contribution becomes larger.

- The larger repulsive force make the radius and tidal deformability larger.

- As a result, the smaller chiral invariant masses cause the larger tidal deformability.
Symmetry energy and Slope parameter

- We include $\rho$ meson into the model and obtain the symmetry energy and the slope parameter.

$$E_{\text{sym}} = \frac{\rho_B}{8} \left( \frac{2\pi^2}{\Sigma_{N,j} k_{FN}^{\text{(i)}} m_{\rho}^{\text{(i)}}} + \frac{g_{\rho NN}^2}{m_{\rho}^2} \right); \quad E_F^{(i)} = \sqrt{\left(k_{FN}^{(i)} \right)^2 + \left(m_{*}^{(i)} \right)^2}$$

$$L = 3\rho_B \left[ \frac{1}{8} \left( \frac{2\pi^2}{kE} + \frac{g_{\rho NN}^2}{m_{\rho}^2} \right) + \frac{\rho_B}{8} \frac{\pi^2}{2k^2} \left( -\frac{2\pi^2}{k^2 E} - \frac{2\pi^2}{E^3} \right) \right]$$

Predictions of the model

- The slope parameter has very little dependence on the chiral invariant mass.
- The symmetry energy does not depend on the choice of chiral invariant masses.
- Both increase linearly with density.
4. Summary

- We constructed an extended parity doublet model including two representations, \( \psi \in [(1,2) \oplus (2,1)] \) and \( \eta \in [(2,3) \oplus (3,2)] \) to study N(939), N(1440), N(1535), N(1650).
- We use masses, decay widths and axial charges to constrain 2 chiral invariant masses, and found that possible combinations are categorized into 5 groups.
- We exclude some values of the chiral invariant masses by requiring the saturation properties of normal nuclear matter indicated by black dots.
- We further obtain more constraints from the tidal deformability determined by the observation of the gravitational waves from neutron star merger GW170817.

N(939) is dominated by \((1,2) \oplus (2,1)\), and the chiral partner to N(939) is a mixture of N(1535) and N(1650).

N(939) is dominated by \((2,3) \oplus (3,2)\), and the chiral partner to N(939) is a mixture of N(1440), N(1535) and N(1650).

N(939) is dominated by \((2,3) \oplus (3,2)\), and the chiral partner to N(939) is N(1535).
The End
Mixing Rates of Nucleons

(1,2)⊕(2,1) : $\psi_1 \blacktriangledown \psi_2 \blacklozenge$

(2,3)⊕(3,2) : $\eta_1 \bigcirc \eta_2 \blacktriangle$

Group 1

N(939) is dominated by [(2,3)⊕(3,2)] representation. $\Rightarrow$ Chiral partner to N(939) = N(1440)

N(1440) is also dominated by [(2,3)⊕(3,2)] representation. $[> 80 \%]$ $\Rightarrow$ Chiral partner to N(939) = mixture of N(1440) + N(1535) + N(1650)

Group 2

N(1440) includes large amount of (1,2)+(2,1) representation. $[[2,3)⊕(3,2)]< 60\%]$ $\Rightarrow$ Chiral partner to N(939) = mixture of N(1440) + N(1535) + N(1650)
• Small $m_0^{(2)}$: N(939) is dominated by $(1,2)\oplus(2,1)$.  
• Large $m_0^{(2)}$: N(939) is dominated by $(2,3)\oplus(3,2)$.  
  – Chiral Partner $\approx$ N(1535)
Mixing Rate of Nucleons NO.3

\[(1,2) \oplus (2,1) : \psi_1 \quad \psi_2 \]
\[(2,3) \oplus (3,2) : \eta_1 \quad \eta_2 \]

Group 4

N(939) is dominated by \( (1,2) \oplus (2,1) \).

\(-\) Chiral Partner \( \approx \) a mixture of N(1535) and N(1650)
Transition Axial-Charges

- We calculated transition axial-charges:
  - $g_A(N_2(1440) - N_3(1535))$, $g_A(N_2(1440) - N_4(1650))$,
  - $g_A(N_1(939) - N_3(1535))$, $g_A(N_3(1535) - N_4(1650))$,
  - $g_A(N_1(939) - N_4(1650))$, $g_A(N_1(939) - N_2(1440))$,
- We find some features.

Predictions of Group 1 are separated from those from other groups.

$g_A(N_3 N_4)$ [for Group 2 and Group 5] ≈ 2
$g_A(N_1 N_3)$ for Group 2 is large.

No Group 3 inside
Neutron Star Properties

• M-R relation

• Tidal deformability
Neutron Star Properties 2

- **M-R relation**

- **Central density**