Locating the fixed point in the phase plane

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In cooperation with

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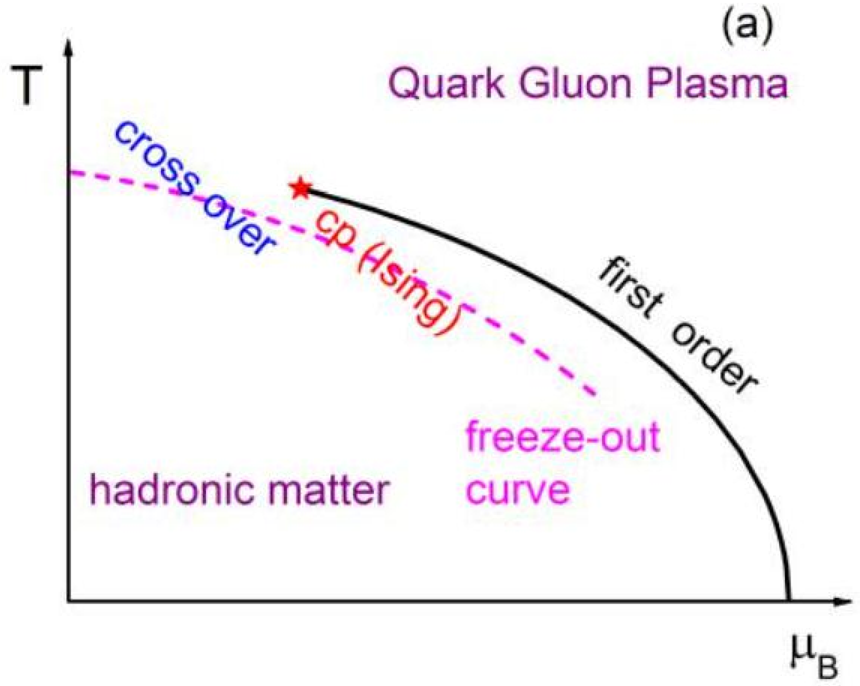
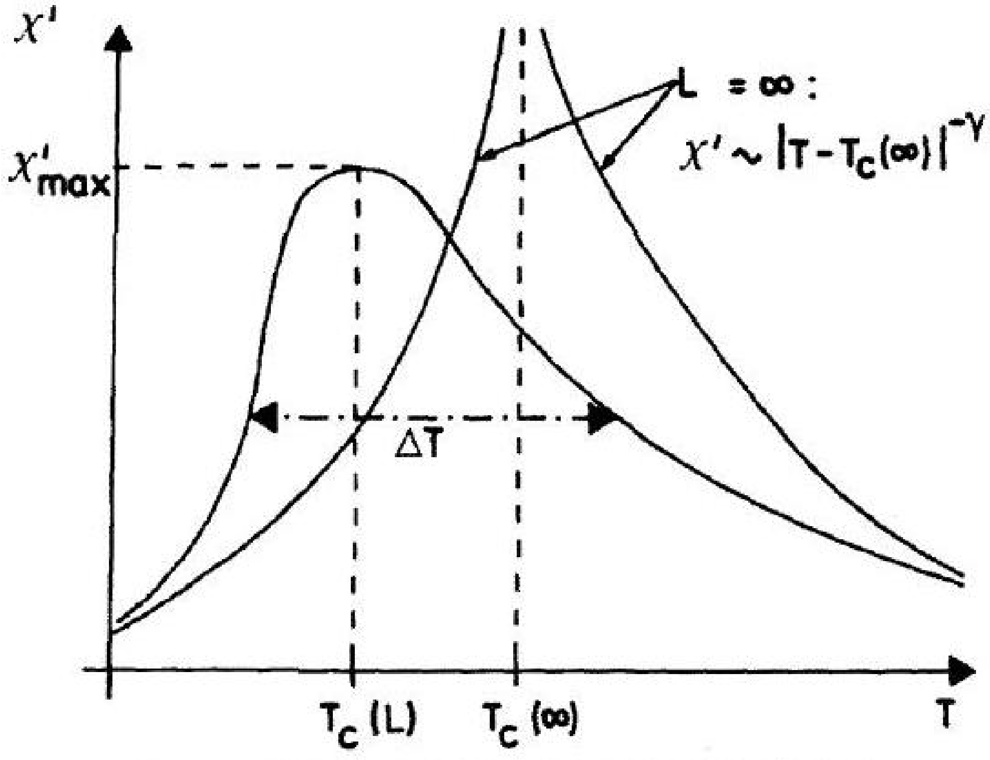
(arXiv:1905.11388)

Section

- Motivation
- The description of fixed point
- Three special samples generated by the Potts model
- Locating the fixed point by defined width
- Summary

1. Motivation





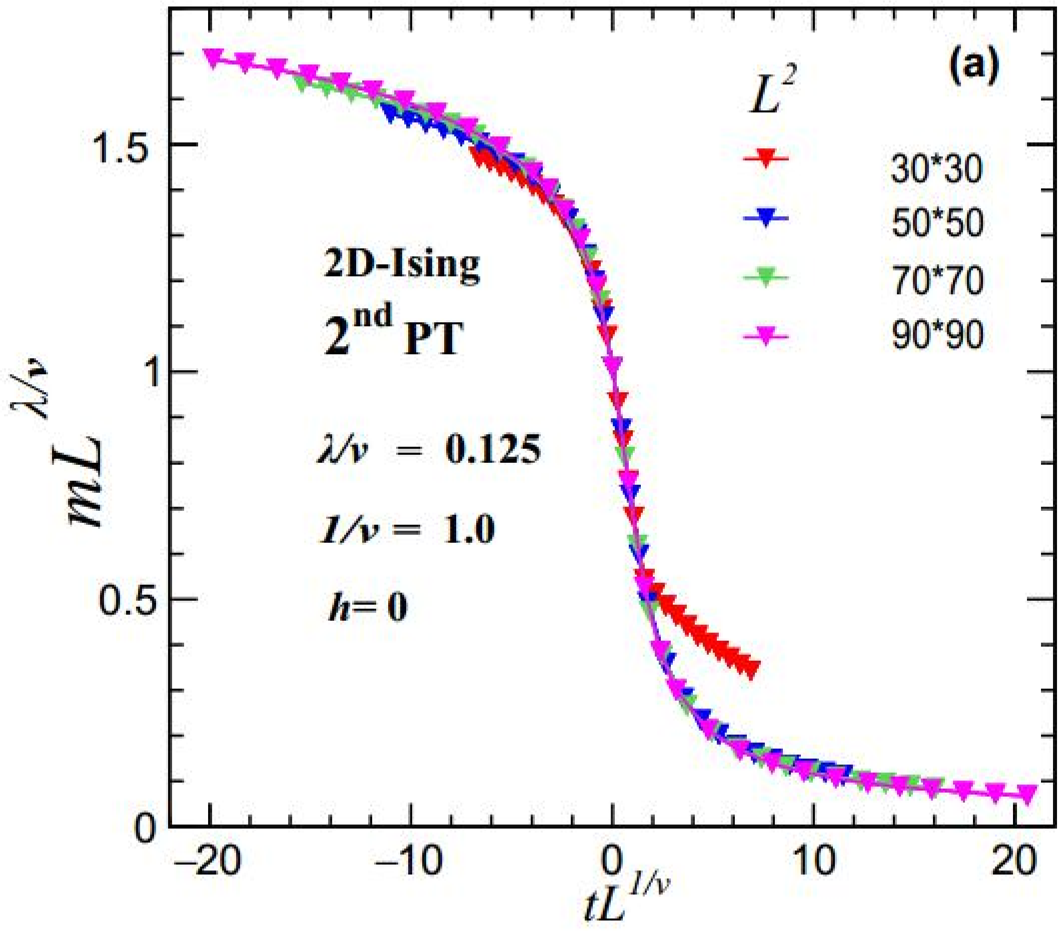
- The peak position of the distribution of susceptibility can define the critical temperature on the condition that the system is infinite or large enough.

- Critical related fluctuations are severely influenced by finite volume. The singularities smeared into finite peaks and the position of CEP shifts into some region . The critical temperature we defined is Pseudo-critical temperature.

- We have to consider the finite size effect and utilize the finite size scaling .

★ Finite-size scaling

✓ **For a finite-size system, the observable Q(T,L) around T_C follows as the form below**



$$Q(T,L) = L^{-\lambda/\nu}\phi_Q\left(tL^{1/\nu}\right)$$

- $t = (T - T_c)/T_c$: reduced temperature
- ν : the critical exponent of correlation length

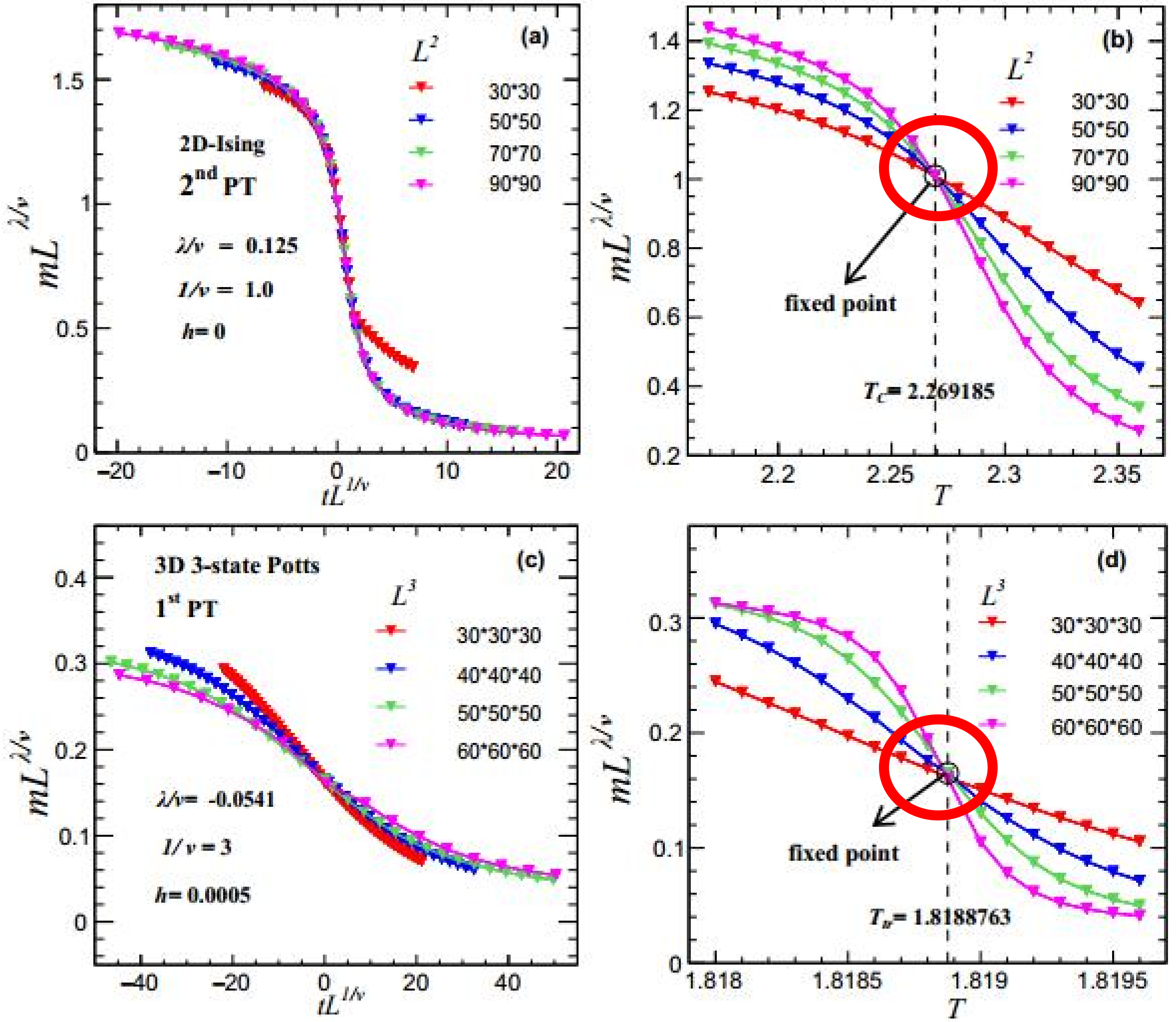
  λ : the critical exponent of Q(t,L)
- $\phi_Q(tL^{1/\nu})$: finite-size scaling function

★ Finite-size scaling and fixed point

At $T=T_C$,

$$f_Q(0) = Q(T_C, L)L^{\lambda/\nu}$$

→ **Constant** → **Fixed point**



2_{nd}-order: finite-size scaling func., and scaling exponent ratio is a fraction.

1^{st}-order: finite-size scaling func., and scaling exponent is determined by spatial dimension (integer).

Crossover: observable is size independent → generalized FSS,where scaling exponent ratio is 0.

Both of the second and first order phase transition show the feature of fixed point. all size curves intersect at critical (transition) temperature. Any deviation from the fixed point, the points with different system sizes would go away from each other.

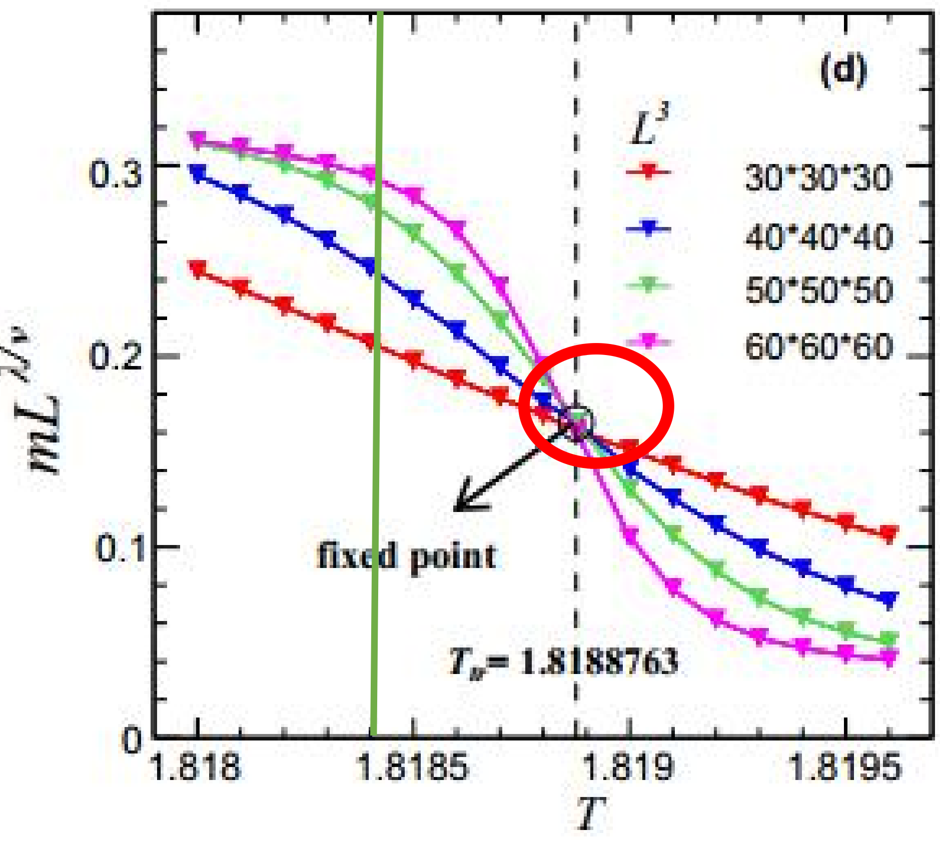
Solution:

If the behavior of fixed point can be described quantitatively,

the location of the phase boundary will be precisely given out.

2. Description of fixed point

★ Quantify



At Tc, Dmin,around the unity. Deviating from Tc,D increases.

Define the width of all size scaled observables Q(T, L)L^a to be the square root of their variance ,ie

$$D(T,a)=\sqrt{\frac{\Delta S_{Q(T,a)L^a}}{N_L-1}} \quad (2)$$

similar to the definition of χ^2 in curve-fitting

Where $\Delta S_{Q(T,L)L^a}$ is the **error weighted variance of all size points to their mean positions**, i.e.,

$$\Delta S_{Q(T,L)L^a}=\sum_{i=1}^{N_L}\frac{\left[Q(T,L_i)L_i^a-\left\langle Q(T,L)L^a\right\rangle\right]^2}{\omega_i^2}$$

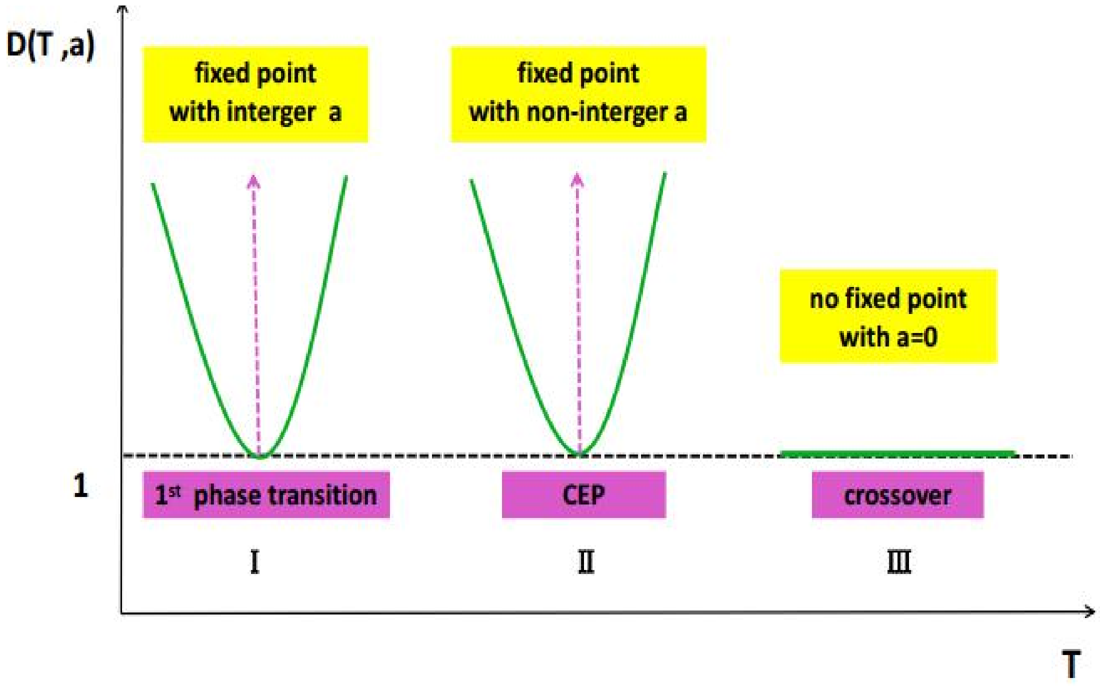
.

exponent ratio: a = λ/ν N$_L$:the number of system' size

$\omega_i=\delta\left[Q(T,L_i)L_i^a\right]$ is the error of $Q(T,L_i)L_i^a$

$$\left\langle Q(T,L)L^a\right\rangle=\frac{\sum_{i=1}^{i=N_L}Q(T,L_i)L_i^a/\omega_i^2}{\sum_{i=1}^{N_L}1/\omega_i^2}$$ the weighted mean

★ The behavior of defined width along the phase boundary



1st order PT :

(I) **T** is low. **D(T, a)** has a **minimum** at **phase transition temperature**, where the ratio **a** is around (n − 1)d, **an integer**. n is the order of susceptibility.

2nd order PT:

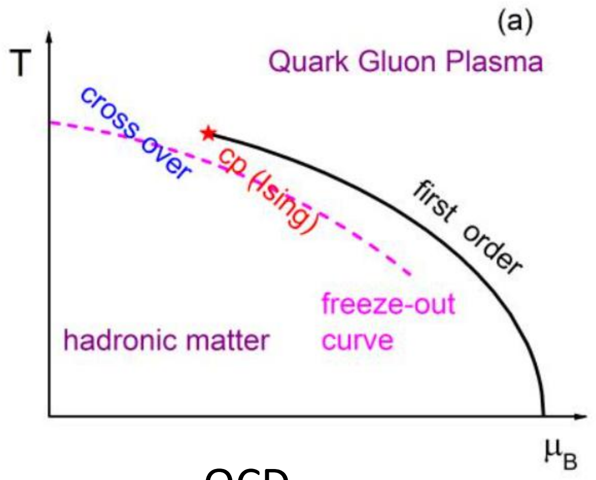
(II) **T** is in the middle. **D(T, a)** has also **a minimum** at **the critical temperature**, and **a** is a **fraction**, in contrast to the case of I.

crossover region :

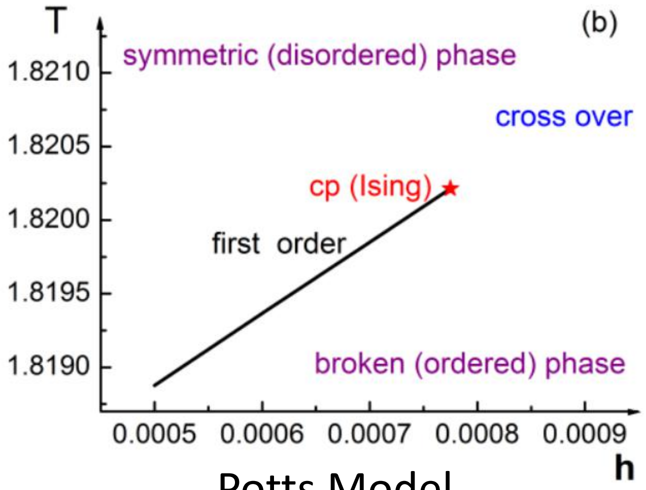
(III) T is high. **D(T, a)** keeps in the same **minimum at all temperature**, and ratio a is around **zero.** This implies **D(T,a**) is a constant**, independent of T.** The observable is the system size independent.

3. 3D three-state Potts model

★ Why Potts model?



QCD



Potts Model

- **Same Z(3) global symmetry**
- **Similar phase structure**
- CP belongs to the same 3D Ising universal class

★ Potts model

------one of the standard paradigms of lattice QCD !

In terms of spin variable $S_i \in 1, 2, 3$, which is located at sites i of a cubic lattice of size $V = L^3$. The Hamiltonian, energy and order parameter of the model are,

$$H = \beta E - hM \qquad E = -J\sum_{<i,j>}\delta(S_i, S_j) \qquad M = \sum_i \delta(S_i, S_g)$$

(**s_g is the direction of ghost spin**)

★ Observable

The order parameter defined as: $$m(T,h) = \frac{3\langle M \rangle}{2V} - \frac{1}{2}$$

At CP, the observable taken as the mean of absolute order parameter:

$$m(T,h) = \left\langle \left| \frac{1}{L^3} \left[\tilde{M}(T,h) - \langle \tilde{M}(T_c, h_c) \rangle \right] \right| \right\rangle$$

★ Three specific samples

Three samples at fixed external fields: h = 0.0005 (the first order phase transition line),
0.000775 (CEP),
0.002 (crossover).

18 T-values starting from $T_0 = 1.8180$ with $\Delta T = 0.0001$

4 system sizes L = 30, 40, 50, 60.

Total 100,000 configurations.

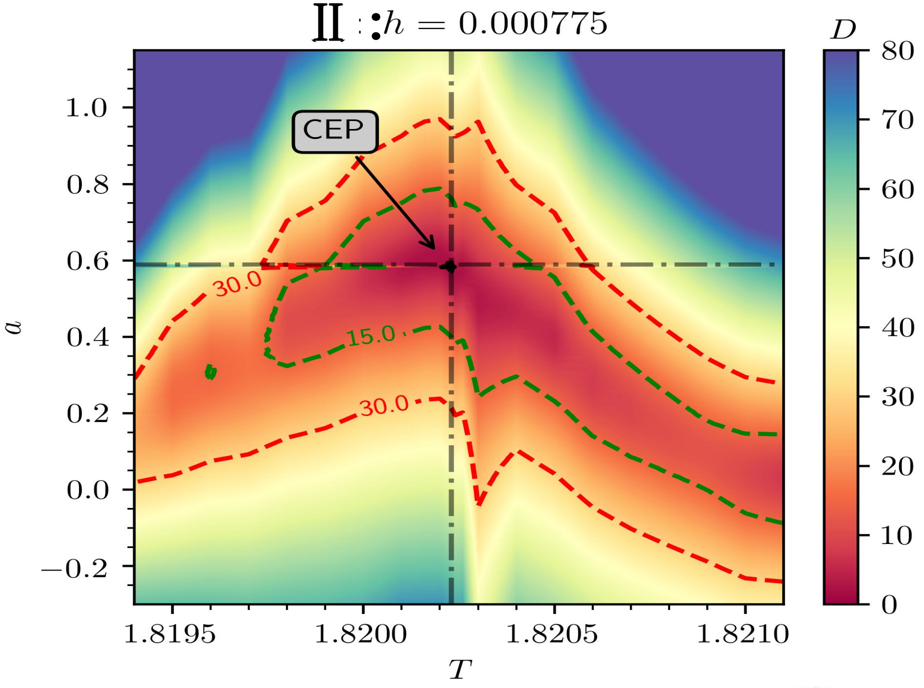
4. Locating the fixed point by defined width

★ The width of scaled observable

$$D(T,a) = \sqrt{\frac{\Delta S_{m(T,L)L^{a}}}{N_L - 1}}$$

- The exponent ratio a = λ/ν is unknown parameter.
- Change T and the ratio a to see where they makes D(T, a) minimum.
- So draw the contour of D(T, a) in the plane of T and a the contour will show where D(T, a) is a minimum.

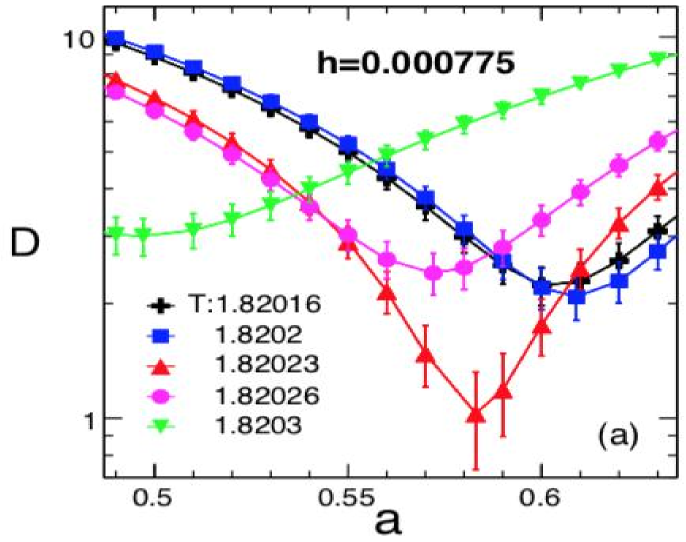
★ Locating the fixed point at h= 0.000775 sample(For CEP)

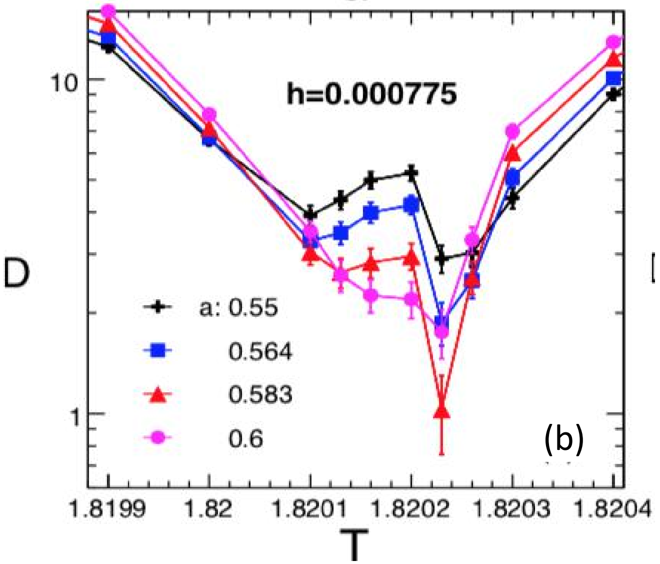


Contour plot :

- D(T, a) gradually **converges to a minimum red area**
- The minimum at a specified T and a, which is near to a fraction, around 0.6.

➢ **It is the nature of fixed point of CEP!**

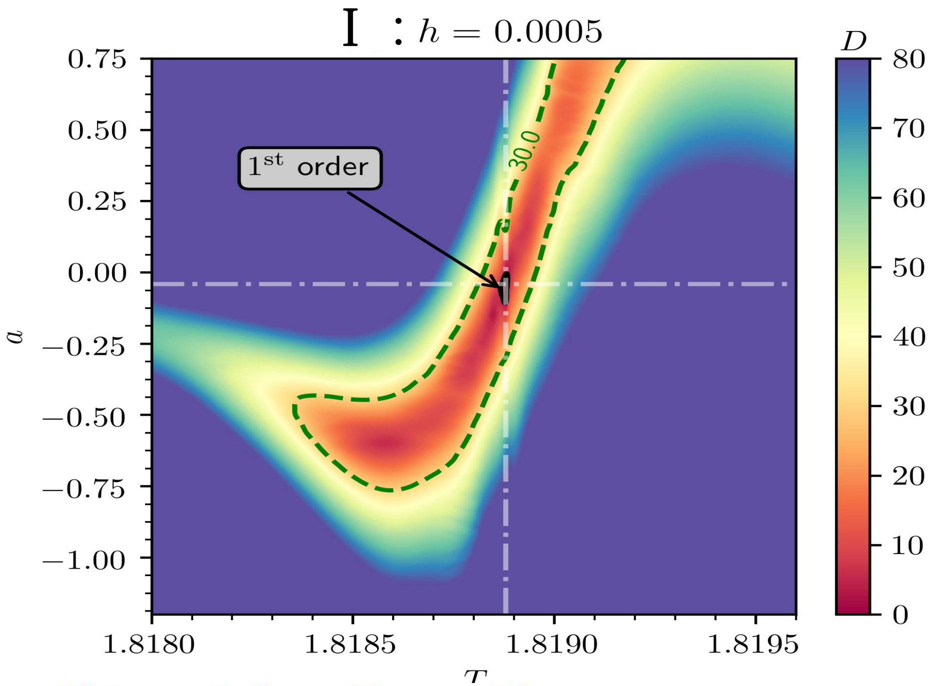




Projection :

- A given T, an **a** makes the D(T, a) min.
- Dmin = 1.0291 ± 0.2946(red one), and T = 1.82023 and a =0.583.
- The original T_c=1.82023372 and ratio a_c=0.564

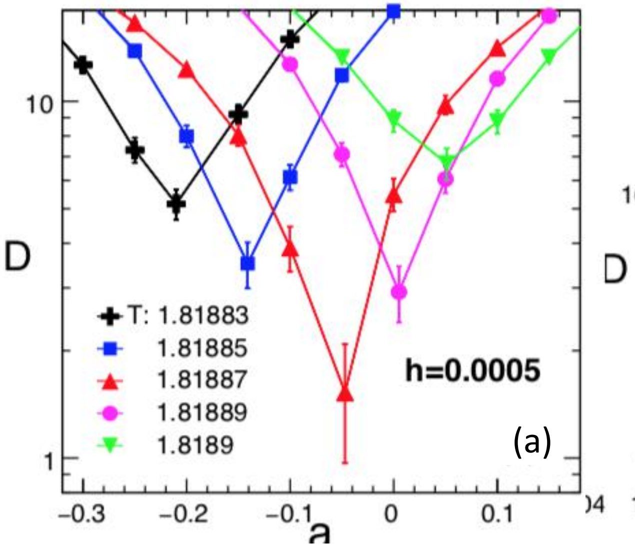
★ Locating the fixed point at h= 0.0005 sample (For 1_{st} PT)

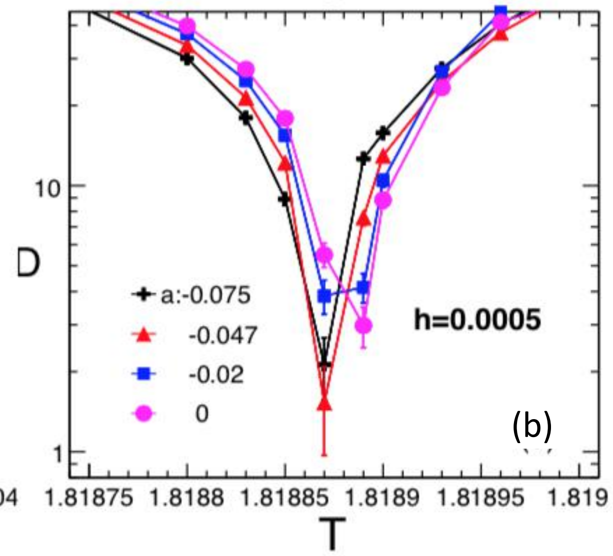


Contour plot :

- D(T, a) gradually **converges to a minimum**
- The minimum at a specific T and **a**, close to 0
- It is the nature of the fixed point of 1st order PT

Projection :



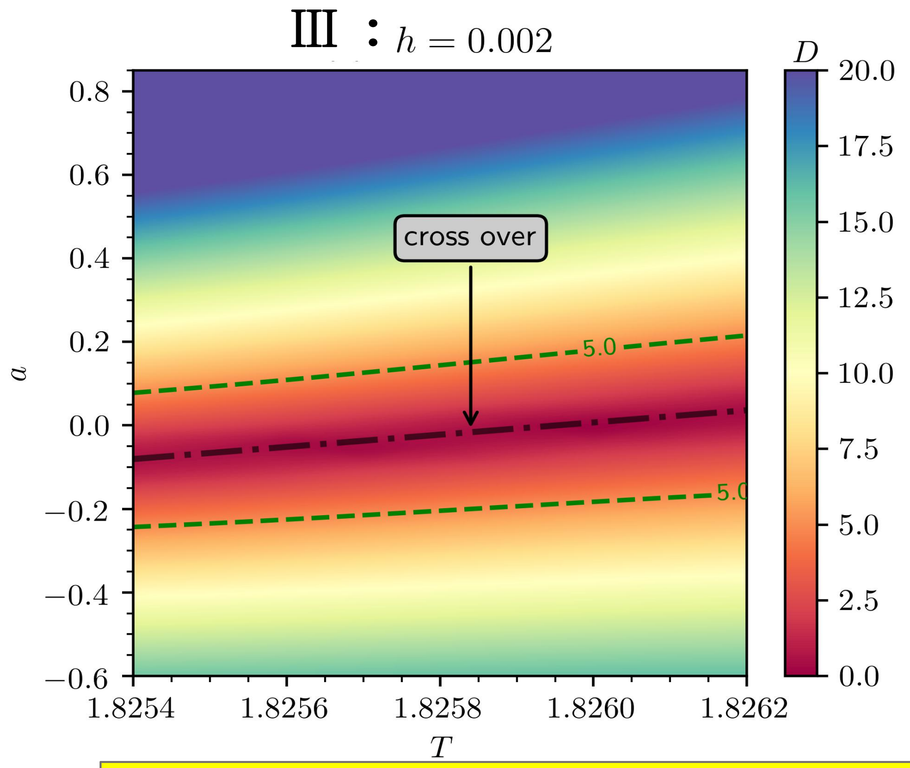


- A given T , an **a** makes the D(T, a) min.

  A given **a**, a T makes the D(T,a) min.
- Dmin = 1.5287 ± 0.5591(red one),

  and T = 1.81887 and a =-0.047.
- The original T_c=1.8188763 and ratio a_T=-0.0541

★ Locating the fixed point at h= 0.002 sample(For crossover)



Contour plot :

- **D(T, a): the band lines parallel to the T-axis**
- Independent of T, and determined by ratio **a** only.
- The red band is close to **zero**.

➢ Characteristics of the crossover

✓ So the width D(T, a) beautifully quantifies the features of the fixed point .
✓ The CP, the 1_{st} phase transition line and crossover can be well located by defined width!

★Discussions

There are still numbers of uncertainties in applying the FSS in relativistic heavy-ion collisions :

- ✓ **an appropriate observable in heavy-ion collisions**
- ✓ **whether the phase boundary is covered by the BES**
- ✓ **whether the system size can be correctly estimated**

5. Summary

- The fluctuations near criticality should follow the finite-size scaling(FSS). It implies that the CEP corresponds to a fixed point. It can be generalized to the first order, and crossover .

- To quantify the feature of the fixed point, at a given T, we define the width of a set of points with different system sizes. The minimum of the width corresponds to the position of the fixed point.

- Using the 3D three-state Potts model, the samples at three external fields corresponding to the CEP, the first order phase transition, and crossover, are generated. It is demonstrated that the minimum of the contour plot of defined width precisely locate the CP, the phase transition line, and crossover region.

- The contour plot of defined width well quantify the feature of fixed point. It provides a practical way to map the QCD phase boundary by scanning the related observable in the phase plane.

Thanks!