

Second-order off-diagonal cumulants of net-proton, net-kaon and net-charge multiplicity distributions at RHIC



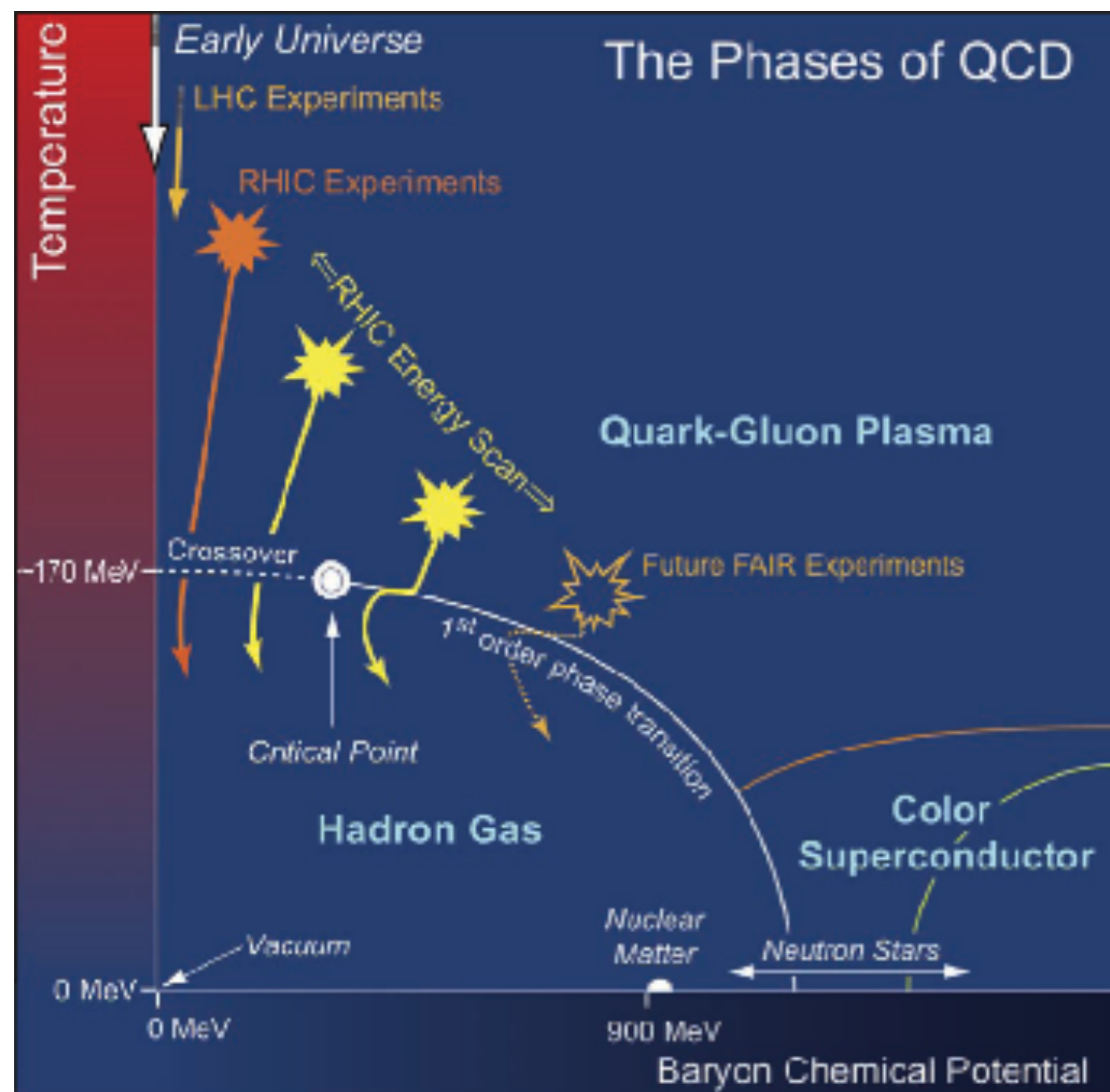
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QPT 2019
Enshi, China

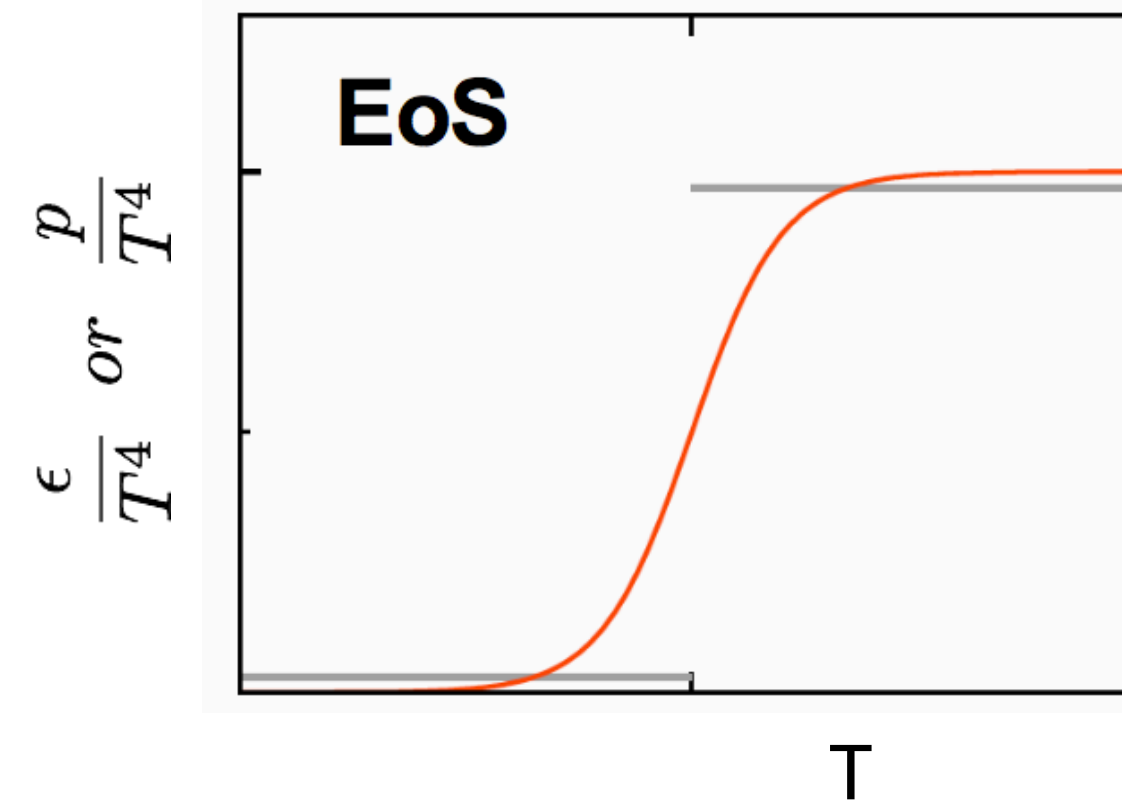
Introduction: QCD phase diagram

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► To map the QCD phase diagram ? Susceptibilities are sensitive to phase boundary



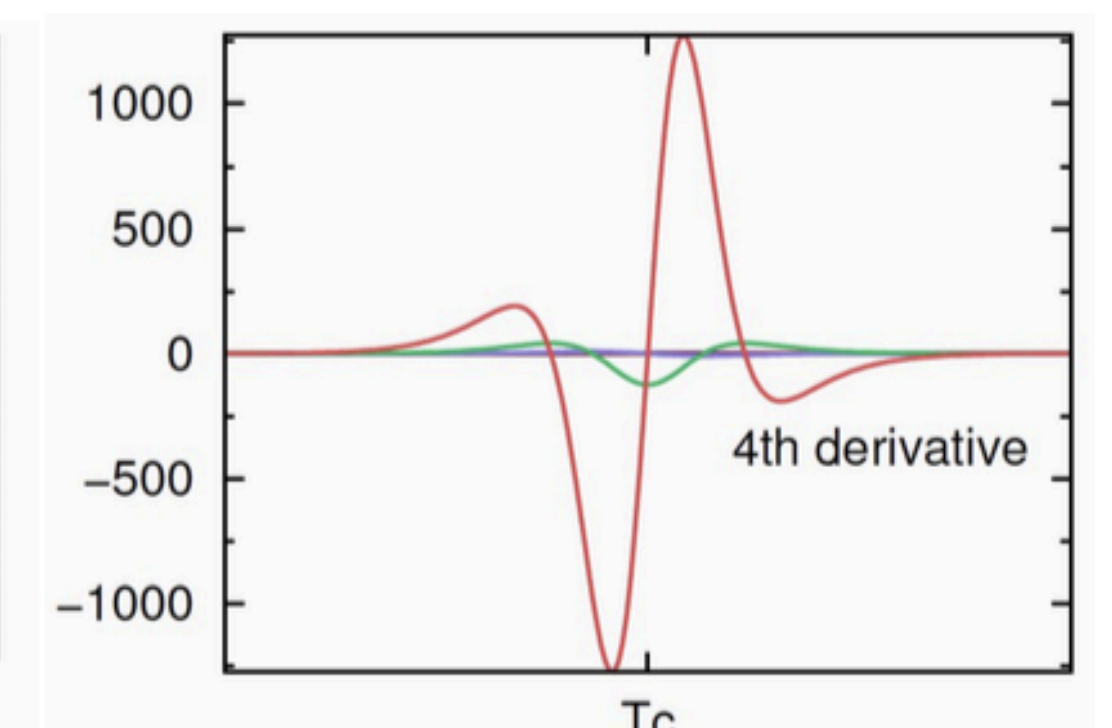
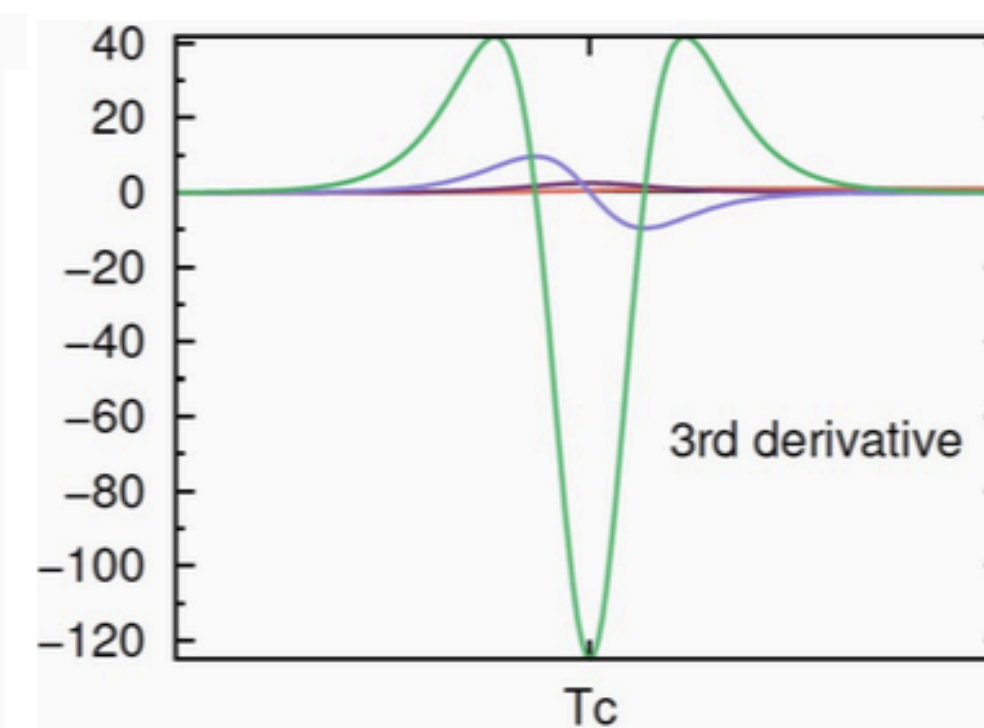
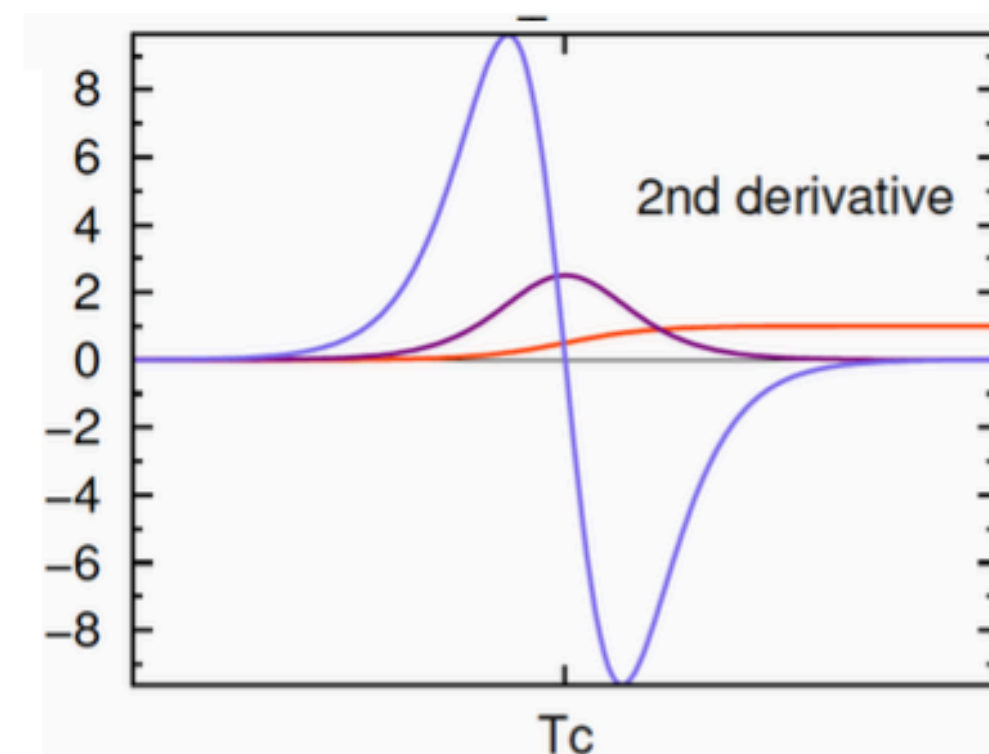
arXiv:0809.3137



$$\text{EoS: } \frac{P}{T^4} = \frac{1}{VT^3} \ln[Z(V, T, \mu_X)]$$

derivative w.r.t. number potential

$$\text{Susceptibilities: } \chi_{mn} = \frac{\partial^{m+n}(P/T^4)}{\partial^m(\mu_{X(1)}/T) \partial^n(\mu_{X(2)}/T)}$$



X. Luo, STAR Analysis Meeting, LBNL

S. Ejiri and M. Kitazawa, Phys. Rev. Lett. 103, 262301 (2009). M. Stephanov, K. Rajagopal, E. Shuryak, Phys. Rev. Lett. 81, 4816 (1998). Y. Hatta, M. Stephanov, Phys. Rev. Lett. 91, 102003 (2003). V. Koch, A. Majumder, J. Randrup, Phys. Rev. Lett. 95, 182301 (2005). M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009). M. Asakawa, M. A. Stephanov, Phys. Rev. Lett. 107, 052301 (2011).

Susceptibilities: Sign change and diverge

Motivation: Susceptibilities and number fluctuation

► Susceptibilities can be estimated by e-by-e fluctuations in HIC

In GCE:

$$\langle \Delta E^n \rangle \sim A \frac{\partial^n Z}{\partial^n T} \longrightarrow \langle \Delta E^n \rangle \sim KT^2 C_V$$

$$\langle \Delta N^n \rangle \sim A \frac{\partial^n Z}{\partial^n \mu} \longrightarrow \langle \Delta N^n \rangle \sim VT^3 \chi_n$$

$$VT^3 \chi_1 \Rightarrow \langle N \rangle = \kappa_1$$

$$VT^3 \chi_2 \Rightarrow \langle (\Delta N)^2 \rangle = \kappa_2$$

$$VT^3 \chi_3 \Rightarrow \langle (\Delta N)^3 \rangle = \kappa_3$$

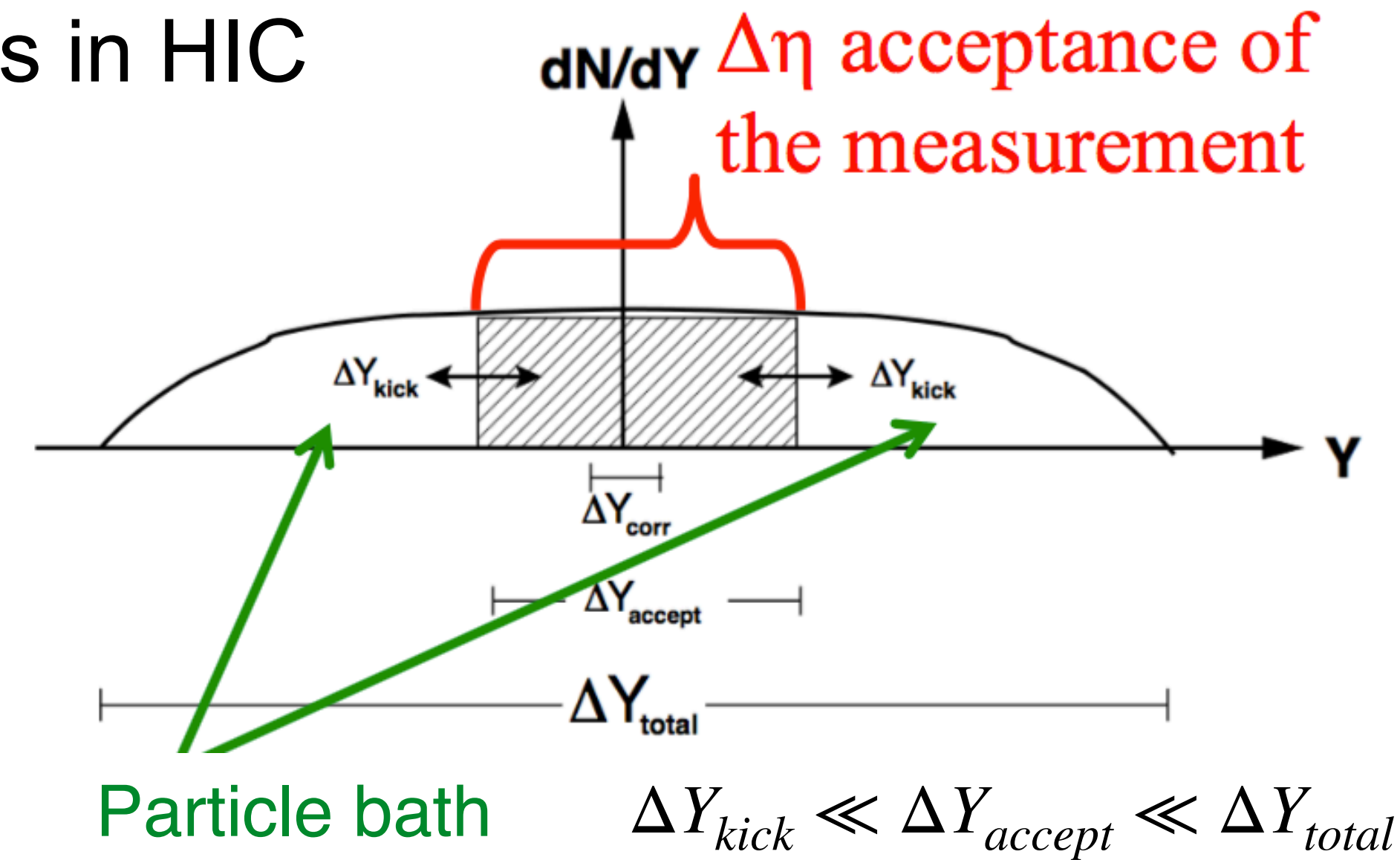
$$VT^3 \chi_{1,1} \Rightarrow \langle (\Delta N)(\Delta M) \rangle = \kappa_{1,1}$$

where $\Delta N = N - \langle N \rangle$ and
 $N = N^+ - N^-$

Net-charge,
 Net-baryon,
 Net-strangness

Experimental proxy

Net-proton,
 Net-kaon,



Theory: susceptibilities

S. Borsanyi, et. al, Phys. Rev. Lett. 111, 062005 (2013),
 R. P. Adak, et. al, Phys. Rev. C 96, 014902 (2017)

- ✓ Net-proton: STAR Collaboration, PRL. 112, 032302 (2014).
- ✓ Net-charge: STAR Collaboration, PRL. 112, 032302 (2014).
- ✓ Net-kaon: STAR Collaboration, PLB. 785, 551 (2018)

✓ off-diagonal: STAR Collaboration, PRC 100, 014902 (2019)

Off-diagonal cumulants and ratio fluctuation:

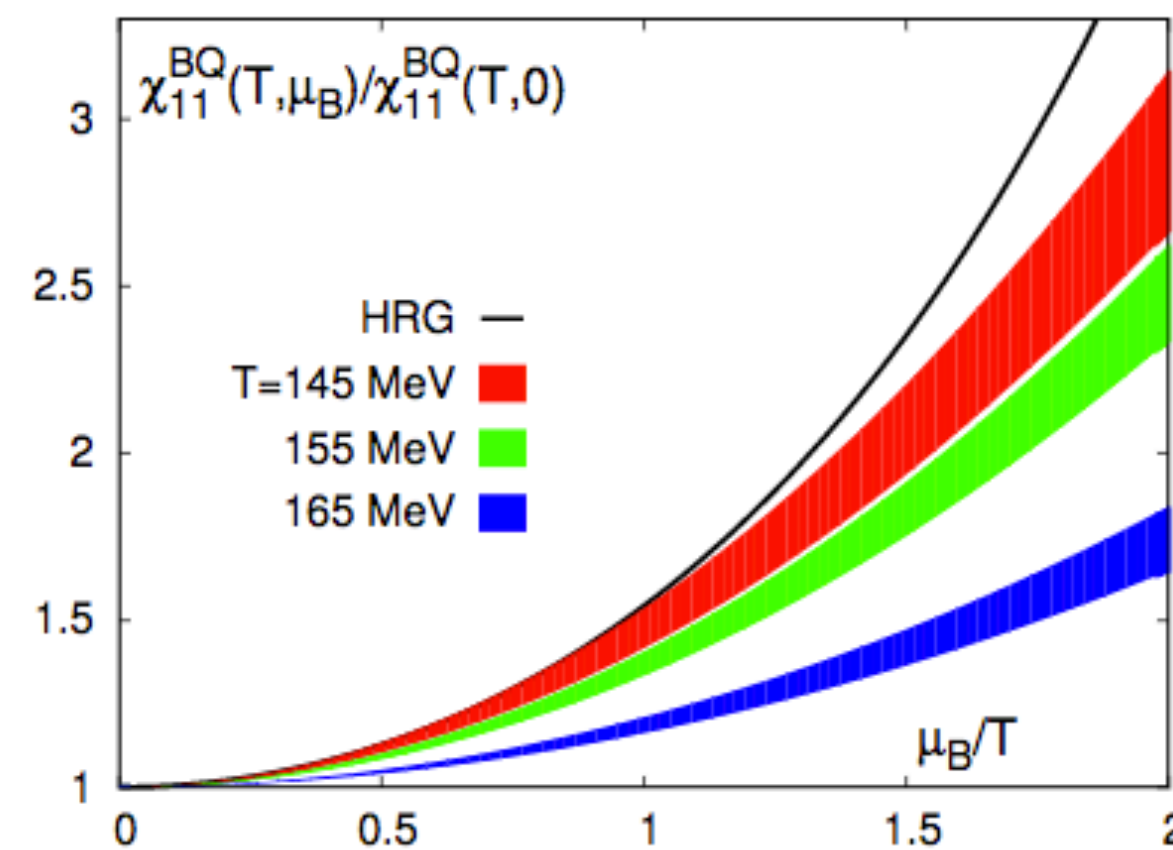
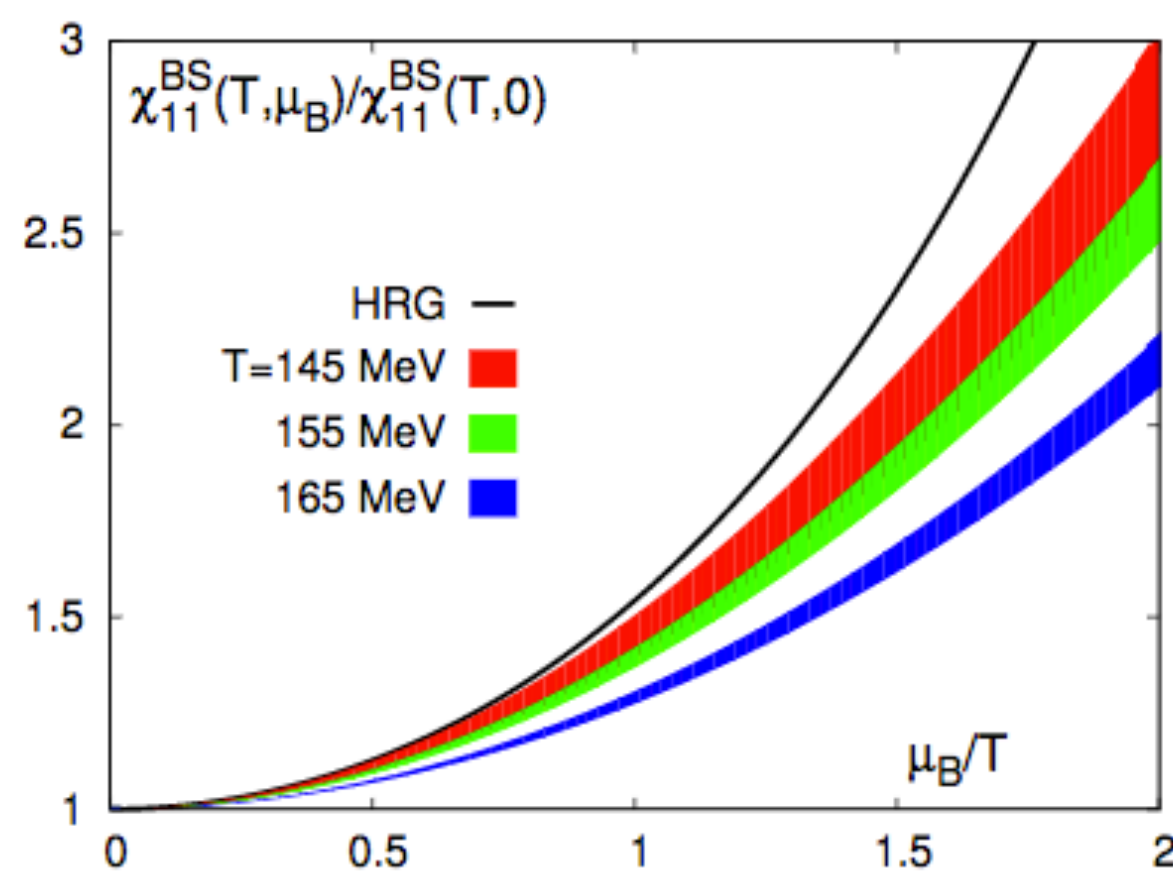
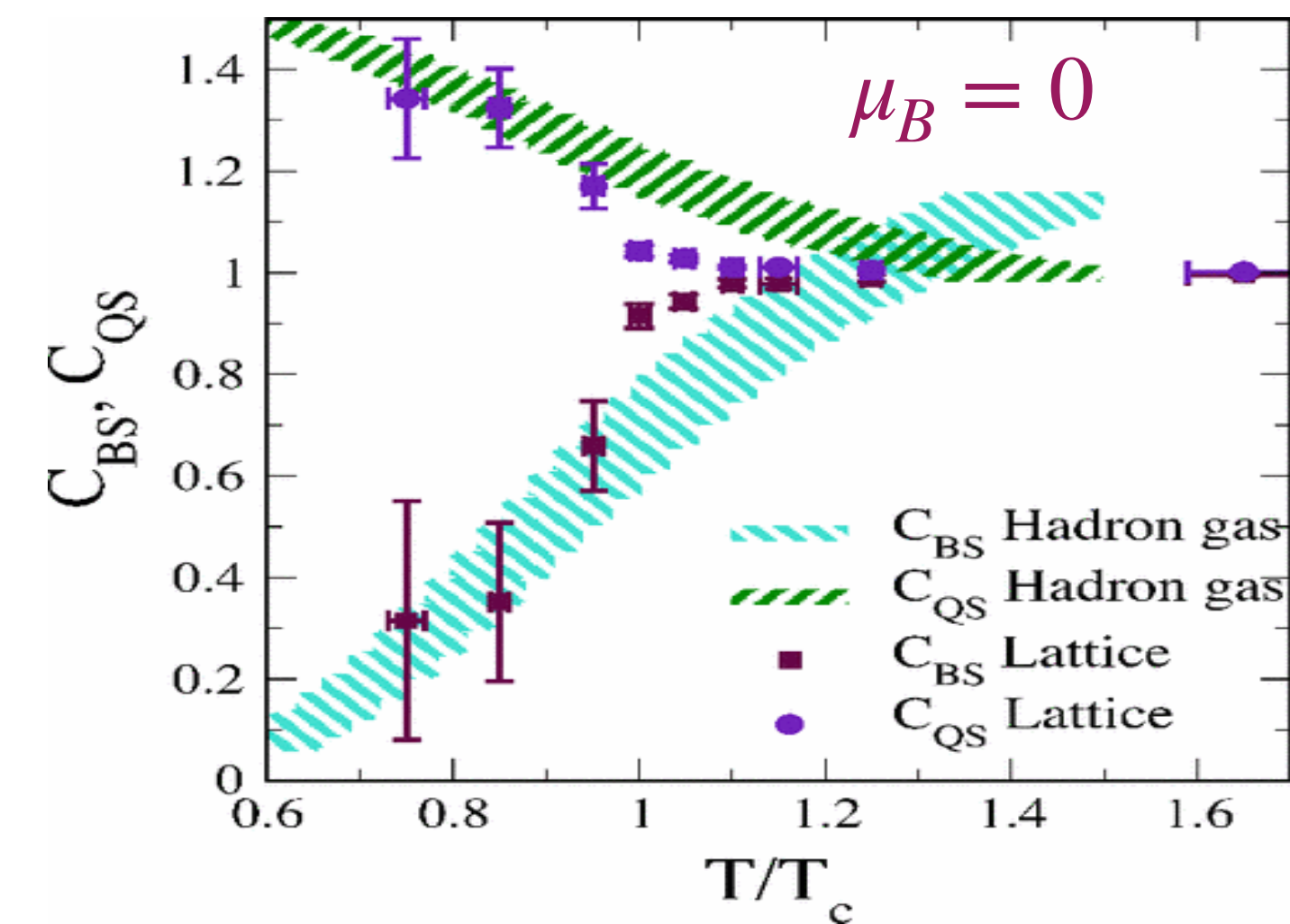
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$$\chi_{XY}^{ij} = \frac{1}{VT^3} M_{XY} = \langle (X - \langle X \rangle)^i (Y - \langle Y \rangle)^j \rangle$$

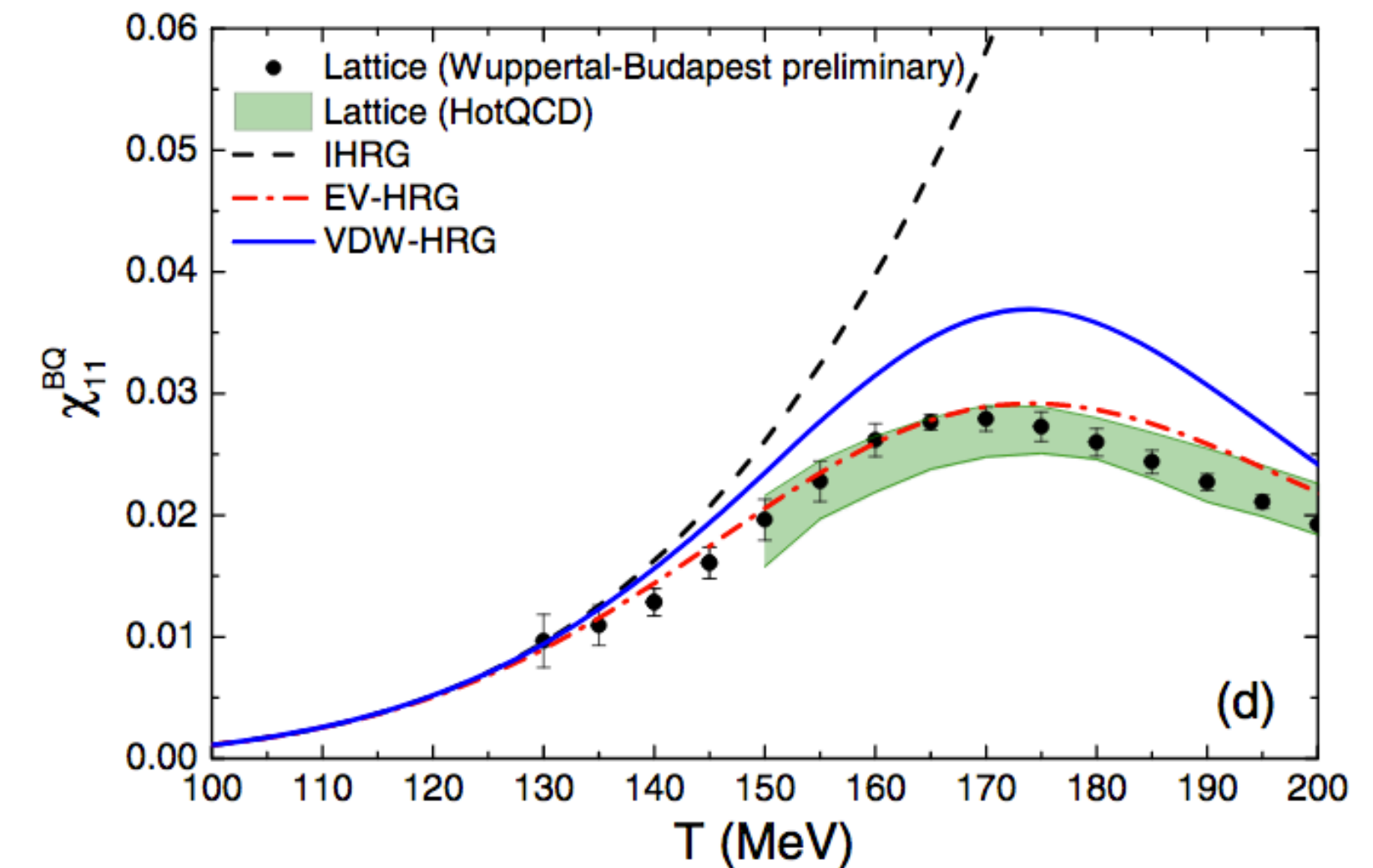
► Different T-dependent for partonic and hadronic phases.

V. Koch et al. PRL.95.182301 (2005),
F. Karsch and K. Redlich, PLB 695 ,
136142 (2011).

► Sensitivity to the difference between
HRG and lattice calculations at the
lowest order. New constrain on **freeze-**
out condition.



F. Karsch Nuclear Physics A 00 (2017) 1–4



A. Bazavov et al. Phys. Rev. D. 86. 034509, PRL.109.192302 (2012),
V. Vovchenko phys. Rev. Lett. 118, 182301

Observables:

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✓ “Variance”

$$c_2 = \sigma^2 = \langle (\delta X)^2 \rangle$$

✓ “Co-variance” – Cross correlation

$$c_{1,1} = \sigma^{1,1} = \langle (\delta X)(\delta Y) \rangle$$

Ratio : $C_{XY} = \frac{\sigma_{XY}^{11}}{\sigma_Y^2}$

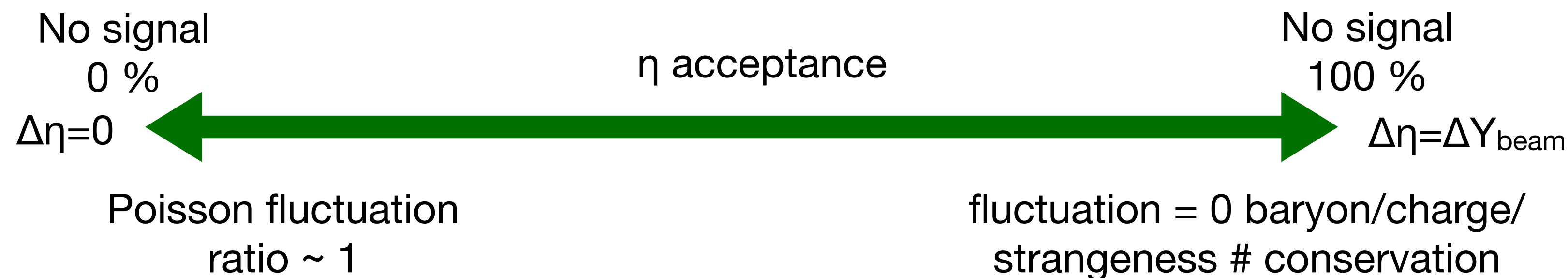
Volume independent correlation compared to self correlation

V. Koch et al. PRL.95.182301 (2005),

We study

► Beam energy and centrality dependence variance, covariance and ratio.

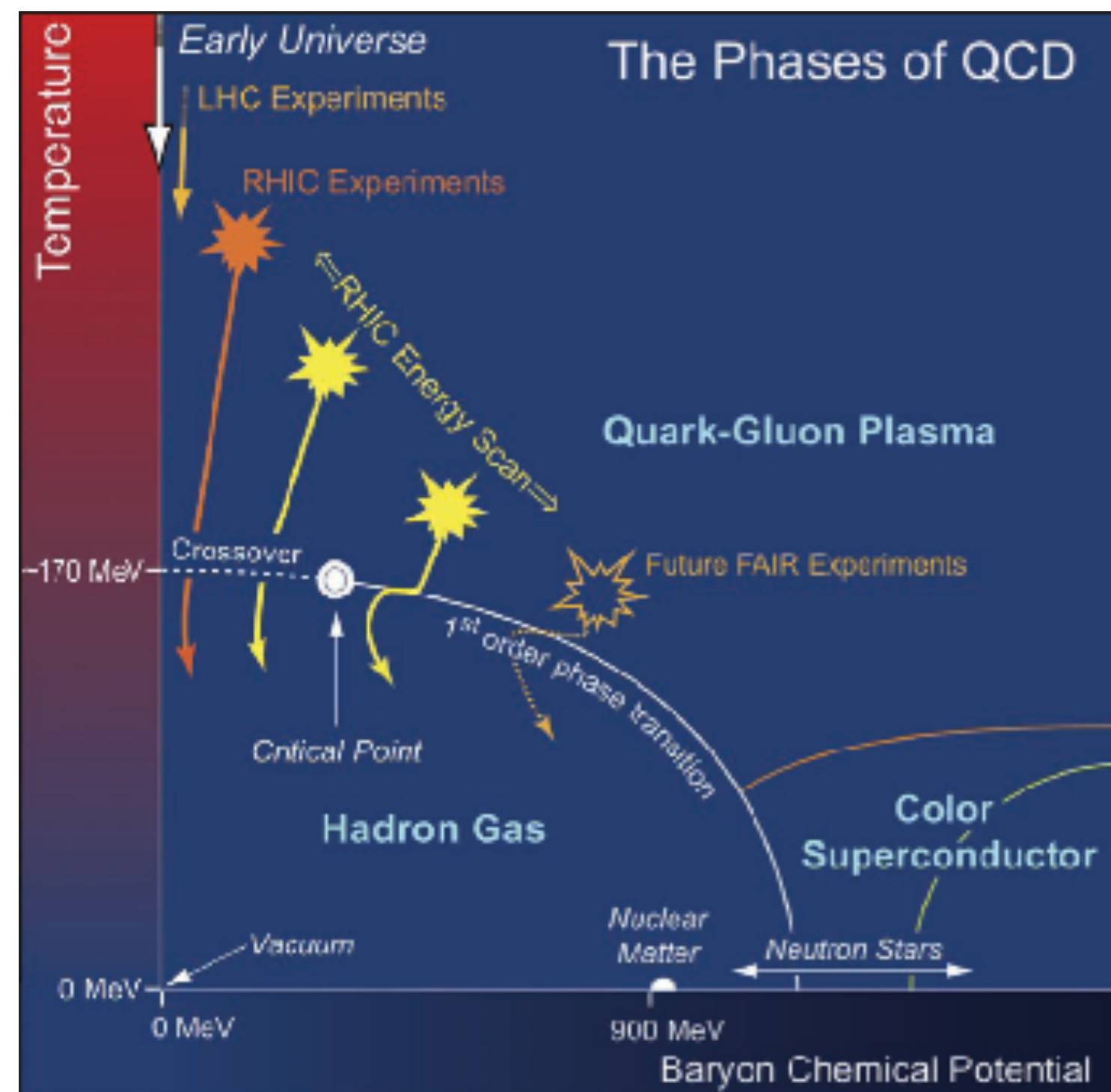
► acceptance ($|\eta|$) dependence:



J. Brewer et al, arXiv:1804.10215 [hep-ph]
A. Chatterjee, et. al, J. Phys. G43, 125103 (2016)

RHIC beam energy scan:

6



arXiv:0809.3137

$\sqrt{s_{NN}}$ (GeV)	μ_B (MeV)	T (MeV)	μ_B/T	Statistics (M) (0-80%)
7.7	422	140	3.020	~4
11.5	316	152	2.084	~12
14.5	316	152	1.639	~20
19.6	206	160	1.287	~36
27	156	163	0.961	~70
39	112	164	0.684	~130
62.4	73	165	0.439	~67
200	25	166	0.150	~350

T, μ_B values from J. Cleymans, et al. Phys. Rev. C 73, 034905

- Varying beam energy, we can access broad region of the QCD phase diagram.
- QCD phase diagram can be mapped between μ_B values 20 to 425 MeV.

Few details:

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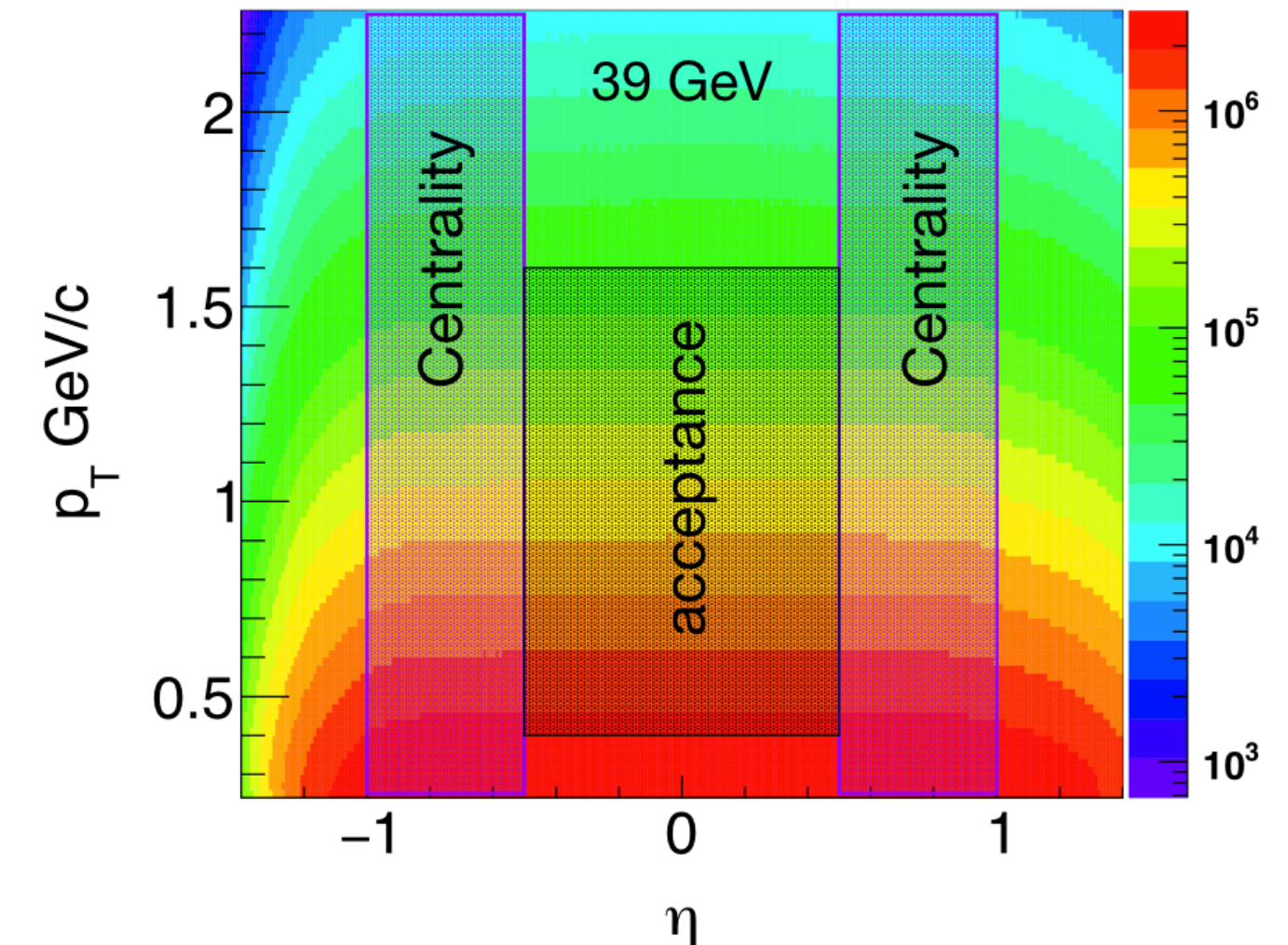
► Centrality definition

Charge particle within $0.5 < |\eta| < 1.0$ and $-1.0 < |\eta| < -0.5$ (avoid track from analysis region), to avoid auto-correlation.

- Centrality bin width (**CBWC**) averaging : To suppress the artificial fluctuation due to initial variation in volume.

$$\kappa^{XY} = \sum_{i=N_1}^{N_2} n_i \kappa_i^{XY} / \sum_{i=N_1}^{N_2} n_i$$

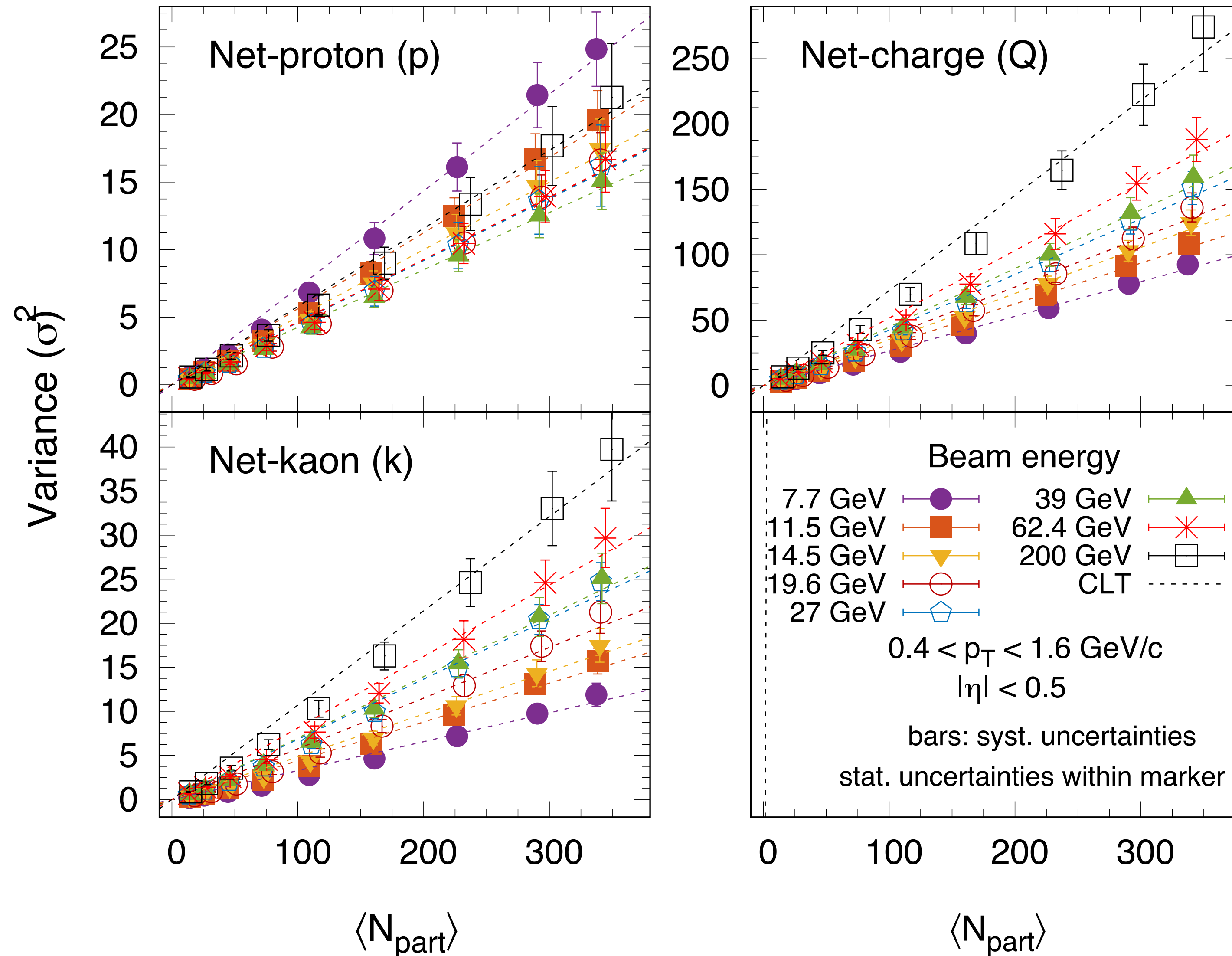
X. Luo, et.al, PRC91, 034907 (2015); JPG40, 105104 (2013), Journal of Physics: Conf. Ser. 316, 012003, N. Sahoo et. al. Phys. Rev. C 87.044906 (2013)



- **Efficiency correction** on cumulants have been done using binomial detection response.
- **Uncertainty estimation:** Statistical uncertainty based on numerical error propagation of multivariate cumulants and systematic uncertainty by varying different track selection criteria and tracking efficiency values.

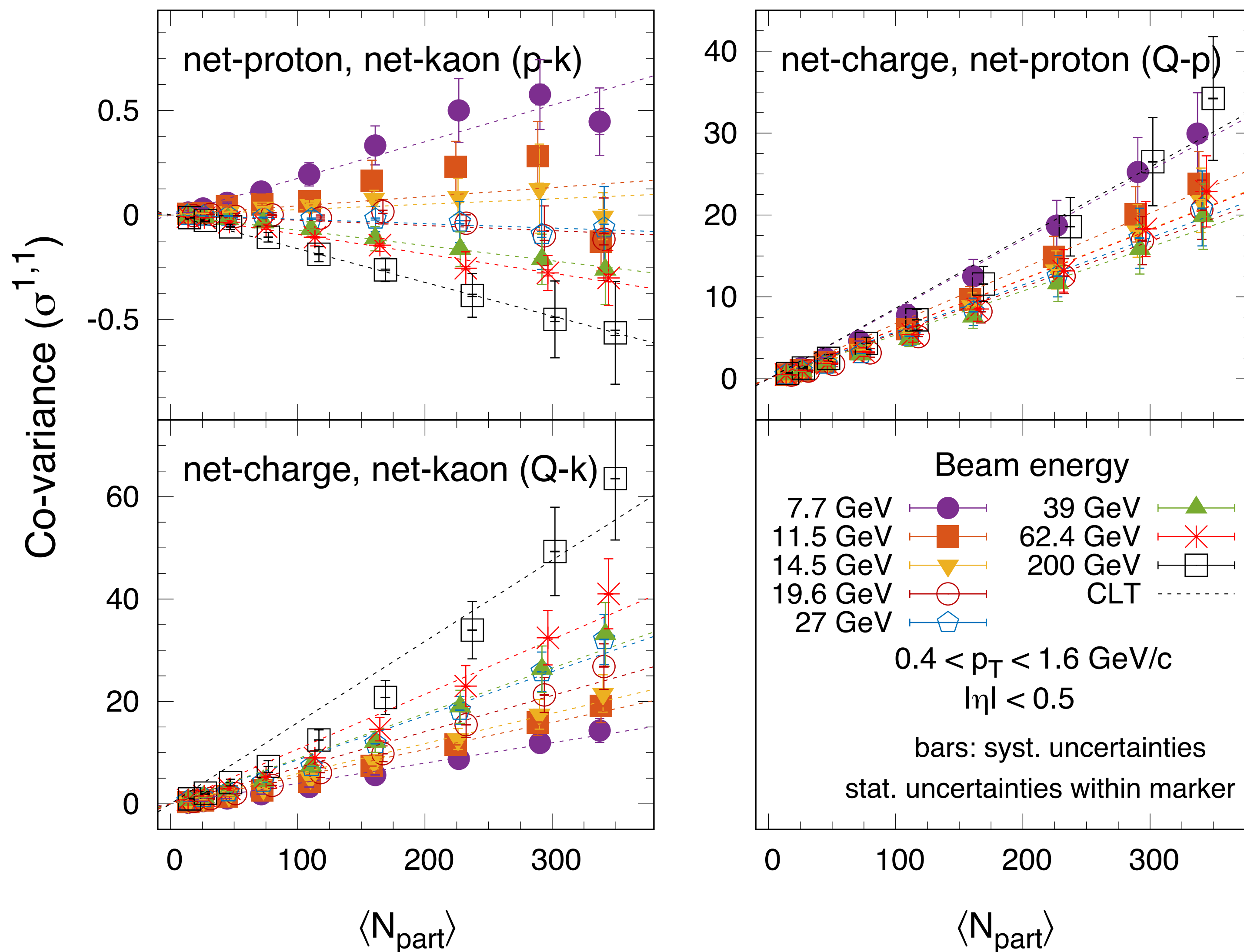
M. Kendall, The Advanced Theory of Statistics No. v. 1 (1943),
A. Bzdak and V Koch, PRC 91, 027901(2015),
X. Luo, PRC91, 034907 (2015)
X. Luo and Nu Xu Nucl. Sci. Tech. 28, 112 (2017)

Diagonal cumulant:



- ▶ Variances shows linear dependence with respect to centrality.
- ▶ Covariance follows the linear dependence: $\sigma^{11} \propto \langle N_{part} \rangle$

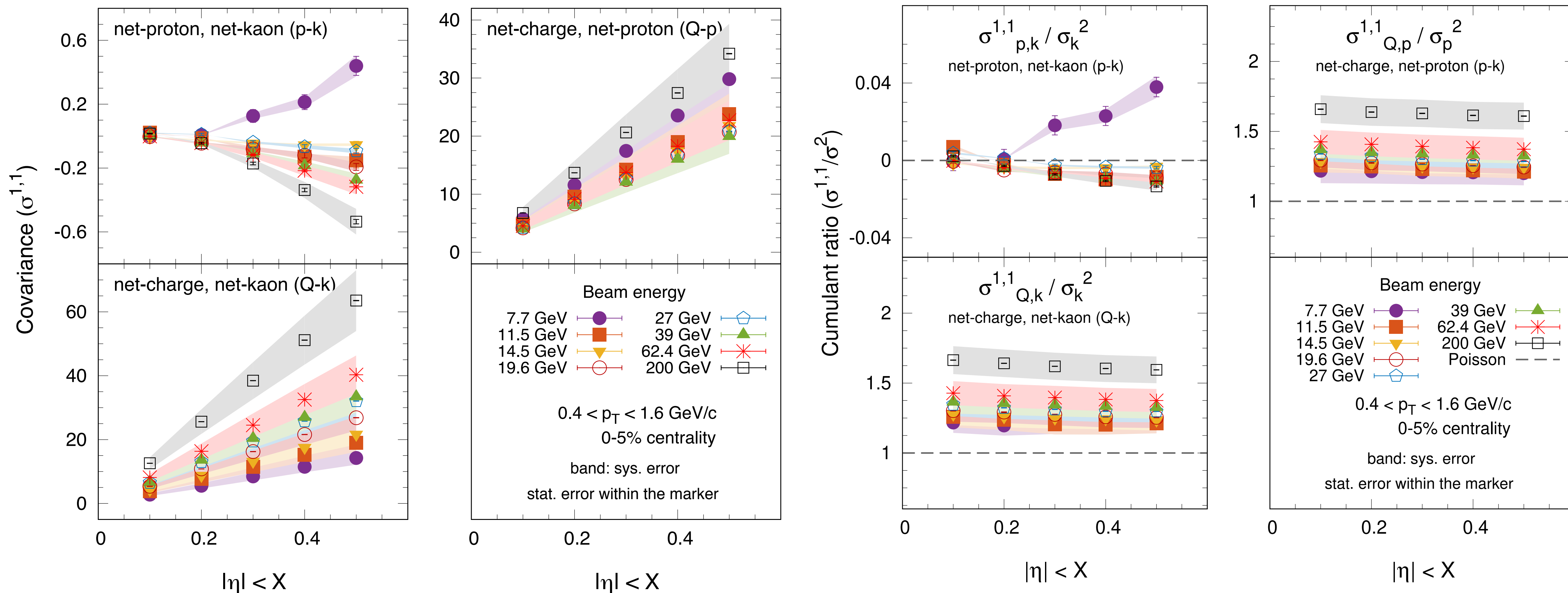
Off-diagonal cumulant:



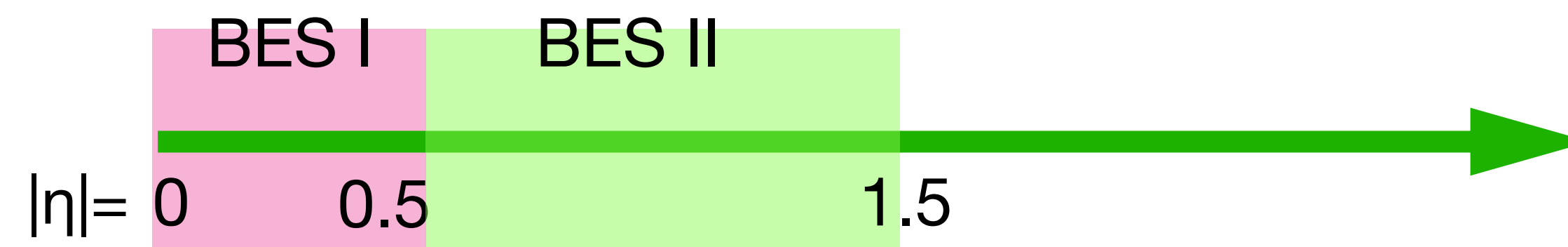
- σ_{Qp} and $\sigma_{Q,k}$ shows linear dependence with respect to centrality.
- The covariance between net-proton and net-kaon is positive at low energy and negative at higher energy — Indicating net-p and net-k are anti-correlated at high energy.

Acceptance dependence:

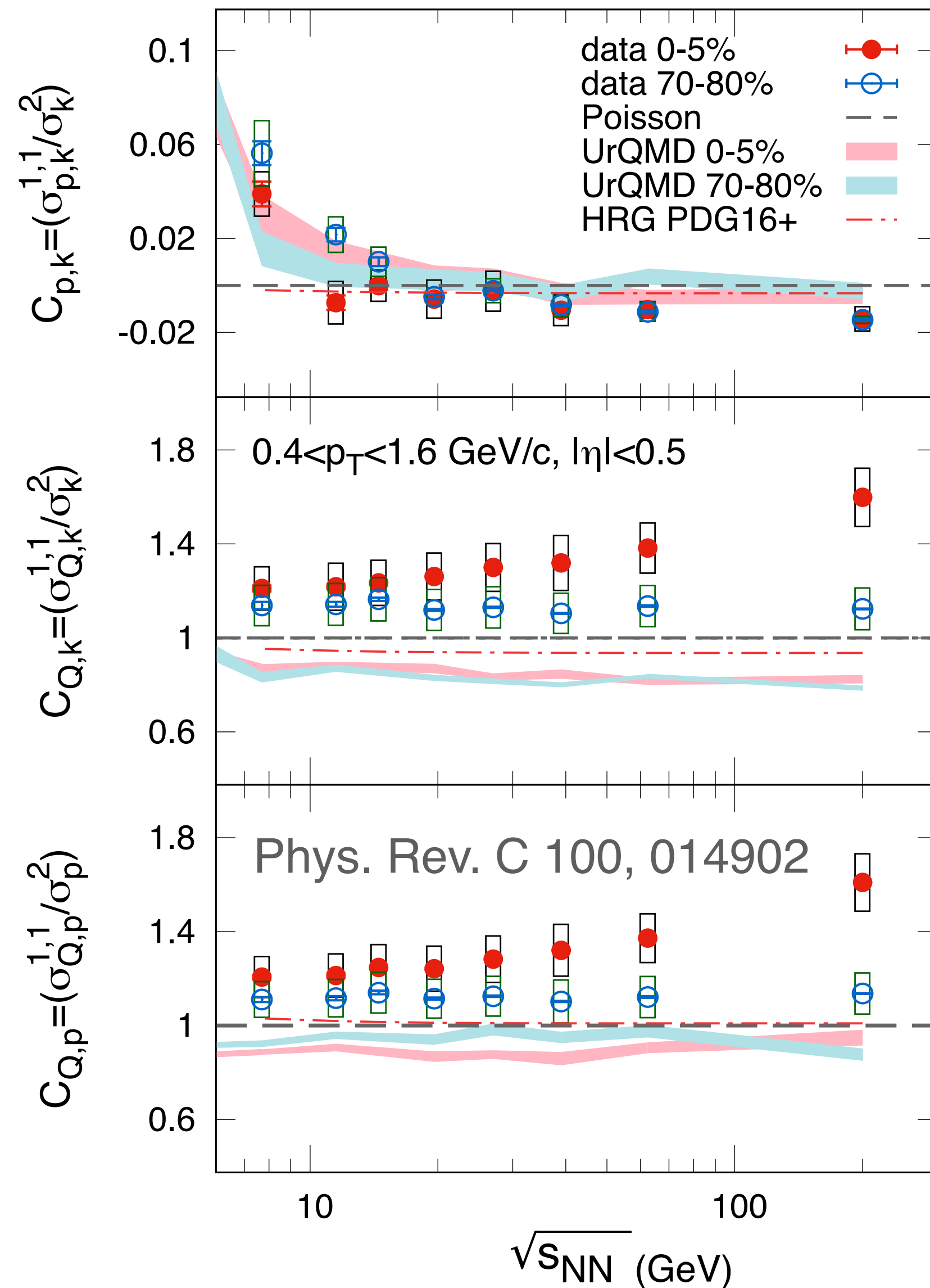
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Phys. Rev. C 100, 014902



Beam energy dependence ratio:



► Volume independent correlation net-p and net-k are positive at lower energy and negative at higher energy.

❖ @ lower energy, $pp \rightarrow p\Lambda K^+$ process may lead positive correlation
J. T. Balewski et. al, PLB 420, 211 (1998)

❖ negative correlation @ 200 GeV cannot explain by models

❖ In QGP phase, B-S correlation is negative. **We also observed negative p-K correlation.** Although direct quantitate comparison is not possible.

V. Koch et al. PRL.95.182301 (2005),

► C_{Qp} and C_{Qk} both shows significantly higher correlation in central compare to peripheral collision. The excess correlation not observed in both UrQMD and HRG.

- ▶ First measurement of all off-diagonal cumulant as a function of collision centrality for Au+Au collisions $\sqrt{s_{NN}} = 7.7-200$ GeV are presented. Results are shown for the kinematic range of $|\eta| < 0.5$ and $0.4 < p_T < 1.6$ GeV/c as well as different $|\eta|$ -windows.
- ▶ An excess correlations observed in Q-p and Q-k for central collisions with respect to the peripheral ones. The correlations increase with the increase of beam energy and different from the model predictions.
- ▶ Net-proton and net-kaon shows anti-correlations for central collisions and for higher energies. In QGP phase, B-S correlation is negative. We also observed negative p-K correlation. Although direct quantitative comparison is not possible.
- ▶ Both diagonal and off-diagonal cumulants of net-charge, net-kaon and net-proton show linear dependence with the $\Delta\eta$ acceptance window.

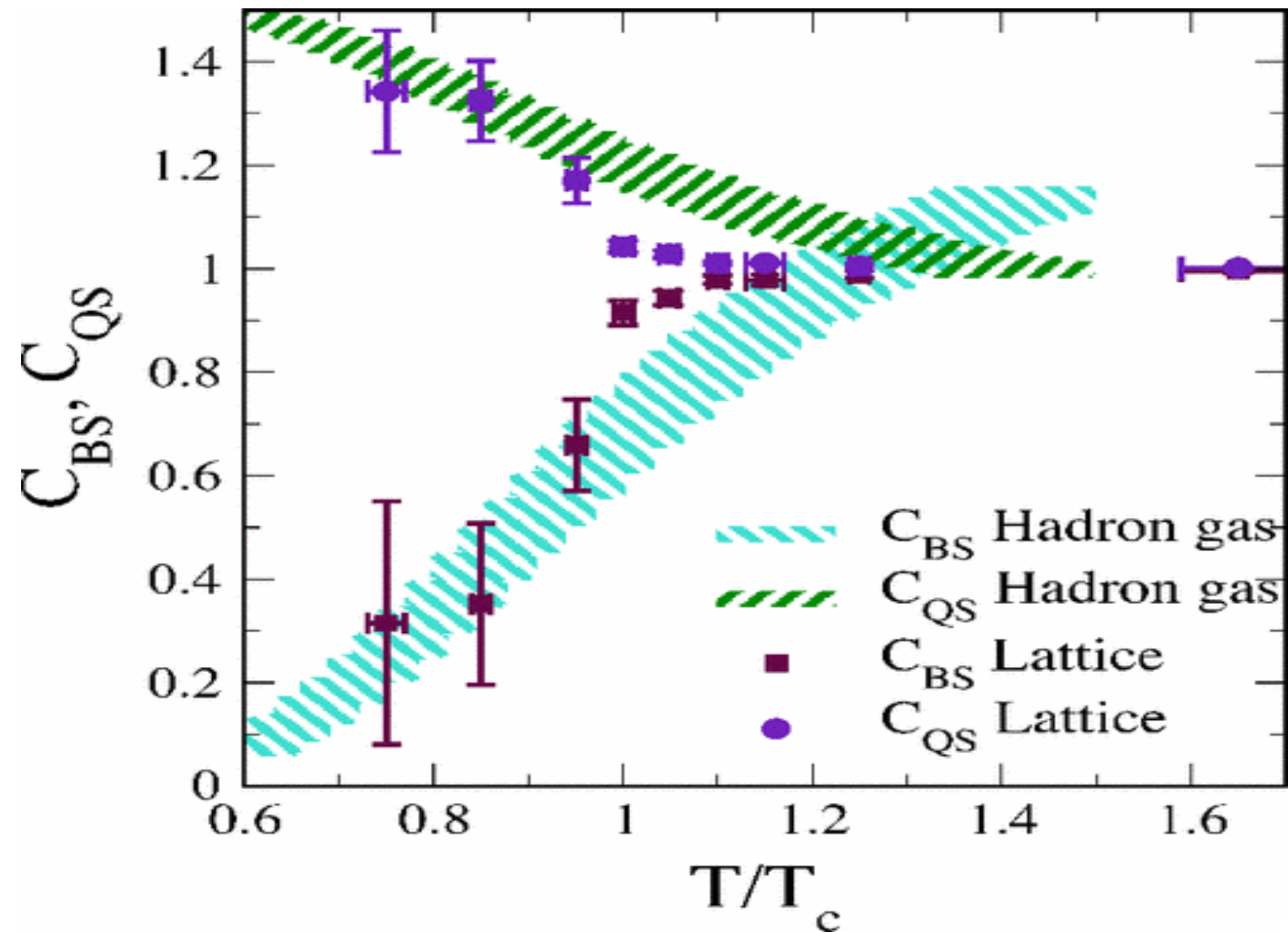
Outlook

- ▶ FXT program proposed during RHIC BES-II will extend the energy down to 3 GeV
- ▶ Working on Higher-order off-diagonal cumulants.

Thank You

Back up slides

B-S correlation in Partonic and Hadronic medium



V. Koch et al. PRL.95.182301 (2005),

	Q	B	S
u	+2/3	1/3	0
d	-1/3	1/3	0
s	-1/3	1/3	-1

$$B = \frac{1}{3}(\Delta u + \Delta d + \Delta s),$$

$$Q = \frac{2}{3}\Delta u - \frac{1}{3}\Delta d - \frac{1}{3}\Delta s,$$

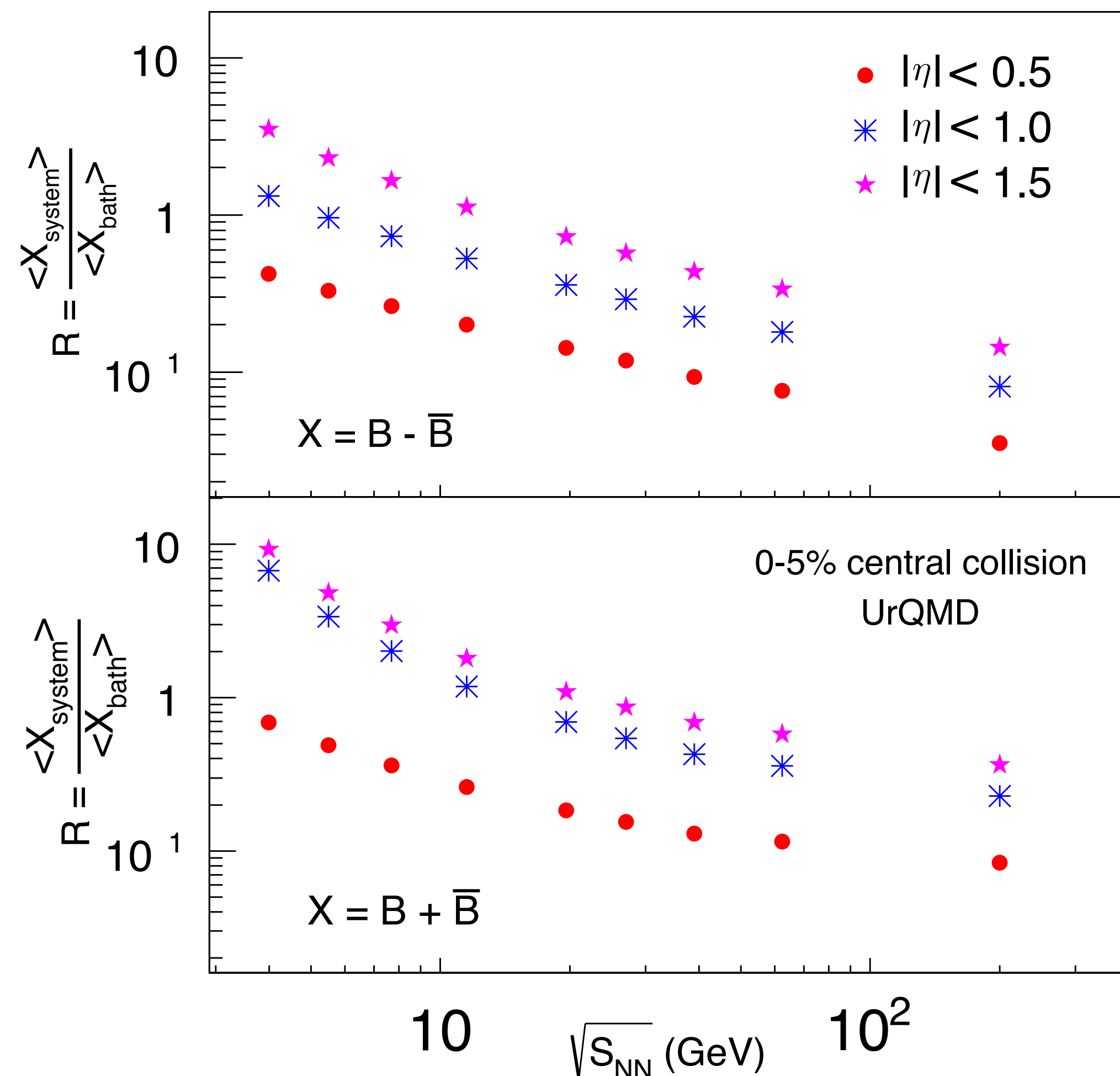
$$S = -\Delta s.$$

Partonic:

$$C_{BS} = -3 \frac{\langle \delta B \delta S \rangle}{\langle \delta S^2 \rangle} = -3 \times \frac{\frac{1}{3}(\Delta u + \Delta d + \Delta s)(-\Delta s)}{(\Delta s)^2} = 1$$

Hadronic: $C_{BS} = R_{11}^{BS} \approx 3 \frac{\langle \Lambda \rangle + \langle \bar{\Lambda} \rangle + \dots + 3\langle \Omega^- \rangle + 3\langle \bar{\Omega}^+ \rangle}{\langle K^0 \rangle + \langle \bar{K}^0 \rangle + \dots + 9\langle \Omega^- \rangle + 9\langle \bar{\Omega}^+ \rangle}.$

Acceptance effect:



$$\Delta Y_{\text{accept}} \ll \Delta Y_{\text{total}}$$

- In order to observe grand canonical fluctuations of the conserved charges, the ratio of the net-quantity within the observation window to the total conserved net-charges in the bath should be $\ll 1$

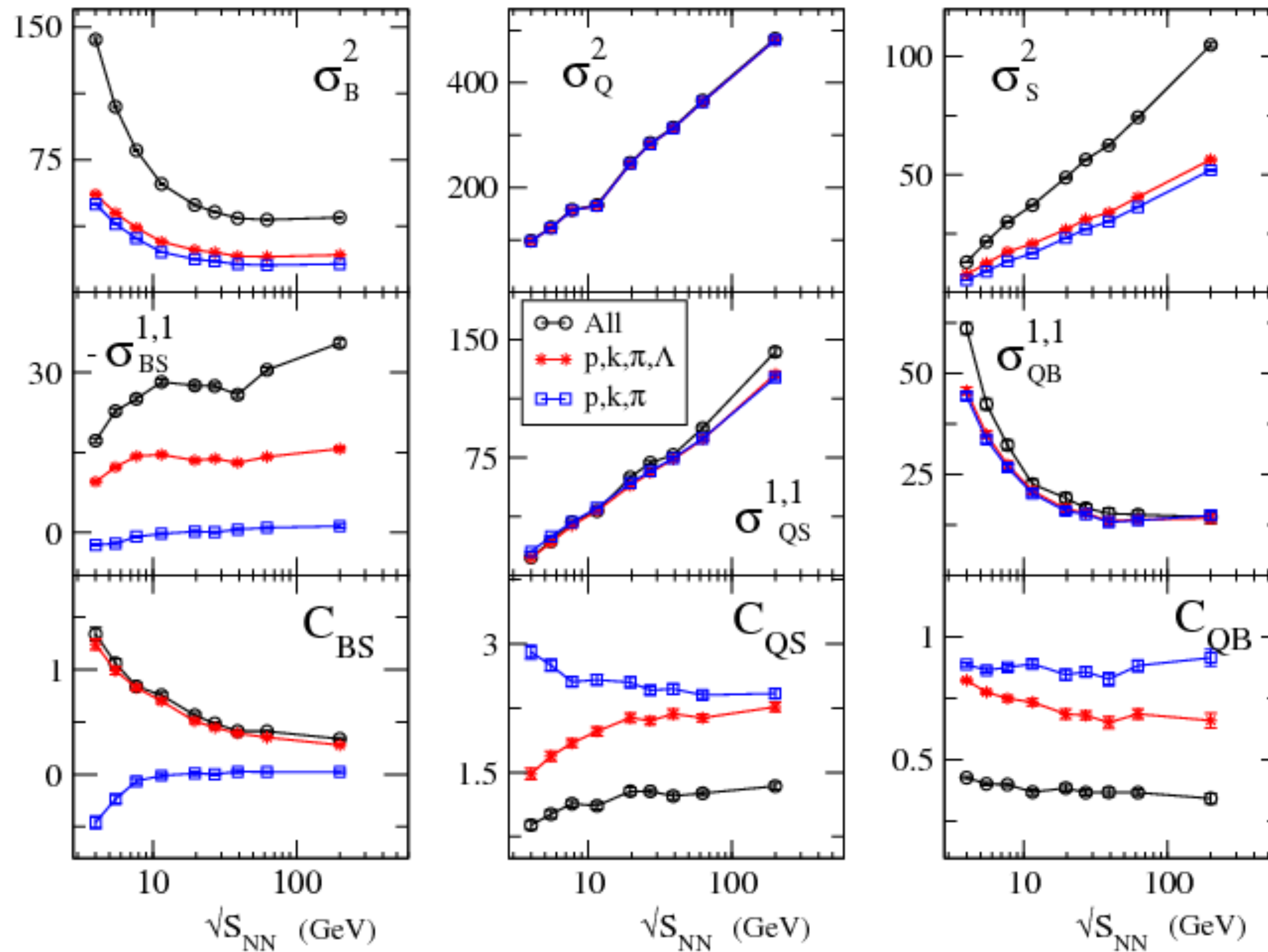
$$R = \frac{\text{system size}}{\text{bath size}} \ll 1$$

- For lower energy larger acceptance window may rise non-thermal behavior.
- A fixed eta window across all beam energy does not correspond to the same system to bath effective volume ratio.

S. Gupta, arXiv: 1508.01136 (2016)

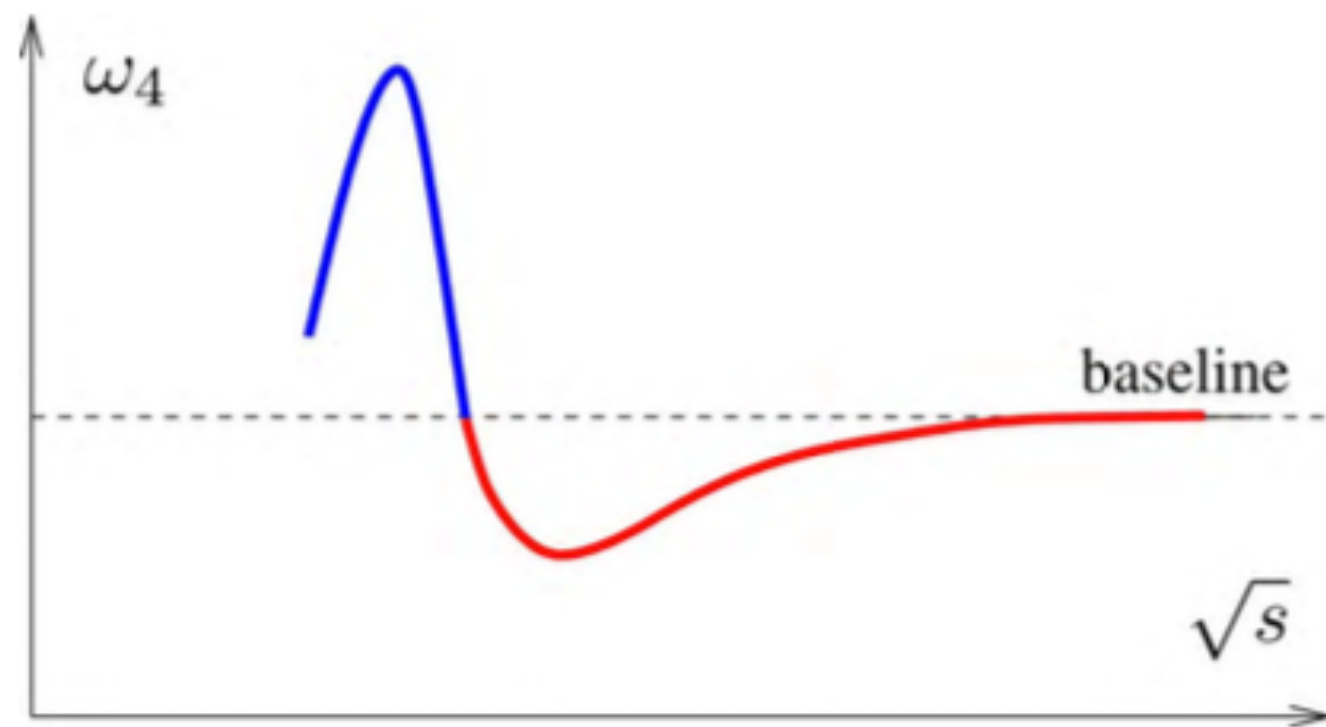
Particle set dependence cumulants:

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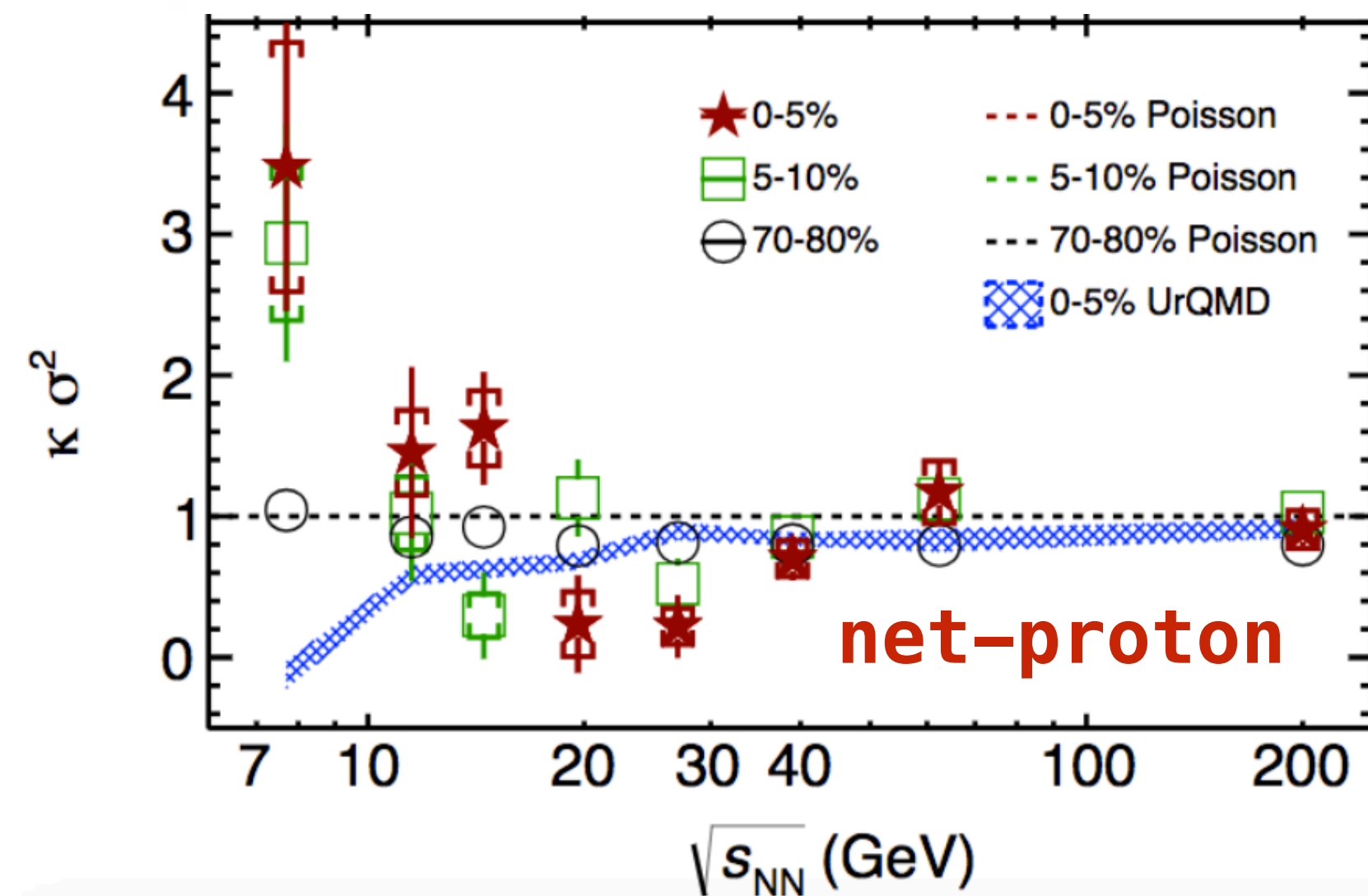


Results from diagonal cumulants analysis:

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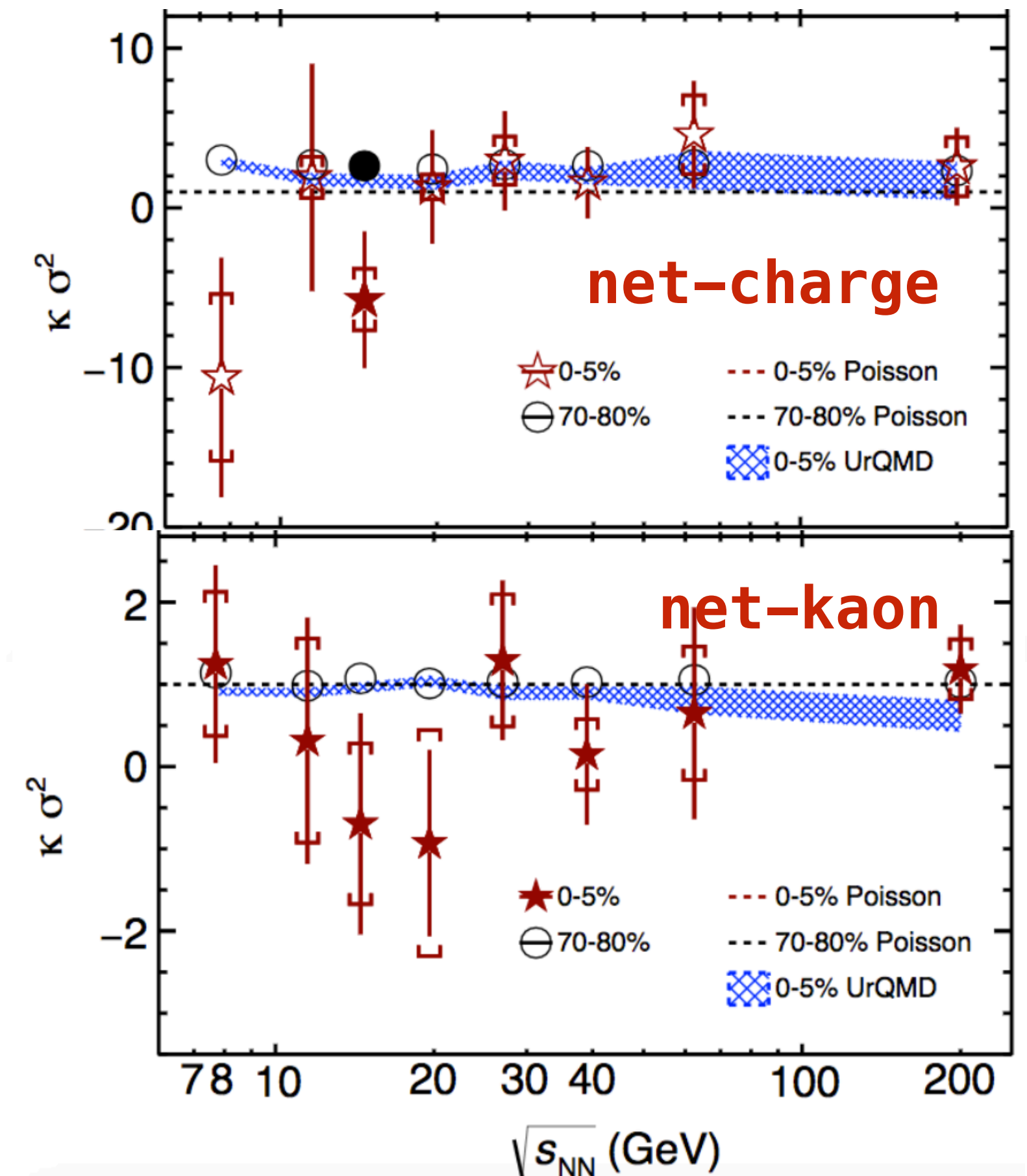


► The expected non-monotonic energy dependence for the normalized fourth order cumulant of multiplicity distributions (ω_4) when the chemical freeze-out line passes the critical region.



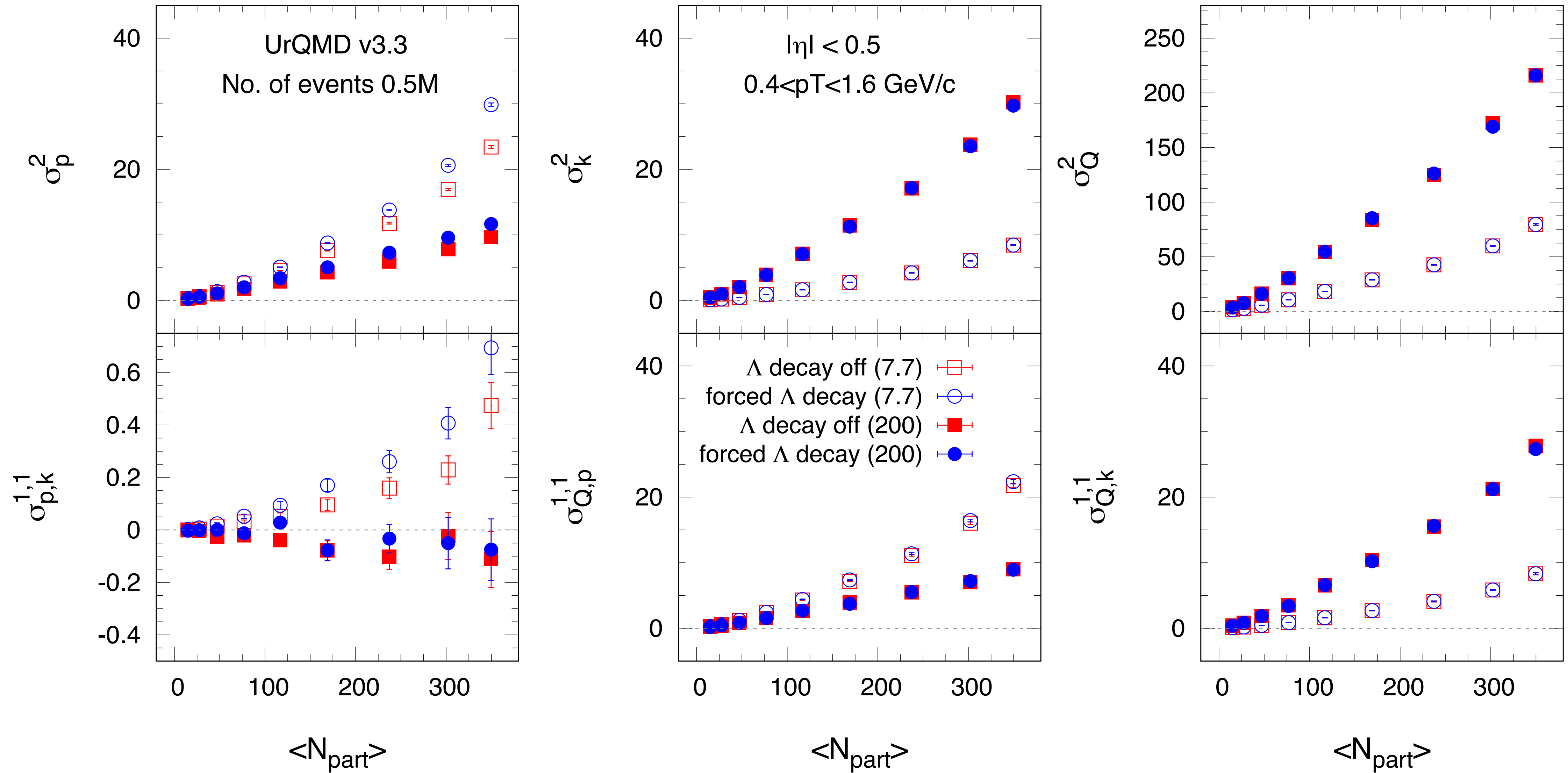
► some non-monotonic behavior observed in net-proton higher moments with large statistical error-bars.

► Need to scan lower energy also



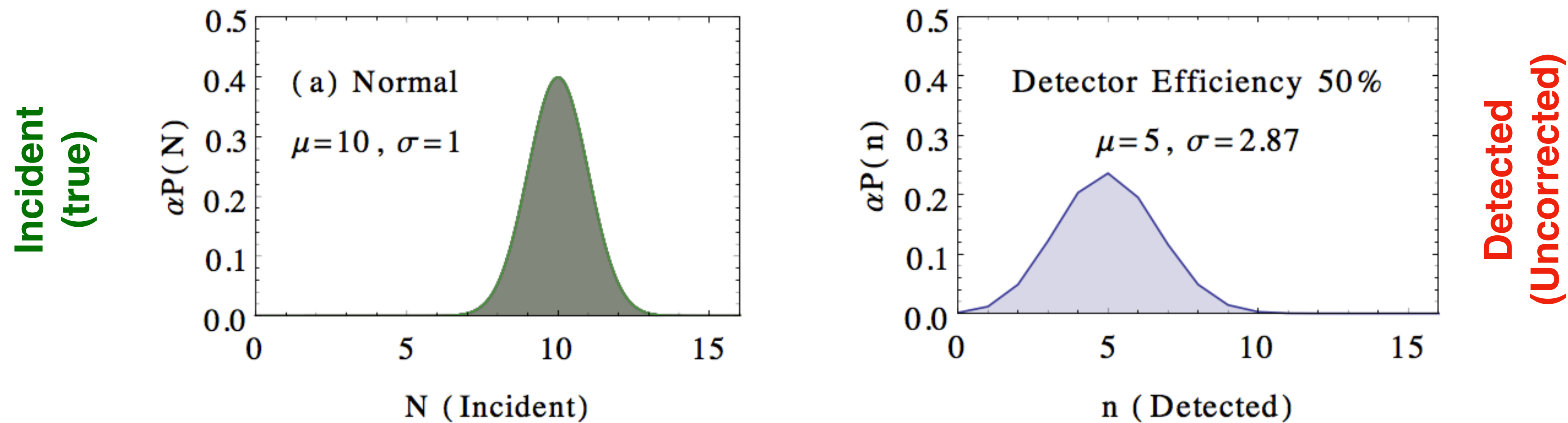
L. Adamczyk (STAR Collaboration), et al., Phys. Rev. Lett. 112, 032302 (2014).
 L. Adamczyk (STAR Collaboration), et al., Phys. Rev. Lett. 113, 092301 (2014)
 arXiv:1709.00773
 STAR, PoS(CPOD14)019; QM (15).

Effect of Λ decay:



Efficiency correction:

► **Efficiency correction** on cumulants have been done using binomial detection response.



$$\underbrace{p(n)}_{\text{Detected}} = \sum_{N=n}^{\infty} \frac{N!}{n!(N-n)!} (\epsilon)^n (1-\epsilon)^{N-n} \underbrace{P(N)}_{\text{Incident (true)}} = \sum_{N=n}^{\infty} B(n | N, \epsilon) P(N)$$

$$\begin{aligned} \mu_1^{obs} &= \epsilon \mu_1^{inc} \\ \mu_2^{obs} &= \epsilon^2 \mu_2^{inc} + \epsilon(1-\epsilon) \mu_1^{inc} \\ \mu_3^{obs} &= \epsilon^3 \mu_3^{inc} + 3\epsilon^2(1-\epsilon) \mu_2^{inc} + \epsilon(1-2\epsilon) \mu_1^{inc} \end{aligned}$$

Convert to **factorial moment**

$$\begin{aligned} f_1^{obs} &= \epsilon f_1^{inc} \\ f_2^{obs} &= \epsilon^2 f_2^{inc} \\ f_3^{obs} &= \epsilon^3 f_3^{inc} \\ f_n^{obs} &= \epsilon^n f_n^{inc} \end{aligned}$$

No clear pattern: **hard to handle**

clear pattern: **easy** to correct

- **Efficiency correction** on cumulants have been done using binomial detection response.
- Strategy is to convert cumulants to Irreducible **factorial moments** and correct it

$$F_{N_{(p,1)}, N_{(p,2)}, N_{(\bar{p},1)}, N_{(\bar{p},2)}, N_{(k_+,1)}, N_{(k_+,2)}, N_{(k_-,1)}, N_{(k_-,2)}}^{s,t,u,v,w,x,y,z} \quad \text{Corrected}$$

$$= \frac{f_{n_{(p,1)}, n_{(p,2)}, n_{(\bar{p},1)}, n_{(\bar{p},2)}, n_{(k_+,1)}, n_{(k_+,2)}, n_{(k_-,1)}, n_{(k_-,2)}}^{s,t,u,v,w,x,y,z}}{\varepsilon_{(p,1)}^s, \varepsilon_{(p,2)}^t, \varepsilon_{(\bar{p},1)}^u, \varepsilon_{(\bar{p},2)}^v, \varepsilon_{(k_+,1)}^w, \varepsilon_{(k_+,2)}^x, \varepsilon_{(k_-,1)}^y, \varepsilon_{(k_-,2)}^z} \quad \text{Uncorrected}$$

A. Bzdak and V. Koch, PRC 86 044904,
PRC 91 027901

TPC (1) and TPC*ToF matching (2) efficiencies

M. Kendall, The Advanced Theory of Statistics No. v. 1 (1943)

$$\varepsilon_{TPC} = \frac{\text{Reconstructed tracks having associated MC tracks}}{\text{MC tracks } (\pi^\pm, K^\pm, p, \bar{p})}$$

$$\varepsilon_{TOF} = \frac{\text{track quality cut \&\& } |n\sigma_{particle}| < 2 \text{ \&\& TOF matched } > 0}{\text{track quality cut \&\& } |n\sigma_{particle}| < 2}$$

