The 13th Workshop on QCD Phase Transition and Relativistic Heavy-Ion Physics (QPT 2019)

Vorticity and Λ spin polarization in heavy ion collisions

Hui Li

Fudan University, Shanghai

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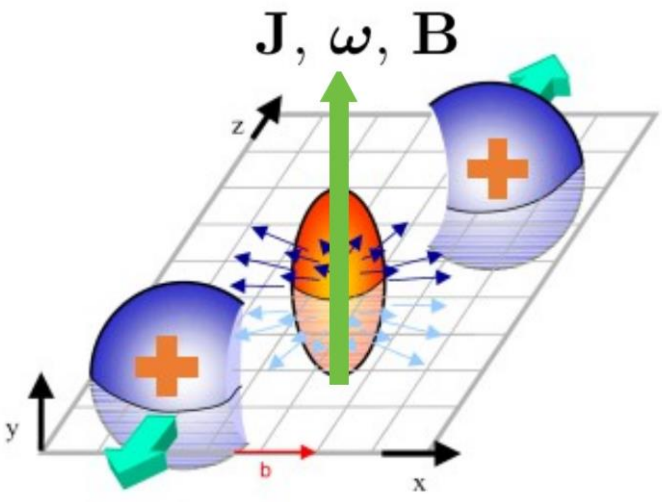
X. L. Xia, H. Li, X. G. Huang, H. Z. Huang,
Phys. Rev. C 100, 014913

Outline

- Introduction
- Status and puzzles on Λ spin polarization
- Feed-down effect on Λ spin polarization
- Summary

Introduction



$J \sim 10^6 \hbar \qquad B \sim 10^{18}\mathrm{G}$

- Huge angular momentum and magnetic field are generated in heavy ion collisions. ω and B can induce chiral anomaly effect, such as CME, CVE, CMW, etc.

- Magnetic and vorticity field can lead to the spin polarization of final hadrons.

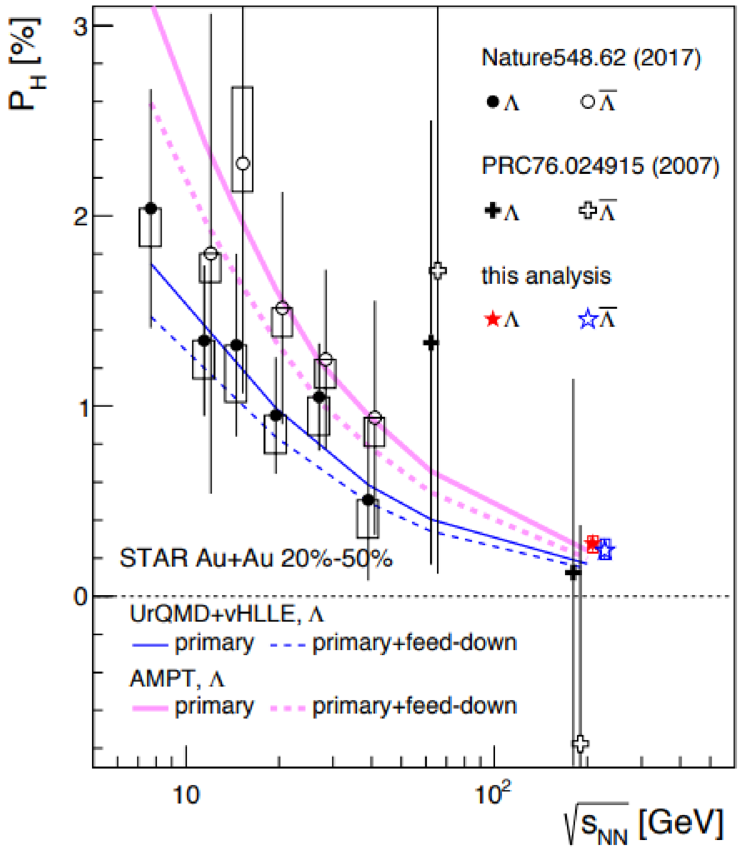
- From the polarization measurement, information of ω and B can be extracted.

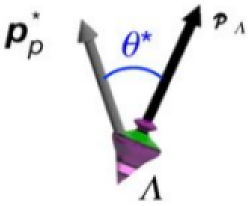
See talks by
J. F. Liao, X. G. Huang, S. Lin, J. H. Gao, Q. Y. Shou, S. Q. Feng, Y. C. Liu, L. Yin …

Global Λ polarization

Angular momentum or vorticity can lead to global polarization

Liang, Wang, PRL 94, 102301 (2005);
Voloshin nucl-th/0410089





$$\frac{dN}{d\cos\theta^*} = \frac{1}{2}\left(1 + \alpha_{\rm H}|\vec{\mathcal{P}}_{\rm H}|\cos\theta^*\right).$$

Measurements:

STAR, Nature 548, 62 2017;
STAR, Phys. Rev. C 98, 014910 (2018).

Theoretical calculation:

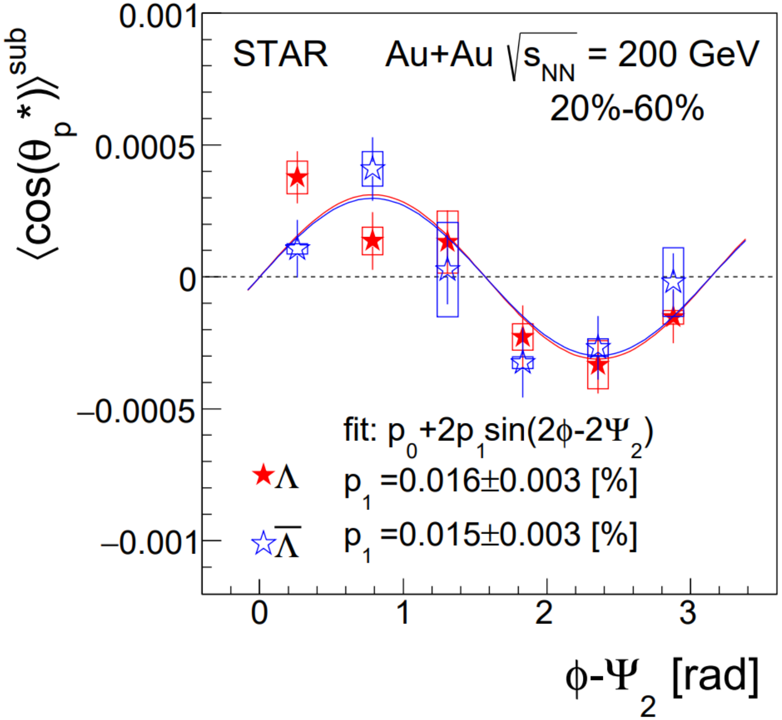
Karpenko-Becattini 2017;
Xie-Wang-Csernai 2017;
Li-Pang-Wang-Xia 2017;
Sun-Ko 2017;
Shi-Li-Liao 2019;
Wei-Deng-Huang 2019; ...

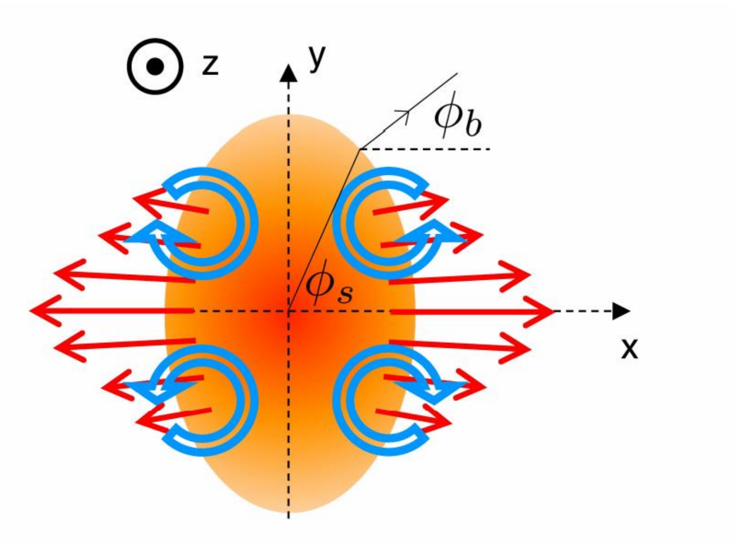
The model calculations agree with the data

Local Λ polarization

- Velocity gradients due to elliptic flow can produce vorticity along the beam direction.

STAR, 1905.11907



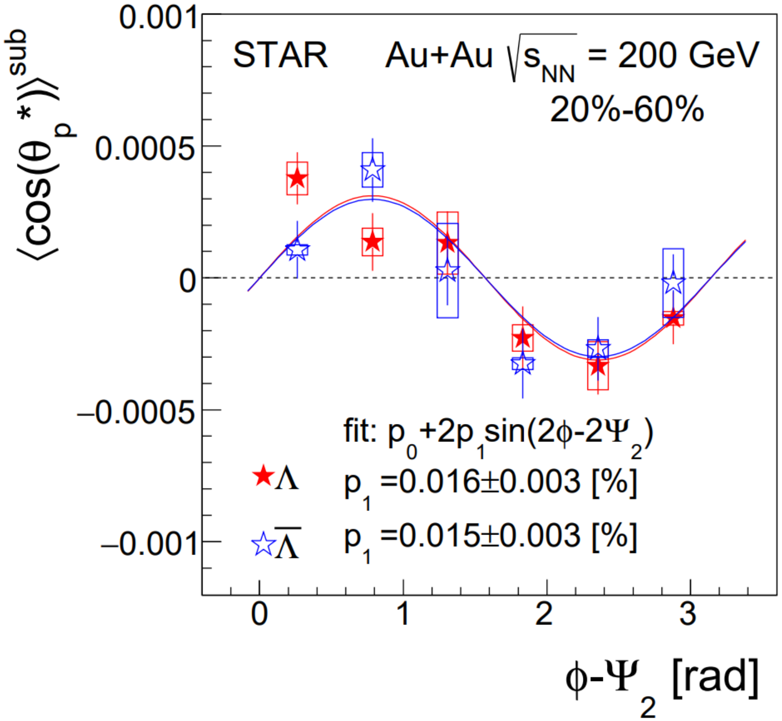


$$P_z = F_z \sin(2\phi)$$

Local Λ polarization

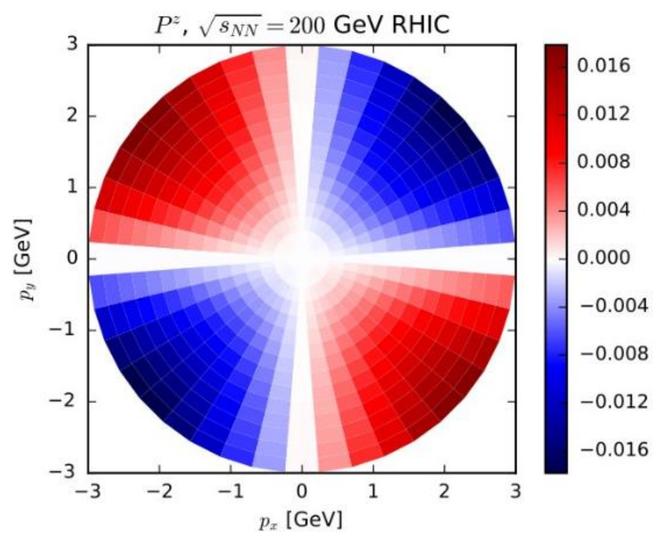
- Local velocity gradients due to elliptic flow may produce vorticity along beam direction
- Sin(2ϕ) signal has been observed, but the sign is opposite between STAR data and hydro and transport models

STAR, 1905.11907

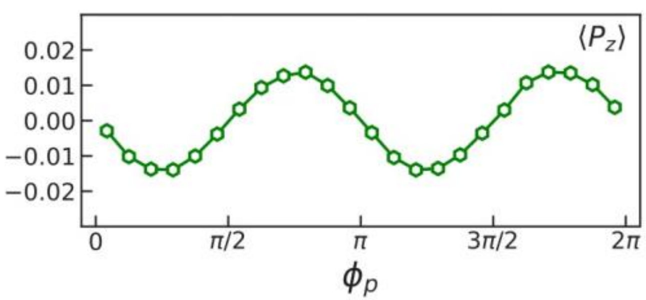


$$P_z = F_z \sin(2\phi)$$

Becattini-Karpenko, 2018



Xia-Li-Tang-Wang, 2018

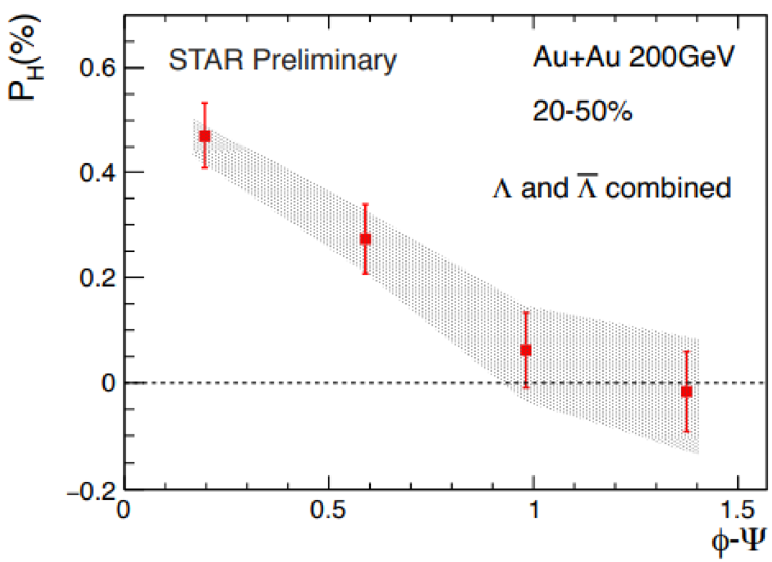


In-plane to out-of-plane difference

Global polarization also has an azimuthal-angle dependence.

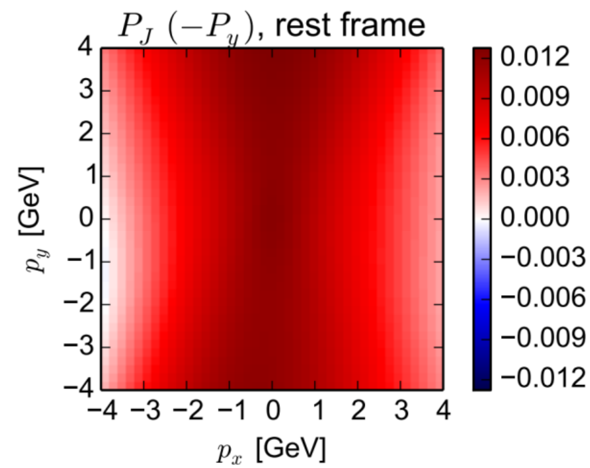
- **STAR data**: the spin polarization is stronger in the reaction plane
- **Model calculation**: the spin polarization is stronger out of plane

STAR, 1808.10482

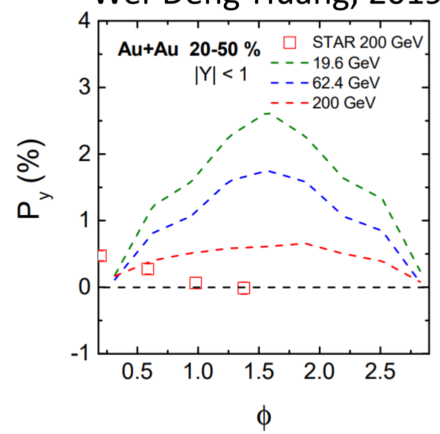


$$P_y = F_0 + F_2 \cos(2\phi)$$

Karpenko-Becattini, 2017

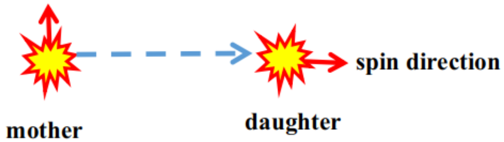


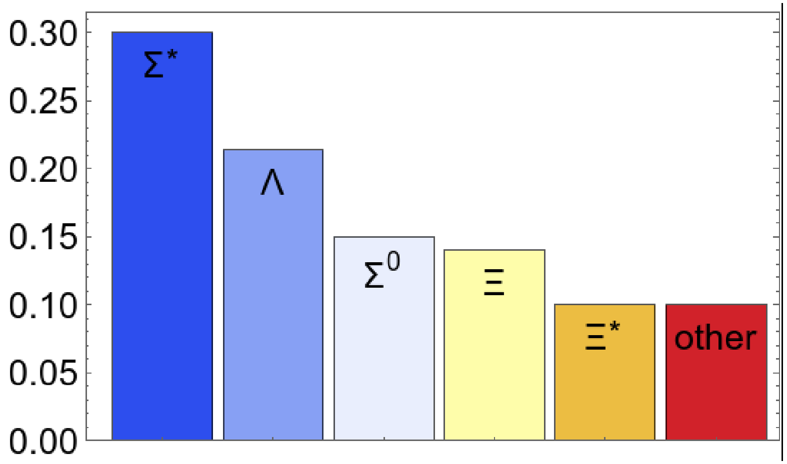
Wei-Deng-Huang, 2019



Motivations

- The spin polarization calculated in the above models is the polarization of primordial Λs, which are thermally produced.





- In experimental measurement, more than 75% Λs are produced by decay of heavier states.

- The spin transfer in decay should be considered!

Motivations

- For the global polarization, i.e. the mean value of spin polarization.

$$\langle \mathbf{P}_D \rangle = C\mathbf{P}_P$$

	spin and parity	$\langle \mathbf{P}_D \rangle / \mathbf{P}_P$
strong decay	$1/2^+ \to 1/2^+ 0^-$	-1/3
strong decay	$1/2^- \to 1/2^+ 0^-$	1
strong decay	$3/2^+ \to 1/2^+ 0^-$	1
strong decay	$3/2^- \to 1/2^+ 0^-$	-3/5
weak decay	$1/2 \quad \to 1/2 \quad 0$	$(2\gamma + 1)/3$
EM decay	$1/2^+ \to 1/2^+ 1^-$	-1/3

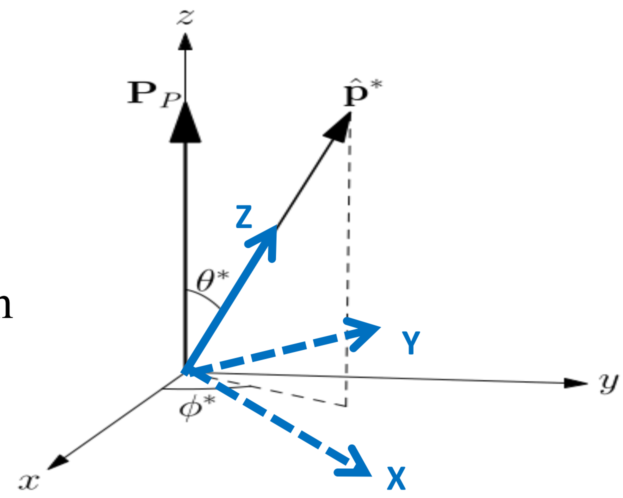
- The global polarization was estimated to be suppressed by a factor of 80%-95%, compared to the primordial polarization.

However, to study the azimuthal-angle dependence, linear equation does not apply.

F. Becittini, I. Karpenko, M. Lisa, I. Upsal, S. Voloshin (2017)

Two-body decay

Decay process $$P \to D + X$$



With given spin state of P, we want:
(1) angular distribution of D,
(2) polarization of D as a function of D's momentum direction.

Resonance Decay

In the helicity basis, the spin density matrix of D is related to that of P by:

$$\rho^f_{\lambda_D\lambda_X;\lambda'_D\lambda'_X}(\theta^*,\phi^*) = \sum_{M_P,M'_P} H_{\lambda_D\lambda_X;M_P}\,\rho^i_{M_P;M'_P}\,H^\dagger_{M'_P;\lambda'_D\lambda'_X}$$

$$H_{\lambda_D\lambda_X;M_P} = \sqrt{\frac{2S_P+1}{4\pi}}\,D^{S_P*}_{M_P;\lambda_D-\lambda_X}(\phi^*,\theta^*,0)A_{\lambda_D;\lambda_X}$$

$\Rightarrow$ Angular distribution

$$Tr(\rho^f) = \frac{1}{N}\frac{dN}{d\Omega^*}$$

Polarization vector

$$\mathbf{P}_D = tr_D(\hat{\mathbf{P}}\rho^D)/tr_D(\rho^D)$$

$$\rho^D = tr_X(\rho^f)$$

Ex1: Polarization vector and angular distribution

Strong decay $1/2^{\pm} \to 1/2^{+}0^{-}$

Parent density matrix:

$$\rho^{i}_{M_P;M'_P} = \mathrm{diag}\left(\frac{1+P_P}{2}, \frac{1-P_P}{2}\right)$$

Daughter density matrix:

$$\rho^{D}_{\lambda_D;\lambda'_D} = \frac{1}{4\pi}\begin{pmatrix} |A_{1/2}|^2(1+P_P\cos\theta^*) & -A_{1/2}A^*_{-1/2}P_P\sin\theta^* \\ -A^*_{1/2}A_{-1/2}P_P\sin\theta^* & |A_{-1/2}|^2(1-P_P\cos\theta^*) \end{pmatrix}$$

Since parity is conserved, amplitude is constrained by

$$A_{\lambda_D;\lambda_X} = \pi_P\pi_D\pi_X(-1)^{S_P-S_D-S_X}A_{-\lambda_D;-\lambda_X}$$

$$\rho^{D}_{\lambda_D;\lambda'_D} = \frac{1}{8\pi}\begin{pmatrix} 1+P_P\cos\theta^* & \pm P_P\sin\theta^* \\ \pm P_P\sin\theta^* & 1-P_P\cos\theta^* \end{pmatrix} \Longrightarrow \frac{1}{N}\frac{dN}{d\Omega^*} = \frac{1}{4\pi}$$

$$\begin{aligned} P_X(\theta^*,\phi^*) &= \pm P_P\sin\theta^*, \\ P_Y(\theta^*,\phi^*) &= 0, \\ P_Z(\theta^*,\phi^*) &= P_P\cos\theta^*. \end{aligned} \Longrightarrow$$

$$\mathbf{P}_D = 2(\mathbf{P}_P\cdot\hat{\mathbf{p}}^*)\hat{\mathbf{p}}^* - \mathbf{P}_P$$

$$\mathbf{P}_D = \mathbf{P}_P$$

Ex2: Polarization vector and angular distribution

Weak decay $1/2 \to 1/2\ 0$ $\Xi \to \Lambda\pi$

Parent density matrix:

$$\rho^{i}_{M_P;M'_P} = \mathrm{diag}\left(\frac{1+P_P}{2}, \frac{1-P_P}{2}\right)$$

Daughter density matrix:

$$\rho^{D}_{\lambda_D;\lambda'_D} = \frac{1}{4\pi}\begin{pmatrix} |A_{1/2}|^2\,(1+P_P\cos\theta^*) & -A_{1/2}A^*_{-1/2}\,P_P\sin\theta^* \\ -A^*_{1/2}A_{-1/2}\,P_P\sin\theta^* & |A_{-1/2}|^2\,(1-P_P\cos\theta^*) \end{pmatrix}$$

parity is not conserved, amplitude can be decomposed into parity-odd and parity-even terms:

$$A_{\pm 1/2} = \frac{A_s \pm A_p}{\sqrt{2\,(|A_s|^2+|A_p|^2)}}$$

$$\mathbf{P}_D = \frac{(\alpha+\mathbf{P}_P\cdot\hat{\mathbf{p}}^*)\,\hat{\mathbf{p}}^* + \beta\,(\mathbf{P}_P\times\hat{\mathbf{p}}^*) + \gamma\hat{\mathbf{p}}^*\times(\mathbf{P}_P\times\hat{\mathbf{p}}^*)}{1+\alpha\mathbf{P}_P\cdot\hat{\mathbf{p}}^*}$$

$$\frac{1}{N}\frac{dN}{d\Omega^*} = \frac{1}{4\pi}\,(1+\alpha P_P\cos\theta^*)$$

T.D. Lee and C.N. Yang 1957

weak decay parameter

$$\begin{cases} \alpha = \dfrac{2\mathrm{Re}(A_s^*A_p)}{|A_s|^2+|A_p|^2}, \\ \beta = \dfrac{2\mathrm{Im}(A_s^*A_p)}{|A_s|^2+|A_p|^2}, \\ \gamma = \dfrac{|A_s|^2-|A_p|^2}{|A_s|^2+|A_p|^2}. \end{cases}$$

- $\alpha = 0, \beta = 0, \gamma = 1 \Longrightarrow \mathbf{P}_D = 2(\mathbf{P}_P\cdot\hat{\mathbf{p}}^*)\hat{\mathbf{p}}^* - \mathbf{P}_P$
- $\alpha = 0, \beta = 0, \gamma = -1 \Longrightarrow \mathbf{P}_D = \mathbf{P}_P$

Polarization vector and angular distribution

For all decay channels, the polarization vector are obtained.

TABLE I. Daughter angular distribution and polarization vector $\mathbf{P}_D$ in different decay channels.

	Spin and parity	$(1/N)dN/d\Omega^*$	$\mathbf{P}_D$	$\langle \mathbf{P}_D \rangle / \mathbf{P}_P$
Strong decay	$1/2^+ \to 1/2^+ 0^-$	$1/(4\pi)$	$2(\mathbf{P}_P \cdot \hat{\mathbf{p}}^*)\hat{\mathbf{p}}^* - \mathbf{P}_P$	$-1/3$
Strong decay	$1/2^- \to 1/2^+ 0^-$	$1/(4\pi)$	$\mathbf{P}_P$	1
Strong decay	$3/2^+ \to 1/2^+ 0^-$	$3[1 - 2\Delta/3 - (1 - 2\Delta)\cos^2\theta^*]/(8\pi)$	Eq. (42)	1
Strong decay	$3/2^- \to 1/2^+ 0^-$	$3[1 - 2\Delta/3 - (1 - 2\Delta)\cos^2\theta^*]/(8\pi)$	Eq. (43)	$-3/5$
Weak decay	$1/2 \to 1/2\, 0$	$(1 + \alpha P_P \cos\theta^*)/(4\pi)$	Eq. (29)	$(2\gamma + 1)/3$
EM decay	$1/2^+ \to 1/2^+ 1^-$	$1/(4\pi)$	$-(\mathbf{P}_P \cdot \hat{\mathbf{p}}^*)\hat{\mathbf{p}}^*$	$-1/3$

$$\mathbf{P}_D = \frac{-4\delta(\mathbf{P}_P \cdot \hat{\mathbf{p}}^*)\hat{\mathbf{p}}^* + [1 - 2\delta - (1 - 10\delta)(\hat{\mathbf{P}}_P \cdot \hat{\mathbf{p}}^*)^2]\mathbf{P}_P}{1 - 2\Delta/3 - (1 - 2\Delta)(\hat{\mathbf{P}}_P \cdot \hat{\mathbf{p}}^*)^2} \qquad (42)$$

$$\mathbf{P}_D = \frac{2[1 - 4\delta - (1 - 10\delta)(\hat{\mathbf{P}}_P \cdot \hat{\mathbf{p}}^*)^2](\mathbf{P}_P \cdot \hat{\mathbf{p}}^*)\hat{\mathbf{p}}^* - [1 - 2\delta - (1 - 10\delta)(\hat{\mathbf{P}}_P \cdot \hat{\mathbf{p}}^*)^2]\mathbf{P}_P}{1 - 2\Delta/3 - (1 - 2\Delta)(\hat{\mathbf{P}}_P \cdot \hat{\mathbf{p}}^*)^2} \qquad (43)$$

$$\mathbf{P}_D = \frac{(\alpha + \mathbf{P}_P \cdot \hat{\mathbf{p}}^*)\hat{\mathbf{p}}^* + \beta(\mathbf{P}_P \times \hat{\mathbf{p}}^*) + \gamma \hat{\mathbf{p}}^* \times (\mathbf{P}_P \times \hat{\mathbf{p}}^*)}{1 + \alpha \mathbf{P}_P \cdot \hat{\mathbf{p}}^*} \qquad (29)$$

Use Monte Carlo simulation to estimate the feed down effect on the polarization.

See also: F. Becattini, G. Cao, E. Speranza, arXiv: 1905.03123

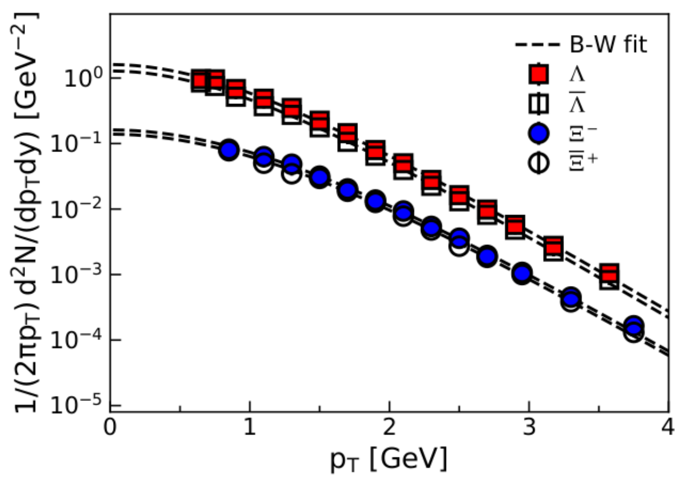
Transverse momentum spectrum and elliptic flow

- Primordial yield ratio are gennerated from THERMUS model
- p_T spectrum and v_2 are fitted to data, and then, are used to sample parent particles.

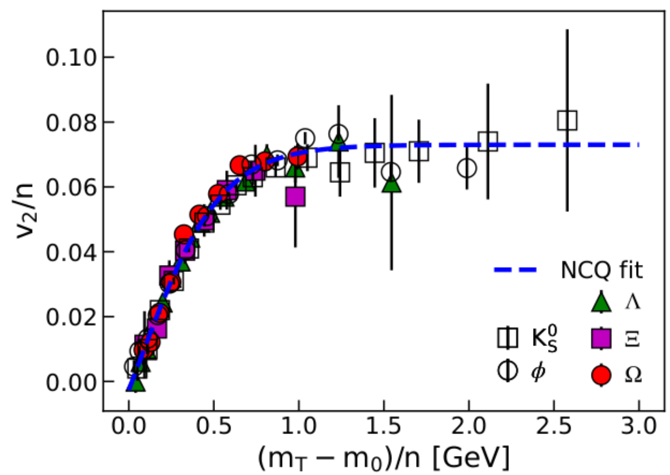
TABLE II. The primordial yield ratio N_i/N_Λ, spin, parity, and decay channels of strange particles.

	N_i/N_Λ	Spin and parity	Decay channel
Λ	1	$1/2^+$	
$\Lambda(1405)$	0.236	$1/2^-$	$\Sigma^0\pi$
$\Lambda(1520)$	0.265	$3/2^-$	$\Sigma^0\pi$
$\Lambda(1600)$	0.098	$1/2^+$	$\Sigma^0\pi$
$\Lambda(1670)$	0.061	$1/2^-$	$\Sigma^0\pi$, $\Lambda\eta$
$\Lambda(1690)$	0.112	$3/2^-$	$\Sigma^0\pi$
Σ^0	0.686	$1/2^+$	$\Lambda\gamma$
Σ^{*0}	0.533	$3/2^+$	$\Lambda\pi$
Σ^{*+}	0.535	$3/2^+$	$\Lambda\pi$, $\Sigma^0\pi$
Σ^{*-}	0.524	$3/2^+$	$\Lambda\pi$, $\Sigma^0\pi$
$\Sigma(1660)$	0.068	$1/2^+$	$\Lambda\pi$, $\Sigma^0\pi$
$\Sigma(1670)$	0.125	$3/2^-$	$\Lambda\pi$, $\Sigma^0\pi$
Ξ^0	0.343	$1/2^+$	$\Lambda\pi$
Ξ^-	0.332	$1/2^+$	$\Lambda\pi$
Ξ^{*0}	0.228	$3/2^+$	$\Xi\pi$
Ξ^{*-}	0.224	$3/2^+$	$\Xi\pi$

Data taken from STAR

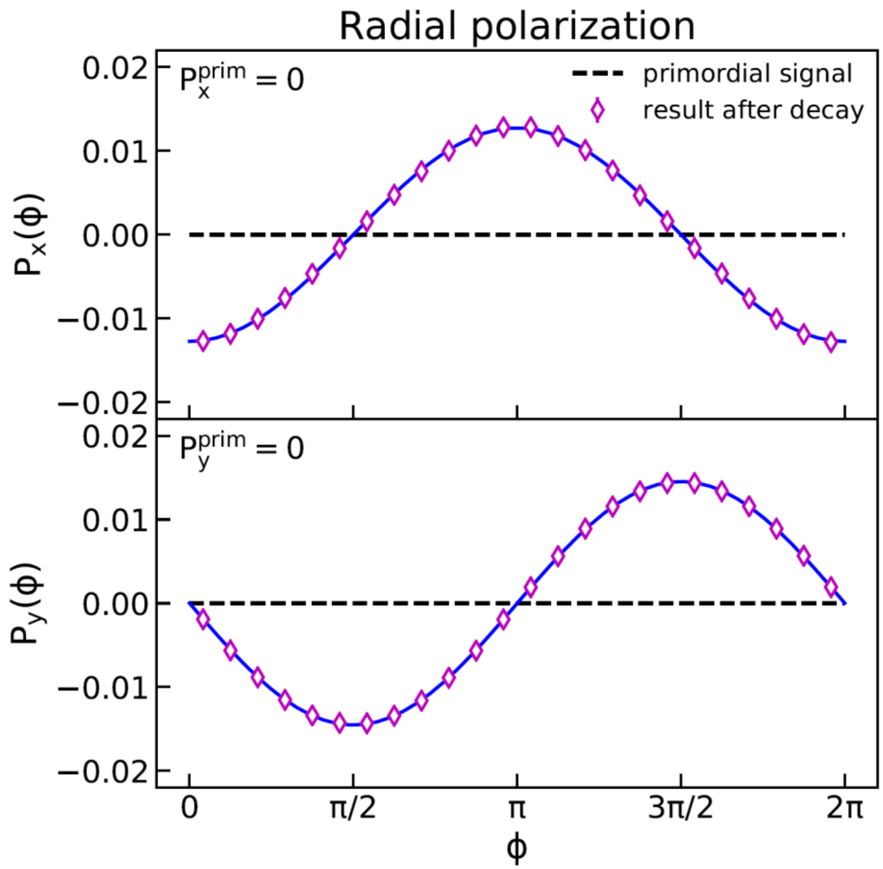


Data taken from STAR



Contribution from Ξ

- We first input zero polarization to parent particles



Weak decay can contribute a radial polarization.

$$P_x(\phi) = K_{1x}\cos\phi,$$
$$P_y(\phi) = K_{1y}\sin\phi\ ,$$

For $\mathbf{P}_P = 0$

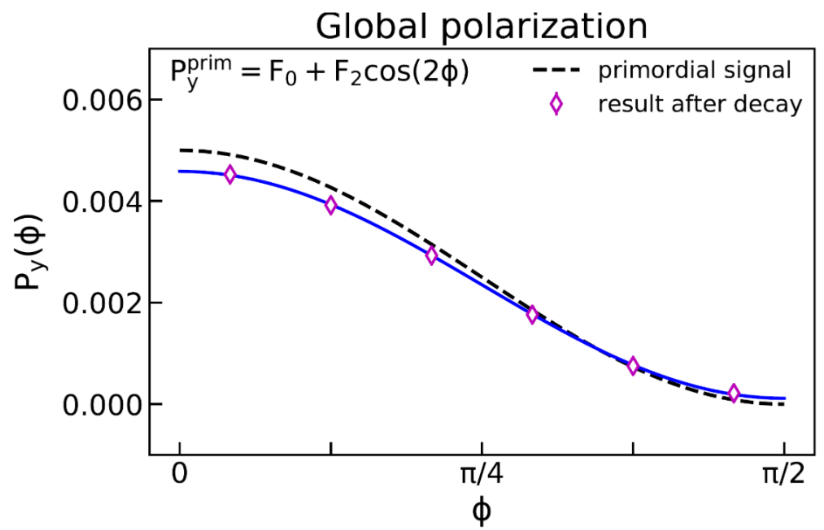
$$\mathbf{P}_D = \frac{(\alpha + \mathbf{P}_P \cdot \hat{\mathbf{p}}^*)\,\hat{\mathbf{p}}^* + \beta\,(\mathbf{P}_P \times \hat{\mathbf{p}}^*) + \gamma\hat{\mathbf{p}}^* \times (\mathbf{P}_P \times \hat{\mathbf{p}}^*)}{1 + \alpha\mathbf{P}_P \cdot \hat{\mathbf{p}}^*}$$

$$\Longrightarrow \quad \mathbf{P}_D = \alpha\hat{\mathbf{p}}^*$$

$$\alpha_\Xi < 0$$

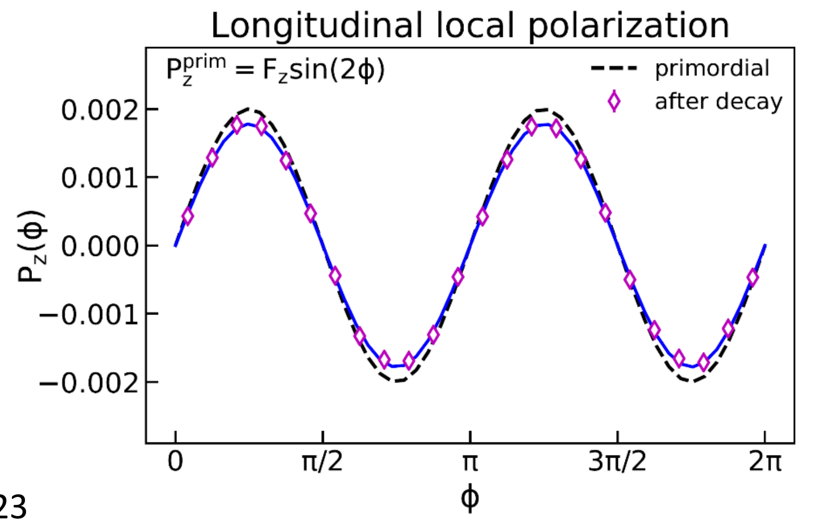
Simulation on the global polarization

- We then input non-zero polarization to parent particles
  - $P_y(\phi) = F_0 + F_2 \cos(2\phi)$
  - $P_z(\phi) = F_z \sin(2\phi)$



- Conclusion:

Feed-down decays suppress around 10% the primordial polarization, but it does not significant change the original harmonic behaviors.



See also: F. Becattini, G. Cao, E. Speranza, arXiv: 1905.03123

Summary

Spin polarization in heavy ion collisions:

- Global polarization---initial angular momentum
- Local polarization---the inhomogeneous expansion

Some puzzles exist in the present data and theoretical calculation.

Feed down effect on the Λ spin polarization is studied

- Spin transfer in decay can not flip the sign of the polarization signal.

- The Ξ weak decay can contribute a radial polarization.

Thanks for your attention!

Ensemble average of the spin vector

$$S^\mu(x,p) = -\frac{1}{8m}(1-n_F)\,\epsilon^{\mu\nu\rho\sigma}p_\nu\varpi_{\rho\sigma}(x)$$

Becattini, Piccinini, Ann. Phys.323 (2008) 2452
Becattini, Chandra, Zanna, Grossi, Annals Phys. 338, 32 (2013)
Becattini, Inghirami, Eur.Phys.J. C75 (2015) no.9, 406
Fang, Pang, Wang, Wang, PRC 94, 024904(2016)

- thermodynamic equilibrium
- Thermal vorticity: $\varpi_{\mu\upsilon} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu) \quad \beta_\mu = u_\mu/T.$

$$S^\mu = \frac{\int \frac{d^3p}{E}\int d\Sigma_\lambda p^\lambda\, f(x,p) S^\mu(x,p)}{\int \frac{d^3p}{E}\int d\Sigma_\lambda p^\lambda\, f(x,p)}.$$