Quark Mass Effect and Transient Field Effect on Chiral Transport

Lixin Yang

Sun Yat-Sen University

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S.Lin, L.X.Yang, 1810.02979, 1908.#

Outline

Quantify Mass Correction to CSE & ACVE

- CSE & ACVE in chiral limit

$$\vec{j}_5 = \frac{1}{2\pi^2}\mu e\vec{B} + \frac{1}{2\pi^2}\left[(\mu^2 + \mu_5^2) + \frac{\pi^2 T^2}{3}\right]\vec{\omega}$$

gravitational anomaly? Landsteiner, Megias, Pena-Benitez, PRL 2011

- Mass correction exists in CSE
  Metlitski, Zhitnitsky, PRD 2005; Gorbar, et al. PRD 2013; E.d.Guo, S.Lin, JHEP 2017 $\cdots$

- Aim: quantify mass correction to CSE & ACVE
  to what extent chiral limit is valid ?
  clearer view of μ independent part

Equal results from two independent methods

By axial anomaly equation & Kubo formulas

$$\sigma_B = \frac{1}{2\pi^2} e \int_0^\infty dq \Big(f_{FD}(E_q - \mu) - f_{FD}(E_q + \mu) \Big)$$

$$\sigma_V = \frac{1}{2\pi^2} \int_0^\infty \Big(f_{FD}(E_q - \mu) + f_{FD}(E_q + \mu) \Big) \frac{2q^2 + m^2}{E_q} dq$$

Zero m, T, μ Limits

Limits	$m=0$	$T=0$	$\mu=0$
σ_B/Ce	μ	$\sqrt{\mu^2-m^2}$	0
σ_V/C	$\mu^2+\frac{\pi^2 T^2}{3}$	$\mu\sqrt{\mu^2-m^2}$	$2\int_0^\infty \tilde{n}(E_q)\frac{2q^2+m^2}{E_q}dq$

For ACVE coefficient

$$\sigma_V(\mu=0)=2\int_0^\infty \tilde{n}(E_q)\frac{2q^2+m^2}{E_q}dq$$

Origin? gravitational anomaly $\rightarrow \frac{\pi^2 T^2}{3} \sim \sigma_V(\mu=0)$

Small m expansion at finite T & μ

Mass correction $\propto m^2$ only, $m^2 \ln m$ is vanishing

$$\sigma_V/C = \frac{1}{2\pi^2}\left(\mu^2 + \frac{\pi^2 T^2}{3} - \frac{m^2}{2}\right) + O(m^4)$$ agree with Flachi, Fukushima, PRD 2018

$$\sigma_B/Ce = \mu + \frac{m^2}{2T}\frac{\partial}{\partial s}\left(\mathrm{Li}_s(-e^{\mu/T}) - \mathrm{Li}_s(-e^{-\mu/T})\right)\Big|_{s=-1} + O(m^4)$$

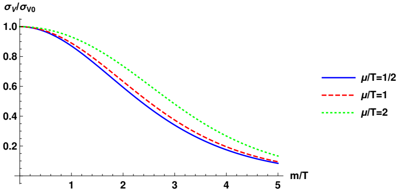
Radiative corrections to CSE Gorbar, Miransky, Shovkovy, X.Wang, PRD 2013

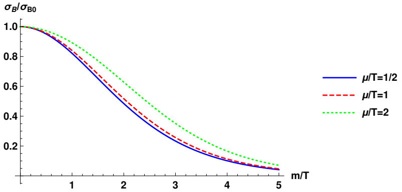
$$\langle j_5^3\rangle_\alpha = -\frac{\alpha e B\mu}{2\pi^3}\left(\ln\frac{2\mu}{m} + \ln\frac{m_\gamma^2}{m^2} + \frac{4}{3}\right) - \frac{\alpha e B m^2}{2\pi^3\mu}\left(\ln\frac{2^{3/2}\mu}{m_\gamma} - \frac{11}{12}\right)$$

In weakly interacting massive theory at $T = 0$

Medium dependent part only $\rightarrow$
Possible logarithmic corrections from vacuum contributions ?

Plot general m-dependence & Generalization to QCD





$$\sigma_V = \sum_f CN_c \int_0^\infty dq \tilde{f}_+(\sqrt{q^2+m_f^2}) \frac{2q^2+m_f^2}{E_q},$$

$$\sigma_B = \sum_f CeN_c \int_0^\infty dq \tilde{f}_-(\sqrt{q^2+m_f^2})$$

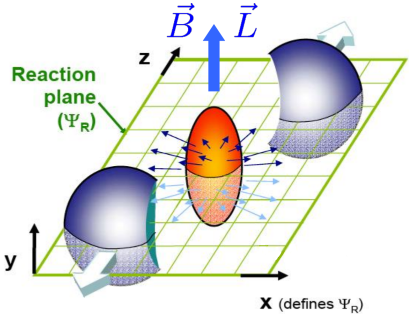
$m_u = m_d \simeq 0$, $m_s = 100\mathrm{MeV} \rightarrow$ Mass correction to σ_B & $\sigma_V < 1\%$
with $50\mathrm{MeV} < \mu < 200\mathrm{MeV}$ & $200\mathrm{MeV} < T < 400\mathrm{MeV}$

Summary for mass effect

- Mass suppresses CSE & ACVE coefficients with less suppression at larger μ
- For phenomenologically relevant case of QGP, the correction is negligible

Frequency dependent conductivity

Strong & Rapid decaying B $\xrightarrow{-\partial_t \vec{B} = \nabla \times \vec{E}}$ Rapid changing E
$\rightarrow$ Frequency dependent conductivity



Noncentral heavy ion collisions

CKT with Landau level basis

- Wigner function at homogeneous B for right-handed chiral fermion

$$W(X,P)=\int \frac{d^4X'}{(2\pi)^4}e^{-iP\cdot X'}\left\langle \psi(X-\frac{1}{2}X')U(A,X+\frac{1}{2}X',X-\frac{1}{2}X')\psi^\dagger(X+\frac{1}{2}X')\right\rangle$$
$$=F\sigma^0+j_i\sigma^i$$

- CKT with LL basis

$$\begin{array}{ccccc}
\text{Wigner transformation} & \leftarrow & \text{Dirac equations} & \rightarrow & \text{eigenvalue\&state} \\
\downarrow & & & & \downarrow \\
\end{array}$$

$$(\Delta_\mu-2ip_\mu)\,\sigma^\mu W=0$$
$$(\Delta_\mu+2ip_\mu)\,W\sigma^\mu=0$$
$$\downarrow$$
$$\Delta_0 F+\Delta_i j_i=0 \quad \text{Transport eqns}$$
$$\Delta_i F+\Delta_0 j_i-2\epsilon^{ijk}p_j j_k=0$$
$$P_0F-p_ij_i=0 \quad \text{Constraint eqns}$$
$$\frac{1}{2}\epsilon^{ijk}\Delta_j j_k+p_iF-P_0j_i=0$$

Landau level basis

$$E^{(n)}_{p_z s},\ \psi^{(n)}_{rs}(t,\vec{r})$$
$$\downarrow$$
$$W=\sum_{n,s}\{f_{FD}(E^{(n)}_{p_z s}-\mu)\delta(p_0+\mu-E^{(n)}_{p_z s})W^{(n)}_{+,s}(\vec{p})$$
$$+\left[1-f_{FD}(E^{(n)}_{p_z s}+\mu)\right]\delta(p_0+\mu+E^{(n)}_{p_z s})W^{(n)}_{-,s}(\vec{p})\}$$

X.L.Sheng, Rischke, Vasak, Q.Wang, EPJA 2018

Wigner function in lowest Landau level(LLL)

- Wigner function LLL, medium dependent part

$$W = \sum_{r=\pm} r f_{FD}(E_{p_3} - r\mu)\delta(P_0 - rE_{p_3}) \frac{E_{p_3} + rp_3}{(2\pi)^3 E_{p_3}} \exp\left(-\frac{p_T^2}{eB}\right) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

where $r = \pm$ for particle/anti-particle $\Rightarrow \quad E_{p_3} = rp_3$

- Components of Wigner function $W = F\sigma^0 + j_i\sigma^i \quad \Rightarrow$

$$(F, j_i) = (1, 0, 0, 1)\frac{1}{(2\pi)^3}\exp\left(-\frac{p_T^2}{eB}\right)\sum_{r=\pm}\frac{r\delta(P_0 - p_3)\theta(rp_3)}{e^{r\beta(p_3-\mu)} + 1}$$

$\downarrow \qquad$ CKT for components

$$\Delta_0 F + \Delta_i j_i = 0, \qquad \Delta_i F + \Delta_0 j_i - 2\epsilon^{ijk} p_j j_k = 0$$

$$P_0 F - p_i j_i = 0, \qquad \frac{1}{2}\epsilon^{ijk}\Delta_j j_k + p_i F - P_0 j_i = 0$$

Perturbative Wigner function & RTA

- Perturbative homogeneous $E \perp B$ where $B \gg T\&\mu$ and $E \propto e^{i\omega t}$
- LL off-shell: $F \to F + \delta F$, $j_i \to j_i + \delta j_i$ with $\delta F \sim \delta j_i \sim \mathcal{O}(E)$

$$\Delta_0 F + \Delta_i j_i = -\frac{\delta F}{\tau}, \qquad \Delta_i F + \Delta_0 j_i - 2\epsilon^{ijk} p_j j_k = -\frac{\delta j_i}{\tau}$$

$$\delta j_{1,2} \propto \exp\left(-\frac{p_T^2}{eB}\right) \quad \to \quad \delta F \,\&\, \delta j_3 \text{ odd in } p_T \quad \to \quad \delta\rho = \delta J_3 = 0$$

where $\tau_\omega \equiv \frac{\tau}{1-i\omega\tau}$ and τ is from Relaxation Time Approximation

- Electric charge & current density

$$\delta\rho = 2\int d^4p\,\delta F, \qquad \delta J_i = 2\int d^4p\,\delta j_i \qquad \Rightarrow$$

$$\delta\vec{J} \overset{B\gg p^2}{=\!=\!=} \frac{e\vec{E}}{8\pi^2\tau_\omega}\left[1 + \frac{1}{|e\vec{B}|}\left(\frac{\mu^2}{2} + \frac{\pi^2 T^2}{6} - \frac{1}{2\tau_\omega^2}\right)\right] - \frac{e^2\mu\vec{B}\times\vec{E}}{4\pi^2|e\vec{B}|}$$

qualitatively agree with $\mathcal{O}(\hbar B)$ CKT with Berry curvature Gorbar, Shovkovy, Vilchinskii, Rudenok, Boyarsky, Ruchayskiy, PRD 2016

Transverse & Hall conductivity

$$\sigma_\perp = \frac{e}{8\pi^2\tau_\omega}\left[1+\frac{1}{|e\vec{B}|}\left(\frac{\mu^2}{2}+\frac{\pi^2 T^2}{6}-\frac{1}{2\tau_\omega^2}\right)\right]$$

$$\sigma_H = -\frac{e^2\mu}{4\pi^2}$$

Generalize to QCD with an overall factor $N_c \sum_f q_f^2$

- $\sigma_\perp \propto \frac{1}{\tau}$ in contrast to $\sigma_\| \propto \tau$
- High ω enhancement of $\sigma_\perp$
- For large B, $\sigma_\perp \sim \mathcal{O}(B^0)$
  qualitatively agree with holographic study W.Li, S.Lin, J.J.Mei, PRD 2018
- HIC: $\frac{eB}{T^2} \lesssim 1.6$ $\rightarrow$ higher LL to be included talk by S.Lin

Summary & Outlook

- Electric conductivity in CKT with LLL & RTA
- Transverse conductivity is inversely proportional to τ
- High ω enhancement of transverse conductivity
- For large B, transverse conductivity approaches a constant
- Outlook: Contribution from higher LL

Thank you!

Backup - Frequency sum & Dirac trace

- Dirac trace

$$\epsilon_{ijk}Tr[\gamma_\mu\gamma^i\gamma_5\gamma_\nu\gamma^0]a^\mu b^\nu = 4i(a_j b_k - a_k b_j)$$
$$\epsilon_{ijk}Tr[\gamma_\mu\gamma^i\gamma_5\gamma_\nu\gamma^j]a^\mu b^\nu = 8i(a^0 b^k - a^k b^0) = 8i(a_k b_0 - a_0 b_k)$$

- Sum over fermionic Matsubara frequencies

$$\frac{1}{\beta}\sum_{\tilde{\omega}_m}\Delta_u(Q)\Delta_v(P+Q) = \frac{u\,\tilde{n}(E_q - u\mu) - v\,\tilde{n}(E_{p+q} - v\mu) + \frac{1}{2}(v-u)}{i\omega_n + uE_q - vE_{p+q}}$$
$$\frac{1}{\beta}\sum_{\tilde{\omega}_m}\Delta_u(Q)\Delta_v(P+Q)i\tilde{\omega}_m = \frac{(u\,E_q - \mu)(\frac{1+u}{2} - u\tilde{n}(E_q - u\mu))}{i\omega_n + uE_q - vE_{p+q}} - \frac{(v\,E_{p+q} - \mu - i\omega_n)(\frac{1+v}{2} - v\tilde{n}(E_{p+q} - vi\omega_n - v\mu))}{i\omega_n + uE_q - vE_{p+q}}$$

Backup - Linear response in a restricted setting

- Static inhomogeneous sources (gauge fields & metric perturbation)

$$u^\mu = (1,0,0,0) \Rightarrow B^i = -\epsilon^{ijk}\partial_j A_k,\ \omega^i = -\frac{1}{2}\epsilon^{ijk}\nabla_j u_k = -\frac{1}{2}\epsilon^{ijk}\partial_j h_{0k}$$

- Absence of pseudoscalar condensate

$$\begin{matrix}\text{Vanishing}\\ \text{mass term}\end{matrix} \xleftarrow[\nabla T=0]{\nabla\mu=0} \begin{cases}\text{symmetries: P-odd, T-odd, C-even}\\ \text{possibile forms: } \nabla\mu\cdot\vec{B},\ \mu\nabla\mu\cdot\vec{\omega},\ T\nabla T\cdot\vec{\omega}\end{cases}$$

- Constitutive equation for axial current

$$j_5^i(x) = -\sigma_B\epsilon^{ijk}\partial_j A_k(x) - \frac{1}{2}\sigma_V\epsilon^{ijk}\partial_j h_{0k}(x)$$

$$\xrightarrow[\text{transform}]{\text{Fourier}} j_5^i(p) = -\sigma_B\epsilon^{ijk}ip_jA_k(p) - \frac{1}{2}\sigma_V\epsilon^{ijk}ip_jh_{0k}(p)$$

Backup-Perturbative Wigner function & RTA

- Background field $B^i \equiv (0,0,B) = -\frac{1}{2}\epsilon^{ijk}F_{jk}$
  Perturbative field $E^i \equiv (E,0,0) = F_{0i} \propto e^{-i\omega t}$
- $F \to F + \delta F,\ j_i \to j_i + \delta j_i$ with $\delta F \sim \delta j_i \sim \mathcal{O}(E)$

$$\Delta_0 F + \Delta_i j_i = -\frac{\delta F}{\tau}, \qquad \Delta_i F + \Delta_0 j_i - 2\epsilon^{ijk} p_j j_k = -\frac{\delta j_i}{\tau}$$

$$\Rightarrow \quad \delta j_1 = eE\frac{2p_3\tau_\omega F - \left(2eB\tau_\omega + \frac{1}{\tau_\omega}\right)\frac{\partial F}{\partial p_0}}{\left(2eB\tau_\omega + \frac{1}{\tau_\omega}\right)^2 + 2p_3^2} \qquad \delta F\,\&\,\delta j_3 \text{ odd in } p_T$$

$$\delta j_2 = -eE\frac{p_3\frac{\partial F}{\partial p_0} + 2\left(2eB\tau_\omega + \frac{1}{\tau_\omega}\right)\tau_\omega F}{\left(2eB\tau_\omega + \frac{1}{\tau_\omega}\right)^2 + 2p_3^2} \qquad p_T \equiv (p_1, p_2)$$

where $\tau_\omega \equiv \frac{\tau}{1-i\omega\tau}$ and we've taken Relaxation Time Approximation (RTA) and $\delta j_{1,2} \propto \exp\left(-\frac{p_T^2}{eB}\right)$ to obtain specific solutions.

CSE & ACVE coefficients in massless case

- Axial anomaly equation in massless QED

$$\partial_\mu j_5^\mu = -Ce^2 E\cdot B \leftarrow \nabla_\mu j_A^\mu = \epsilon^{\mu\nu\rho\lambda}\left(\frac{d_{ABC}}{32\pi^2}F^B_{\mu\nu}F^C_{\rho\lambda} + \frac{b_A}{768\pi^2}\mathcal{R}^{\alpha}{}_{\beta\mu\nu}\mathcal{R}^{\beta}{}_{\alpha\rho\lambda}\right)$$

with fluid velocity u^μ, external fields

$$E^\mu = F^{\mu\nu}u_\nu, \quad B^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}u_\nu F_{\rho\sigma}, \quad \omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}u_\nu \nabla_\rho u_\sigma$$

- Divergence form in static case $\rightarrow$ CSE & ACVE coefficients

$$e\vec{E} = -\nabla\mu,\ \nabla\cdot\vec{B} = 2\vec{E}\cdot\vec{\omega},\ \nabla\cdot\vec{\omega} = 0,\ \nabla T = 0 \quad \text{in medium field}$$

$$\xrightarrow[\text{by parts}]{\text{integration}} \frac{1}{C}\nabla\cdot\vec{j}_5 = \nabla\cdot\left(e\mu\vec{B}\right) + \nabla\cdot\left(\mu^2\vec{\omega} + \# T^2\vec{\omega}\right) = \nabla\cdot\left(\sigma_B\vec{B} + \sigma_V\vec{\omega}\right)$$

Wigner function from Landau level basis(LLB)

- Background field $B^i \equiv (0,0,B) = -\frac{1}{2}\epsilon^{ijk}F_{jk}$
- LLB from Dirac equations

$$E^{(0)}_{p_z} = \sqrt{m^2 + (p_z - \mu_5)^2}, \quad E^{(n)}_{p_z s} = \sqrt{m^2 + \left(\sqrt{p_z^2 + 2neB} - s\mu_5\right)^2}$$

$$\psi^{(0)}_r(t,\vec{r}) = \exp[-irE^{(0)}_{p_z}t + i\mu t + ip_x x + ip_z z]\xi^{(0)}_r(p_x,p_z,y)$$

$$\psi^{(n)}_{rs}(t,\vec{r}) = \exp[-irE^{(n)}_{p_z s}t + i\mu t + ip_x x + ip_z z]\xi^{(0)}_{rs}(p_x,p_z,y)$$

- Wigner function from LLB

$$W(P) = \sum_{n,s}\{f_{FD}(E^{(n)}_{p_z s} - \mu)\delta(p_0 + \mu - E^{(n)}_{p_z s})W^{(n)}_{+,s}(\vec{p}) + \left[1 - f_{FD}(E^{(n)}_{p_z s} + \mu)\right]\delta(p_0 + \mu + E^{(n)}_{p_z s})W^{(n)}_{-,s}(\vec{p})\}$$

X.L.Sheng, Rischke, Vasak, Q.Wang, EPJA 2018