



三峡大学  
CHINA THREE GORGES UNIVERSITY

# Chiral Electromagnetic Current and Charge Separation in Relativistic Heavy-Ion Collisions

1. Introductions
2. The Magnetic Fields in Relativistic HIC
3. Chiral Electromagnetic Current in Relativistic HIC
4. Charge Separation in Relativistic HIC

Sheng-Qin Feng (冯笙琴)      Three Gorges University (三峡大学)

Collab. with: X. Ai (艾鑫), B.-X. Chen(陈帮祥), Y. J. Mo (莫玉俊), D. She (佘端) and Y. Zhong (钟洋)

The 13th Workshop on QCD Phase Transitions and Relativistic HIC, Enshi, August 16-20, 2019

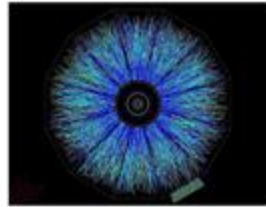
# 1. Introduction

The 13th Workshop on QCD Phase Transitions and Relativistic HIC, Enshi, August 16-20, 2019

D. Kharzeev, L. D. McLerran, H. J. Warringa, NPA 803, 227 (2008)

# Strong

Strong matter produced  
in heavy ion collisions



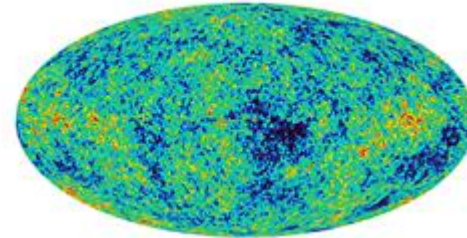
Topological charge changing transitions

induce difference between number of left- and right-handed fermions

Parity to be violated locally in microscopic domains in QCD at finite temperature

# ElectroWeak Matter

Electroweak matter produced  
in the early universe



Topological charge changing transitions

induce nonzero baryon + lepton number

Parity to be violated globally of weak interactions of the standard model

At high temperatures these transitions are unsuppressed (Sphalerons)  
Manton ('83), Manton and Klinkhamer ('84), McLerran and Shaposhnikov ('85)

How to observe topological charge changing transitions in hot quark matter?

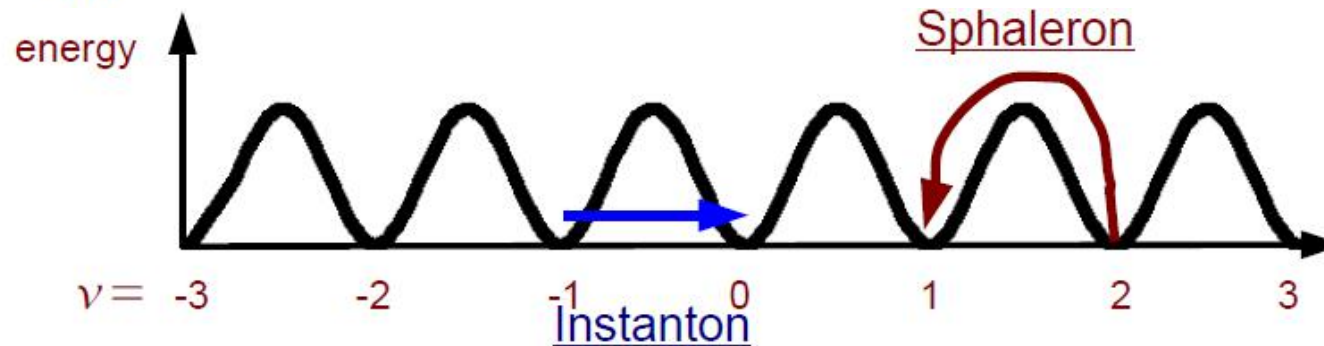
An asymmetry between matter and antimatter is observed  
Kuzmin, Shaposhnikov ('85)

Kharzeev, McLerran, Warringa, ('08)

# Instantons and Sphaleron

$$Q_w = \frac{g^2}{8\pi^2} \int d^4x \vec{E}_a \cdot \vec{B}_a = 0, \pm 1, \pm 2, \dots$$

Stable under smooth deformations  
Change topological charge vacuum



**Instantons:** Configuration with finite action. **Tunneling through barrier**

**Suppression of rate at T=0,** 't Hooft ('76), Pisarski and Yaffe ('80)

Sphaleron: Configuration with finite energy. Go over barrier.

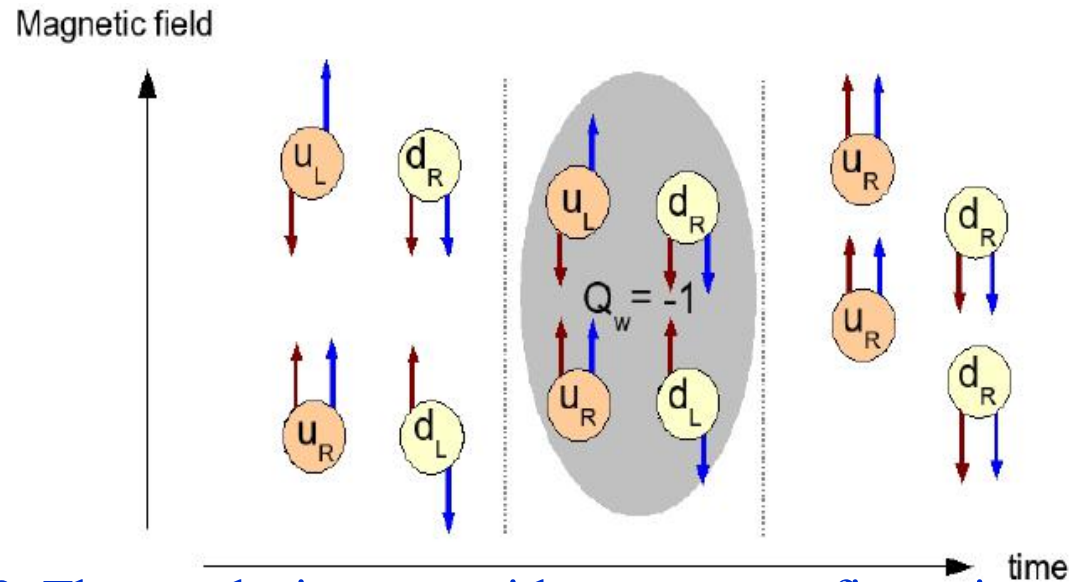
Only possible at finite temperature, rate not suppressed, look for it in QGP!

Manton ('83), Manton and Klinkhamer ('84), McLerran, Mottola and Shaposhnikov ('88)

$$\frac{d N_t^\pm}{d^3x d t} \sim 385 \alpha_s^5 T^4$$

Bödeker, Moore and Rummukainen ('00),  
several transitions per fm<sup>-3</sup> per fm/c

# The Chiral Magnetic Effect (CME)



1. Due to very large magnetic field, the up and down quarks in the lowest Landau level can only move along the direction of the magnetic field. Initially there are as many left-handed as right-handed quarks.

2. The quarks interact with a gauge configuration with non-zero  $Q_w$ . Assuming  $Q_w = -1$ , this will convert a left-handed up/down quark into a right-handed up/down quark by reversing the direction of momentum.
3. The right-handed up quarks will move upwards, the right-handed down quarks will move downwards. A charge of  $q = 2e$  will be created between two sides of a plane perpendicular to the magnetic fields.

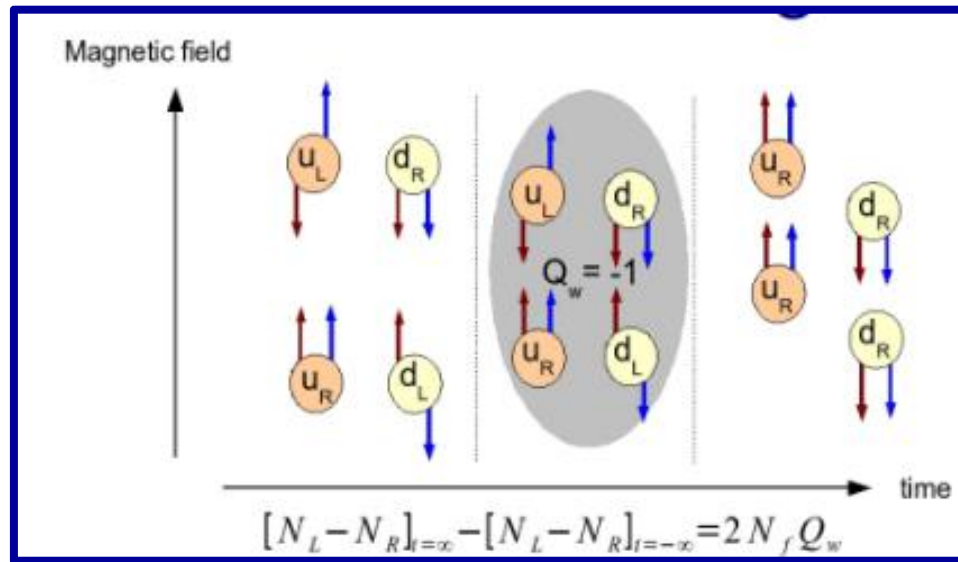
**In finite volume this causes separation of positive from negative charge**

**In presence of magnetic field, this induces a chiral electromagnetic current**

D. Kharzeev, L. D. McLerran, H. J. Warringa, NPA 803, 227 (2008)

The 13th Workshop on QCD Phase Transitions and Relativistic HIC, Enshi, August 16-20, 2019

# The Chiral Magnetic Effect



Charge difference:

$$Q = 2 Q_w \sum_f |q_f|$$

Same sign for  
antiparticles!

Topological charge changing transitions induces chirality

In finite volume this causes separation of positive from negative charge

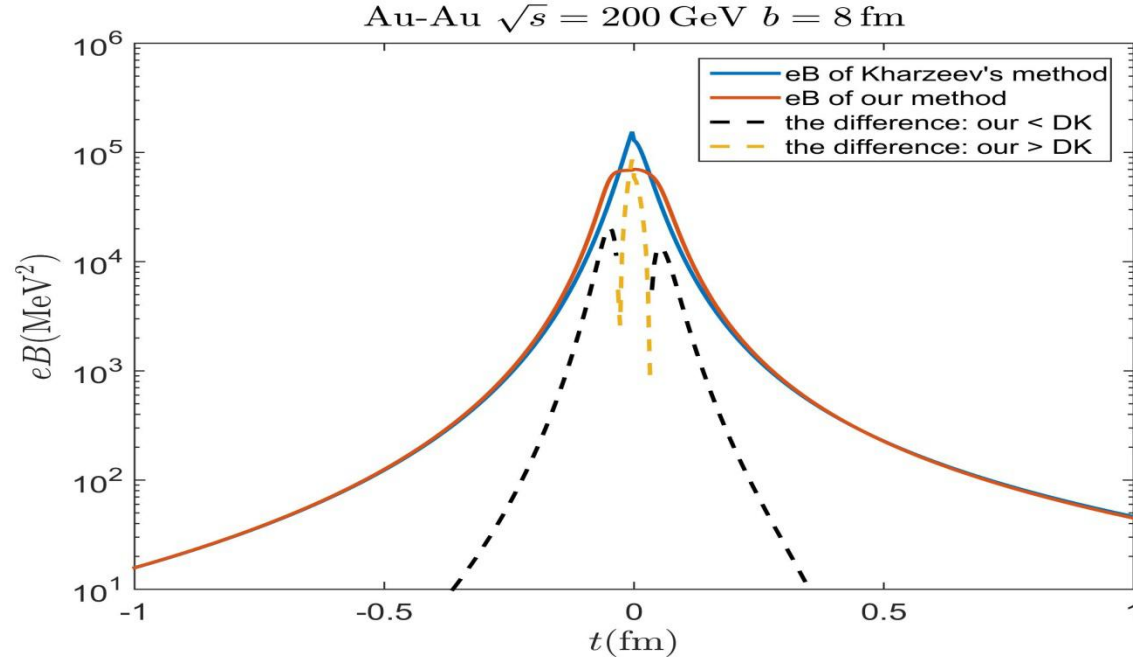
Reasonable polarization of quarks requires:  $e B \sim \frac{1}{\rho^2} \sim \alpha_s^2 T^2 \sim 10^3 - 10^4 \text{ MeV}^2$

D. Kharzeev, L. D. McLerran, H. J. Warringa, NPA 803, 227 (2008)

## 2. Magnetic Field in HIC



# Consider the thickness of the Lorentz contraction of the collision nuclear in the Z directions



KMW's model --**pancake approximation**( $z \rightarrow 0$ ), we consider the thickness in the  $z$  direction

RHIC@BNL

$$eB(\tau = 0.2 \text{ fm}) = 10^3 \sim 10^4 \text{ MeV}^2 \sim 10^{17} \text{ G}$$

Reasonable polarization of quarks requires:  $eB \sim 1/\rho^2 \sim 10^3 - 10^4 \text{ MeV}^2$

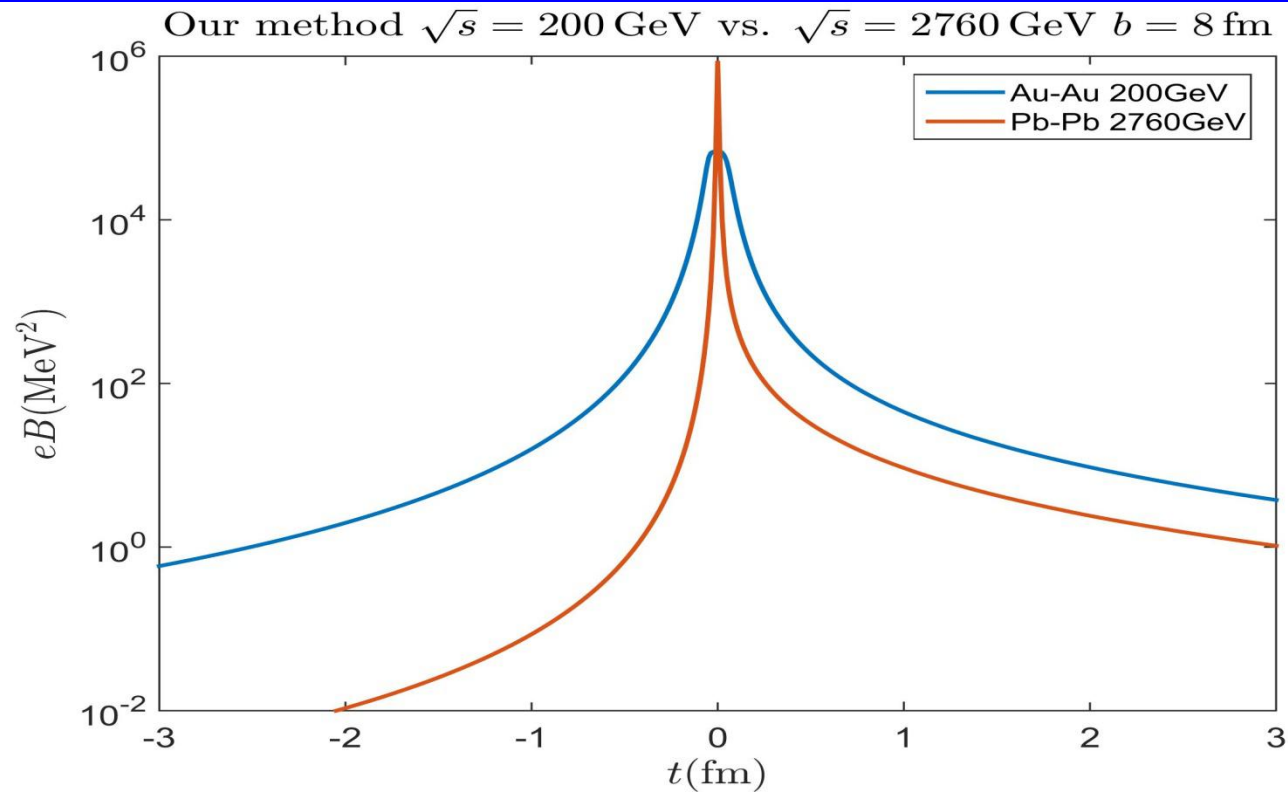
Y. -J. Mo, S. -Q. Feng and Y. -F. Shi, Phys. Rev. C 024901 (2013);

Y. Zhong, C.-B. Yang, X. Cai and S. -Q. Feng, Adv. High Energy Phys. 2014 (2014) 193039

D. Kharzeev, L. D. McLerran, H. J. Warringa, NPA 803, 227 (2008)



# Comparison of Magnetic fields of RHIC with LHC



At LHC, magnetic fields falls off rapidly with time than that at RHIC, **Chiral Magnetic Effect Is early time dynamics**

$$eB(\tau = 0.2\text{fm}) = 10^3 \sim 10^4 \text{ MeV}^2 \sim 10^{17} \text{ G} \quad (200\text{GeV})$$

$$eB(\tau = 0.2\text{fm}) = 10^2 \sim 10^3 \text{ MeV}^2 \sim 10^{16} \text{ G} \quad (2760\text{GeV})$$

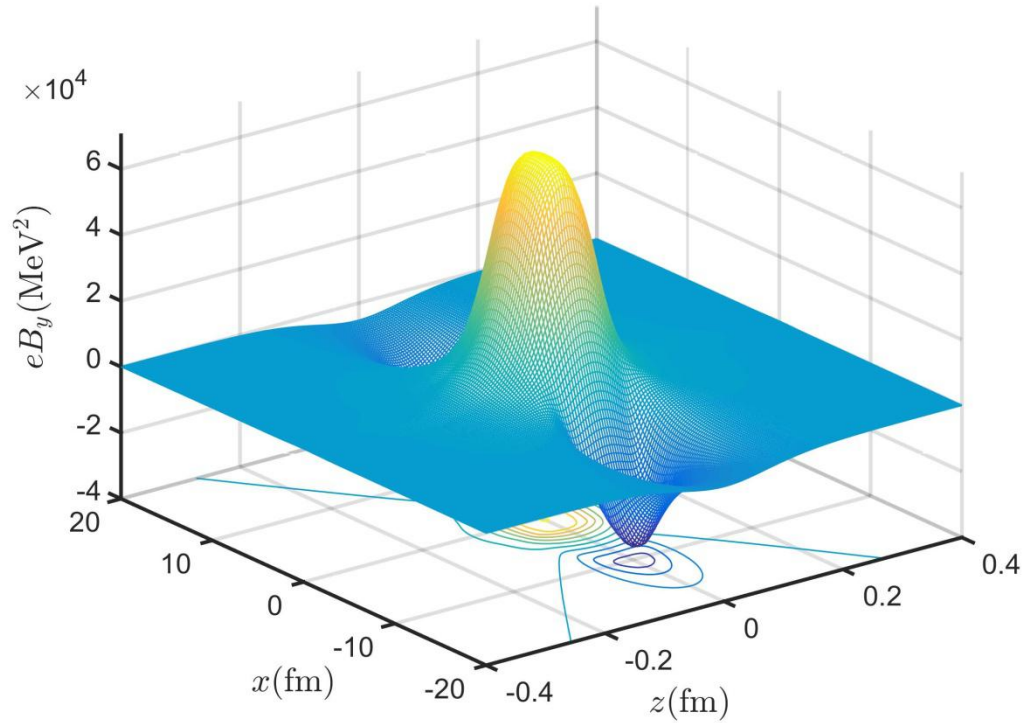
Low energy quarks which are produced in early stages will be polarized in the direction perpendicular to reaction plane

Y. -J. Mo, S. -Q. Feng and Y. -F. Shi, Phys. Rev. C 024901 (2013);

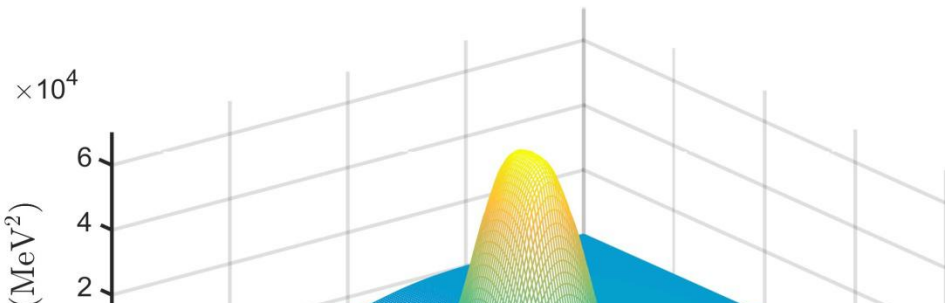
Y. Zhong, C. -B. Yang, X. Cai and S. -Q. Feng, Adv. High Energy Phys. 2014 (2014) 193039

# Magnetic field distributes with space and time

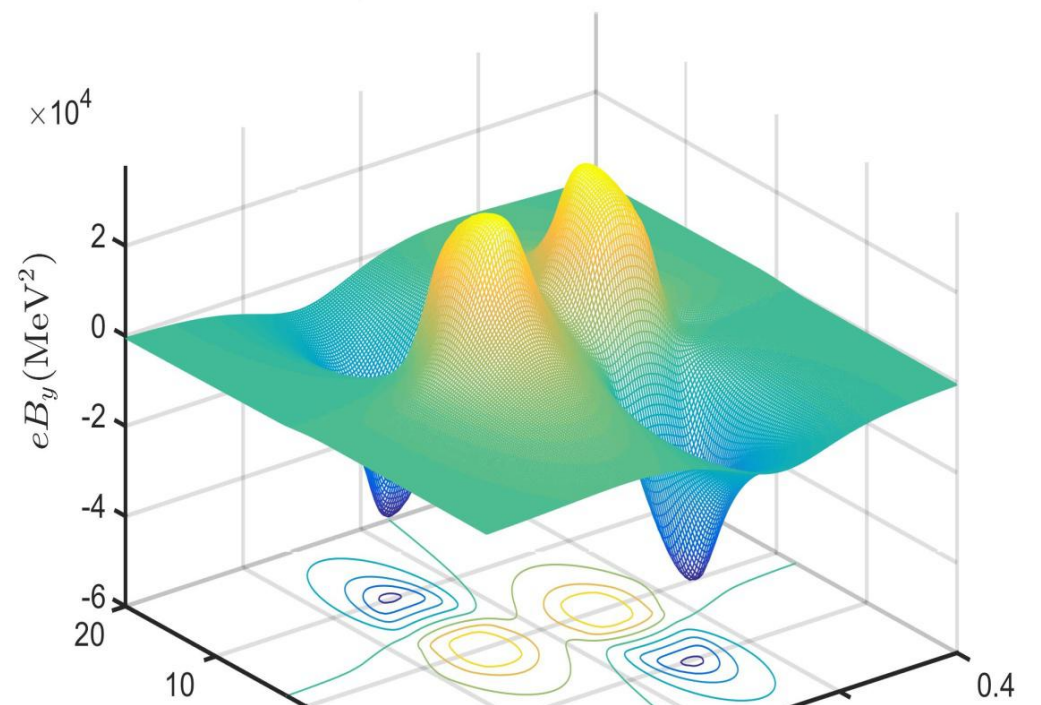
Au-Au  $\sqrt{s} = 200$  GeV  $b = 8$  fm  $t = 0.001$  fm



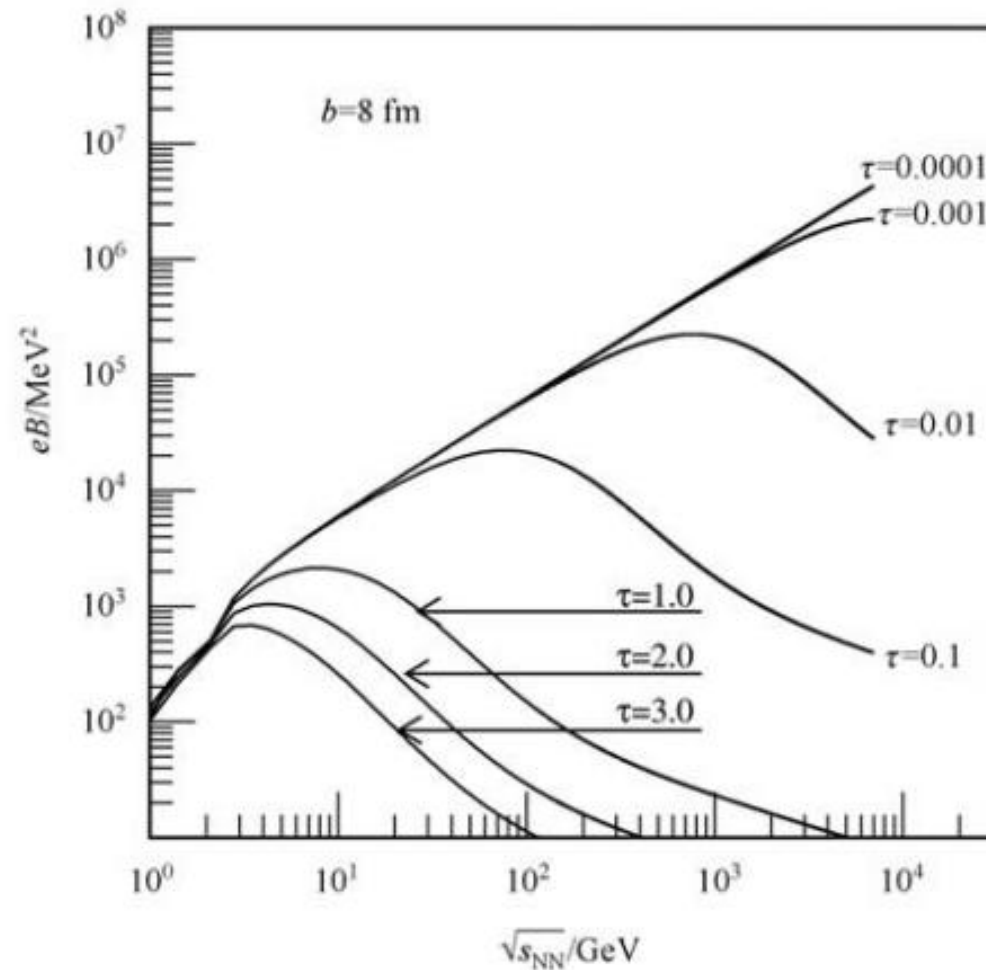
Au-Au  $\sqrt{s} = 200$  GeV  $b = 8$  fm  $t = 0.01$  fm



Au-Au  $\sqrt{s} = 200$  GeV  $b = 8$  fm  $t = 0.1$  fm



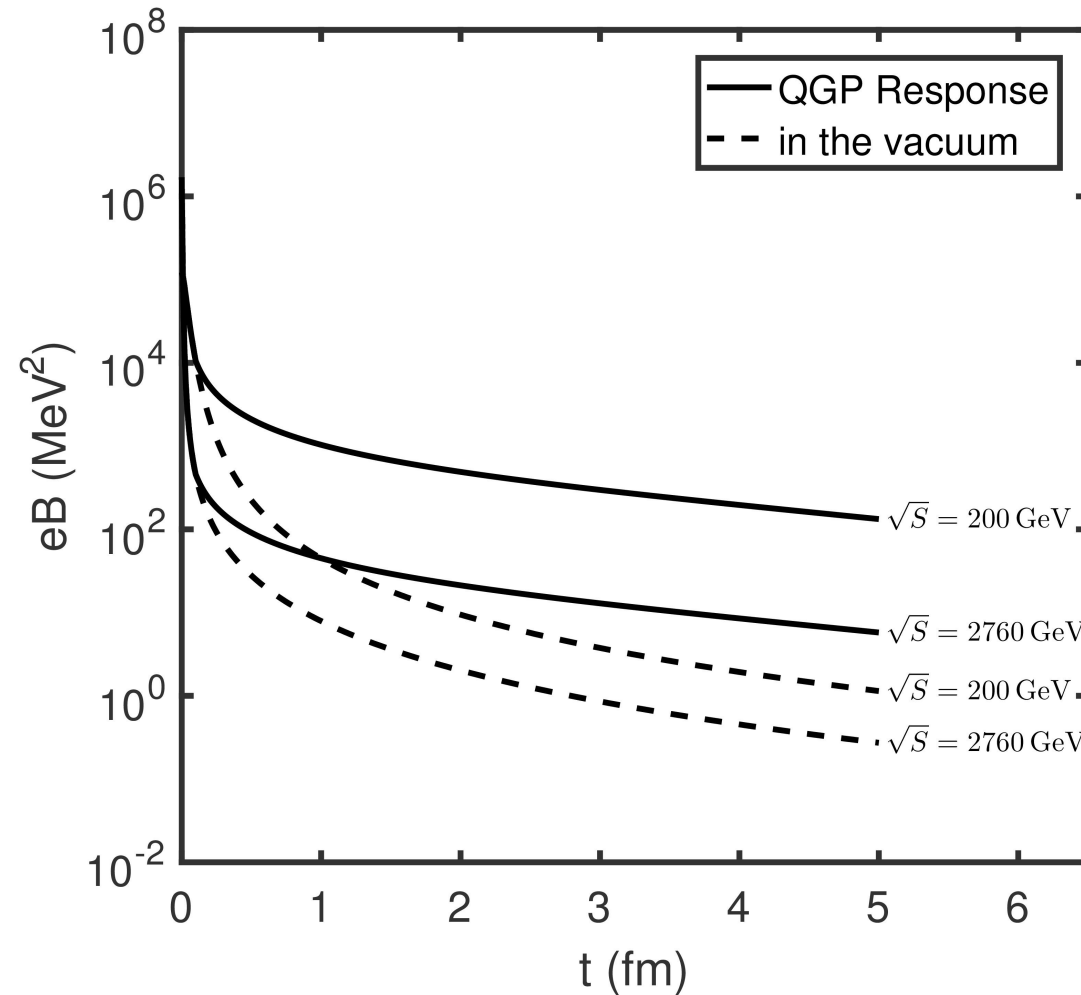
# Magnetic fields change with collision energy at different time



Y. Zhong, C.-B. Yang, X. Cai and S. -Q. Feng , Adv. High Energy Phys. 2014 (2014) 193039

The 13th Workshop on QCD Phase Transitions and Relativistic HIC, Enshi, August 16-20 2019

# Consider the response of QGP in relativistic heavy-ion collisions

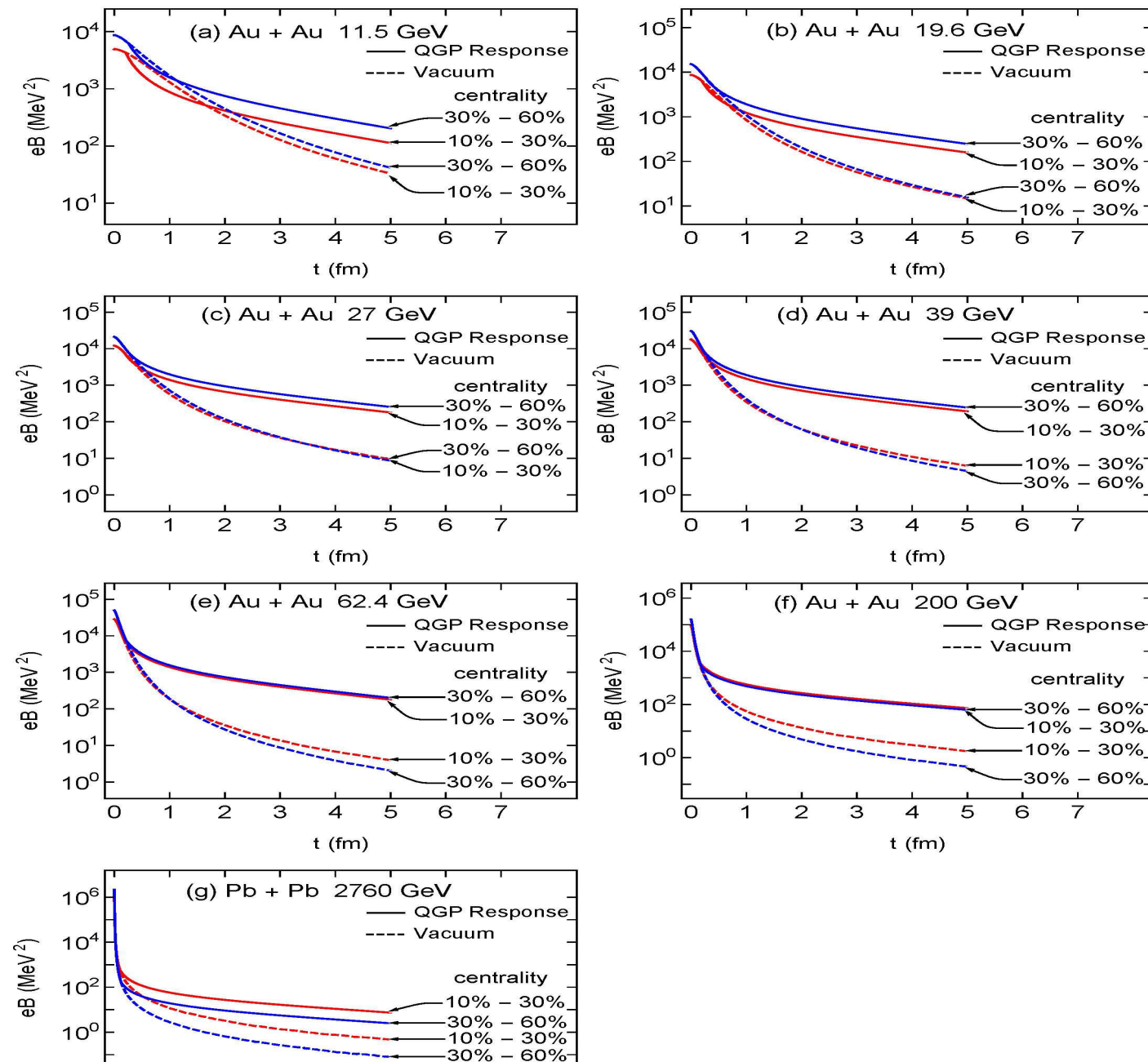


Coupled Maxwell +hydro: W. T. Deng, and X. G. Huang, Phys. Rev. C 85, 044907 (2012).

D. She, S. -Q. Feng, et al., European Physical Journal A 54: 48 (2018)

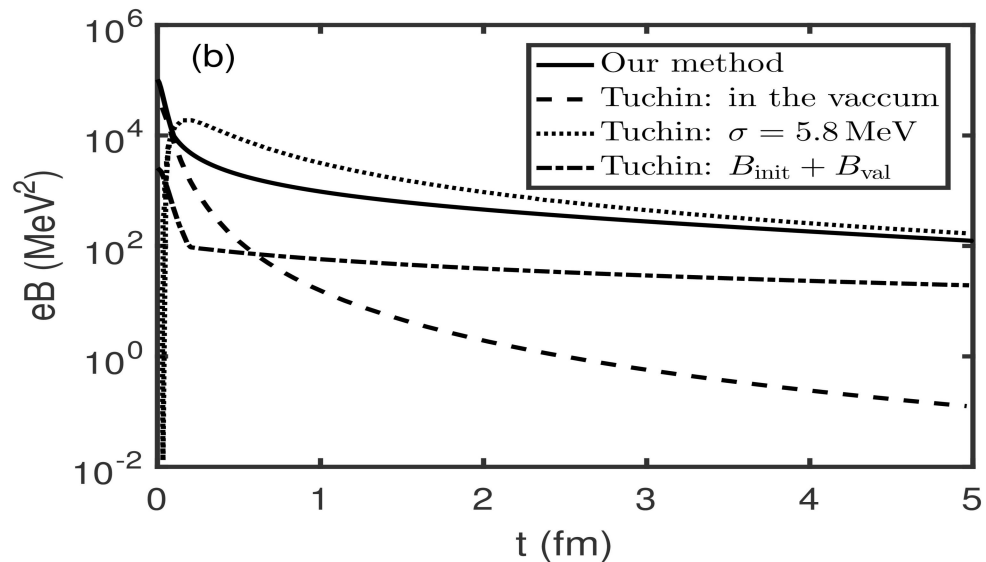
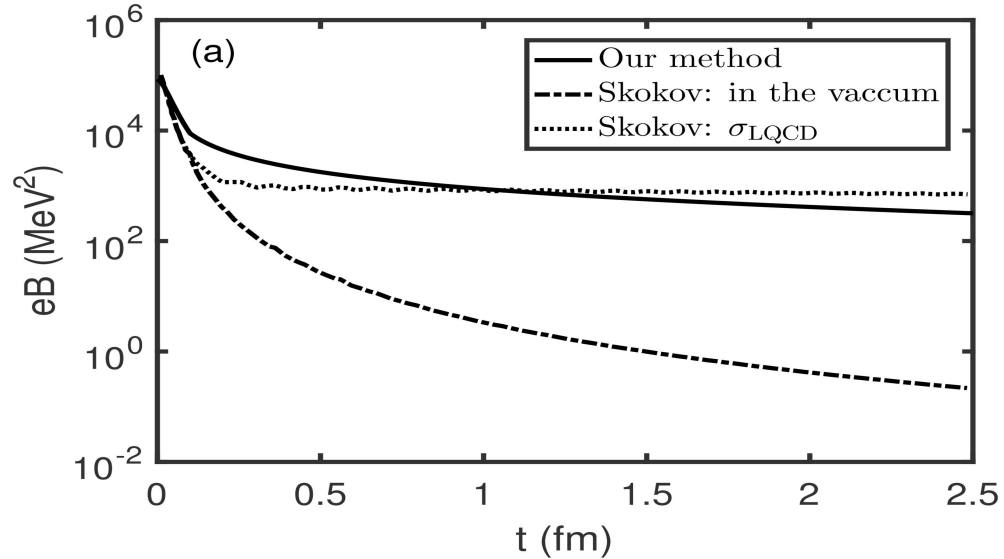
The 13th Workshop on QCD Phase Transitions and Relativistic HIC, Enshi, August 16-20 2019

# Magnetic field by considering the response of QGP



B.-X.Chen and S.-Q. Feng, in preparation

# Comparison with other group calculation results



1. L. McLerran, V. Skokov, Nucl. Phys. A929, 184 (2014).
2. K. Tuchin, Phys. Rev. C 93, 014905 (2016).

D. She, S. -Q. Feng, et al., European Physical Journal A 54 : 48 (2018)

The 13th Workshop on QCD Phase Transitions and Relativistic HIC, Enshi, August 16-20, 2019

# 3. Chiral Electromagnetic Current

Based on: D. She, S. -Q. Feng, et al., European Physical Journal A 54:48 (2018)

The 13th Workshop on QCD Phase Transitions and Relativistic HIC, Enshi, August 16-20, 2019



# Chiral Magnetic Conductivity (CMC)

---

The induced vector current can be calculated using the Kubo formula. This formula is provided by using first order in the time-dependent perturbation:

$$\langle j^\mu(x) \rangle = \int d^4x' \Pi_R^{\mu\nu}(x, x') A_\nu(x')$$

Where  $j^\mu(x) = e \bar{\psi}(x) \gamma^\mu \psi(x)$ , and the retarded response function is given by

$$\Pi_R^{\mu\nu}(x, x') = i \langle [j^\mu(x), j^\nu(x')] \rangle \theta(t - t').$$

Select vector field:  $A_x = A_z = 0, A = A_y$

Then:  $B_z = \partial_x A_y(x)$

The induced vector current in the magnetic field direction:

$$\langle j_z(x) \rangle = \sigma_\chi(p) \tilde{B}_z(p) e^{-ipx} \quad \sigma_\chi(p) = \frac{1}{ip^1} \tilde{\Pi}_R^{23}(p) = \frac{1}{2ip^i} \tilde{\Pi}_R^{jk}(p) \varepsilon^{ijk}$$

Chiral magnetic conductivity(CMC):

$$\sigma_\chi(p) = \frac{1}{ip^i} G_R^i(p) \quad G_R^i(p) = \frac{1}{2} \varepsilon^{ijk} \tilde{\Pi}_R^{jk}(p)$$

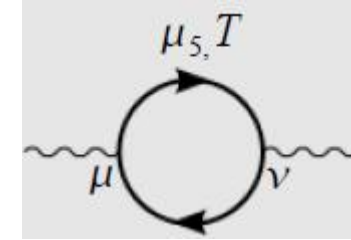
# Retarded Correlator

Retarded correlator can be given by Euclidean correlator:

$$G_R^i(p_0, \mathbf{p}) = G_E^i(\omega_n, \mathbf{p})|_{i\omega_n \rightarrow p_0 + i\epsilon}$$

At very high temperatures, Euclidean correlator:

$$G_E^i(P) = \frac{e^2}{2\beta} \sum_{\tilde{\omega}_m} \int \frac{d^3 q}{(2\pi)^3} \epsilon^{ijk} \text{tr}[\gamma^k S(Q) \gamma^j S(P + Q)]$$



The bare fermion propagator as a function of Euclidean momentum  $Q$  in the presence of a chiral chemical potential

$$S(Q) = \frac{1}{i\gamma^0(\tilde{\omega}_m - i\mu - i\mu_s\gamma^5) - \gamma \cdot q}$$

From fermion propagator and chirality projection operators, the integrand function can be written:

$$G_R^i(p) = \frac{ie^2}{16\pi^2} \frac{p^i}{p} \frac{p^2 - p_0^2}{p^2} \int_0^\infty dq f(q) \sum_{t=\pm} (2q + tp_0) \times \log \left[ \frac{(p_0 + i\epsilon + tq)^2 - (q + p)^2}{(p_0 + i\epsilon + tq)^2 - (q - p)^2} \right]$$

$$f(q) \equiv \sum_{s=\pm} s[\tilde{n}(q - \mu_s) - \tilde{n}(q + \mu_s)]$$

# Real and Imaginary Part of CMC

---

At zero temperature:

$$\sigma''_{\chi}(\omega) = \frac{e^2}{3\pi} \omega \delta(\omega) \mu_5 - \frac{e^2 \omega^2}{96\pi} \sum_{s,t=\pm} st \delta(\omega/2 + t\mu_s).$$

For large temperature:

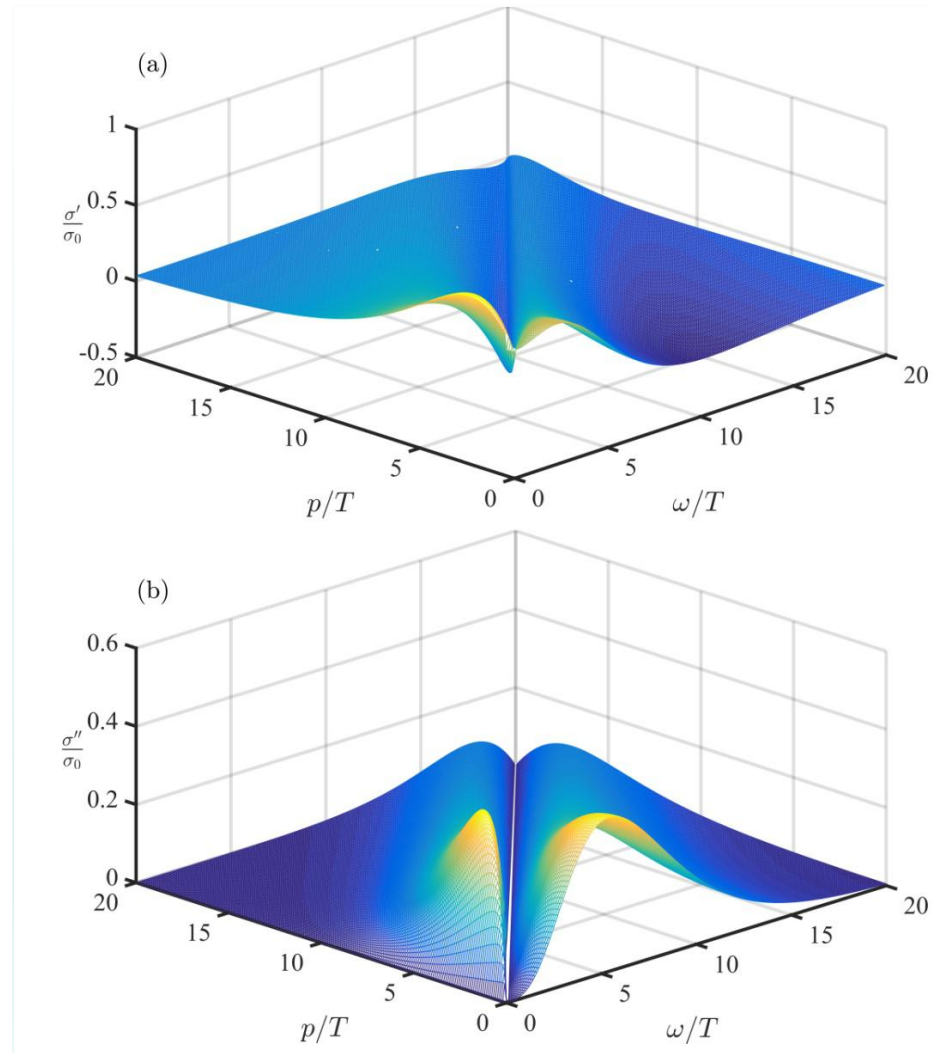
$$\begin{aligned} \sigma''_{\chi}(\omega) = & \frac{e^2}{3\pi} \omega \delta(\omega) \mu_5 + \frac{e^2 \omega |\omega|}{24\pi T^2} \tilde{n}(|\omega|/2)^3 \\ & \times [e^{|\omega|/T} - e^{|\omega|/(2T)}] \mu_5. \end{aligned}$$

By Kramers-Kronig relation:

$$\sigma'_{\chi}(\omega) = \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} dq_0 \frac{\sigma''_{\chi}(q_0)}{q_0 - \omega},$$

$$\sigma''_{\chi}(\omega) = -\frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} dq_0 \frac{\sigma'_{\chi}(q_0)}{q_0 - \omega}.$$

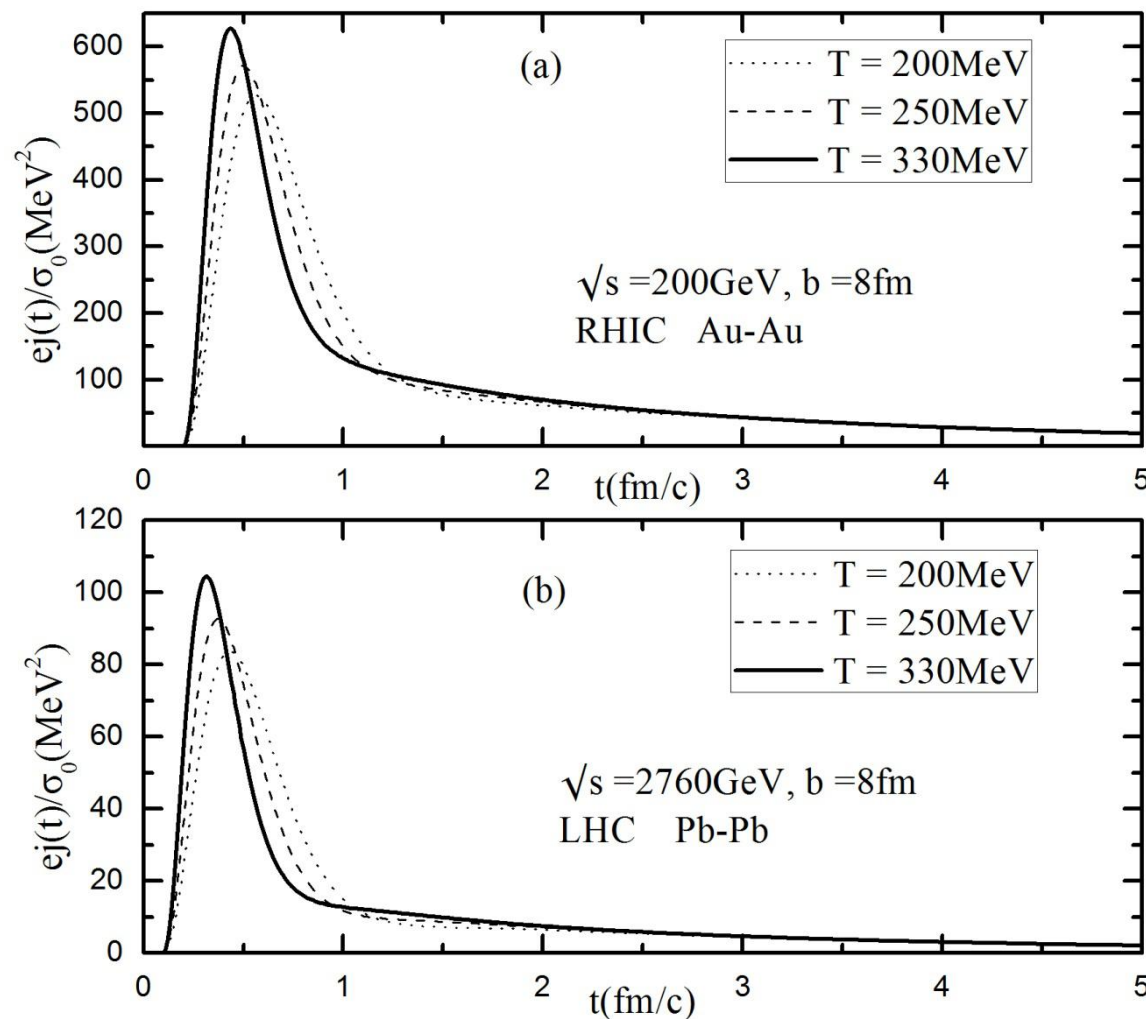
# Frequency and momentum dependence of the chiral magnetic conductivity



$$\mu = 10\text{MeV}, \mu_5 = 1\text{MeV}, T = 200\text{MeV}$$

D. She, S. -Q. Feng, et al., European Physical Journal A 54:48 (2018)

# Chiral Electromagnetic Current (CEC) in different temperatures



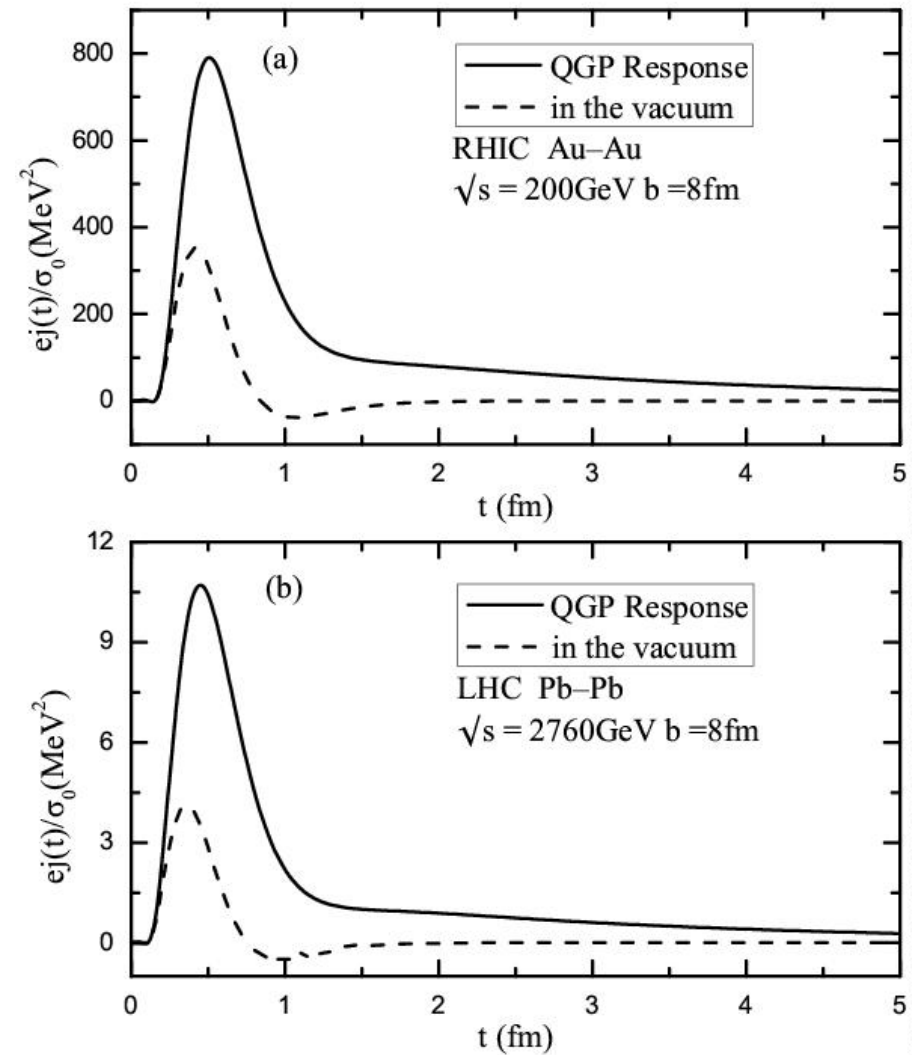
$$j(t) = \int_0^\infty \frac{d\omega}{\pi} [\sigma'(\omega) \cos(\omega t) + \sigma''(\omega) \sin(\omega t)] \tilde{B}(\omega).$$

$$\tilde{B}(\omega) = \int_{-\infty}^\infty dt e^{i\omega t} B(t).$$

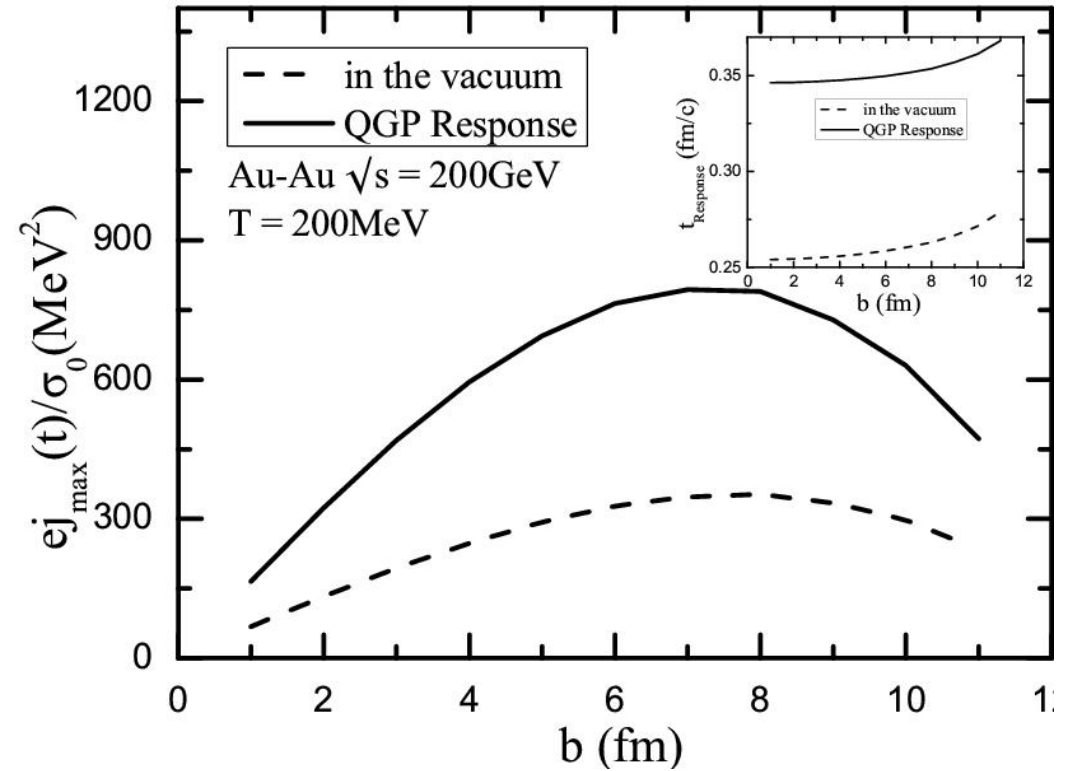
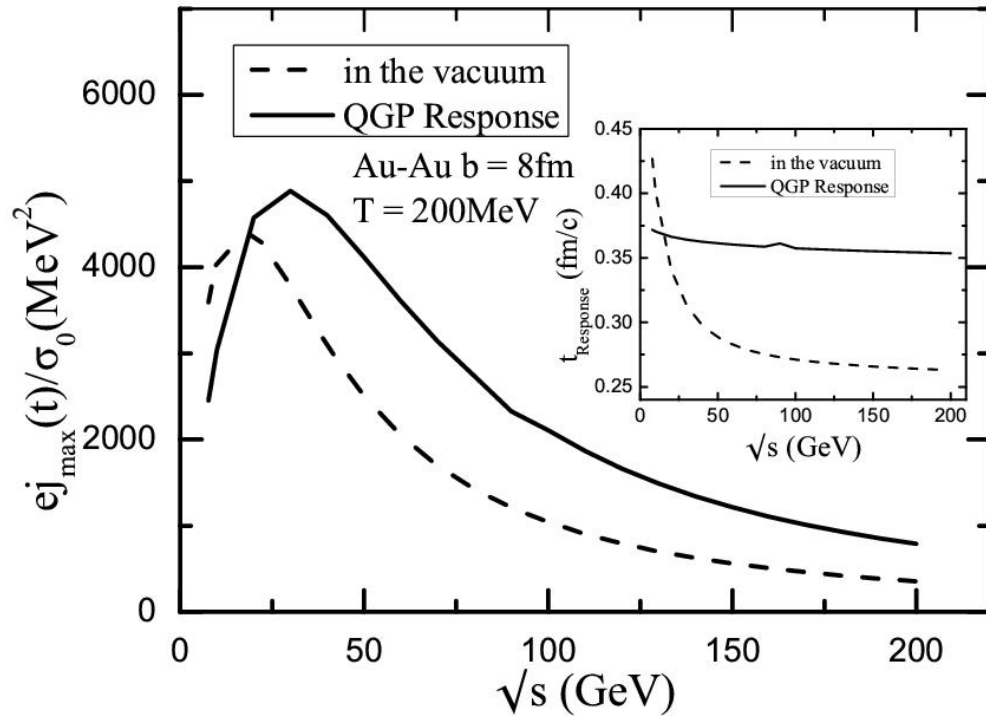
D. She, S. -Q. Feng, et al., European Physical Journal A 54:48 (2018)

The 13th Workshop on QCD Phase Transitions and Relativistic HIC, Enshi, August 16-20, 2019

## Comparison by QGP response with in the vacuum



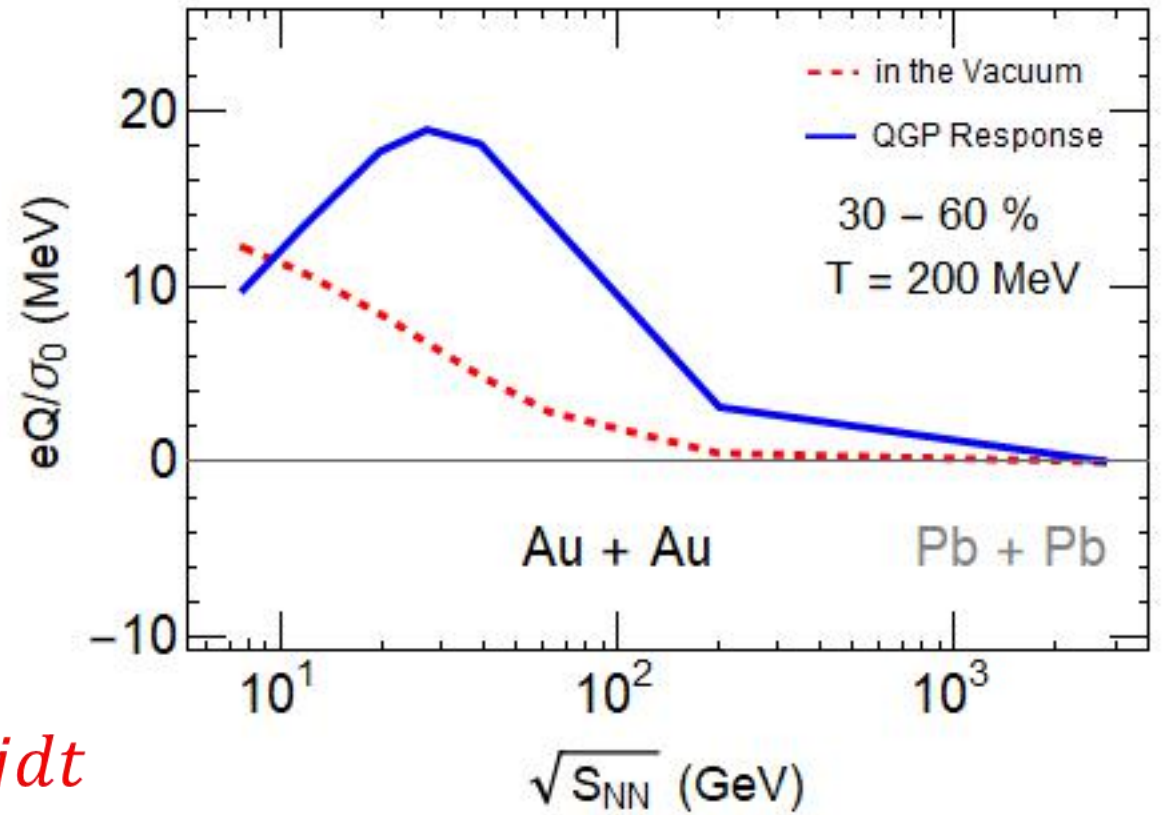
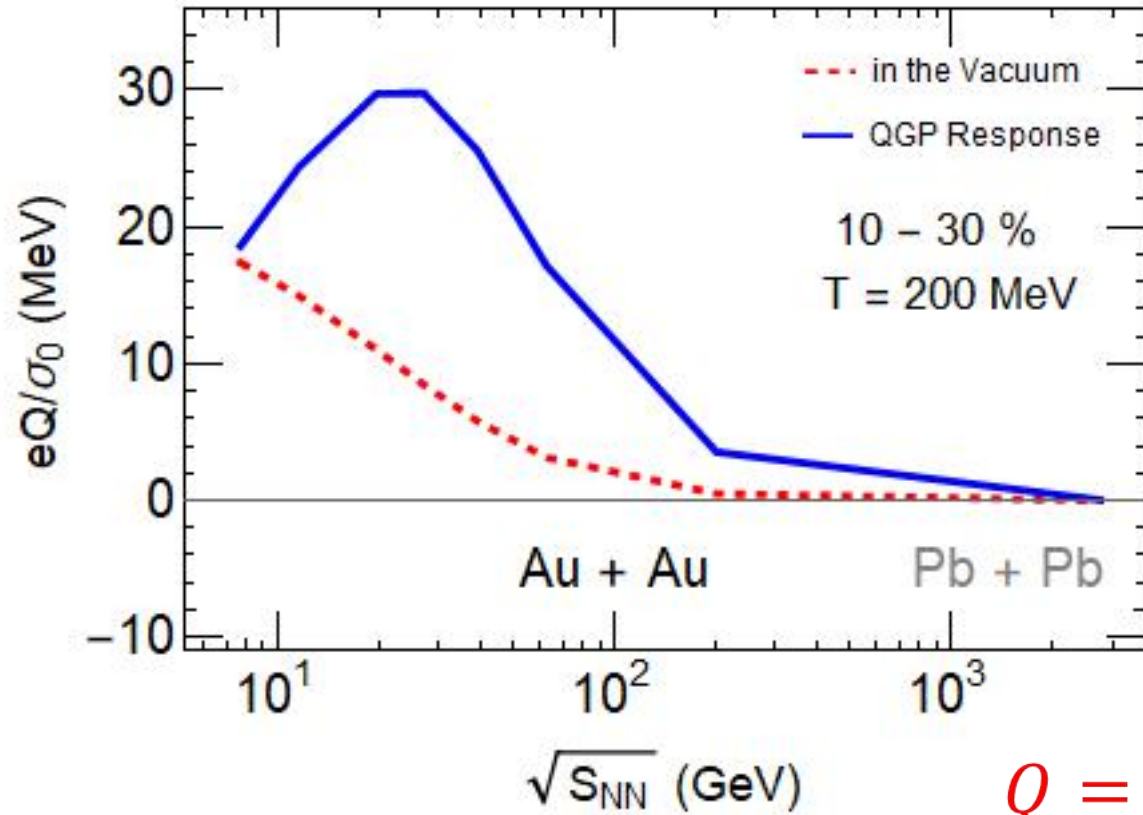
# Maximum Chiral Electromagnetic Current (CEC) on energy and impact parameter



D. She, S. -Q. Feng, et al., European Physical Journal A 54:48 (2018)



# Chiral electromagnetic charge with different centrality



$$Q = \int j dt$$

## 4. Charge Separation features in Relativistic HIC

Based on: S.-Q. Feng, X. Ai et al., Chinese Physics, C 42 054102 (2018)

The 13th Workshop on QCD Phase Transitions and Relativistic HIC, Enshi, August 16-20, 2019

# CME Signal at RHIC

PRL **103**, 251601 (2009)

Selected for a *Viewpoint in Physics*  
PHYSICAL REVIEW LETTERS

week ending  
18 DECEMBER 2009

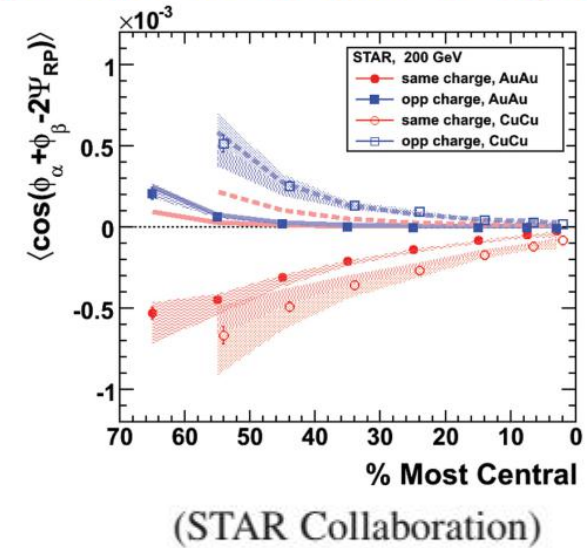
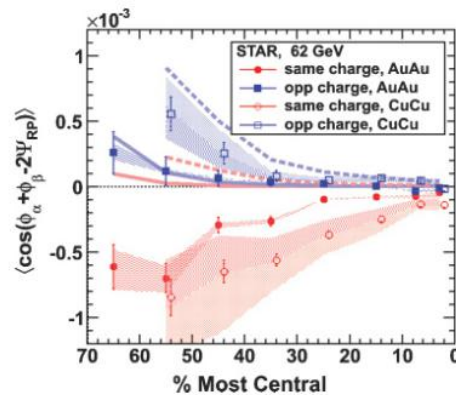
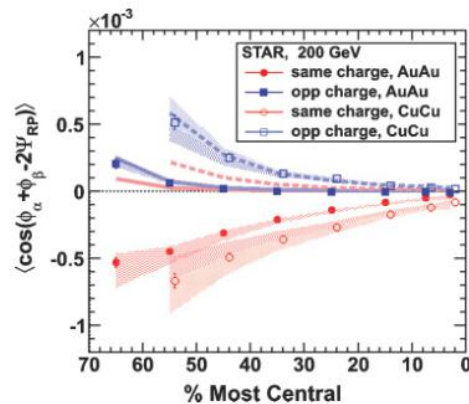


## Azimuthal Charged-Particle Correlations and Possible Local Strong Parity Violation

PHYSICAL REVIEW C **81**, 054908 (2010)

### Observation of charge-dependent azimuthal correlations and possible local strong parity violation in heavy-ion collisions

(STAR Collaboration)



# CME Signal at LHC

PRL **110**, 012301 (2013)

PHYSICAL REVIEW LETTERS

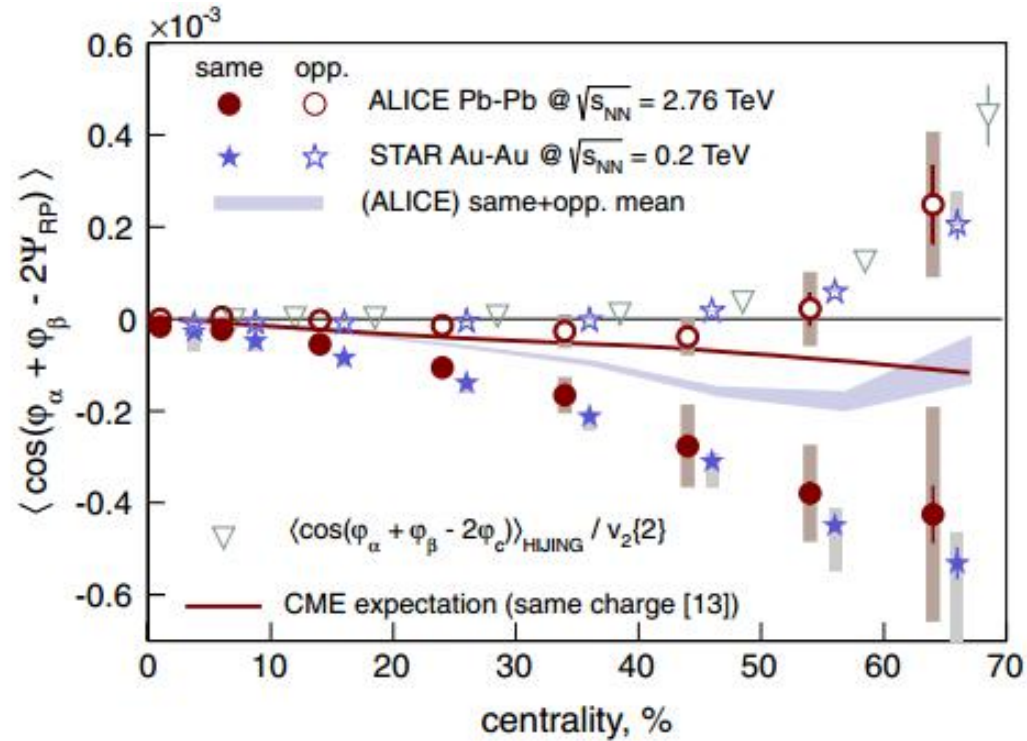
week ending  
4 JANUARY 2013

## Charge separation relative to the reaction plane in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV

B. Abelev *et al.*\*

(ALICE Collaboration)

(Received 5 July 2012; published 2 January 2013)



## Calculate the charge separations between both sides of reaction plane

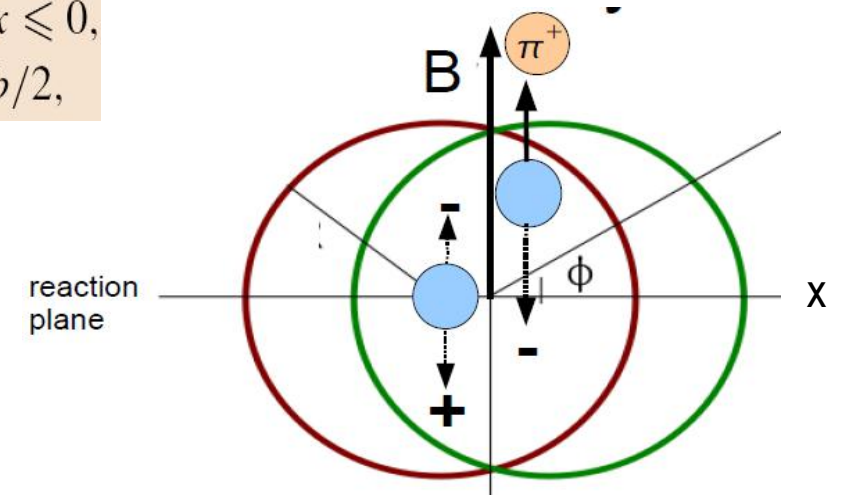
$$\langle \Delta_{\pm}^2 \rangle = 2\kappa\alpha_S \left[ \sum_f q_f^2 \right]^2 \int_{V_{\perp}} d^2x_{\perp} \left[ (\xi_{-}(x_{\perp}))^2 + (\xi_{+}(x_{\perp}))^2 \right] \int d\eta \int_{\tau_i}^{\tau_f} d\tau \tau [eB(\tau, \eta, x_{\perp})]^2$$

$$\langle \Delta_{+}\Delta_{-} \rangle = -4\kappa\alpha_S \left[ \sum_f q_f^2 \right]^2 \int_{V_{\perp}} d^2x_{\perp} \xi_{-}(x_{\perp}) \xi_{+}(x_{\perp}) \int d\eta \int_{\tau_i}^{\tau_f} d\tau \tau [eB(\tau, \eta, x_{\perp})]^2$$

$$\xi_{\pm}(x_{\perp}) = \exp(-|y_{\pm}(x) - y|/\lambda)$$

$$y_{+}(x) = -y_{-}(x) = \begin{cases} \sqrt{R^2 - (x - b/2)^2} & -R + b/2 \leq x \leq 0, \\ \sqrt{R^2 - (x + b/2)^2} & 0 \leq x \leq R - b/2, \end{cases}$$

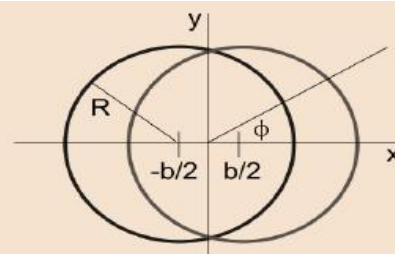
**The Chiral Magnetic Effect is  
a near the surface effect**



## Calculate the charge separations between both sides of reaction plane

$$a_{++} = a_{--} = \frac{1}{N_+^2} \frac{\pi^2}{16} \langle \Delta_{\pm}^2 \rangle,$$

$$a_{+-} = a_{-+} = \frac{1}{N_+ N_-} \frac{\pi^2}{16} \langle \Delta_+ \Delta_- \rangle$$



$\phi$  : angle between particle and reaction plane

$$\frac{dN_{\pm}}{d\phi} = \frac{N_{\pm}}{2\pi} + a_{\pm} \sin \phi + v_2 \cos 2\phi + \dots$$

Average over many equivalent events  
(to cancel statistical fluctuations) can give us

$$\langle a_+^2 \rangle \sim \langle \Delta_+^2 \rangle \quad \text{Pref. emission positive on one side}$$

$$\langle a_-^2 \rangle \sim \langle \Delta_-^2 \rangle \quad \text{Pref. emission negative on one side}$$

$$\langle a_+ a_- \rangle \sim \langle \Delta_+ \Delta_- \rangle \quad \text{Correlations between positive on one and negative on other side}$$



# Beam-Energy Dependence of Charge Separation along the Magnetic Field in Au + Au Collisions at RHIC

three particle correlator  $\gamma$ , two particle correlator  $\delta$ ,

$$\gamma \equiv \langle \cos(\phi_1 + \phi_2 - 2\Psi_{\text{RP}}) \rangle = \kappa v_2 F - H,$$

$$\delta \equiv \langle \cos(\phi_1 - \phi_2) \rangle = F + H,$$

where, H and F represent signal from CME and background, F mainly comes from elliptic flow.

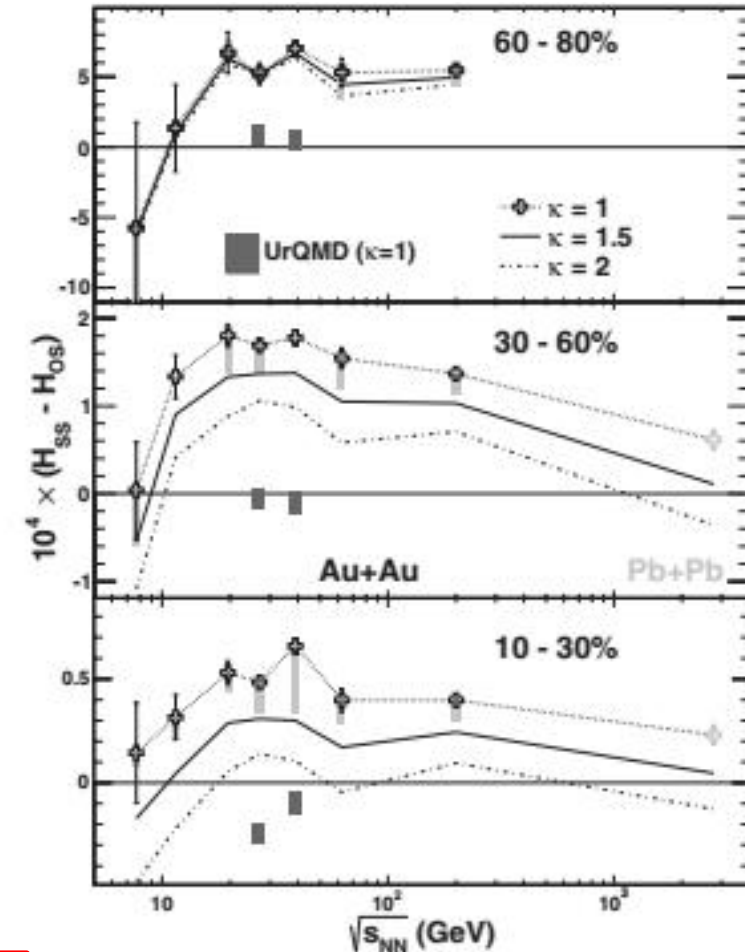
$$H^\kappa = \frac{\kappa v_2 \delta - \gamma}{1 + \kappa v_2}.$$

Star's (14) used:  $H_{\text{SS}}, H_{\text{OS}}$

can subtract non-flow background

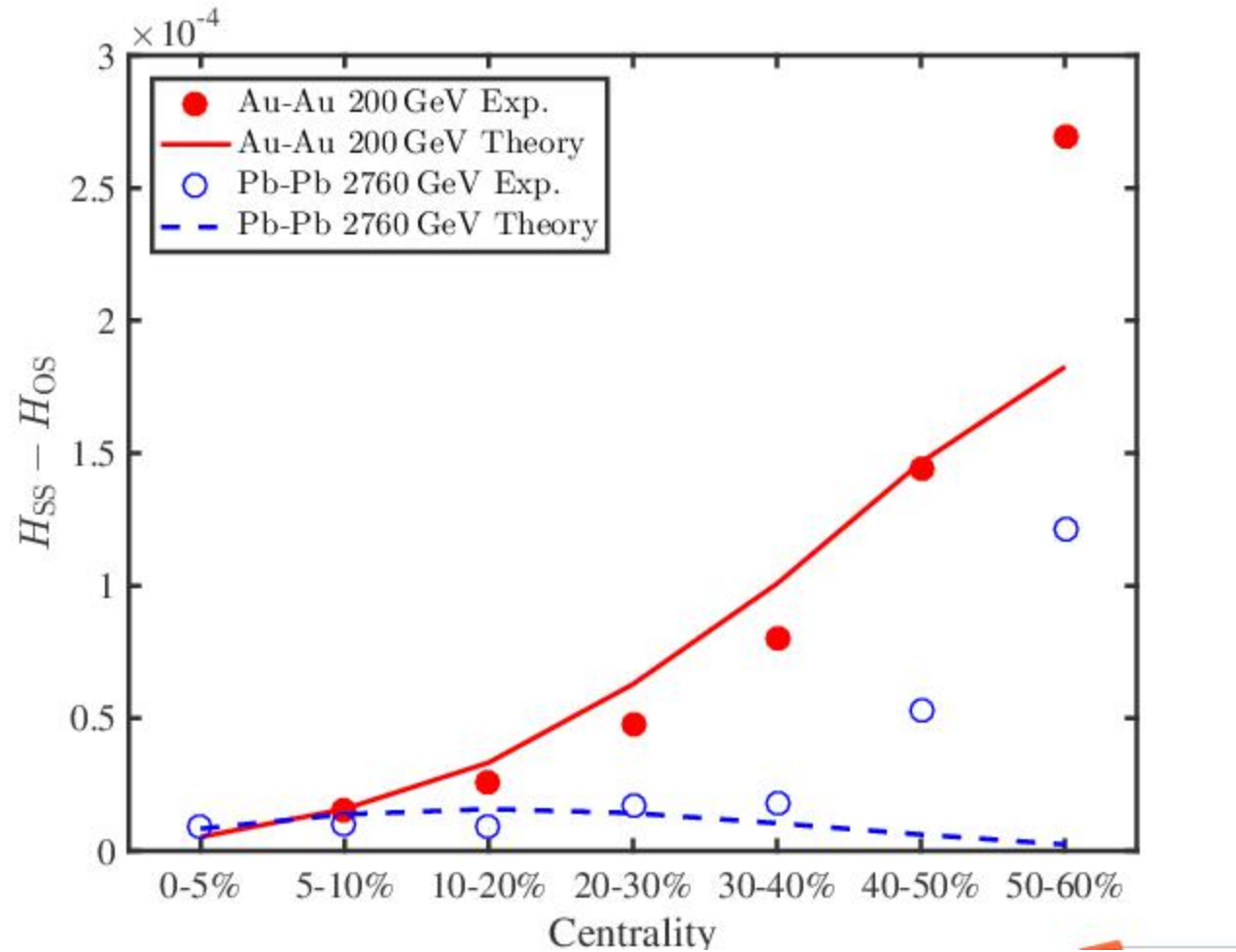
A. Bzdak, V. Koch, and J. Liao, Phys. Rev. C 83, 014905 (2011).

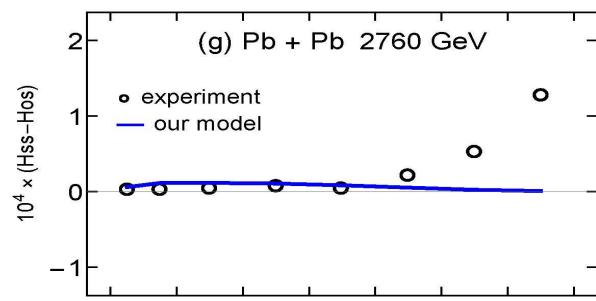
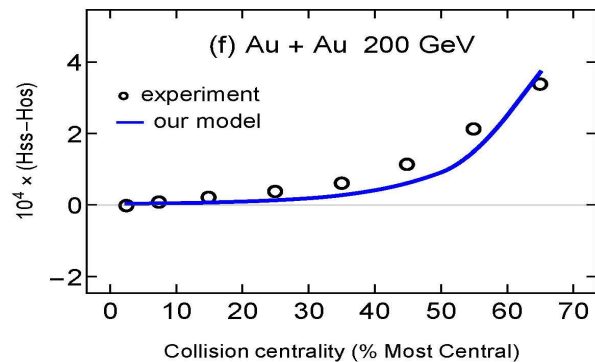
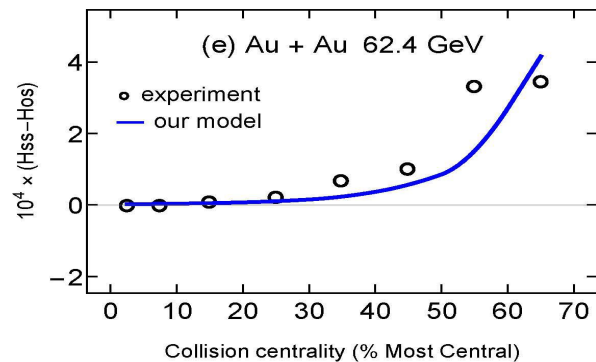
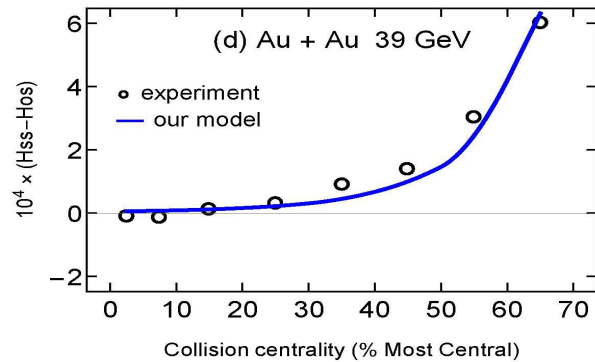
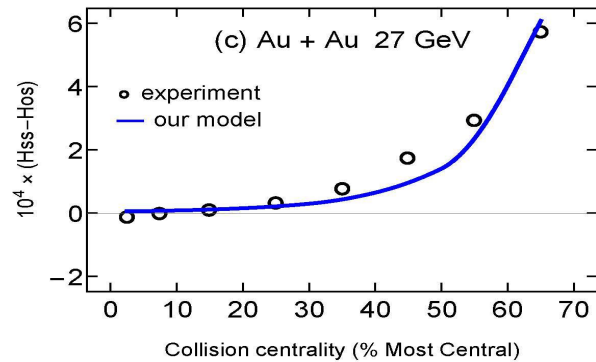
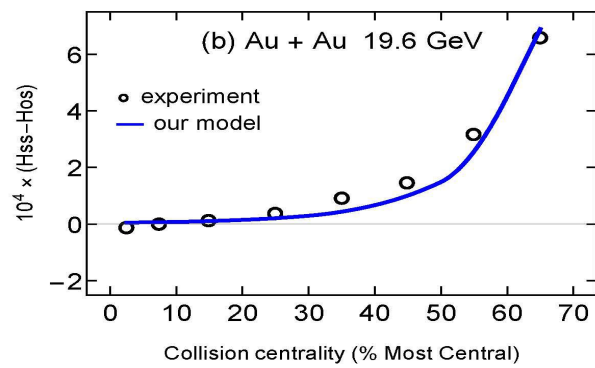
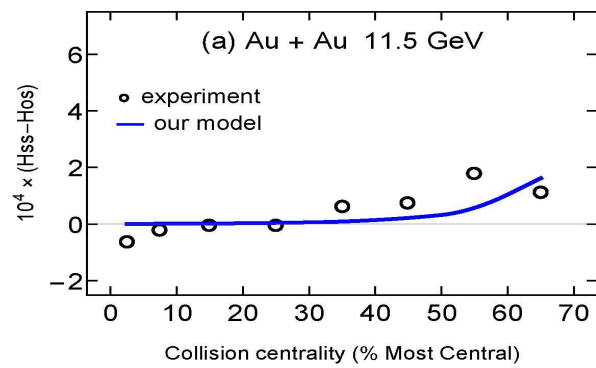
A. Bzdak, V. Koch, and J. Liao, Lect. Notes Phys. 871, 503 (2013)



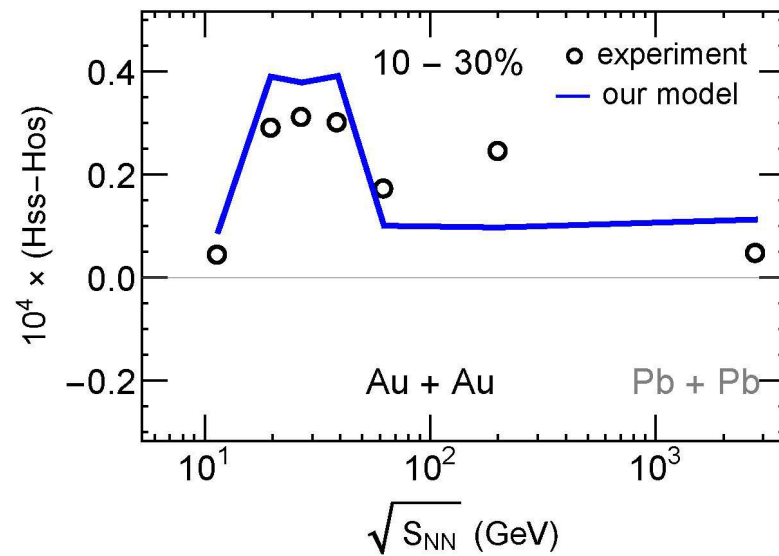
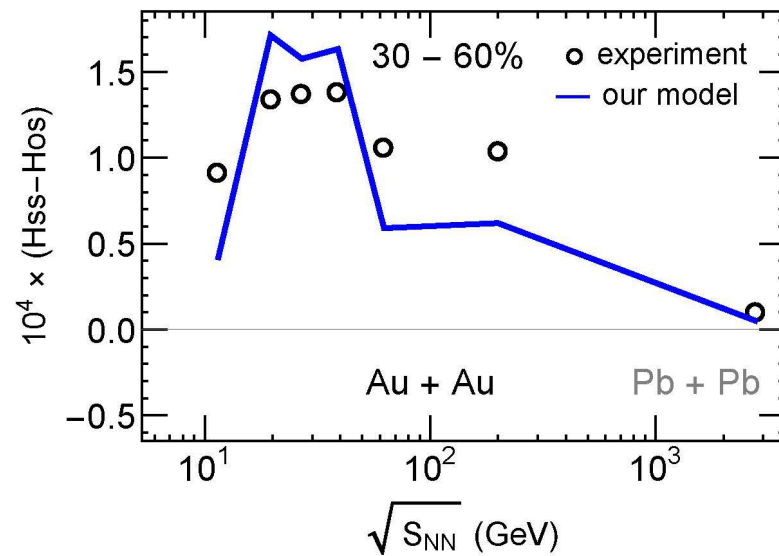


# Fit with experimental data



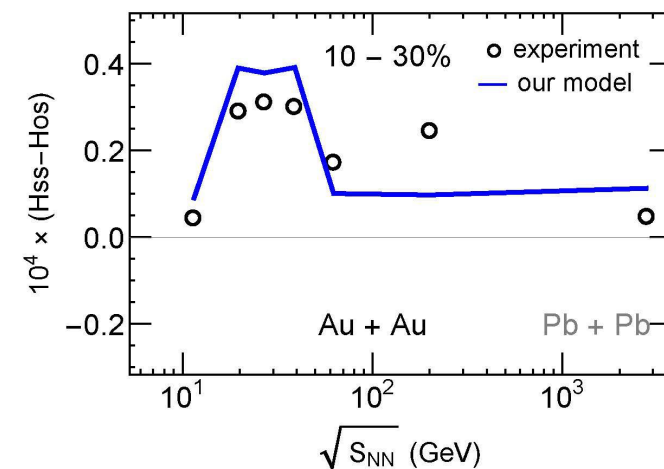
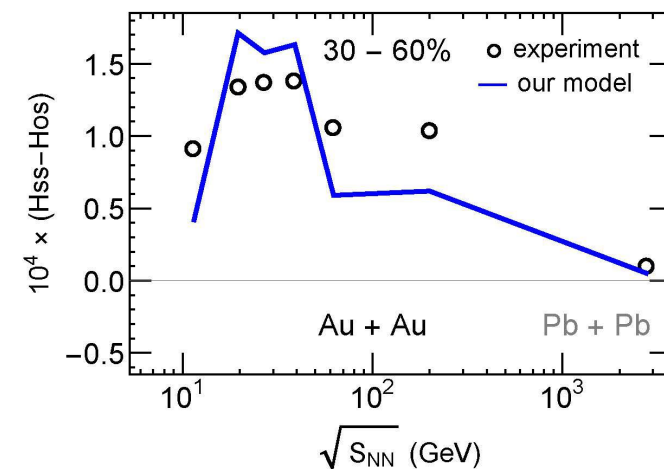
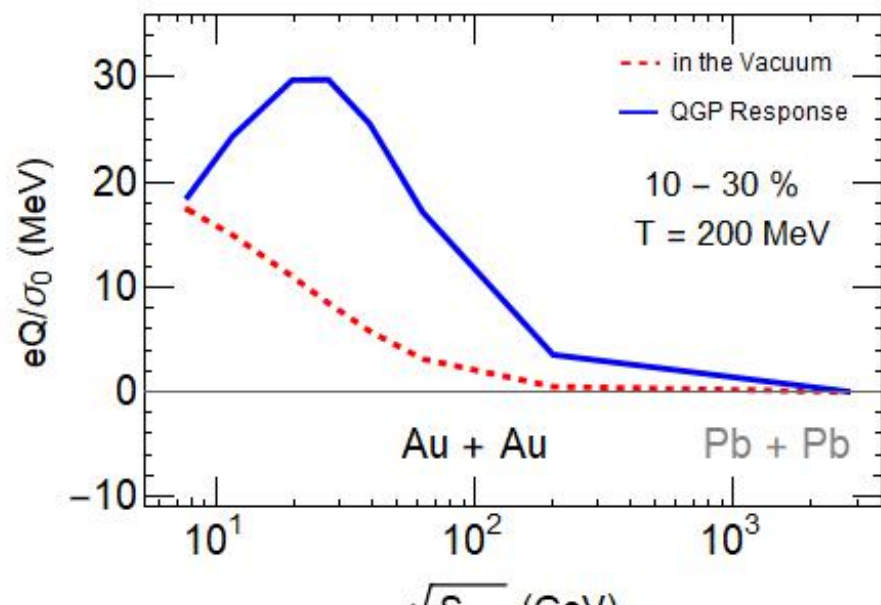
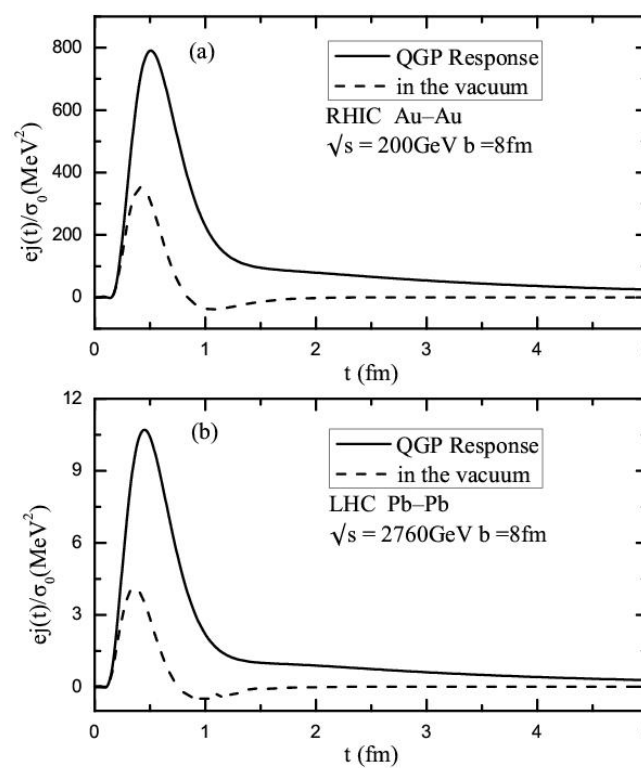
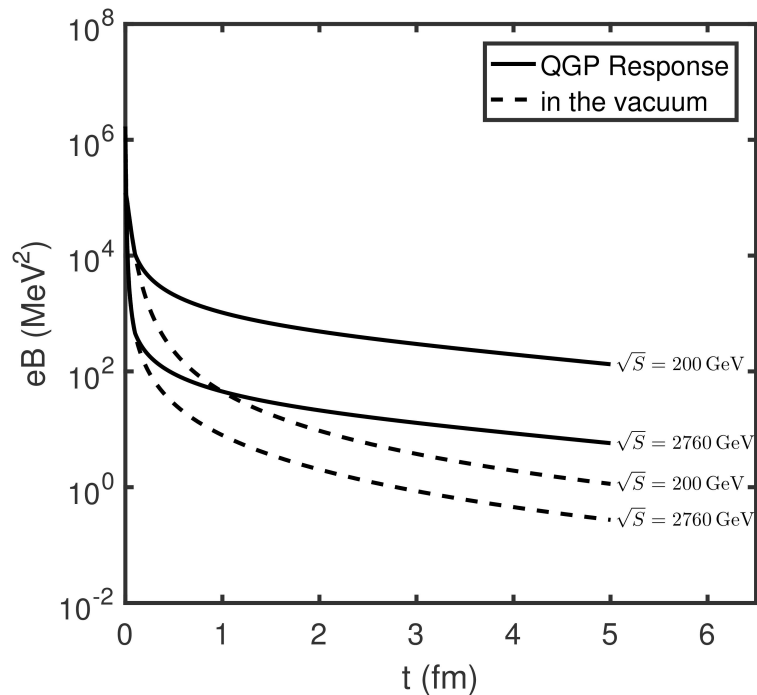


B.-X.Chen and S.-Q. Feng, in preparation



B.-X.Chen and S.-Q. Feng, in preparation

# Summary and Conclusions



# Thank your attentions

The 13th Workshop on QCD Phase Transitions and Relativistic HIC, En Shi, August 16-20, 2019