Nuclear Science Computing Center at CCNU

Meson masses in external magnetic fields from Lattice QCD

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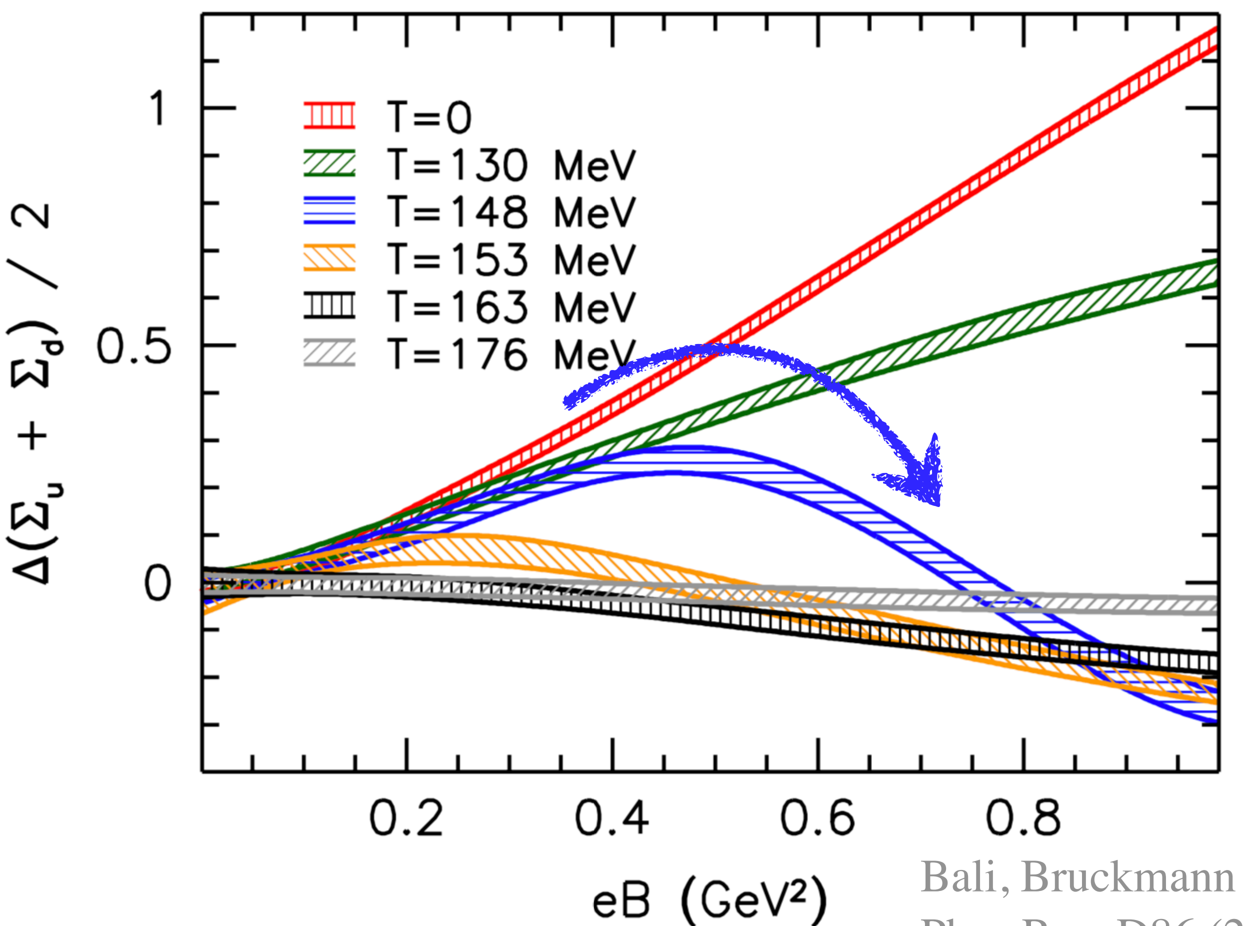
Outline

- Motivation
- Current Lattice Status
- Results
- Summary and Outlook

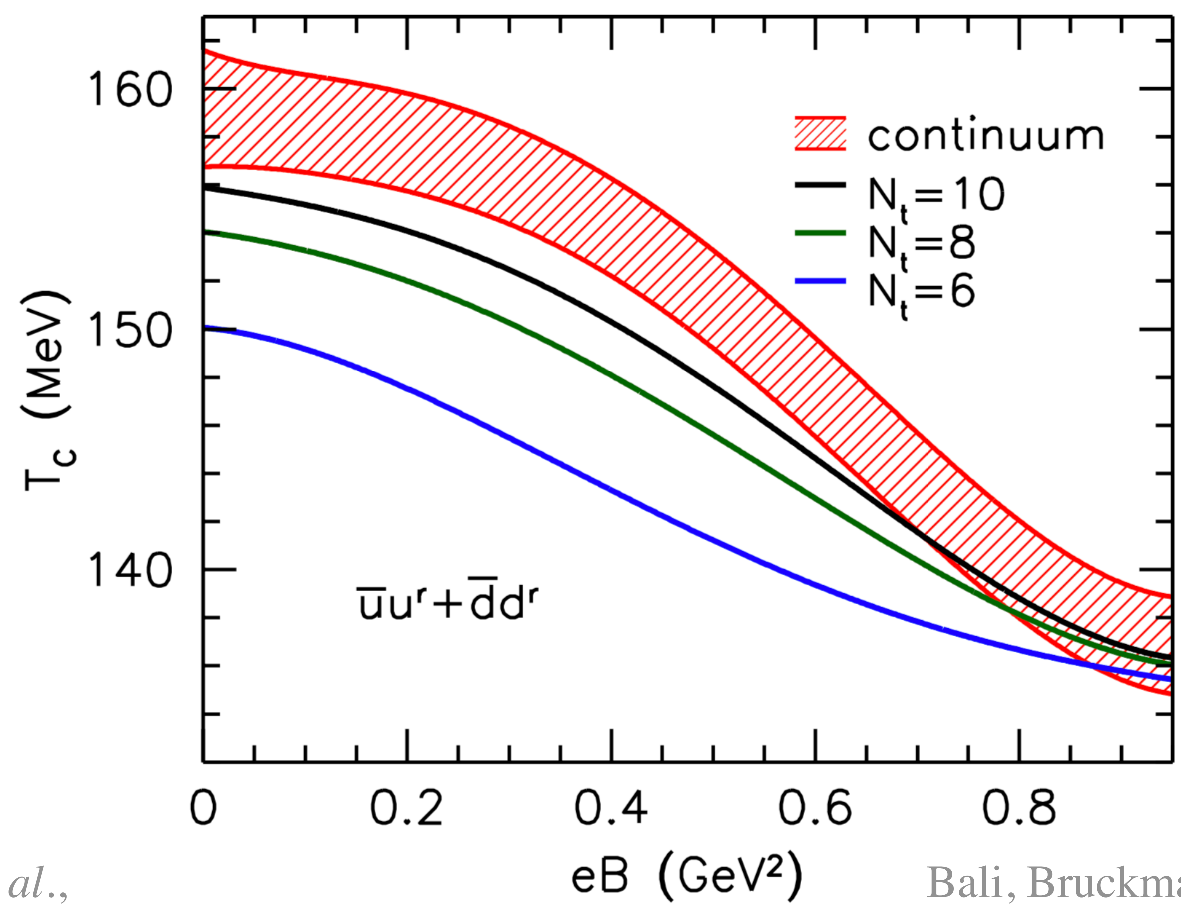
Motivation

- Inverse Magnetic catalysis

Stout Staggered fermion (@physical quark mass)



Bali, Bruckmann *et al.*,
Phys.Rev. D86 (2012) 071502



Bali, Bruckmann *et al.*,
JHEP 1202 (2012) 044

- Inverse magnetic catalysis near critical temperature
- Critical temperature decrease as magnetic field increase

➡ **eB-dependence of π^0 meson mass ?**

Motivation

- Superconductivity
  - NJL model and GL approach:

M. N. Chernodub Phys. Rev. Lett. **106**, 142003 (2011)
M. N. Chernodub, Phys. Rev. D 82, 085011 (2010)

- magnetic field leads to a vacuum instability $\longrightarrow$ color superconductivity
- superconducting states signaled by condensation of charged ρ-meson.

  - Vafa-Witten theorem and QCD inequality:

- charged ρ-meson cannot condensate in QCD
- $m_{\rho^\pm} \geq \left(m_{\pi_u^0} + m_{\pi_d^0}\right)/2$

Y. Hidaka and A.Yamamoto Phys. Rev. D 87, 094502 (2013)

➡ **eB-dependence π^0 meson mass ?**

- Lowest Landau Level

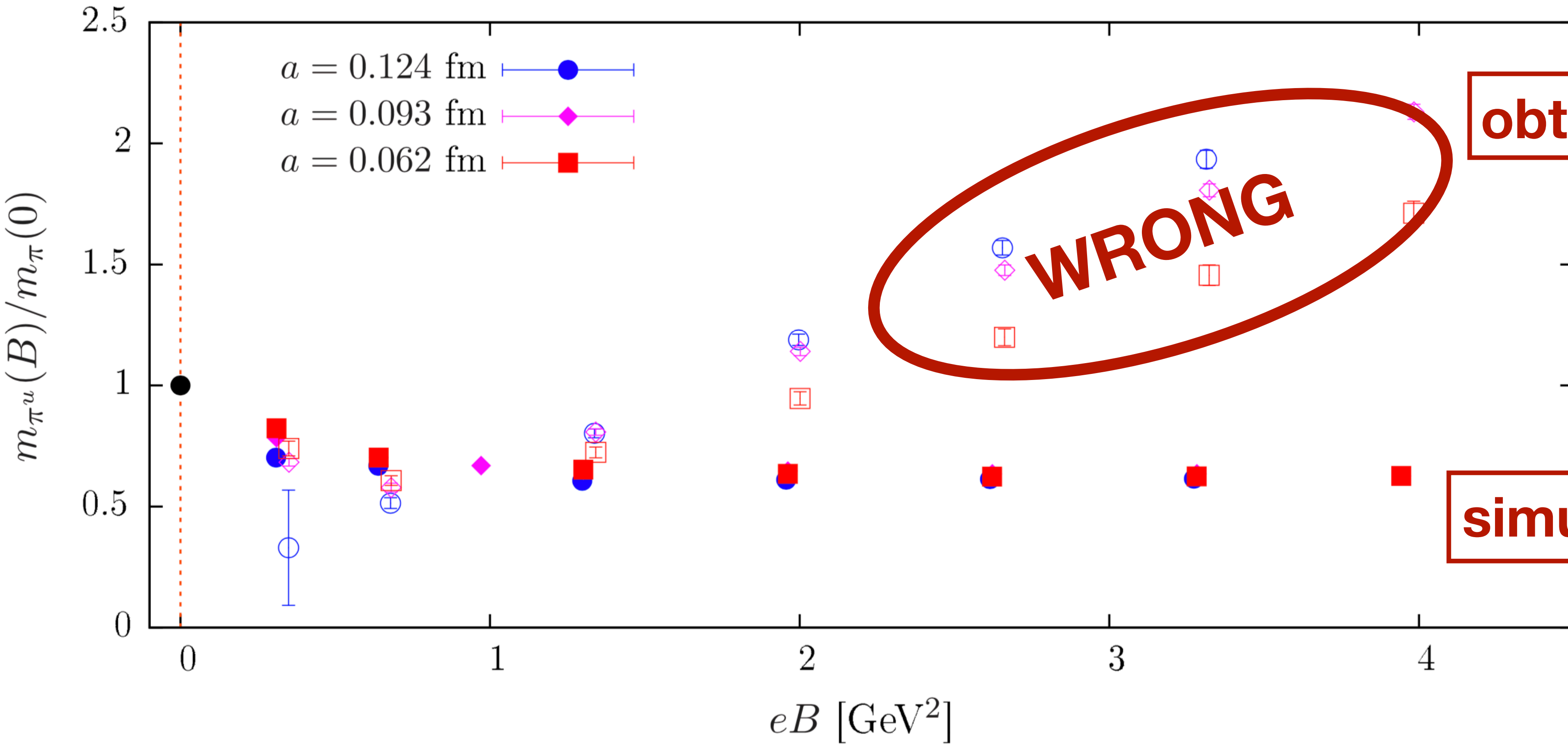
$$E^2 = p_z^2 + (2n+1)|qB| - g s_z qB + m^2 . \qquad (g=2)$$

Current lattice status

(Further study using **quenched Wilson** fermion)

Bali, Brandt *et al.*, PHYS. REV. D 97, 034505 (2018)

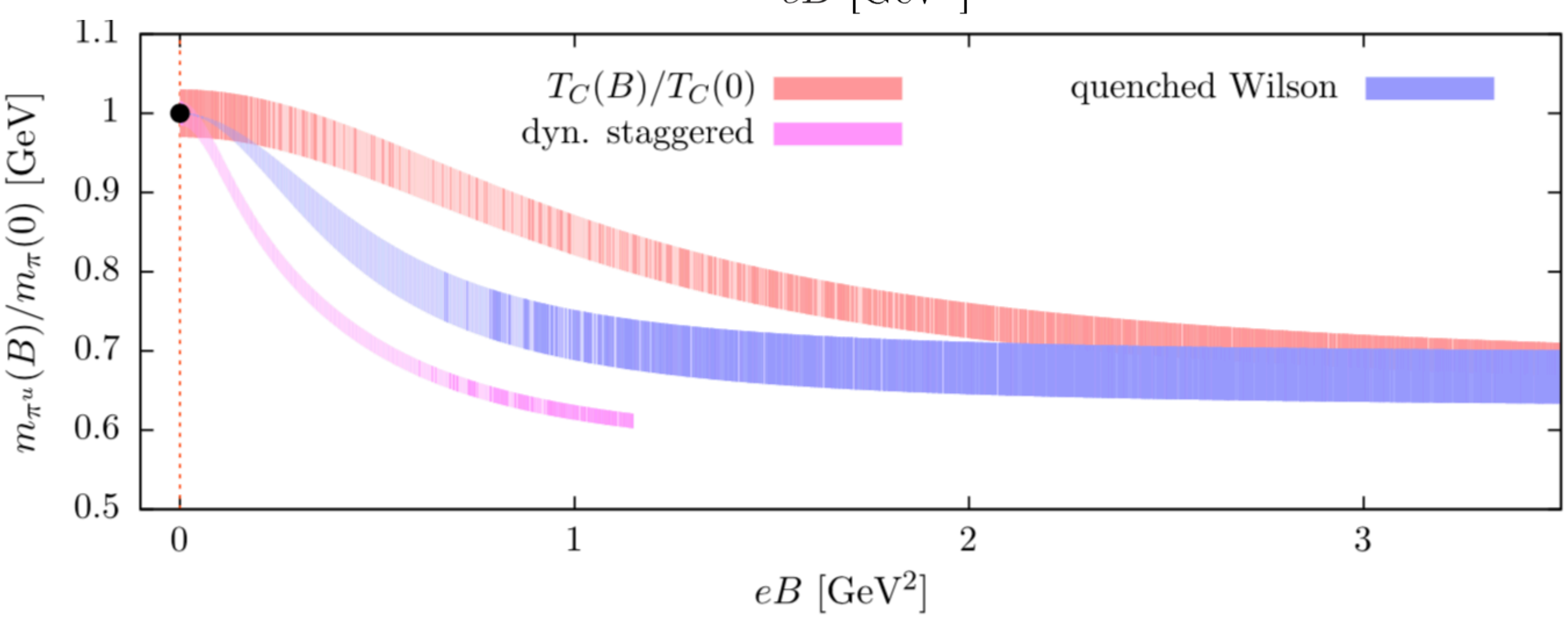
Y. Hidaka and A. Yamamoto Phys. Rev. D 87, 094502 (2013)



obtained with constant κ

- field–dependent quark mass renormalization need to be considered

simulated on LCP



- neutral pion mass decreases and saturates at nonzero value in large eB
- no charged ρ-meson condensation
- dynamical result π^0 mass has the same tendency

Summary on lattice results

Particle Mass		quenched Wilson/Overlap Bali *et al* (2018) Luschevskaya *et al* (2014)	dynamical stout Bali *et al* (2018)
π^0		↘	↘
π^+		↗	?
ρ^0	ρ_0^0	↘	?
	ρ_+^0	↗	?
ρ^+	ρ_0^+	↗	?
	ρ_+^+	↘	?
	ρ_-^+	↗	?

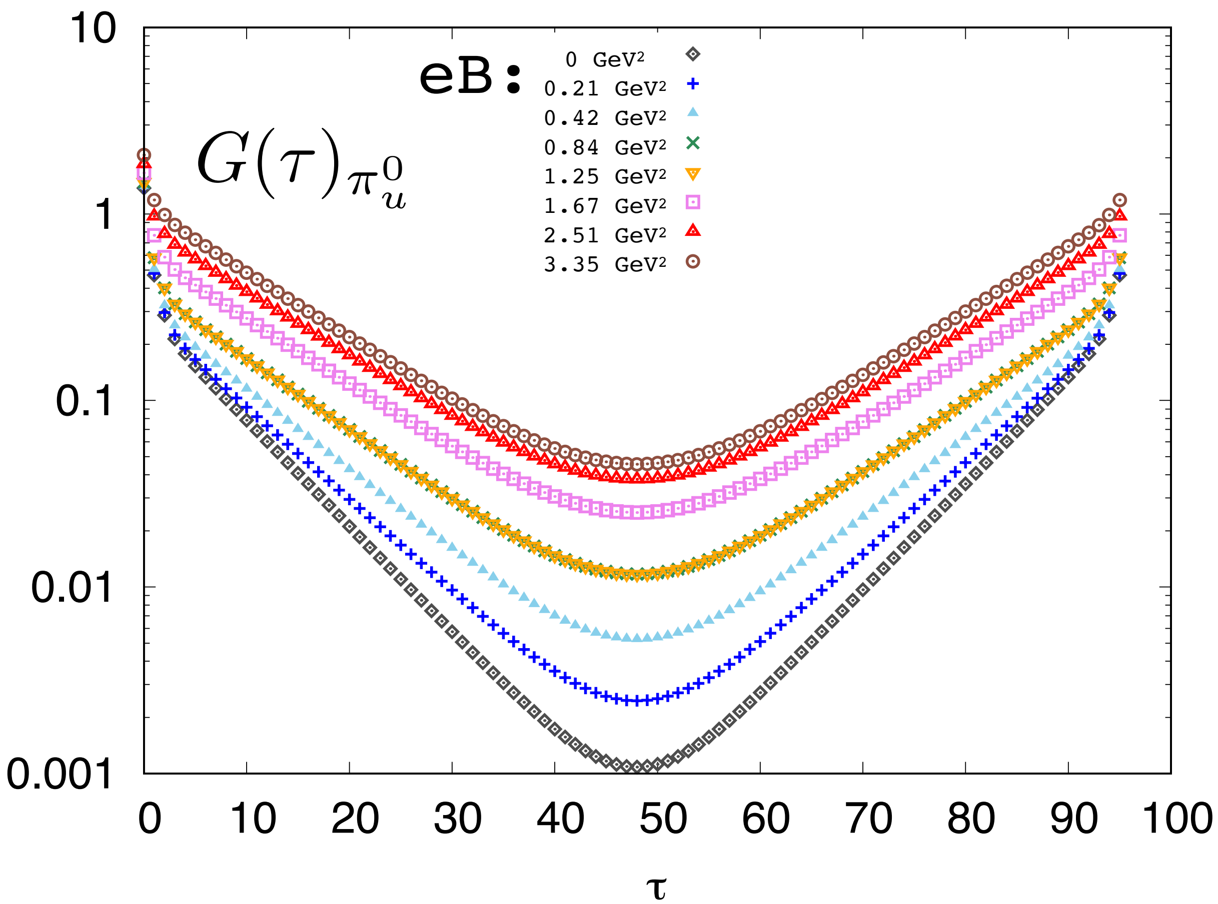
Y. Hidaka and A.Yamamoto Phys. Rev. D 87, 094502 (2013)
Bali, Brandt *et al*., PHYS. REV. D 97, 034505 (2018)
O. Larina ,E.V. Luschevskaya *et al*., POS Lattice 2014

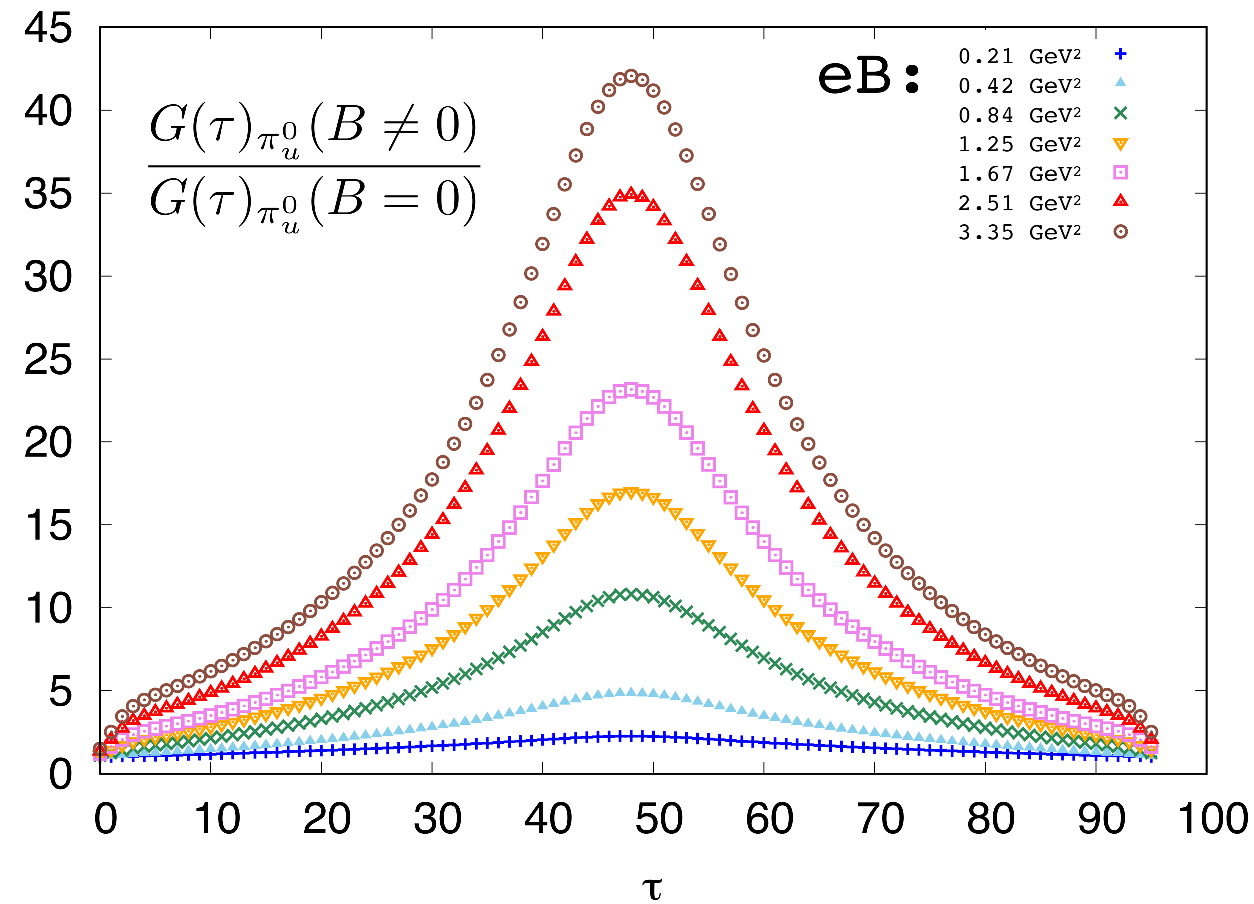
Correlator

- Correlator on lattice $G(\tau) \equiv \langle O(\tau) O^\dagger(0) \rangle = \sum_n \langle 0 | O^\dagger | n \rangle \langle n | O | 0 \rangle e^{-\tau E_n}$

$$\left\langle O_H(m) O_H^\dagger(n) \right\rangle = -\frac{1}{Z} \int \mathcal{D}[U] e^{-S_G[U]} \det D_u \det D_d \times \mathrm{Tr}\left[\Gamma_H D_u^{-1}(n,m) \Gamma_H D_d^{-1}(m,n)\right]$$

$$\left\langle O_H(n) O_H^\dagger(m) \right\rangle_F = \langle \overline{d}(n) \Gamma_H u(n) \overline{u}(m) \Gamma_H d(m) \rangle_F = -\mathrm{Tr}\left[\Gamma_H D_u^{-1}(n,m) \Gamma_H D_d^{-1}(m,n)\right]$$



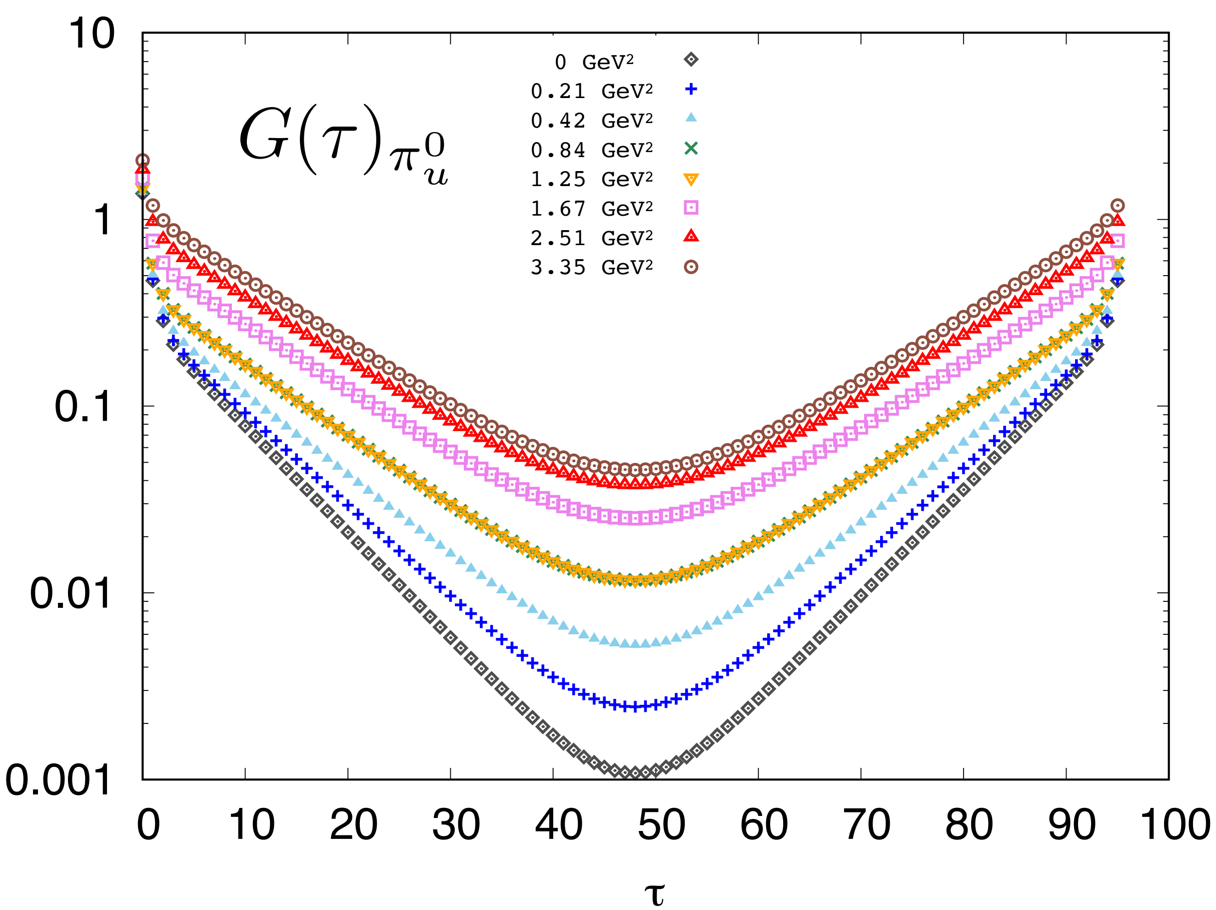


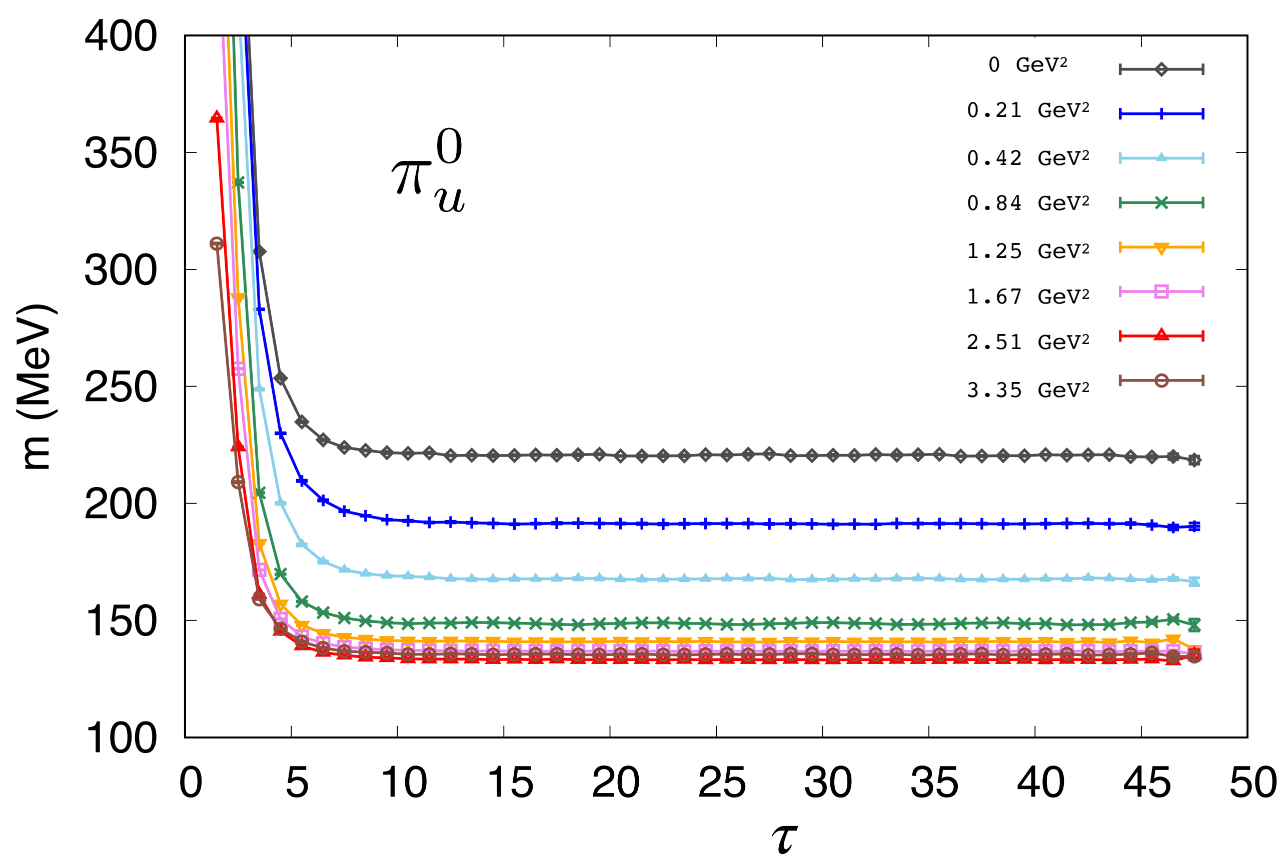
- $G(\tau)_{\pi_u^0}$ increase as the magnetic field grows

Correlator and Mass

- Effective mass $G(\tau) \equiv \langle O(\tau) O^\dagger(0) \rangle = \sum_n \langle 0 | O^\dagger | n \rangle \langle n | O | 0 \rangle e^{-\tau E_n}$

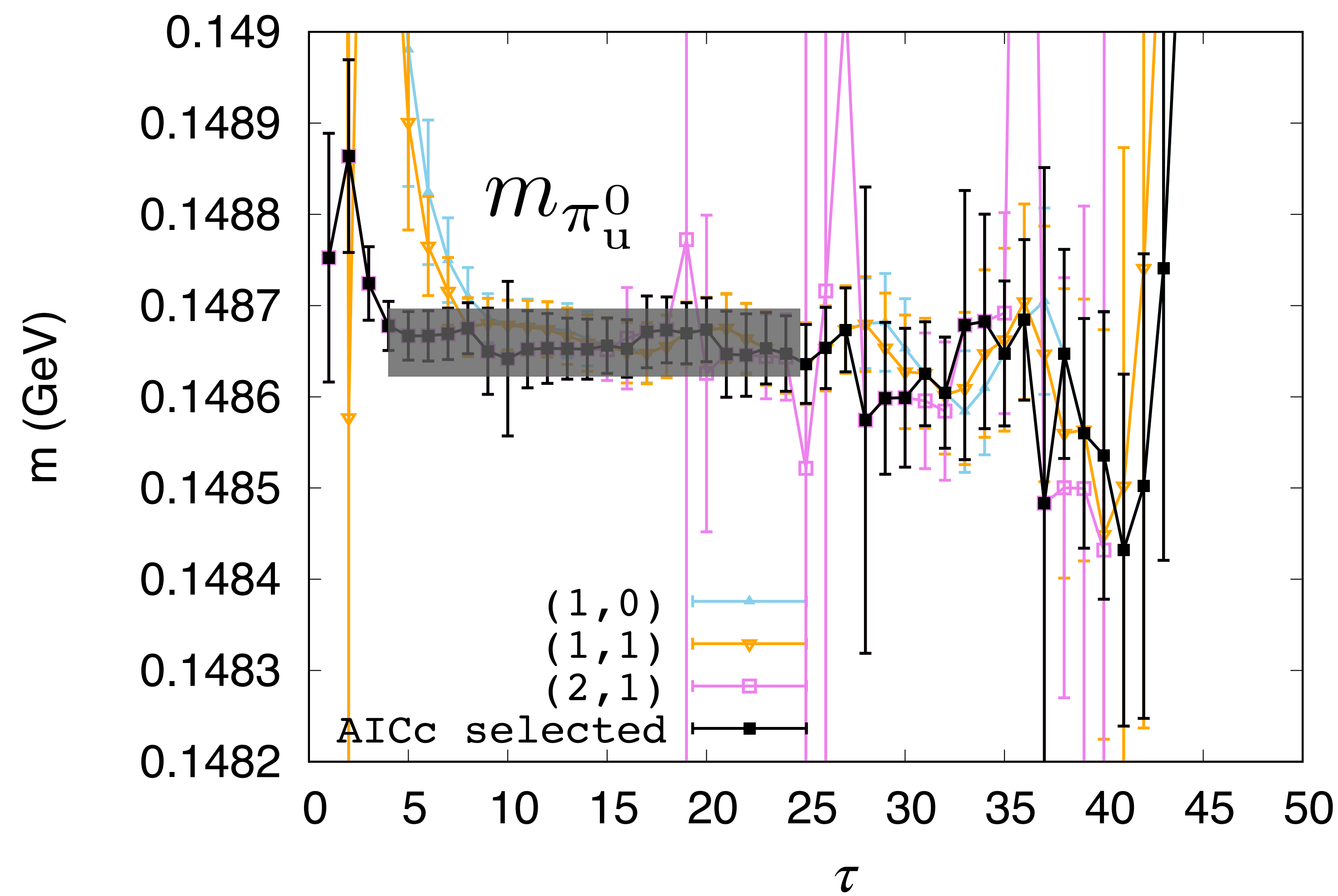
$$\frac{G(\tau)}{G(\tau+1)} = \frac{\cosh\left(m_{\text{eff}}\left(\tau - N_T/2\right)\right)}{\cosh\left(m_{\text{eff}}\left(\tau + 1 - N_T/2\right)\right)}$$ (with boundary condition)





Correlator and Mass

- Correlator fit



- fit ansatz for different number of states

$$G_{\text{fit}}(\tau) = \sum_{i=1}^{N_s,no} A_{no,i} \exp(-m_{no,i}\tau) - (-1)^{\tau} \sum_{i=0}^{N_s,osc} A_{osc,i} \exp(-m_{osc,i}\tau)$$

- choose the best fit by AICc (corrected Akaike Information criterion)

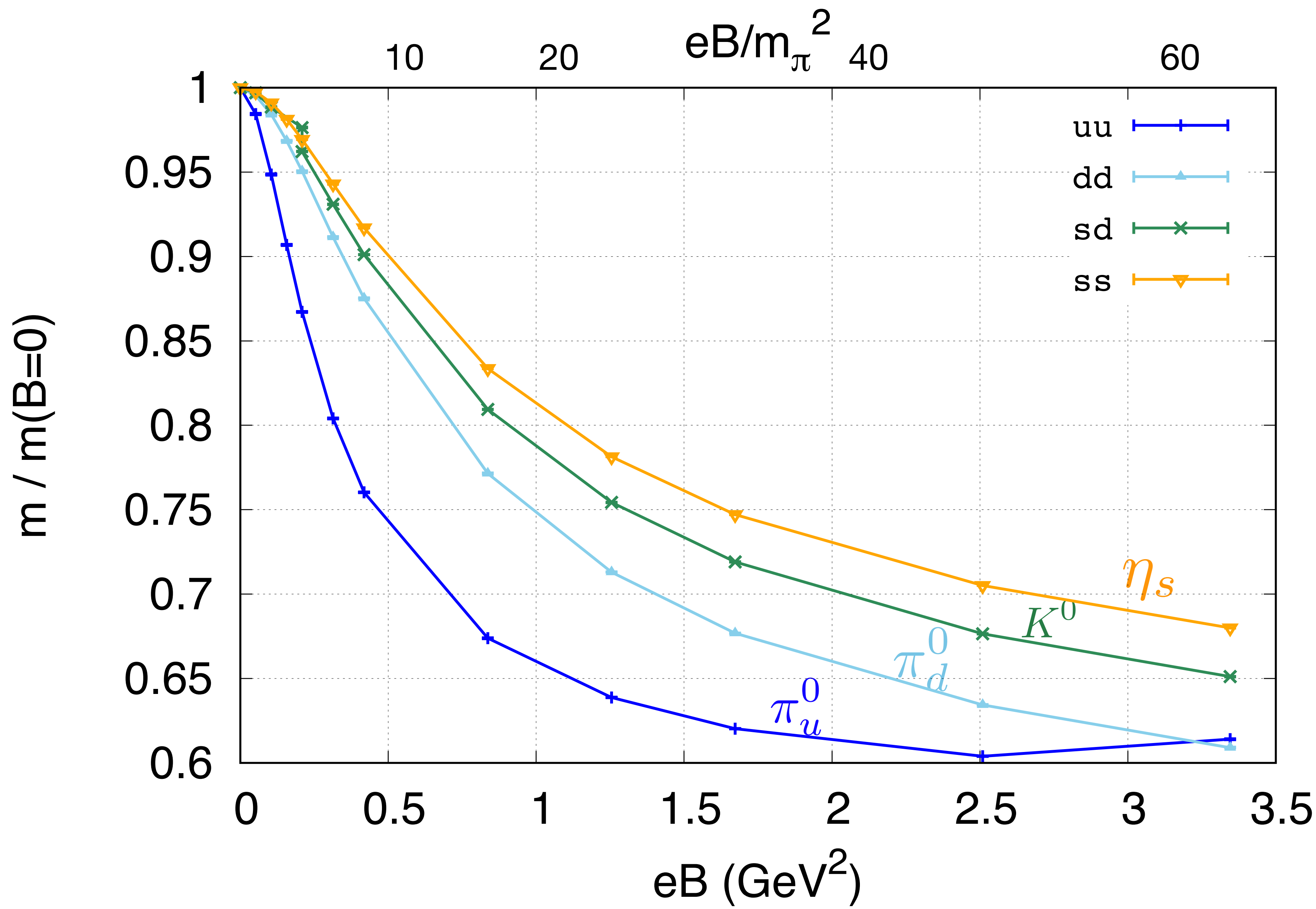
$$\text{AICc} = 2k - \ln(\hat{L}) + \frac{2k^2 + 2k}{n - k - 1}$$

- weighted average

H. Akaike 1997

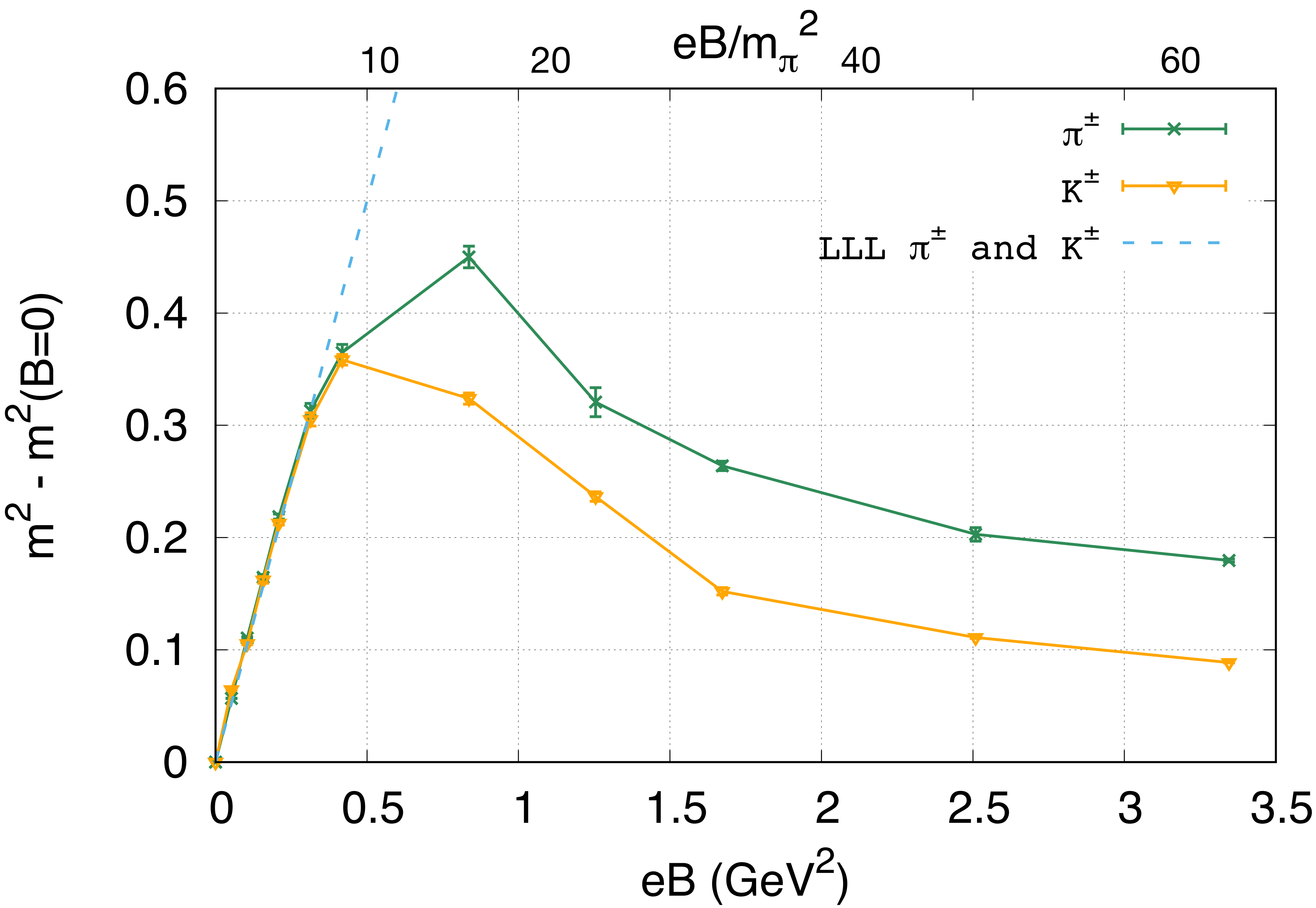
J. E. Cavanaugh 1997

Mass of Neutral Pseudo-scalar Particle



- Neutral PS particles' masses decrease as eB grows and saturate at large eB.
- Lighter hadrons more affected by magnetic field.
- Neutral PS particles have quite large (30~40%) mass reduction.

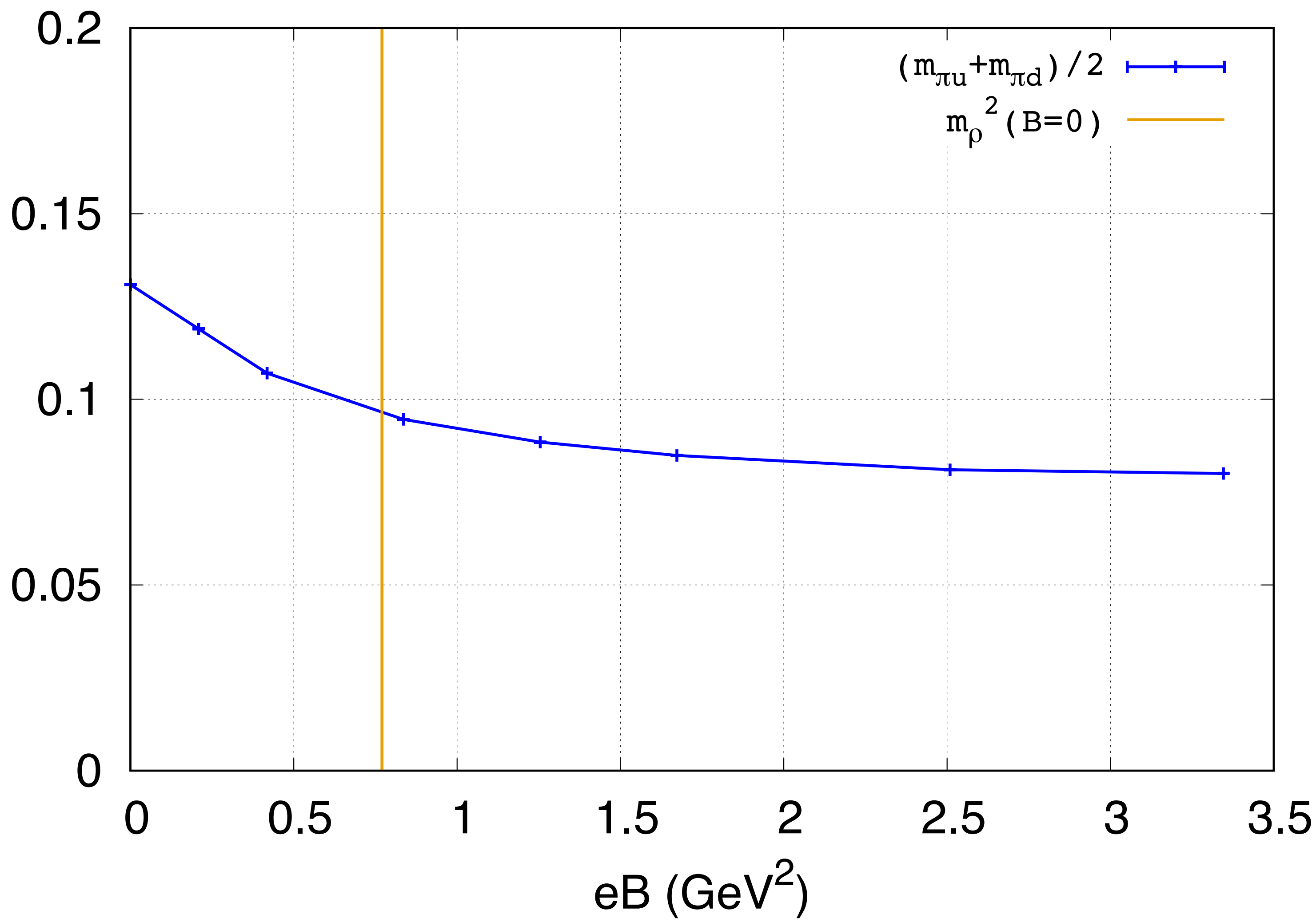
Charged PS Particle Mass



- In general, charged PS particles' masses have non-monotonous behavior, which is different from quenched lattice result.
- At $0 < \mathrm{eB} \lesssim 0.3\mathrm{GeV}^2$, PS particles' masses can be well described by Lowest Landau level approximation.

$$LLL\ approx: E^2 = p_z^2 + (2n+1)|qB| - gs_z qB + m^2 \longrightarrow \quad m_{\pi^\pm}^2(B) = m_{\pi^\pm}^2(B=0) + eB$$

ρ-meson condensation



- QCD inequality:

$$m_{\rho^\pm} \geq \left(m_{\pi_u^0} + m_{\pi_d^0}\right)/2$$

- NJL model

$$\mathrm{eB}_c \sim m_\rho^2(\mathrm{B}=0)$$

At $\mathrm{eB}_c \sim m_\rho^2(\mathrm{B}=0)$ point, π mass stays nonzero; the lower bound of ρ^+ meson mass stays nonzero.

➡ No ρ-meson condensation observed.

Summary and Outlook

Particle Mass		quenched Wilson/Overlap Bali *et al* (2018) Luschevskaya *et al* (2014)	dynamical stout Bali *et al* (2018)	dynamical HISQ
π^0		↘	↘	↘
π^+		↗		⤵
ρ^0	ρ_0^0	↘		?
	ρ_+^0	↗		
ρ^+	ρ_0^+	↗		
	ρ_+^+	↘		
	ρ_-^+	↗		

- eB-dependence of polarized vector channel particle mass
- pion decay constant
- magnetic polarizability...

Y. Hidaka and A.Yamamoto Phys. Rev. D 87, 094502 (2013)
Bali, Brandt *et al*., PHYS. REV. D 97, 034505 (2018)
O. Larina ,E.V. Luschevskaya *et al*., POS Lattice 2014

Lattice Setup

- (2+1) flavor Dynamical HISQ fermion at T=0
- Lattice size: 32^3 x 96, a^{-1} = 1.6852 GeV
  - eB-dependence
- m_π = 230 MeV
- $N_b = 0, 4, 8, 16, 24, 32, 48, 64$
  - $qB = a^{-2}\frac{2\pi N_b}{N_x N_y}$ with $\begin{cases} 0 \le N_b \le \frac{N_x N_y}{4} \\ N_b \in \mathbb{Z} \end{cases}$