Mass Spectra and Decay of Mesons under Strong External Magnetic Field

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August 19, 2019

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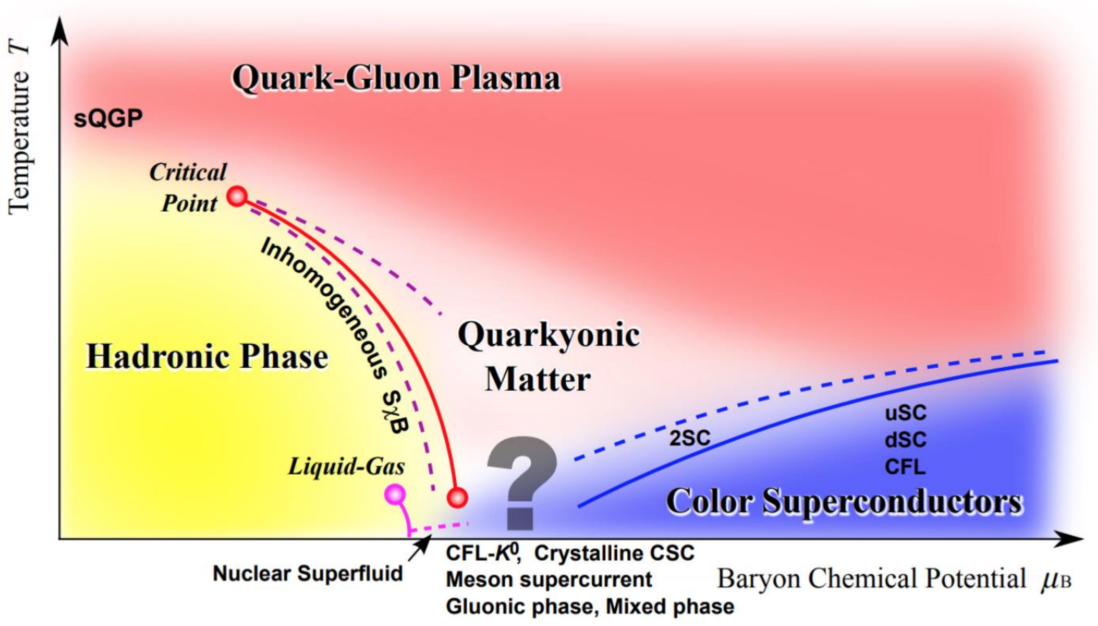
Motivation



The study of the state of quark matter under extreme conditions has attracted much attention . Extreme conditions include high temperatures and finite baryon chemical potentials, as well as strong magnetic field.

Here, we use two-flavor NJL model to study the effect of strong magnetic field on the properties of mesons.

Method （Considering the effect of magnetic field）

1. Ritus Scheme

$$S_f(x,y) = \sum_n \int \frac{d^3\tilde{p}}{(2\pi)^3} e^{-i\tilde{p}(x-y)} P_n(x_1,p_2) D_f(\bar{p}) P_n(y_1,p_2)$$

Shijun Mao, arXiv:1808.10242vl [nucl-th] 30 Aug 2018

M. Coppola, D. Gomez Dumm, N.N. Scoccola,arXiv:1802.08041vl[hep-ph]22 Feb 2018

2. Schwinger Scheme

$$S_f(x,y) = e^{i\Phi_f(x_\perp,y_\perp)} \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \widetilde{S}_f(p_\perp,p_\parallel)$$

Mei Huang, Pengfei Zhuang, Weiqin Chao,arXiv:9903304v1[hep-ph] 10 Mar 1999

Hao Liu, Xinyang Wang, Lang Yu, and Mei Huang，arXiv:1801.02174v1 [hep-ph] 7 jan 2018

Shijun Mao, arXiv:1808.10242vl [nucl-th] 30 Aug 2018

Difference: The way of definition of propagator is different, there are differences in calculation methods, but the theoretical results are basically same.

NJL model on mean field approximation

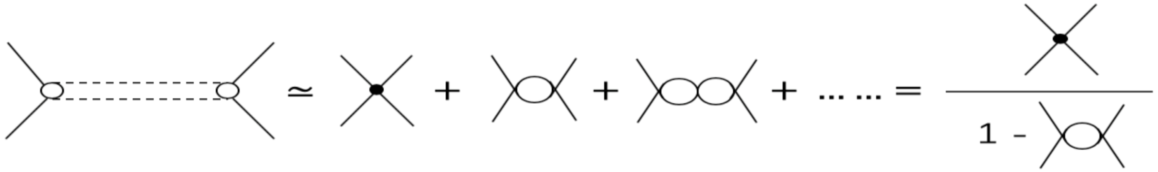
Two flavor Nambu--Jona-Lasinio model

$$\mathcal{L} = \bar{\psi}\left(i\gamma_\nu D^\nu - m_0\right)\psi + \frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right]$$

$$\mathbf{B} = (0, 0, B)$$

(1)Quark： mean field approximation

(2)Meson： RPA renormalization



$$\mathcal{D}_M(k) = \frac{2G}{1 - 2G\Pi_M(k)}$$

$$M = \sigma, \pi_0, \pi_+, \pi_-$$

Mean field approximation

Order parameters :

$$\mathcal{L} = \bar{\psi}\left(i\gamma_\mu D^\mu - m_0\right)\psi + G\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right]$$

Effective quark mass:

$$m_q = m_0 - 2G\langle\bar{\psi}\psi\rangle$$

Thermodynamic potential :

$$\Omega_{mf} = \frac{\left(m_q - m_0\right)^2}{4G} + \Omega_q$$

$$\Omega_q = -3\sum_{f=u,d}\sum_n \alpha_n \int \frac{dp_z}{2\pi}\frac{|Q_f B|}{2\pi}\left[\frac{E_f^+ + E_f^-}{2} + T\ln\left(\left(1 + e^{-\frac{E_f^+}{T}}\right)\left(1 - e^{-\frac{E_f^+}{T}}\right)\right)\right]$$

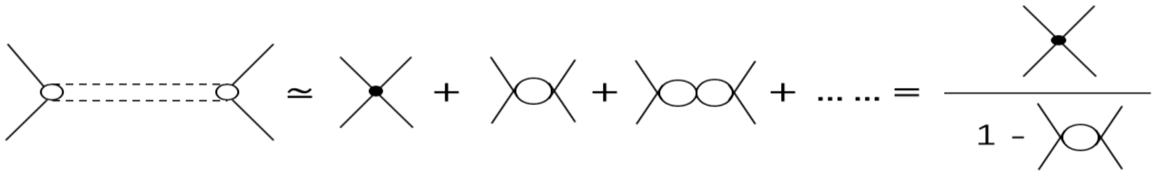
Gap equation:

$$\frac{\partial\Omega_{mf}}{\partial m_q} = m_q(1 - 2GJ_1) - m_0 = 0$$

$$J_1 = N_c \sum_{f,n} \alpha_n \frac{|Q_f B|}{2\pi}\int \frac{dp_3}{2\pi}\frac{\tanh\left(\frac{E_f}{2T}\right)}{E_f}$$

Meson

Meson propagator（Random Phase Approximation）



$M = \sigma, \pi_0, \pi_+, \pi_-$

Momentum space :

$$\mathcal{D}_M(k) = \frac{2G}{1 - 2G\Pi_M(k)}$$

Pole equation :

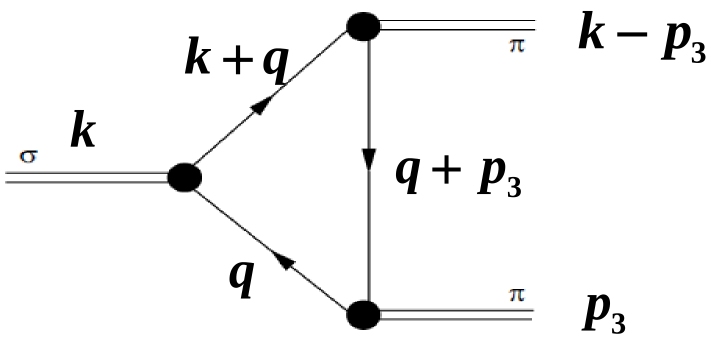
$$1 - 2G\Pi_M(\omega = m_M, \mathbf{0}) = 0$$

Finite temperature approximation :

$$\int \frac{d^4k}{(2\pi)^4} = iT\sum_n \int d^3 \frac{\mathbf{k}}{(2\pi)^3}$$

Decay process

$\sigma \to \pi\pi$ Decay



$$\frac{d\Gamma_{\sigma\to 2\pi}}{d\Omega} = \frac{1}{32\pi^2}\frac{|\mathbf{p}|}{m_\sigma^2}|M|^2$$

π momentum： $|\mathbf{p}| = \sqrt{\frac{m_\sigma^2}{4} - m_\pi^2}$

M matrix： $M = g_\sigma g_\pi^2 A_{\sigma\pi\pi}$

$$\Gamma_{\sigma\to 2\pi}(T,\mu) = \Gamma_{\sigma\to 2\pi^0}(T,\mu) + \Gamma_{\sigma\to\pi^+\pi^-}(T,\mu)$$

The coupling strength of π-quark and Sigma-quark in the equation

$$g_\pi^{-2}(T,\mu) = \frac{\partial\Pi_\pi(k_0,\mathbf{0};T,\mu)}{\partial k_0^2}\Big|_{k_0^2=m_\pi^2} \;,$$

$$g_\sigma^{-2}(T,\mu) = \frac{\partial\Pi_\sigma(k_0,\mathbf{0};T,\mu)}{\partial k_0^2}\Big|_{k_0^2=m_\sigma^2} \;,$$

Decay process

- $m_\pi = 2m_q$，The temperature is defined as Mott temperature： T_{Mott}
- $m_\sigma = 2m_\pi$，Temperature is defined as dissociation temperature： T_{diss}
- In the chiral limit，$T_{Mott} = T_{diss} = T_c$，$T_c$ is the critical temperature of phase transition
- When $T < T_{diss}$，σ meson can decay into two π mesons
- When $T > T_{Mott}$，π meson can decay into a pair of positive and negative quarks

Ritus scheme

Triangle diagram factor： $A_{\sigma\pi\pi}$

$$iA_{\sigma\pi\pi}(x,y,z) = -Tr[g_{\sigma qq} * \Gamma_\sigma * iS_u(x,y) * g_{\pi qq} * \Gamma_\pi * iS_u(y,z) * g_{\pi qq} * \Gamma_\pi * iS_u(z,x)]$$

Meson vertices：

$$\Gamma_M = \begin{cases} 1 & M = \sigma \\ i\tau + \gamma_5 & M = \pi_+ \\ i\tau - \gamma_5 & M = \pi_- \\ i\tau_3\gamma_5 & M = \pi_0 \end{cases} \quad \Gamma_M^* = \begin{cases} 1 & M = \sigma \\ i\tau - \gamma_5 & M = \pi_+ \\ i\tau + \gamma_5 & M = \pi_- \\ i\tau_3\gamma_5 & M = \pi_0 \end{cases}$$

Propagator in coordinate ：

$$S_f(x,y) = \sum_n \int \frac{d^3\tilde{p}}{(2\pi)^3} e^{-i\tilde{p}(x-y)} P_n(x_1,p_2) D_f(\bar{p}) P_n(y_1,p_2)$$

$$D_f^{-1}(\bar{p}) = \gamma \cdot \bar{p} - m_q,$$

$$P_n(x_1,p_2) = \frac{1}{2}[g_n^{sf}(x_1,p_2) + I_n g_{n-1}^{sf}(x_1,p_2)] + \frac{is_f}{2}[g_n^{sf}(x_1,p_2) - I_n g_{n-1}^{sf}(x_1,p_2)]\gamma_1\gamma_2$$

Ritus conservation momentum： $\bar{p} = (p_0, 0, -s_f\sqrt{2n|Q_fB|}, p_3)$

(n is quark energy level in a magnetic field)

$$S_f = \mathrm{sgn}(Q_f B)$$

$$g_n^{sf}(x_1,p_2) = \emptyset_n(x_1 - s_f p_2/|Q_f B|)$$

$$\emptyset_n(\zeta) = (2^n n! \sqrt{\pi}|Q_f B|^{-1/2})^{-\frac{1}{2}} e^{-\frac{\zeta^2|Q_f B|}{2}} H_n(\zeta/|Q_f B|^{-1/2})$$

$H_n(\zeta)$ is Hermit polynomial

Calculation process

$$iA_{\sigma\pi\pi}(x,y,z) = -Tr[g_{\sigma qq} * \Gamma_\sigma * iS_u(x,y) * g_{\pi qq} * \Gamma_\pi * iS_u(y,z) * g_{\pi qq} * \Gamma_\pi * iS_u(z,x)]$$

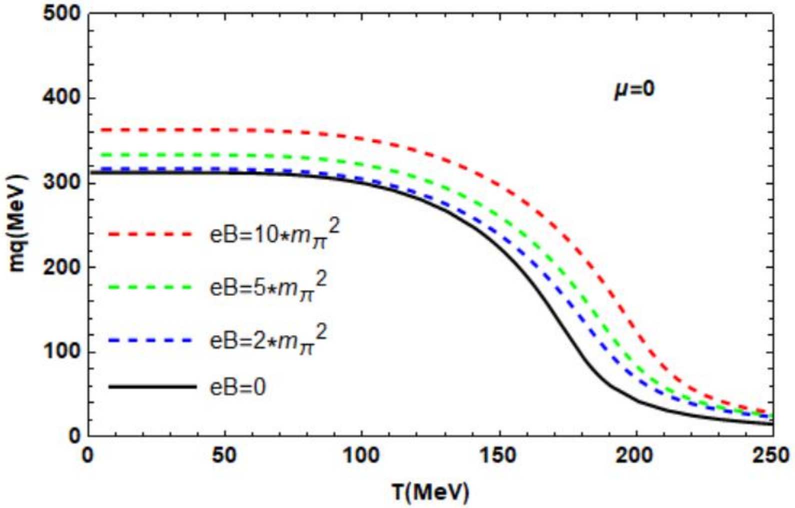
$$\therefore \quad A_{\sigma\pi\pi}(x,y,z) = 2i^6 N_c g^3 Tr[\mathbf{1} \cdot S_u(x,y) \cdot r_5 \cdot S_u(y,z) \cdot r_5 \cdot S_d(z,x)]$$

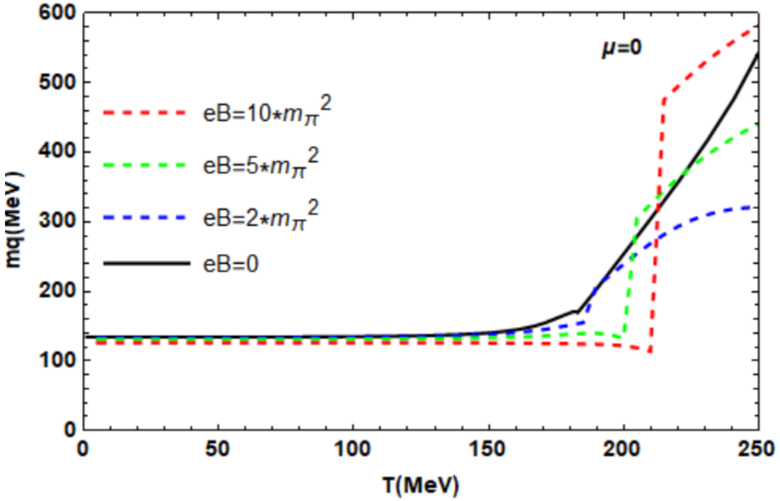
$$= 2i^2 N_c g^3 \sum_{n,n',n''} \int \frac{d^3\tilde{\mathrm{p}} d^3\tilde{t} d^3\tilde{q}}{(2\pi)^9} e^{-i\tilde{q}(x-y)-i\tilde{t}(y-z)-i\tilde{\mathrm{p}}(z-x)}$$

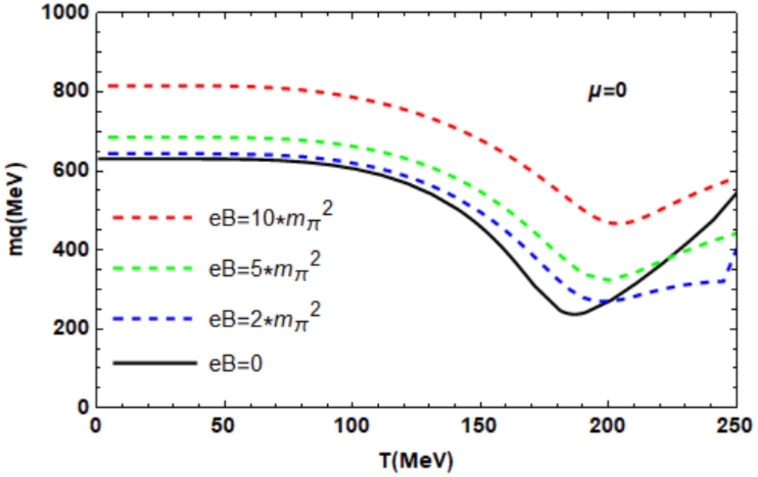
$$\cdot Tr[P_n(x_1,q_2) \frac{r \cdot \bar{q} + m_q}{\bar{q}^2 - m_q^2} P_n(y,q_2) \cdot P_{n'}(y_1,t_2) \frac{-r \cdot \bar{t} + m_q}{\bar{q}^2 - m_q^2} P_{n'}(z_1,t_2)$$

$$\cdot P_{n''}(z_1,\mathrm{p}_2) \frac{r \cdot \bar{p} + m_q}{\bar{q}^2 - m_q^2} P_{n''}(x_1,p_2)]$$

Neutral meson mass spectra under strong magnetic field





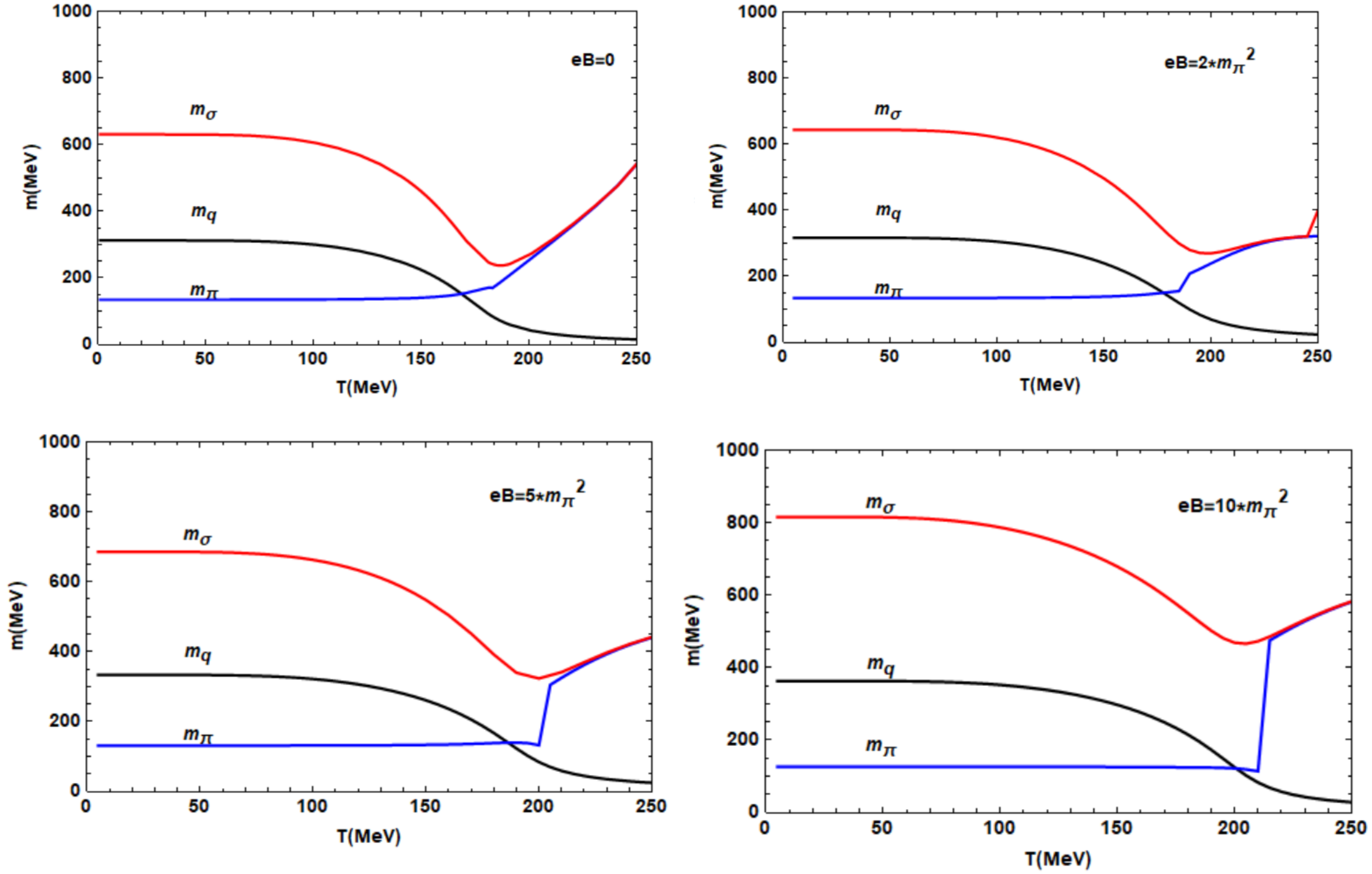


Since the mass of the flowing quark is not zero, the symmetry is obviously broken, and the constituent quarks have mass. Near the critical temperature, the quark mass and meson mass change significantly.

The presence of the magnetic field breaks the isospin symmetry and causes the mass splitting between the neutral and charged mesons. In this case, we're only thinking about neutral mesons.

The mass transition of the neutral pions in the figure is caused by the discrete quark energy levels in the magnetic field. The mass transition of the neutral pions at the T_{Mott} point is a non-trivial magnetic field effect. When the magnetic field disappears, the meson mass becomes continuous.

Neutral meson mass spectra under strong magnetic field

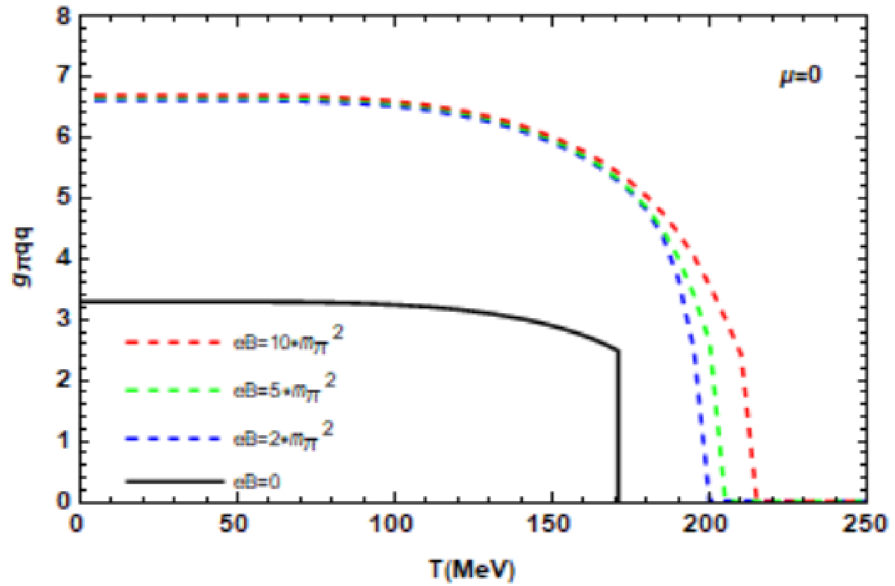


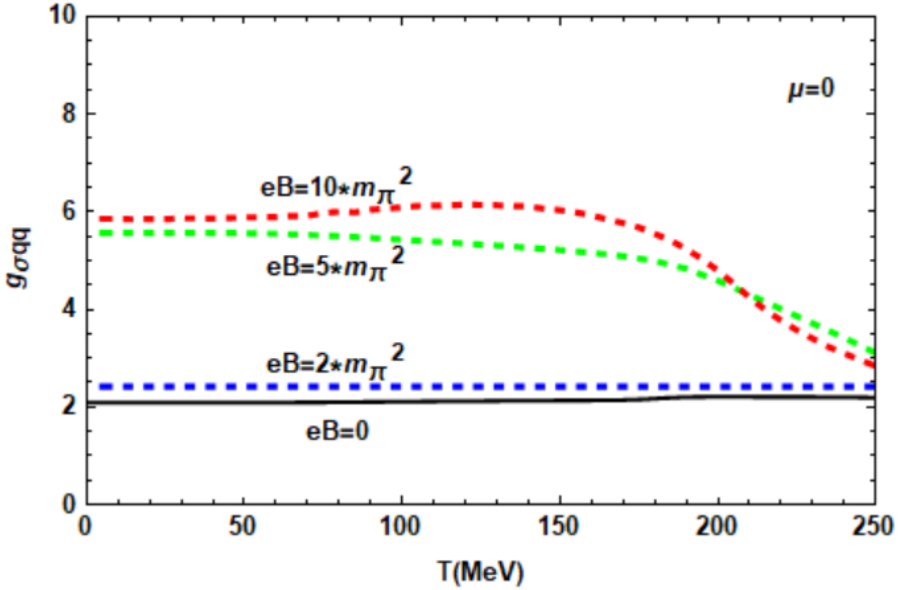
The temperature-dependent transition is a crossover.

In the crossover region, the sigma particle mass increases with the magnetic field and the π mass decreases slightly with the magnetic field.

In the break recovery region, the sigma meson mass degenerates to the π mass.

Coupling constant

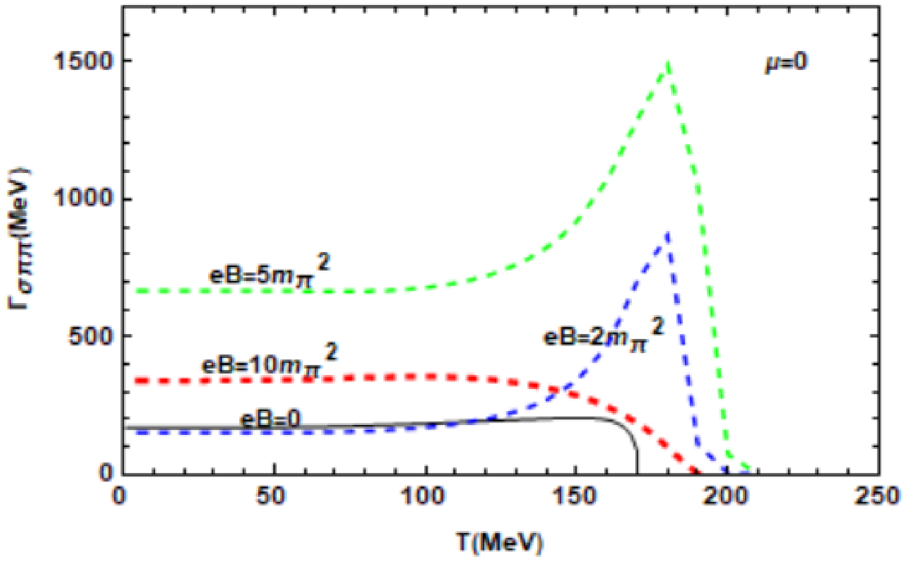




With the addition of the external magnetic field, the coupling constant increases significantly, and with the increase of the magnetic field, the coupling constant increases gradually, and the critical temperature also increases gradually.

The results here are different from those of lattice point QCD, mainly due to the effect of anti-magnetic catalysis.

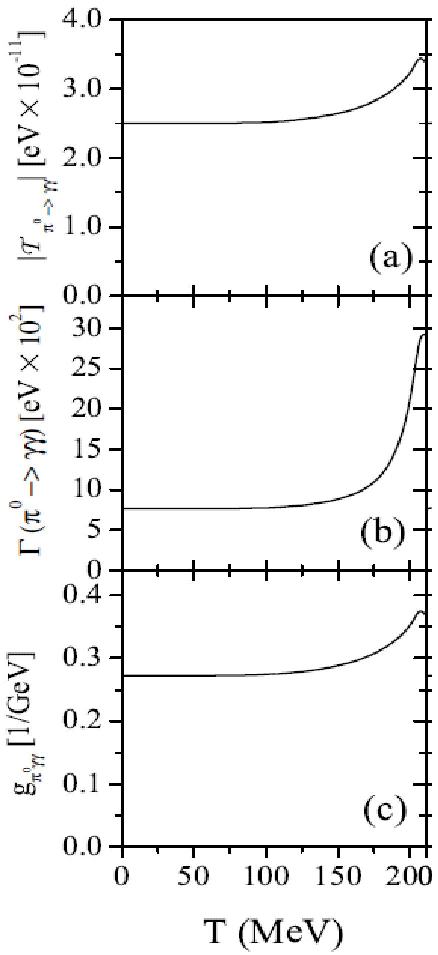
Decay width



Under the corresponding magnetic field, the decay width at T_{Mott} shows a sharp increase, and the sharp increase in the decay width reflects the trend of chiral symmetry restoration.

When the decay width is zero, the time that it takes to decay is infinite, which means that the decay process stops.

Decay width



arXiv:hep-ph/0403263

Our results on the decay width can be obtained from the study of the width of the neutral pions decay into photons in [arXiv:hep-ph/0403263]. Similar results were obtained for different neutral mesons.

However, when the external magnetic field is large, it can be found that the decay width is restricted and the decay range becomes smaller.

Here, the explanation of the disappearance of Mott peak can be referred to [NPA 622(1997) 478-496]. It is that Mott peak changes significantly with the change of phase transition order, and the peak value disappears when the phase transition order becomes weaker.

Summary and Outlook

1. The neutral meson is also affected by the external magnetic field, which is mainly due to the "magnetization" of the charged quark that makes up the meson, which affects the properties of the meson, such as meson mass, coupling constant, decay width and so on.

2. Here, the effect of the magnetic field is to increase the critical temperature in the mass spectrum, thereby increasing the symmetry breaking region.

3. At the same time, the existence of magnetic field breaks the isospin symmetry, and the separation of quark energy levels leads to the mass splitting of neutral pions.

4. In this work, a general method is used, which can be extended to other charged mesons, such as K and ρ mesons, and further research will be carried out later.

Thank you！