

Revisiting Chiral Kinetic Theory from High Density Effective Theory

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This talk is based on...

Chiral Kinetic Theory from Effective Field Theory Revisited

Shu Lin, Aradhya Shukla, *JHEP* 06 (2019) 60,
arXiv 1901.01528 [hep-ph]

Plan of the Talk

- Introduction
 - Chiral Kinetic Theory and High Density Effective Theory
- Gauge Covariant Wigner Function and Equations of Motion (EOM)
- Reparametrization Invariance of Classical Action and EOMs
- Chiral Kinetic Theory from High Density Effective Theory
 - Transport and Constitutive Equations
- Equivalence of Chiral Kinetic Theories
- Conclusions

Introduction

Chiral Kinetic Theory and Berry Phase

- Kinetic theory has many applications in nuclear physics, astrophysics, cosmology and condensed matter physics. Earlier it's relativistic version misses the effect triangle anomaly

$$\partial_\mu J_5^\mu = \frac{1}{4\pi^2} \mathbf{E} \cdot \mathbf{B}$$

(Breaking of the axial symmetry at quantum level !)

- For Fermi liquid, this deficiency has been fixed by including the effect of Berry Phase and Berry Curvature¹ $\Omega_p = \pm \frac{\hat{\mathbf{p}}}{2|\mathbf{p}|}$
- The effect of Berry phase and curvature modify the particle number current together with transport equation.

¹Berry, Proc. R. Soc. A (1984)

Son, Yamamoto, PRD (2012) PRL (2012),

Stephanov, Yin, PRL (2012),

Gao, Liang, Pu, Q. Wang, X.-N. Wang, PRL (2012), PRD (2014),

Hidaka, Pu, Yang, PRD (2016)

Introduction

Chiral Anomaly and Chiral Kinetic Theory

Chiral Kinetic Theory is obtained (Talks by Xu-Guang Huang, Shu Lin, Jianhua Gao,...)

- Field Theory Approach²

$$\left[\Delta_0 + \hat{\mathbf{p}}^i \left(1 + \frac{\mathbf{B} \cdot \hat{\mathbf{p}}}{2p^2} \right) \Delta_i - \frac{\epsilon^{ijk} \hat{\mathbf{p}}^j \mathbf{E}^k}{2p^2} \Delta_i \right] n(X, l) = 0$$

- Effective Field Theory Approach

- High density Effective Theory (HDET)

[*Son, Yamamoto, PRD (2014)*]

- On-Shell Effective Field Theory (OSEFT)

$$\left[\Delta_0 + \hat{\mathbf{p}}^i \left(1 + \frac{\mathbf{B} \cdot \hat{\mathbf{p}}}{2p^2} \right) \Delta_i - \frac{\epsilon^{ijk} \hat{\mathbf{p}}^j \mathbf{E}^k + \mathbf{B}_\perp^i}{4p^2} \Delta_i \right] n_v(X, l) = 0.$$

[*Carignano, Manuel, Torres-Rincon, PRD (2018)*]

²Hidaka, Pu, Yang, PRD (2017), PRD (2018),
Gao, Liang, Pu, Q. Wang, X.-N. Wang, PRD (2011),
Huang, Shi, Jiang, Liao, Zhuang, PRD (2018)

Introduction

High Density Effective Theory

- High density effective theory (HDET) valid in the vicinity of Fermi surface.
- Constructed by removing the fast modes from Lagrangian

$$\mathcal{L}_0 = \bar{\psi}(i\gamma^\mu D_\mu)\psi + \mu\bar{\psi}\gamma^0\psi,$$

With $E_\pm = \mu \pm |\mathbf{p}|$: Energy of (anti-)particles

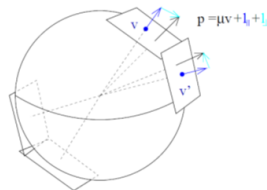
- At low energy, particles with $E_+ \sim 0 \implies$ **slow modes**
antiparticles with $E_- \sim -2\mu \implies$ **fast modes**

High Density Effective Theory

- Decomposing the energy and momentum of fermions as³

$p^0 = \mu + l^0$ and $\mathbf{p} = \mu\mathbf{v} + \mathbf{l}$ with $l^0, \mathbf{l} \ll \mu$ with

$$\psi(x) = \sum_{\mathbf{v}} e^{i\mu\mathbf{v}\cdot\mathbf{x}} [\psi_{+\mathbf{v}}(x) + \psi_{-\mathbf{v}}(x)]$$



Effective Lagrangian:

$$\begin{aligned} \mathcal{L}_{eff} &= \psi_{+v}^\dagger \sum_n D^{(n)} \psi_{+v} \\ &= \psi_{+v}^\dagger \left[i\mathbf{v} \cdot \mathbf{D} + \frac{\not{D}_\perp^2}{2\mu} + \frac{\not{D}_\perp (-i\mathbf{v} \cdot \mathbf{D}) \not{D}_\perp}{4\mu^2} \right] \psi_{+v}, \end{aligned}$$

uto order $O(1/\mu^2)$

³Hong, PLB (1998), NPB (2000)
 Schafer NPA (2003)
 Son, Yamamoto, PRD (2013)

Equations of Motion

- Equation of motion emerging from the effective Lagrangian is satisfied by the two-point function

$$\begin{aligned} \mathcal{D}_x G_v(x, y) &= 0, & G_v(x, y) \mathcal{D}_y^\dagger &= 0, \\ P_- G_v(x, y) &= 0, & G_v(x, y) P_- &= 0 \end{aligned} \quad \text{[Projection Condition]}$$

- The gauge invariant Wigner function is⁴

$$\tilde{G}_v(X, l) = \int_s e^{il \cdot s} G_v(X + s/2, X - s/2) U(X - s/2, X + s/2).$$

with Wilson line defined as

$$U(y, x) = P \exp \left[-i \int_x^y dz^\mu A_\mu(z) \right],$$

path ordering P from y to x .

⁴Elze, NPB (1986)

Vasak, Ann. of Phys., (1987)

Equations of Motion

The terms $I_{\pm}^{(n)} = \int_s e^{il \cdot s} \left(\mathcal{D}_x^{(n)} G_v(x, y) \pm G_v(x, y) \mathcal{D}_y^{(n)\dagger} \right)$
with $n = 0, 1, 2$ result into

$$\begin{aligned} I_+^{(0)} &= 2v \cdot \bar{l} \tilde{G}_v, & I_-^{(0)} &= i v^\mu \Delta_\mu \tilde{G}_v, \\ I_+^{(1)} &= \frac{1}{\mu} \left[-\bar{l}_\perp^2 + \mathbf{B} \cdot \mathbf{v} \right] \tilde{G}_v, & I_-^{(1)} &= \frac{i}{\mu} \bar{l}_\perp^\mu \Delta_\mu \tilde{G}_v, \\ I_+^{(2)} &= \frac{1}{4\mu^2} \left[4\bar{l}_\parallel \bar{l}_\perp^2 - 4\bar{l}_\parallel (\mathbf{B} \cdot \mathbf{v}) + 2\mathbf{B} \cdot \bar{\mathbf{l}}_\perp + 2(\mathbf{E} \times \bar{\mathbf{l}}) \cdot \mathbf{v} \right] \tilde{G}_v \\ I_-^{(2)} &= -\frac{i}{4\mu^2} \left[\left(4\bar{l}_\parallel \bar{l}^\mu - \bar{v}^\mu (\bar{l}_\perp^2 - \mathbf{B} \cdot \mathbf{v}) \right) \Delta_\mu - \left(\varepsilon^{ijk} v^k \bar{v}_\mu F^{i\mu} \right) \Delta_j \right] \tilde{G}_v, \end{aligned}$$

$\Delta_\mu = \partial_\mu - F_{\mu\nu} \frac{\partial}{\partial l_\nu}$ and $\bar{l}^\mu = (l^\mu - A^\mu)$: kinetic momentum of particle.

Disagree with Son and Yamamoto, PRD (2013) at $O(1/\mu^2)$.

Equations of Motion

From $I_+^{(n)}$ terms

$$\begin{aligned}\tilde{G}_v &= 2\pi P_+ \delta\left(l_0 - l_{\parallel} - \frac{1}{2\mu}[l_{\perp}^2 - \mathbf{B} \cdot \mathbf{v}] + \frac{1}{2\mu^2}[l_{\parallel}(l_{\perp}^2 - \mathbf{B} \cdot \mathbf{v})]\right. \\ &\quad \left. + \frac{1}{4\mu^2}[\mathbf{B} \cdot \mathbf{l}_{\perp} + (\mathbf{E} \times \mathbf{l}) \cdot \mathbf{v}]\right) n_v(X, l),\end{aligned}$$

$n_v(X, l)$: distribution function.

- **PUZZLE:** δ -function \Rightarrow dispersion relation depends on \mathbf{v} and not invariant under Reparametrization!

Reparametrization Invariance and Frame Dependence

- The momentum decomposition⁵: $p^\mu = \mu v^\mu + l^\mu$ is not unique.
- $v^\mu \rightarrow v^{\mu'} = v^\mu + \delta v^\mu$, $l^\mu \rightarrow l^{\mu'} = l^\mu - \mu \delta v^\mu$, $v \cdot \delta v = 0$.

Under reparametrization transformation (RT)

$$\begin{aligned}\delta\psi_v &= i\mu\delta v \cdot x\psi_v - \frac{\delta v}{2}\left(1 - \frac{1}{2\mu + i\bar{v} \cdot D}i\not{D}_\perp\right)\psi_v, \\ \delta\psi_v^\dagger &= -i\mu\delta v \cdot x\psi_v^\dagger - \psi_v^\dagger\left(1 - i\not{D}_\perp \frac{1}{2\mu + i\bar{v} \cdot D}\right)\frac{\delta v}{2}.\end{aligned}$$

$$\delta\mathcal{L} = \psi_v^\dagger i v \cdot D \psi_v + \psi_v^\dagger \not{D}_\perp \frac{1}{2\mu + i\bar{v} \cdot D} \not{D}_\perp \psi_v = 0.$$

⁵Killian, Ohl, PRD (1994),
Finkemeier, Georgi, Irvin, PRD (1997)
Sundrum, PRD (1998)

Reparametrization Invariance Equations of Motion

Under RT

$$\text{tr } \delta \tilde{G}_v(X, l) = \frac{1}{4\mu} \delta v_j \Delta_i v^k \varepsilon^{ijk} \text{tr } \tilde{G}_v(X, l) + \frac{1}{2\mu} \delta v_j l_i \Delta_{ij} \text{tr } \tilde{G}_v(X, l)$$

Arises from antiparticle contribution

with $\Delta_{ij} = \delta_{ij} - v_i v_j$. Wigner function is not invariant under RT!

$$\delta I_{\pm}^{(n)} = \int_s e^{il \cdot s} (\mathcal{D}_x^{(n)} G_v(x, y) \pm G_v(x, y) \mathcal{D}_y^{\dagger(n)}) = 0, \quad \text{Invariant under RT}$$

Contributions of Differential operators and Gauge invariant Wigner function cancels each other.

Reparametrization Invariance Equations of Motion

$$\begin{aligned}I_+^{(0)} &= 2\mathbf{v} \cdot \bar{\mathbf{l}} \tilde{G}_v, & I_-^{(0)} &= i v^\mu \Delta_\mu \tilde{G}_v, \\I_+^{(1)} &= \frac{1}{\mu} \left[-\bar{l}_\perp^2 + \mathbf{B} \cdot \mathbf{v} \right] \tilde{G}_v, & I_-^{(1)} &= \frac{i}{\mu} \bar{l}_\perp^\mu \Delta_\mu \tilde{G}_v, \\I_+^{(2)} &= \frac{1}{4\mu^2} \left[4\bar{l}_\parallel \bar{l}_\perp^2 - 4\bar{l}_\parallel (\mathbf{B} \cdot \mathbf{v}) + 2\mathbf{B} \cdot \bar{\mathbf{l}}_\perp + 2(\mathbf{E} \times \bar{\mathbf{l}}) \cdot \mathbf{v} \right] \tilde{G}_v \\I_-^{(2)} &= -\frac{i}{4\mu^2} \left[\left(4\bar{l}_\parallel \bar{l}^\mu - \bar{v}^\mu (\bar{l}_\perp^2 - \mathbf{B} \cdot \mathbf{v}) \right) \Delta_\mu - \left(\varepsilon^{ijk} v^k \bar{v}_\mu F^{i\mu} \right) \Delta_j \right] \tilde{G}_v,\end{aligned}$$

Dispersion relation and CKE depends on \mathbf{v} and not unique

Expected: antiparticle contribution is \mathbf{v} dependent!

Transport Equation

We use a natural scheme by making a choice $\mathbf{l} \parallel \mathbf{v}$, or equivalently $l_{\parallel} = l$, $l_{\perp} = 0$ ⁶

$$\begin{aligned}I_{+}^{(0)} &= 2v \cdot l \tilde{G}_v, & I_{-}^{(0)} &= iv^{\mu} \Delta_{\mu} \tilde{G}_v, \\I_{+}^{(1)} &= \frac{\mathbf{B} \cdot \mathbf{v}}{\mu} \tilde{G}_v, & I_{-}^{(1)} &= 0, \\I_{+}^{(2)} &= -\frac{\mathbf{B} \cdot \mathbf{v} l}{\mu^2} \tilde{G}_v, \\I_{-}^{(2)} &= \frac{1}{4\mu^2} \left[-i\bar{v}^{\mu} \mathbf{B} \cdot \mathbf{v} \Delta_{\mu} + i\bar{v}^{\nu} \epsilon^{ijm} v^m F_{i\nu} \Delta_j \right] \tilde{G}_v.\end{aligned}$$

⁶Hands, PRD (2004)

Transport Equation

- Combining plus equations as

$$I_+^{(0)} + I_+^{(1)} + I_+^{(2)} = \left[2(l_0 - l) + \frac{\mathbf{B} \cdot \mathbf{v}}{\mu} - \frac{\mathbf{B} \cdot \mathbf{v} l}{\mu^2} \right] \tilde{G}_v.$$

In terms of original momentum $p_0 = p - \frac{\mathbf{B} \cdot \hat{\mathbf{p}}}{2p}$

Same as dispersion relation for particle in magnetic field.

- Using $P_+ \tilde{G}_v = \tilde{G}_v P_+ = \tilde{G}_v$, \tilde{G}_v can be parametrize as

$$\tilde{G}_v = 2\pi \delta \left(l_0 - l + \frac{\mathbf{B} \cdot \mathbf{v}}{2\mu} - \frac{\mathbf{B} \cdot \mathbf{v} l}{2\mu^2} \right) n_v(X, l) P_+.$$

n_v is the distribution function

Transport Equation

In terms of full momentum $\mathbf{p} = \mu\mathbf{v} + \mathbf{l}$:

$$\left[\Delta_0 + \hat{\mathbf{p}}^i \left(1 + \frac{\mathbf{B} \cdot \hat{\mathbf{p}}}{2p^2} \right) \Delta_i - \frac{\epsilon^{ijk} \hat{\mathbf{p}}^j \mathbf{E}^k + \mathbf{B}_\perp^i}{4p^2} \Delta_i \right] n_v(X, l) = 0.$$

Agrees with OSEFT approach upon identifying a cut-off between two theory⁷.

CKE from field theory approach⁸

$$\left[\Delta_0 + \hat{\mathbf{p}}^i \left(1 + \frac{\mathbf{B} \cdot \hat{\mathbf{p}}}{2p^2} \right) \Delta_i - \frac{\epsilon^{ijk} \hat{\mathbf{p}}^j \mathbf{E}^k}{2p^2} \Delta_i \right] n(X, l) = 0$$

⁷Cariganano, Manuel, Torres-Rincon, PRD (2018)

⁸Hidaka, Pu, Yang, PRD (2017), PRD (2018)

Constitutive Equation

From $j^\mu = \psi^\dagger \sigma^\mu \psi$, particle number and total current is

$$\begin{aligned}n &= \frac{1}{(2\pi)^4} \int_l \left(1 + \frac{1}{2\mu^2} [\mathbf{B} \cdot \mathbf{v}] \right) \text{tr} \tilde{G}_v(X, l), \\j^i &= \frac{1}{(2\pi)^4} \int_l \left[v^i + \frac{1}{2\mu} \Delta_j v^k + \frac{1}{4\mu^2} \left(-\partial_{Xj} l_\nu \varepsilon^{ijm} v^m \bar{v}^\nu \right. \right. \\&\quad \left. \left. - 2\mathbf{B} \cdot \mathbf{v} v^i + F_{\nu j} \bar{v}^\nu v^m \varepsilon^{ijm} \right) \right] \text{tr} \tilde{G}_v(X, l).\end{aligned}$$

We used scheme condition $\mathbf{l} \parallel \mathbf{v}$ to simplify the expression

Agrees with [Manuel et al. PRD (2018)] after identifying a cut-off.

Equivalence of Chiral Kinetic Equation

Distribution function n and n_v are coefficients of δ -function of \tilde{G} and \tilde{G}_v

$$\tilde{G} = \int_s e^{ip \cdot s} \psi(x) \psi^\dagger(y) U(y, x), \quad \tilde{G}_v = \int_s e^{il \cdot s} \psi_v(x) \psi_v^\dagger(y) U(y, x).$$

using $\psi(x) = e^{i\mu \mathbf{v} \cdot \mathbf{x}} \left(1 + \frac{1}{2\mu} (-i \not{D}_\perp) \right) \psi_v(x)$

ψ_v : Dressed particle

$$\text{tr} \tilde{G} = \text{tr} \tilde{G}_v - \frac{1}{4\mu^2} l_i \Delta_j \text{tr} \tilde{G}_v \epsilon^{ijm} v^m \Rightarrow n = n_v - \frac{1}{4\mu^2} l_i \Delta_j n_v \epsilon^{ijm} v^m$$

Equivalence of Chiral Kinetic Equations

$$\left[\Delta_0 + \hat{\mathbf{p}}^i \left(1 + \frac{\mathbf{B} \cdot \hat{\mathbf{p}}}{2p^2} \right) \Delta_i - \frac{\epsilon^{ijk} \hat{\mathbf{p}}^j \mathbf{E}^k}{2p^2} \Delta_i \right] n(X, l) = 0$$

[CKE from Field Theory formalism]



$$\left[\Delta_0 + \hat{\mathbf{p}}^i \left(1 + \frac{\mathbf{B} \cdot \hat{\mathbf{p}}}{2p^2} \right) \Delta_i - \frac{\epsilon^{ijk} \hat{\mathbf{p}}^j \mathbf{E}^k + \mathbf{B}_\perp^i}{4p^2} \Delta_i \right] n_v(X, l) = 0$$

[CKE from HDET formalism]

Conclusions

- We revisit CKT and find it differs from its counterpart from field theory approach at higher order of $(1/\mu)$. It agrees with the CKE obtained by OSEFT.
- Despite the disagreement, both CKE obtained from field theory and effective field theory formalisms are equivalent with the difference being choices of degrees of freedom.
- Under Reparametrization transformation of Fermi velocity \mathbf{v} , distribution function and CKE both transforms. A specific choice $\mathbf{v} \parallel \mathbf{1}$ results into our CKE.

Thanks...