Nuclear Science
Computing Center at CCNU

Dirac eigenvalue spectrum of Nf =2+1 QCD towards the chiral limit using HISQ action

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Outline

1. Motivation
2. Chiral & $U(1)_A$ symmetry and Dirac eigenvalue spectrum
3. Results
4. Summary

Motivation

- The magnitude of $U(1)_A$ symmetry breaking could affect the nature of phase transition for QCD with two massless quark flavors.

  — Substantial $U(1)_A$ breaking

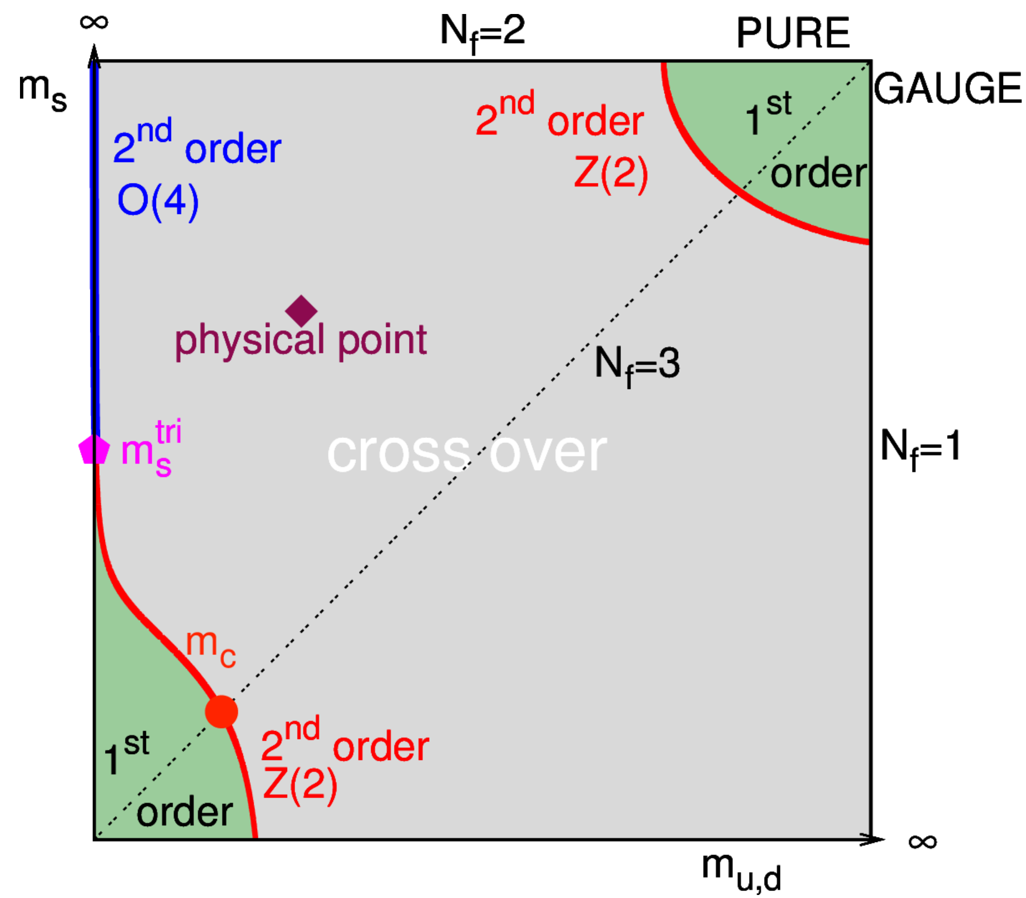
  Critical behavior: O(4)

  Second order phase transition

  — Strong suppression of the anomaly

  Critical behavior: $O(2)\times O(4)$

  First order phase transition



H.-T. Ding et al., arXiv: 1504.05274

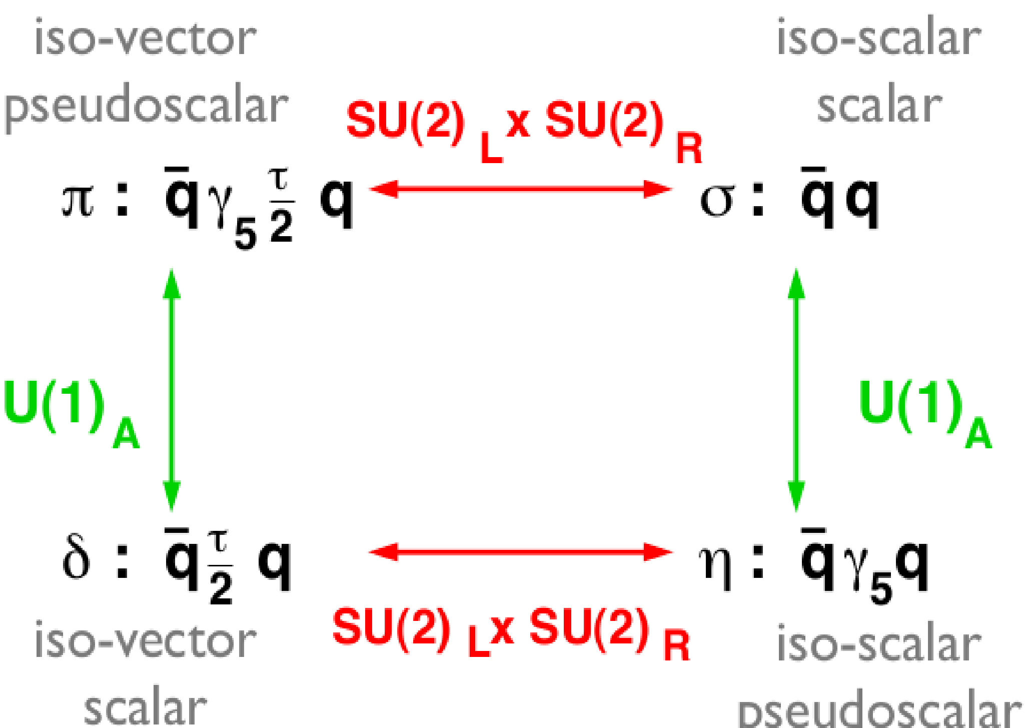
Pisarski and Wilczek, PRD 29(1984)338

Butti, Pelissetto and Vicar, JHEP 08 (2003)029

Understanding the fate of $U(1)_A$ at $T\neq 0$ is important.

Chiral & U(1)$_A$ symmetry and susceptibilities

Susceptibilities defined as integrated two point correlation functions of eight local operators, e.g. $\chi_\pi = \int \mathrm{d}^4x \, \langle \pi^i(x)\pi^i(0) \rangle$



restoration of $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R}$:

$$\chi_\pi - \chi_\sigma = 0$$

$$\chi_\delta - \chi_\eta = 0$$

restoration of $\mathrm{U(1)_A}$:

$$\chi_\pi - \chi_\delta = 0$$

$$\chi_\delta - \chi_\eta = 0$$

The definition of the Dirac eigenvalue spectrum:

$$\rho(\lambda, m) = \frac{1}{V} \sum_k \langle \delta(\lambda - \lambda_k) \rangle \, , D\phi = i\lambda\phi$$

The order parameter of chiral phase transition:

$$\boxed{\langle \bar{\psi}\psi \rangle \overset{V\to\infty}{\longrightarrow} \frac{N_f}{4} \int_0^\infty d\lambda \frac{2m\rho(\lambda, m)}{\lambda^2 + m^2} \overset{m\to 0}{\longrightarrow} \pi\rho(0)}$$

T.Banks and A.Casher (1980)

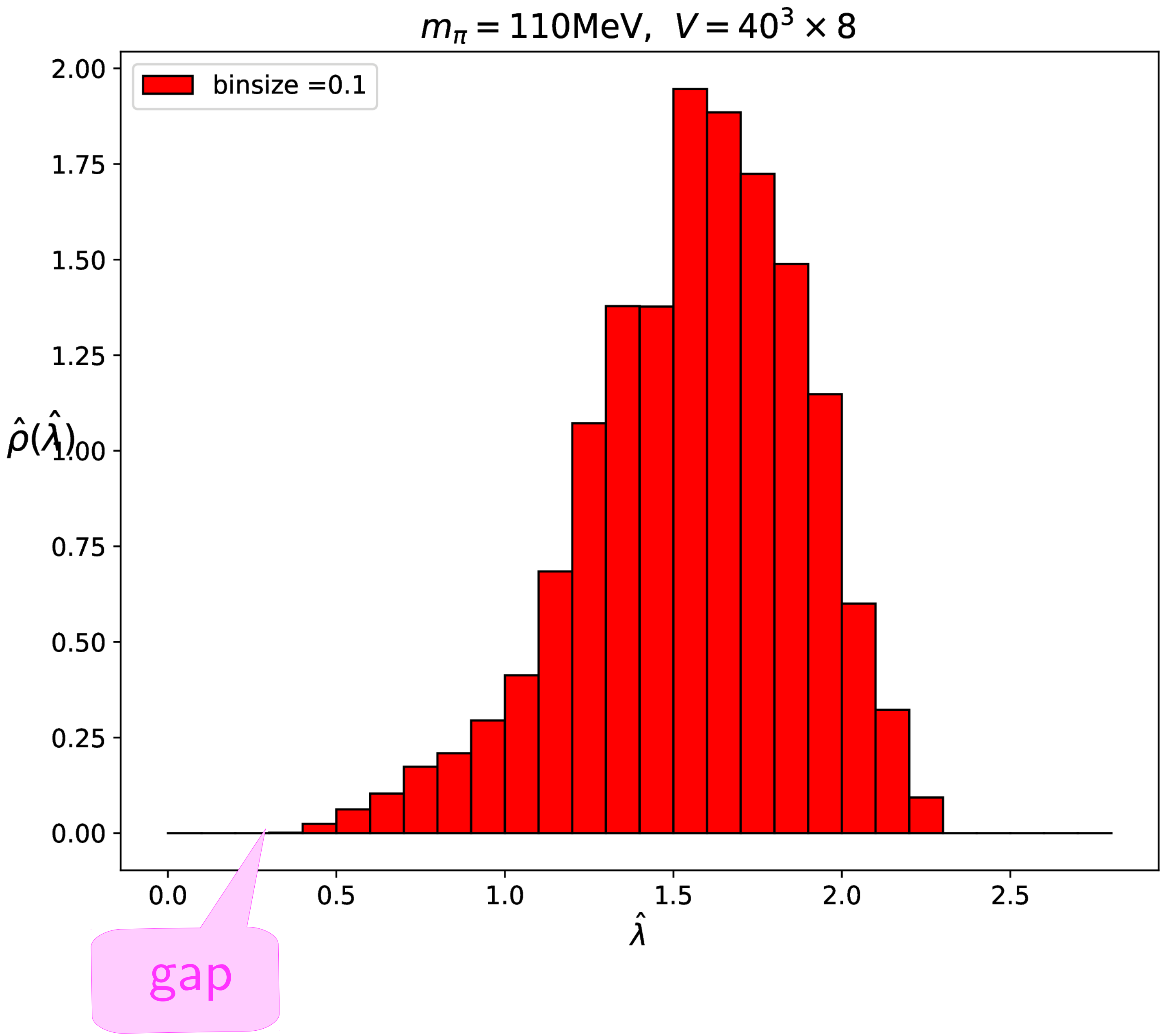
— if the chiral symmetry is restored, $\langle \bar{\psi}\psi \rangle = 0 \to \rho(0) = 0$

An indicator of the U(1)$_A$ symmetry breaking/restoration:

$$\boxed{\chi_\pi - \chi_\delta \overset{V\to\infty}{\longrightarrow} \frac{N_f}{4} \int_0^\infty d\lambda \frac{4m^2\rho(\lambda, m)}{(\lambda^2 + m^2)^2}}$$

— if ρ(λ) has a gap around λ=0 in the chiral limit,
both the chiral and U(1)$_A$ symmetries are restored.

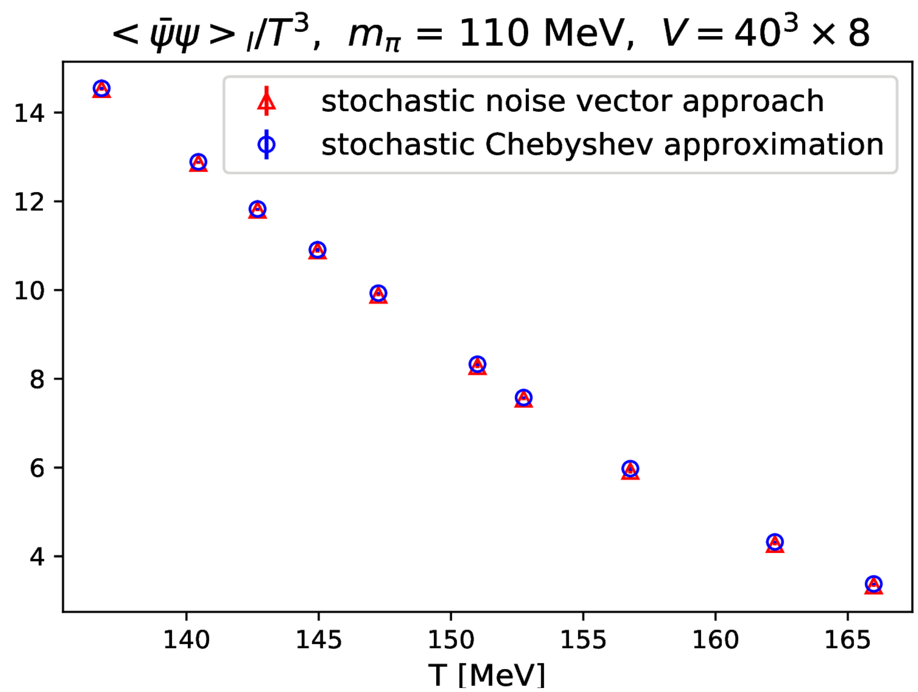
Dirac eigenvalue spectrum in the free case

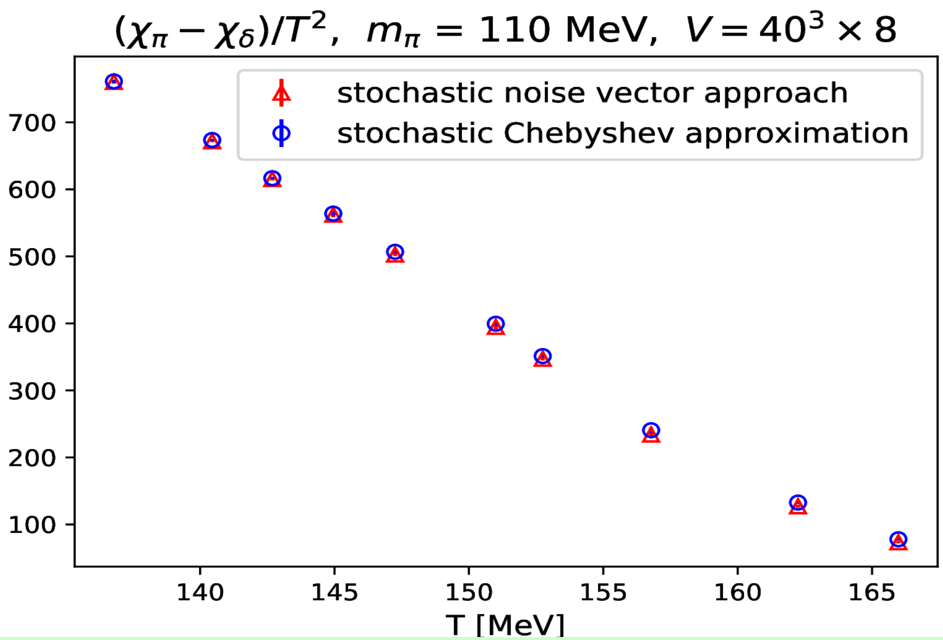


$\lambda < 0.4$, $\rho(\lambda) = 0$: gap

large λ: $\rho(\lambda) \sim \lambda^3$

Reproduction of chiral condensate and susceptibility using Dirac eigenvalue spectrum





$$\langle \bar{\psi}\psi \rangle = \frac{N_f}{4}\frac{1}{V}\langle \mathrm{Tr} M^{-1} \rangle$$

$$\langle \bar{\psi}\psi \rangle \xrightarrow{V\to\infty} \frac{N_f}{4}\int_0^\infty d\lambda \frac{2m\rho(\lambda, m)}{\lambda^2 + m^2}$$

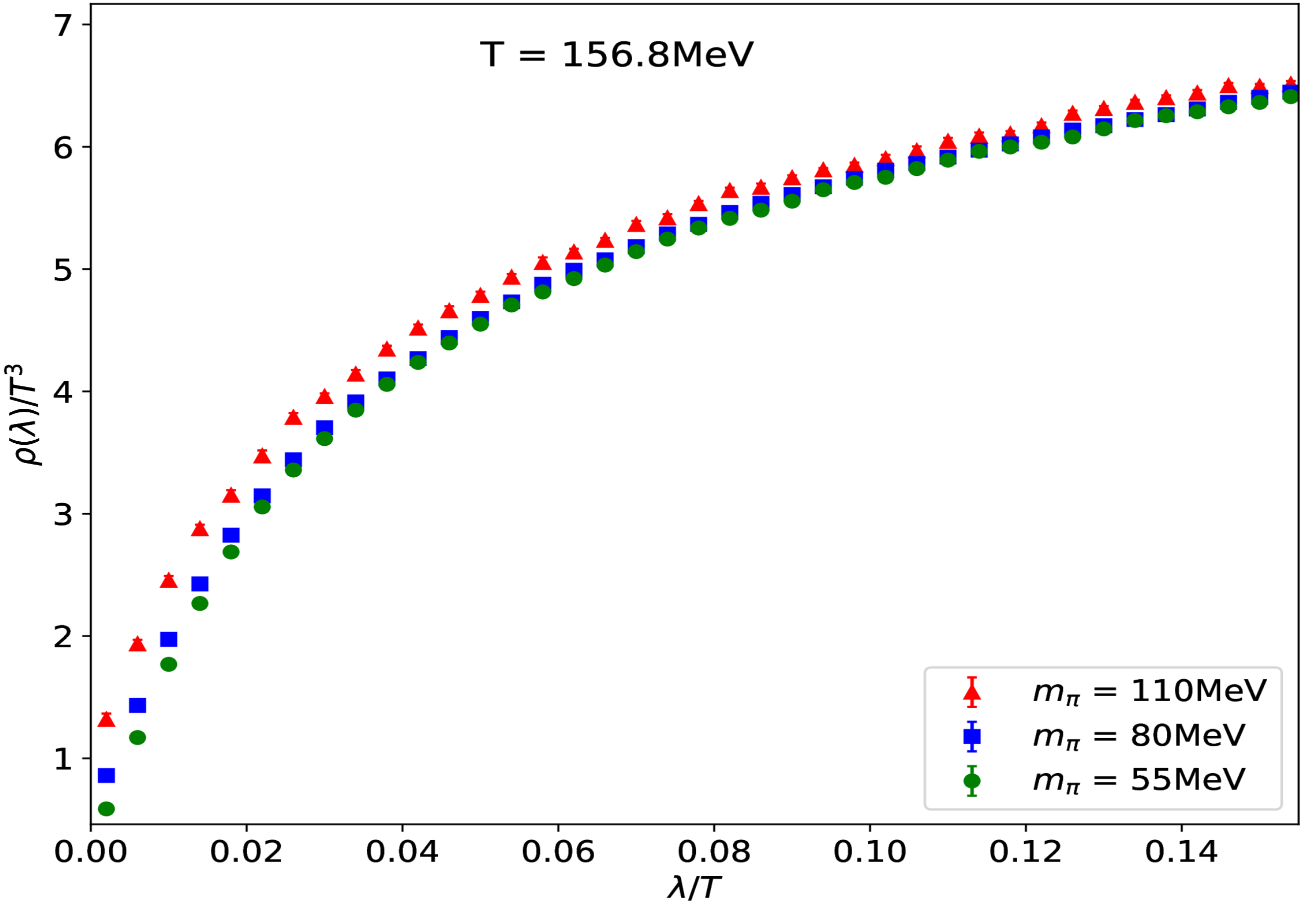
$$\chi_\pi - \chi_\delta = \frac{N_f}{4}\frac{1}{mV}\langle \mathrm{Tr} M^{-1} \rangle + \frac{N_f}{4}\frac{1}{V}\langle \mathrm{Tr} M^{-2} \rangle$$

$$\chi_\pi - \chi_\delta \xrightarrow{V\to\infty} \frac{N_f}{4}\int_0^\infty d\lambda \frac{4m^2\rho(\lambda, m)}{(\lambda^2 + m^2)^2}$$

The Dirac eigenvalue spectrum capture all the chiral information.

Sea quark mass dependence of ρ(λ)

$$\langle\bar{\psi}\psi\rangle \xrightarrow{V\to\infty} \frac{N_f}{4}\int_0^\infty d\lambda \frac{2m\rho(\lambda,m)}{\lambda^2+m^2}$$



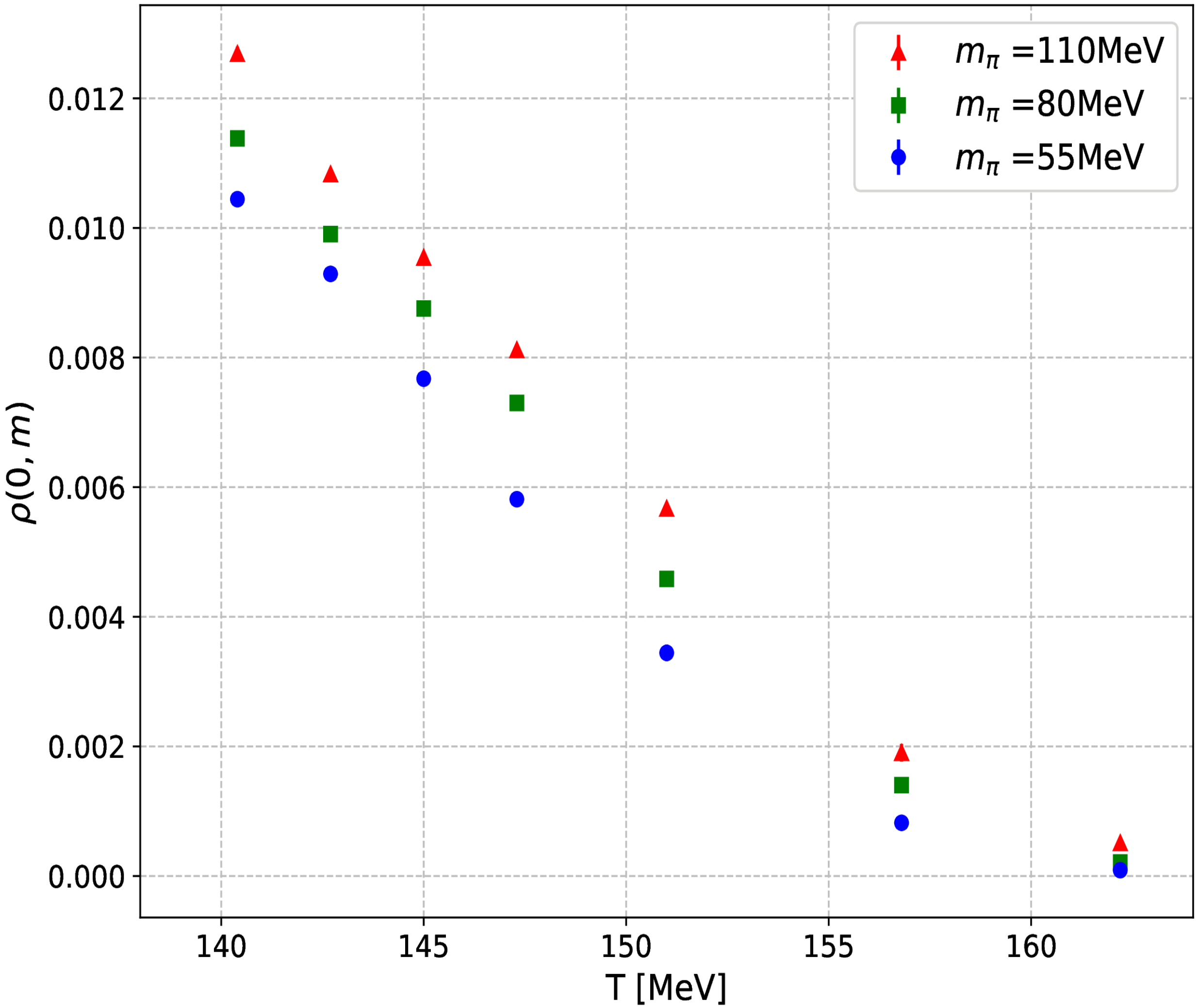
Small λ part: larger quark mass dependence

Large λ part: smaller quark mass dependence

Estimate of $\rho(0,m)$

Fit $\rho(\lambda,m)$ to a cubic polynomial:

$$\rho(\lambda,m) = C_0 + C_1\lambda + C_2\lambda^2 + C_3\lambda^3 \Longrightarrow \rho(0,m) = C_0$$



$T\uparrow \quad \rho(0,m)\downarrow$

$$\langle\bar{\psi}\psi\rangle = \pi\rho(0)\downarrow$$

T_{pc} estimated from the χ_M :

157.8(1) MeV (m_π=110 MeV)

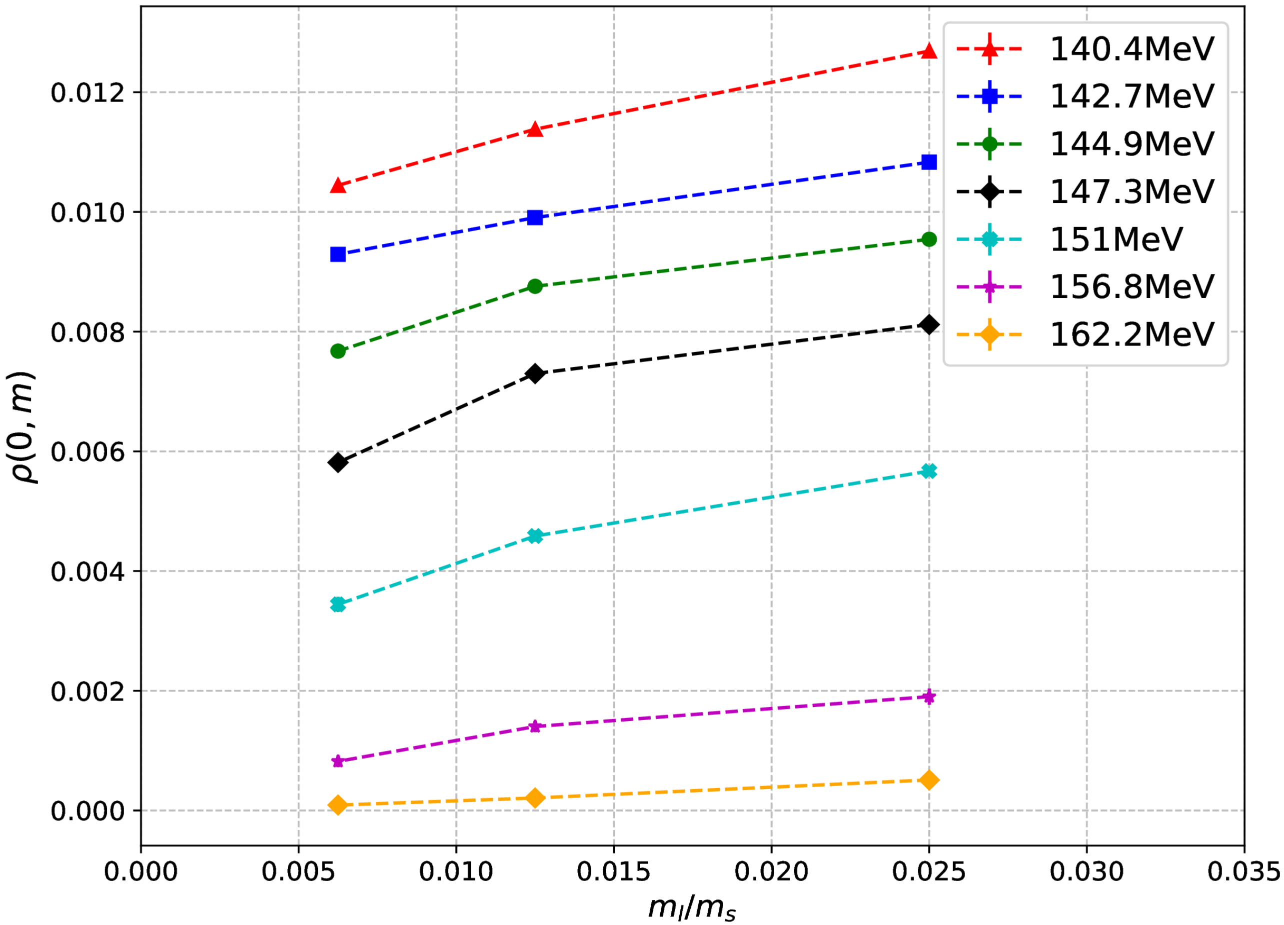
153.7(3) MeV (m_π=80 MeV)

150.7(3) MeV (m_π=55 MeV)

H.-T. Ding et al.,
Phys.Rev.Lett. 123 (2019) 062002

Quark mass dependence of ρ(0,m)

$$T \geq T_c : \text{ If } \rho(0, m) \sim m \overset{m \to 0}{\Longrightarrow} \boxed{\begin{array}{c} \langle \overline{\psi}\psi \rangle = 0 \\ \chi_\pi - \chi_\delta = \chi_{\text{disc}} \neq 0 \end{array}}$$



Bazavov et al., [HotQCD]
Phys.Rev. D86 (2012) 094503

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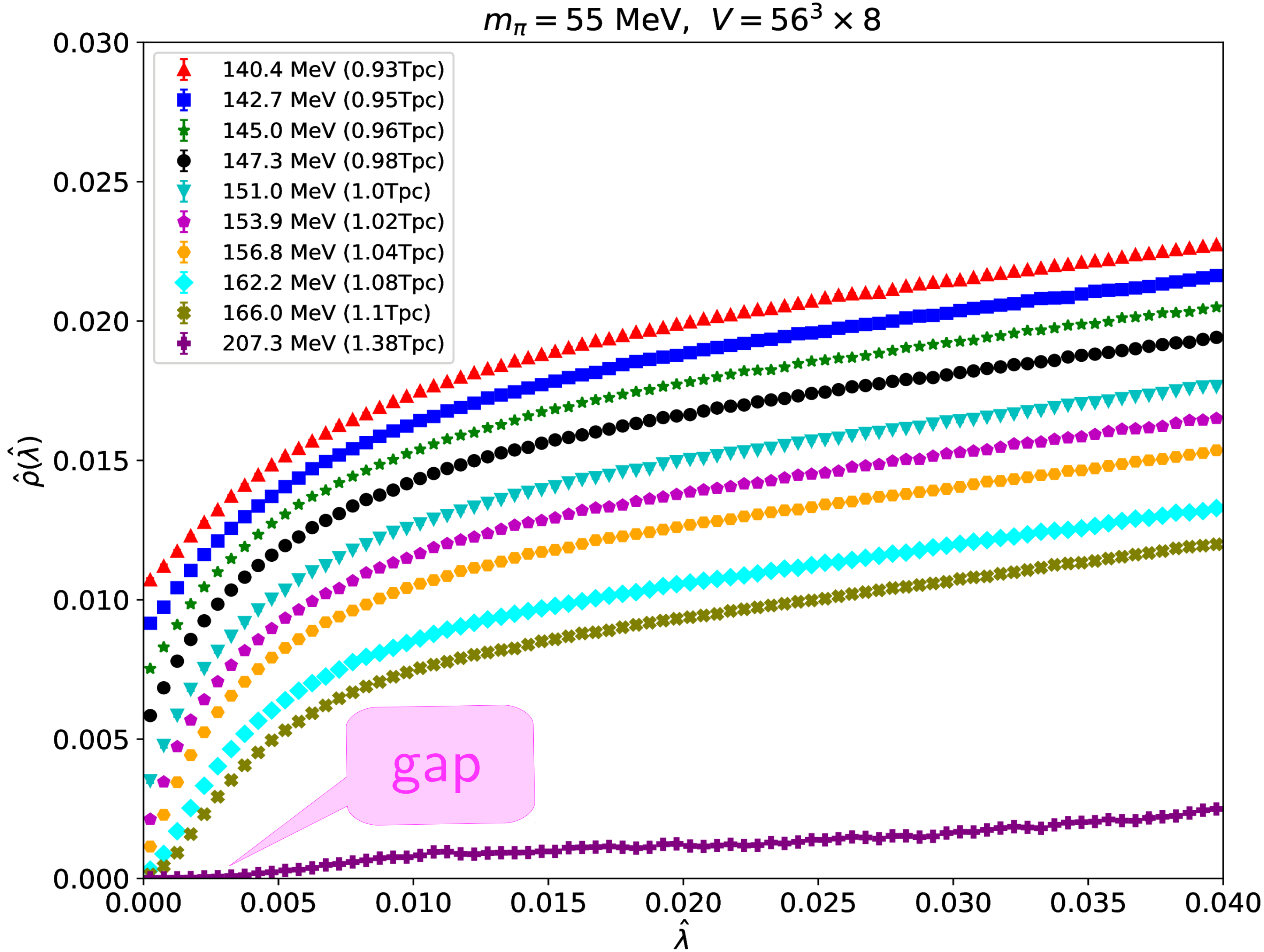
ρ(0,m) has a linear dependence on the mass just above T_{pc}

U(1)$_A$ remains broken just above T_{pc}

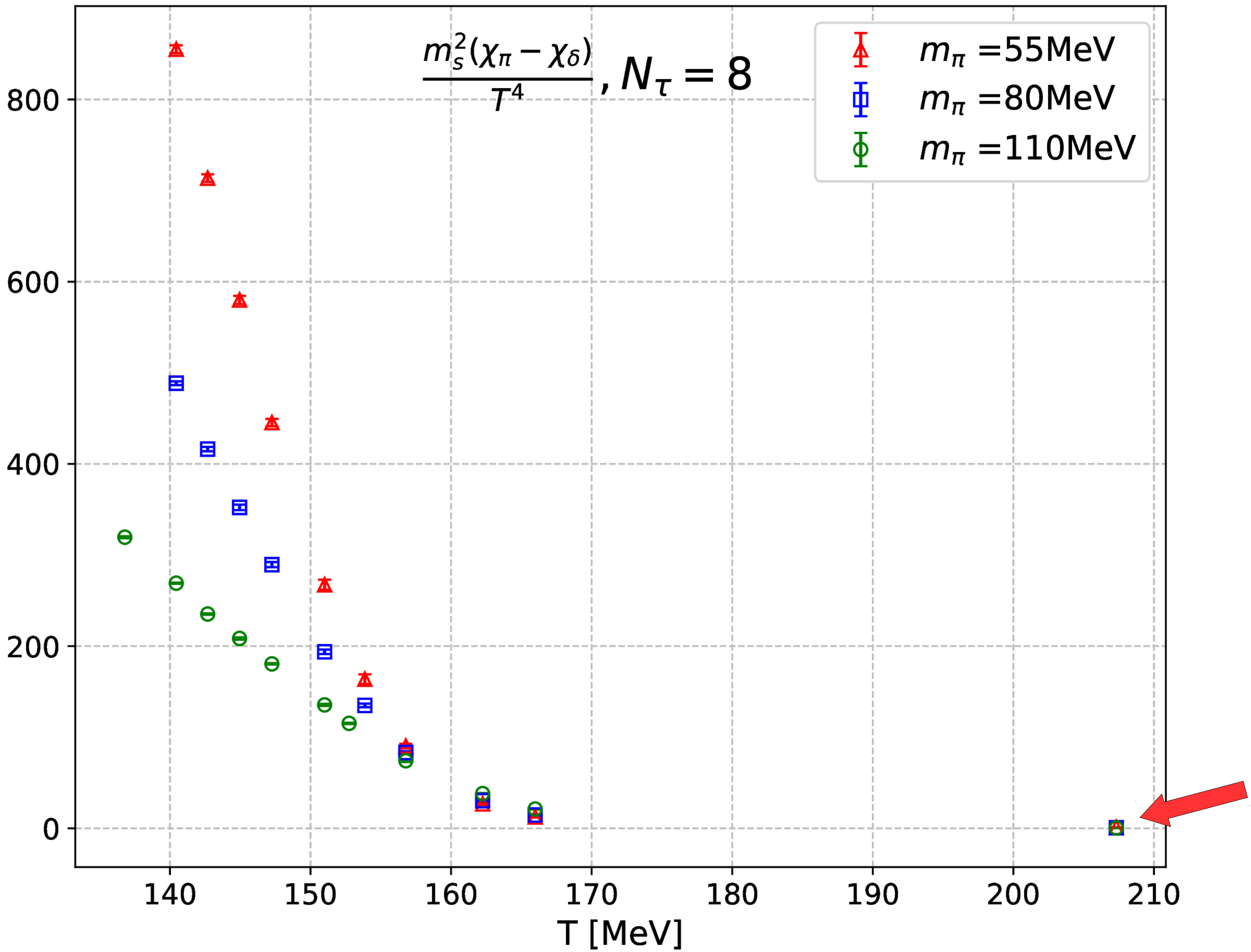
T dependence of low-lying eigenvalue spectrum



- Eigenvalue around zero is suppressed as T increases
- There seems have a gap around zero at T=207.3 MeV ($1.38T_{pc}$)

(C.f. $T_{pc} = 150.7(3)$ MeV)

$U(1)_A$ susceptibility



$U(1)_A$ seems restored at T=207 MeV ($1.38T_{pc}$) for small quark mass

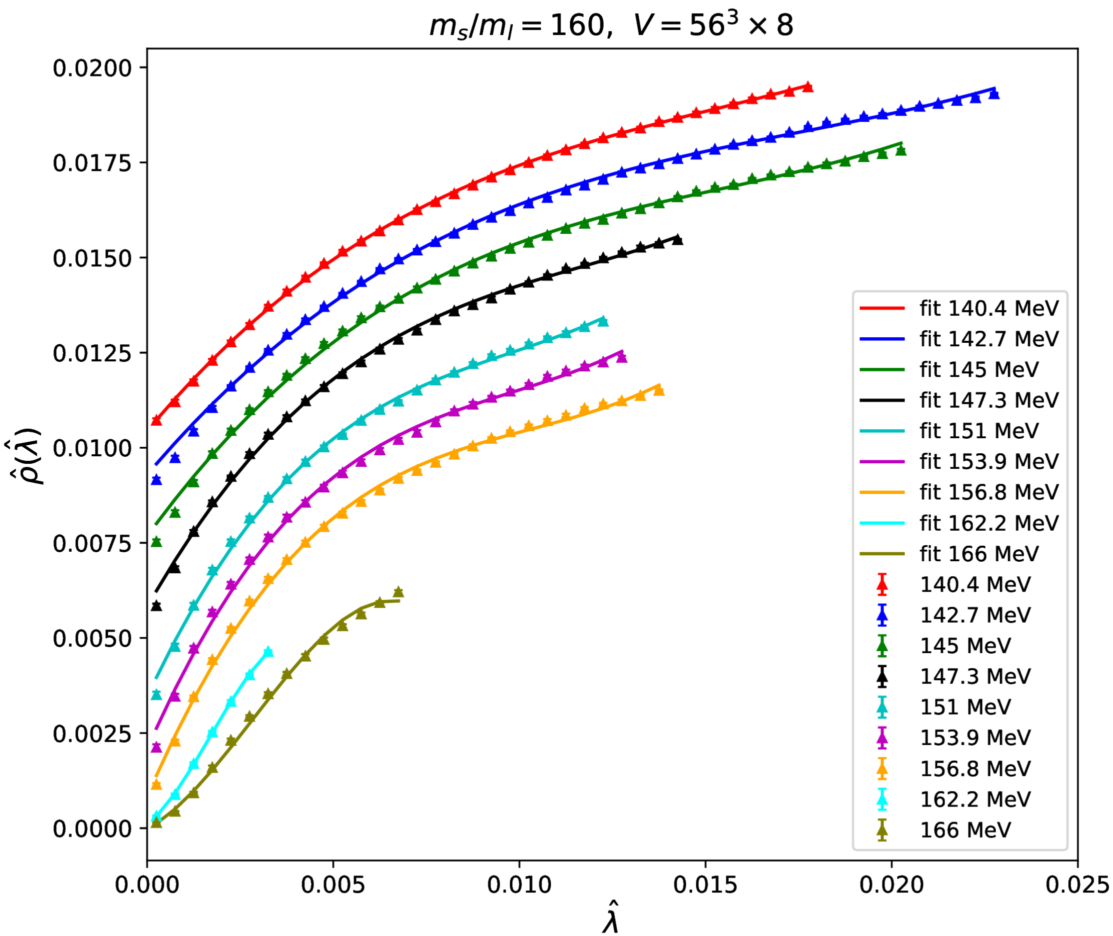
Conclusions

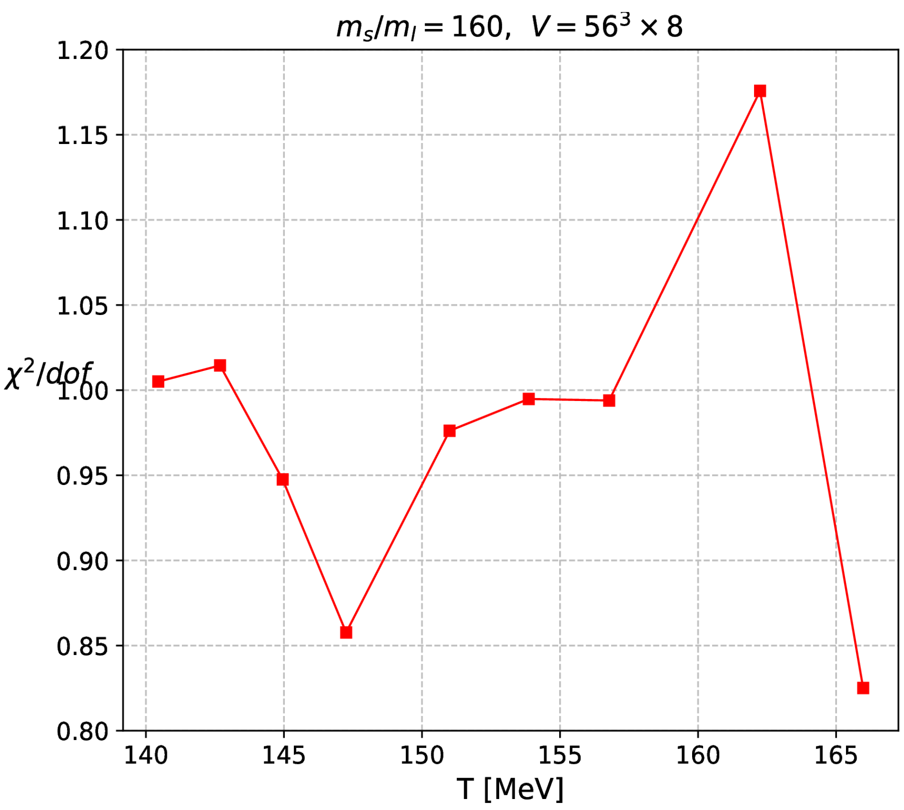
- Chiral observables can be reproduced well from Dirac eigenvalue spectrum.
- Dirac eigenvalue spectrum at zero has a linear quark mass dependence just above $T_{pc.}$
- $U(1)_A$ symmetry seems to be restored at around 207MeV ($1.38T_{pc}$) for small quark mass.

Backup

Fit $\rho(\lambda,m)$ to a cubic polynomial:

$$\rho(\lambda,m) = C_0 + C_1\lambda + C_2\lambda^2 + C_3\lambda^3$$





possible behaviors for $\rho(\lambda, m)$

Instanton

$T \gg T_c$: $\rho(\lambda,m) = C_i m^2 \delta(\lambda) + C_\lambda \lambda + C_m m + O(\lambda m)$

$m \to 0, \lambda \to 0$: $\rho(0,m) = C_m m$

Ansatz	$\langle\bar{\psi}\psi\rangle$	χ_π	χ_δ	$\chi_\pi - \chi_\delta$	χ_{disc}
$m^2\delta(\lambda)$	m	1	-1	2	2
λ	$-2m\ln(m)$	$-2\ln(m)$	$-2\ln(m)$	2	0
m	πm	π	0	π	π

in the chiral limit

If $\rho(0,m) \sim m$ $\Rightarrow$

$$\boxed{\begin{array}{c} \langle\bar{\psi}\psi\rangle = 0 \\ \chi_\pi - \chi_\delta = \chi_{disc} \neq 0 \end{array}}$$

Bazavov et al., [HotQCD] arXiv:1205.3535

$$\gamma_j^{[s,t]} = \begin{cases} \frac{1}{\pi}\Big(\arccos(s) - \arccos(t)\Big) & \text{for } j = 0 \\ \frac{2}{\pi}\frac{\sin(j\arccos(s)) - \sin(j\arccos(t))}{j} & \text{for } j > 0 \end{cases} \tag{1}$$

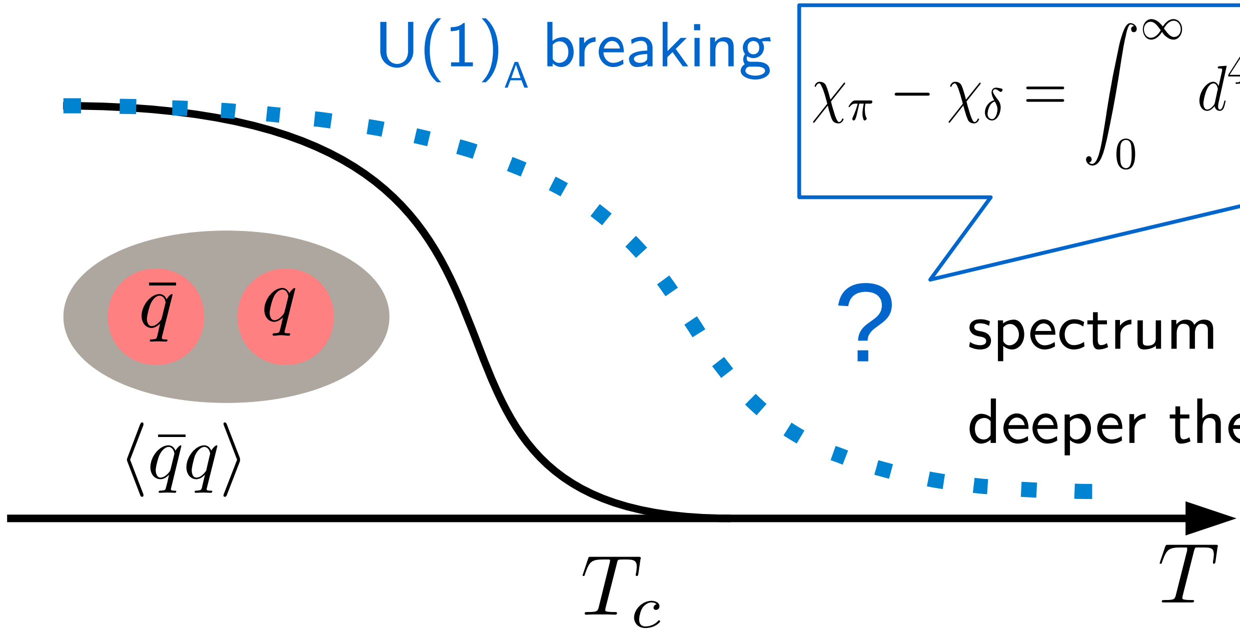
damping factor :

$$g_j^p = \frac{(1 - \frac{j}{p+2})\sin\alpha_p \cos(j\alpha_p) + \frac{1}{p+2}\cos\alpha_p \sin(j\alpha_p)}{\sin\alpha_p}, \alpha_p = \frac{\pi}{p+2}$$

Motivation

For T>T$_c$, chiral symmetry breaking by nonzero $\langle \bar{q}q \rangle$ is restored

how about U(1)$_A$ symmetry ?



$$\chi_\pi - \chi_\delta = \int_0^\infty d^4x \left[\pi^a(x)\pi^a(x) - \delta^a(x)\delta^a(x)\right]$$

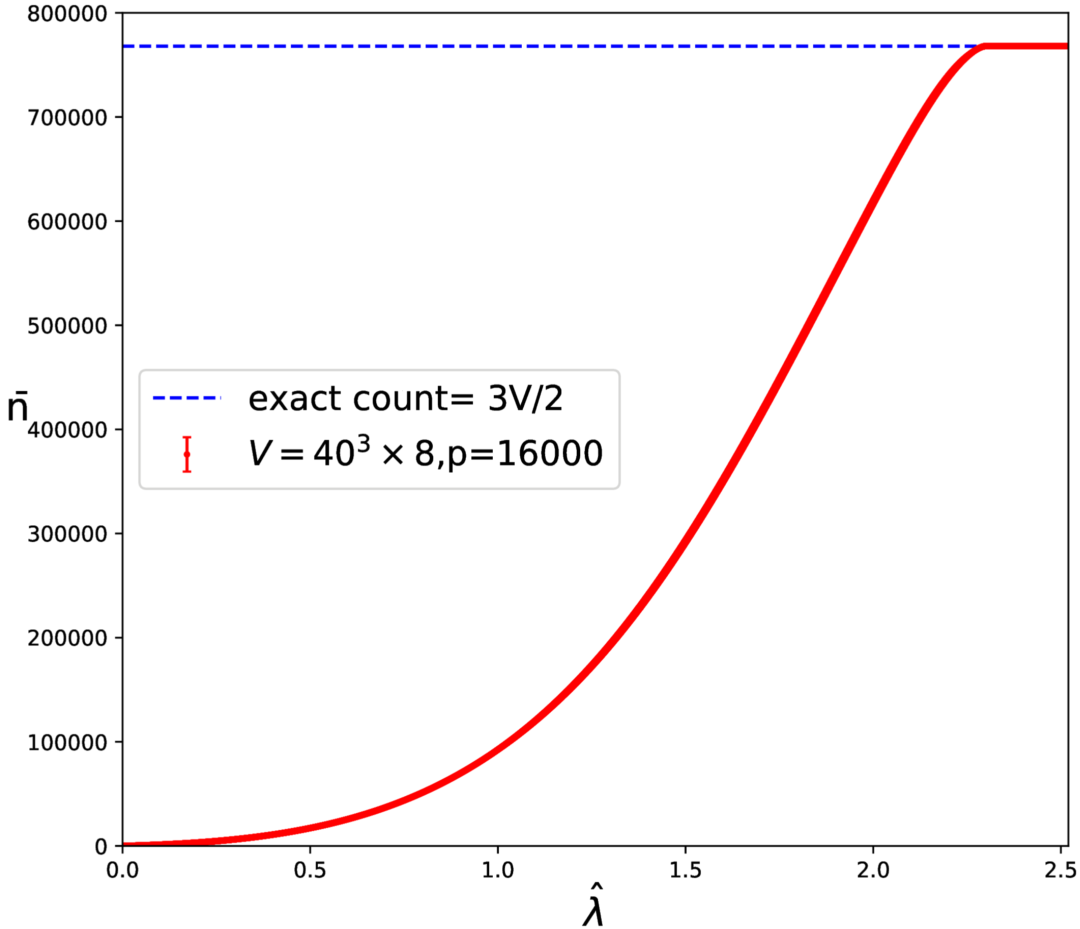
spectrum allow us to understand deeper the microscopic mechanisms

U(1)$_A$ susceptibility from Dirac spectrum

$$\chi_\pi - \chi_\delta = \int_0^\infty d\lambda \rho(\lambda) \frac{2m^2}{(\lambda^2 + m^2)^2}$$

Sanity check by mode number

$$\bar{n}[s,t] \approx \frac{1}{N_r} \sum_{r=1}^{N_r} \sum_{j=0}^{p} g_j^p \gamma_j \langle \xi_r^\dagger T_j(A) \xi_r \rangle$$



The mode number converges to the exact count

How can we get the Dirac spectrum?

- Commonly used method: Lanczos algorithm to calculate the individual low-lying eigenvalues

- Here we utilized the Chebyshev filtering technique combined with a stochastic estimate of the mode number

$$\bar{n}[s,t] \approx \frac{1}{N_r} \sum_{r=1}^{N_r} \sum_{j=0}^{p} g_j^p \boxed{\gamma_j} \langle \xi_r^\dagger \boxed{T_j(A)} \xi_r \rangle$$

T_j : Chebyshev polynomial

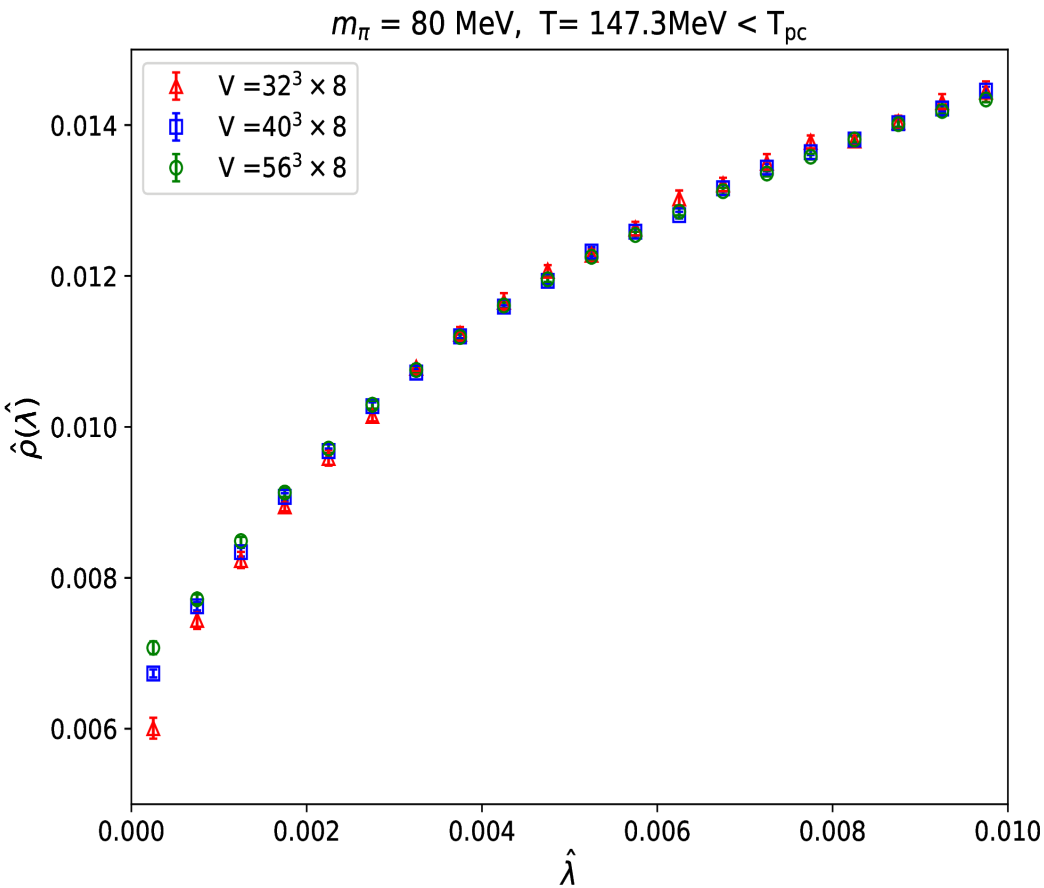
γ_j : coefficient

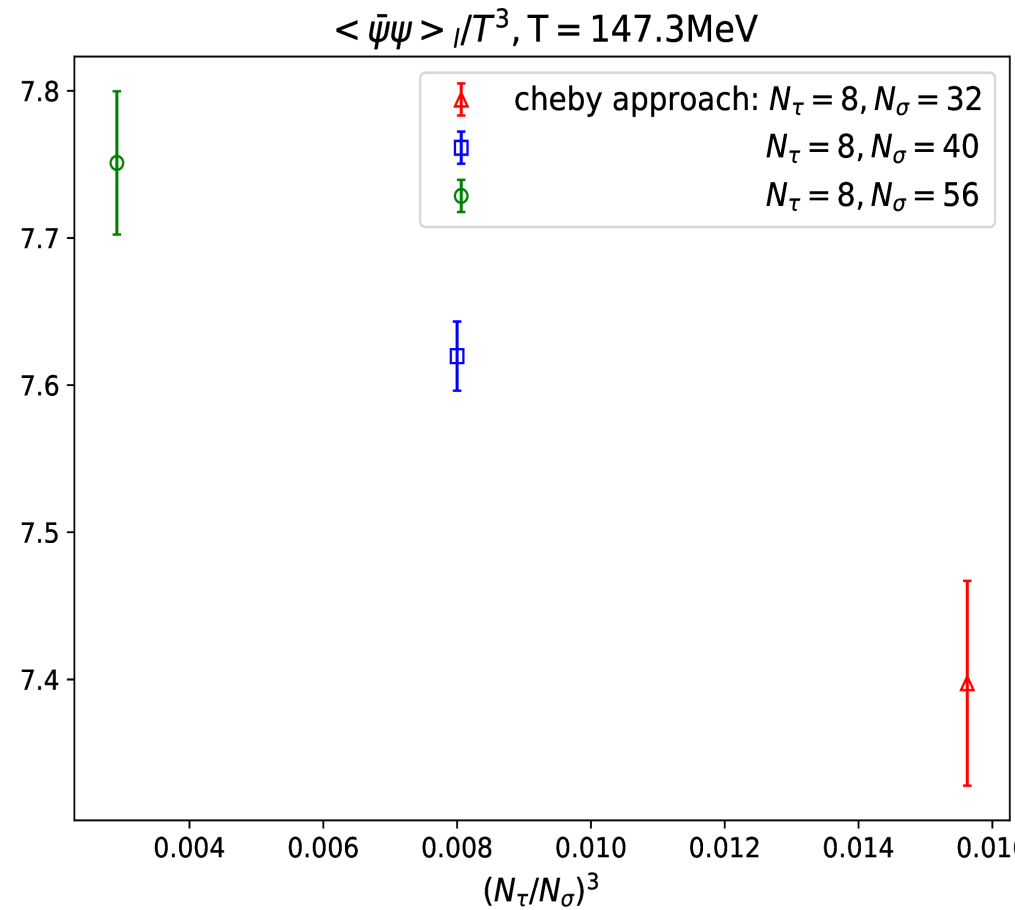
p : polynomial order

$$\rho(\lambda,\delta) = \frac{1}{V} \frac{\bar{n}[\lambda - \delta/2, \lambda + \delta/2]}{\delta} \qquad (\lambda \geq \delta/2)$$

Giusti,Luscher, arXiv: 0812.3638
Di Napoli et al., arXiv: 1308.4275
Fodor et al., arXiv: 1605.08091
Cossu et al., arXiv: 1601.00744

Volume dependence of $\rho(\lambda)$





- The near zero mode show larger volume dependence
- The chiral condensate increases as the volume is increased