

# Chirality production, chiral magnetic effect and Schwinger mechanism

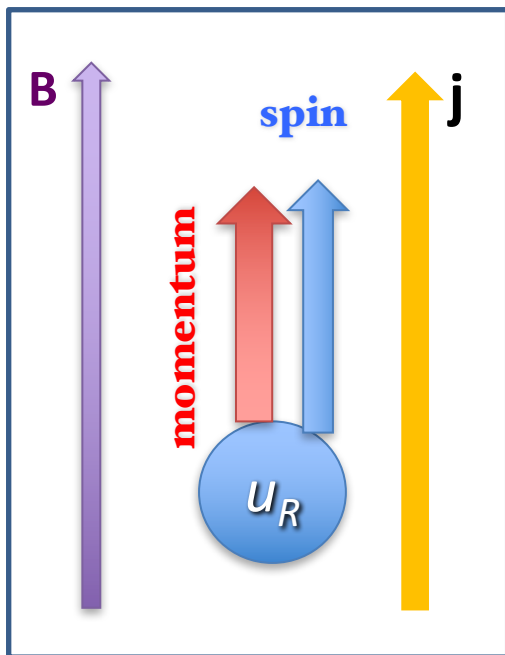
**Shi Pu ( 浦 实 )**

**University of Science and Technology of China**

**The 13<sup>th</sup> workshop on QCD Phase Transition and  
Relativistic Heavy-Ion Physics, QPT 2019**

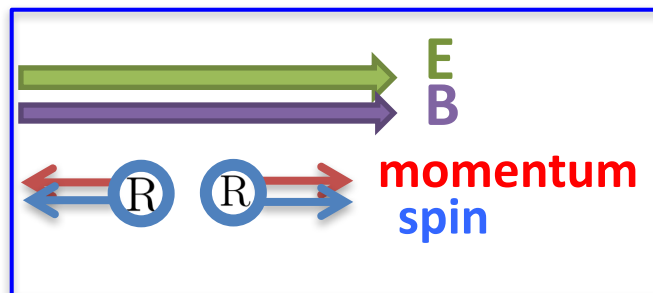
**2019.08.16 – 08.20, Enshi**

# Outlines



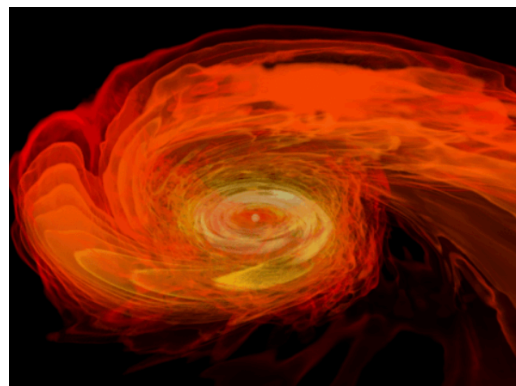
## 1. Chiral magnetic effects

microscopic



## 2. Chirality production, Schwinger mechanism

macroscopic

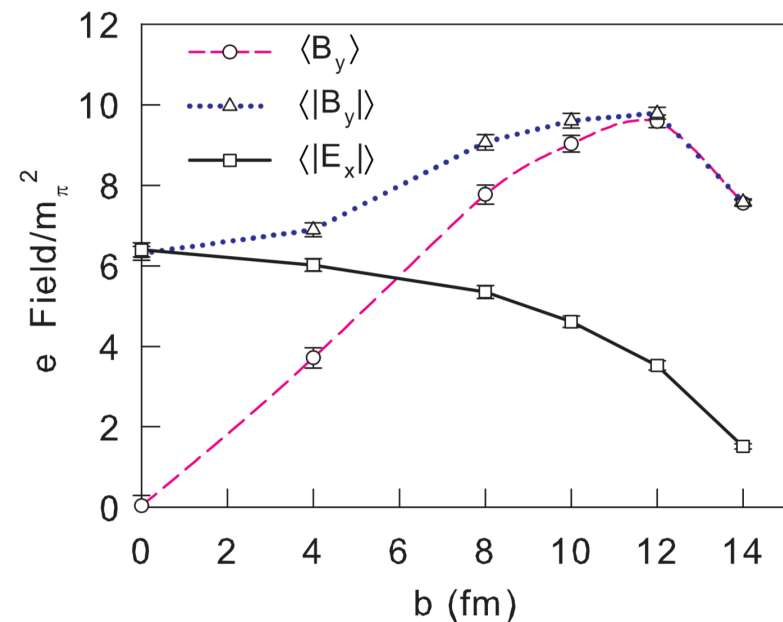
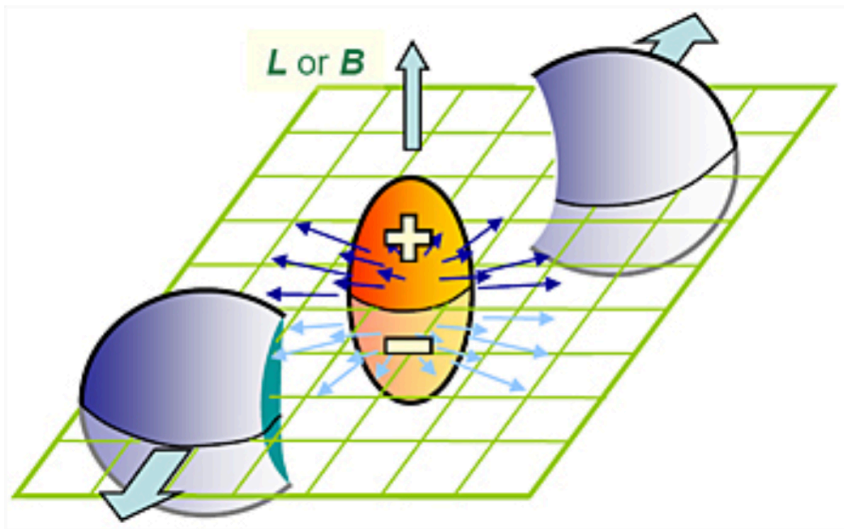


## 3. Anomalous magneto-hydrodynamics

# **1. Chiral Magnetic Effect**

# Strong Electromagnetic fields in HIC

- Theoretical estimation: **Ref : Prof. Jingfeng Liao's talk**  
Lienard-Wiechert potential + Event-by-event

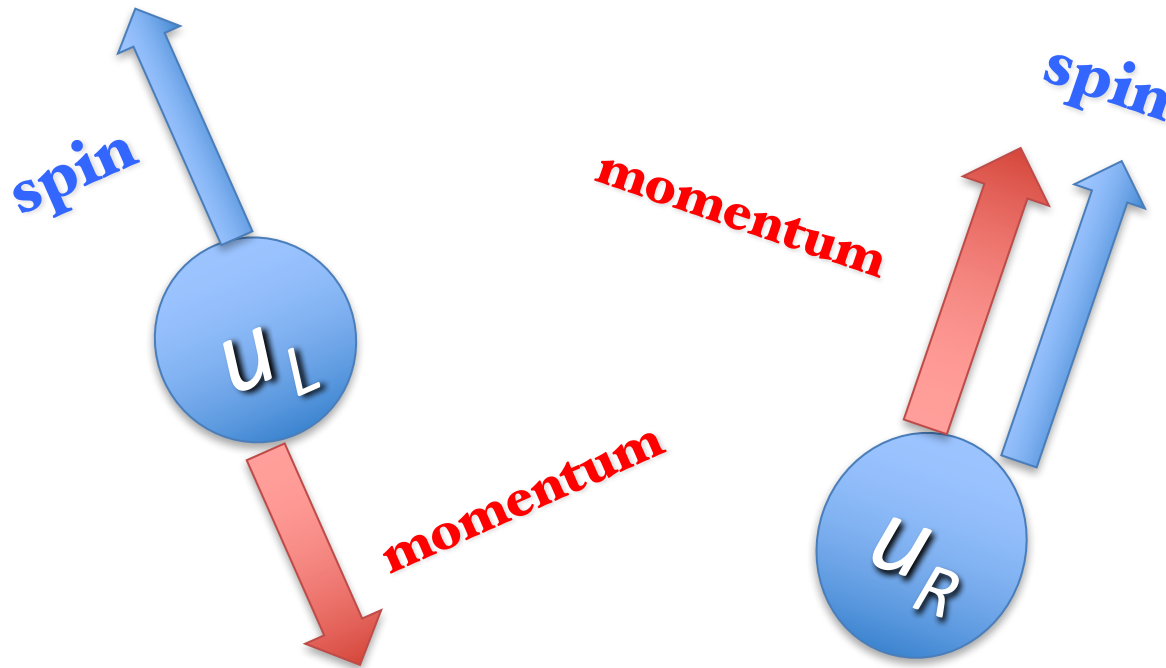


**A. Bzdak, V. Skokov PRC 2012 ;W.T. Deng, X.G. Huang PRC 2012;V.Roy, SP, PRC 2015;  
H. Li, X.I. Sheng, Q.Wang, 2016; etc. / review: K. Tuchin 2013**

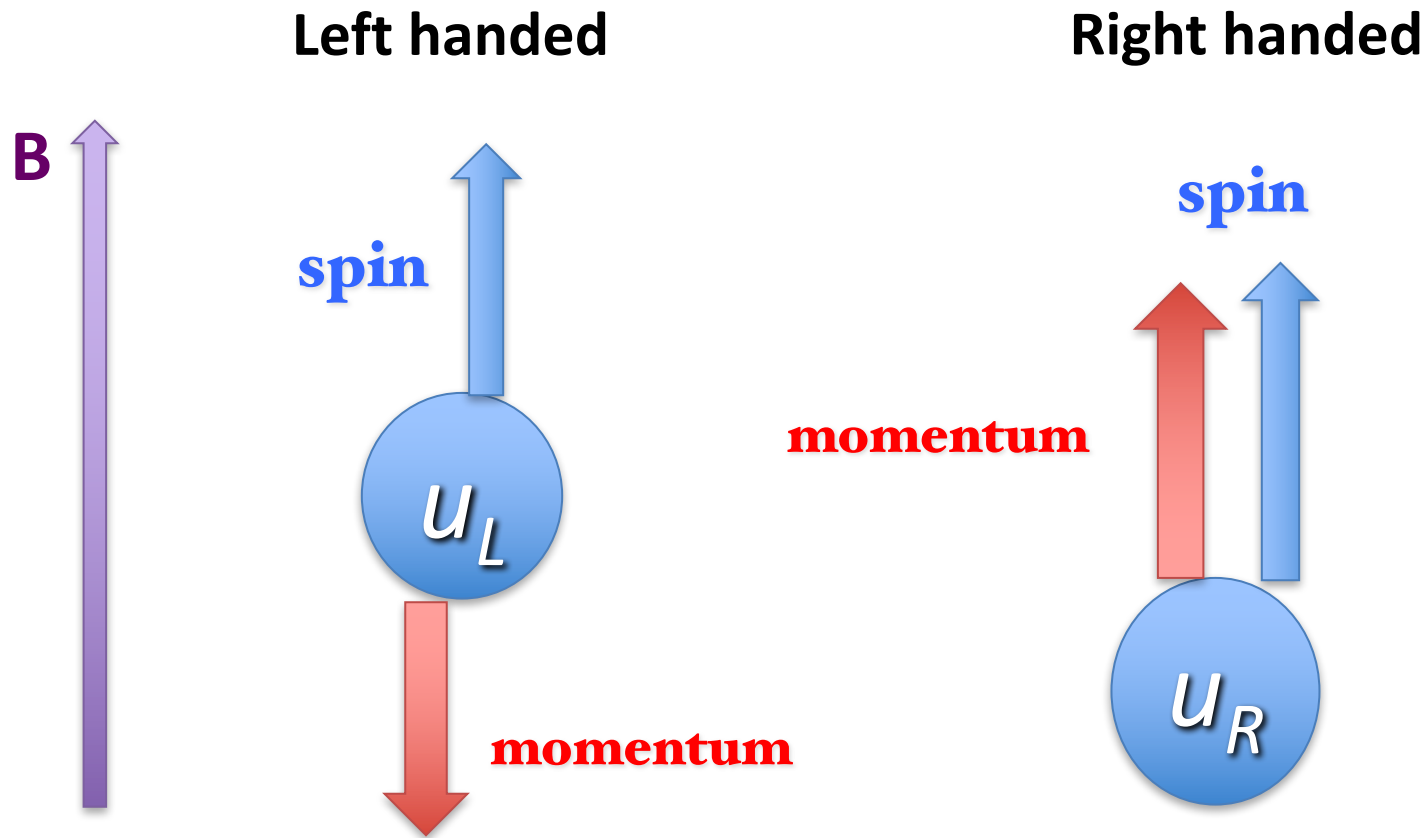
# Chirality and massless fermions

Left handed

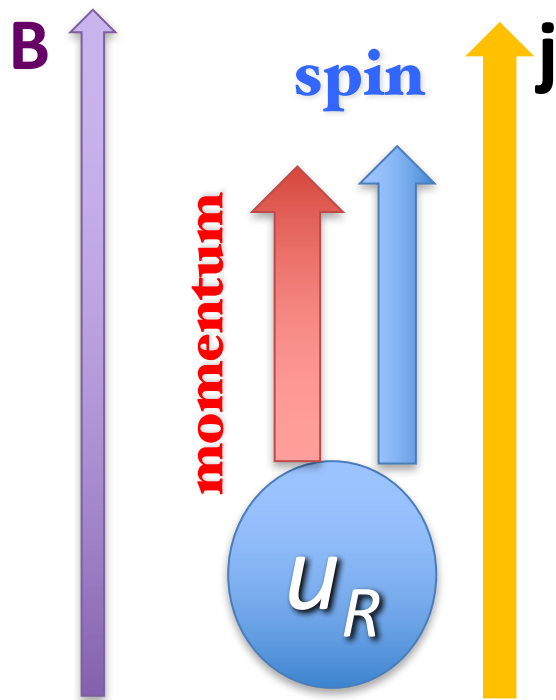
Right handed



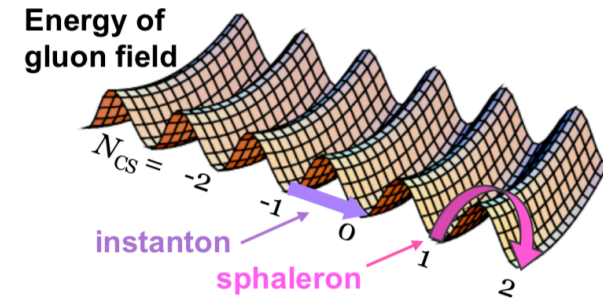
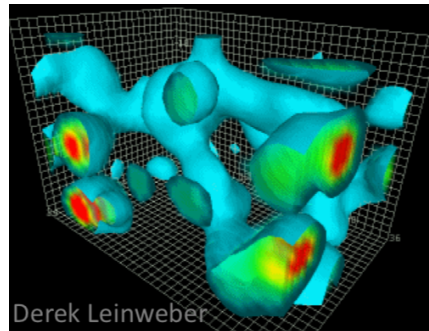
# Polarization by magnetic fields



# Chiral Magnetic Effect



- Magnetic fields
- Nonzero axial chemical potential
- Number of Left handed fermions  $\neq$  Number of Right handed fermions



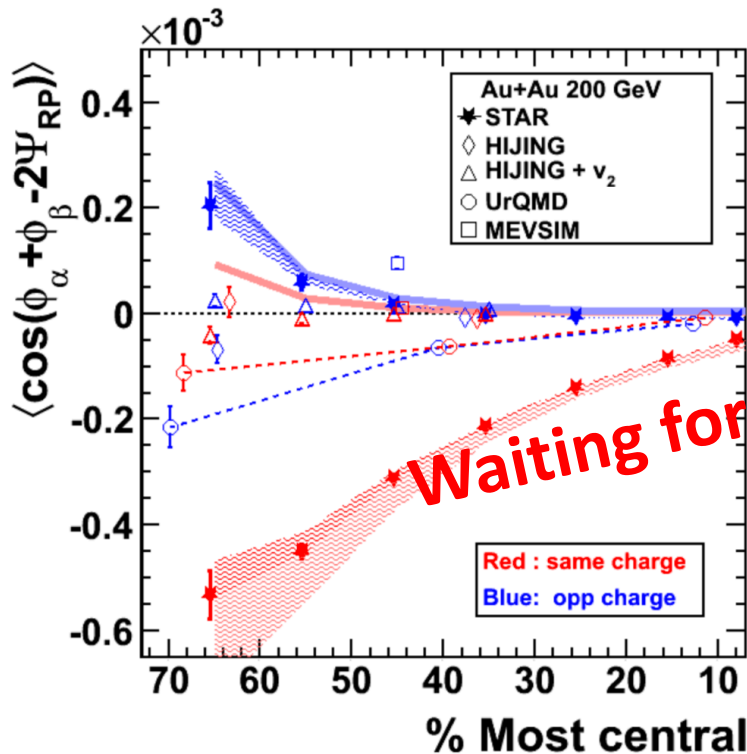
- Charge current: charge separation

$$\mathbf{j} = \frac{e^2}{2\pi^2} \mu_5 \mathbf{B},$$

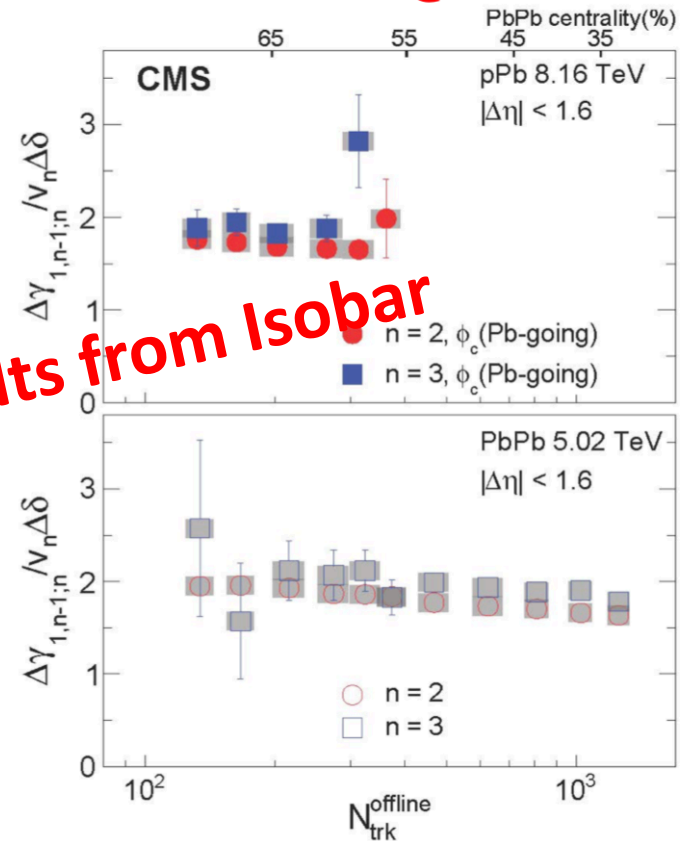
Kharzeev, Fukushima, Warrigna, (08,09), etc. ...

# Heavy ion collisions

- Experiments: signal VS background



Waiting for the results from Isobar



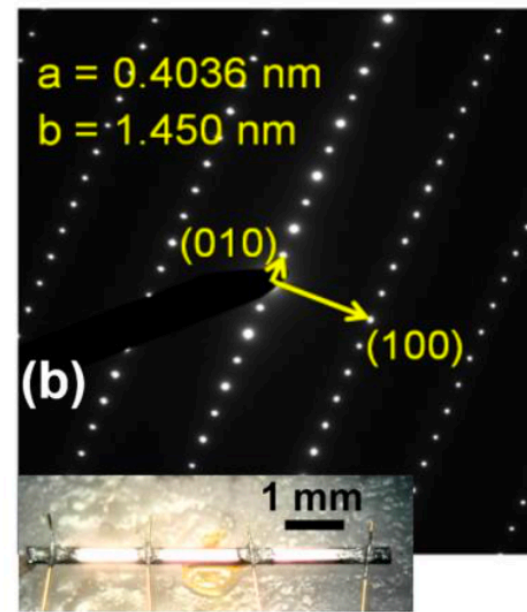
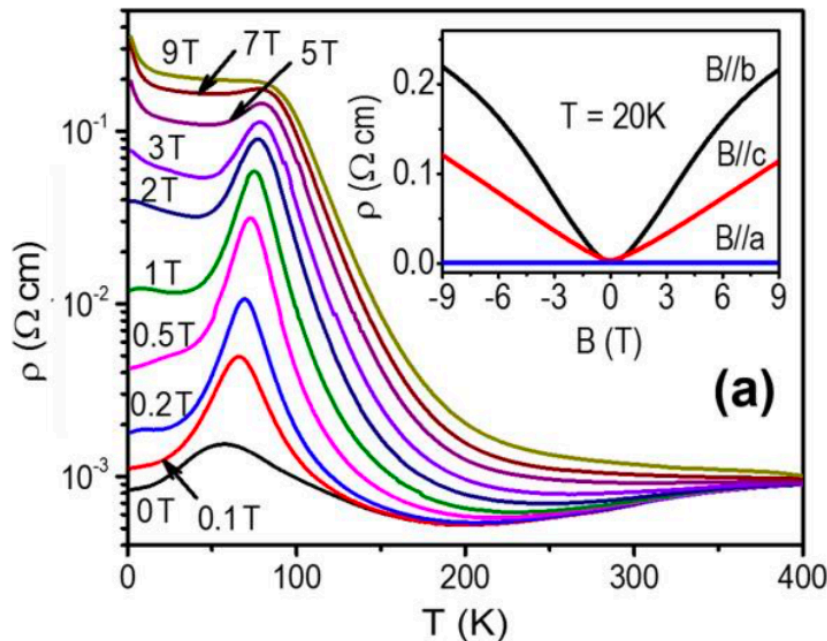
STAR PRL 103, 251601(2009);  
PRC 81, 054908

CMS PRL 118, 122301 (2016);  
PRC 97, 044912



# Condense matter

- **Weyl Semi-metal: new transport effects**



**$\text{ZrTe}_5$ : Nature Physics, 12 , 550–554, (2016)**

## **2. Chirality production and Schwinger Mechanism**

Ref: Patrick Copinger, Kenji Fukshima, SP,  
**Phys.Rev.Lett.** 121 (2018), 261602

# Chirality production VS Schwinger Mechanism

- Axial Ward identity

$$\partial_\mu j_5^\mu = 2im\bar{\psi}\gamma^5\psi - \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$



Chiral current

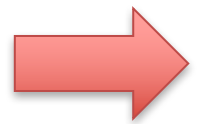


Pseudo-scalar



~ E.B, Chiral anomaly

- Volume integral


$$\frac{d}{dt}N_5 = \int d^3x \left( 2im\bar{\psi}\gamma^5\psi - \frac{e^2}{2\pi^2}E \cdot B \right)$$

# Pseudo-scalar

$$\partial_\mu j_5^\mu = 2im\bar{\psi}\gamma^5\psi - \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

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- **Massless limit:** Pseudo-scalar term  $\rightarrow 0$
- **Method:**

- Perturbative:

JHG, Z.T. Liang, Q. Wang, X.N. Wang, arXiv:1802.06216; Weickgenannt, Sheng, Speranza, Q. Wang, 1902.06513; Hattori, Hidaka, Yang, 1903.01653; Wang, Guo, Shi, Zhuang, 1903.03461

**Ref : Prof. Jianhua Gao' s talk (Aug. 17th )**

- Non-perturbative: World-line formalism


# World-line Formalism

- Spinor Feynman propagator at background fields:

$$S_A(x, y) = \text{---}\!\!\!\rightarrow\!\!\!\text{---} + \text{---}\!\!\!\rightarrow\!\!\!\text{---} \begin{array}{c} \text{wavy line} \end{array} + \text{---}\!\!\!\rightarrow\!\!\!\text{---} \begin{array}{c} \text{wavy line} \\ \text{wavy line} \end{array} + \dots$$

$$= (i\not{D}_x + m) \underline{\Delta(x, y)}$$

- Path integral: (Homogenous, Constant E,B)

$$\Delta(x, y) = \int_0^\infty ds e^{-im^2 s} \frac{e^2 EB}{(4\pi)^2} \frac{\exp \left[ -\frac{i}{2} eF \sigma s + \frac{i}{2} x eF y - \frac{i}{4} z \coth(eFs) eF z \right]}{\sinh(eEs) \sin(eBs)}$$


**s: Schwinger proper time**


M. D. Schwartz, Quantum Field theory and the standard model;  
Christian Schubert: lecture note on the Worldline Formalism

# “Well-known” Result done by Schwinger

- Homogenous Constant E,B at z direction
- Using world-line formulism (or original Schwinger's methods):

$$\langle \partial_\mu j_5^\mu \rangle = -2im \langle \bar{\psi} \gamma^5 \psi \rangle - \frac{e^2}{2\pi^2} \mathbf{E} \cdot \mathbf{B} = 0 \quad ?$$

J. Schwinger, Phys. Rev. 82,5 (1951);  
M. D. Schwartz, Quantum Field  
theory and the standard model;


$$\langle \bar{\psi} \gamma^5 \psi \rangle = i \frac{1}{4\pi^2} \frac{EB}{m}$$

# Puzzle: All vanishing?

$$\langle \partial_\mu j_5^\mu \rangle = -2im \langle \bar{\psi} \gamma^5 \psi \rangle - \frac{e^2}{2\pi^2} \mathbf{E} \cdot \mathbf{B} = 0$$

- Taking  $m \rightarrow 0$  at the very beginning: Weyl fermions

$$\partial_\mu j_5^\mu = -\frac{e^2}{2\pi^2} \mathbf{E} \cdot \mathbf{B}$$

- After all the calculations, taking  $m \rightarrow 0$ .

$$\langle \partial_\mu j_5^\mu \rangle = -2im \langle \bar{\psi} \gamma^5 \psi \rangle - \frac{e^2}{2\pi^2} \mathbf{E} \cdot \mathbf{B} = 0$$

Massless limit of  
massive Dirac  
equation

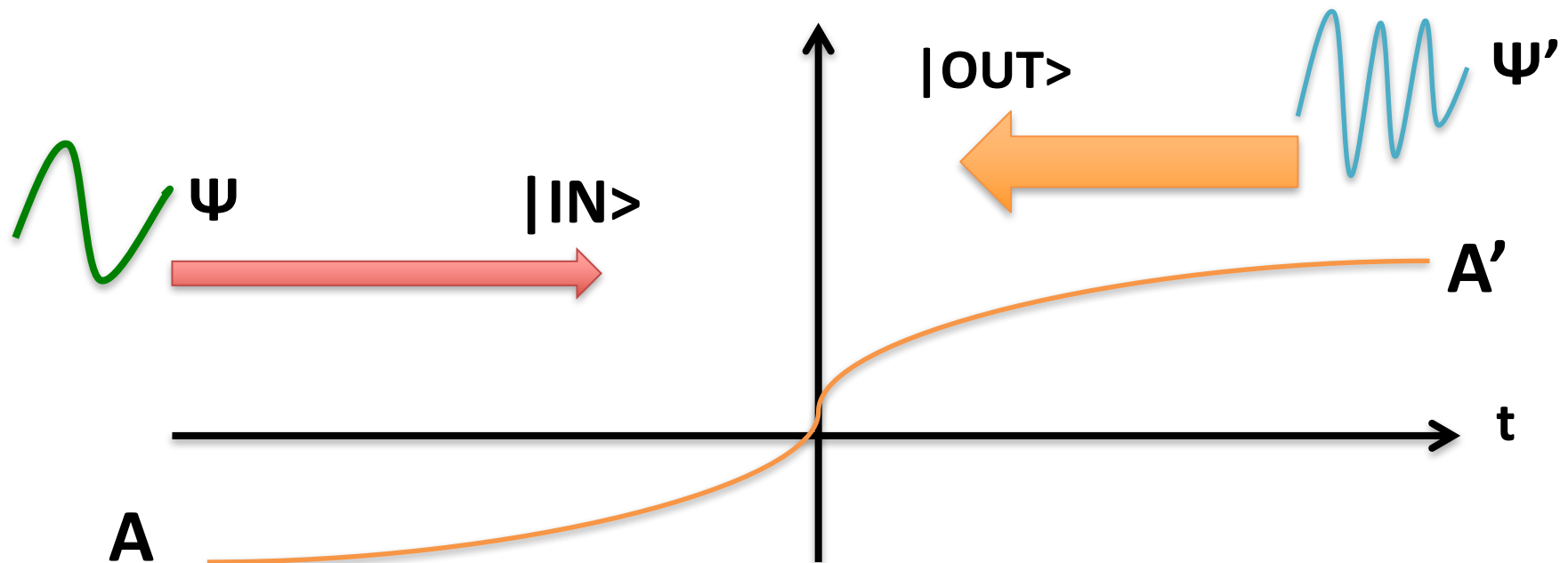
VS

Massless Dirac  
equation

# IN and OUT States

- Homogenous Constant  $E_z, B_z$  field:

$$A^z(t) = eE_z t, \quad H = H(A(t)),$$





# Unstable vacuum

- $|0, IN\rangle$  is **NOT** equal to  $|0, OUT\rangle$

$$| \langle 0, out | 0, in \rangle |^2 \neq 1$$

- Schwinger Pair Production Rate:

$$P_0 = 1 - | \langle 0, out | 0, in \rangle |^2 = \frac{e^2 E_z B_z}{4\pi^2} \coth \left( \frac{B_z}{E_z} \pi \right) \exp \left( - \frac{m^2 \pi}{|e E_z|} \right)$$

(n=1 world-line instanton)

# Expectation Value: IN-IN states

- Transition amplitude

$$\langle 0, out | \partial_\mu j_5^\mu | 0, in \rangle$$

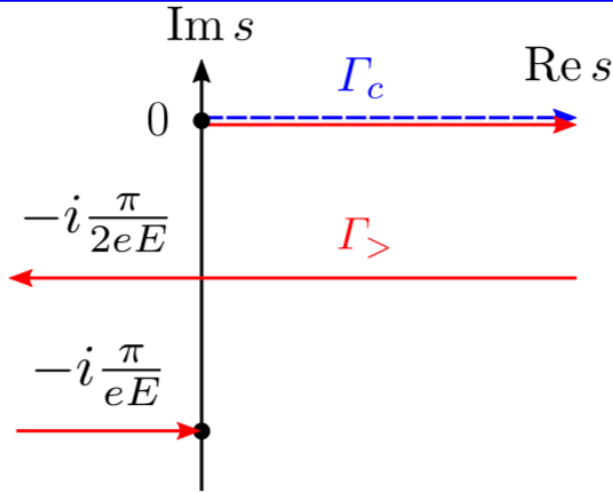
- Expectation value

$$\langle 0, in | \partial_\mu j_5^\mu | 0, in \rangle$$

Review: F. Gelis, N. Tanji 2015; N. Tanji 2009

Textbook: E.S. Fradkin, D.M. Gitman, Sh.M. Shvartsman: Quantum Electrodynamics with Unstable Vacuum, 1991

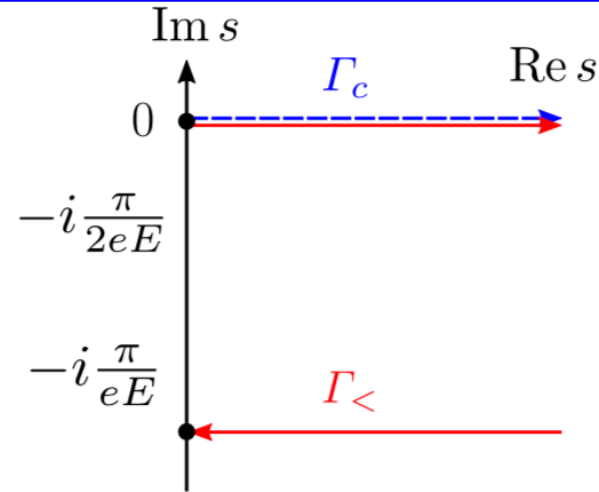
# Feynman Propagator for IN-IN



IN-OUT Propagator Path in Blue

$$S_A(x, y) = (i\not{D}_x + m) \Delta(x, y)$$

$$\Delta(x, y) = \left[ \theta(x_3 - y_3) \int_{\Gamma^C} + \theta(y_3 - x_3) \int_{\Gamma^C} \right] ds \times e^{-im^2 s} g(x, y, s),$$



IN-IN Propagator Path in Red

$$S_{in}(x, y) = (i\not{D}_x + m) \Delta_{in}(x, y)$$

$$\Delta_{in}(x, y) = \left[ \theta(x_3 - y_3) \int_{\Gamma^>} + \theta(y_3 - x_3) \int_{\Gamma^<} \right] ds \times e^{-im^2 s} g(x, y, s),$$

Textbook: E.S. Fradkin, D.M. Gitman, Sh.M. Shvartsman:

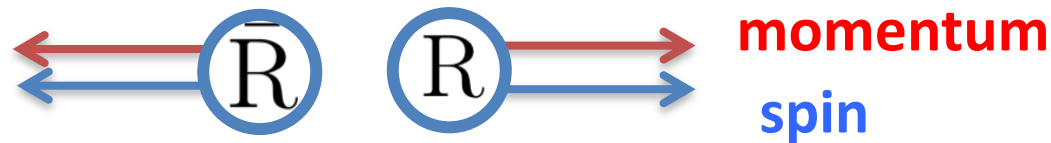
Quantum Electrodynamics with Unstable Vacuum, 1991

Shi Pu(USTC) 浦实 (中国科学技术大学) Chirality production, chiral magnetic effect and Schwinger mechanism

# Chirality Production

$$\partial_\mu j_5^\mu = \frac{e^2 EB}{2\pi^2} \exp\left(-\frac{\pi m^2}{eE}\right)$$

- $m \rightarrow 0$ ,  
Chiral anomaly



$$\frac{1}{2}\partial_t n_5 = \text{Schwinger Pair Production rate}$$

K. Fukushima, D.Kharzeev, H. Warringa PRL 2010

# Mass correction to CME

- E,B at z direction:

$$j^3 = \frac{e^2 EB}{2\pi^2} \coth\left(\frac{B}{E}\pi\right) \exp\left(-\frac{\pi m^2}{eE}\right) t$$

- Non-perturbative:  $\sim 1/(eE)$
- Sum over all Landau levels:  $\coth(B/E \pi)$

# Summary for world-line formalism

- Introduce a new method to compute **non-perturbative dynamical** quantities in **strong EB** fields.

e.g.

- Axial Ward identity, correct mass correction!
- Mass correction to CME
- Dynamical chiral condense

# 3. Anomalous **M**agneto**H**ydro**D**ynamics

Ref: Irfan Siddique, Ren-jie Wang, SP, and Qun Wang,  
**Phys.Rev.D** 99 (2019), 114029

# Anomalous **M**ageto**H**ydro**D**ynamics

- Conservation equations:
  - Energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0, \quad T^{\mu\nu} = \underbrace{T_F^{\mu\nu}}_{\text{Fluid part}} + \underbrace{T_{EM}^{\mu\nu}}_{\text{Electromagnetic part}}.$$

- (anomalous) currents conservation

$$\partial_\mu j_e^\mu = 0, \quad \partial_\mu j_5^\mu = -e^2 C E \cdot B,$$

**Electric Charge current**                      **Chiral current**

- Maxwell's equation:

$$\partial_\mu F^{\mu\nu} = j_e^\nu, \quad \partial_\mu (\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}) = 0.$$



# Previous Studies: ideal MHD without CME

- 1+1 D ideal MHD Bjorken flow

V.Roy, SP, L. Rezzolla, D. Rischke, Phys.Lett. B750, 45

– With magnetization effects

SP, V. Roy, L. Rezzolla, D. Rischke, Phys.Rev. D93, 074022

- 2+1 D ideal MHD Bjorken flow (perturbative)

SP, Di-Lun Yang, Phys.Rev. D93, 054042

- Background Magnetic field: contribution to  $v_2$

V.Roy, SP, L. Rezzolla, D. Rischke, Phys.Rev. C96, 054909

## How to add the CME to Magnetohydrodynamics?

# ideal limit of MHD

- Ideal:

- Electric conductivity is infinite

$$\sigma \rightarrow \infty$$

$$\mathbf{j} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \longrightarrow \mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

Maxwell's  
equation

$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E} = -\nabla \times (\mathbf{v} \times \mathbf{B})$$

- No space for the CME

$$\nabla \times \mathbf{B} = \mathbf{j} + \partial_t \mathbf{E}$$

- Anomalous MHD needs finite conductivity

# Constitution Eqs. for Anomalous MHD

$$\partial_\mu T^{\mu\nu} = 0,$$

$$T^{\mu\nu} = (\varepsilon + p + E^2 + B^2)u^\mu u^\nu - (p + \frac{1}{2}E^2 + \frac{1}{2}B^2)g^{\mu\nu} \\ - E^\mu E^\nu - B^\mu B^\nu - u^\mu \epsilon^{\nu\lambda\alpha\beta} E_\lambda B_\alpha u_\beta - u^\nu \epsilon^{\mu\lambda\alpha\beta} E_\lambda B_\alpha u_\beta,$$

$$\partial_\mu j_e^\mu = 0,$$

**Electric  
Conducting  
flow CME**

$$\partial_\mu j_5^\mu = -e^2 C E \cdot B,$$

$$j_e^\mu = n_e u^\mu + \sigma E^\mu + \xi B^\mu, \\ j_5^\mu = n_5 u^\mu + \sigma_5 E^\mu + \xi_5 B^\mu,$$

$$\partial_\mu F^{\mu\nu} = j_e^\nu,$$

**CSE**

$$\partial_\mu (\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}) = 0.$$

$$F^{\mu\nu} = E^\mu u^\nu - E^\nu u^\mu + \epsilon^{\mu\nu\alpha\beta} u_\alpha B_\beta,$$

# Equation of States (EoS)

- High (chiral) chemical potential:

Energy density  $\rightarrow \varepsilon = c_s^{-2} p, \leftarrow$  pressure

$$n_e = a\mu_e(\mu_e^2 + 3\mu_5^2),$$
$$n_5 = a\mu_5(\mu_5^2 + 3\mu_e^2),$$

$\leftarrow$  Chemical potential

$\leftarrow$  Chiral chemical potential

- High temperature:

$$\varepsilon = c_s^{-2} p,$$
$$n_e = a\mu_e T^2, \leftarrow$$
 temperature
$$n_5 = a\mu_5 T^2,$$

# Bjorken boost invariance

- Profound Bjorken velocity

$$u^\mu = \gamma(1, 0, 0, z/t),$$

- Bjorken invariance: all observed quantity are independent on rapidity.
- Q: could the Bjorken velocity hold in electromagnetic fields?

# Simplification

- Neutral fluid:
  - Electric field will not accelerate the fluid
- Force-free-like magnetic field:
- Configuration of Electromagnetic fields:

$$E^\mu = (0, 0, \chi E(\tau), 0), \quad B^\mu = (0, 0, B(\tau), 0),$$

$$\chi = \pm 1 \quad \tau: \text{proper time}$$

$$E^\mu = F^{\mu\nu} u_\nu, \quad B^\mu = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu F_{\alpha\beta},$$

**Not EM fields  
In lab frame!**

# Results: High temperature EoS

- **Analytic solutions:** (at the order of  $\hbar$ ):

chiral density

$$n_5(\tau) = n_{5,0} \left( \frac{\tau_0}{\tau} \right) \{ 1 + a_2 e^{\sigma \tau_0} [E_1(\sigma \tau_0) - E_1(\sigma \tau)] \},$$

Energy density

$\tau$ : proper time

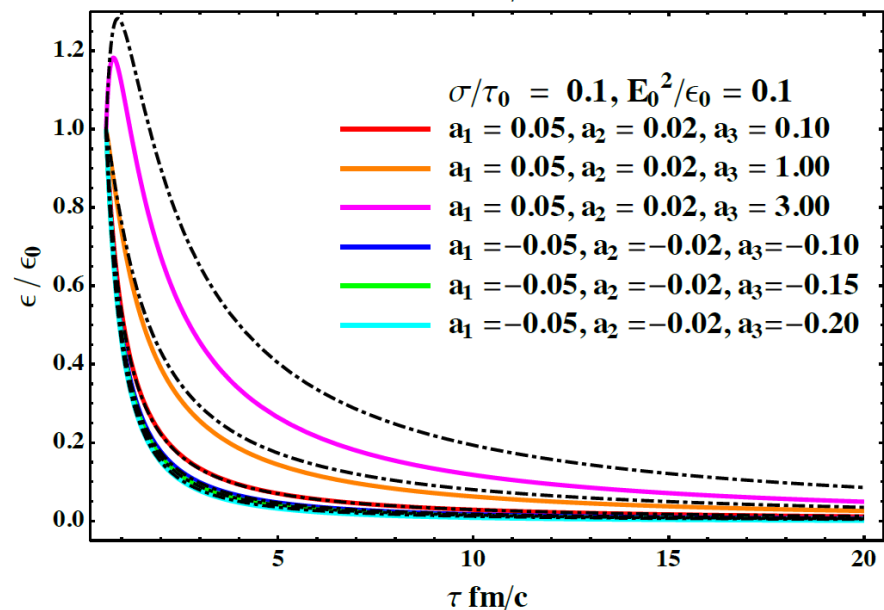
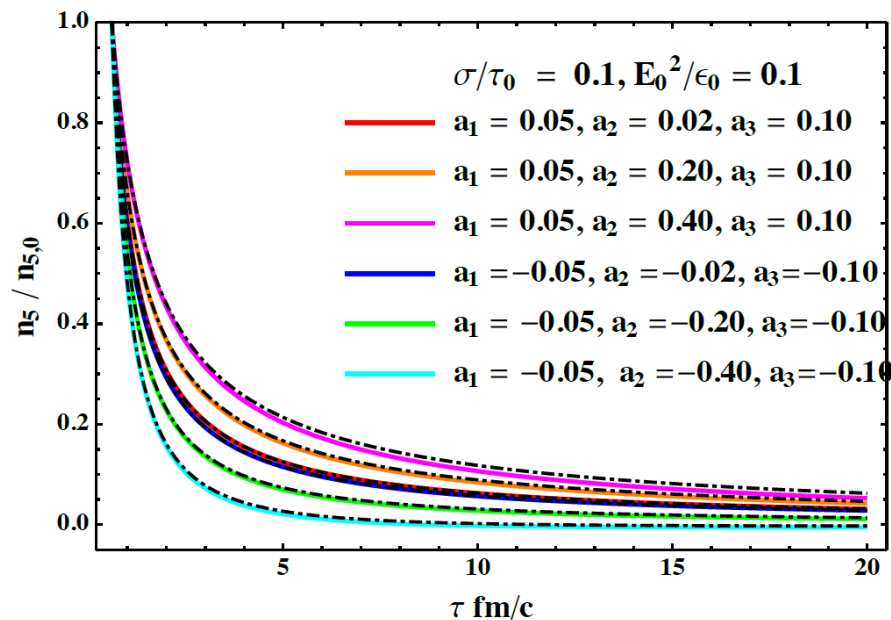
$\sigma$ : electric conductivity

$$\begin{aligned} \varepsilon(\tau) = & \epsilon_0 \left( \frac{\tau_0}{\tau} \right)^{1+c_s^2} \left\{ 1 + \sigma \frac{E_0^2}{\varepsilon_0} e^{2\sigma \tau_0} [\tau_0 E_{1-c_s^2}(2\sigma \tau_0) - \tau \left( \frac{\tau}{\tau_0} \right)^{c_s^2-1} E_{1-c_s^2}(2\sigma \tau)] \right. \\ & \left. + \frac{a_3}{\tau_0} e^{\sigma \tau_0} [\tau_0 E_{2-3c_s^2}(\sigma \tau_0) - \tau \left( \frac{\tau_0}{\tau} \right)^{2-3c_s^2} E_{2-3c_s^2}(\sigma \tau)] \right\}. \end{aligned}$$

**$a_1, a_2, a_3$  are related to the initial EM fields and chirality density**

**$E_n(x)$ : the generalized exponential integral.**  $E_n(z) \equiv \int_1^\infty dt t^{-n} e^{-zt}$

# Analytic solution VS numerical results



Solid line: numerical results / Dashed line: analytic solutions

$$a_1 = eC\chi \frac{B_0 n_{5,0}}{a T_0^2 E_0} \tau_0,$$

$$a_2 = \frac{e^2 C \chi E_0 B_0}{n_{5,0}} \tau_0,$$

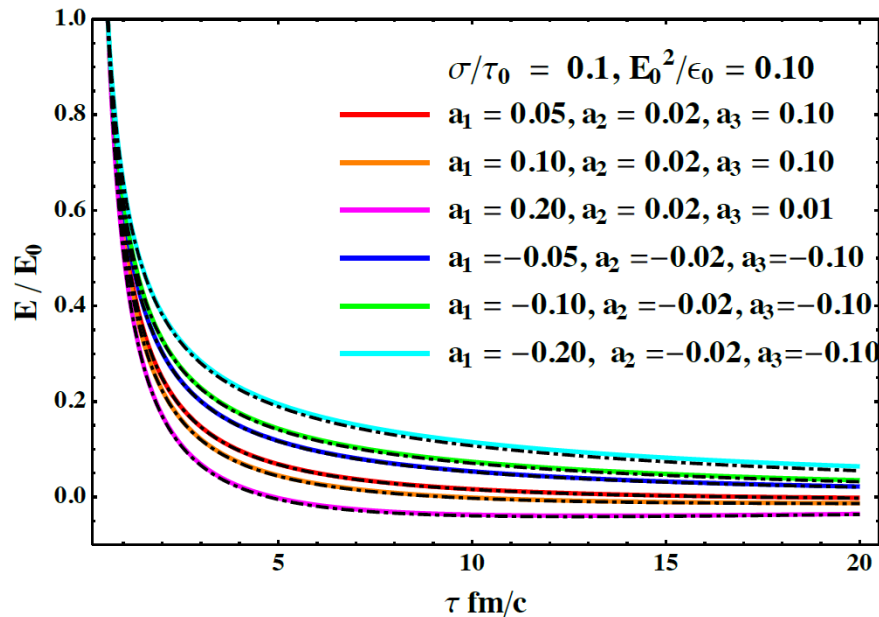
$$a_3 = \frac{eC\chi}{a} \frac{n_{5,0} E_0 B_0}{\epsilon_0 T_0^2} \tau_0.$$

**C: chiral anomaly**

**Coefficients =  $\hbar/(2\pi^2)$**



# Electromagnetic fields in the Lab frame



$$\mathbf{E}_L = (\gamma v^z B(\tau), \chi \gamma E(\tau), 0),$$

$$\mathbf{B}_L = (-\gamma v^z \chi E(\tau), \gamma B(\tau), 0),$$

$\tau$ : proper time

$\sigma$ : electric conductivity

$$E(\tau) = E_0 \left( \frac{\tau_0}{\tau} \right)^{2c_s^2} \left\{ e^{-\sigma(\tau-\tau_0)} - a_1 e^{-\sigma\tau} \left[ E_{1-2c_s^2}(-\sigma\tau_0) - \left( \frac{\tau}{\tau_0} \right)^{2c_s^2} E_{1-2c_s^2}(-\sigma\tau) \right] \right\},$$

$$B(\tau) = B_0 \frac{\tau_0}{\tau},$$

$$a_1 = eC\chi \frac{B_0 n_{5,0}}{aT_0^2 E_0} \tau_0,$$

**In the Lab frame:**

**B** decays  $\sim 1/\tau$

**B**x decays  $\sim \exp(-\sigma\tau)/\tau$

**Much slower than decaying in vacuum**

# Why $B_z$ and $E_z$ vanish?

- We have checked the Maxwell's eq. in Lab frame.  
Space and time derivatives of  $B_z$ ,  $E_z$  are zero.
- Key point: the currents is different with static case!

$$\nabla \times \mathbf{B}_L = \mathbf{j}_e + \partial_t \mathbf{E}_L.$$

$$\mathbf{j}_{e,\parallel} = \sigma \mathbf{E}_{L,\parallel} + \xi \mathbf{B}_{L,\parallel}, \quad \mathbf{v}: \text{three vector of fluid velocity}$$

$$\mathbf{j}_{e,\perp} = \sigma \gamma (\mathbf{E}_L + \mathbf{v} \times \mathbf{B}_L)_\perp + \xi \gamma (\mathbf{B}_L - \mathbf{v} \times \mathbf{E}_L)_\perp,$$

- Similar to the force-free EM fields in classical electrodynamics (e.g. Woltjer states)

Qin, Liu, Li, Squire, PRL 109, 235001 (2012); Xia, Qin, Q. Wang, PRD(2016)

# Summary of Anomalous MHD

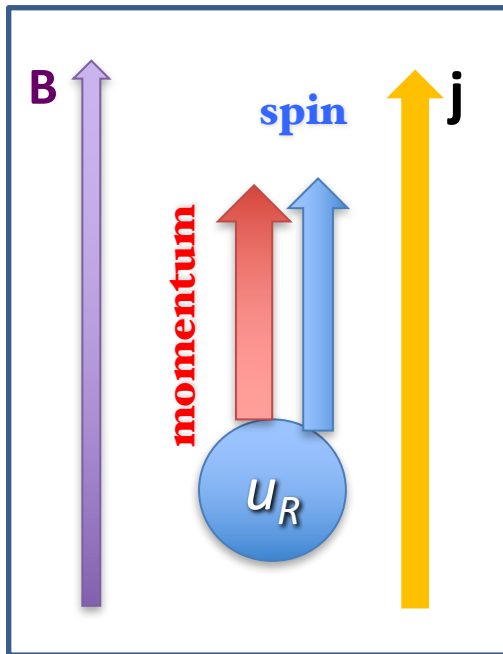
- Anomalous MHD:

Hydrodynamic eq. + Maxwell's eq. + Chiral anomalous currents

- Analytic solutions of anomalous MHD in Bjorken flow with transverse EM fields
- In lab frame, B field decays much slower than in the vacuum

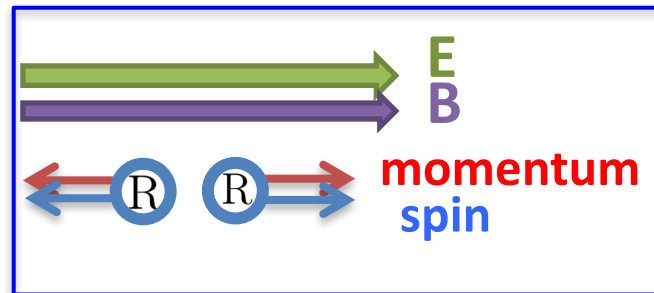
$B_y$  decays like  $\sim 1/\tau$ ,  $B_x$  decays like  $\sim \exp(-\sigma \tau)/\tau$

# Summary



1. Chiral magnetic effects

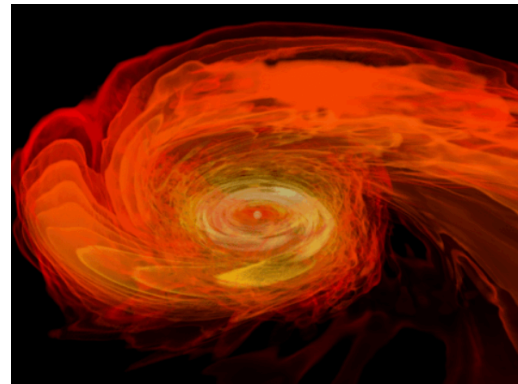
microscopic



2. Chirality production, Schwinger mechanism



macroscopic



3. Anomalous magneto-hydrodynamics

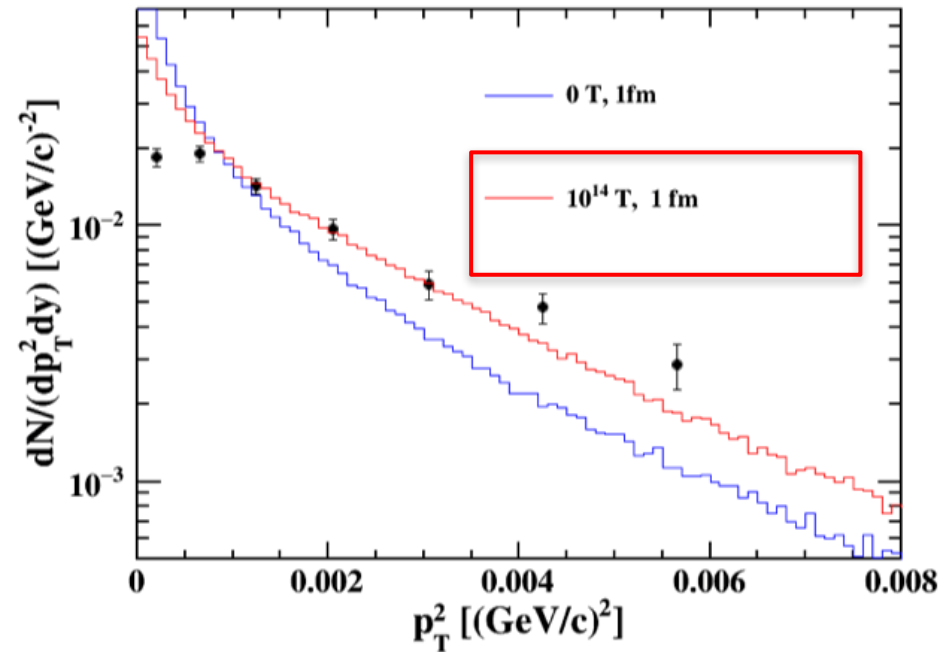
Thank you for your time!

# Backup

# Strong Electromagnetic fields in HIC

- Experiential measurement

By assuming magnetic field  $\sim 10^{14}$  T, it can explain the data.



# World-line Formalism (I)

- Schwinger proper time:

$$\frac{i}{A + i\varepsilon} = \int_0^\infty dT e^{iT(A+i\varepsilon)}$$

- Spinor Feynman propagator at background fields:

$$\begin{aligned}
 G_A(x, y) &= \text{---} \rightarrow \text{---} + \text{---} \rightarrow \text{---} \text{---} + \text{---} \rightarrow \text{---} \text{---} \text{---} + \dots \\
 &= \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot (x-y)} \frac{i}{\not{D} - m + i\varepsilon} \\
 &= \int_0^\infty dT e^{-im^2 T} \langle y | (\not{D} + m) e^{-i\hat{H}_A T} | x \rangle, \\
 &\quad \hat{H}_A = -(\hat{p}^\mu - eA^\mu)^2 + \frac{e}{2} \sigma^{\mu\nu} F_{\mu\nu}.
 \end{aligned}$$

M. D. Schwartz, Quantum Field theory and the standard model;

Christian Schubert: lecture note on the Worldline Formalism



# Lienard-Weichart formula

- Estimations from classic electromagnetic dynamics

$$\vec{E}(\vec{r}, t) = \frac{e}{4\pi} \sum_{i=1}^{N_{\text{proton}}} Z_i \frac{\vec{R}_i - R_i \vec{v}_i}{(R_i - \vec{R}_i \cdot \vec{v}_i)^3} (1 - v_i^2), \quad (1)$$

$$\vec{B}(\vec{r}, t) = \frac{e}{4\pi} \sum_{i=1}^{N_{\text{proton}}} Z_i \frac{\vec{v}_i \times \vec{R}_i}{(R_i - \vec{R}_i \cdot \vec{v}_i)^3} (1 - v_i^2), \quad (2)$$

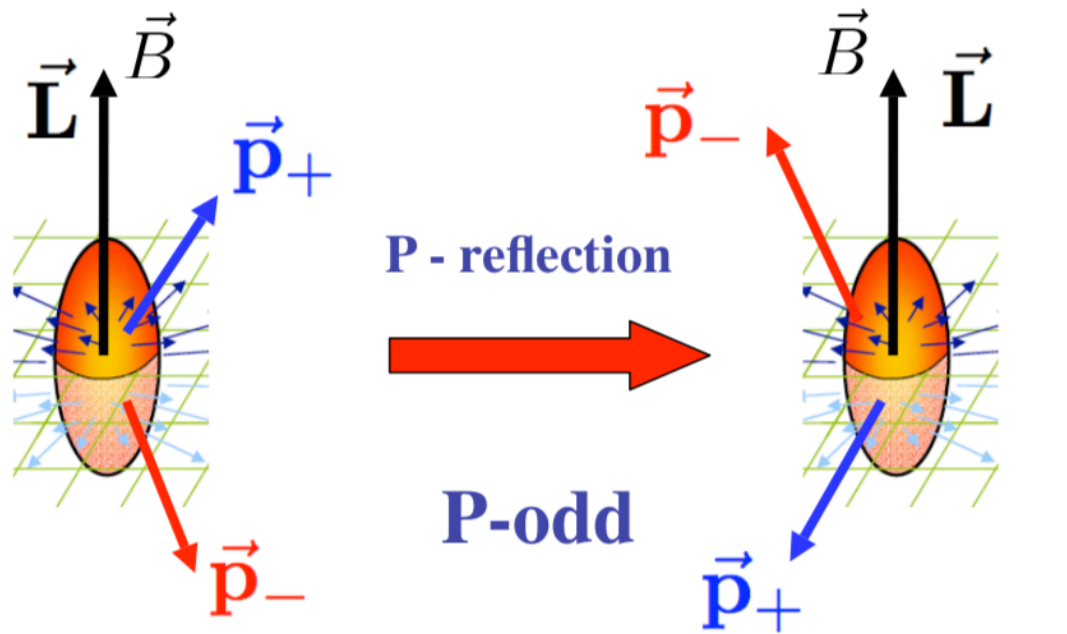
- Position of charged particles:

e.g. MC-Glauber Model + Woods-Saxon nuclear distributions

# Charge separation ?= Parity Violation

Slides from Kharzeev's talk at 26<sup>th</sup> Winter Workshop on Nuclear Dynamics (2010)

Charge separation = parity violation:



$$\mathcal{P} : \quad \vec{p} \rightarrow -\vec{p}; \quad \vec{B} \rightarrow \vec{B}; \quad \vec{L} \rightarrow \vec{L}$$

