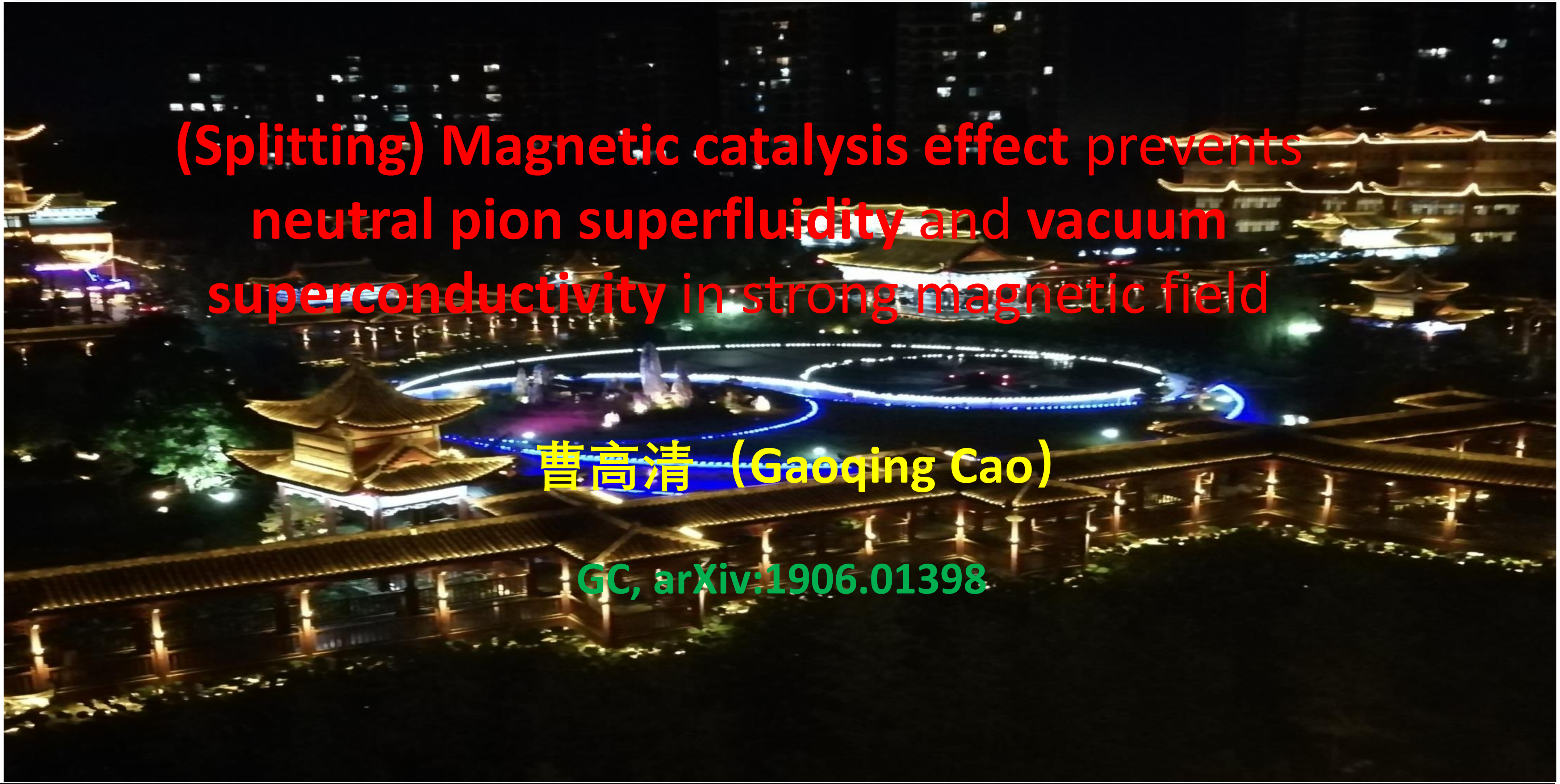
第十三届QCD相变与相对论重离子物理研讨会



(Splitting) Magnetic catalysis effect prevents neutral pion superfluidity and vacuum superconductivity in strong magnetic field

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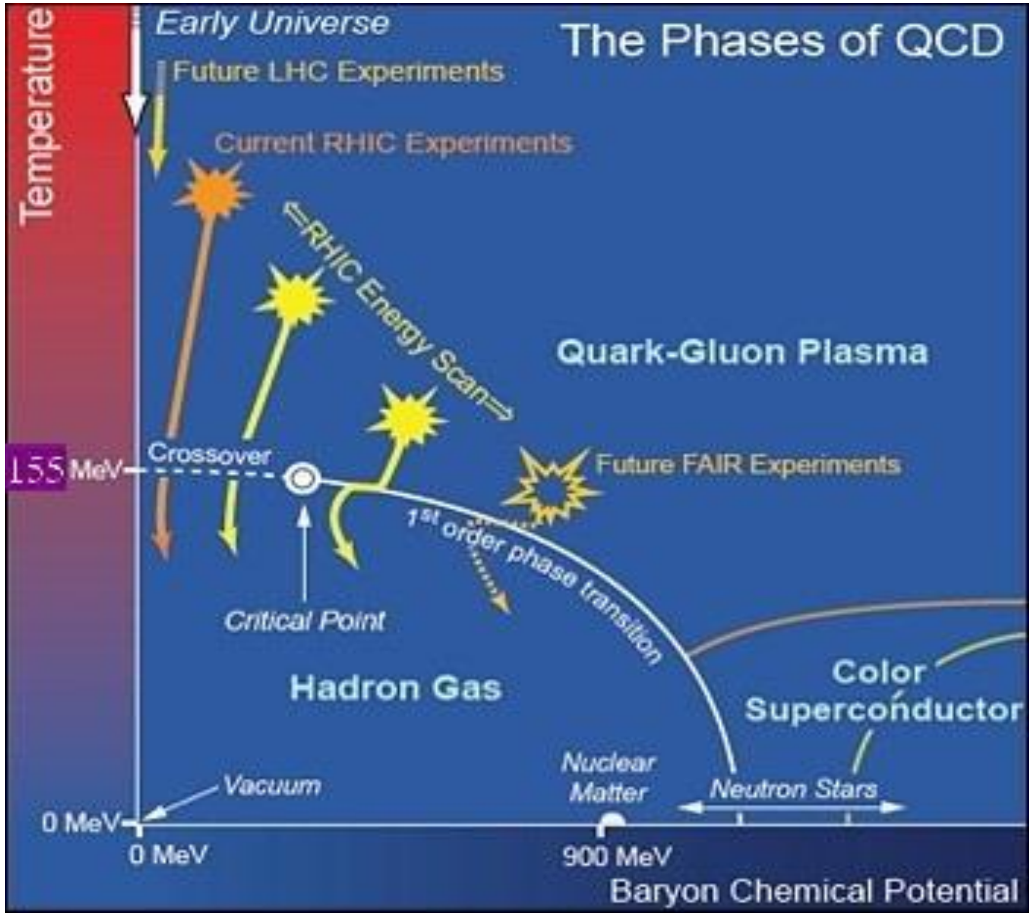
GC, arXiv:1906.01398

2019.8.16—2019.8.20

Outline

- QCD phase diagram
- Extreme conditions in HICs
- Intriguing phenomena in strong magnetic field
- NPSF and VSC in two-flavor NJL model
- NPSF and VSC in three-flavor NJL model
- Summary and prospective

Recent focus: $T-\mu$ phase diagram

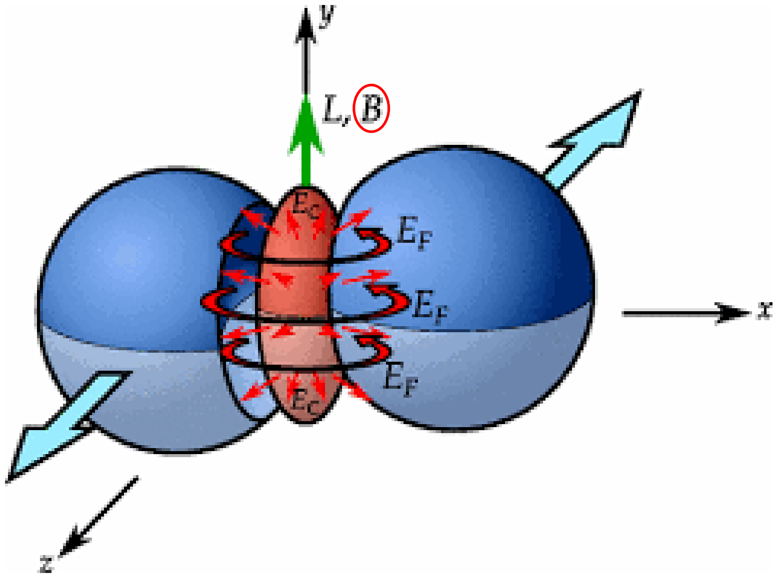


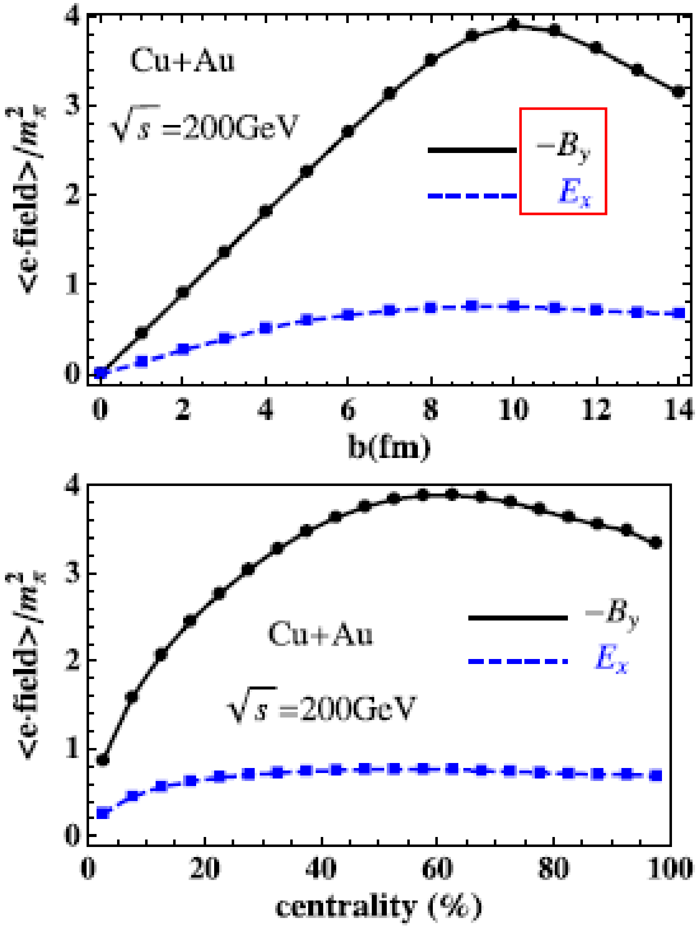
Review

- Crossover at low chemical potential: $T_c \sim 155 MeV$ – LQCD; （*H. Ding's talk*）
- *First-order* chiral transition at moderate chemical potential – effective theories;
- *Critical end point* is now a hot topic –BES II.（*M. Huang, W. Fu, X. Luo* and *D. Li's talk*）
- In neutron stars, high isospin density might be involved – *charged pion superfluidity*.

Extreme conditions in heavy ion collisions

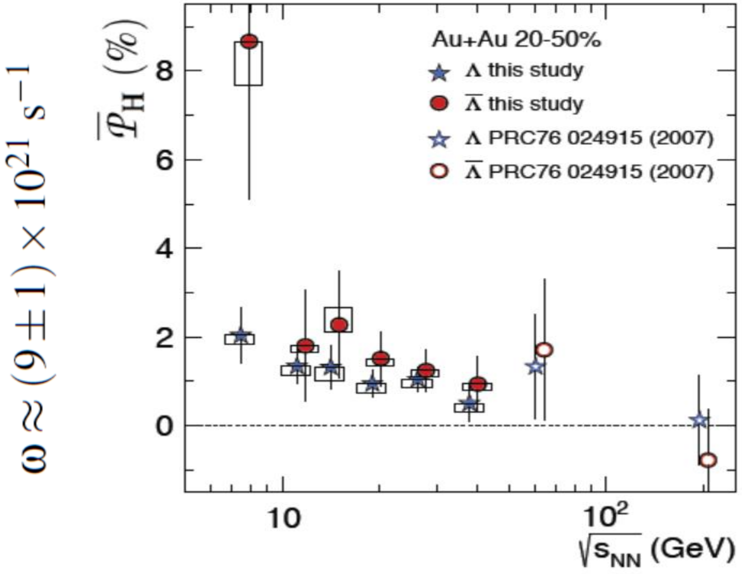
J. Liao, X. Huang , H. Ding, S. Mao , L. Yu, L. He and S. Pu's talks





W.-T. Deng, X.-G. Huang, PLB 742, 296 (2015).

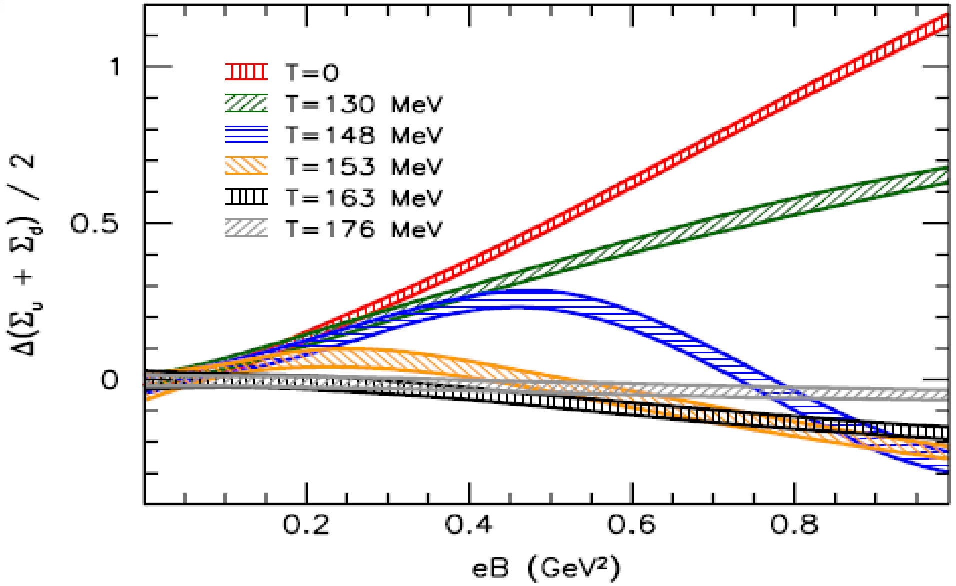
"Sign puzzle", H. Li's talk



~ 6 MeV

L. Adamczyk et al., , Nature 548, 62 (2017)

Intriguing phenomena in strong magnetic field

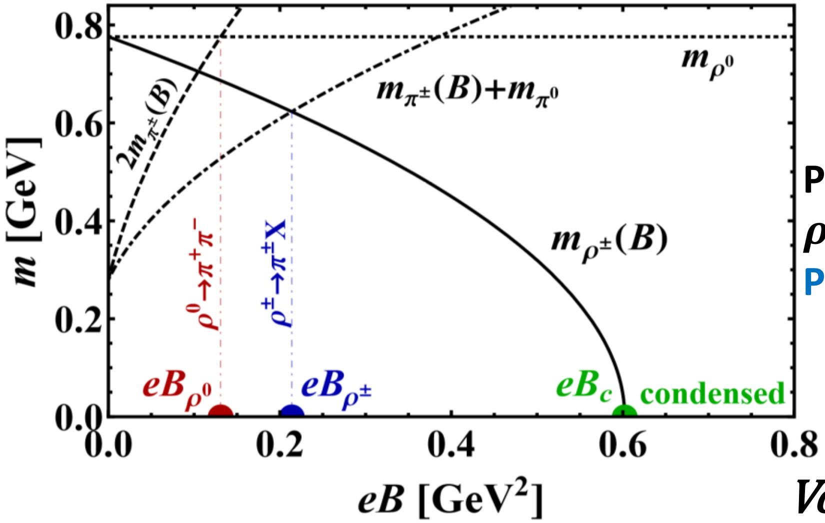


G. Bali, et al., JHEP 1202 (2012) 044; Phys.Rev. D86 (2012) 071502(R).

(1) At $T = 0$, Σ increases with B; around T_c, Σ decreases with B;

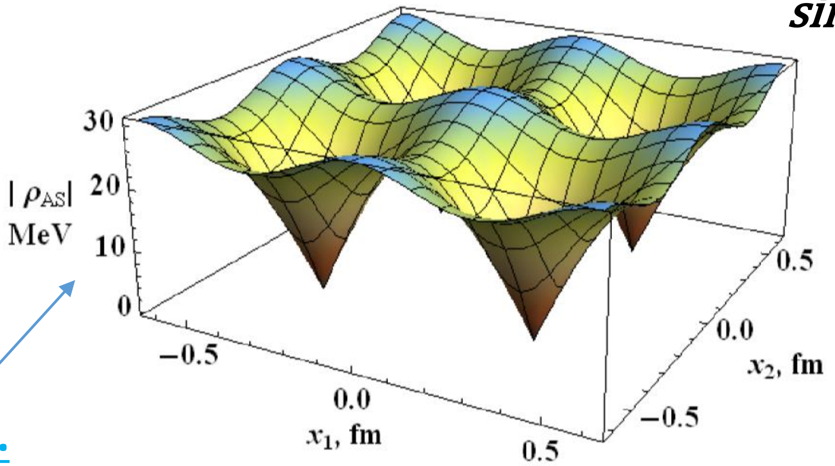
(2) T_c monotonically decreases with B (Inverse magnetic catalysis effect).

Vacuum superconductivity of QCD at $eB = m_\rho^2$.



Positive charged ρ meson with spin Parallel to B.

Vortex structure, similar to $W^\pm$.

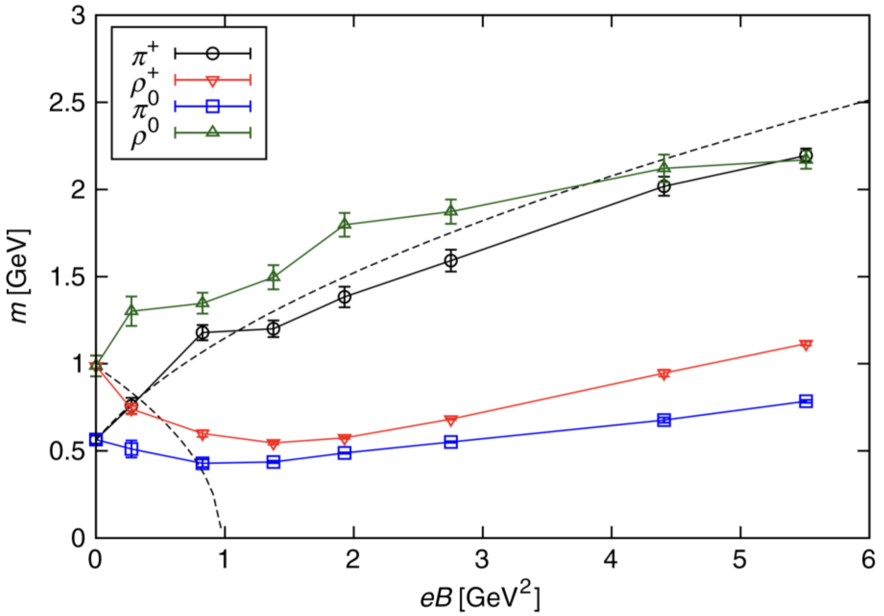


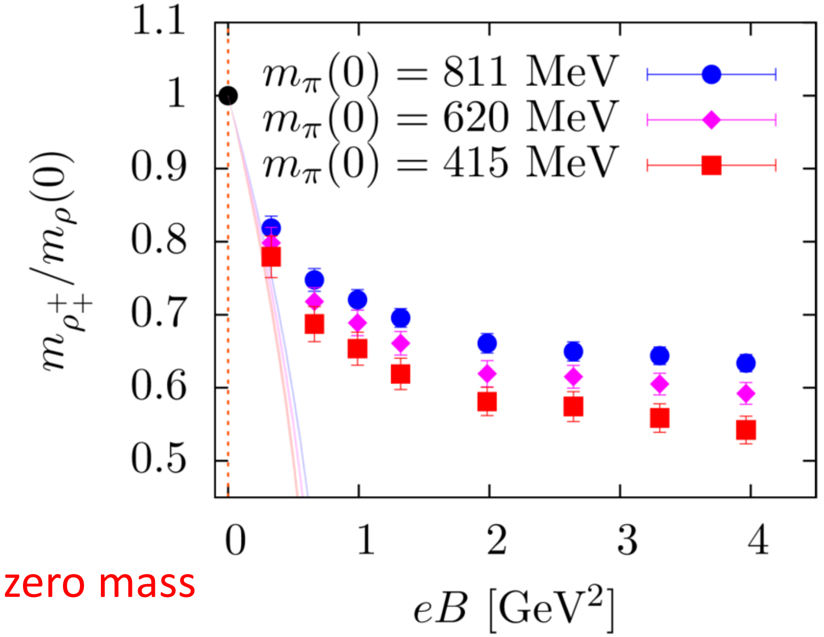
M. N. Chernodub, Phys. Rev. D 82, 085011 (2010).

Vafa-Witten theorem and Lattice QCD results

Wikipedia: vector-like global symmetry such as isospin and baryon number in vector-like gauge theories like QCD cannot be spontaneously broken as long as the theta angle is zero. (VW theorem)

$$\left| \mathrm{Tr} F \frac{1}{\not{D} + m + \epsilon\Gamma} \right| \leq \sum_{n=1}^{\infty} \frac{(\epsilon C)^n}{m^{n+1}} = \frac{\epsilon C}{m} \frac{1}{m - \epsilon C}. \xrightarrow{\epsilon \to 0} 0$$





No zero mass

Y. Hidaka and A. Yamamoto, PRD 87, 094502 (2013).

G. Bali, et al., PRD 97, 034505 (2018).

NPSF and VSC in two-flavor NJL model

Lagrangian: $$\mathcal{L}=\bar{\psi}\left(i\not{D}-m_0\right)\psi+G_S\left[\left(\bar{\psi}\psi\right)^2+\left(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi\right)^2\right]$$
$$\underline{-G_V\left[\left(\bar{\psi}\gamma^\mu\tau^a\psi\right)^2+\left(\bar{\psi}i\gamma^\mu\gamma_5\tau^a\psi\right)^2\right]}.$$

Vector interaction

Hubbard-Stratonovich Transformation:

$$\mathcal{L}=\bar{\psi}\left[i\tilde{\not{D}}-m_0-\sigma-i\gamma_5\left(\tau_3\pi^0+\tau_\pm\pi^\pm\right)\right]\psi-\frac{\sigma^2+(\pi^0)^2+\pi^\mp\pi^\pm}{4G_S}+\frac{(\omega^\mu)^2+(\rho_0^\mu)^2+\boxed{\rho_\mu^\mp\rho^{\pm\mu}}+(A^{a\mu})^2}{4G_V},$$

$$\tilde{D}_\mu=\partial_\mu+i(\underline{qA_\mu}-\omega_\mu-\tau_3\rho_{0\mu}\underline{-\tau_\pm\rho^{\pm\mu}}-i\gamma_5\tau^aA_\mu^a),$$

$$A_\mu=(0,0,-Bx_1,0)$$

Meson spectra:

$$D_{SS}^{-1}(y,x)=-\frac{e^{-iq_S\int_x^y A\cdot dx}}{2G_S}+\frac{i}{V_4}\mathrm{Tr}\,\mathcal{G}\Gamma_{S^*}\mathcal{G}\Gamma_S,$$

$$D_{\bar{V}_\mu\bar{V}_\nu}^{-1}(y,x)=\frac{e^{-iq_V\int_x^y A\cdot dx}g_{\mu\nu}}{2G_V}+\frac{i}{V_4}\mathrm{Tr}\,\mathcal{G}\Gamma_{\bar{V}_\mu^*}\mathcal{G}\Gamma_{\bar{V}_\nu},$$

$$\boxed{\mathcal{G}=\mathrm{diag}(G_\mathrm{u},G_\mathrm{d})}$$

Quark propagator

Vertices: $$\boxed{\Gamma_{\sigma/\sigma^*}=-1,\ \Gamma_{\pi^0/\pi^{0*}}=-i\gamma^5\tau_3,\ \Gamma_{\pi_\pm}=-i\gamma^5\tau_\pm,\ \Gamma_{\bar{\omega}_\mu/\bar{\omega}_\mu^*}=\bar{\gamma}_\mu^\pm,\ \Gamma_{\bar{\rho}_{0\mu}/\bar{\rho}_{0\mu}^*}=\bar{\gamma}_\mu^\pm\tau_3,\ \Gamma_{\bar{\rho}_{\pm\mu}}=\bar{\gamma}_\mu^\pm\tau_\pm,}$$

Intuition in lowest Landau level approximation

GL expansion coefficients:

$$-D^{-1}_{\pi^0\pi^0}(0) = \frac{1}{2G_S} - \frac{N_c}{\pi}\int \frac{d^2k}{(2\pi)^2}\frac{|eB|}{k^2+m^2},$$

$$-D^{-1}_{\bar\rho_1^+\bar\rho_1^+}(0) = \frac{1}{2G_V} - \frac{\boxed{16}N_c}{\boxed{9}\pi}\int \frac{d^2k}{(2\pi)^2}\frac{|eB|}{k^2+m^2}.$$

Discussions

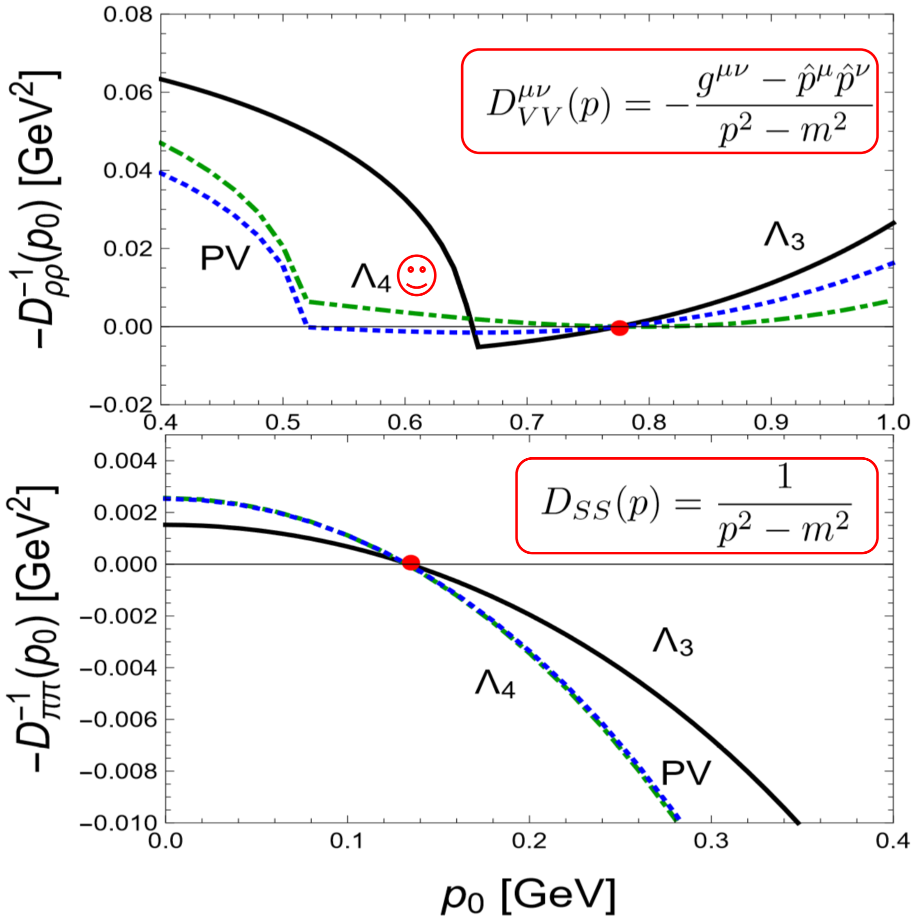
1) GLECs both decrease with magnetic field thus favor mass reductions for small B, where quark mass is almost a constant;

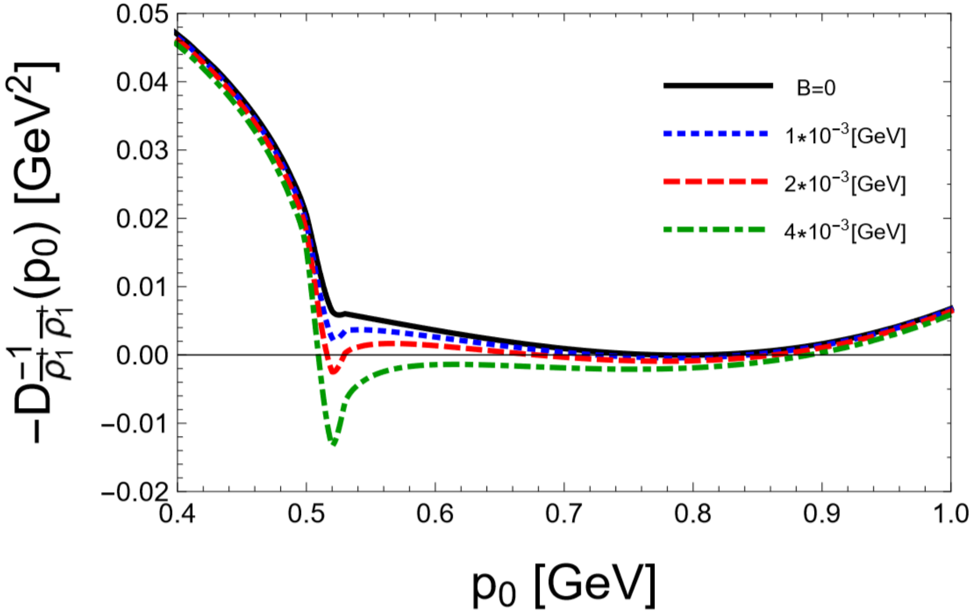
2) ρ meson mass decreases more quickly with a larger coefficient in front of B;

3) Magnetic catalysis effect (MCE) gives $-D^{-1}_{\pi^0\pi^0}(0) = \frac{m_0}{2mG_S}$, thus disfavors NPSF;

4) Both u and d quarks are involved in single polarization loop for rho meson, thus more sensitive to mass splitting.

Invalidity to physical ρ meson with $m_\rho > 2m_q$



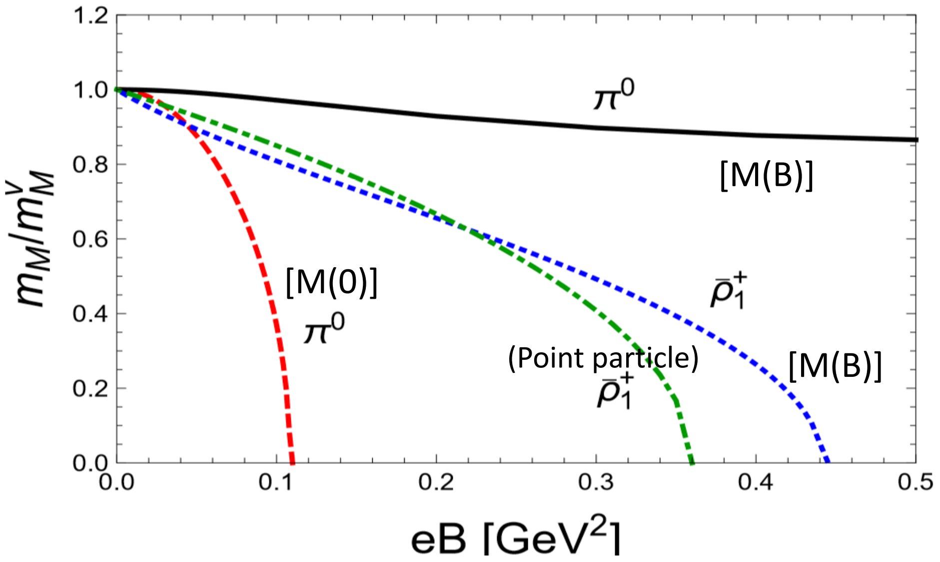


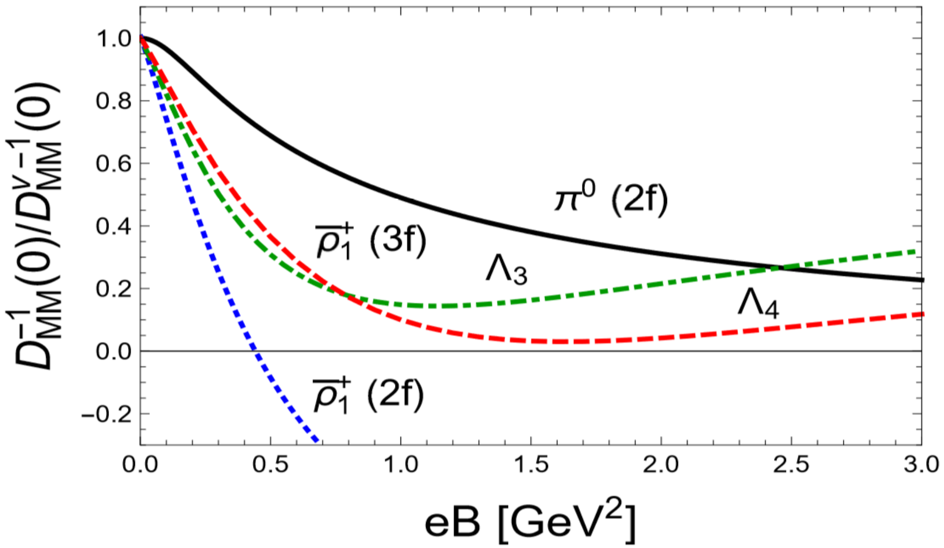
For vanishing B, only four-momentum cutoff gives the correct signs around the poles

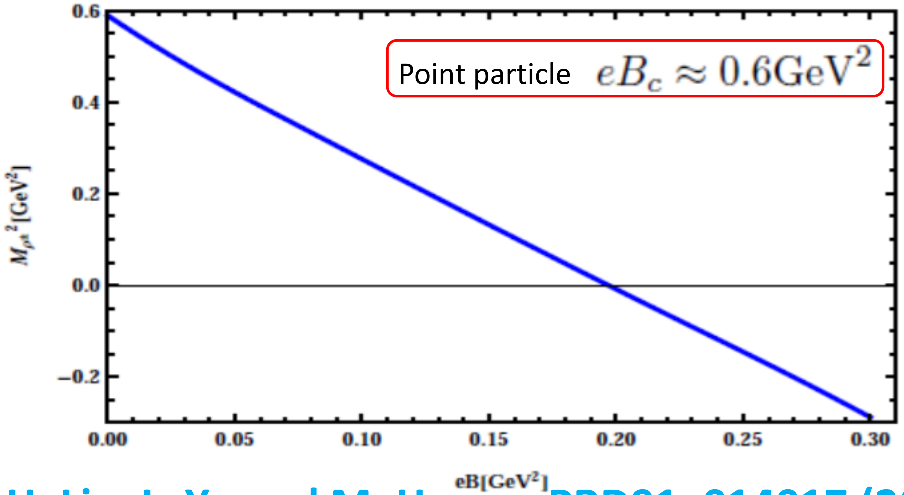
The pole mass of $\bar{\rho}_1^+$ varies quickly and discontinuously with B due to the strong dip around $2m_q$

Lack of confinement

Meson spectra







H. Liu, L. Yu and M. Huang, PRD91, 014017 (2015).

1) MCE disfavors NPSF (consistent with P. Zhuang, M. Huang and S. Mao results) and delays VSC (inconsistent with M. Huang's results),

2) GL expansion coefficients are consistent with the mass spectra.

NPSF and VSC in three-flavor NJL model

Lagrangian: $$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi}(i\not{D} - m_0)\psi + G_S \sum_{a=0}^{8} [(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2]$$

!!! The difference is not s quark

$$+\mathcal{L}_6 - G_V \left[\left(\bar{\psi}\gamma^\mu\tau^a\psi\right)^2 + \left(\bar{\psi}i\gamma^\mu\gamma_5\tau^a\psi\right)^2 \right]$$

Vector interaction

$$\mathcal{L}_6 = -K \sum_{s=\pm} \mathrm{Det}\bar{\psi}\Gamma^s\psi$$

t' Hooft determinant

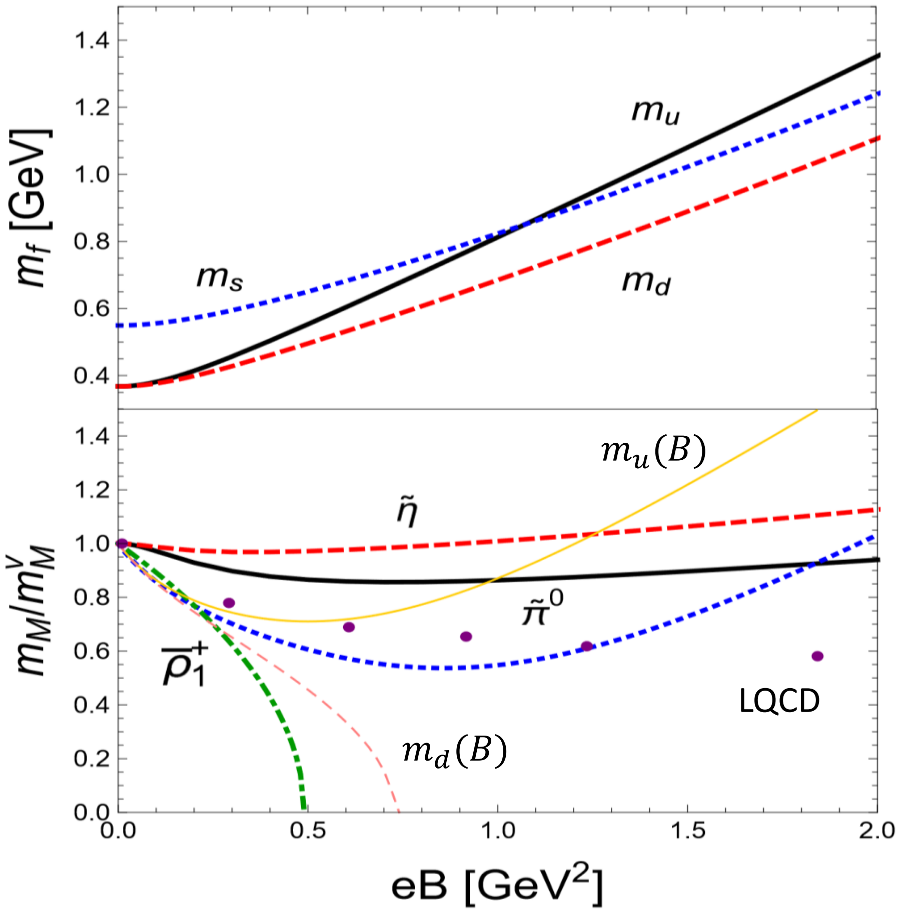
Reduced Four-fermion interaction theory:

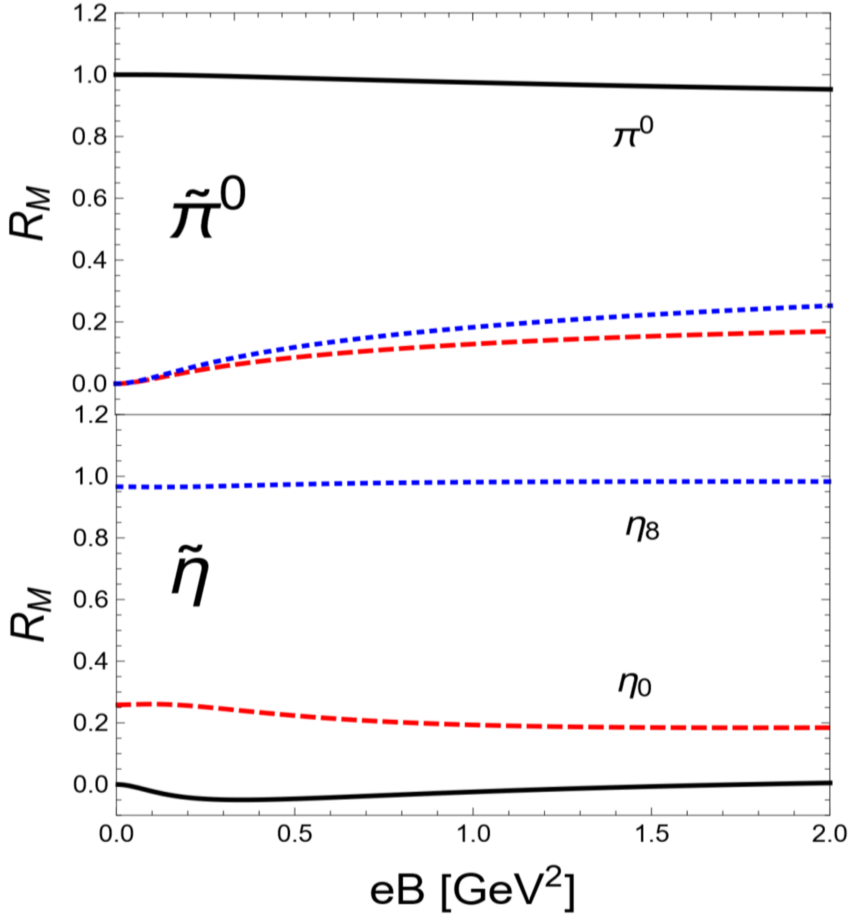
$$\mathcal{L}^4_{\mathrm{NJL}} = \bar{\psi}(i\not{D} - m_0)\psi + \sum_{a,b=0}^{8} [G^-_{ab}(\bar{\psi}\lambda^a\psi)(\bar{\psi}\lambda^b\psi) + G^+_{ab}(\bar{\psi}i\gamma_5\lambda^a\psi)(\bar{\psi}i\gamma_5\lambda^b\psi)]$$
$$-G_V\left[\left(\bar{\psi}\gamma^\mu\tau^a\psi\right)^2 + \left(\bar{\psi}i\gamma^\mu\gamma_5\tau^a\psi\right)^2\right]$$

Coupling matrices:

$$G^{\mp}_{00} = G_S \mp \frac{K}{3}\sum_{\mathrm{f=u,d,s}}\sigma_{\mathrm{f}},\ G^{\mp}_{11} = G^{\mp}_{22} = G^{\mp}_{33} = G_S \pm \frac{K}{2}\sigma_s,\ G^{\mp}_{44} = G^{\mp}_{55} = G_S \pm \frac{K}{2}\sigma_{\mathrm{d}},\ G^{\mp}_{66} = G^{\mp}_{77} = G_S \pm \frac{K}{2}\sigma_{\mathrm{u}},$$
$$G^{\mp}_{88} = G_S \mp \frac{K}{6}(\sigma_s - 2\sigma_{\mathrm{u}} - 2\sigma_{\mathrm{d}}),\ G^{\mp}_{08} = \mp\frac{\sqrt{2}K}{12}(2\sigma_s - \sigma_{\mathrm{u}} - \sigma_{\mathrm{d}}),\ G^{\mp}_{38} = -\sqrt{2}G^{\mp}_{03} = \mp\frac{\sqrt{3}K}{6}(\sigma_{\mathrm{u}} - \sigma_{\mathrm{d}}).$$

Meson spectra





Splitting MCE to quarks with different charges, which disfavors VSC. Semi-quantitatively consistent with LQCD.

Flavor mixings of pseudoscalar mesons, no flavor separation

Summary and prospective

- The fate of neutral pion superfluidity and vacuum superconductivity in strong magnetic field are rechecked and compared within two- and three-flavor NJL model;
- We found similar natures for the reductions of π^0 and $\bar{\rho}_1^+$ masses in weak B region;
- NPSF never happens due to MCE, VSC gets delayed in 2f-NJL model and is disfavored in 3f-NJL model due to splitting MCE;
- Extension to system with parallel magnetic field and rotation – charged ρ meson condensation similar to charged pion.

Thanks!

Backup I: Vacuum regularization for ρ meson

$$-D^{-1}_{\bar\rho_1^+\bar\rho_1^+} = \frac{1}{2G_V} + \underline{\Delta\Pi_{\bar\rho_1^+\bar\rho_1^+}} - 8N_c\int^{\mathrm{reg}}\frac{d^4k}{(2\pi)^4}\left(1+\frac{eB}{k_4^2+E_{\mathbf{k}}^2}\right)\frac{m^2+k_4(k_4+p_4)+k_3^2}{(k_4^2+E_{\mathbf{k}}^2)[(k_4+p_4)^2+E_{\mathbf{k}}^2]}$$

$$= \Pi_{\bar\rho_1^+\bar\rho_1^+}(p_4) - \Pi^{o(B^2)}_{\bar\rho_1^+\bar\rho_1^+}(p_4)$$

$$\boxed{\Pi_{\bar\rho_1^+\bar\rho_1^+}(p_4)} = -\frac{N_c}{4\pi^2}\int\frac{ds}{s}\int_{-1}^{1}du\; e^{-s(m^2+u^+u^-p_4^2)}\left(m^2+\frac{1}{s}-u^+u^-p_4^2\right)\frac{[1+\tanh B_u^{s+}][1-\tanh B_d^{s-}]}{\tanh B_u^{s+}/B_u^s+\tanh B_d^{s-}/B_d^s}$$

$$\boxed{\Pi^{o(B^2)}_{\bar\rho_1^+\bar\rho_1^+}(p_4)} = -\frac{N_c}{4\pi^2}\int\frac{ds}{s}\int_{-1}^{1}du\; e^{-s\left(m^2+\frac{1-u^2}{4}p_4^2\right)}\left(m^2+\frac{1}{s}-\frac{1-u^2}{4}p_4^2\right)\left(1+\frac{eBs}{2}\right)$$

The introduced regularization has no interplay with magnetic field B.

! Proper-time integral is ultraviolet divergent for $p_4 = ip_0$ with $p_0 > 2m_q$

Mathematical artifact

Solution: variable transfromation $s\left(m^2+\frac{1-u^2}{4}p_4^2\right) \to s$

Backup II: Landau-level presentations

Ultraviolet divergence can be eliminated by using Landau-level presentation:

$$S_{\rm f}(k) = -i\ e^{-\frac{\mathbf{k}_\perp^2}{|q_{\rm f}B|}} \sum_{n=0}^{\infty} (-1)^n \frac{D_n(q_{\rm f}B,k)}{k_4^2+k_3^2+m^2+2n|q_{\rm f}B|},$$

$$D_n(q_{\rm f}B,k) = (m - \not{k}_4 - \not{k}_3)\left[\mathcal{P}_+^{\rm f} L_n\left(\frac{2\mathbf{k}_\perp^2}{|q_{\rm f}B|}\right) - \mathcal{P}_-^{\rm f} L_{n-1}\left(\frac{2\mathbf{k}_{\rm f}^2}{|q_{\rm f}B|}\right)\right] + 4(\not{k}_1 + \not{k}_2) L_{n-1}^1\left(\frac{2\mathbf{k}_\perp^2}{|q_{\rm f}B|}\right)$$

Polarization becomes:

$$\Pi_{\bar\rho_1^+\bar\rho_1^+}(B,p_4) = -32N_c \sum_{n=0}^{\infty}\sum_{n'=0}^{\infty}\int \frac{\mathrm{d}^4k}{(2\pi)^4} e^{-\frac{\mathbf{k}_\perp^2}{|q_{\rm u}B|}-\frac{\mathbf{k}_\perp^2}{|q_{\rm d}B|}} \frac{(m^2+k_3^2+(k_4+p_4)k_4)L_n\left(\frac{2\mathbf{k}_\perp^2}{|q_{\rm u}B|}\right)L_{n'}\left(\frac{2\mathbf{k}_\perp^2}{|q_{\rm d}B|}\right)}{((k_4+p_4)^2+E_{\rm u}^{B\,2})(k_4^2+E_{\rm d}^{B\,2})}$$

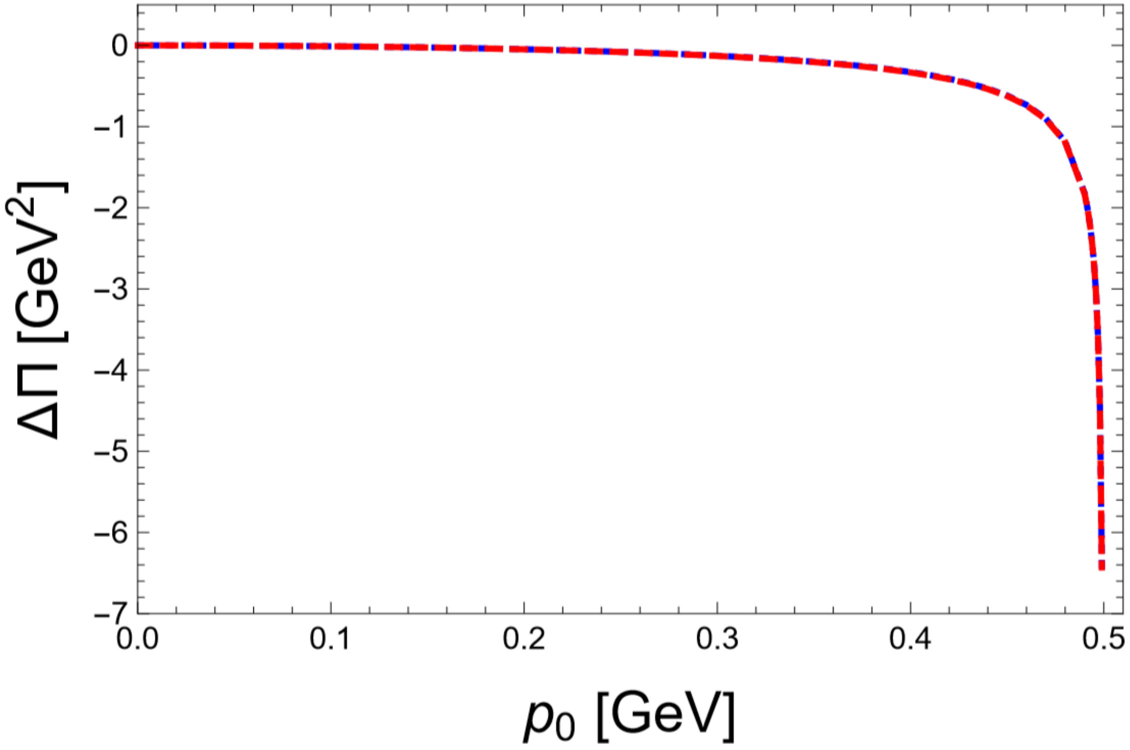
$$= -4N_c \sum_{n=0}^{\infty}\sum_{n'=0}^{\infty} \frac{eB}{\pi}\int \frac{\mathrm{d}\mathbf{k}_3}{(2\pi)}\left[\frac{(m^2+E_{\rm u}^B E_{\rm d}^B + k_3^2) G_{nn'}}{p_4^2+(E_{\rm u}^B+E_{\rm d}^B)^2}\left(\frac{1}{E_{\rm u}^B}+\frac{1}{E_{\rm d}^B}\right)\right],$$

$$\boxed{E_{\rm u}^B \equiv \sqrt{k_3^2+m^2+2n|q_{\rm u}B|} \qquad E_{\rm d}^B \equiv \sqrt{k_3^2+m^2+2n'|q_{\rm d}B|}}$$

$$G_{nn'} \equiv \int_0^\infty \mathrm{d}x\ e^{-\left(\frac{1}{|\tilde q_{\rm u}|}+\frac{1}{|\tilde q_{\rm d}|}\right)x} L_n\left(\frac{2x}{|\tilde q_{\rm u}|}\right) L_{n'}\left(\frac{2x}{|\tilde q_{\rm d}|}\right) = \frac{1}{4}\sum_{k=0}^{n}\sum_{k'=0}^{n'}\binom{n}{n-k}\binom{n'}{n'-k'}\binom{k+k'}{k}(-2|\tilde q_{\rm d}|)^{k+1}(-2|\tilde q_{\rm u}|)^{k'+1}$$

Backup II: equality between proper-time and Landau-level presentations

$$\boxed{\Delta\Pi \equiv [\Pi_{\bar{\rho}_1^+\bar{\rho}_1^+}(B_2, i\,p_0) - \Pi_{\bar{\rho}_1^+\bar{\rho}_1^+}(B_2, 0)] - (B_2 \to B_1)}$$



They are precisely consistent with each other up to $2m_q$