Chiral Phase Transition from Holographic QCD

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Department of Physics, Jinan University

Based on : arxiv:1908.02000, JHEP 1901(2019)165
JHEP 1702 (2017) 042 , JHEP 1702 (2017) 030,
JHEP 1604 (2016) 036, Phys.Rev. D93 (2016), 101901.

Collaborators: Kaddour Chelabi, Jianwei Chen, Xun Chen, Zhen Fang, Song He, Defu Hou, Mei Huang, Yue-Liang Wu, Yi Yang, Pei-Hung Yuan

QPT2019@Enshi, 2019.08.16-20

QCD vacuum

- **Confinement:**

Polyakov Loop: $L \equiv Tr[T(e^{i\int A_t dt})]$

$\langle L \rangle = 0 \quad confinement$

$\langle L \rangle \neq 0 \quad deconfinement$

- **Chiral symmetry breaking:**

Global symmetry in Lagrangian level in chiral limit:

$U_L(N_f) \times U_R(N_f)$

$q_L \to e^{-i\theta_L^a t_L^a} q_L, \quad q_R \to e^{-i\theta_R^a t_R^a} q_R$

Spontaneously broken in vacuum:

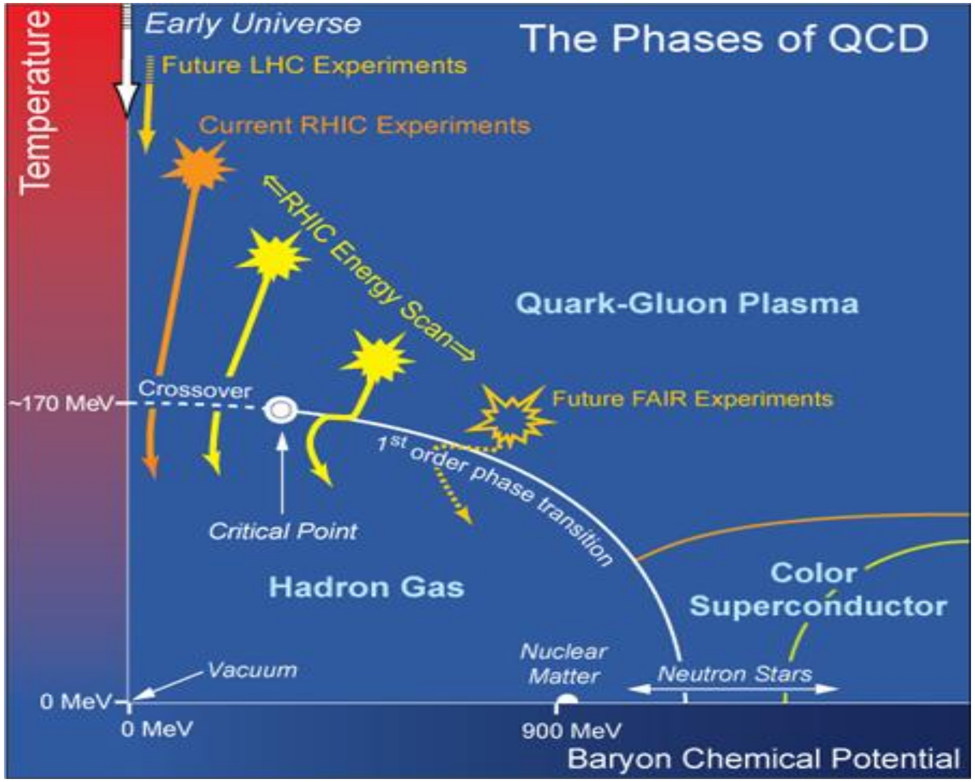
$$\boxed{m_u \approx m_d = 3-5MeV, m_{P(uud)} \approx 1GeV}$$

$$\boxed{m_\rho \simeq 770\mathrm{MeV}, m_{a_1} \simeq 1260\mathrm{MeV}}$$

Described by non-vanishing chiral condensate

$$\boxed{\langle \bar{q}q \rangle \neq 0}$$

QCD phase transitions

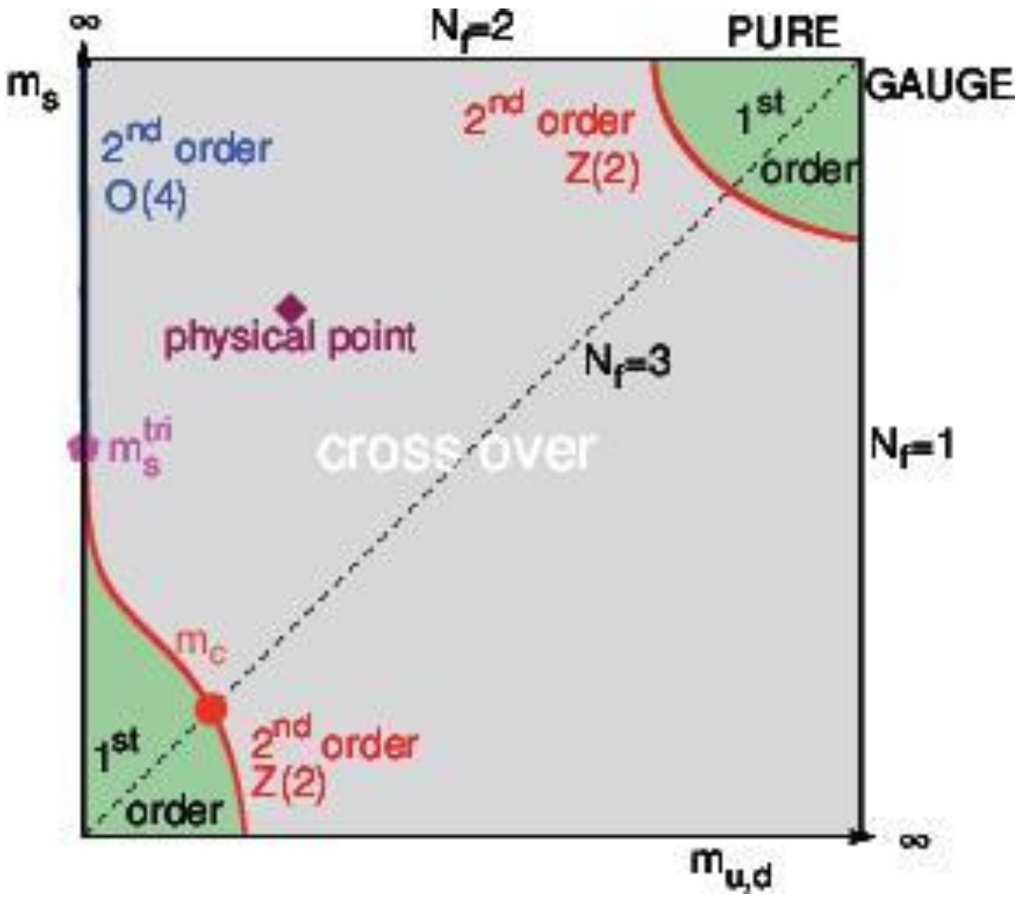


Chiral phase transition breaking and restoration of chiral symmetry:

$SU(N_f)_L \times SU(N_f)_R$

Deconfinement phase transition

Mass Diagram: Columbia Plot



From H.-T. Ding, F. Karsch and S. Mukherjee, Int. J. Mod. Phys. E 24 (2015) 1530007

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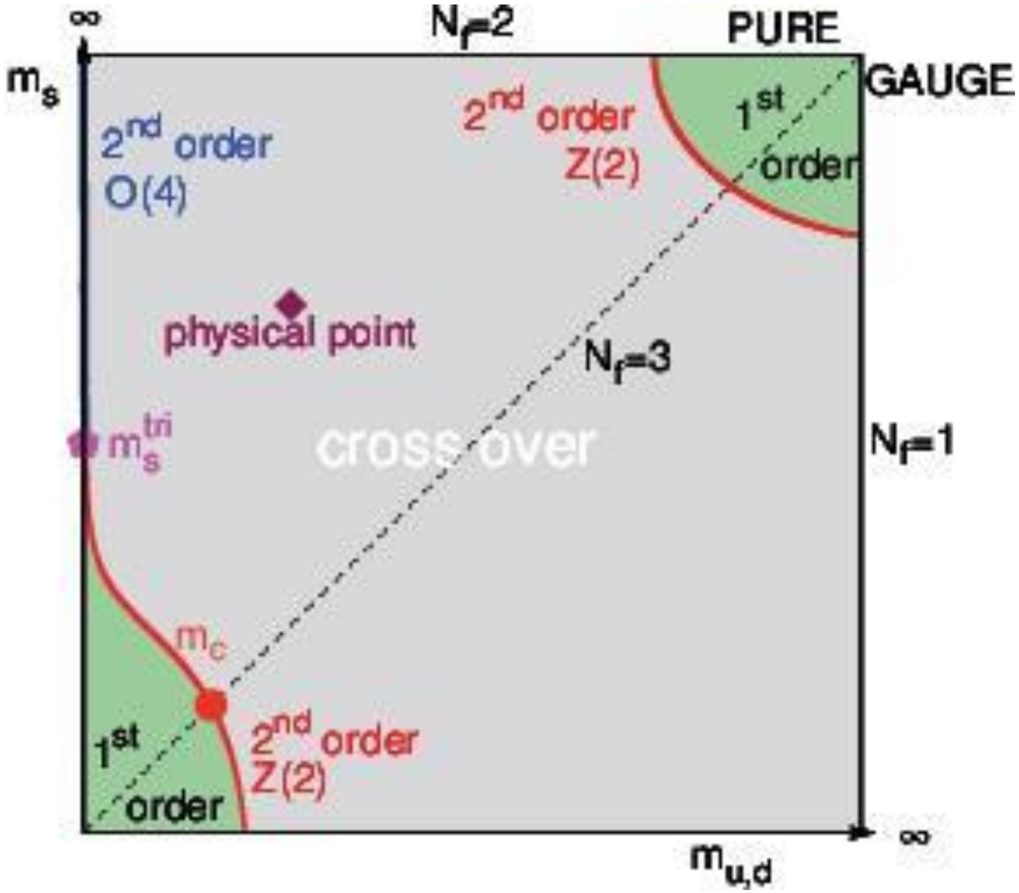

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Upper right:
1^{st} order phase transition
Left part:
chiral symmetry as good approximation

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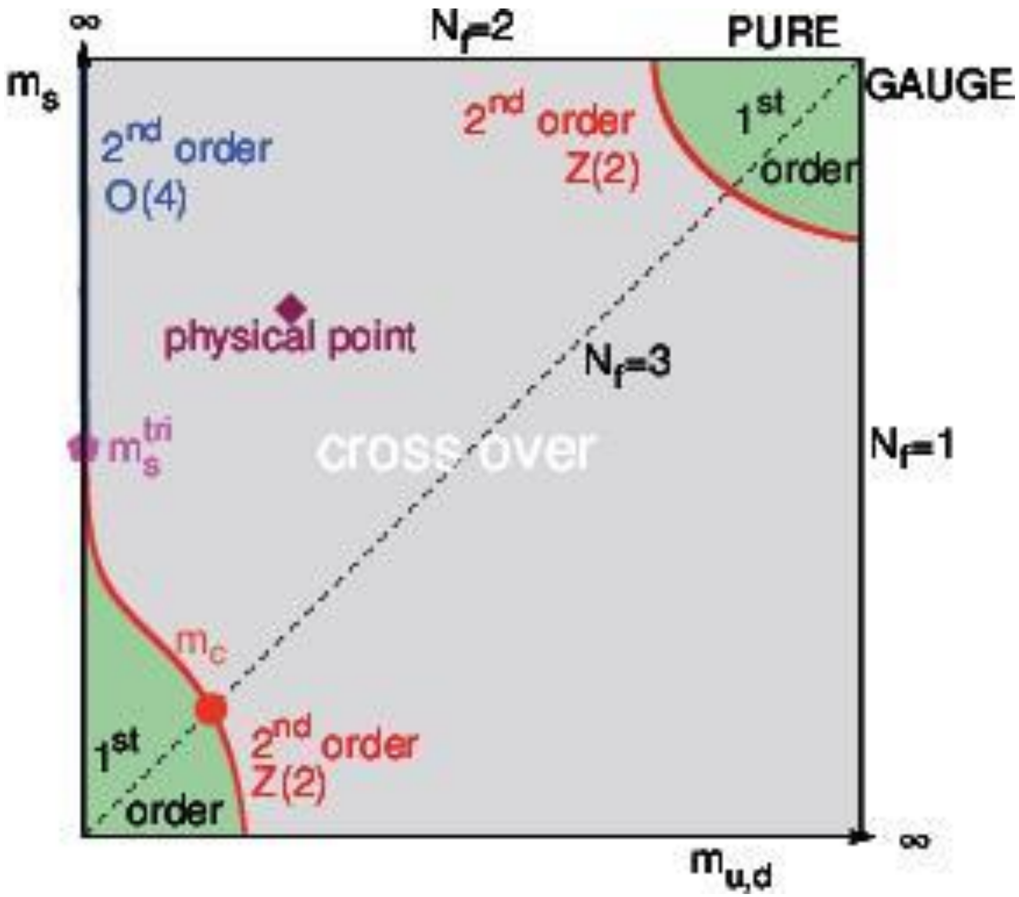

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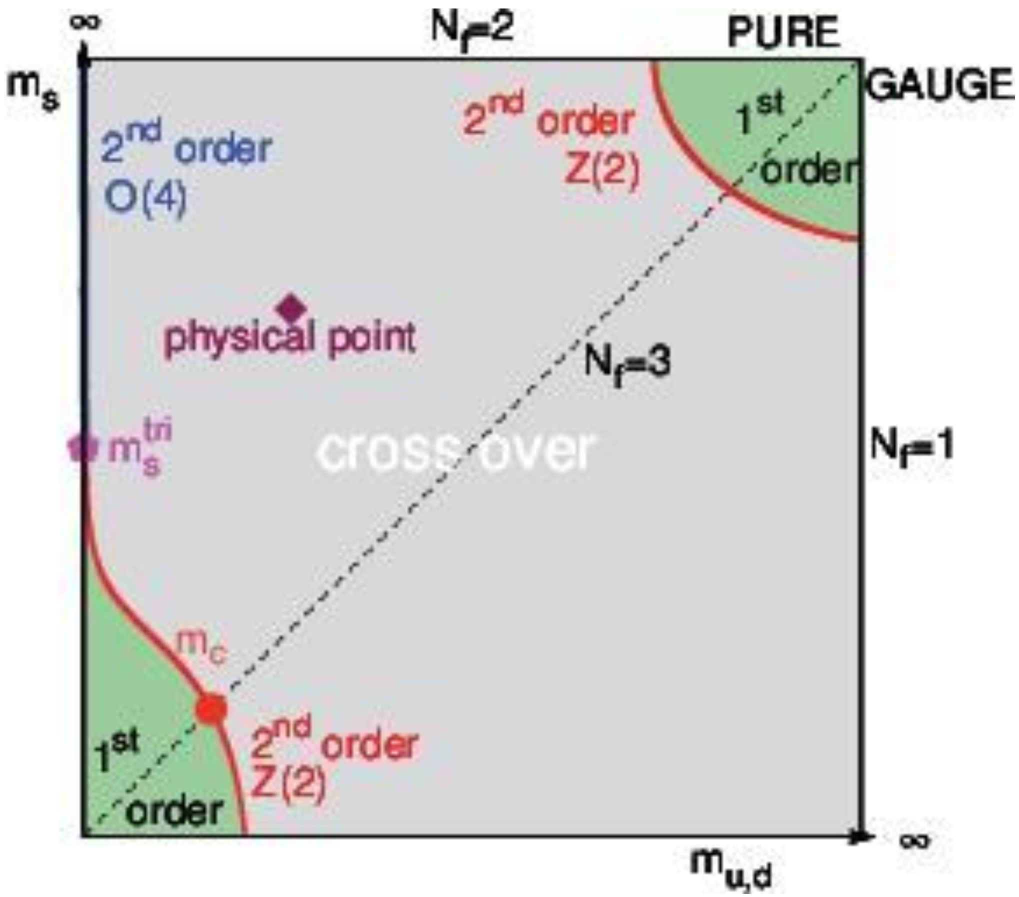
Two flavor chiral limit: 2^{nd} order
Any finite quark mass: crossover

$N_f = 2 + 1$ chiral limit: 1^{st} order
Sufficient large mass: crossover

Mass Diagram: Columbia Plot



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Sufficient large mass: crossover

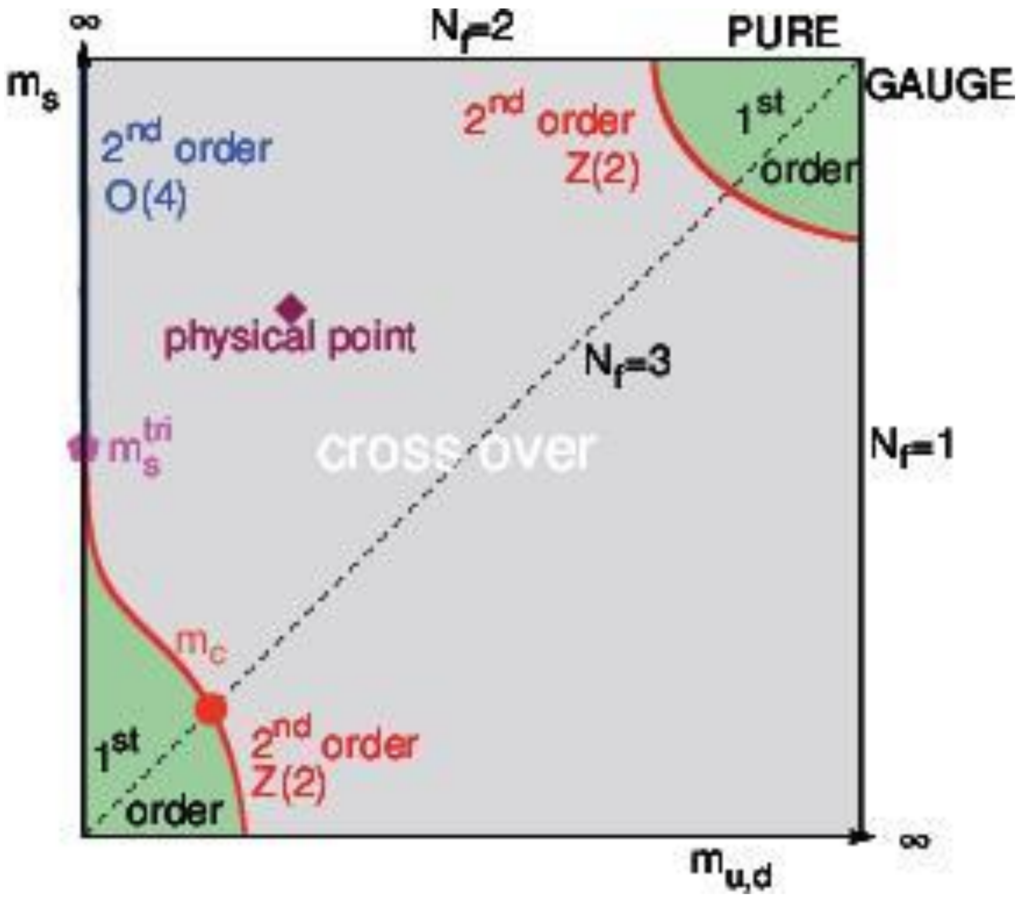
Universality classes:

$\langle\bar{\psi}\psi\rangle \simeq t^{\beta}$ $\quad \langle\bar{\psi}\psi\rangle \simeq (m - m_c)^{1/\delta}$

$O(4), Z(2)$

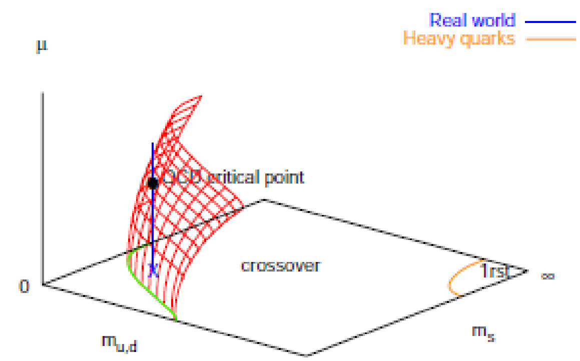
$m_s^{tric} > (or <) m_s^{phys}$

Mass Diagram: Columbia Plot



From H.-T. Ding, F. Karsch and S. Mukherjee, Int. J. Mod. Phys. E 24 (2015) 1530007

P.D.Forcrandab and O.Philipsenc, JHEP11(2008)012



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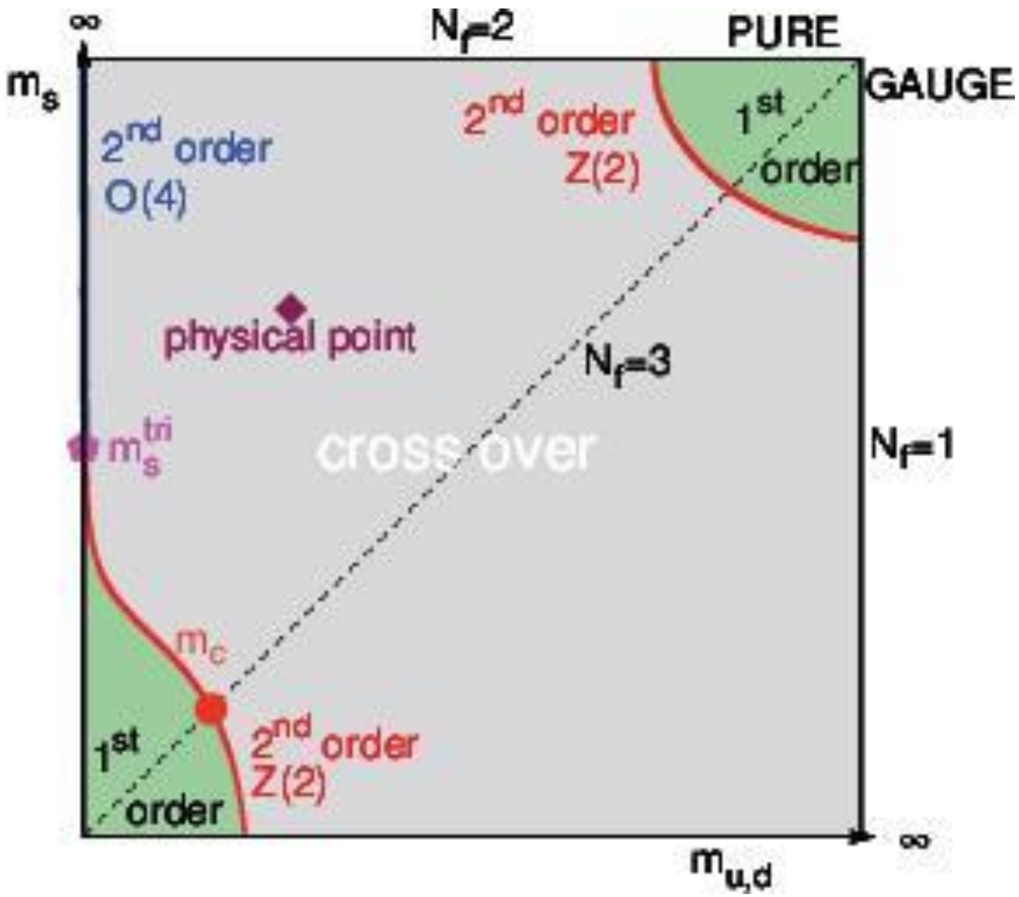

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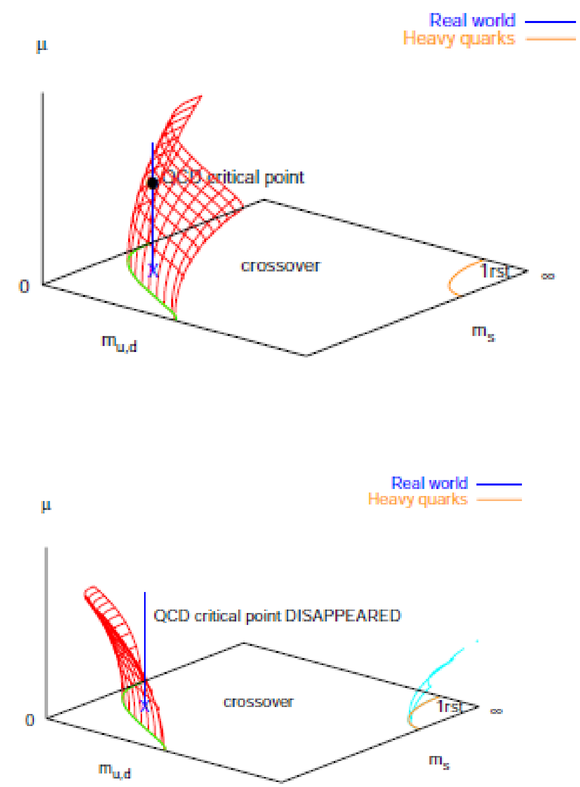
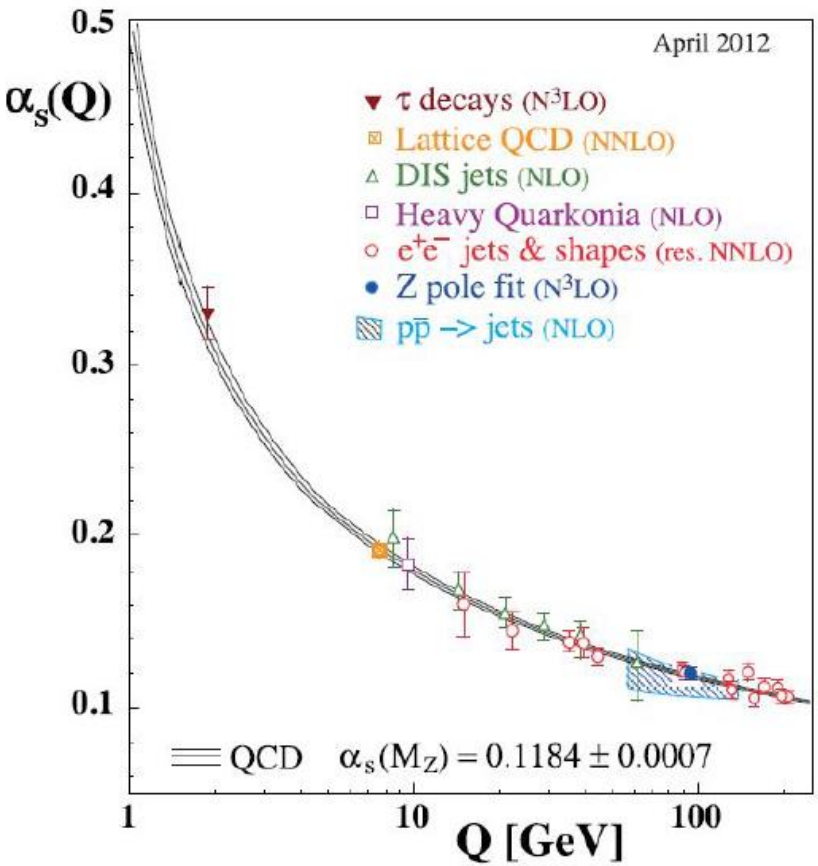


Non-perturbative Method



Non-pertubative Method:
Lattice Simulation, DSE, FRG,
Effective models: NJL, PNJL, QM…

Lattice: sign problem @ finite chemical potential

Holography

Near transition Point:
perturbative methods become invalid

Holographic Method

Closed strings in AdS background

$$\frac{R^4}{l_s^4} = 4\pi g_s N_c >> 1$$

J.Maldacena
Adv.Theor.Math.Phys.2:231-252,1998

SYM in Minkovski Space Time

$$\lambda = g_{YM}^2 N = 4\pi g_s N >> 1$$

application to QCD:
try to break the conformal symmetry

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Holography: 5th dimension(r)
plays as energy scale

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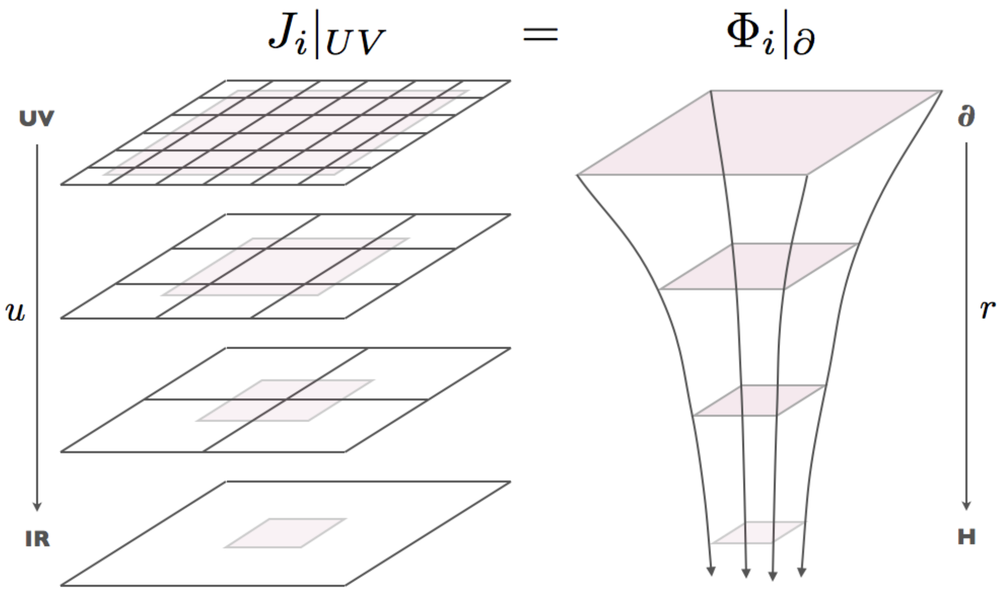

Correspondence:

$$\Phi(t,x,r) = J(t,x) r^{\Delta_1} + \ldots + O(t,x) r^{\Delta_2} + \ldots$$

$$Z_{Gauge} \sim Z_{Gravity} \sim e^{-S_{Gravity}} \sim e^{-\int a(z) JO}$$

Holographic Method

Closed strings in AdS background ⟷ SYM in Minkovski Space Time

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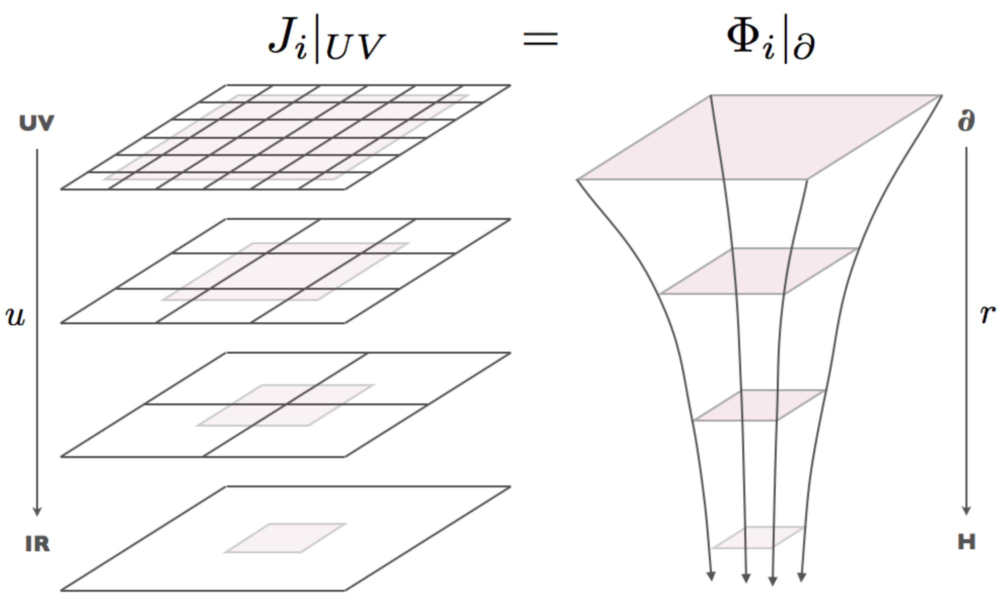


application to QCD:
try to break the conformal symmetry

Current status: model studies

e.g. : $\frac{\eta}{s} = \frac{1}{4\pi}$

P. Kovtun, D.T. Son and A.O. Starinets,
Phys.Rev.Lett.94(2005)111601

bottom-up:
Einstein-Dilaton-Maxwell system,
Soft-Wall model, Hard-Wall model…

top-down: SS model, Dp-Dq system…

Phase Transitions from Holographic

bottom-up models:

Mainly on confinement/deconfinement phase transition

Einstein-Dilaton System: deconfinemet transition at zero μ

Einstein-Dilaton-Maxwell System: $T-\mu$ phase diagram

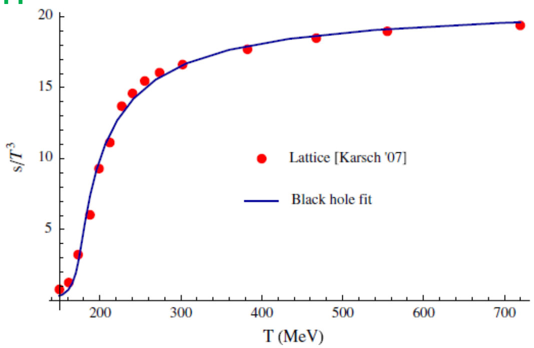
……

Different phases are distinguished by <L>

$$\mathcal{L} = \frac{1}{2\kappa^2}\left[R - \frac{f(\phi)}{4}F_{\mu\nu}^2 - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

Lattice data at $\mu = 0$ as input

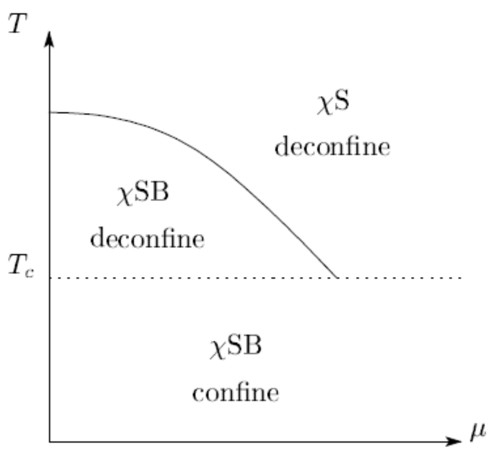
O.Steven, S. Gubser and C.Rosen, PRD 84, 126014 (2011)



$$(T_c, \mu_c) \approx (143\ \mathrm{MeV}, 783\ \mathrm{MeV})$$

top-down models:

More convenient for chiral phase transition, different way of embedding to realize symmetry breaking and restoration

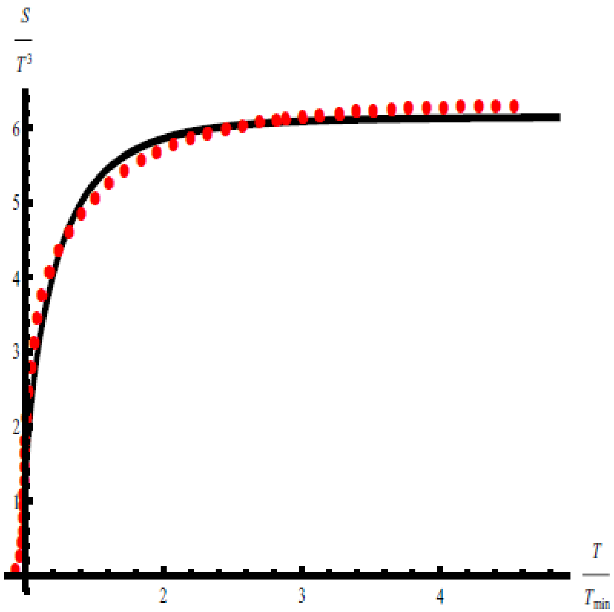


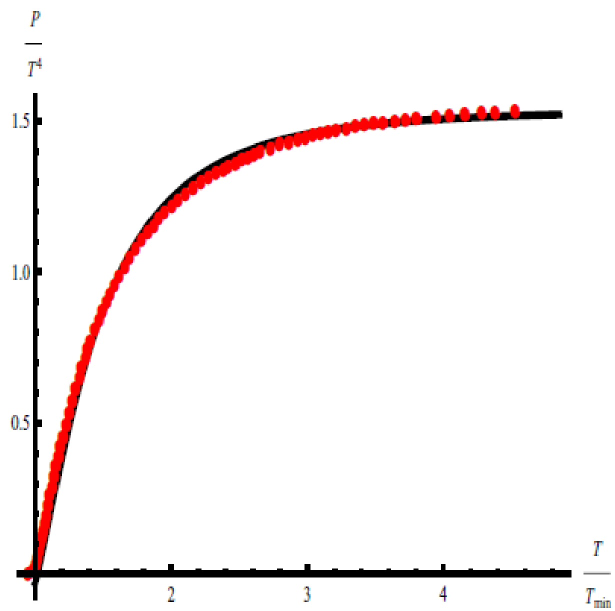
N.Horigome and Y. Tanii,JHEP01(2007)072

Phase Transitions from Holographic

bottom-up models: Quenched Dynamical HQCD

$$S_{5D} = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g^E}\left(R - \frac{4}{3}\partial_\mu\phi\partial^\mu\phi - V_E(\phi)\right)$$





DL , S.He, M.Huang and Q.S.Yan, JHEP 1109 (2011) 041

Soft-Wall Model

A.Karch, E.Katz, D.T.Son, M.A.Stephonov, Phys.Rev.D74:015005,2006

Promote 4D global chiral symmetry to 5D:

$$I = \int d^5x \, e^{-\Phi(z)} \sqrt{g} \left\{ -|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right\} \qquad SU_L(N_f) \times SU_R(N_f)$$

Field and Operator Correspondence:

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	Δ	$(m_5)^2$
$\bar{q}_L \gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R \gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

$$A^a_{L,\mu}(z) = j^a_{L,\mu}(x) + J^a_{L,\mu}(x) z^2 + \ldots$$

$$A^a_{R,\mu}(z) = j^a_{R,\mu}(x) + J^a_{R,\mu}(x) z^2 + \ldots$$

$$X(z) = m_q z + \langle \bar{q} q \rangle z^3 + \cdots$$

$$Z_{QCD} \sim Z_{Gravity} \sim e^{-S_{Gravity}} \sim e^{-\int c(z) j_i J^i + d(z) m \bar{q} q}$$

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$$Z_{CFT} \sim Z_{Gravity} \sim e^{-S_{Gravity}} \sim e^{-\int c(z) j_i J^i + d(z) m \bar{q} q}$$

$$\int_x e^{iqx} \langle J^a_\mu(x) J^b_\nu(0) \rangle = \delta^{ab} (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi_V(Q^2)$$

$$\Pi_V(-q^2) = -\frac{1}{g_5^2} \sum_\rho \frac{[\psi'_\rho(\epsilon)/\epsilon]^2}{(q^2 - m_\rho^2 + i\varepsilon) m_\rho^2}$$

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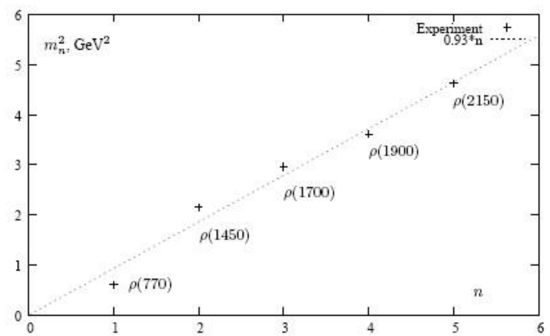
AdS_5+quadratic dilaton

$$dS^2 = e^{2A_s}\left(-dt^2 + dz^2 + dx^i dx_i\right)$$

$$A_s = -\log(z), \qquad \Phi = \mu^2 z^2$$

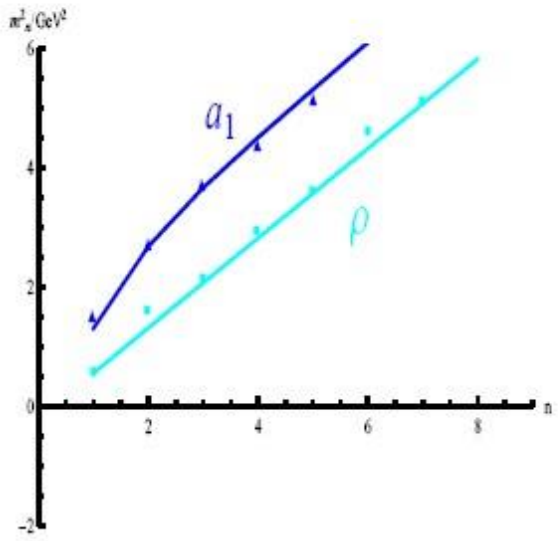


Linear confinement $m_n^2 \propto n$

Hadronic physics in Soft-Wall Model

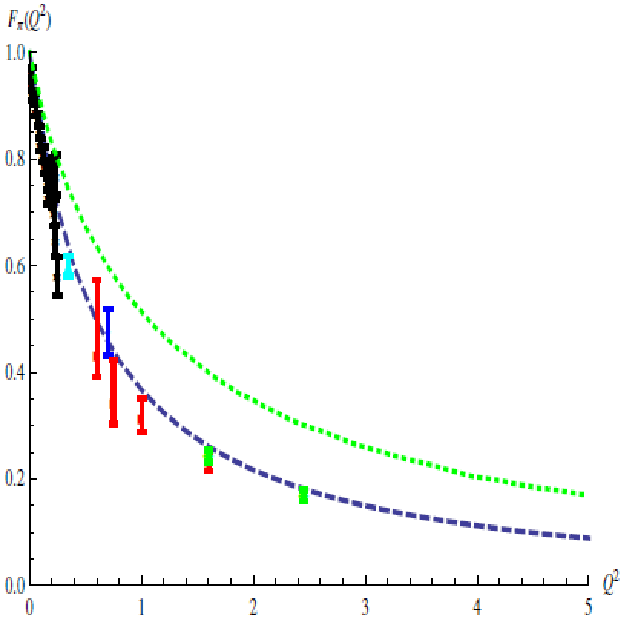
Modified metric, dilaton, scalar potential
Describe the scalar, pseudo scalar, vector, axial vector meson spectra in good agreement with experimental data.

T. Gherghetta, J. I. Kapusta and T. M. Kelley,Phys. Rev. D **79** (2009) 076003
T. M. Kelley, S. P. Bartz and J. I. Kapusta, Phys. Rev. D **83** (2011) 016002
Y. -Q. Sui, Y. -L. Wu, Z. -F. Xie and Y. -B. Yang, Phys. Rev. D **81**(2010) 014024
S.He, S.Y.Wu, Y.Yang and P.H.Yuan,JHEP 1304 (2013) 093
DL, M.Huang, Q.S.Yan, Eur.Phys.J. C73 (2013) 2615
....



quadratic dilaton: linear confinement
non-zero X: chiral symmetry breaking

SU(2)： $u\&d$ quarks



DL, M.Huang, Q.S.Yan, Eur.Phys.J. C73 (2013) 2615
DL, M.Huang, JHEP11(2013)088

Extend to $N_f > 2$

Y.Q.Sui, Y.L.Wu, Y.B.Yang, Phys.Rev. D83 (2011), 065030

SU(3)：including strange quark

n	ρ exp. (MeV)	Theory	K^* exp. (MeV)	Theory
0	775.5 ± 1	791	891.66 ± 0.26	797
1	1465 ± 25	1570	1414 ± 15	1573
2	1720 ± 20	1786	1717 ± 27	1788
3	1909 ± 30	1951	$\cdots$	1955
4	2149 ± 17	2137	$\cdots$	2140
5	2265 ± 40	2375	$\cdots$	2377

ϕ exp. (MeV)	Theory	ω exp. (MeV)	Theory
1019.455 ± 0.020	791	782.65 ± 0.12	791
1680 ± 20	1570	1400–1450	1570
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A.B.Bayona, G.Krein, C.Miller, Phys.Rev. D96 (2017), 014017

SU(4)： including charm quark

Mass	Model (MeV)	Measured (MeV)
m_ρ	775.6	775.3 ± 0.3 [50]
m_π	142.5	139.6 [50]
m_{a_1}	1232	1230 ± 40 [50]
m_K	489.2	493.7 [50]
m_{K^*}	803.7	891.7 ± 0.3 [50]
m_{K_1}	1359	1272 ± 7 [50]
$m_{K_0^*}$	674.9	682 ± 29 [50]
m_D	1831	1870 [50]
m_{D_s}	1987	1968 [50]
m_{D^*}	2161	2010 [50]
$m_{D_s^*}$	2006	2112 [50]

differences less than 5%

Chiral Condensate in Soft-Wall Model

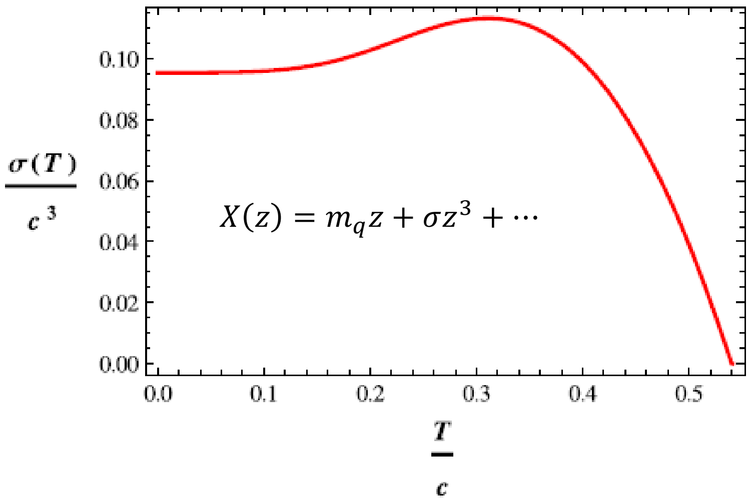
Extend to finite temperature:
AdS-SW black hole solution

$$dS^2 = e^{2A_s}\left(f(z)dt^2 - \frac{1}{f(z)}dz^2 - dx^i dx_i\right)$$

$$A_s = -\log(z)\,, \qquad \Phi = \mu^2 z^2$$

$$f(z) = 1 - \frac{z^4}{z_h^4}, \qquad T = \frac{1}{\pi\, z_h}$$

P. Colangelo, F. Giannuzzi, S. Nicotri, V. Tangorra , Eur. Phys. J. C (2012) 72:2096



Chiral Condensate in Soft-Wall Model

Extend to finite temperature:
AdS-SW black hole solution

$\sigma \propto m_q,$
When $m_q \to 0, \sigma \to 0$
not spontaneous breaking!!

P. Colangelo, F. Giannuzzi, S. Nicotri, V. Tangorra ,
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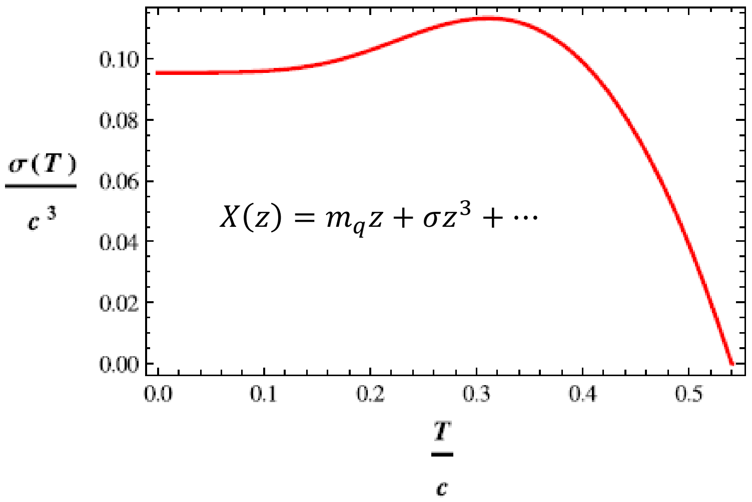

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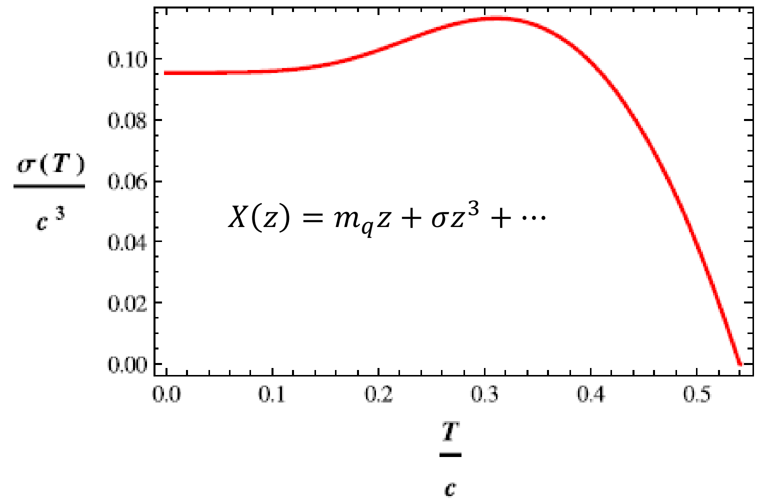
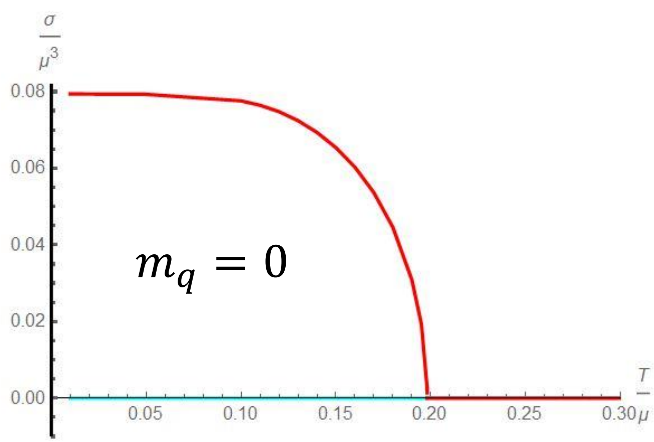


If takeing $\Phi = -\mu^2 z^2$

e.g. F.Zuo, Phys.Rev. D82 (2010) 086011
S.J.Brodsky, G.F.Teramond and A.Deur,
Phys.Rev. D81 (2010) 096010



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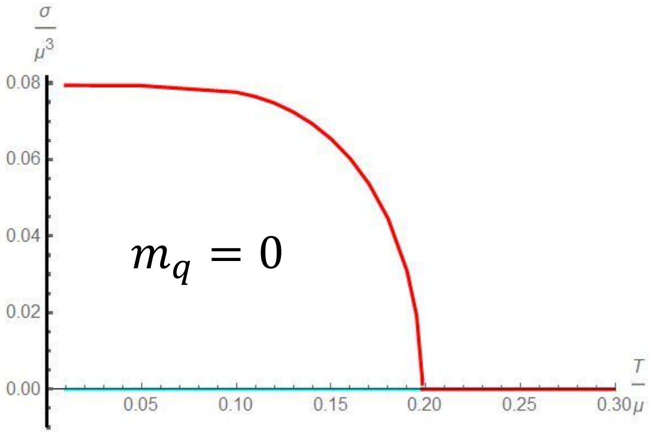
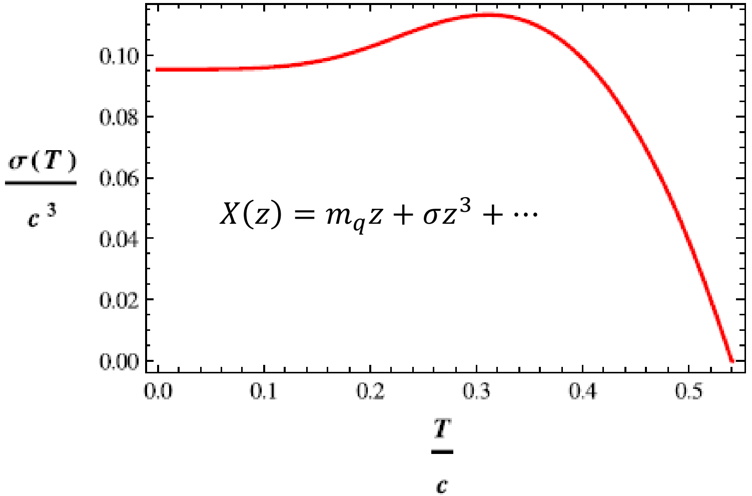


P. Colangelo, F. Giannuzzi, S. Nicotri, V. Tangorra , Eur. Phys. J. C (2012) 72:2096



However, massless scalar meson would appear in the spectra if $\int_0^\infty e^{\Phi - A_s} dz < \infty$

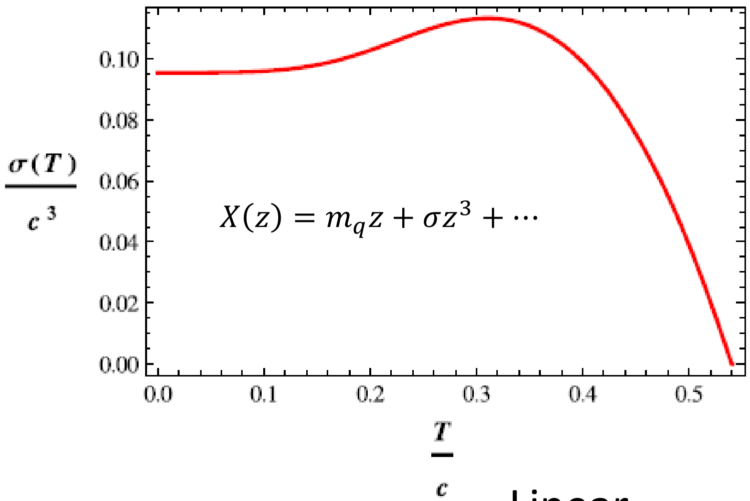
A.Karch, E.Katz, D.T.Son, M.A.Stephonov , JHEP 1104 (2011) 066

Chiral Condensate in Soft-Wall Model

Extend to finite temperature:
AdS-SW black hole solution

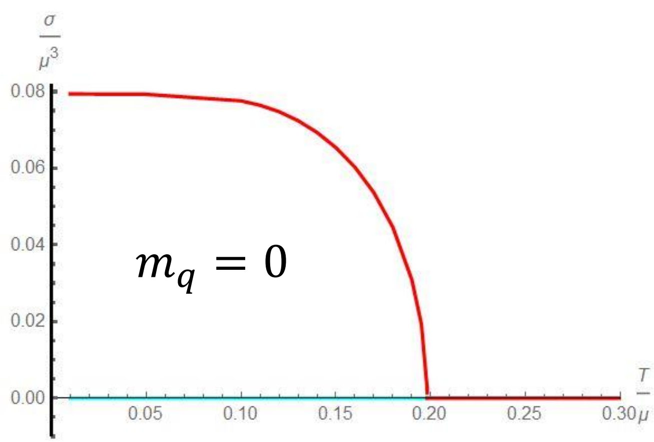
$\sigma \propto m_q$,
When $m_q \to 0, \sigma \to 0$
not spontaneous breaking!!

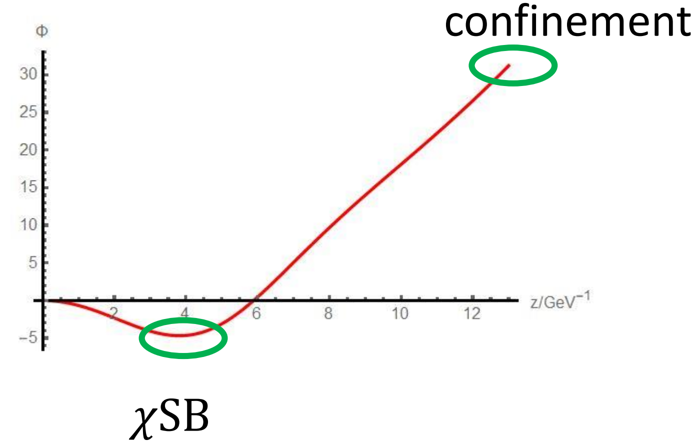
P. Colangelo, F. Giannuzzi, S. Nicotri, V. Tangorra ,
Eur. Phys. J. C (2012) 72:2096



If takeing $\Phi = -\mu^2 z^2$

e.g. F.Zuo, Phys.Rev. D82 (2010) 086011
S.J.Brodsky, G.F.Teramond and A.Deur,
Phys.Rev. D81 (2010) 096010





Two Scales: Confinement and Chiral

E.V.Shuryak and T.Schfer, Ann. Rev. Nucl.Part. Sci. 47 (1997) 359

Shuryak and Schafer's work about instanton liquid model

In addition to that, one can argue that the scales for chiral symmetry breaking and confinement are very different (2): $\Lambda_{\chi\mathrm{SB}} \gg \Lambda_{\mathrm{conf}} \sim \Lambda_{\mathrm{QCD}}$. In particular,

$$\Lambda_{\chi SB} \sim 1GeV, \Lambda_{conf} \sim 200-300\mathrm{MeV}$$

Consider the interpolation of positive and negative model

$$\Phi(z) = -\mu_1^2 z^2 + (\mu_1^2 + \mu_0^2) z^2 Tanh(\mu_2^2 z^2)$$

$z \to 0, \quad \Phi(z) \to -\mu_1^2 z^2,$ negative

$z \to \infty, \quad \Phi(z) \to \mu_0^2 z^2$ positive, linear confinement

$\mu_0 = 0.43GeV$ to pruduce correct slope for higher states

Three parameters: μ_1, μ_2, v_4

Qualitative results would not depend on the exact form of dilaton

Two Scales: Confinement and Chiral

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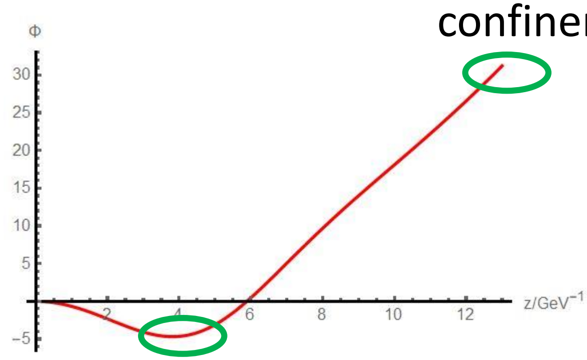
Linear confinement



χSB

Qualitative results would not depend on the exact form of dilaton
Exact form of dilaton might be fixed by hadron data

Towards $N_f = 2 + 1$

integrate instantons in QCD lagarangian, t'Hooft, 1976

Consider $SU(3) \times SU(3)$ symmetry of

$$I = \int d^5x\sqrt{-g}e^{-\Phi}\{Tr\,[-|DX|^2 - (-3|X|^2 + \lambda\,|X|^4)] + \gamma\,Re[\det(X)]\}$$

χSB:
$$X = \begin{pmatrix} \chi_u & 0 & 0 \\ 0 & \chi_d & 0 \\ 0 & 0 & \chi_s \end{pmatrix}$$

If $m_u = m_d = m_s$, we have $\chi_u = \chi_d = \chi_s = \chi$, $SU(3)$ case:

$$\chi'' + (3A_s' - \Phi' + \frac{f'}{f})\chi' - \frac{e^{2A_s}}{f}V'(\chi) = 0$$

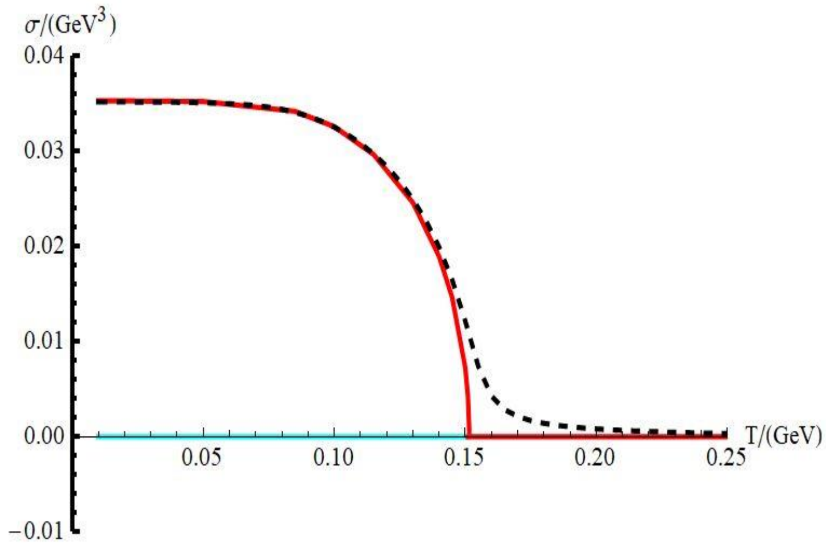
with $V(\chi) = -\frac{3}{2}\chi^2 + v_3\,\chi^3 + v_4\chi^4$, v_3 comes from the determinant term

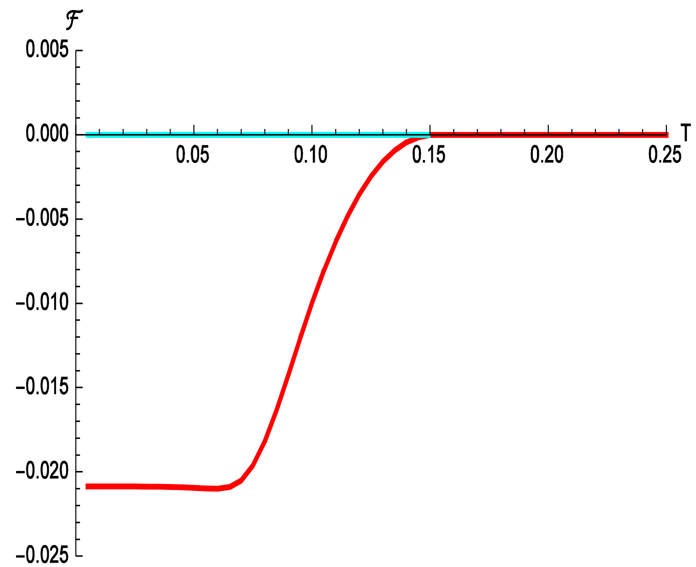
If $m_u = m_d \neq m_s$, we have $\chi_u = \chi_d \neq \chi_s$, $N_f = 2 + 1$ case:

$$\chi_l'' + \left(3A_s' - \Phi' + \frac{f'}{f}\right)\chi_l' + \frac{e^{2A_s}}{f}(3\chi_l - v_3\chi_l\chi_s - v_4\chi_l^3) = 0$$

$$\chi_s'' + (3A_s' - \Phi' + \frac{f'}{f})\chi_s' + \frac{e^{2A_s}}{f}(3\chi_s - v_3\chi_l^2 - v_4\chi_s^3) = 0$$

Two degenerate quarks





On-shell action:

$$\mathcal{F} = -v_4 \int_0^{z_h} dz\, e^{5A_s-\Phi}\chi^4 - \frac{1}{2}(\chi e^{5A_s-\Phi} f \chi')|_0$$

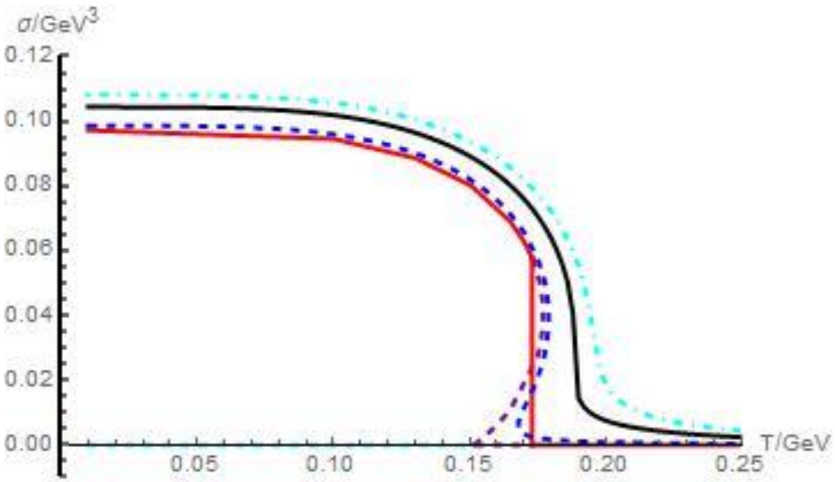
$\sigma_0 = (320MeV)^3, T_C = 150MeV$
agrees with lattice results for light flavor

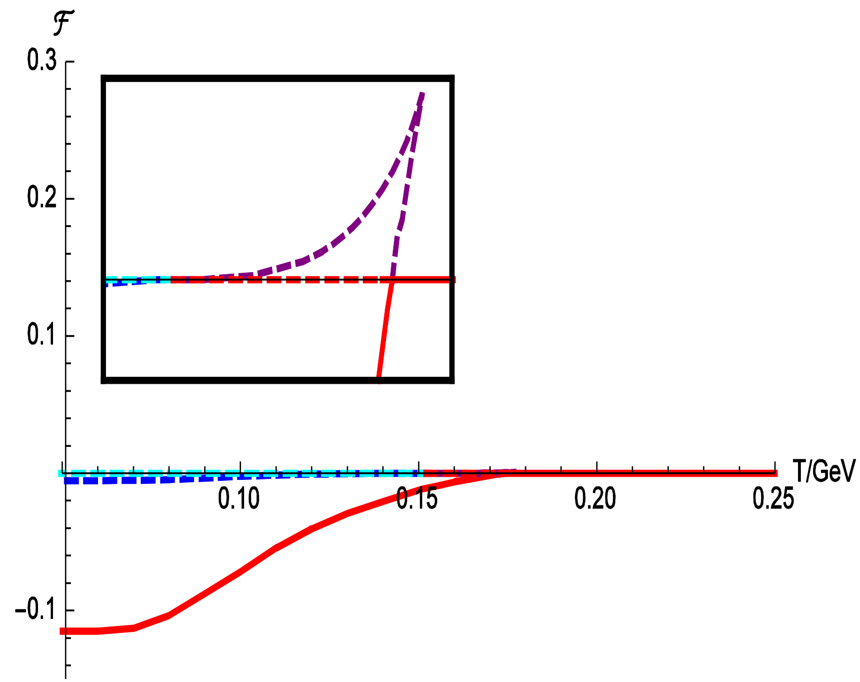
Comparing to pure negative results, the negative part dominant at scale higher than the positive part
Consistent with the instanton liquid model study

Non-trivial branch is tangent to the trivial branch
2^{nd} order phase transition in chiral limit

at finite quark mass, the transition become crossover

Three degenerate quarks





multi-solutions in at small quark masses
Only at sufficient high quark masses, it turns to crossover

On-shell action:

$$\mathcal{F} = -\int_0^{z_h} dz\, e^{5A_s-\Phi}(\frac{1}{2}v_3\chi^3 + v_4\chi^4) - \frac{1}{2}(\chi e^{5A_s-\Phi} f\chi')|_0$$

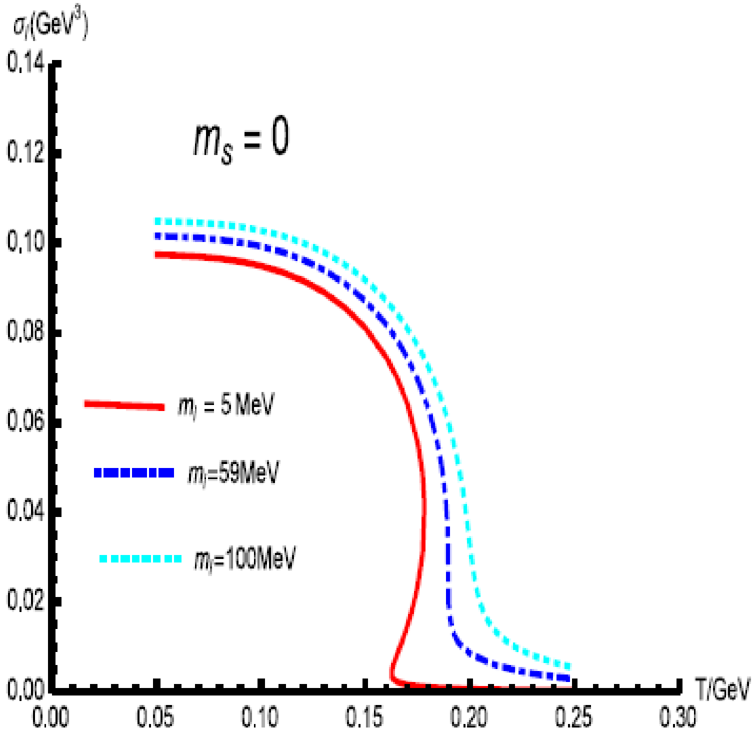
Positive non-trivial solutions are favored

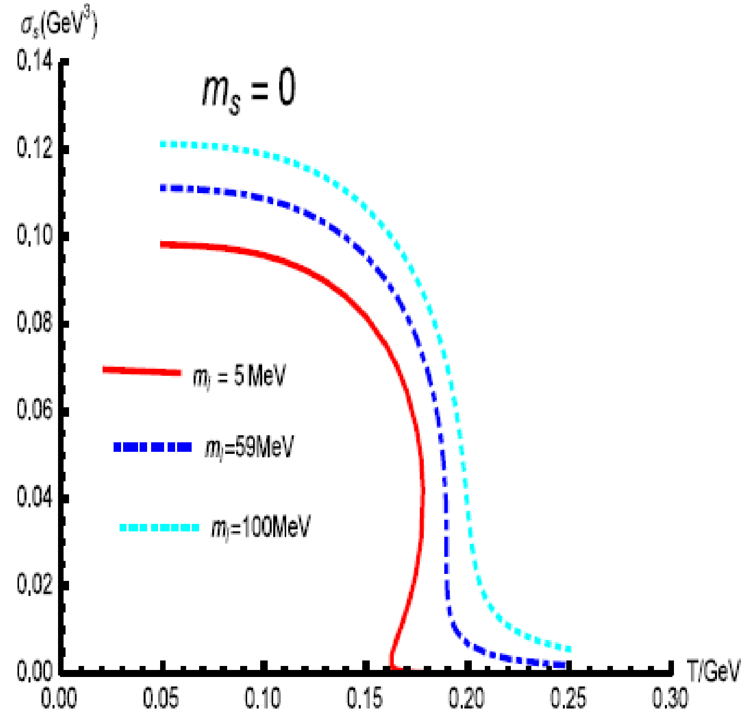
$$m_c = 37MeV$$

1st Order Phase Transition in chiral limit

$$\boldsymbol{T_C = 173MeV}$$

Chiral Condensate for $N_f = 2 + 1$

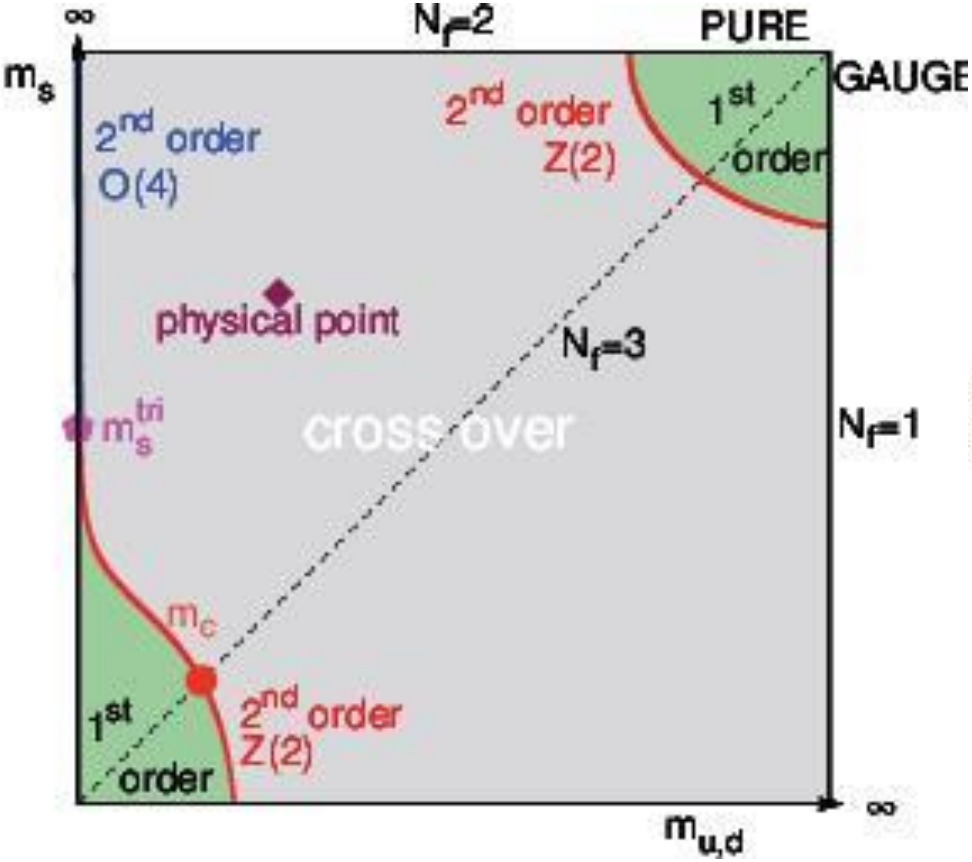


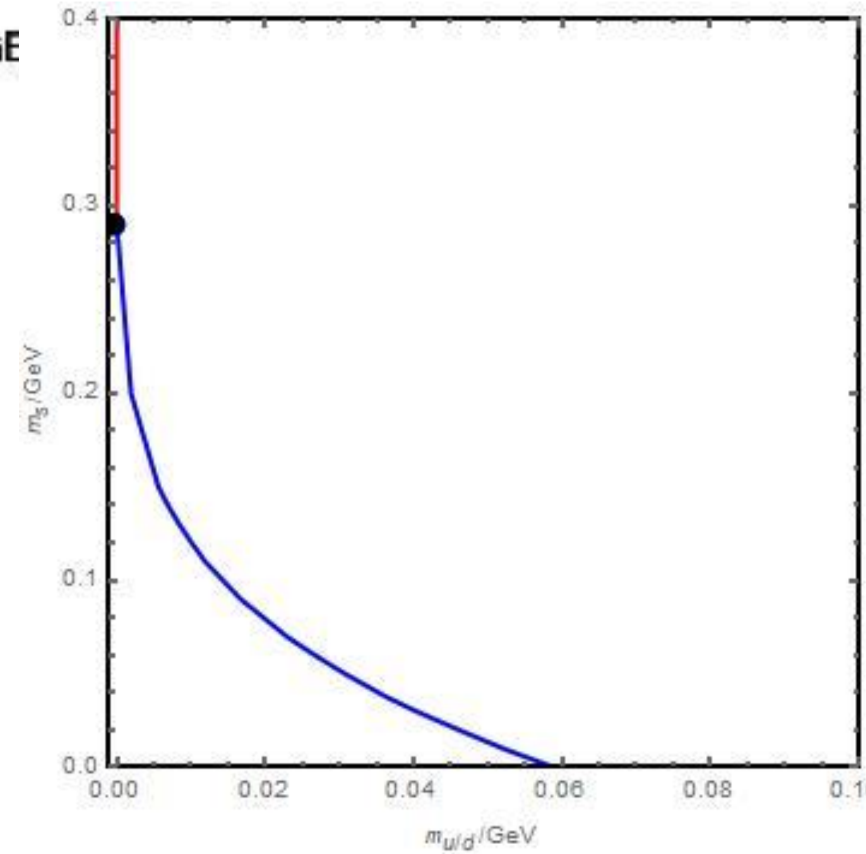


$$X = \begin{pmatrix} \chi_l & 0 & 0 \\ 0 & \chi_l & 0 \\ 0 & 0 & \chi_s \end{pmatrix}$$

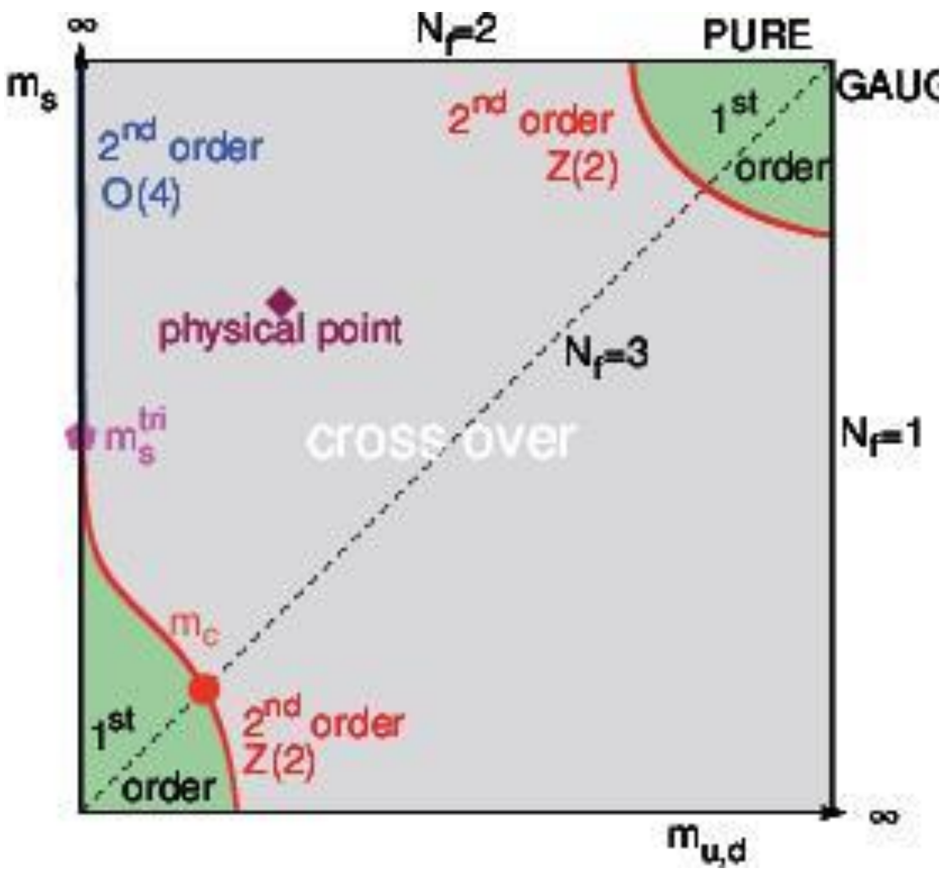
$m_{l,c} \simeq 55MeV$

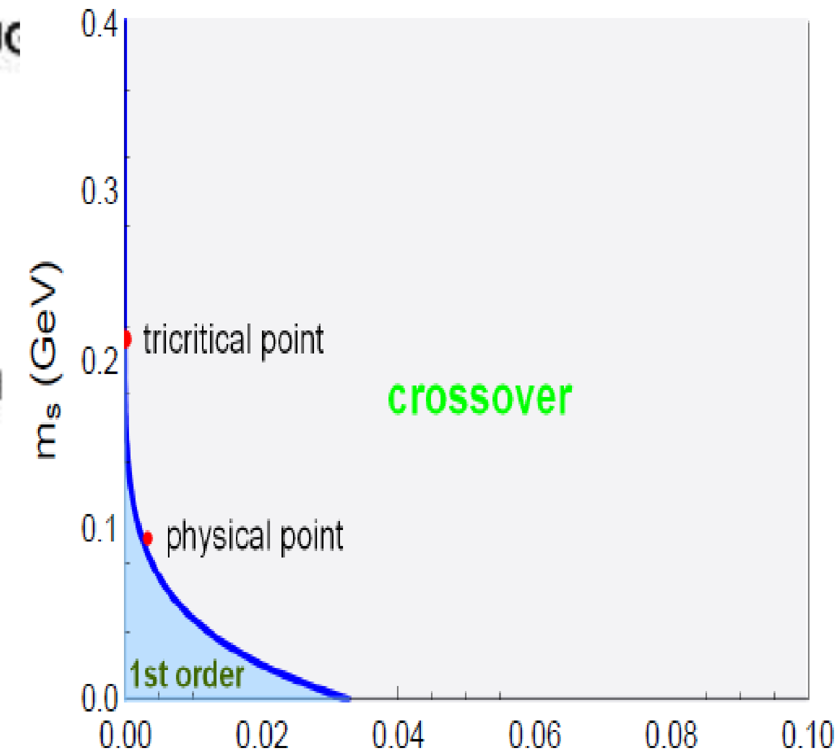
Phase Diagrame





Phase Diagrame

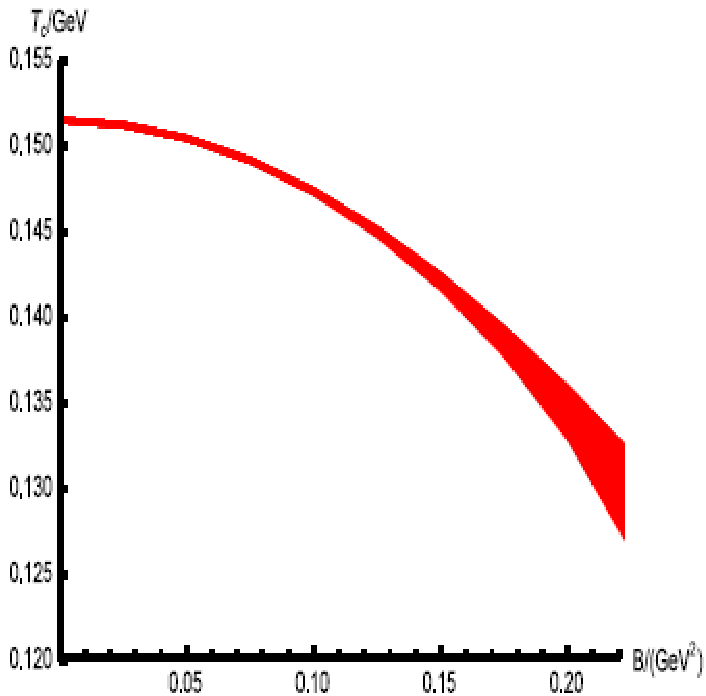




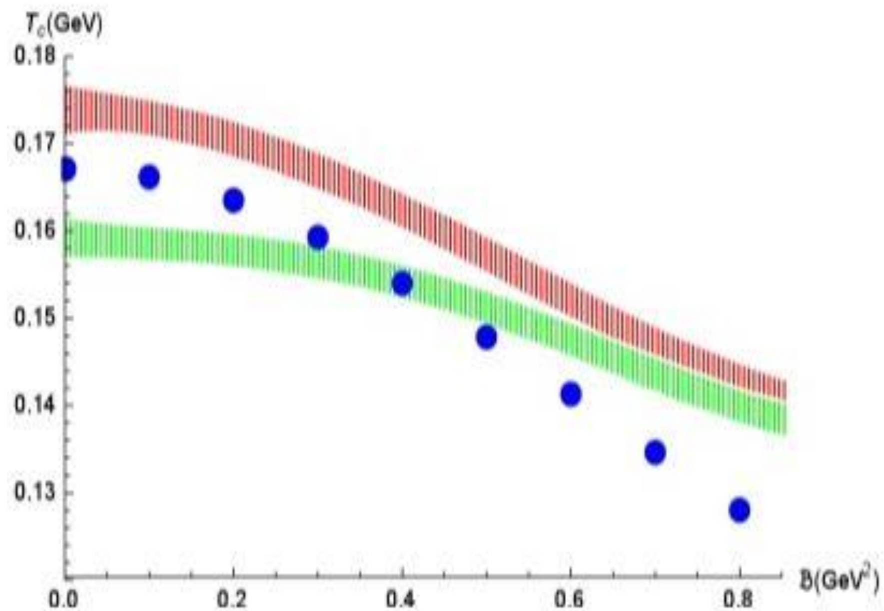
Z.Fang, Y.L.Wu, L.Zhang, Phys.Rev. D98 (2018) no.11, 114003

Similar structure from different models

Inverse magnetic catalysis



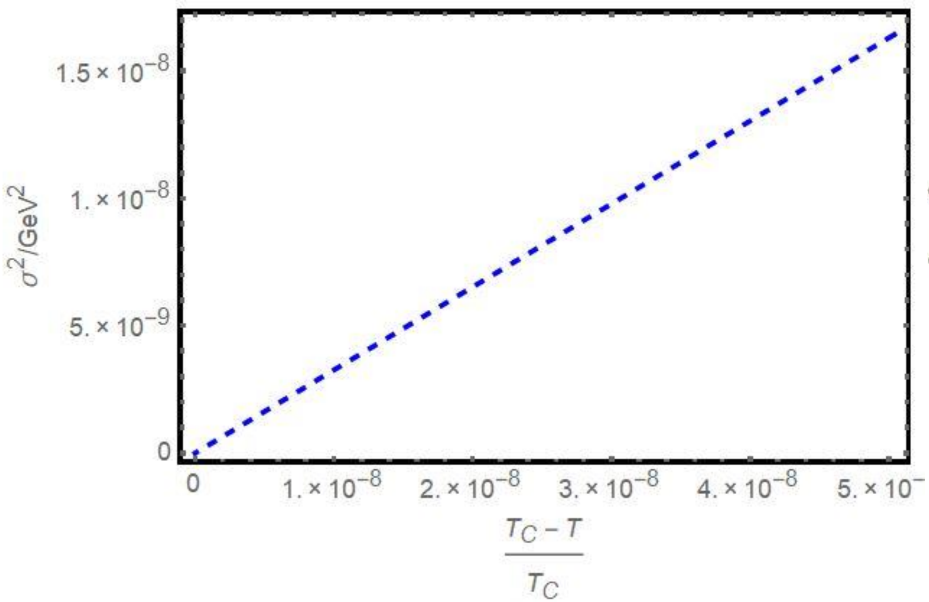
DL, M.Huang, Y.Yang, P.H.Yuan, JHEP 1702 (2017) 030



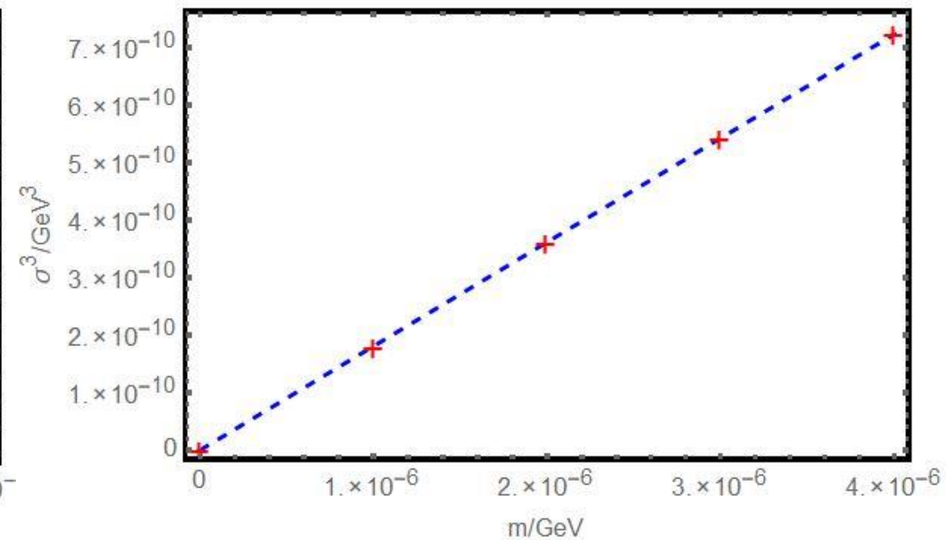
Z.Fang, Phys.Lett. B758 (2016) 1-8

Critical exponents: $N_f = 2$

$\langle \bar{\psi}\psi \rangle \sim (T_C - T)^{\beta}$



$\langle \bar{\psi}\psi \rangle \sim (m - m_C)^{1/\delta}$



Both numerical and analytical results: $\beta = \frac{1}{2}, \quad \delta = 3$ 3D Ising model, mean field results

analytical derivation(expansion near critical point)

$$If \quad \Phi = \Phi_0 + \Phi_1 (T - T_C)^{\varepsilon}, \qquad \beta = \frac{\varepsilon}{n-2} \qquad \left(V(\chi) = -\frac{3}{2}\chi^2 + \lambda\chi^n + \cdots \right)$$

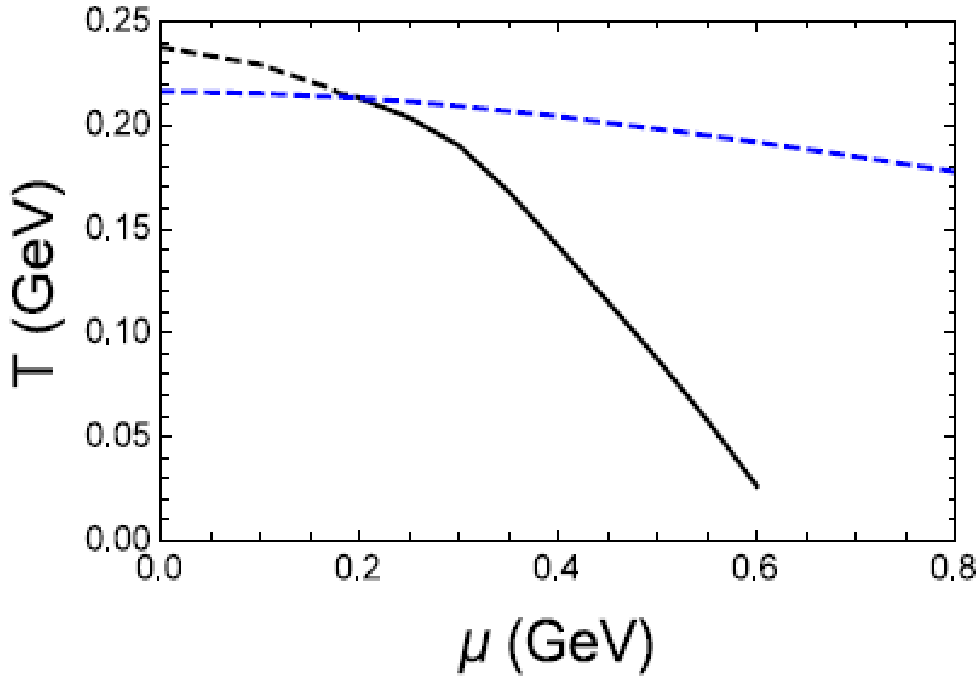
Critical exponents: $N_f = 2 + 1$

	$N_f = 2$	extended $N_f = 2$	$N_f = 3$
β	$\frac{1}{2}$	$\frac{\gamma}{n-2}$	$\frac{1}{3}$
δ	3	$n-1$	3
$N_f = 2+1$	$m_s > m_{s,t}$	$m_s = m_{s,t}$	$m_s < m_{s,t}$
β	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{3}$
δ	3	5	3

O(4): $\boldsymbol{\beta} \approx$ 0.0.385, $\boldsymbol{\delta} \approx 4.824$
Z(2): $\boldsymbol{\beta} \approx$ 0.0.327, $\boldsymbol{\delta} \approx 4.789$

Towards a dynamical dual background

$$
\begin{aligned}
S^s_{total} &= S^s_G + S^s_M, \\
S^s_G &= \frac{1}{16\pi G_5}\int d^5x\sqrt{-g^s}e^{-2\Phi}[R^s + 4\partial_\mu\Phi\partial^\mu\Phi - V^s_G(\Phi) - \frac{h(\Phi)}{4}e^{\frac{4\Phi}{3}}F_{\mu\nu}F^{\mu\nu}], \\
S^s_M &= -\int d^5x\sqrt{-g^s}e^{-\Phi}Tr[\nabla_\mu X^\dagger\nabla^\mu X + V^s_X(|X|, F_{\mu\nu}F^{\mu\nu})].
\end{aligned}
$$



Summary

- **Chiral phase transition could be well described in Soft-wall models. The phase diagram is in agreement with the Columbia plot.**
- **The critical exponents require a dynamical holographic QCD model to take the RG flow into account correctly.**
- **From the study of DHQCD model, a quarkyonic phase might exist at intermediate region.**
- **In the future: effects of $\mu_I, B, \ldots$**

Thanks for your attention!