The electromagnetic properties of viscous QGP

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Outline

- **Motivation**
- **The viscous chromohydrodynamics**
- **The induced current, polarization tensor, (chromo)electric permittivity, (chromo)magnetic permeability, (chromo)refractive index,energy loss and wakes induced by the fast partons in viscous QGP.**
- **Electric conductivity**
- **Summary**

Motivation

- Some striking findings in HIC
  - → the strong jet quenching
  - → A nearly perfect fluid with a small viscosity

  Nucl.Phys.A 757(2005)1;28;102;184….

- Casual viscous hydrodynamics simulation: focus on collective flows, etc.

  Huichao Song and Heinz, et al; Romatschke and Baier, et al
  Teaney and Dusling,et al; Sangyong Jeon ,Schenke , Shen Chun et al
  Blaizot and Yan Li……

Initial stage in heavy-ion collisions

Color-electric flux tube →strong color electric field

Glasma stage(early stage in HIC):

Lappi & McLerran, Nucl.Phys.A 772-200; Dumitru, Gelis, McLerran and Venugopalan, Nucl.Phys.A 810-91;

Magnetic scenario for QGP(late stage near T_c region):

J.F.Liao and Shuryak, PRC 75-054907, PRL 101-162302

Strong magnetic field

Fukushima, Kharzeev and Warringa PRD 78-074033; Kharzeev, McLerran and Warringa, NPA 803-227; Badak and Skokov, PLB 710-171; Skokov et al, Int.J.Mod.Phys. A 24-5925; W.T.Deng and X.G.Huang, PRC 85-044907….

How about electromagnetic properties in viscous QGP？

- **The electric permittivity and magnetic permeability are physics quantities to describe the difference of the electric and magnetic properties of the vector field in the medium and in the vacuum.**

- **In general, medium polarization is a function of the distribution of the background particles**
  N.Krall and A.Trivilpiece, Principles of Plasma Physics, 1973

- **Shear viscosity will modify the distribution functions of the constituents of the QGP, thus it will affect the induced current, gluon polariziation tensor, through which electric permittivity , magnetic permeability will be affacted.**

- The chromohydrodynamics can describe the polarization effect in the same way as the QGP kinetic theory

  M.Mannarelli and C.Manuel PRD 77-054018

- Extend Ideal chromohydrodynamic equations to the viscous ones → induced current and gluon self-enery with shear viscosity → electric permittivity , magnetic permeability , refraction index and electric conductivity in viscous QGP

Induced current and polarization tensor with viscous chromohydrodynamics

QGP transport equations

$$p^\mu D_\mu Q(p,x) + \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial_p^\nu Q(p,x)\} = C[Q,\bar{Q},G],$$

$$p^\mu D_\mu \bar{Q}(p,x) - \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial_p^\nu \bar{Q}(p,x)\} = \bar{C}[Q,\bar{Q},G],$$

$$p^\mu \mathcal{D}_\mu G(p,x) + \frac{g}{2} p^\mu \{\mathcal{F}_{\mu\nu}(x), \partial_p^\nu G(p,x)\} = C_g[Q,\bar{Q},G].$$

$$D_\mu = \partial_\mu - ig[A_\mu(x),\cdots], \qquad \mathcal{D}_\mu = \partial_\mu - ig[\mathcal{A}_\mu(x)\cdots].$$

$$A_\mu(X) = A_{\mu,a}(X)\tau^a, \qquad \mathcal{A}_\mu(X) = T^a A_{\mu,a}(X),$$

Yang-Mills equation $D_\mu F^{\mu\nu}(x) = j^\nu(x),$ $\displaystyle\int_p = \int \frac{d^4p}{(2\pi)^3} 2\Theta(p_0)\delta(p^2),$

$$j^\mu(x) = -\frac{g}{2}\int_p p^\mu [Q(p,x) - \bar{Q}(p,x) - \frac{1}{3}Tr[Q(p,x) - \bar{Q}(p,x)] + 2\tau^a Tr[T^a G(p,x)]],$$

Elze and Heinz, Phys.Rep 183-81; S.Mrowczynski, Phys.Rev.D 39-1940

The modified distribution function due to shear viscosity

$$Q = Q_o + \delta Q = Q_o + \frac{c'}{2T^3}\frac{\eta}{s} Q_o(1 \pm Q_o) p^\mu p^\nu \langle \nabla_\mu u_\nu \rangle$$

De Groot et al, Relativistic Kinetic Theory; D.Teaney, PRC 68 (2003)034913,

Hui-chao Song, arXiv: 0908.3656 [nucl-th]; P.Arnold, et al JHEP 0011-001,

K.Dusling and T.Schäfer, PRC 85(2012)044909,

Asakawa, Bass and Muller, Prog.Theor.Phys 116-725

For massless Boson and Fermion $c' = \pi^4/90\zeta(5)$ and $c' = 14\pi^4/1350\zeta(5)$

Dusling, NPA 809-246, PRC 77-034905

$$p^\mu D_\mu Q(p,x) + \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial_p^\nu Q(p,x)\} = C[Q, \bar{Q}, G],$$

Integrating the equation with $\int_p = \int \frac{d^4p}{(2\pi)^3} 2\Theta(p_0)\delta(p^2),$

$$D_\mu n^\mu = 0, \qquad n^\mu(x) = \int_p p^\mu Q(p,x)$$

Multiplying the equation by p and perform the same integral

$$D_\mu T^{\mu\nu} - \frac{g}{2}\{F_\mu^\nu, n^\mu(x)\} = 0, \qquad T^{\mu\nu} = \int_p p^\mu p^\nu Q(p,x).$$

$$n^{\mu} = n(x) u^{\mu}$$

$$T^{\mu\nu} = \frac{1}{2}\big(\epsilon(x) + p(x)\big)\{u^{\mu}, u^{\nu}\} - p(x) g^{\mu\nu} + \pi^{\mu\nu}, \quad \pi^{\mu\nu} = \eta\langle\nabla^{\mu} u^{\nu}\rangle$$

$$j^{\mu}(x) = -\frac{g}{2}\left(n u^{\mu} - \frac{1}{3}\mathrm{Tr}[n u^{\mu}]\right)$$

$$D_{\mu} n^{\mu} = 0, \qquad D_{\mu} T^{\mu\nu} - \frac{g}{2}\{F_{\mu}^{\nu}, n^{\mu}(x)\} = 0,$$

If ideal distribution function is used, $\pi^{\mu\nu} = \eta\langle\nabla^{\mu} u^{\nu}\rangle$ will be absent in the second conservation equation, we will recover the ideal chromohydrodynamic equations

Manuel & Mrowczynski, PRD 74-105003;
Mannarelli & Manuel， Phys.Rev.D 76-094007; Phys.Rev.D 77-054018

Around the stationary and colorless state $\bar{n}$, $\bar{u}^{\mu}$, $\bar{p}$ and $\bar{\epsilon}$, **linearization of the hydrodynamic quantities**

$$n(x) = \bar{n} + \delta n(x), \qquad u^{\mu}(x) = \bar{u}^{\mu} + \delta u^{\mu}(x),$$

$$p(x) = \bar{p} + \delta p(x), \qquad \epsilon(x) = \bar{\epsilon} + \delta\epsilon(x),$$

Diagonalized fluctuation quantities to compare with the stationary ones

$$\delta n \ll \bar{n}, \qquad \delta u^{\mu} \ll \bar{u}^{\mu}, \qquad \delta p \ll \bar{p}, \qquad \delta\epsilon \ll \bar{\epsilon}.$$

In stationary state

$$D_{\mu}\bar{n} = 0, \qquad D_{\mu}\bar{u}^{\nu} = 0, \qquad D_{\mu}\bar{p} = 0, \qquad D_{\mu}\bar{\epsilon} = 0,$$

$$\bar{j}^{\mu}(x) = -\frac{g}{2}\left(\bar{n}\bar{u}^{\mu} - \frac{1}{3}\mathrm{Tr}\left[\bar{n}\bar{u}^{\mu}\right]\right) = 0.$$

Substituting the linearized hydrodynamic quantities in $T^{\mu\nu}$ **and** n^{μ}

$$T^{\mu\nu} = \frac{1}{2}\left(\epsilon(x) + p(x)\right)\left\{u^{\mu}, u^{\nu}\right\} - p(x)g^{\mu\nu} + \pi^{\mu\nu}$$

$$n^{\mu} = n(x)u^{\mu} \qquad j^{\mu}(x) = -\frac{g}{2}\left(nu^{\mu} - \frac{1}{3}\mathrm{Tr}\left[nu^{\mu}\right]\right)$$

$$n^{\mu} = \bar{n}\bar{u}^{\mu} + \bar{n}\delta u^{\mu} + \delta n\,\bar{u}^{\mu},$$

$$T^{\mu\nu} = (\bar{\epsilon} + \bar{p})\bar{u}^{\mu}\bar{u}^{\nu} - \bar{p}g^{\mu\nu} + (\delta\epsilon + \delta p)\bar{u}^{\mu}\bar{u}^{\nu} + (\bar{\epsilon} + \bar{p})\big(\bar{u}^{\mu}\delta u^{\nu} + \delta u^{\mu}\bar{u}^{\nu}\big) - \delta p g^{\mu\nu}$$
$$+ \eta\left\{\big(g^{\mu\sigma} - \bar{u}^{\mu}\bar{u}^{\sigma}\big)\partial_{\sigma}\delta u^{\nu} + \big(g^{\nu\sigma} - \bar{u}^{\nu}\bar{u}^{\sigma}\big)\partial_{\sigma}\delta u^{\mu} - \frac{2}{3}\big(g^{\mu\nu} - \bar{u}^{\mu}\bar{u}^{\nu}\big)\partial_{\sigma}\delta u^{\sigma}\right\}.$$

$$D_{\mu}n^{\mu} = 0, \qquad D_{\mu}T^{\mu\nu} - \frac{g}{2}\big\{F^{\nu}_{\mu}, n^{\mu}(x)\big\} = 0,$$

Hydrodynamic quantities and corresponding fluctuation parts have in general both colorless and colored parts

$$n_{\alpha\beta} = n_0 I_{\alpha\beta} + \frac{1}{2}n_a\tau^a_{\alpha\beta} \qquad \delta n_{\alpha\beta} = \delta n_0 I_{\alpha\beta} + \frac{1}{2}\delta n_a\tau^a_{\alpha\beta}$$

Projecting the conservation equations on $\bar{u}^{\mu}$ and $(g^{\mu\nu} - \bar{u}^{\mu}\bar{u}^{\nu})$,

$$\bar{n}k_{\mu}\delta u^{\mu}_a + k_{\mu}\delta n_a\bar{u}^{\mu} = 0,$$

$$\bar{u}^{\mu}k_{\mu}\delta\epsilon_a + (\bar{\epsilon} + \bar{p})k_{\mu}\delta u^{\mu}_a = 0,$$

$$(\bar{\epsilon} + \bar{p})(\bar{u}\cdot k)\delta u^{\nu}_a + \big(-k^{\nu} + \bar{u}^{\nu}(\bar{u}\cdot k)\big)\delta p_a$$
$$+ \eta\left\{\big(k^2 - (k\cdot\bar{u})\big)\delta u^{\nu}_a + \big(k^{\mu}k^{\nu} - k^{\mu}\bar{u}^{\nu}\big)\delta u_{\mu,a} + \frac{2}{3}\big(\bar{u}^{\nu}(k\cdot\bar{u}) - k^{\nu}\big)k_{\sigma}\delta u^{\sigma}_a\right\}$$
$$= ig\bar{n}\bar{u}_{\mu}F^{\mu\nu}_a(k),$$

Introducing an EoS $\delta p_a = c_s^2 \delta\epsilon_a$

$$\delta n_a = -\frac{\bar{n} k_\mu \delta u_a^\mu}{k \cdot \bar{u}},$$

$$\delta u_{\sigma,a} = \frac{i}{1 + D(k^2 - (k \cdot \bar{u})^2)} \cdot \frac{g\bar{n}}{(\bar{\epsilon} + \bar{p})(k \cdot \bar{u})} \left\{ g_{\sigma\nu} + (B + E)\left[k_\sigma k_\nu - \bar{u}_\sigma k_\nu (k \cdot \bar{u}) \right] \right\} \cdot \bar{u}_\mu F_a^{\mu\nu},$$

and parameters B、D、E

$$B = -\frac{1}{k^2 + \left(\frac{1}{c_s^2} - 1\right)(k \cdot \bar{u})^2}, \qquad D = \frac{\eta}{(\bar{\epsilon} + \bar{p})(k \cdot \bar{u})},$$

$$E = -\frac{D(1 + 4B(k^2 - (k \cdot \bar{u})^2))}{3 + 3c_s^2\left(\frac{k^2}{(k \cdot \bar{u})^2} - 1\right) + 4D(k^2 - (k \cdot \bar{u})^2)}.$$

The color current due to colored fluctuation of hydro-dynamic quantities

$$\delta j_a^\mu = -\frac{g}{2}\left(\bar{n}\delta u_a^\mu + \delta n_a \bar{u}^\mu - \frac{1}{3}\mathrm{Tr}\left[\bar{n}\delta u_a^\mu + \delta n_a \bar{u}^\mu\right]\right)$$

Linearized $F_a^{\mu\nu}(k) = -ik^\mu A_a^\nu(k) + ik^\nu A_a^\mu(k)$ **and** $\mathrm{Tr}[F^{\mu\nu}] = 0$

$$\delta j_a^\mu = -\frac{g}{2}\left(\bar{n}\delta u_a^\mu + \delta n_a \bar{u}^\mu\right).$$

Substituting δu_a^μ δn_a **and in terms of** $\delta j_a^\mu(k) = -\Pi_{ab}^{\mu\nu}(k) A_{\nu,b}(k),$

We can extract the gluon polarization tensor

$$\begin{aligned}\Pi_{ab}^{\mu\nu}(\omega,k) = -\delta_{ab}\Big\{\omega_p^2 \cdot \frac{1}{1 + D(k^2 - (k\cdot\bar{u})^2)} \cdot \frac{1}{(k\cdot\bar{u})^2} \cdot \big[(k\cdot\bar{u})(\bar{u}^\mu k^\nu + k^\mu \bar{u}^\nu) \\ - k^2 \bar{u}^\mu \bar{u}^\nu - (k\cdot\bar{u})^2 g^{\mu\nu} + (B+E)\cdot\big[k^2(k\cdot\bar{u})(\bar{u}^\mu k^\nu + k^\mu \bar{u}^\nu) \\ - k^\mu k^\nu (k\cdot\bar{u})^2 - k^4 \bar{u}^\mu \bar{u}^\nu\big]\big]\Big\}, \quad \omega_p^2 = \frac{g^2\bar{n}^2}{2(\bar{\epsilon}+\bar{p})}\end{aligned}$$

Viscous correction to vector meson thermal self-energy
G.Vujanovic, C.Young, B.Schenke, R.Rapp,Sangyong Jeon and C.Gale,
PRC 89-034904

$$c_s = \sqrt{\frac{1}{3(1+\frac{1}{2y}\log\frac{1-y}{1+y})} + \frac{1}{y^2}} \; (y = \frac{k}{\omega}). \quad \varepsilon^{ij}(\omega, k) = \delta^{ij} + \frac{\Pi^{ij}(\omega, k)}{\omega^2}$$

$$\begin{aligned}
\varepsilon_L(\omega, k) = 1 + \frac{3\omega_p^2}{k^2}\left[1 - \frac{\omega}{2k}\left(\log\left|\frac{\omega+k}{\omega-k}\right| - i\pi\Theta(k^2-\omega^2)\right)\right] \\
- \frac{12\omega_p^2}{k^2}\frac{\eta\omega}{sT} \times \left\{1 - \frac{\omega}{k}\log\left|\frac{\omega+k}{\omega-k}\right| + \frac{\omega^2}{4k^2}\left(\log\left|\frac{\omega+k}{\omega-k}\right|\right)^2\right. \\
\left. - \frac{\omega^2}{4k^2}\pi^2\Theta(k^2-\omega^2) + i\left(\frac{\omega}{k}\pi - \frac{\omega^2}{2k^2}\pi\log\left|\frac{\omega+k}{\omega-k}\right|\right)\Theta(k^2-\omega^2)\right\}.
\end{aligned}$$

$$\varepsilon_T(\omega, k) = 1 - \frac{\omega_p^2}{\omega^2}\frac{1}{1 - \frac{\eta}{s}\frac{k^2}{\omega T}}$$

Bing-feng Jiang and Jia-rong Li , NPA 847(2010)268;

J.Peralta-Ramos and E.Calzetta, PRD 86(2012)125024

$(\varepsilon_L, \varepsilon_T)$ → **Electric permittivity** ε **and magnetic permeability** μ_M

$$\varepsilon = \varepsilon_L$$

$$\frac{1}{\mu_M} = 1 + (\frac{\omega^2}{k^2})[\varepsilon_L(\omega, k) - \varepsilon_T(\omega, k)]$$

refractive index in viscous quark-gluon plasma

$n^2 = \varepsilon(\omega, k)\mu_M(\omega, k)$ **is a square definition and is not sensitive to**

ε and μ_M → $-\varepsilon$ and $-\mu_M$ **Corresponding a transformation of n from**

$n = \sqrt{\varepsilon(\omega, k)\mu_M(\omega, k)}$ **general refraction index to a negative one** $-\sqrt{\varepsilon(\omega, k)\mu_M(\omega, k)}$

The physical nature: the electromagnetic phase velocity propagates opposite to the energy flow

Veselago, Sov.Phys.Usp 10-509; Agranovich et al, Phys.Usp 49-1029,

Ramakrishna, Rep.Prog.Phys 68-449; Pendry, PRL 85-3966; Smith et al, PRL 85-2933

The criterion for negative refraction

Real permittivity and permeability medium, $\varepsilon < 0$ and $\mu < 0$ simultaneously

Dissipative medium, $\varepsilon(\omega, k) = \varepsilon_r + i\varepsilon_i, \ \mu_M(\omega, k) = \mu_r + i\mu_i$

Depine-Lakhtakia index (DL index) $n_{\text{eff}} = \varepsilon_r|\mu_M| + \mu_r|\varepsilon|$ $n_{\text{eff}} < 0$ → $\text{Re}\,n < 0$

McCall et al, Eur.J.Phys 23-353; Depine and Lakhtakia, Microw.Opt.Technol.Lett 41-315

Holographic method

A.Amariti, D.Forcella, A.Mariotti and G.Policastro, JHEP 04 (2011)036
A.Amariti, D.Forcella, A.Mariotti and G.Policastro and M.Siani, JHEP10(2011)104
A.Amariti, D.Forcella and A.Mariotti, JHEP01(2013)105
Xian-hui Ge, Kwanghyun Jo and Sang-Jin Sin, JHEP 03(2011)104

Xin Gao and Hong-bao Zhang, JHEP 08(2010)075

P.Phukon and T.Sarkar, JHEP 09(2013)102

S.Mahapatra, P.Phukon and T.Sarkar, JHEP 01(2014)135.

…….

HTL perturbative theory

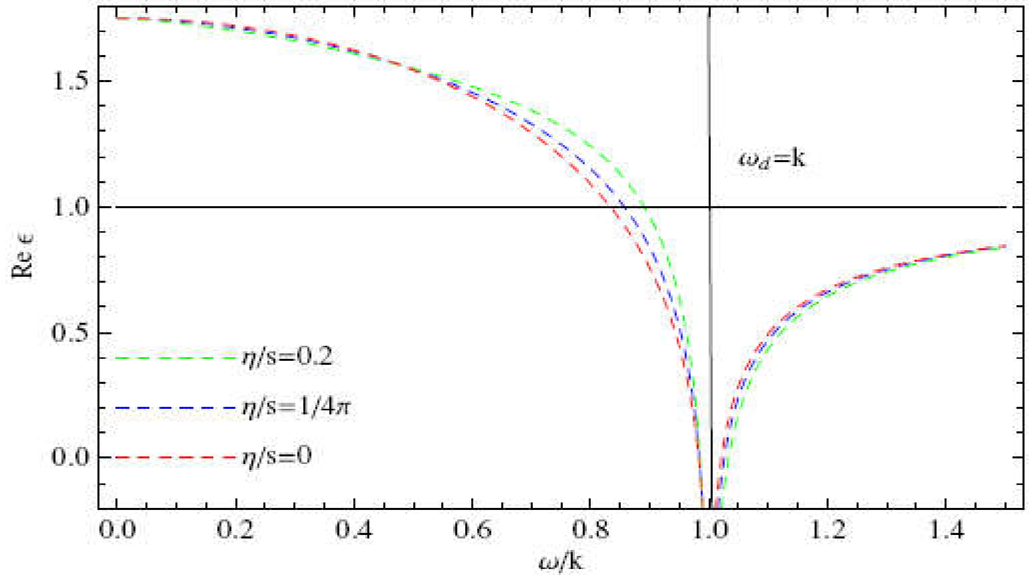
Juan Liu, M.J.Luo, Qun Wang and Hao-jie Xu, PRD 84,125027(2011).

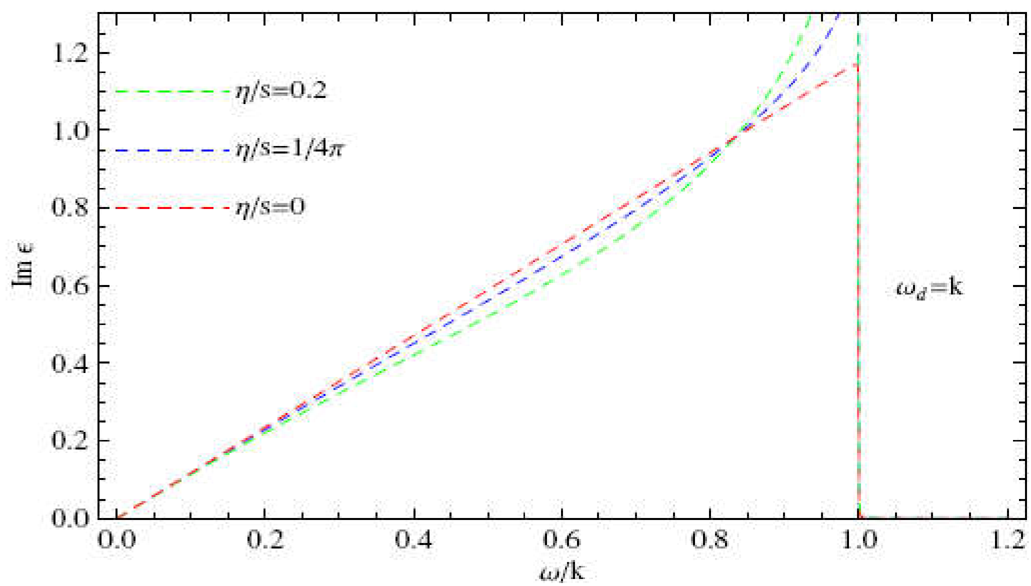
Our consideration: viscous effect on the refractive index

$k = 0.2\omega_p$ and $T = \omega_p$

Electric permittivity pole does not change

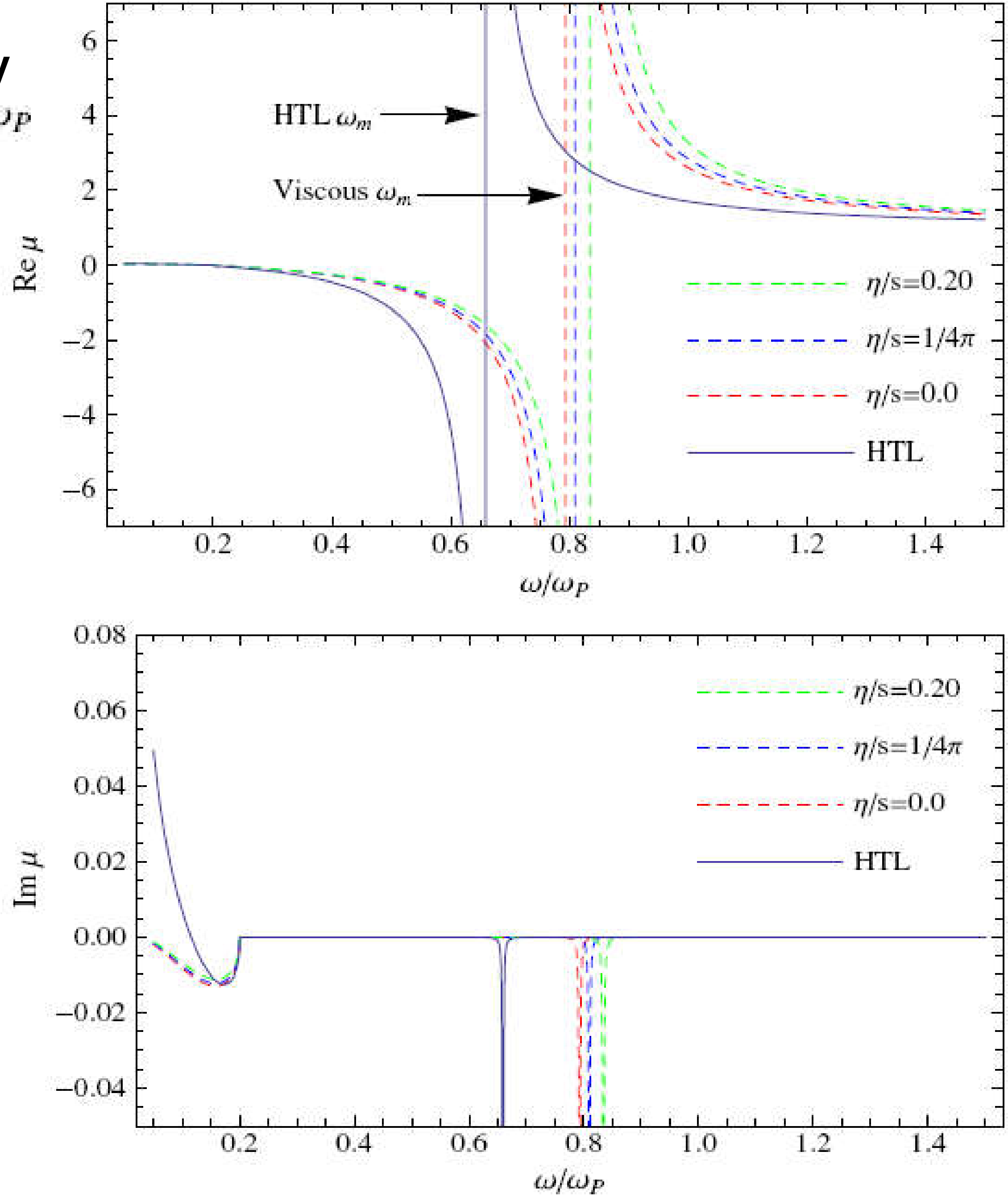
as the increase of shear viscosity



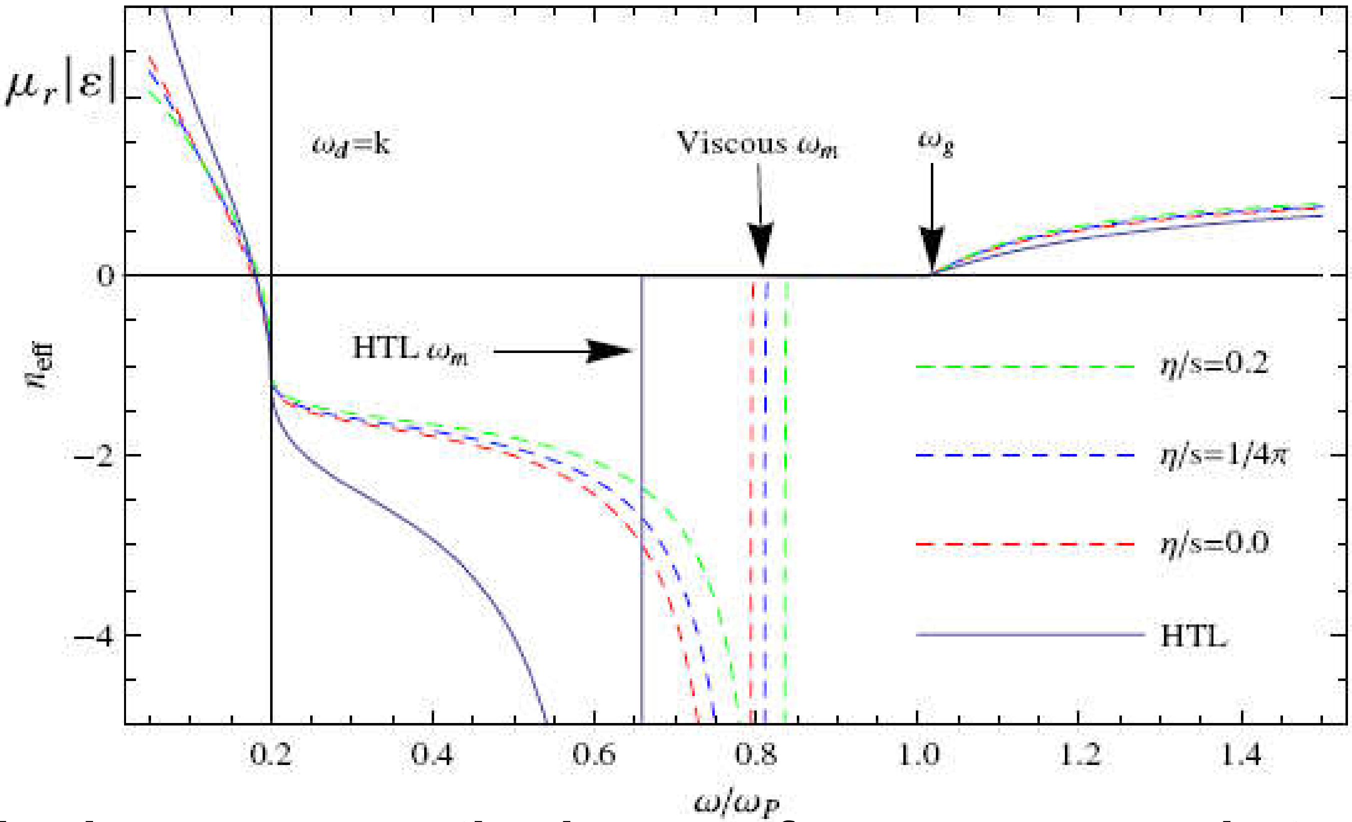


Magnetic permeability pole shift to large ω/ω_P

as the increase of shear viscosity



$$n_{\text{eff}} = \varepsilon_r|\mu_M| + \mu_r|\varepsilon|$$



1.The DL index becomes negative in some frequency range, but propagation modes do not exist.

2.The starting point of frequency range $n_{\text{eff}} < 0$ is around the electric permittivity pole, and the magnetic permeability pole determines the endpoint

3. As η/s increases, the frequency range for $n_{\text{eff}} < 0$ becomes wider.

Bing-feng Jiang, De-fu Hou and Jia-rong Li , PRD 88,045014

Energy loss in Thoma-Gyulassy formalism

- **Retarding force from the induced chromoelectric field acting on the fast parton lead to energy loss**

$$-\frac{dE}{dx} = -\frac{\mathbf{v}}{v} q^a \cdot \mathrm{Re}\mathbf{E}^{\mathrm{a}}_{\mathrm{ind}}(\mathbf{x} = \mathbf{v}\mathrm{t}, \mathrm{t})$$

E^a_{ind} induced chromoelectric field, q^a color charge of the fast parton,due to the external current of the fast parton $\mathbf{j}^a_{ext}$,the total chromoelectric field

$$[\varepsilon_{ij}(\omega, k) - \frac{k^2}{\omega^2}(\delta_{ij} - \frac{k_i k_j}{k^2})]E^a_{tot,j}(\omega, k) = \frac{4\pi}{i\omega} j^a_{ext,i}(\omega, k)$$

- **In isotropic and homogeneous medium, dielectric tensor $\varepsilon_{ij}(\omega, k)$ can be decomposed as**

$$\varepsilon_{ij}(\omega, k) = \varepsilon_L(\omega, k)\frac{k_i k_j}{k^2} + \varepsilon_T(\delta_{ij} - \frac{k_i k_j}{k^2})$$

- **The external current due to the fast quark**

$$\mathbf{j}^a_{ext}(\omega, k) = 2\pi q^a \mathbf{v}\delta(\omega - \mathbf{k}\cdot\mathbf{v})$$

- **After working out the induced chromoelectric field in terms of foregoing equations , we can get the energy loss formula**

$$-\frac{dE}{dx} = -\frac{C_F \alpha_s}{2\pi^2 v} \int d^3k \{ \frac{\omega}{k^2} [\mathrm{Im}\varepsilon_{\mathrm{L}}^{-1} + (v^2k^2 - \omega^2)\mathrm{Im}(\omega^2\varepsilon_{\mathrm{T}} - \mathrm{k}^2)^{-1}] \}_{\omega=\mathbf{k}\cdot\mathbf{v}}$$

M.Thoma and M.Gyulassy, NPB 351 (1991)491

The formula is applicable for the soft momentum transfer relating to the plasma polarization effect → polarization energy loss

Our consideration: viscous effect on polarization energy loss

$$\varepsilon_L(\omega,k) = 1 + \frac{3\omega_p^2}{k^2}\left[1 - \frac{\omega}{2k}\left(\log\left|\frac{\omega+k}{\omega-k}\right| - i\pi\Theta(k^2-\omega^2)\right)\right]$$
$$- \frac{12\omega_p^2}{k^2}\frac{\eta\omega}{sT} \times \left\{1 - \frac{\omega}{k}\log\left|\frac{\omega+k}{\omega-k}\right| + \frac{\omega^2}{4k^2}\left(\log\left|\frac{\omega+k}{\omega-k}\right|\right)^2\right.$$
$$\left. - \frac{\omega^2}{4k^2}\pi^2\Theta(k^2-\omega^2) + i\left(\frac{\omega}{k}\pi - \frac{\omega^2}{2k^2}\pi\log\left|\frac{\omega+k}{\omega-k}\right|\right)\Theta(k^2-\omega^2)\right\}.$$

$$\varepsilon_T(\omega,k) = 1 - \frac{\omega_p^2}{\omega^2}\frac{1}{1-\frac{\eta}{s}\frac{k^2}{\omega T}}.$$

The transverse dielectric function $\varepsilon_T(\omega,k)$ is a pure real function of ω, k, in Thoma-Gyulassy formalism, it has no contribution to the energy loss

S.Mrowczynski, PLB 269(1991)383; Y.Koike and T.Matsui, PRD 45(1992)3237
A.Sitenko, Electromagnetic Fluctuations in Plasma, 1967
A.Akhiezer, et al. Plasma Electrodynamics, 1975

main contribution comes from longitudinal dielectric function

According to the longitudinal dielectric function

$$\mathrm{Im}\varepsilon_{\mathrm{L}}^{-1}(\omega,\mathrm{k})|_{\omega=\mathbf{k}\cdot\mathbf{v}} = -\frac{\mathrm{m}_{\mathrm{D}}^{2}\pi \mathrm{v}\cos\theta}{2}\cdot\frac{\mathrm{k}^{2}-\mathrm{w}_{1}\cdot\mathrm{k}^{3}}{[\mathrm{k}^{2}+\mathrm{w}_{3}\cdot\mathrm{k}+\mathrm{w}_{2}]^{2}+\mathrm{w}_{4}\cdot[1-\mathrm{w}_{1}\cdot\mathrm{k}]^{2}}$$

$$\begin{aligned}
w_1 &= \frac{8v\cos\theta}{T}\frac{\eta}{s}\left(1-\frac{v\cos\theta}{2}\ln\left|\frac{1+v\cos\theta}{1-v\cos\theta}\right|\right),\\
w_2 &= m_D^2\left(1-\frac{v\cos\theta}{2}\left(\ln\left|\frac{1+v\cos\theta}{1-v\cos\theta}\right|\right)\right),\\
w_3 &= -\frac{4m_D^2 v\cos\theta}{T}\frac{\eta}{s}\times\left\{1-v\cos\theta\ln\left|\frac{1+v\cos\theta}{1-v\cos\theta}\right|\right.\\
+ &\quad \left.\frac{v^2\cos^2\theta}{4}\left(\ln\left|\frac{1+v\cos\theta}{1-v\cos\theta}\right|\right)^2-\frac{v^2\cos^2\theta}{4}\pi^2\right\},\\
w_4 &= \frac{m_D^4\pi^2 v^2\cos^2\theta}{4},
\end{aligned}$$

$$-\frac{dE}{dx} = \frac{m_D^2 C_F\alpha_s v}{2}\int_{-1}^{1} d(\cos\theta)\cos^2\theta\int_0^{k_{max}} dk\cdot\left\{\frac{k^3-w_1\cdot k^4}{[k^2+w_3\cdot k+w_2]^2+w_4\cdot[1-w_1\cdot k]^2}\right\}$$

In numerical analysis, assuming quark mass of light, charm and bottom

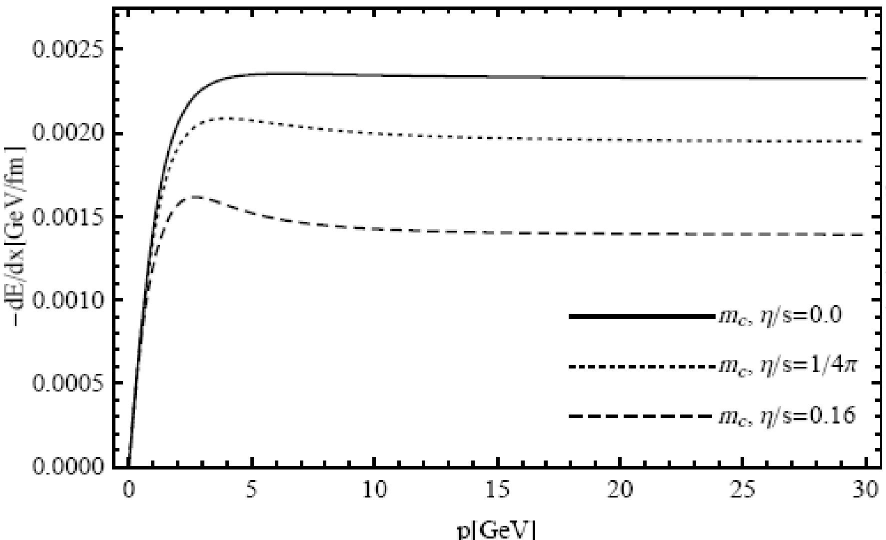
$$m_0 = 0,\ m_c = 1.5\text{GeV and } m_b = 5.0\text{GeV}$$

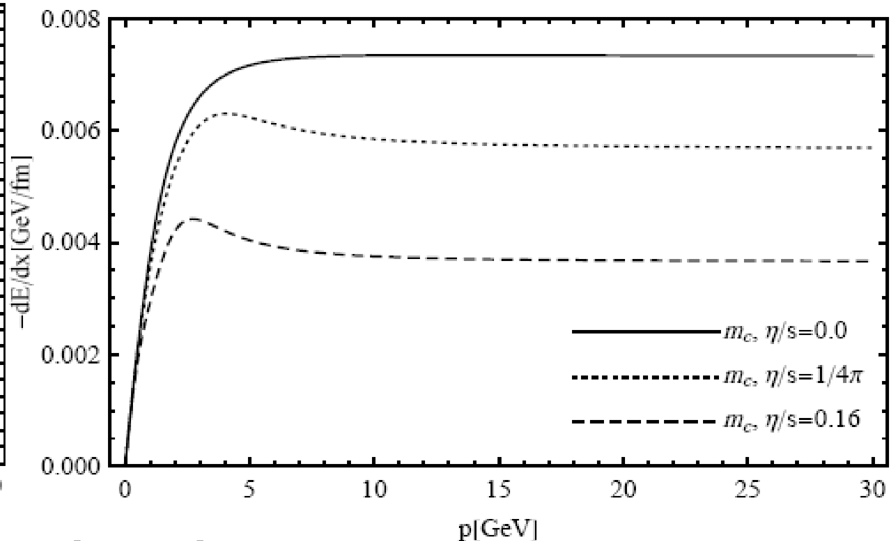
$$T = 0.3\text{GeV},\ \alpha_s = 0.3 \text{ and } N_f = 2 \qquad m_D \sim 0.7\text{GeV}$$

$$k_{max} = 0.5\text{GeV and } 0.7\text{GeV}$$

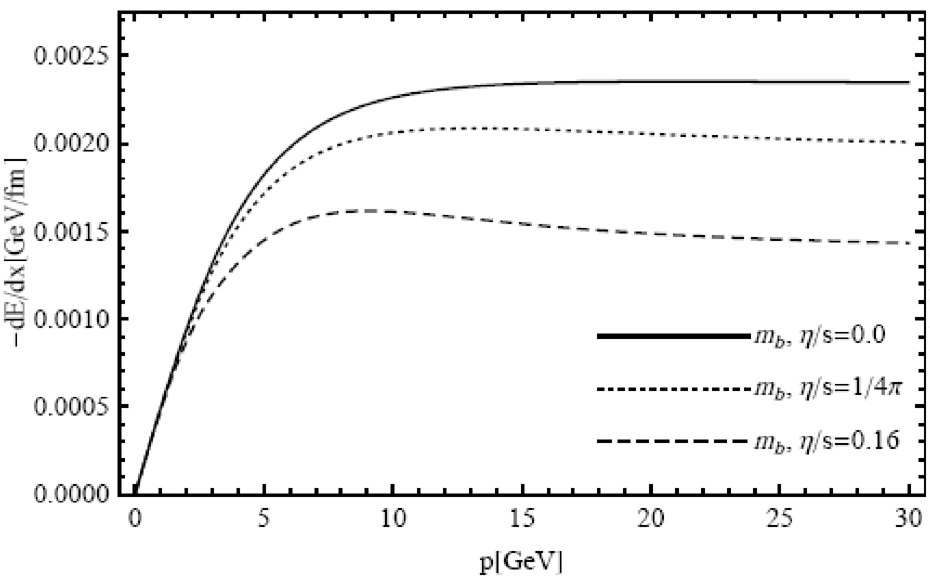
$$\eta/s = 0 \qquad k_{max} = 0.5\text{GeV} \qquad k_{max} = 0.7\text{GeV}$$

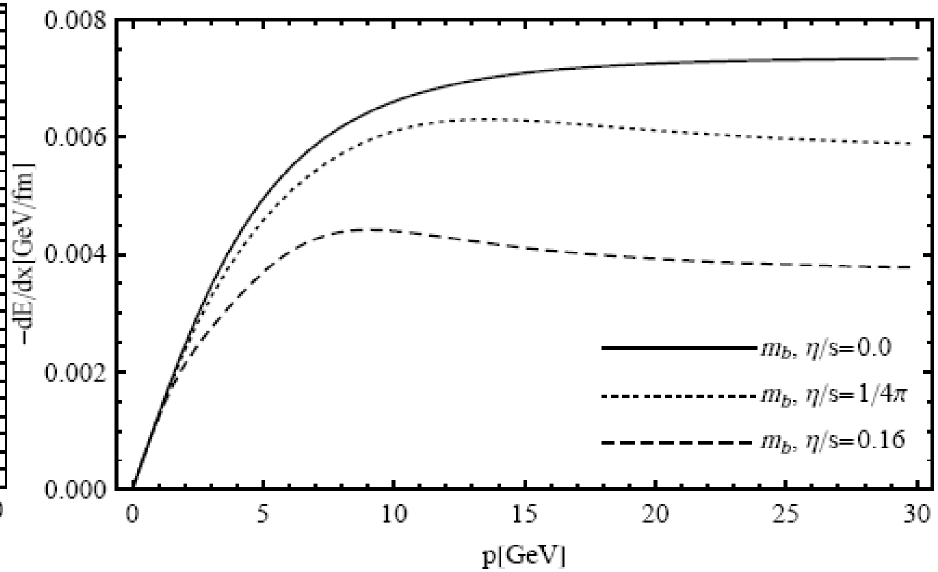
For charm quark with different shear viscosity





For bottom quark with different shear viscosity





Main results: Polarization energy loss mainly comes from longitudinal dielectric function, shear viscosity reduces the soft collisional energy loss

Bing-feng Jiang, De-fu Hou and Jia-rong Li , J.Phys.G 42(2015)085107, arxiv:1405.0083

M.Elias, J.Peralta-Ramos and E.Calzetta, PRD 90(2014)014038, arXiv: 1404.7790

Based on the viscous dielectric function, we also investigate Fluctuation energy loss due to color-electric field fluctuation

NPA 953,176

Wakes induced by the fast parton travels through the QGP medium

JPG 39,025007

NPA 856,121

Electric conductivity in a viscous quark-gluon plasma

Electric conductivity: charge transport coefficient, reflect the electro-magnetic response of the medium to the external electric field.

Dominate the space-time evolution energy-momentum , quark-gluon chemical equilibration rate, Gatoff et al, PRD 36-114, Eskola et al, PRC 47-2329.

Determine duration of tranisient strong magnetic field and strength for the CME, Fukushima, PRD 78-074033, Tuchin, AHEP 2013,490495…….

Influence the soft photon and low mass dilepton yield
Eskola et al, PRC 47-2329, Kapusta, Moore….
Yi Yin, PRC 90,044903

Method for Electric conductivity

Kinetic theory 80s-90s,Heinz, Ann.Phys 168-148;Mrowczynski,PRD 39-1940;Blaizot,PR 359-355….

Resummation QCD perturbation theory, Effective kinetic theory Manuel,Hou Defu, Heiselberg, Arnold,Bodeker,Blaizot…….

Numerical Boltzmann equation,Pulisi,Greif, XuZhe…

The quasiparticle model…

DS equation…

Lattice…

Our consideration: viscous chromohydrodynamics

viscous effect on the electric conductivity

Transport

Temperature gradient → heat transport → heat diffision or conductivity
Velocity gradient →momentum transport →shear or bulk viscosity
External field(charge) distrubance→charge transport →electric conductivity

Correlation between different typy of the transport…

Wiedemann-Franz law: heat conductivity over electric conductivity
Quark matter: G.S.Denicol, et al, Phys. Rev. D 99, 056017

Analogy to the W-F law: shear viscosity over thermal conductivity
Quantum Hall state

How about electric conductivity and viscosity?

Shear viscosity and the electric conductivity are separately calculated under the same physical conditions with Kinetic theory, and then the ratio of shear viscosity over electric conductivity is performed to address the relative importance of charge and momentum transport.

Puglisi et al, PLB 751,326
Thakur et al, PRD 95,096009
Mitra et al, PRD 96,094003
Sahoo et al, PRD 98,054005
Muller et al, PRD 91,125010

An alternative approach to study the connection between shear viscosity and electric conductivity: viscous chromohydrodynamics

The color current

$$\delta j_a^\mu = -\frac{g}{2}\left(\bar{n}\delta u_a^\mu + \delta n_a \bar{u}^\mu\right).$$

Substituting δu_a^μ **and** δn_a **in terms o** $j_a^\mu = \sigma_{ab}^{\mu\nu} F_{\nu\lambda}^b u^\lambda,$

We can extract the conductivity tensor

$$\begin{aligned}\sigma_{ab}^{\mu\nu} &= -\frac{i}{\omega}\frac{\delta^{ab}\omega_p^2}{1-Dk^2}\{g^{\mu\nu} + (B+E)(K^\mu K^\nu - \overline{u}^\mu K^\nu (K\cdot\overline{u})) \\ &- \frac{\overline{u}^\mu K^\nu}{(K\cdot\overline{u})} - (B+E)(\frac{K^2\overline{u}^\mu K^\nu}{K\cdot\overline{u}} - \overline{u}^\mu K^\nu (K\cdot\overline{u}))\}. \quad (21)\end{aligned}$$

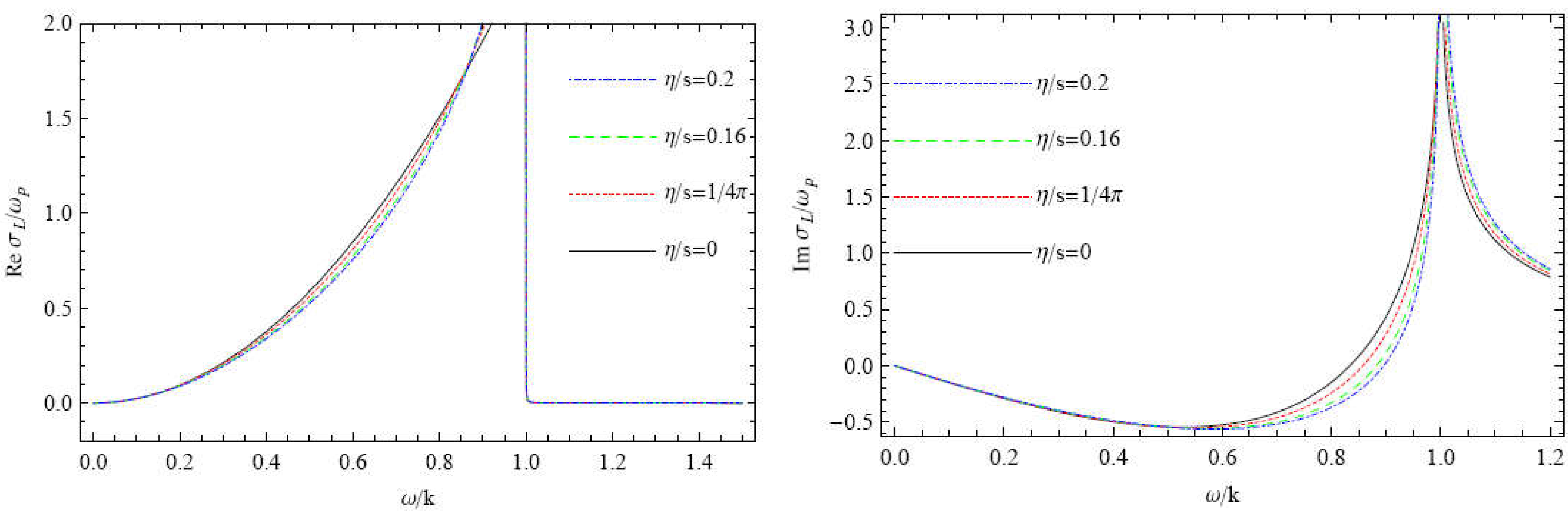
The spatial part is

$$\sigma^{ij} = -\frac{i}{\omega}\frac{\omega_p^2}{1-Dk^2}\{g^{ij} + (B+E)K^i K^j\}.$$

According to the project operators $\sigma^{ij} = \sigma_L \frac{k^i k^j}{k^2} + \sigma_T(\delta^{ij} - \frac{k^i k^j}{k^2}),$

$$
\begin{aligned}
\frac{\sigma_L(\omega,k)}{\omega_P} &= -\frac{i\omega}{1-\frac{\eta k^2}{sT\omega}}\frac{3\omega_P}{k^2}(1-\frac{\omega}{2k}\log\frac{\omega+k+i\xi}{\omega-k+i\xi}) \\
&+ \frac{i\omega_P}{1-\frac{\eta k^2}{sT\omega}}\frac{\eta}{sT}\frac{1}{1+4\frac{\eta\omega}{sT}(1-\frac{\omega}{2k}\log\frac{\omega+k+i\xi}{\omega-k+i\xi})} \quad (28) \\
&\cdot \{3(1-\frac{\omega}{2k}\log\frac{\omega+k+i\xi}{\omega-k+i\xi}) \\
&+ \frac{12\omega^2}{k^2}(1-\frac{\omega}{2k}\cdot\log\frac{\omega+k+i\xi}{\omega-k+i\xi})^2\}
\end{aligned}
$$

$$
\frac{\sigma_T}{\omega_p} = \frac{i\omega_p}{\omega}\frac{1}{1-\frac{\eta k^2}{s\omega T}}
$$



$$\sigma_L = -i\omega\{\varepsilon_L - 1\}$$

The Onsager relations:

Medium polarization is a function of the distribution of the background particles.Shear viscosity will modify the distribution functions of the constituents of the QGP, thus it will affect the induced current, with the viscous chromo-hydrodynamics,one can derive the viscous induced current,through which we can study the viscous effect on the electric conductivity.

summary

- **Deriving the viscous chromohydrodynamics**
- **Deriving the viscous induced current**
- **The viscous dielectric functions**
- **Refractive index in viscous QGP**
- **Polarization energy loss in viscous QGP**
- **The electric conductivity in viscous QGP**

Thanks for your attention !