Chiral kinetic theory from Landau level basis

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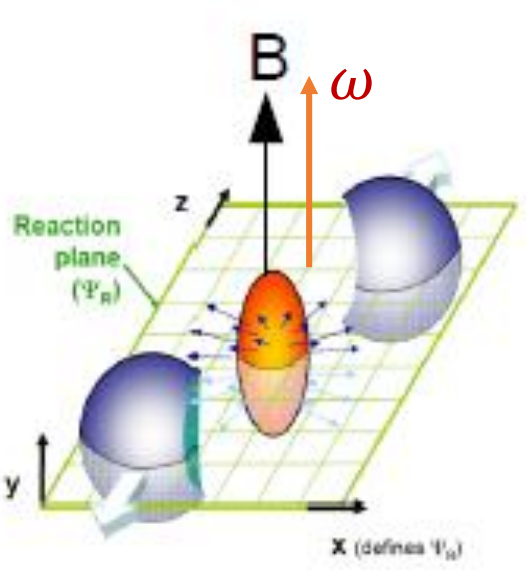


SL, L.X.Yang, 1908.XXXXX

Outline

- Anomalous effect and Chiral kinetic theory (CKT)
- CKT as hbar expansion and its difficulty at higher order
- Chiral kinetic theory with Landau level basis
- Simple examples: longitudinal conductivity
- Application: transverse conductivity of QGP
- Summary and outlook

Anomalous effects in heavy ion collisions



Chiral Magnetic/Separation Effect(CME/CSE)

$$\boldsymbol{j} = C\mu_5 e\boldsymbol{B} \qquad \boldsymbol{j}_5 = C\mu e\boldsymbol{B}$$

Kharzeev, Zhitnitsky, NPA 2007
Kharzeev, McLerran, Warringa, NPA 2008
Metlitski, Zhitnitsky, PRD 2005

Chiral Vortical Effect(VCVE/ACVE)

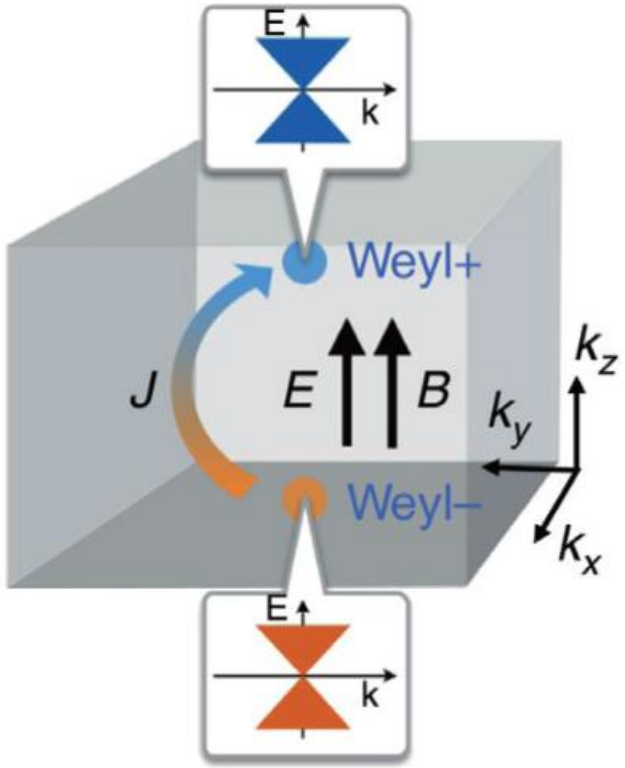
$$\boldsymbol{j} = C\mu_5\mu\boldsymbol{\omega} \qquad \boldsymbol{j}_5 = C(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3})\boldsymbol{\omega}$$

talk by Shou

Vilenken, PRD 1980
Erdmenger et al, JHEP 2009
Banerjee et al, JHEP 2011

Anomalous effect in Weyl semimetal

Weyl Semi-metal



Li, Kharzeev et al Nature. Phys. (2016)
Gooth et al Nature. (2017)
Son, Spivak PRB (2013)

Theoretical frameworks for anomalous effect

Anomalous hydrodynamics: near equilibrium, can be strongly coupled.

talk by Liao

Chiral kinetic theory: can be far from equilibrium, weakly coupled

talks by X.G.Huang, J.H.Gao

Chiral kinetic theory

Transport equation

$$(1+\hbar sQ\boldsymbol{\Omega}_p\cdot\mathbf{B})\,\partial_t f(x,E_p,\mathbf{p})$$
$$+\left[\mathbf{v}+\hbar sQ(\mathbf{E}\times\boldsymbol{\Omega}_p)+\hbar sQ\frac{1}{2|\mathbf{p}|^2}\mathbf{B}\right]\cdot\boldsymbol{\nabla}_x f(x,E_p,\mathbf{p})$$
$$+\left[Q\tilde{\mathbf{E}}+Q\mathbf{v}\times\mathbf{B}+\hbar sQ^2(\mathbf{E}\cdot\mathbf{B})\boldsymbol{\Omega}_p\right]\cdot\boldsymbol{\nabla}_p f(x,E_p,\mathbf{p}) \;=\; 0$$

Berry curvature $\quad \boldsymbol{\Omega}_{\mathbf{p}}\equiv\pm\dfrac{\hat{\mathbf{p}}}{2|\mathbf{p}|^2}$

compared to Boltzmann equation

$$\partial_t f+\mathbf{v}\cdot\boldsymbol{\nabla}_x f+Q(\boldsymbol{E}+\mathbf{v}\times\boldsymbol{B})\cdot\boldsymbol{\nabla}_p f=0$$

Son, Yamamoto, PRD (2012) PRL (2012)
Stephanov, Yin, PRL (2012)
Gao, Liang, Pu, Q. Wang, X.-N. Wang, PRL (2012), PRD (2014)
Manuel, Torres-Rincol, PRD (2014)
Hidaka, Pu, Yang, PRD (2016)
Huang, Shi, Jiang, Liao, Zhuang PRD (2018)
Liu, Gao, Mameda, Huang, PRD (2019)

CKT as a perturbation theory in hbar

$O(\hbar^0)$: spinless particle $\quad \partial_t f + \mathbf{v}\cdot\boldsymbol{\nabla}_x f + Q(\boldsymbol{E}+\mathbf{v}\times\boldsymbol{B})\cdot\boldsymbol{\nabla}_p f = 0 \quad$ $\delta(p^2)$

$O(\hbar)$: particle with magnetic moment + Berry curvature

$$
\begin{aligned}
&(1+\hbar sQ\boldsymbol{\Omega}_p\cdot\mathbf{B})\,\partial_t f(x,E_p,\mathbf{p}) \\
&+\left[\mathbf{v}+\hbar sQ(\mathbf{E}\times\boldsymbol{\Omega}_p)+\hbar sQ\frac{1}{2|\mathbf{p}|^2}\mathbf{B}\right]\cdot\boldsymbol{\nabla}_x f(x,E_p,\mathbf{p}) \\
&+\left[Q\tilde{\mathbf{E}}+Q\mathbf{v}\times\mathbf{B}+\hbar sQ^2(\mathbf{E}\cdot\mathbf{B})\boldsymbol{\Omega}_p\right]\cdot\boldsymbol{\nabla}_p f(x,E_p,\mathbf{p}) \;=\; 0
\end{aligned}
$$

$\delta(p^2) \to \delta(p^2 + Q\boldsymbol{B}\cdot\boldsymbol{p}/p_0)$

valid when $\hbar E, \hbar B \ll p^2, \hbar\partial_X \ll p$

Difficulty of CKT at higher order

$O(\hbar^0)$: spinless particle $\partial_t f + \mathbf{v}\cdot\boldsymbol{\nabla}_x f + Q(\boldsymbol{E}+\mathbf{v}\times\boldsymbol{B})\cdot\boldsymbol{\nabla}_p f = 0$

$O(\hbar)$: particle with magnetic moment + Berry curvature

$$\begin{aligned}
&\left(1+\hbar sQ\boldsymbol{\Omega}_p\cdot\mathbf{B}\right)\partial_t f(x,E_p,\mathbf{p})\\
&+\left[\mathbf{v}+\hbar sQ(\mathbf{E}\times\boldsymbol{\Omega}_p)+\hbar sQ\frac{1}{2|\mathbf{p}|^2}\mathbf{B}\right]\cdot\boldsymbol{\nabla}_x f(x,E_p,\mathbf{p})\\
&+\left[Q\tilde{\mathbf{E}}+Q\mathbf{v}\times\mathbf{B}+\hbar sQ^2(\mathbf{E}\cdot\mathbf{B})\boldsymbol{\Omega}_p\right]\cdot\boldsymbol{\nabla}_p f(x,E_p,\mathbf{p}) \;=\; 0
\end{aligned}$$

$O(\hbar^2)$: particle no longer on-shell, simple picture lost $\delta(p^2) \nrightarrow \delta(\tilde{p}^2)$

$$p_\mu G^\mu_{(0)}\left[f\delta(\tilde{p}^2)\right]+\frac{\hbar s}{2}\mathbf{G}^{(0)}\cdot\left\{\frac{1}{p_0}\mathbf{G}^{(0)}\times\left[\mathbf{p}f\delta(\tilde{p}^2)\right]\right\}+\hbar^2 C(f)=0$$

$C(f)$: off-shell effect

Gao, Liang, Q. Wang, X.N. Wang PRD (2018)
Wang, Guo, Shi, Zhuang PRD (2019)

Two alternative formulations

free particle basis: subject to weak electromagnetic field $\hbar B \ll \boldsymbol{p}^2$

with increasing B, CKT more IR singular, Landau quantization effect becomes important.

Landau level basis: subject to strong magnetic field $\hbar B \gg \boldsymbol{p}^2$

with increasing B, lower Landau levels truncation more accurate

Landau level basis

Wigner function at constant B for right-handed chiral fermion

$$W(X,P)=\int\frac{d^4X'}{(2\pi)^4}\exp(-ip\cdot X')\left\langle\psi\left(X-\frac{1}{2}X'\right)U\left(A,X+\frac{1}{2}X',X-\frac{1}{2}X'\right)\psi^{\dagger}\left(X-\frac{1}{2}X'\right)\right\rangle$$

Example: lowest Landau level

$$W(P)=f(p_0)[\delta(p_0-|p_3|)\theta(p_3)W_{+}^{(0)}(p)+\delta(p_0+|p_3|)\theta(-p_3)W_{-}^{(0)}(p)]$$

$$W_r^{(0)}(P)=\frac{2}{(2\pi)^3}\exp(-\frac{p_T^2}{eB})\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\qquad r=+/-$$

transverse size: $\frac{1}{\sqrt{eB}}$

LLL spin: aligns with B

Sheng, Rischke, Wang, Vasak EPJA (2018)

CKT with Landau level basis from perturbation

$$W(X,P)=\int\frac{d^4X'}{(2\pi)^4}\exp(-ip\cdot X')\left\langle\psi\left(X-\frac{1}{2}X'\right)U\left(A,X+\frac{1}{2}X',X-\frac{1}{2}X'\right)\psi^\dagger\left(X-\frac{1}{2}X'\right)\right\rangle$$

$$A_\mu\to A_\mu(B)+a_\mu(e,b)\qquad F_{\mu\nu}\to F_{\mu\nu}(B)+f_{\mu\nu}(e,b)$$

$$\left(\frac{\hbar}{2}\Delta_\mu-ip_\mu\right)\sigma^\mu W=0$$
$$\left(\frac{\hbar}{2}\Delta_\mu+ip_\mu\right)W\sigma^\mu=0$$

$$\Delta_\mu=\partial_\mu-\frac{\partial}{\partial p_\nu}(F_{\mu\nu}+f_{\mu\nu})$$

Valid to all order in background $\hbar$B and O($\hbar\partial_X$) and O($\hbar f$)

CKT with LL basis for components

W Hermitian $W = F \cdot 1 + j_i \sigma^i$

$$J^0 \propto \int \frac{d^4p}{(2\pi)^4} F \qquad J^i \propto \int \frac{d^4p}{(2\pi)^4} j_i$$

$$\begin{cases} \Delta_0 F + \Delta_i j_i = 0 \\ \Delta_0 j_i + \Delta_i F + 2\epsilon^{ijk} p_j j_k = 0 \\ p_0 F + p_i j_i = 0 \\ -p_0 j_i - p_i F + \frac{1}{2}\epsilon^{ijk}\Delta_j j_k = 0 \end{cases}$$

Transport equations

Constraint equations

In the absence of ∂_X and f, automatically satisfied by the background

LLL: $F = j_3 \propto f_{FD}(p_0)\delta(p_0 - p_3)\exp\left(-\frac{p_T^2}{eB}\right), \quad j_1 = j_2 = 0$

simple example with $O(E)$: longitudinal conductivity

$$W = F \cdot 1 + j_i \sigma^i$$

For homogeneous E ∥ B , components on LL shell and p_T dependence unchanged

$$j_3 = \frac{p_3}{p_0} F \propto f(p_0)\delta(p_0 - p_3)\exp(-\frac{p_T^2}{eB}) \qquad j_1 = j_2 = 0$$

$$\Delta_0 F + \Delta_i j_i = 0$$



$$\frac{\partial f_\pm}{\partial t} + \dot{z}\frac{\partial f_\pm}{\partial z} + \dot{p_z}\frac{\partial f_\pm}{\partial p_z} = C[f_\pm, f_g]$$

Hattori, Li, Satow, Yee PRD (2018)

Application with $O(E)$: transverse conductivity

$$W = F \cdot 1 + j_i \sigma^i$$

For homogeneous $E \perp B$, components off LL shell and p_T dependence changed

$$\Delta_0 F + \Delta_i j_i = -\frac{\delta F}{\tau}$$
$$\Delta_i F + \Delta_0 j_i - 2\epsilon^{ijk} p_j j_k = -\frac{\delta j_i}{\tau}$$

relaxation time approximation

$eB \gg T\&\mu$

talk by L.X. Yang

$$\delta\vec{J} = \frac{e\vec{E}}{8\pi^2\tau}\left[1 + \frac{1}{eB}\left(\frac{\mu^2}{2} + \frac{\pi^2 T^2}{6} - \frac{1}{2\tau^2}\right)\right] - \frac{e^2\mu\vec{B}\times\vec{E}}{4\pi^2 eB}$$

$\sigma_\perp \sim O(B^0)$, qualitative agreement with holographic study

Li, SL, Mei, PRD 2019

$\sigma_\perp \sim O(1/\tau)$ in contrast to $\sigma_\parallel \sim O(\tau)$

Application with $O(E)$ and $O(\partial_t)$: frequency dependent transverse conductivity

$$\vec{E} \propto e^{-i\omega t}$$

$$\tau \to \tau_\omega \equiv \frac{\tau}{1 - i\omega\tau}$$ effective relaxation time

$$\delta\vec{J} = \frac{e\vec{E}}{8\pi^2\tau_\omega}\left[1 + \frac{1}{eB}\left(\frac{\mu^2}{2} + \frac{\pi^2T^2}{6} - \frac{1}{2\tau_\omega^2}\right)\right] - \frac{e^2\mu\vec{B}\times\vec{E}}{4\pi^2 eB}$$

$$\sigma_\perp = \frac{e^2}{8\pi^2\tau_\omega}\left[1 + \frac{1}{eB}\left(\frac{\mu^2}{2} + \frac{\pi^2T^2}{6} - \frac{1}{2\tau_\omega^2}\right)\right]$$

$$\sigma_H = -\frac{e^2\mu}{4\pi^2}$$

Enhancement of transverse conductivity at large ω!

transverse conductivity of QGP

$$\sigma_{\perp} = \sum_f N_c Q_f^2 \frac{e^2}{8\pi^2 \tau_\omega}\left[1 + \frac{1}{eB}\left(\frac{\mu^2}{2} + \frac{\pi^2 T^2}{6} - \frac{1}{2\tau_\omega^2}\right)\right]$$

$$\sigma_H = -\sum_f N_c Q_f^2 \frac{e^2 \mu}{4\pi^2}$$

Estimate parameters in HIC

$eB \sim m_\pi^2 - 10 m_\pi^2$,
T=350MeV
$\frac{e^2\tau}{3}\chi = 0.37 e^2 \sum_f Q_f^2 T$ Ding et al, PRD 2011
$\omega \sim 0.2\text{fm} - 1\text{fm}$ from lifetime of B

$\tau T \sim 0.3$
$\frac{eB}{T^2} \sim 0.2 - 1.6$ higher LL to be included
$\omega\tau \sim 1 - 5$ transient effect maybe relevant

Summary

- CKT with Landau level basis
- Application: CKT with homogeneous E field
- Transverse conductivity approaches constant at large B
- Transverse conductivity enhancement at high frequency

Outlook

- Higher Landau levels contribution
- Self-energy of photon in magnetized QGP

Thank you!

Solution to δj_i

$$\delta j_1 = \frac{4p_3\tau F - \left(2eB\tau + \frac{1}{\tau}\right)\frac{\partial F}{\partial p_0}}{\left(2eB\tau + \frac{1}{\tau}\right)^2 + 4p_3^2}$$

$$\delta j_2 = \frac{-2p_3\frac{\partial F}{\partial p_0} - 2\left(2eB\tau + \frac{1}{\tau}\right)\tau F}{\left(2eB\tau + \frac{1}{\tau}\right)^2 + 4p_3^2}$$

$$B \gg p_3^2, 1/\tau^2$$

$$\delta j_1 = \frac{-\frac{\partial F}{\partial p_0}}{2eB\tau}$$

$$\delta j_2 = \frac{-2\tau F}{2eB\tau}$$