Theories of singly and doubly heavy-flavor Baryon spectroscopy

Bing Chen

Anyang Normal University

---

in collaboration with:

Xiang Liu (Lanzhou U.), Ailin Zhang (Shanghai U.), Ke-wei Wei (AYNU),
and Takayuki Matsuki (TKU)

第二届理论实验联合研讨会：重子谱和衰变（兰州大学）

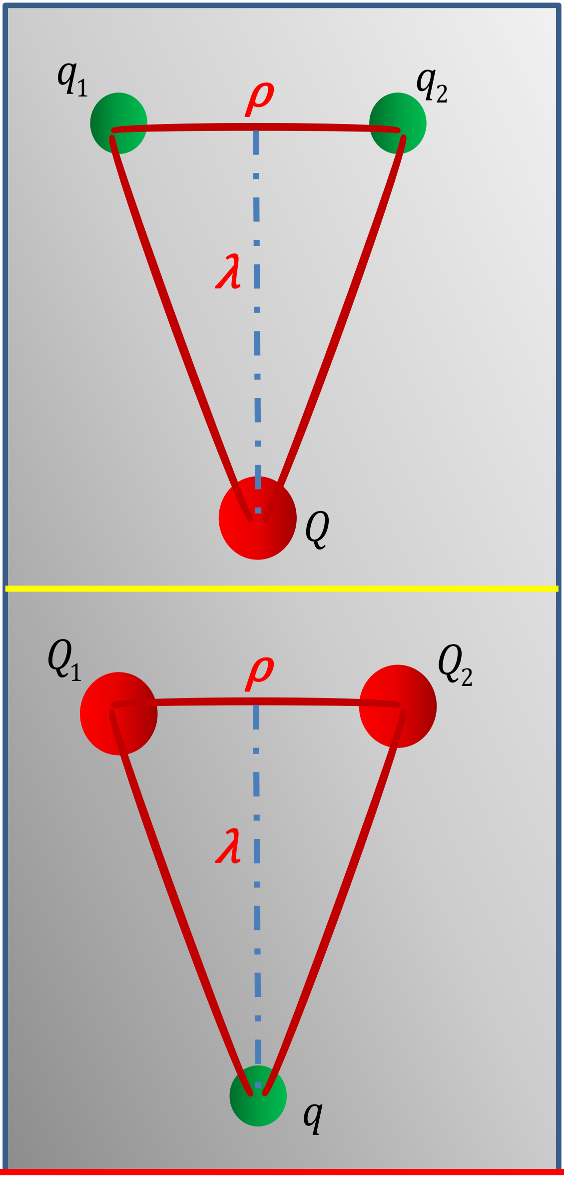
Outline

- 1. Experimental status of the singly and doubly heavy baryons
- 2. Study of singly and doubly heavy baryons in diquark picture
- 3. Study the masses of heavy baryon in three-body picture

Section Ⅰ

Experimental status of the singly and doubly heavy baryons

The terminology for the singly and doubly heavy baryons



With the Jacobi coordinates, the excited baryon systems can be classified as the ρ-mode, λ-mode, and mixing mode excitations.

For singly heavy baryons, we consider the symmetry between two light quarks;
For doubly heavy baryons, we consider the symmetry between two heavy quarks;

$$\Psi_{\text{diquark}} = C_A \otimes \underline{\phi_{\text{flavor}} \otimes \chi_{\text{spin}} \otimes \psi_{\text{spatial}}}$$

symmetric !

Take the charm baryon as an example,

❶. for Λ_Q, $\phi_{\text{flavor}}[A]$, then even l_ρ ($\psi_{\text{spatial}}[S]$) $\Leftrightarrow S_{\text{diquark}} = 0$
then odd l_ρ ($\psi_{\text{spatial}}[A]$) $\Leftrightarrow S_{\text{diquark}} = 1$;

❷. for Σ_Q, $\phi_{\text{flavor}}[S]$, then even l_ρ ($\psi_{\text{spatial}}[S]$) $\Leftrightarrow S_{\text{diquark}} = 1$
then odd l_ρ ($\psi_{\text{spatial}}[A]$) $\Leftrightarrow S_{\text{diquark}} = 0$.

The classification of the singly and doubly heavy baryons

For $L = l_\rho + l_\lambda \leq 2$, the allowed singly heavy baryon states in the three-body picture are listed in the following table. The parity of a state is determined by $P = (-1)^L$.

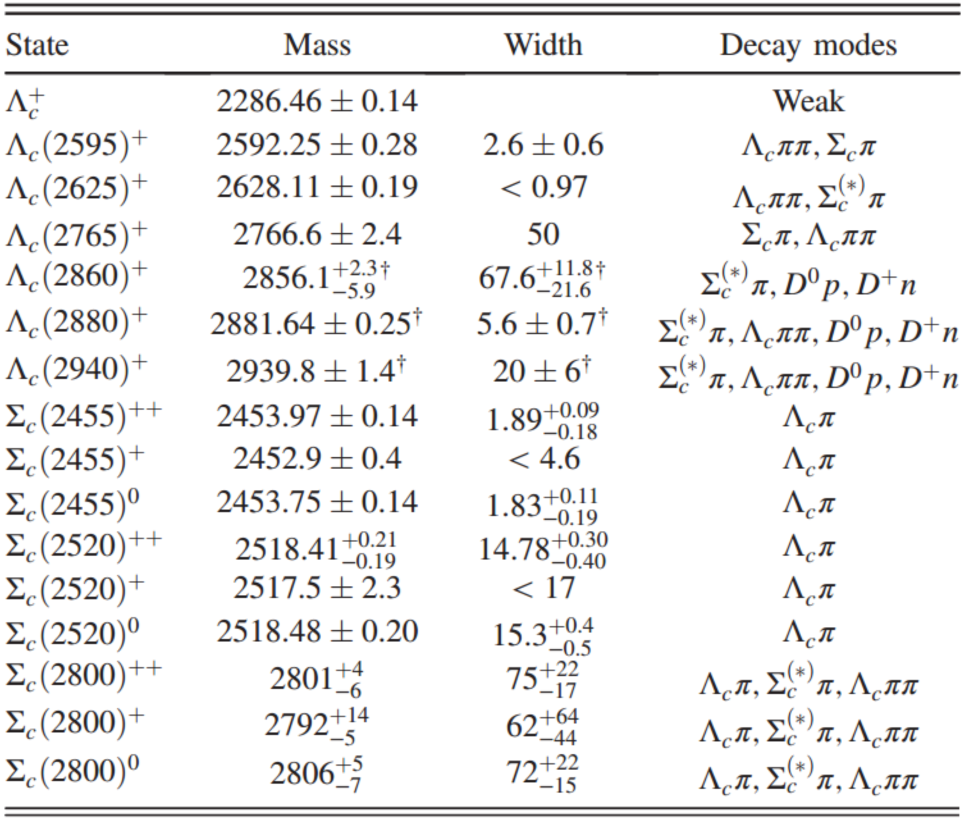
l_ρ	l_λ	type	spin	j_l	s_Q	J^P
0	0	Λ_Q	0	0	1/2	$1/2^+$
		Σ_Q	1	1	1/2	$1/2^+$, $3/2^+$
	1	Λ_Q	0	1	1/2	$1/2^-$, $3/2^-$
		Σ_Q	1	0,1,2	1/2	$1/2^-$, $1/2'^-$, $3/2^-$, $3/2'^-$, $5/2^-$
	2	Λ_Q	0	2	1/2	$3/2^+$, $5/2^+$
		Σ_Q	1	1,2,3	1/2	$1/2^+$, $3/2^+$, $3/2'^+$, $5/2^+$, $5/2'^+$, $7/2^+$
1	0	Λ_Q	1	0,1,2	1/2	$1/2^-$, $1/2'^-$, $3/2^-$, $3/2'^-$, $5/2^-$
		Σ_Q	0	1	1/2	$1/2^-$, $3/2^-$
	1	Λ_Q	1	1	1/2	$1/2^+$, $3/2^+$
				0,1,2	1/2	$1/2^+$, $1/2'^+$, $3/2^+$, $3/2'^+$, $5/2^+$
				1,2,3	1/2	$1/2^+$, $3/2^+$, $3/2'^+$, $5/2^+$, $5/2'^+$, $7/2^+$
		Σ_Q	0	0,1,2	1/2	$1/2^+$, $1/2'^+$, $3/2^+$, $3/2'^+$, $5/2^+$
2	0	Λ_Q	0	2	1/2	$3/2^+$, $5/2^+$
		Σ_Q	1	1,2,3	1/2	$1/2^+$, $3/2^+$, $3/2'^+$, $5/2^+$, $5/2'^+$, $7/2^+$

$$\left|\left(\left([n_\rho l_\rho]\otimes[n_\lambda l_\lambda]\right)_L \otimes s_\rho\right)_{j_l} \otimes s_Q\right\rangle_J$$

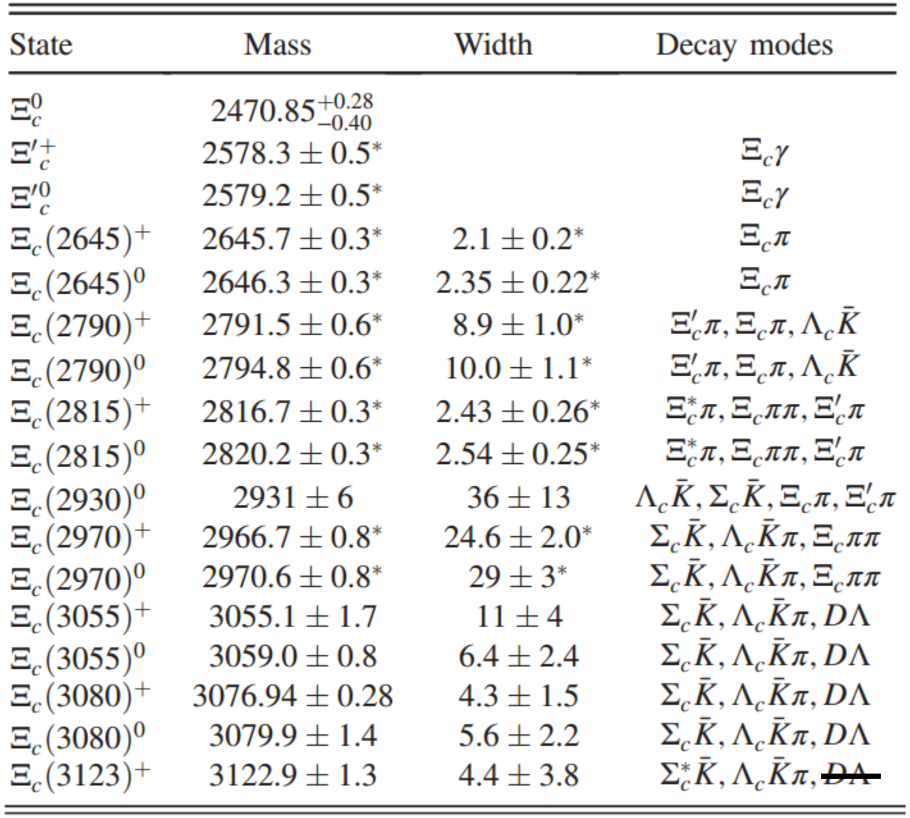
Table (1)

Experimental results of the singly heavy baryon states

Status of charmed baryons



State	Mass	Width	Decay modes
Λ_c^+	2286.46 ± 0.14		Weak
$\Lambda_c(2595)^+$	2592.25 ± 0.28	2.6 ± 0.6	$\Lambda_c\pi\pi, \Sigma_c\pi$
$\Lambda_c(2625)^+$	2628.11 ± 0.19	< 0.97	$\Lambda_c\pi\pi, \Sigma_c^{(*)}\pi$
$\Lambda_c(2765)^+$	2766.6 ± 2.4	50	$\Sigma_c\pi, \Lambda_c\pi\pi$
$\Lambda_c(2860)^+$	$2856.1^{+2.3\dagger}_{-5.9}$	$67.6^{+11.8\dagger}_{-21.6}$	$\Sigma_c^{(*)}\pi, D^0p, D^+n$
$\Lambda_c(2880)^+$	$2881.64 \pm 0.25^\dagger$	$5.6 \pm 0.7^\dagger$	$\Sigma_c^{(*)}\pi, \Lambda_c\pi\pi, D^0p, D^+n$
$\Lambda_c(2940)^+$	$2939.8 \pm 1.4^\dagger$	$20 \pm 6^\dagger$	$\Sigma_c^{(*)}\pi, \Lambda_c\pi\pi, D^0p, D^+n$
$\Sigma_c(2455)^{++}$	2453.97 ± 0.14	$1.89^{+0.09}_{-0.18}$	$\Lambda_c\pi$
$\Sigma_c(2455)^+$	2452.9 ± 0.4	< 4.6	$\Lambda_c\pi$
$\Sigma_c(2455)^0$	2453.75 ± 0.14	$1.83^{+0.11}_{-0.19}$	$\Lambda_c\pi$
$\Sigma_c(2520)^{++}$	$2518.41^{+0.21}_{-0.19}$	$14.78^{+0.30}_{-0.40}$	$\Lambda_c\pi$
$\Sigma_c(2520)^+$	2517.5 ± 2.3	< 17	$\Lambda_c\pi$
$\Sigma_c(2520)^0$	2518.48 ± 0.20	$15.3^{+0.4}_{-0.5}$	$\Lambda_c\pi$
$\Sigma_c(2800)^{++}$	2801^{+4}_{-6}	75^{+22}_{-17}	$\Lambda_c\pi, \Sigma_c^{(*)}\pi, \Lambda_c\pi\pi$
$\Sigma_c(2800)^+$	2792^{+14}_{-5}	62^{+64}_{-44}	$\Lambda_c\pi, \Sigma_c^{(*)}\pi, \Lambda_c\pi\pi$
$\Sigma_c(2800)^0$	2806^{+5}_{-7}	72^{+22}_{-15}	$\Lambda_c\pi, \Sigma_c^{(*)}\pi, \Lambda_c\pi\pi$

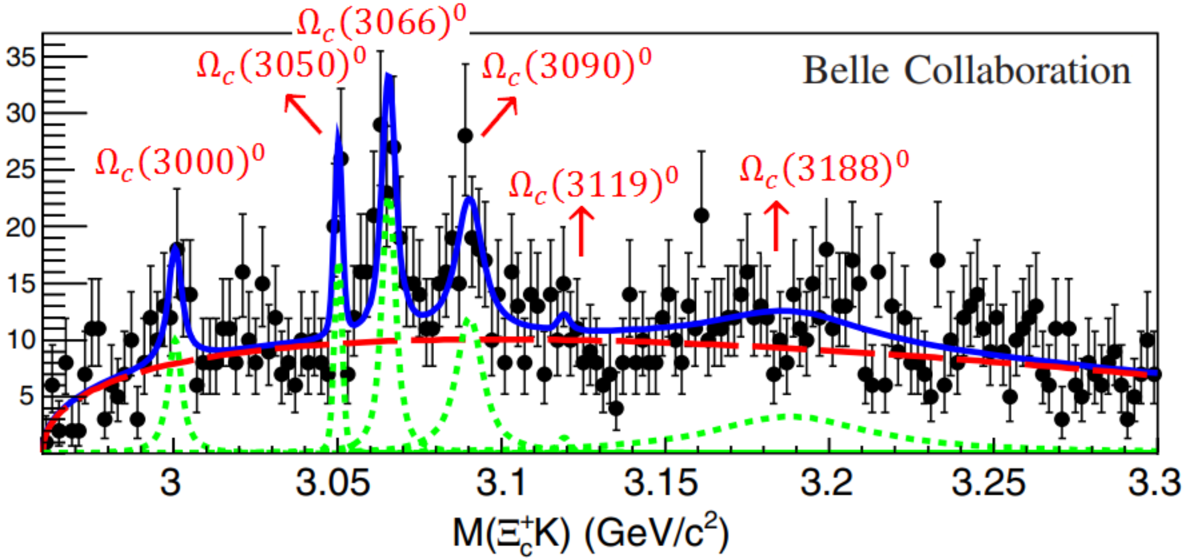


State	Mass	Width	Decay modes
Ξ_c^0	$2470.85^{+0.28}_{-0.40}$		
$\Xi_c'^+$	$2578.3 \pm 0.5^*$		$\Xi_c\gamma$
$\Xi_c'^0$	$2579.2 \pm 0.5^*$		$\Xi_c\gamma$
$\Xi_c(2645)^+$	$2645.7 \pm 0.3^*$	$2.1 \pm 0.2^*$	$\Xi_c\pi$
$\Xi_c(2645)^0$	$2646.3 \pm 0.3^*$	$2.35 \pm 0.22^*$	$\Xi_c\pi$
$\Xi_c(2790)^+$	$2791.5 \pm 0.6^*$	$8.9 \pm 1.0^*$	$\Xi_c'\pi, \Xi_c\pi, \Lambda_c\bar{K}$
$\Xi_c(2790)^0$	$2794.8 \pm 0.6^*$	$10.0 \pm 1.1^*$	$\Xi_c'\pi, \Xi_c\pi, \Lambda_c\bar{K}$
$\Xi_c(2815)^+$	$2816.7 \pm 0.3^*$	$2.43 \pm 0.26^*$	$\Xi_c^*\pi, \Xi_c\pi\pi, \Xi_c'\pi$
$\Xi_c(2815)^0$	$2820.2 \pm 0.3^*$	$2.54 \pm 0.25^*$	$\Xi_c^*\pi, \Xi_c\pi\pi, \Xi_c'\pi$
$\Xi_c(2930)^0$	2931 ± 6	36 ± 13	$\Lambda_c\bar{K}, \Sigma_c\bar{K}, \Xi_c\pi, \Xi_c'\pi$
$\Xi_c(2970)^+$	$2966.7 \pm 0.8^*$	$24.6 \pm 2.0^*$	$\Sigma_c\bar{K}, \Lambda_c\bar{K}\pi, \Xi_c\pi\pi$
$\Xi_c(2970)^0$	$2970.6 \pm 0.8^*$	$29 \pm 3^*$	$\Sigma_c\bar{K}, \Lambda_c\bar{K}\pi, \Xi_c\pi\pi$
$\Xi_c(3055)^+$	3055.1 ± 1.7	11 ± 4	$\Sigma_c\bar{K}, \Lambda_c\bar{K}\pi, D\Lambda$
$\Xi_c(3055)^0$	3059.0 ± 0.8	6.4 ± 2.4	$\Sigma_c\bar{K}, \Lambda_c\bar{K}\pi, D\Lambda$
$\Xi_c(3080)^+$	3076.94 ± 0.28	4.3 ± 1.5	$\Sigma_c\bar{K}, \Lambda_c\bar{K}\pi, D\Lambda$
$\Xi_c(3080)^0$	3079.9 ± 1.4	5.6 ± 2.2	$\Sigma_c\bar{K}, \Lambda_c\bar{K}\pi, D\Lambda$
$\Xi_c(3123)^+$	3122.9 ± 1.3	4.4 ± 3.8	$\Sigma_c^*\bar{K}, \Lambda_c\bar{K}\pi,$ ~~$D\Lambda$~~

Mass spectra, widths (in units of MeV), and decay modes of the observed charmed baryons

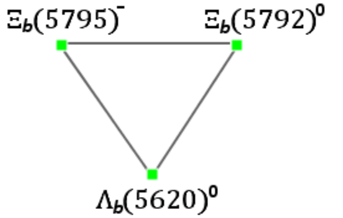
Experimental results of the singly heavy baryon states

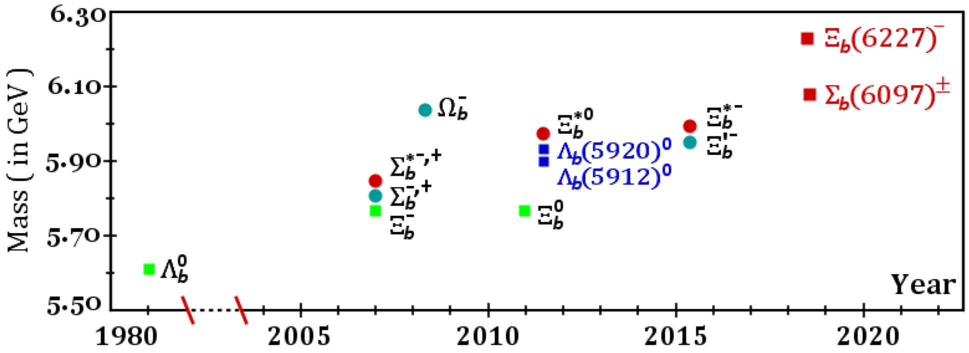
State	Decay mode	Mass (MeV)	Width (MeV)
$\Omega_c(2700)^0$	week	2695.2 ± 1.7	
$\Omega_c(2770)^0$	$\Omega_c^0\gamma$	2765.9 ± 2.0	
$\Omega_c(3000)^0$	$\Xi_c K$	$3000.4 \pm 0.2 \pm 0.1^{+0.3}_{-0.5}$	$4.5 \pm 0.6 \pm 0.3$
$\Omega_c(3050)^0$	$\Xi_c K$	$3050.2 \pm 0.1 \pm 0.1^{+0.3}_{-0.5}$	$0.8 \pm 0.2 \pm 0.1$
$\Omega_c(3066)^0$	$\Xi_c K$, $\Xi_c' K$	$3065.6 \pm 0.1 \pm 0.3^{+0.3}_{-0.5}$	$3.5 \pm 0.4 \pm 0.2$
$\Omega_c(3090)^0$	$\Xi_c K$, $\Xi_c' K$	$3090.2 \pm 0.3 \pm 0.5^{+0.3}_{-0.5}$	$8.7 \pm 1.0 \pm 0.8$
$\Omega_c(3119)^0$	$\Xi_c K$, $\Xi_c' K$	$3119.1 \pm 0.3 \pm 0.9^{+0.3}_{-0.5}$	$1.1 \pm 0.8 \pm 0.4$
$\Omega_c(3188)^0$	$\Xi_c K$	$3188 \pm 5 \pm 13$	$60 \pm 15 \pm 11$



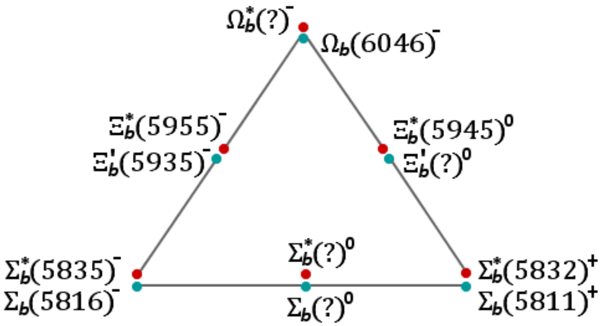
Experimental results of the singly heavy baryon states

Status of bottom baryons





Names	Status	Mass	Width
$\Lambda_b(5619)^0$	$* * *$	5619.4 ± 0.6	–
$\Lambda_b(5912)^0$	$* * *$	5912.0 ± 0.6	< 0.66
$\Lambda_b(5920)^0$	$* * *$	5919.8 ± 0.8	< 0.63
$\Sigma_b(5815)^-$	$* * *$	5815.5 ± 1.8	$4.9^{+3.3}_{-2.4}$
$\Sigma_b^*(5835)^-$	$* * *$	5835.1 ± 1.9	7.5 ± 2.3
$\Sigma_b(6097)^-$		6098.0 ± 2.2	28.9 ± 5.1
$\Xi_b(5790)^-$	$* * *$	5791.1 ± 2.2	–
$\Xi_b'(5935)^-$	$**$	5935.02 ± 0.53	< 0.08
$\Xi_b^*(5955)^-$	$* * *$	5955.33 ± 0.68	1.65 ± 0.41
$\Xi_b(6227)^-$		6226.9 ± 2.5	18.1 ± 7.2

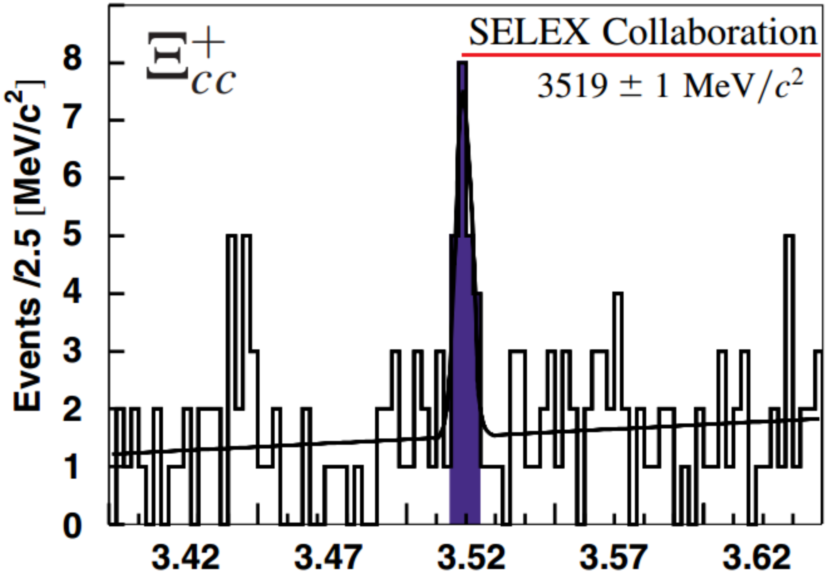


Bing Chen, Ke-wei Wei, Xiang Liu, and Ailin Zhang,
Phys. Rev. D 98, 031502 (2018)

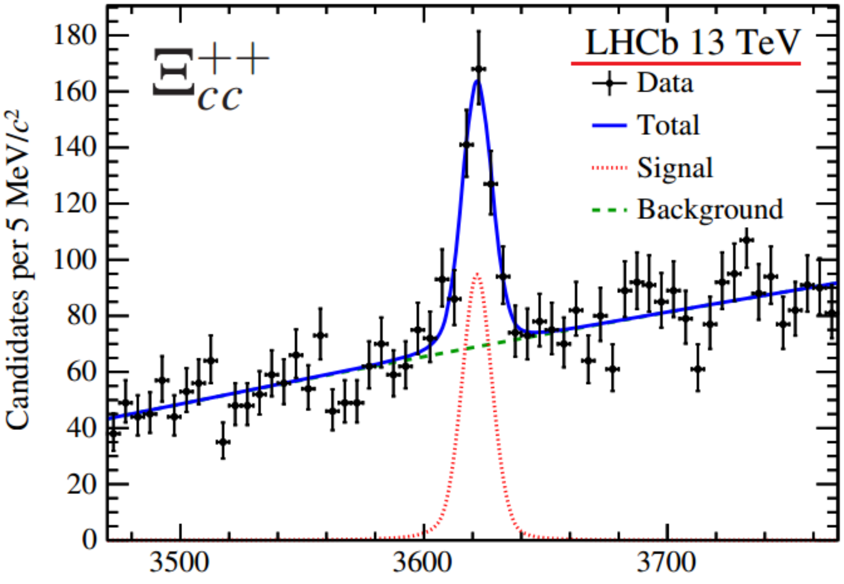
Mass spectra, widths (in units of MeV), and decay modes of the observed bottom baryons

Experimental results of the doubly heavy baryon states

its lifetime is less than 33 fs at 90% confidence.



$3621.40 \pm 0.72 \pm 0.27 \pm 0.14$ MeV



The Ξ_{cc}^{++} lifetime: $0.256^{+0.024}_{-0.022}(\mathrm{stat}) \pm 0.014(\mathrm{syst})$ ps.

1. First Observation of the Doubly Charmed Ξ_{cc}^{+}, SELEX Collaboration, Phys. Rev. Lett. **89**, 112001 (2002) .
2. Observation of the doubly charmed baryon Ξ_{cc}^{++}, LHCb Collaboration, Phys. Rev. Lett. **119**, 112001 (2017) .
3. First Observation of the Doubly Charmed Baryon Decay $\Xi_{cc}^{++} \to \Xi_{c}^{+}\,\pi^{+}$, LHCb Collaboration, Phys. Rev. Lett. **121**, 162002 (2018) .
4. Measurement of the Lifetime of the Doubly Charmed Baryon Ξ_{cc}^{++}, LHCb Collaboration, Phys. Rev. Lett. **121**, 052002 (2018) .

Section Ⅱ

Study of singly and doubly heavy baryons in diquark picture

Experimental and theoretical foundation for the diquark picture

Within the diquark picture, the mass difference between the corresponding Ξ_c and Λ_c states is about 180~200 MeV, while the corresponding Ξ_c' and Σ_c states is about 120~130 MeV

doublet	s_l^P	J^P	Λ_c (cud)	Ξ_c (csq)	δ_M
$1S$	0^+	$\frac{1}{2}^+$	$\Lambda_c(2286)$	$\Xi_c(2470)$	184
$2S$	0^+	$\frac{1}{2}^+$	$\Lambda_c(2760)$	$\Xi_c(2970)$	200
$1P$	1^-	$\frac{1}{2}^-$	$\Lambda_c(2595)$	$\Xi_c(2790)$	200
		$\frac{3}{2}^-$	$\Lambda_c(2625)$	$\Xi_c(2815)$	188
$1D$	2^+	$\frac{3}{2}^+$	$\Lambda_c(2860)$	$\Xi_c(3055)$	201
		$\frac{5}{2}^+$	$\Lambda_c(2880)$	$\Xi_c(3080)$	198
$2P$	1^-	$\frac{1}{2}^-$			
		$\frac{3}{2}^-$	$\Lambda_c(2940)$	$\Xi_c(3123)$	184

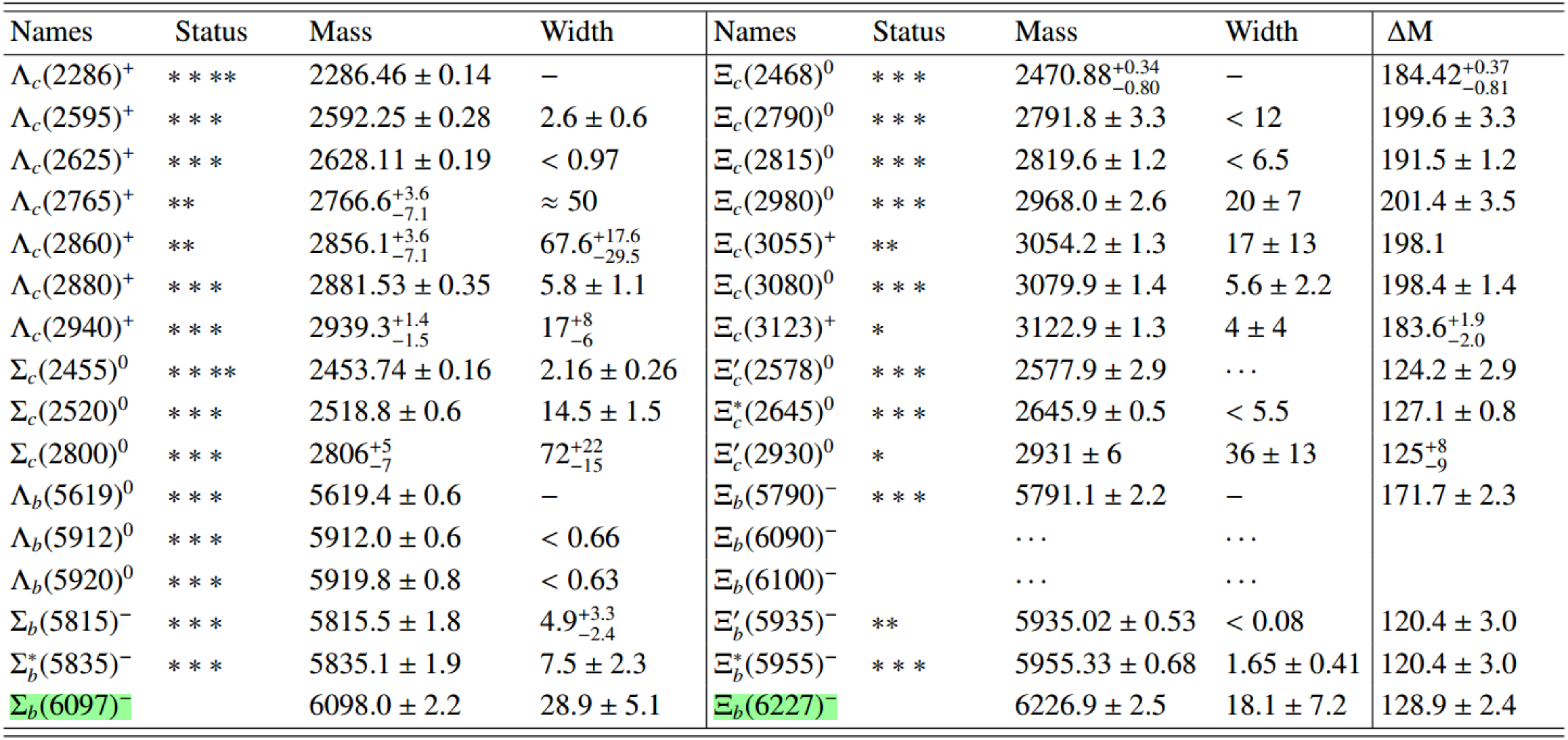
$$\frac{16\pi\alpha}{9m_i m_j}\delta^3(\vec{r}_{ij}^{\,3})\,\vec{S}_i\cdot\vec{S}_j$$

$$\delta_{\text{"good"}} = (m_s - m_q) + \frac{3}{4}\lambda;$$
$$\delta_{\text{"bad"}} = (m_s - m_q) - \frac{1}{4}\lambda;$$

$$m_{\Xi_c'(2930)} - m_{\Sigma_c(2800)} \approx$$
$$m_{\Xi_c'(2570)} - m_{\Sigma_c(2455)} \approx$$
$$m_{\Xi_c'(2645)} - m_{\Sigma_c(2520)} \approx$$
125 MeV

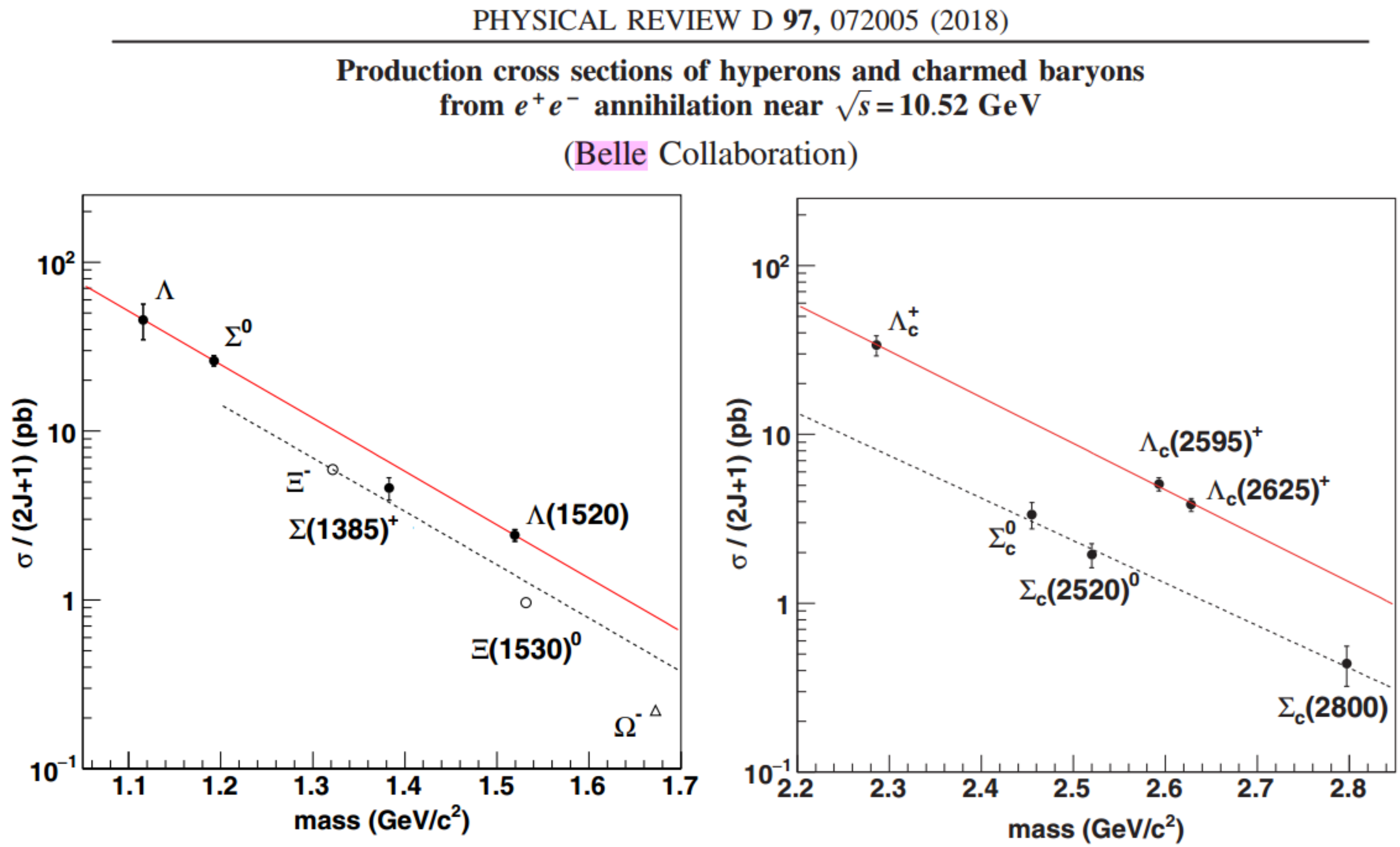
Experimental and theoretical foundation for the diquark picture

Within the diquark picture, the mass difference between the corresponding Ξ_Q and Λ_Q states is about 180~200 MeV, while the corresponding Ξ_Q' and Σ_Q states is about 120~130 MeV



Names	Status	Mass	Width	Names	Status	Mass	Width	ΔM
$\Lambda_c(2286)^+$	$* * **$	2286.46 ± 0.14	–	$\Xi_c(2468)^0$	$* * *$	$2470.88^{+0.34}_{-0.80}$	–	$184.42^{+0.37}_{-0.81}$
$\Lambda_c(2595)^+$	$* * *$	2592.25 ± 0.28	2.6 ± 0.6	$\Xi_c(2790)^0$	$* * *$	2791.8 ± 3.3	< 12	199.6 ± 3.3
$\Lambda_c(2625)^+$	$* * *$	2628.11 ± 0.19	< 0.97	$\Xi_c(2815)^0$	$* * *$	2819.6 ± 1.2	< 6.5	191.5 ± 1.2
$\Lambda_c(2765)^+$	$**$	$2766.6^{+3.6}_{-7.1}$	≈ 50	$\Xi_c(2980)^0$	$* * *$	2968.0 ± 2.6	20 ± 7	201.4 ± 3.5
$\Lambda_c(2860)^+$	$**$	$2856.1^{+3.6}_{-7.1}$	$67.6^{+17.6}_{-29.5}$	$\Xi_c(3055)^+$	$**$	3054.2 ± 1.3	17 ± 13	198.1
$\Lambda_c(2880)^+$	$* * *$	2881.53 ± 0.35	5.8 ± 1.1	$\Xi_c(3080)^0$	$* * *$	3079.9 ± 1.4	5.6 ± 2.2	198.4 ± 1.4
$\Lambda_c(2940)^+$	$* * *$	$2939.3^{+1.4}_{-1.5}$	17^{+8}_{-6}	$\Xi_c(3123)^+$	$*$	3122.9 ± 1.3	4 ± 4	$183.6^{+1.9}_{-2.0}$
$\Sigma_c(2455)^0$	$* * **$	2453.74 ± 0.16	2.16 ± 0.26	$\Xi_c'(2578)^0$	$* * *$	2577.9 ± 2.9	$\cdots$	124.2 ± 2.9
$\Sigma_c(2520)^0$	$* * *$	2518.8 ± 0.6	14.5 ± 1.5	$\Xi_c^*(2645)^0$	$* * *$	2645.9 ± 0.5	< 5.5	127.1 ± 0.8
$\Sigma_c(2800)^0$	$* * *$	2806^{+5}_{-7}	72^{+22}_{-15}	$\Xi_c'(2930)^0$	$*$	2931 ± 6	36 ± 13	125^{+8}_{-9}
$\Lambda_b(5619)^0$	$* * *$	5619.4 ± 0.6	–	$\Xi_b(5790)^-$	$* * *$	5791.1 ± 2.2	–	171.7 ± 2.3
$\Lambda_b(5912)^0$	$* * *$	5912.0 ± 0.6	< 0.66	$\Xi_b(6090)^-$		$\cdots$	$\cdots$	
$\Lambda_b(5920)^0$	$* * *$	5919.8 ± 0.8	< 0.63	$\Xi_b(6100)^-$		$\cdots$	$\cdots$	
$\Sigma_b(5815)^-$	$* * *$	5815.5 ± 1.8	$4.9^{+3.3}_{-2.4}$	$\Xi_b'(5935)^-$	$**$	5935.02 ± 0.53	< 0.08	120.4 ± 3.0
$\Sigma_b^*(5835)^-$	$* * *$	5835.1 ± 1.9	7.5 ± 2.3	$\Xi_b^*(5955)^-$	$* * *$	5955.33 ± 0.68	1.65 ± 0.41	120.4 ± 3.0
$\Sigma_b(6097)^-$		6098.0 ± 2.2	28.9 ± 5.1	$\Xi_b(6227)^-$		6226.9 ± 2.5	18.1 ± 7.2	128.9 ± 2.4

Experimental and theoretical foundation for the diquark picture



PHYSICAL REVIEW D **97,** 072005 (2018)

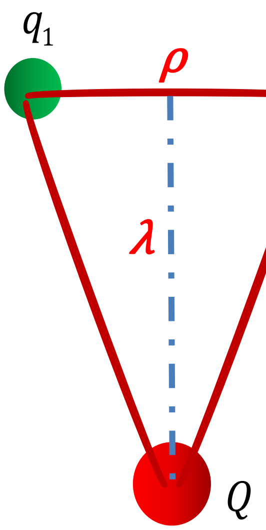
Production cross sections of hyperons and charmed baryons from e^+e^- annihilation near $\sqrt{s} = 10.52$ GeV

(Belle Collaboration)

This observation supports the theory that the diquark production is the main process of charmed baryon production from e^+e^- annihilation and that the diquark structure exists in the ground state and low-lying excited states of Λ_c^+ baryons.

Experimental and theoretical foundation for the diquark picture

If we assume that two light quarks in the charmed baryon systems develop into a quark cluster, then the mass differences between Ξ_c and Λ_c states can be understood well. In the quark cluster picture (or diquark picture), the degree of freedom of two light quarks in a diquark system is frozen. Only the degree of freedom between the center of mass of two light quarks and the c quark can be excited.



$$H_0 = \sum_{i=1}^{3} \frac{p_i^2}{2m_i} + \sum_{i<j} \left(\frac{1}{2} \kappa r_{ij}^2 + U(r_{ij}) \right) - \frac{\sum_i p_i^2}{2M}$$

In terms of Jacobi relative coordinates

$$\vec{\rho} = \frac{1}{\sqrt{2}} (\vec{r}_2 - \vec{r}_1); \quad \vec{\lambda} = \frac{1}{\sqrt{6}} (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)$$

$$H_0 = \frac{p_\rho^2}{2m_\rho} + \frac{p_\lambda^2}{2m_\lambda} + \frac{1}{2} m_\rho \omega_\rho^2 \rho^2 + \frac{1}{2} m_\lambda \omega_\lambda^2 \lambda^2 + \langle U \rangle$$

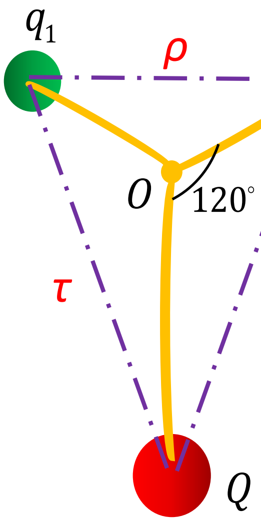
$$E_{\rho,\lambda} = \left(2n_\rho + l_\rho + \frac{3}{2} \right) \omega_\rho + \left(2n_\lambda + l_\lambda + \frac{3}{2} \right) \omega_\lambda$$

Experimental and theoretical foundation for the diquark picture

Since the oscillator frequencies ω_ρ and ω_λ are different in the charmed baryons ($m_Q \gg m_q$)

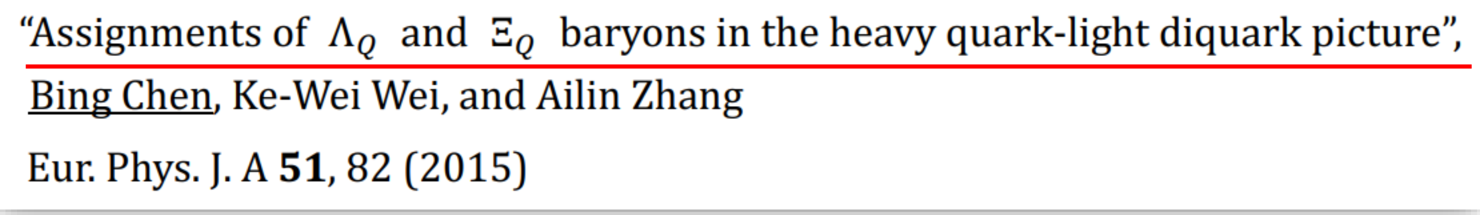
$$\frac{\omega_\lambda}{\omega_\rho} = \sqrt{\frac{1}{3}\left(1 + \frac{2m_q}{m_Q}\right)} < 1$$

which indicates that the λ mode excited charmed baryons would have lower excited energies.



$$\left[\begin{array}{l} -\frac{\nabla_\rho^2}{2m_\rho} + \left(-\frac{2}{3}\frac{\alpha}{\rho} + \sqrt{\frac{3}{4}}\, b\rho\right) + \frac{16\pi\alpha}{9m^2}\vec{s}_1\cdot\vec{s}_2\,\delta^3(\vec{\rho}) \\ -\frac{\nabla_\lambda^2}{2m_\lambda} + \left(-\frac{4}{3}\frac{\alpha}{\sqrt{\lambda^2+\frac{\rho^2}{4}}} + b\lambda\right) + \frac{16\pi\alpha}{9mm_Q}\vec{s}_{di}\cdot\vec{s}_Q\,\delta^3(\vec{\tau}) \end{array}\right]\Psi(\vec{\rho},\vec{\lambda}) = E\Psi(\vec{\rho},\vec{\lambda})$$

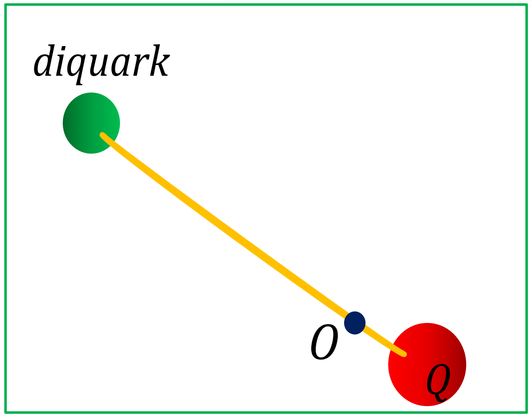
Masses and decays of charmed baryons in diquark picture



“Assignments of Λ_Q and Ξ_Q baryons in the heavy quark-light diquark picture”,

Bing Chen, Ke-Wei Wei, and Ailin Zhang

Eur. Phys. J. A **51**, 82 (2015)



In the relativistic flux tube (RFT) model, Selem and Wilczek have obtained the following mass formula,

$$E = M + \sqrt{\frac{\sigma L}{2}} + 2^{1/4}\kappa L^{-1/4}\mu^{3/2}$$

A. Selem, F. Wilczek, arXiv:hep-ph/ 0602128

We have extended equation above to the radial excited heavy baryons as

Chew-Frautschi formula

$$\left(\varepsilon_{nL} - m_Q\right)^2 = \frac{1}{2}\sigma(\kappa n + L) + \left(m_l + \zeta_Q\right)^2$$

where $\zeta_Q = m_Q u_1^2$。

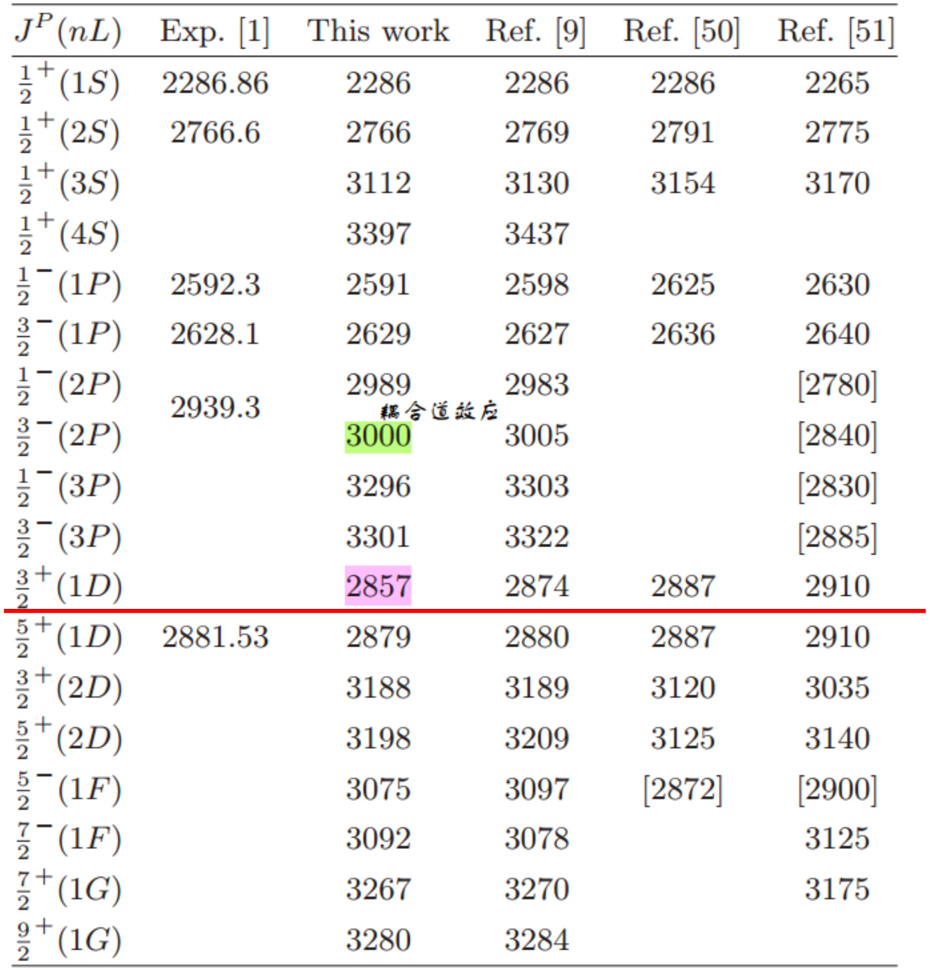
Finally, we added the $\vec{s}_Q \cdot \vec{L}$ couplings term as

$$H_{nL}^{so} = \frac{1}{3\times 2^{5/2}} \frac{\alpha_s}{(u_1+u_2)^3} \left(\frac{\sigma}{\kappa n+L}\right)^{3/2} \frac{1}{m_l m_Q} \vec{s}_Q \cdot \vec{L}$$

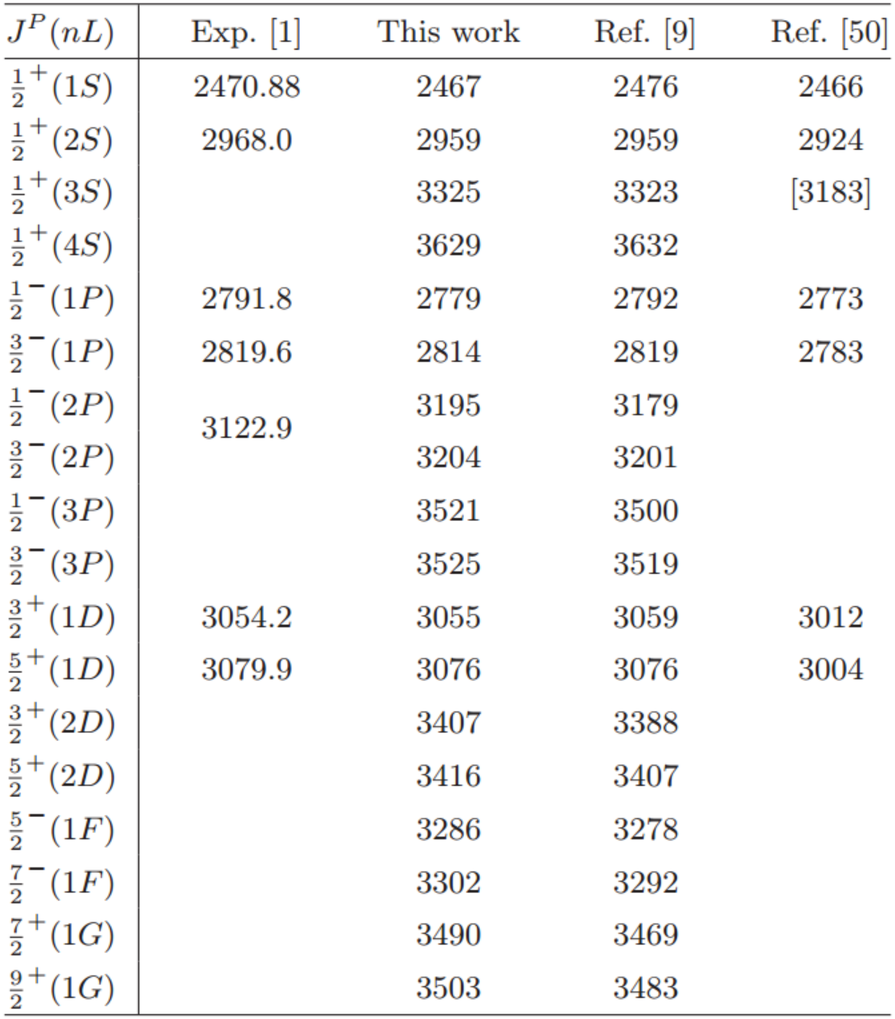
Masses and decays of charmed baryons in diquark picture

Results of the excited Λ_c^+ and Ξ_c excited states:

Bing Chen, Ke-Wei Wei, Ailin Zhang,
Eur.Phys.J. A **51** (2015) 82



$J^P(nL)$	Exp. [1]	This work	Ref. [9]	Ref. [50]	Ref. [51]
$\frac{1}{2}^+(1S)$	2286.86	2286	2286	2286	2265
$\frac{1}{2}^+(2S)$	2766.6	2766	2769	2791	2775
$\frac{1}{2}^+(3S)$		3112	3130	3154	3170
$\frac{1}{2}^+(4S)$		3397	3437		
$\frac{1}{2}^-(1P)$	2592.3	2591	2598	2625	2630
$\frac{3}{2}^-(1P)$	2628.1	2629	2627	2636	2640
$\frac{1}{2}^-(2P)$	2939.3	2989	2983		[2780]
$\frac{3}{2}^-(2P)$		耦合道效应 3000	3005		[2840]
$\frac{1}{2}^-(3P)$		3296	3303		[2830]
$\frac{3}{2}^-(3P)$		3301	3322		[2885]
$\frac{3}{2}^+(1D)$		2857	2874	2887	2910
$\frac{5}{2}^+(1D)$	2881.53	2879	2880	2887	2910
$\frac{3}{2}^+(2D)$		3188	3189	3120	3035
$\frac{5}{2}^+(2D)$		3198	3209	3125	3140
$\frac{5}{2}^-(1F)$		3075	3097	[2872]	[2900]
$\frac{7}{2}^-(1F)$		3092	3078		3125
$\frac{7}{2}^+(1G)$		3267	3270		3175
$\frac{9}{2}^+(1G)$		3280	3284		



$J^P(nL)$	Exp. [1]	This work	Ref. [9]	Ref. [50]
$\frac{1}{2}^+(1S)$	2470.88	2467	2476	2466
$\frac{1}{2}^+(2S)$	2968.0	2959	2959	2924
$\frac{1}{2}^+(3S)$		3325	3323	[3183]
$\frac{1}{2}^+(4S)$		3629	3632	
$\frac{1}{2}^-(1P)$	2791.8	2779	2792	2773
$\frac{3}{2}^-(1P)$	2819.6	2814	2819	2783
$\frac{1}{2}^-(2P)$	3122.9	3195	3179	
$\frac{3}{2}^-(2P)$		3204	3201	
$\frac{1}{2}^-(3P)$		3521	3500	
$\frac{3}{2}^-(3P)$		3525	3519	
$\frac{3}{2}^+(1D)$	3054.2	3055	3059	3012
$\frac{5}{2}^+(1D)$	3079.9	3076	3076	3004
$\frac{3}{2}^+(2D)$		3407	3388	
$\frac{5}{2}^+(2D)$		3416	3407	
$\frac{5}{2}^-(1F)$		3286	3278	
$\frac{7}{2}^-(1F)$		3302	3292	
$\frac{7}{2}^+(1G)$		3490	3469	
$\frac{9}{2}^+(1G)$		3503	3483	

Masses and decays of charmed baryons in diquark picture

LHCb Collaboration (R. Aaij (CERN) *et al.*), JHEP 1705 (2017) 030

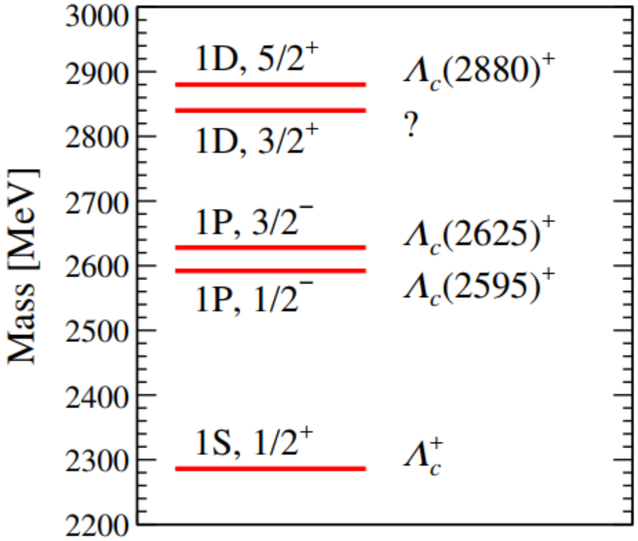


Figure 1. Expected spectrum of the Λ_c^+ ground state and its orbital excitations from a study based on the nonrelativistic heavy quark-light diquark model [21], along with the observed resonances corresponding to those states [23].

$\frac{3}{2}^{+}$

$$m(\Lambda_c(2860)^+) = 2856.1^{+2.0}_{-1.7}(\text{stat}) \pm 0.5(\text{syst})^{+1.1}_{-5.6}(\text{model})\ \text{MeV},$$

$$\Gamma(\Lambda_c(2860)^+) = 67.6^{+10.1}_{-8.1}(\text{stat}) \pm 1.4(\text{syst})^{+5.9}_{-20.0}(\text{model})\ \text{MeV}$$

The assignments of singly heavy baryons in diquark picture

Bing Chen, Ke-Wei Wei, Xiang Liu, Takayuki Matsuki, Eur. Phys. J. C 77, 154 (2017)
Bing Chen, Xiang Liu, and Ailin Zhang, Phys. Rev. D 95, 074022 (2017)
Bing Chen and Xiang Liu, Phys. Rev. D 95, 074022 (2017)

doublet	s_l^P	J^P	Λ_c (cud)	Ξ_c (csq)	Λ_b (bud)	Ξ_b (bsq)	comment
1S	0^+	$1/2^+$	$\Lambda_c(2286)$	$\Xi_c(2470)$	$\Lambda_b(5620)$	$\Xi_b(5795)$	****
2S	0^+	$1/2^+$	$\Lambda_c(2760)$	$\Xi_c(2980)$			*
1P	1^-	$1/2^-$	$\Lambda_c(2595)$	$\Xi_c(2790)$	$\Lambda_b(5912)$		***
		$3/2^-$	$\Lambda_c(2625)$	$\Xi_c(2815)$	$\Lambda_b(5920)$		
1D	2^+	$3/2^+$	$\Lambda_c(2860)$	$\Xi_c(3055)$			**~***
		$5/2^+$	$\Lambda_c(2880)$	$\Xi_c(3080)$			
2P	1^-	$1/2^-$					*~**
		$3/2^-$	$\Lambda_c(2940)$	$\Xi_c(3123)$			

The assignments of singly heavy baryons in diquark picture

Bing Chen, Ke-Wei Wei, Xiang Liu, Takayuki Matsuki, Eur. Phys. J. C 77, 154 (2017)
Bing Chen, Xiang Liu, and Ailin Zhang, Phys. Rev. D 95, 074022 (2017)
Bing Chen and Xiang Liu, Phys. Rev. D 95, 074022 (2017)

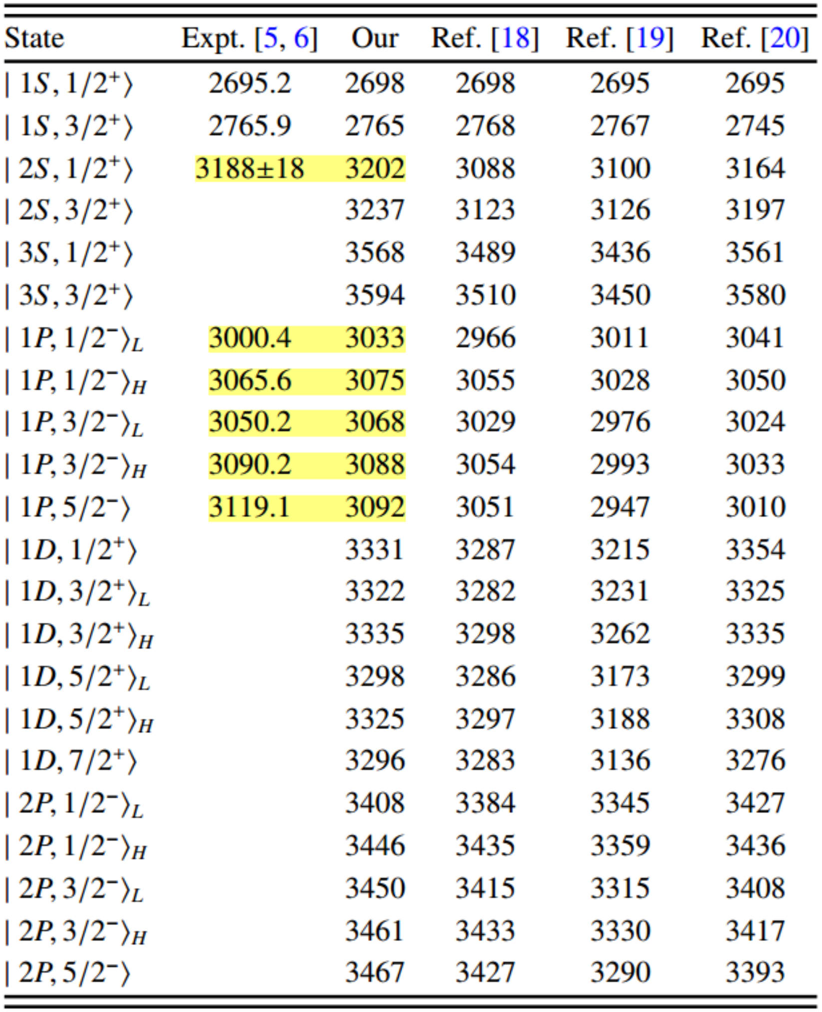
For bottom baryons

Bing Chen, Ke-wei Wei, Xiang Liu, and Ailin Zhang, Phys. Rev. D 98, 031502 (2018)
Bing Chen and Xiang Liu, Phys. Rev. D 98, 074032 (2018)

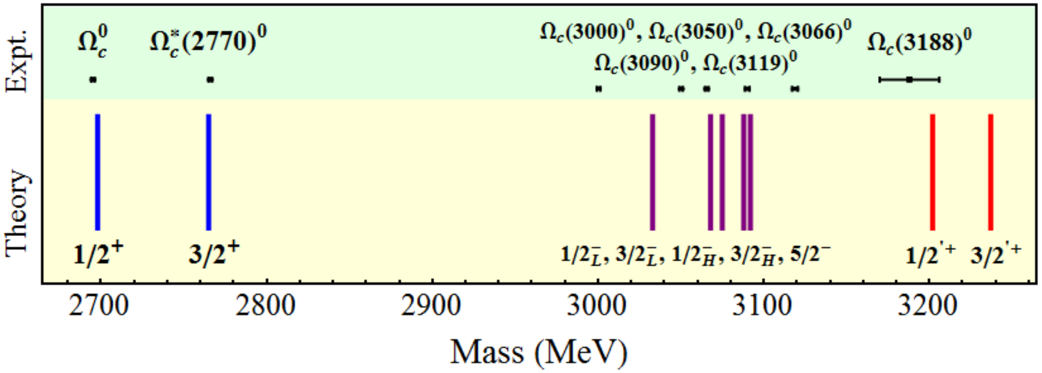
n*L*	**1*S***		**1*P***					**2*S***	
J^P	$1/2^+$	$3/2^+$	$1/2^-$	$1/2'^-$	$3/2^-$	$3/2'^-$	$5/2^-$	$1/2^+$	$3/2^+$
Σ_c	$\Sigma_c(2455)$	$\Sigma_c(2520)$				$\Sigma_c(2800)$			
Ξ'_c	$\Xi'_c(2570)$	$\Xi'_c(2645)$				$\Xi'_c(2930)$			
Ω_c	$\Omega_c(2700)$	$\Omega_c(2760)$	$\Omega_c(3000)$	$\Omega_c(3090)$	$\Omega_c(3050)$	$\Omega_c(3119)$	$\Omega_c(3066)$	$\Omega_c(3188)$	
Σ_b	$\Sigma_b(5815)$	$\Sigma_b(5835)$				**$\Sigma_b(6097)^\pm$**			
Ξ'_b	$\Xi'_b(5935)$	$\Xi'_b(5955)$				**$\Xi'_b(6227)^-$**			
Ω_b	$\Omega_b(6045)$						$\Omega_b(6360)^?$		

Explanation of the five narrow Ω_c states

Bing Chen and Xiang Liu, Phys. Rev. D 96, 094015 (2017)



State	Expt. [5, 6]	Our	Ref. [18]	Ref. [19]	Ref. [20]
$\vert 1S,1/2^+\rangle$	2695.2	2698	2698	2695	2695
$\vert 1S,3/2^+\rangle$	2765.9	2765	2768	2767	2745
$\vert 2S,1/2^+\rangle$	3188±18	3202	3088	3100	3164
$\vert 2S,3/2^+\rangle$		3237	3123	3126	3197
$\vert 3S,1/2^+\rangle$		3568	3489	3436	3561
$\vert 3S,3/2^+\rangle$		3594	3510	3450	3580
$\vert 1P,1/2^-\rangle_L$	3000.4	3033	2966	3011	3041
$\vert 1P,1/2^-\rangle_H$	3065.6	3075	3055	3028	3050
$\vert 1P,3/2^-\rangle_L$	3050.2	3068	3029	2976	3024
$\vert 1P,3/2^-\rangle_H$	3090.2	3088	3054	2993	3033
$\vert 1P,5/2^-\rangle$	3119.1	3092	3051	2947	3010
$\vert 1D,1/2^+\rangle$		3331	3287	3215	3354
$\vert 1D,3/2^+\rangle_L$		3322	3282	3231	3325
$\vert 1D,3/2^+\rangle_H$		3335	3298	3262	3335
$\vert 1D,5/2^+\rangle_L$		3298	3286	3173	3299
$\vert 1D,5/2^+\rangle_H$		3325	3297	3188	3308
$\vert 1D,7/2^+\rangle$		3296	3283	3136	3276
$\vert 2P,1/2^-\rangle_L$		3408	3384	3345	3427
$\vert 2P,1/2^-\rangle_H$		3446	3435	3359	3436
$\vert 2P,3/2^-\rangle_L$		3450	3415	3315	3408
$\vert 2P,3/2^-\rangle_H$		3461	3433	3330	3417
$\vert 2P,5/2^-\rangle$		3467	3427	3290	3393



Mass:

$$\begin{pmatrix} \vert\Omega_c(3033)\rangle \\ \vert\Omega_c(3075)\rangle \end{pmatrix} = \begin{pmatrix} \mathrm{Cos}158^\circ & -\mathrm{Sin}158^\circ \\ \mathrm{Sin}158^\circ & \mathrm{Cos}158^\circ \end{pmatrix} \begin{pmatrix} \Omega_c\vert 0,1/2^-\rangle \\ \Omega_c\vert 1,1/2^-\rangle \end{pmatrix}$$

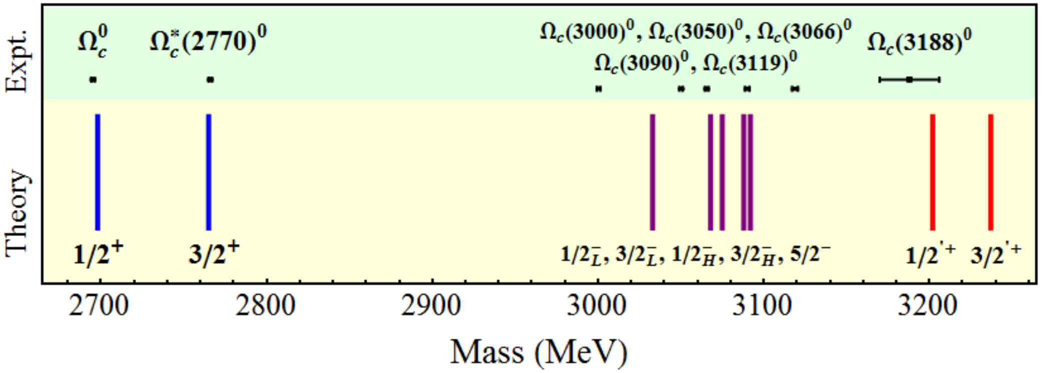
Decay:

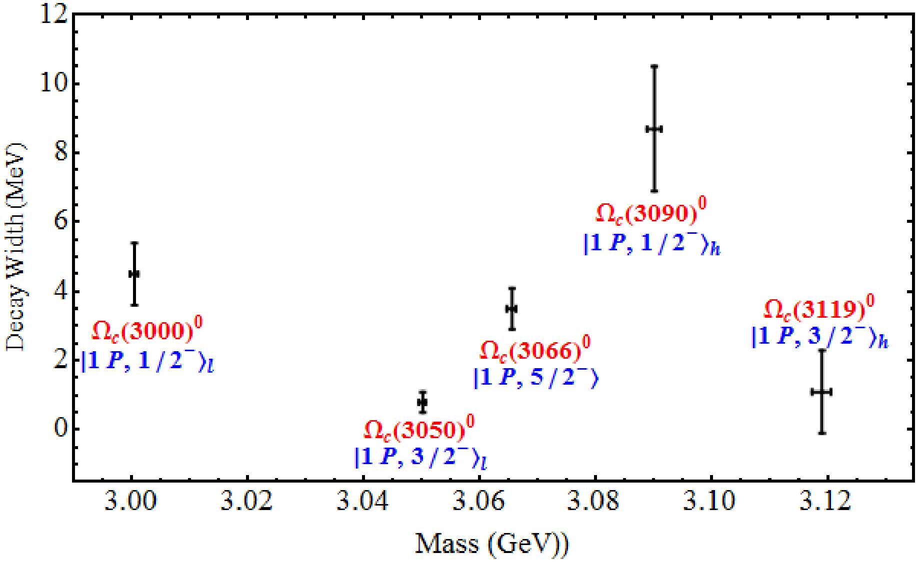
$$\begin{pmatrix} \vert\Omega_c(3000)\rangle \\ \vert\Omega_c(3090)\rangle \end{pmatrix} = \begin{pmatrix} \mathrm{Cos}116.3^\circ & -\mathrm{Sin}116.3^\circ \\ \mathrm{Sin}116.3^\circ & \mathrm{Cos}116.3^\circ \end{pmatrix} \begin{pmatrix} \Omega_c\vert 0,1/2^-\rangle \\ \Omega_c\vert 1,1/2^-\rangle \end{pmatrix}$$

Explanation of the five narrow Ω_c states

Bing Chen and Xiang Liu, Phys. Rev. D 96, 094015 (2017)

State	Expt. [5, 6]	Our	Ref. [18]	Ref. [19]	Ref. [20]
$\|1S, 1/2^+\rangle$	2695.2	2698	2698	2695	2695
$\|1S, 3/2^+\rangle$	2765.9	2765	2768	2767	2745
$\|2S, 1/2^+\rangle$	3188±18	3202	3088	3100	3164
$\|2S, 3/2^+\rangle$		3237	3123	3126	3197
$\|3S, 1/2^+\rangle$		3568	3489	3436	3561
$\|3S, 3/2^+\rangle$		3594	3510	3450	3580
$\|1P, 1/2^-\rangle_L$	3000.4	3033	2966	3011	3041
$\|1P, 1/2^-\rangle_H$	3065.6	3075	3055	3028	3050
$\|1P, 3/2^-\rangle_L$	3050.2	3068	3029	2976	3024
$\|1P, 3/2^-\rangle_H$	3090.2	3088	3054	2993	3033
$\|1P, 5/2^-\rangle$	3119.1	3092	3051	2947	3010
$\|1D, 1/2^+\rangle$		3331	3287	3215	3354
$\|1D, 3/2^+\rangle_L$		3322	3282	3231	3325
$\|1D, 3/2^+\rangle_H$		3335	3298	3262	3335
$\|1D, 5/2^+\rangle_L$		3298	3286	3173	3299
$\|1D, 5/2^+\rangle_H$		3325	3297	3188	3308
$\|1D, 7/2^+\rangle$		3296	3283	3136	3276
$\|2P, 1/2^-\rangle_L$		3408	3384	3345	3427
$\|2P, 1/2^-\rangle_H$		3446	3435	3359	3436
$\|2P, 3/2^-\rangle_L$		3450	3415	3315	3408
$\|2P, 3/2^-\rangle_H$		3461	3433	3330	3417
$\|2P, 5/2^-\rangle$		3467	3427	3290	3393





Explanation of the newly discovered bottom baryon: $\Xi_b(6227)^-$

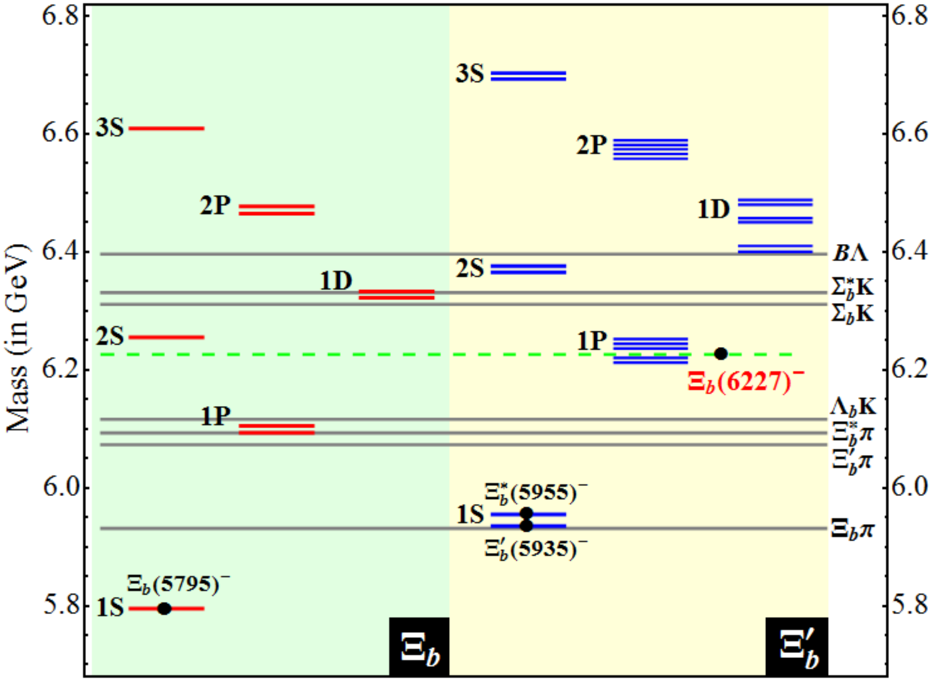
$\Xi_b(6227)^-$

Bing Chen, Ke-wei Wei, Xiang Liu, and Ailin Zhang, Phys. Rev. D 98, 031502 (2018)

Phys. Rev. Lett. ,**121**, 072002 (2018)

$$M = 6226.9 \pm 2.0(\mathrm{stat}) \pm 0.3(\mathrm{syst}) \pm 0.2(\Lambda_b^0)\ \mathrm{MeV},$$
$$\Gamma = 18.1 \pm 5.4(\mathrm{stat}) \pm 1.8(\mathrm{syst})\ \mathrm{MeV}.$$

Decay channels	Prediction	Experiments [PDG]
$\Sigma_b(5815)^- \to \Lambda_b^0\pi^-$	5.12	$4.9^{+3.3}_{-2.4}$
$\Sigma_b(5835)^- \to \Lambda_b^0\pi^-$	9.13	7.5 ± 2.3
$\Xi_b'(5935)^- \to \Xi_b\pi$	0.05	< 0.08, CL=95%
$\Xi_b^*(5955)^- \to \Xi_b\pi$	1.09	1.65 ± 0.33



Decay	$1/2^-$		$3/2^-$		$5/2^-$
modes	$\Xi_{b0}'(6249)$	$\Xi_{b1}'(6239)$	$\Xi_{b1}'(6244)$	$\Xi_{b2}'(6213)$	$\Xi_{b2}'(6217)$
$\Lambda_b K$	9.1	×	×	10.2	11.0
$\Xi_b\pi$	0.2	×	×	11.4	11.7
$\Xi_b'(5935)\pi$	×	15.1	0.9	1.0	0.5
$\Xi_b^*(5955)\pi$	×	2.0	23.7	1.0	1.7
$\Xi_b(6096)\pi$	0.3	0.1	0.1	–	–
$\Xi_b(6102)\pi$	0.3	–	0.1	–	–
Theory	9.9	17.2	24.8	23.6	24.9
Expt. [9]				$18.1 \pm 5.4 \pm 1.8$	

Explanation of the newly discovered bottom baryon: $\Sigma_b(6097)^\pm$

$\Sigma_b(6097)^\pm$

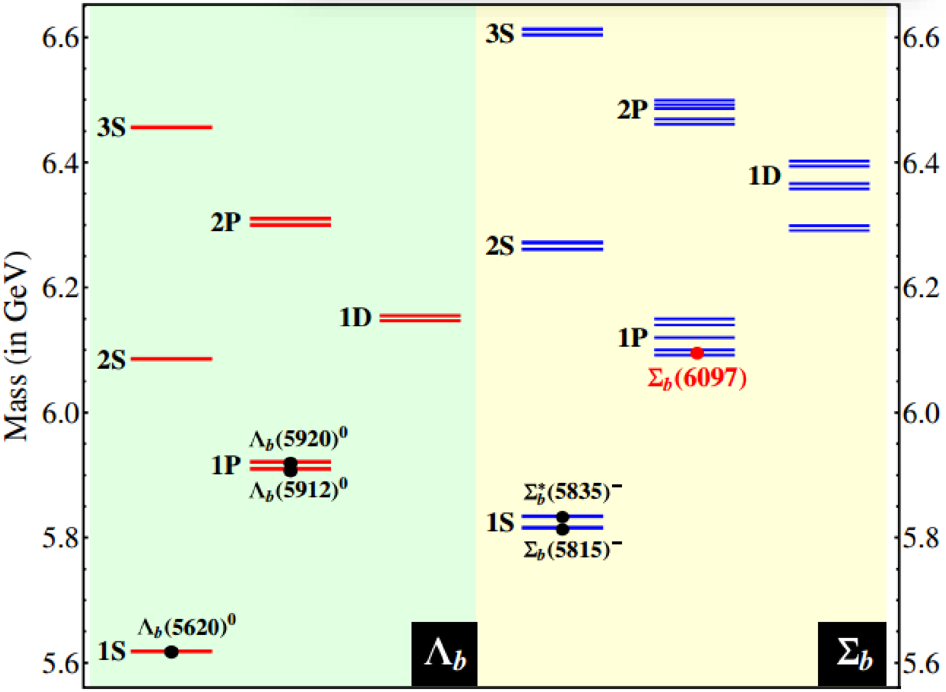
Bing Chen and Xiang Liu, Phys. Rev. D 98, 031502 (2018)

arXiv:1809.07752, Submitted to Phys. Rev. Lett.

$\Sigma_b(6097)^-$:

$M = 6098.0 \pm 1.7 \pm 0.5$ MeV;

$\Gamma = 28.9 \pm 4.2 \pm 0.9$ MeV.



Decay modes	$1/2^-$		$3/2^-$		$5/2^-$
	$\Omega_{b0}(6361)$	$\Omega_{b1}(6352)$	$\Omega_{b1}(6363)$	$\Omega_{b2}(6344)$	$\Omega_{b2}(6349)$
$\Xi_b K$	33.4	×	×	2.3	2.7

Decay modes	$1/2^-$		$3/2^-$		$5/2^-$
	$\Sigma_{b0}(6150)$	$\Sigma_{b1}(6134)$	$\Sigma_{b1}(6139)$	$\Sigma_{b2}(6094)$	$\Sigma_{b2}(6098)$
$\Lambda_b(5620)\pi$	10.4	×	×	35.2	35.8
$\Sigma_b(5815)\pi$	×	83.8	4.5	3.7	1.8
$\Sigma_b^*(5835)\pi$	×	6.6	90.8	2.6	4.4
$\Lambda_b(5912)\pi$	5.9	13.3	3.3	0.7	0.0
$\Lambda_b(5920)\pi$	10.9	5.6	15.8	0.1	0.8
Theory	27.2	109.3	114.4	42.3	42.8
Expt. [1]				$31.0 \pm 5.5 \pm 0.7$	

The measured branching ratios

Belle [2007]
Phys. Rev. Lett. 98, 262001 (2007)

$$\frac{B(\Lambda_c(2880)\to\Sigma_c(2520)\pi)}{B(\Lambda_c(2880)\to\Sigma_c(2455)\pi)} = \mathbf{0.225} \pm 0.062 \pm 0.025;$$

BABAR [2008]
Phys. Rev. D **77**, 012002 (2008)

$$\frac{B(\Xi_c(3080)\to\Sigma_c(2520)K)}{B(\Xi_c(3080)\to\Sigma_c(2455)K)} = \frac{0.55\pm0.05\pm0.05}{0.45\pm0.05\pm0.05} = 0.82 \sim 1.86;$$

Belle [2016]
Phys. Rev. D **94**, 032002 (2016)

$$\frac{B(\Xi_c(3080)\to\Sigma_c(2520)K)}{B(\Xi_c(3080)\to\Sigma_c(2455)K)} = 1.07 \pm 0.27 \pm 0.04;$$

$$\frac{B(\Xi_c(3080)\to\Lambda D\ )}{B(\Xi_c(3080)\to\Sigma_c(2455)K)} = 1.29 \pm 0.30 \pm 0.15;$$

$$\frac{B(\Xi_c(3055)\to\Lambda D\ )}{B(\Xi_c(3055)\to\Sigma_c(2455)K)} = 5.09 \pm 1.01 \pm 0.76;$$

Hai-Yang Cheng, Chun-Khiang Chua, Phys.Rev. D **75** (2007) 014006
Bing Chen, Xiang Liu, and Ailin Zhang, Phys.Rev. D **95**, 074022 (2017)
Ya-Xiong Yao, Kai-Lei Wang, Xian-Hui Zhong, arXiv:1803.00364

A short summary : success and difficulty

$\Lambda_c(2940)^+$

states	Expt.	CWLM	EFG	CWZ	CI
$\|2P, 1/2^- \rangle$	——	2980	2983	2989	3030
$\|2P, 3/2^- \rangle$	2939.3	**3004**	**3005**	**3000**	**3035**

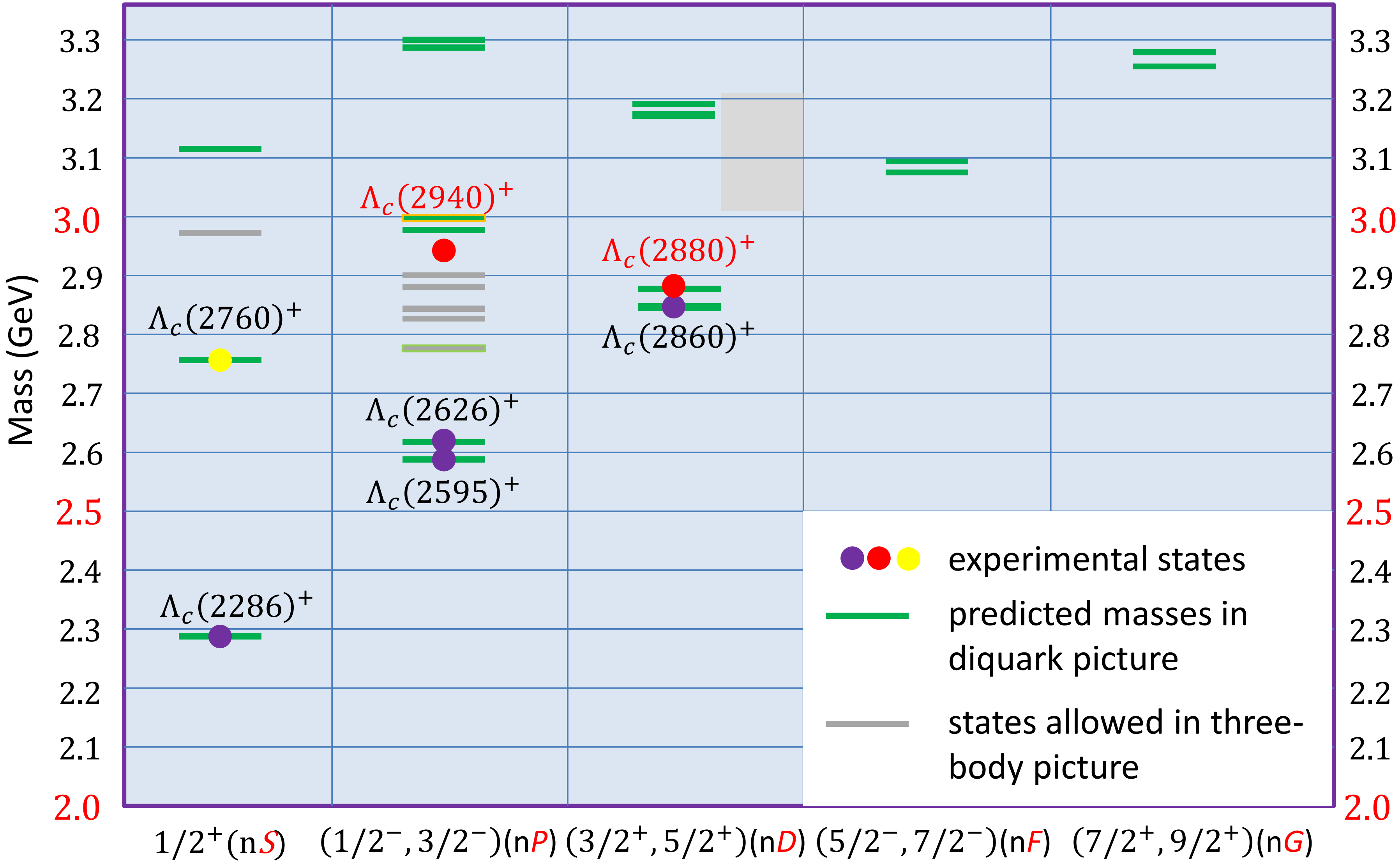
Mass (in MeV)

Some thresholds in an S wave plays an important dynamical role in making the $|2P, 3/2^- \rangle$ state a notable mass shift.

$D^* p$	$D^* n$
2950	**2950**

So the coupled-channel effects should be considered for the $2P$ Λ_c^+states.

A short summary : success and difficulty



A short summary: success and difficulty

In the heavy quark-light diquark picture, most of the observed charmed baryon states can be explained including their mass spectrum and strong decays. As discussed above, however, there are some questions which should be investigated deeply.

(1) The branching ratios of $\Lambda_c(2880)^+$and $\Xi_c(3080)$ pose challenges to the diquark picture,

(2) The coupled-channel effects should be studied for the 2*P* Λ_c^+ states,

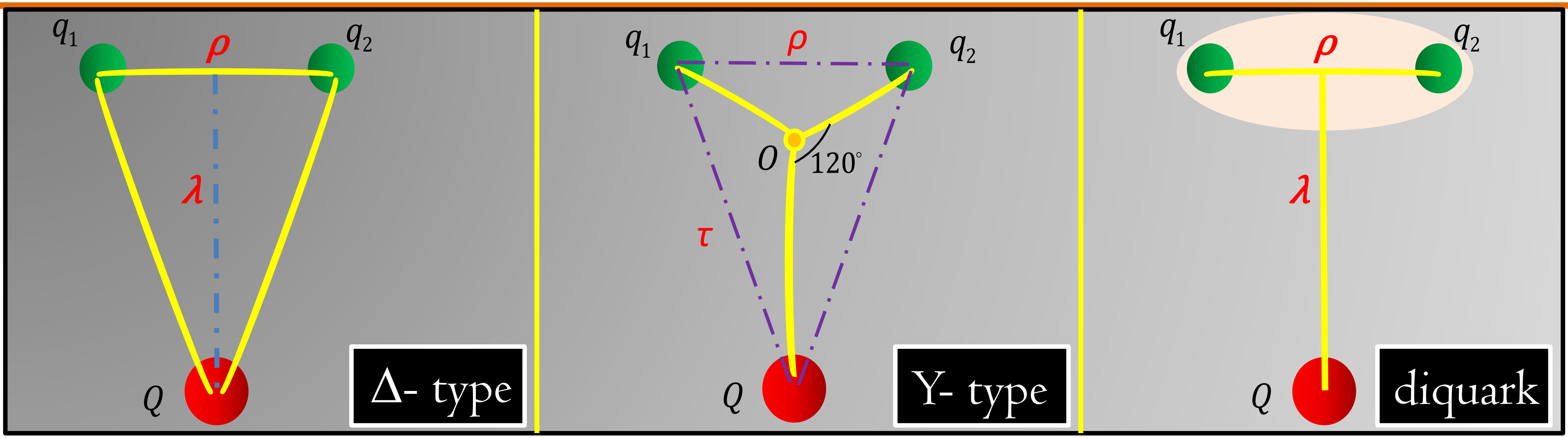
(3) The spin-parity of $\Lambda_c(2760)^+$ [or a $\Sigma_c(2760)^+$ state] and $\Xi_c^{(\prime)}(2980)$ should be measured,

(4) We may ask why no ρ-mode excited heavy baryon state has been detected by any experiment. If they exist, how are we going to find them?

Section III

Study the masses of heavy baryon in three-body picture

Different confinement mechanism

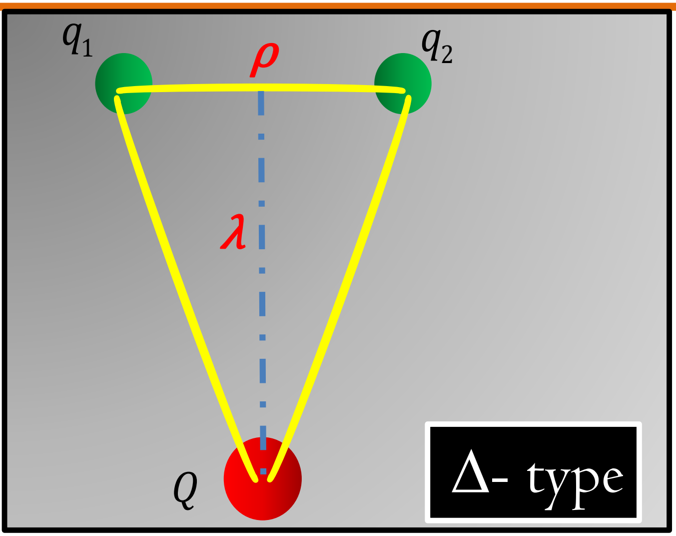


1. “Heavy baryons in a quark model”,
   W. Roberts and M. Pervin, Int. J. Mod. Phys. A **23**, 2817 (2008).

2. “Baryons in a Relativized Quark Model with Chromodynamics”,
   S. Capstick and N. Isgur, Phys. Rev. D **34**, 2809 (1986)

3. “Precise determination of the three-quark potential in SU(3) lattice gauge”,
   Y. Koma and M. Koma, Phys. Rev. D **95** , 094513 (2017).

A baryon system in the harmonic oscillator potential



The Hamiltonian could be written as

$$\hat{H}=\underbrace{\frac{p_1^2}{2m_1}+\frac{p_2^2}{2m_2}+\frac{p_3^2}{2m_3}+\sum_{i=1,i<j}^{3}\frac{1}{2}K\left(\vec{r}_i-\vec{r}_j\right)^2}_{\widehat{H}_0}+\sum_{i=1,i<j}^{2}g\left(r_{ij}\right)\times\hat{H}\left[\vec{S},\vec{L}\right]$$

In the Jacobi coordinates, we may define

$$\vec{\rho}=\vec{r}_1-\vec{r}_2;\qquad \vec{\lambda}=\frac{m_1\vec{r}_1+m_2\vec{r}_2}{m_1+m_2}-\vec{r}_3;\qquad \vec{R}=\frac{m_1\vec{r}_1+m_2\vec{r}_2+m_3\vec{r}_3}{m_1+m_2+m_3};$$

Then the part of $\widehat{H}_0$ could be reduced as

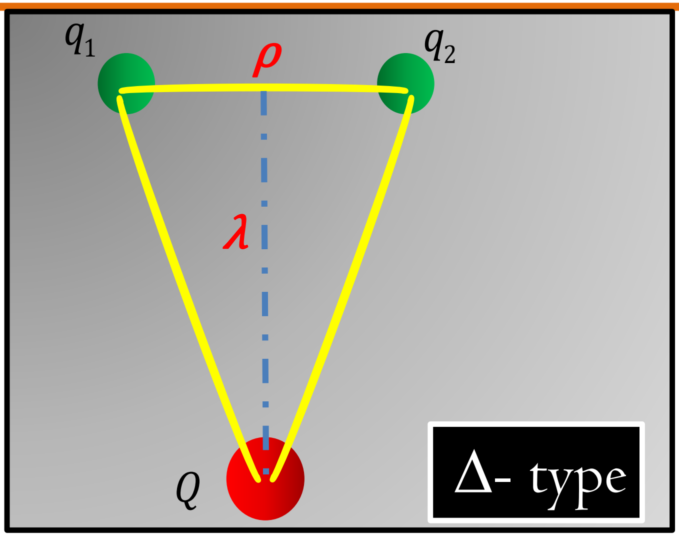
$$\hat{H}_0=\sum_{i=1}^{3}\frac{p_i^2}{2m_i}+\sum_{i=1,i<j}^{3}\frac{1}{2}K\left(\vec{r}_i-\vec{r}_j\right)^2=\frac{p_\rho^2}{2m_\rho}+\frac{p_\lambda^2}{2m_\lambda}+\frac{P^2}{2M}+K\left[1-\frac{m_1m_2}{\left(m_1+m_2\right)^2}\right]\rho^2+K\frac{m_2-m_1}{m_1+m_2}\vec{\rho}\cdot\vec{\lambda}+K\lambda^2$$

where

$$m_\rho=\frac{m_1m_2}{m_1+m_2};\qquad m_\lambda=\frac{m_3\left(m_1+m_2\right)}{m_1+m_2+m_3};\qquad M=m_1+m_2+m_3;$$

$$\vec{p}_\rho=\frac{m_2}{m_1+m_2}\vec{p}_1-\frac{m_1}{m_1+m_2}\vec{p}_2;\qquad \vec{p}_\lambda=\frac{m_3}{m_1+m_2+m_3}\vec{p}_{12}-\frac{m_1+m_2}{m_1+m_2+m_3}\vec{p}_3;\qquad \vec{P}=\vec{p}_1+\vec{p}_2+\vec{p}_3.$$

A baryon system contains at least two identical quarks



The Hamiltonian could be written as

$$\hat{H}=\underbrace{\frac{p_1^2}{2m_1}+\frac{p_2^2}{2m_2}+\frac{p_3^2}{2m_3}+\sum_{i=1,i<j}^{3}\frac{1}{2}K\left(\vec{r}_i-\vec{r}_j\right)^2}_{\hat{H}_0}+\sum_{i=1,i<j}^{2}g\left(r_{ij}\right)\times\hat{H}\left[\vec{S},\vec{L}\right]$$

Now, we define

$$\vec{\rho}=\frac{1}{\sqrt{2}}\left(\vec{r}_1-\vec{r}_2\right);\qquad \vec{\lambda}=\frac{1}{\sqrt{6}}\left(\vec{r}_1+\vec{r}_2-2\vec{r}_3\right);\qquad \vec{R}=\frac{m\vec{r}_1+m\vec{r}_2+\mu\vec{r}_3}{2m+\mu};$$

Then the part of $\hat{H}_0$ could be reduced as

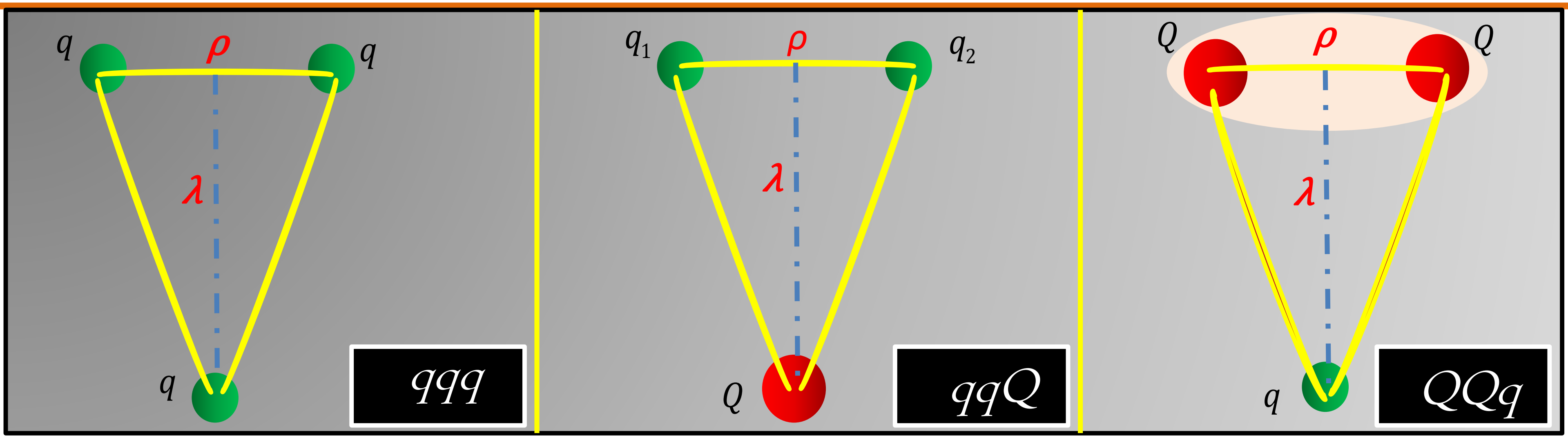
$$\hat{H}_0=\sum_{i=1}^{3}\frac{p_i^2}{2m_i}+\sum_{i=1,i<j}^{3}\frac{1}{2}K\left(\vec{r}_i-\vec{r}_j\right)^2=\frac{p_\rho^2}{2m}+\frac{p_\lambda^2}{2m_\lambda}+\frac{P^2}{2M}+\frac{3}{2}K\left(\rho^2+\lambda^2\right)$$

where

$$m_\lambda=\frac{3m\mu}{2m+\mu};\qquad M=2m+\mu;$$

$$\vec{p}_\rho=\frac{1}{\sqrt{2}}\left(\vec{p}_1-\vec{p}_2\right);\qquad \vec{p}_\lambda=\sqrt{\frac{3}{2}}\frac{\mu}{2m+\mu}\vec{p}_{12}-\sqrt{6}\frac{m}{2m+\mu}\vec{p}_3;\qquad \vec{P}=\vec{p}_1+\vec{p}_2+\vec{p}_3.$$

The frequencies of the ρ mode and λ mode



$$\frac{\omega_\rho}{\omega_\lambda} = \sqrt{\frac{\frac{3}{2}K/m}{\frac{3}{2}K/m_\lambda}} = \sqrt{\frac{m_\lambda}{m}} = \sqrt{\frac{3\mu}{2m+\mu}}$$

❶ qqq baryon

$\mu = m$

$\omega_\rho = \omega_\lambda$

❷ qqQ baryon

$\mu > m$

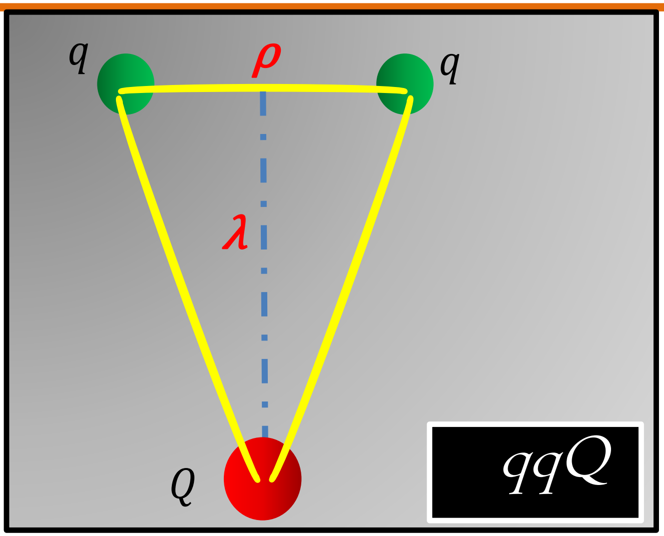
$\omega_\rho > \omega_\lambda$

❸ QQq baryon

$\mu < m$

$\omega_\rho < \omega_\lambda$

How to obtain the masses of singly heavy baryons in three-body picture



We try to solve the following Schrödinger equation

$$\hat{H}_0=\frac{p_1^2}{2m}+\frac{p_2^2}{2m}+\frac{p_3^2}{2\mu}+\sum_{i=1,i<j}^{3}\left(-\frac{\alpha}{r_{ij}}+br_{ij}\right)+C_{qqQ}$$

with

$$\vec{\rho}=\frac{1}{\sqrt{2}}\left(\vec{r}_1-\vec{r}_2\right);\qquad \vec{r}_1=\vec{R}+\frac{\vec{\rho}}{\sqrt{2}}+\sqrt{\frac{3}{2}}\frac{\mu}{2m+\mu}\vec{\lambda};$$

$$\vec{\lambda}=\frac{1}{\sqrt{6}}\left(\vec{r}_1+\vec{r}_2-2\vec{r}_3\right);\qquad \vec{r}_2=\vec{R}-\frac{\vec{\rho}}{\sqrt{2}}+\sqrt{\frac{3}{2}}\frac{\mu}{2m+\mu}\vec{\lambda};$$

$$\vec{R}=\frac{m\vec{r}_1+m\vec{r}_2+\mu\vec{r}_3}{2m+\mu};\qquad \vec{r}_3=\vec{R}-\sqrt{6}\frac{m}{2m+\mu}\vec{\lambda};$$

We have

$$\hat{H}_0=\frac{p_\rho^2}{2m}+\frac{p_\lambda^2}{2m_\lambda}+\left(-\frac{\alpha}{\sqrt{2}\rho}+\sqrt{2}\rho b\right)+\left(-\frac{4\alpha}{\sqrt{2\left(3\lambda^2+\rho^2\right)}}+\sqrt{2\left(3\lambda^2+\rho^2\right)}b\right)+C_{qqQ}^{'}$$

Comment

To solve the Schrödinger equation

$$\hat{H}_0 = \frac{p_\rho^2}{2m} + \frac{p_\lambda^2}{2m_\lambda} + \left(-\frac{\alpha}{\sqrt{2}\rho} + \sqrt{2}\rho b\right) + \left(-\frac{4\alpha}{\sqrt{2\left(3\lambda^2+\rho^2\right)}} + \sqrt{2\left(3\lambda^2+\rho^2\right)}\, b\right) + C'_{qqQ}$$

To solve the equation above, one may define the wave function of a heavy baryon state as follows

$$\Psi_{JM} = \sum_{M_L,m} \langle JM|LM_L, SM-M_L\rangle \langle LM_L|l_\rho m, l_\lambda M_L - m\rangle\, \psi^{n_\rho}_{l_\rho m}(\boldsymbol{\rho})\psi^{n_\lambda}_{l_\lambda m}(\boldsymbol{\lambda})\chi_S(M-M_L)$$

The Ψ_{JM} can be denoted in in abbreviation, i.e.,

$$\Psi_{JM} \equiv \left[\psi^{(n_\rho l_\rho)(n_\lambda l_\lambda)}_{LM_L}(\rho,\lambda)\chi_S(M-M_L)\right]_{JM} \equiv \left|\left(l_\rho \otimes l_\lambda\right)^{(n_\rho n_\lambda)}_L \otimes \left(s_\rho \otimes s_Q\right)_S\right\rangle_J$$

The harmonic oscillator wave function could be used to denote the spatial part of a heavy baryon state, i.e.,

$$\psi^n_{lm}(\mathbf{r}) = \alpha^{3/2}\sqrt{\frac{2(n-1)!}{\mathrm{Gamma}[n+l+\frac{1}{2}]}}\,(\alpha r)^l \mathrm{L}^{l+\frac{1}{2}}_{n-1}(\alpha^2 r^2)\, e^{-\frac{\alpha^2 r^2}{2}}\, Y_{lm}(\mathbf{r})$$

An example

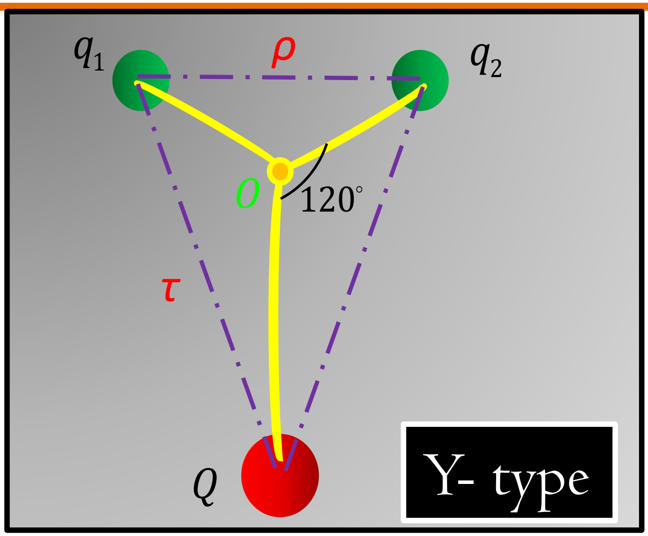
If we try to calculate the mass of $\Lambda_c(2286)^+$ by the method above, we should first construct the spin-space wave functions as following (see Roberts & Pervin, Int. J. Mod. Phys. A **23** (2008) 2817)

$$\begin{aligned}
\Psi^{\bar{3}}_{\frac{1}{2}^+ M} = & ([\eta_1 \overset{2.30}{\psi_{000000}}(\boldsymbol{\rho},\boldsymbol{\lambda}) + \eta_2 \overset{3.0}{\psi_{001000}}(\boldsymbol{\rho},\boldsymbol{\lambda}) + \eta_3 \overset{2.8}{\psi_{000010}}(\boldsymbol{\rho},\boldsymbol{\lambda})]\chi^{\rho}_{1/2}(M) \\
& + \eta_4 \overset{3.0\sim3.2}{\psi_{000101}}(\boldsymbol{\rho},\boldsymbol{\lambda})\chi^{\lambda}_{1/2}(M) + \eta_5[\psi_{1M_L0101}(\boldsymbol{\rho},\boldsymbol{\lambda})\chi^{S}_{3/2}(M-M_L)]_{1/2,M} \\
& + \eta_6[\psi_{1M_L0101}(\boldsymbol{\rho},\boldsymbol{\lambda})\chi^{\lambda}_{1/2}(M-M_L)]_{1/2,M} \\
& + \eta_7[\psi_{2M_L0101}(\boldsymbol{\rho},\boldsymbol{\lambda})\chi^{S}_{3/2}(M-M_L)]_{1/2,M}) \, .
\end{aligned} \tag{14}$$

If we want to calculate the masses of higher excited baryon states, we should construct a more complicated wave function;

The obtained wave function is very difficult to be used to calculate the strong decays.

"Y-type" confinement mechanism

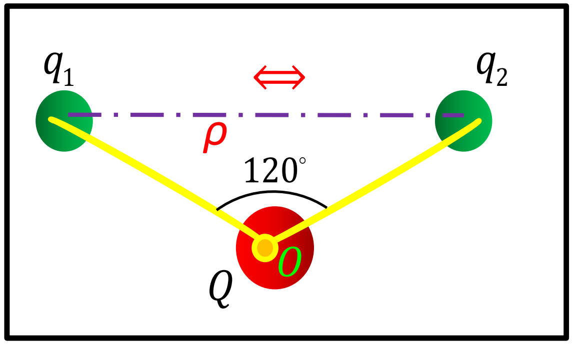


Here we take the work by Capstick and Isgur to discuss this question:

"Baryons in a Relativized Quark Model with Chromodynamics"
[Phys. Rev. D **34**, 2809 (1986)]

$$V_{q_1q_2q_3} = \sum_{i<j}\left(-\frac{2}{3}\frac{\alpha_s}{r_{ij}} + fbr_{ij}\right) + b\left(\sum_{i} r_{iO} - f\sum_{i<j} r_{ij}\right)$$

where the junction ***O*** should minimise the sum of r_{iO}.



$$V_{q_1q_2q_3} = \left(-\frac{2}{3}\frac{\alpha}{\rho} + \sqrt{\frac{3}{4}}b\rho\right) + \left(-\frac{4}{3}\frac{\alpha}{\sqrt{\lambda^2 + \frac{\rho^2}{4}}} + b\lambda\right)$$

Comment

Spin-dependent interactions with OGE mechanism

❶. The magnetic-dipole-magnetic-dipole interaction

$$\boldsymbol{V_{hyp}} = \sum_{i<j}^{3} \frac{2\alpha_s}{3m_i m_j}\left[\frac{8\pi}{3}\mathbf{s}_i \cdot \mathbf{s}_j \delta^3(\mathbf{r}_{ij}) + \frac{1}{r_{ij}^3}\left(\frac{\mathbf{s}_i \cdot \mathbf{r}_{ij}\mathbf{s}_j \cdot \mathbf{r}_{ij}}{r_{ij}^2} - \mathbf{s}_i \cdot \mathbf{s}_j\right)\right]$$

❷. The color-magnetic piece

$$\boldsymbol{V_{so}^{cm}} = \sum_{i<j}^{3} \frac{2\alpha_s}{3r_{ij}^3}\left[\frac{\mathbf{r}_{ij} \times \mathbf{p}_i \cdot \mathbf{s}_i}{m_i^2} + \frac{\mathbf{r}_{ij} \times \mathbf{p}_i \cdot \mathbf{s}_j}{m_i m_j} - \frac{\mathbf{r}_{ij} \times \mathbf{p}_j \cdot \mathbf{s}_j}{m_j^2} - \frac{\mathbf{r}_{ij} \times \mathbf{p}_j \cdot \mathbf{s}_i}{m_i m_j}\right]$$

❸. The Thomas piece

$$\boldsymbol{V_{so}^{\mathrm{TP}}} = -\sum_{i<j}^{3} \frac{1}{2r_{ij}} \frac{\partial V_{si}}{\partial r_{ij}}\left(\frac{\mathbf{r}_{ij} \times \mathbf{p}_i \cdot \mathbf{s}_i}{m_i^2} - \frac{\mathbf{r}_{ij} \times \mathbf{p}_j \cdot \mathbf{s}_j}{m_j^2}\right)$$

here $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$.

Spin-dependent interactions with OGE mechanism

Wigner-Eckhart Theorem

In the center of mass coordinate of the heavy baryon, we have

$$\begin{cases} \mathbf{r}_1 = \dfrac{\boldsymbol{\rho}}{\sqrt{2}} + \sqrt{\dfrac{3}{2}} \dfrac{\mu}{2m+\mu} \boldsymbol{\lambda} \\ \mathbf{r}_2 = -\dfrac{\boldsymbol{\rho}}{\sqrt{2}} + \sqrt{\dfrac{3}{2}} \dfrac{\mu}{2m+\mu} \boldsymbol{\lambda} \\ \mathbf{r}_3 = -\sqrt{6} \dfrac{m}{2m+\mu} \boldsymbol{\lambda} \end{cases} \qquad \begin{cases} \mathbf{p}_1 = \dfrac{1}{\sqrt{2}} \mathbf{p}_\rho + \sqrt{\dfrac{1}{6}} \mathbf{p}_\lambda \\ \mathbf{p}_2 = -\dfrac{1}{\sqrt{2}} \mathbf{p}_\rho + \sqrt{\dfrac{1}{6}} \mathbf{p}_\lambda \\ \mathbf{p}_3 = -\sqrt{\dfrac{2}{3}} \mathbf{p}_\lambda \end{cases}$$

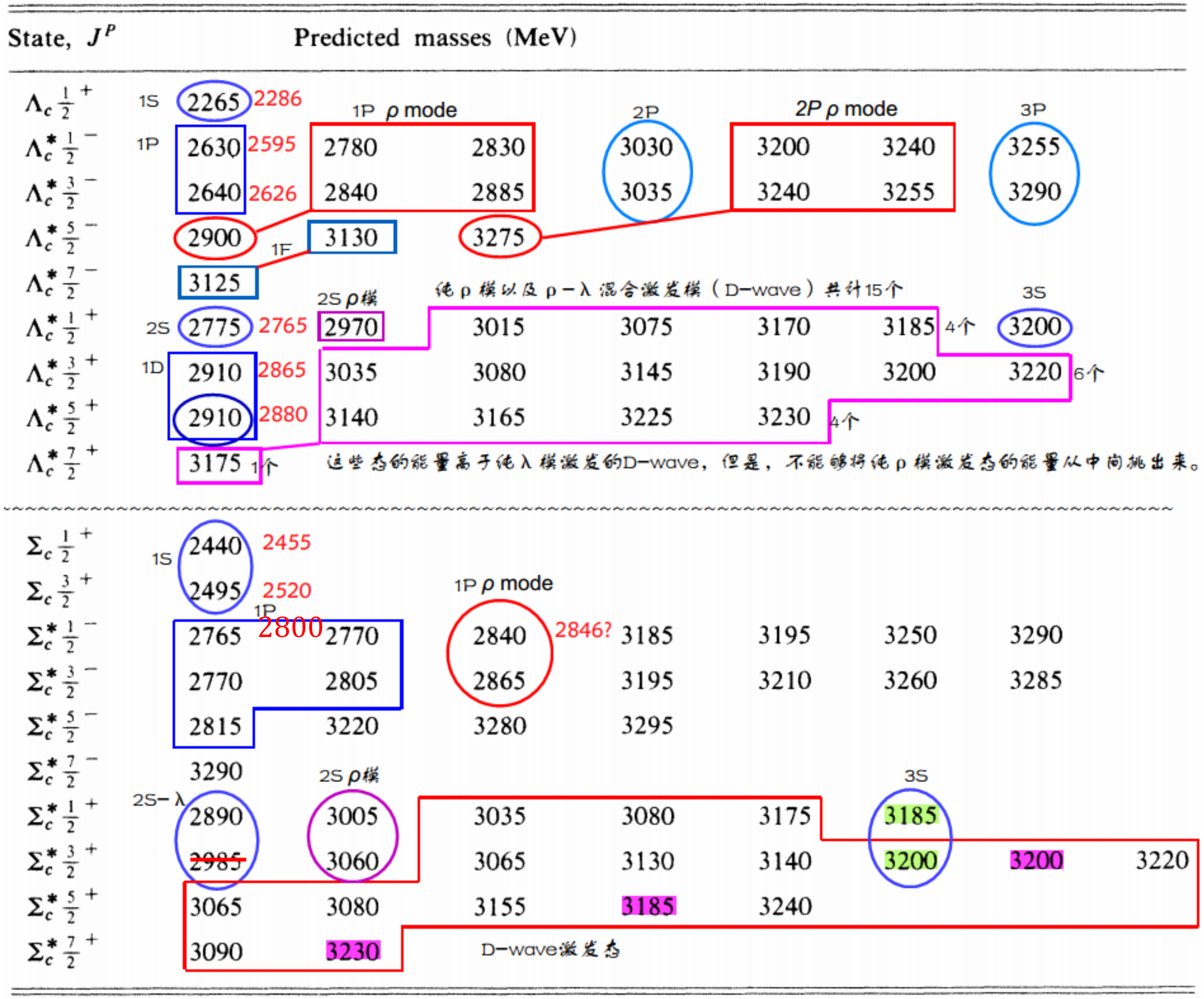
With the relationships above, we can deal with the spin-dependent interaction before.

Goldstone boson exchange

"The Spectrum of the nucleons and the strange hyperons and chiral dynamics"
L.Ya. Glozman and D.O. Riska, Phys. Rep. **268**, 263 (1996)

The predicted masses given by Capstick and Isgur

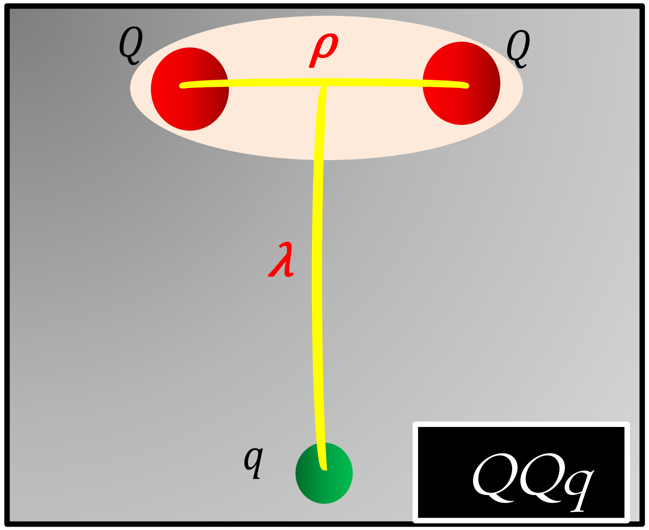
"Baryons in a Relativized Quark Model with Chromodynamics" [Phys. Rev. D **34**, 2809 (1986)]



State, J^P		Predicted masses (MeV)								
$\Lambda_c\ \frac{1}{2}^+$	1S	2265	2286	1P ρ mode		2P	2P ρ mode		3P	
$\Lambda_c^*\ \frac{1}{2}^-$	1P	2630	2595	2780	2830	3030	3200	3240	3255	
$\Lambda_c^*\ \frac{3}{2}^-$		2640	2626	2840	2885	3035	3240	3255	3290	
$\Lambda_c^*\ \frac{5}{2}^-$		2900	1F	3130	3275					
$\Lambda_c^*\ \frac{7}{2}^-$		3125								
				2S ρ模	纯 ρ 模以及 ρ－λ 混合激发模（D-wave）共计15个				3S	
$\Lambda_c^*\ \frac{1}{2}^+$	2S	2775	2765	2970	3015	3075	3170	3185	3200	4个
$\Lambda_c^*\ \frac{3}{2}^+$	1D	2910	2865	3035	3080	3145	3190	3200	3220	6个
$\Lambda_c^*\ \frac{5}{2}^+$		2910	2880	3140	3165	3225	3230			4个
$\Lambda_c^*\ \frac{7}{2}^+$		3175	1个	这些态的能量高于纯 λ 模激发的D-wave，但是，不能够将纯 ρ 模激发态的能量从中间挑出来。						

$\Sigma_c\ \frac{1}{2}^+$	1S	2440	2455								
$\Sigma_c\ \frac{3}{2}^+$		2495	2520		1P ρ mode						
$\Sigma_c^*\ \frac{1}{2}^-$	1P	2765	2800	2770	2840	2846?	3185	3195	3250	3290	
$\Sigma_c^*\ \frac{3}{2}^-$		2770		2805	2865		3195	3210	3260	3285	
$\Sigma_c^*\ \frac{5}{2}^-$		2815		3220	3280		3295				
$\Sigma_c^*\ \frac{7}{2}^-$		3290		2S ρ模					3S		
$\Sigma_c^*\ \frac{1}{2}^+$	2S− λ	2890		3005	3035		3080	3175	3185		
$\Sigma_c^*\ \frac{3}{2}^+$		~~2985~~		3060	3065		3130	3140	3200	3200	3220
$\Sigma_c^*\ \frac{5}{2}^+$		3065		3080	3155		3185	3240			
$\Sigma_c^*\ \frac{7}{2}^+$		3090		3230	D-wave激发态						

A approximate "T-type" confinement mechanism
—diquark model



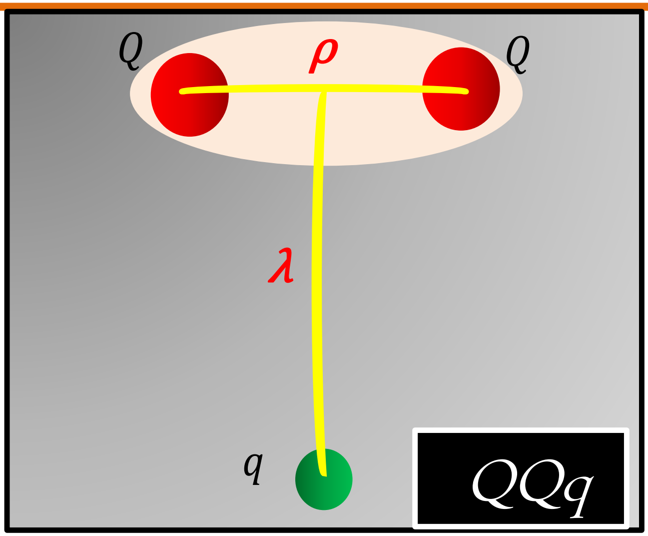
❶. For the singly heavy baryons, two light quarks in a λ-mode excited state can be treated as a block with the antitriplet ($\bar{3}$) color structure and peculiar size.

❷. For the doubly heavy baryons, the velocity of two heavy quarks is much lower than the light quark. Then we may divide the dynamics of QQq system into two parts (Born-Oppenheimer approximation):

$$\hat{H}_{QQq} \simeq \hat{H}_{QQ} + \hat{H}_q$$

1. "Spectroscopy of doubly heavy baryons", S. S. Gershtein, et. al., Phys. Rev. D **62**, 054021 (2000) ;
2. "Mass spectra of doubly heavy $\Omega_{QQ'}$ baryons", V.V. Kiselev, et. al., Phys. Rev. D **66**, 034030 (2002) ;
3. "Mass spectra of doubly heavy baryons in the relativistic quark model", D. Ebert, et. al., Phys. Rev. D **66**, 014008 (2002) ;
4. "Mass spectra and radiative transitions of doubly heavy baryons in a relativized quark model", Qi-Fang Lü, Kai-Lei Wang, Li-Ye Xiao, and Xian-Hui Zhong, Phys. Rev. D **96**, 114006 (2007) ;

A approximate "T-type" confinement mechanism —diquark model



Based on the heavy quark symmetry, we may choose the following the basis

$$\left|\left(\left(l_\rho \otimes l_\lambda\right)_L \otimes s_q\right)_{j_l} \otimes s_{QQ}\right\rangle_J$$

to study the strong decays of QQq baryons.

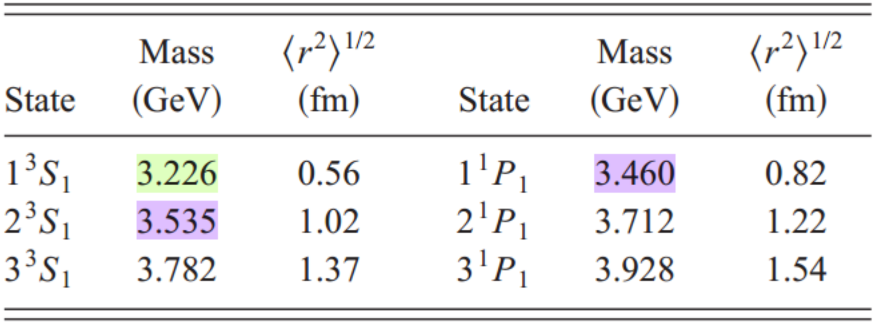
$$\left|\left(l_\rho \otimes s_{QQ}\right)_{J_\rho} \otimes \left(l_\lambda \otimes s_q\right)_{J_q}\right\rangle_J$$

l_λ	l_ρ	L	j_l	S_{QQ}	J^P
0	0	0	1/2	1	$1/2^+$, $3/2^+$
	1	1	1/2, 3/2	0	$1/2^-$, $3/2^-$
	2	2	3/2, 5/2	1	$(1/2^+,3/2^+,5/2^+)$, $(3/2^+,5/2^+,7/2^+)$
1	0	1	1/2,3/2	1	$(1/2^-,3/2^-)$, $(1/2^-,3/2^-,5/2^-)$
	1	0	1/2	0	$1/2^+$
		1	1/2, 3/2		$1/2^+$, $3/2^+$
		2	3/2, 5/2		$3/2^+$, $5/2^+$,
2	0	2	3/2, 5/2	1	$(1/2^+,3/2^+,5/2^+)$, $(3/2^+,5/2^+,7/2^+)$

The obtained masses of Ξ_{cc} baryons in the diquark picture

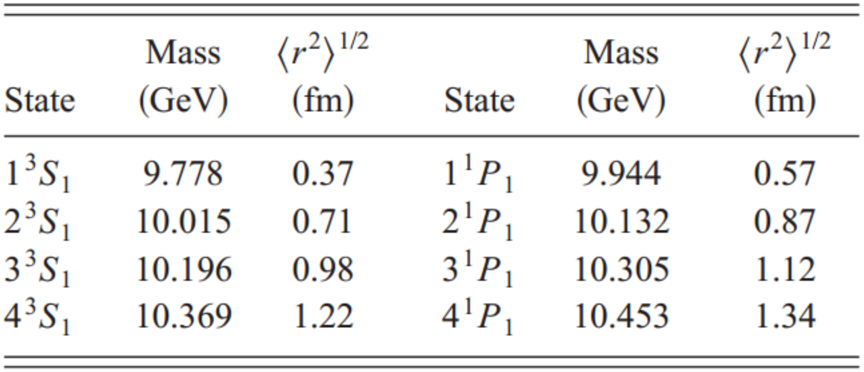
"Spectroscopy of doubly heavy baryons", S. S. Gershtein, et. al., Phys. Rev. D **62**, 054021 (2000) ;
"Mass spectra of doubly heavy baryons in the relativistic quark model", D. Ebert, et. al., Phys. Rev. D **66**, 014008 (2002) ;

Mass spectrum and mean squared radii of the cc diquark.



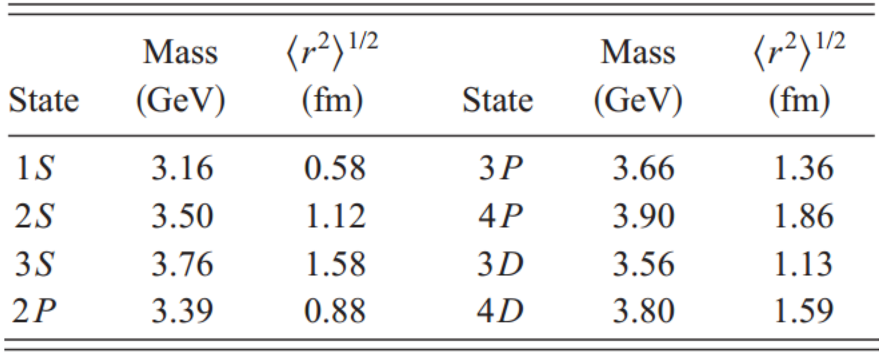
State	Mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)	State	Mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)
1^3S_1	3.226	0.56	1^1P_1	3.460	0.82
2^3S_1	3.535	1.02	2^1P_1	3.712	1.22
3^3S_1	3.782	1.37	3^1P_1	3.928	1.54

Mass spectrum and mean squared radii of bb diquark.



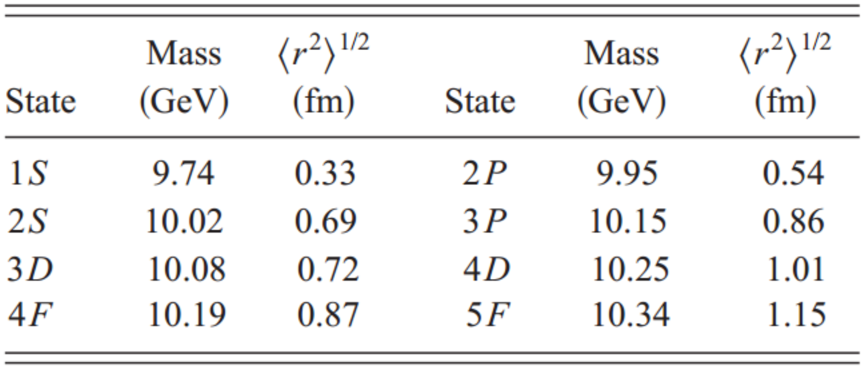
State	Mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)	State	Mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)
1^3S_1	9.778	0.37	1^1P_1	9.944	0.57
2^3S_1	10.015	0.71	2^1P_1	10.132	0.87
3^3S_1	10.196	0.98	3^1P_1	10.305	1.12
4^3S_1	10.369	1.22	4^1P_1	10.453	1.34

Mass spectrum and mean squared radii of the cc diquark.



State	Mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)	State	Mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)
$1S$	3.16	0.58	$3P$	3.66	1.36
$2S$	3.50	1.12	$4P$	3.90	1.86
$3S$	3.76	1.58	$3D$	3.56	1.13
$2P$	3.39	0.88	$4D$	3.80	1.59

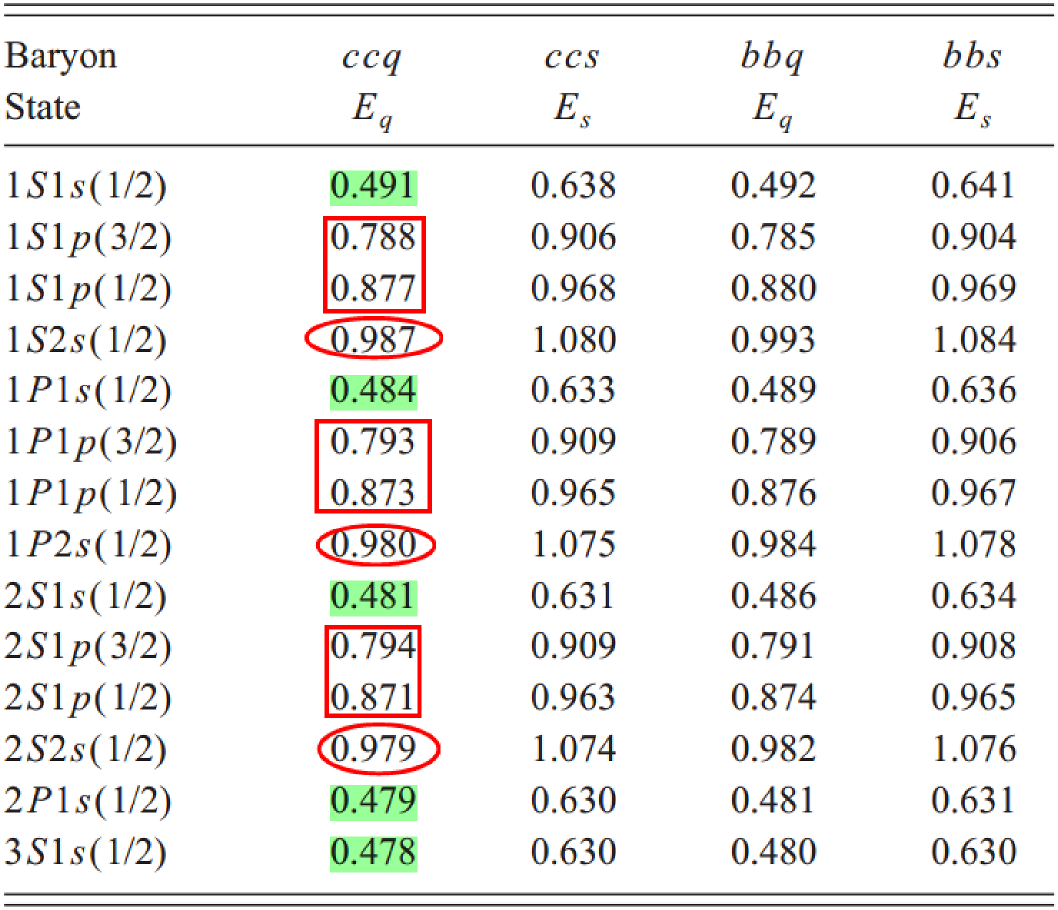
Mass spectrum and mean squared radii of bb diquark.



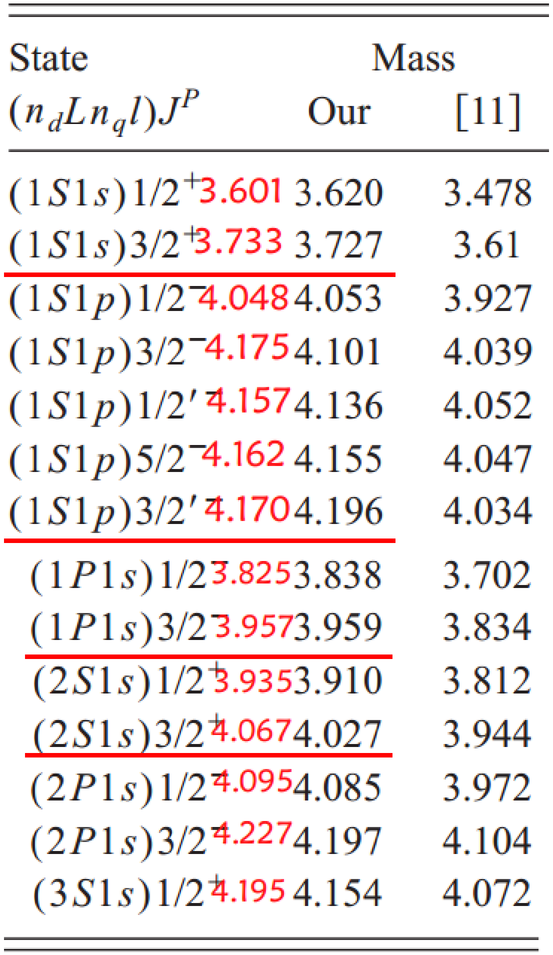
State	Mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)	State	Mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)
$1S$	9.74	0.33	$2P$	9.95	0.54
$2S$	10.02	0.69	$3P$	10.15	0.86
$3D$	10.08	0.72	$4D$	10.25	1.01
$4F$	10.19	0.87	$5F$	10.34	1.15

The obtained masses of Ξ_{cc} baryons in the diquark picture

"Spectroscopy of doubly heavy baryons", S. S. Gershtein, et. al., Phys. Rev. D **62**, 054021 (2000) ;
"Mass spectra of doubly heavy baryons in the relativistic quark model", D. Ebert, et. al., Phys. Rev. D **66**, 014008 (2002) ;



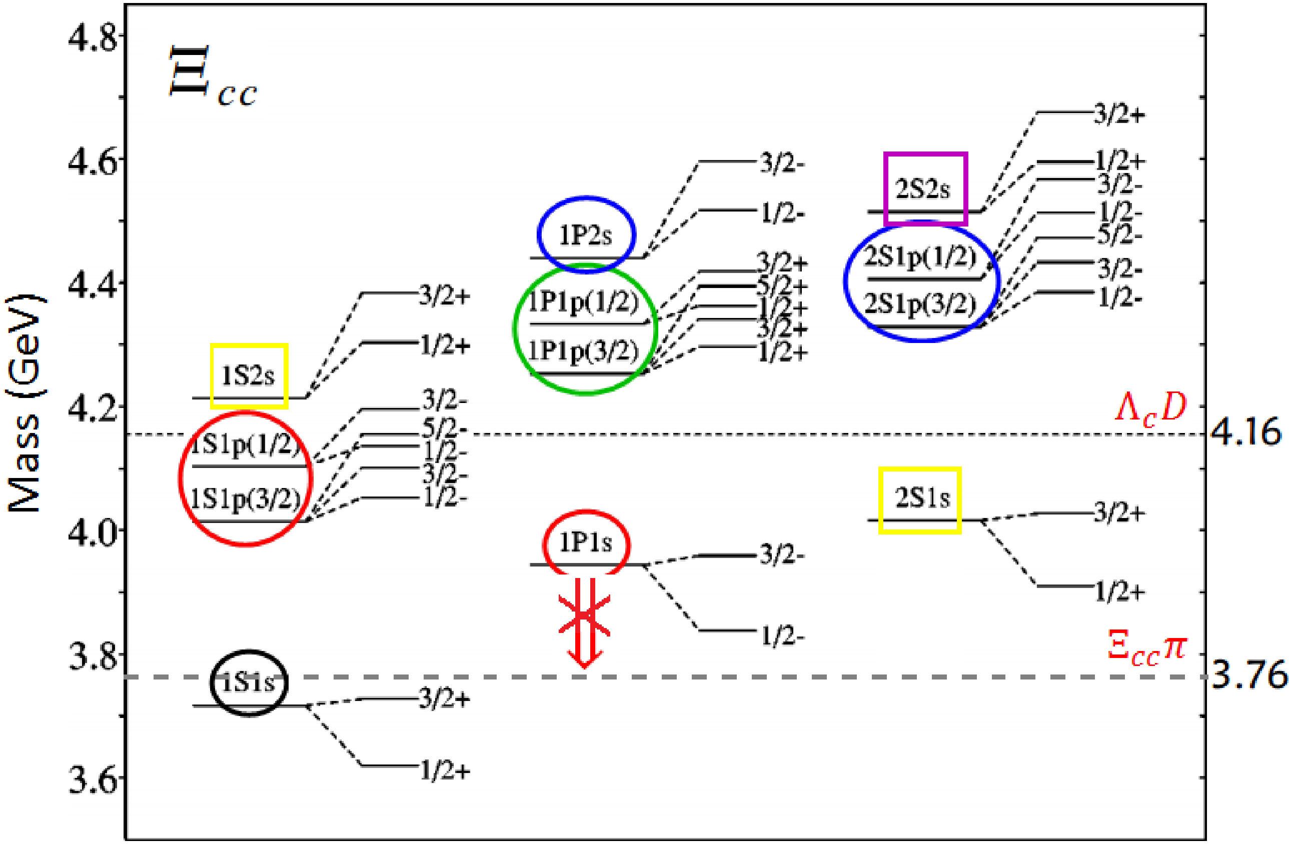
Baryon State	ccq E_q	ccs E_s	bbq E_q	bbs E_s
$1S1s(1/2)$	0.491	0.638	0.492	0.641
$1S1p(3/2)$	0.788	0.906	0.785	0.904
$1S1p(1/2)$	0.877	0.968	0.880	0.969
$1S2s(1/2)$	0.987	1.080	0.993	1.084
$1P1s(1/2)$	0.484	0.633	0.489	0.636
$1P1p(3/2)$	0.793	0.909	0.789	0.906
$1P1p(1/2)$	0.873	0.965	0.876	0.967
$1P2s(1/2)$	0.980	1.075	0.984	1.078
$2S1s(1/2)$	0.481	0.631	0.486	0.634
$2S1p(3/2)$	0.794	0.909	0.791	0.908
$2S1p(1/2)$	0.871	0.963	0.874	0.965
$2S2s(1/2)$	0.979	1.074	0.982	1.076
$2P1s(1/2)$	0.479	0.630	0.481	0.631
$3S1s(1/2)$	0.478	0.630	0.480	0.630



State $(n_dLn_ql)J^P$		Mass Our	[11]
$(1S1s)1/2^+$	3.601	3.620	3.478
$(1S1s)3/2^+$	3.733	3.727	3.61
$(1S1p)1/2^-$	4.048	4.053	3.927
$(1S1p)3/2^-$	4.175	4.101	4.039
$(1S1p)1/2'^-$	4.157	4.136	4.052
$(1S1p)5/2^-$	4.162	4.155	4.047
$(1S1p)3/2'^-$	4.170	4.196	4.034
$(1P1s)1/2$	3.825	3.838	3.702
$(1P1s)3/2$	3.957	3.959	3.834
$(2S1s)1/2^+$	3.935	3.910	3.812
$(2S1s)3/2^+$	4.067	4.027	3.944
$(2P1s)1/2$	4.095	4.085	3.972
$(2P1s)3/2$	4.227	4.197	4.104
$(3S1s)1/2^+$	4.195	4.154	4.072

The obtained masses of Ξ_{cc} baryons in the diquark picture

"Mass spectra of doubly heavy baryons in the relativistic quark model", D. Ebert, et. al., Phys. Rev. D **66**, 014008 (2002) ;



Summary and outlook

(1) More and more heavy baryon states have been discovered by experiments. However, most of all can be explained as the λ-mode excitations;

(2) The definite ρ- mode excited heavy baryons may help us to deeply understand the confinement mechanism.

(3) The five narrow Ω_c states have never been investigated thoroughly in the three-body picture,

(4) How to distinguish the different confinement mechanism? Can we give some definite criteria which can be tested by experiments?

(5) Here we just give a scheme for studying the masses and strong decays in the conventional quenched quark model. In fact, The effects of hadron loops may play an important role for some (heavy) baryon states. The “unquenched” quark mode is still in the development stage.

(6) If we want to disentangle the puzzles of the baryon dynamics, maybe we should try to study the whole baryon family in a suite way.

Thank you！