Topological amplitude and its $SU(N)$ decomposition

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arXiv: 1812.XXXX

Contents

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Motivation

- Heavy hadron decays in flavor *SU*(3) symmetry:
  - The topological diagram amplitude (TDA)
    - H. Y. Cheng and C. W. Chiang, Phys. Rev. D **85**, 034036 (2012)
    - H. Y. Cheng and C. W. Chiang, Phys. Rev. D **86**, 014014 (2012)
    - H. Y. Cheng, C. W. Chiang and A. L. Kuo, Phys. Rev. D **93**, 114010 (2016)
  - The *SU*(3) irreducible representation amplitude (IRA)
    - W. Wang, Z. P. Xing and J. Xu, Eur. Phys. J. C 77, no. 11, 800 (2017)
    - C. Q. Geng, Y. K. Hsiao, C. W. Liu and T. H. Tsai, JHEP 1711, 147 (2017)
    - Y. J. Shi, W. Wang, Y. Xing and J. Xu, Eur. Phys. J. C 78, no. 1, 56 (2018)
    - W. Wang and J. Xu, Phys. Rev. D 97, no. 9, 093007 (2018)
    - C. Q. Geng, Y. K. Hsiao, Y. H. Lin and L. L. Liu, Phys. Lett. B 776, 265 (2018)
    - C. Q. Geng, Y. K. Hsiao, C. W. Liu and T. H. Tsai, Phys. Rev. D 97, no. 7, 073006 (2018)
    - C. Q. Geng, Y. K. Hsiao, C. W. Liu and T. H. Tsai, Eur. Phys. J. C 78, no. 7, 593 (2018)
    - C. Q. Geng, Y. K. Hsiao, C. W. Liu and T. H. Tsai, arXiv:1810.01079 [hep-ph]

Motivation

- Flavor *SU*(3) Topological Diagram and Irreducible Representation Amplitudes for Heavy Meson Charmless Hadronic Decays: Mismatch and Equivalence
  - X. G. He and W. Wang, Chin. Phys. C 42, 103108 (2018)
  - X. G. He, Y. J. Shi and W. Wang, arXiv:1811.03480 [hep-ph]
    - A bridge between topological diagram and tensor contraction is built in *B* and *D* meson decays
    - Some new topological diagrams are identified in order to give a more reasonable topological description of heavy meson decays
    - The equivalence relations between the topological amplitudes and the *SU*(3) irreducible representation amplitudes for one type of decay can be derived by some simple calculations.

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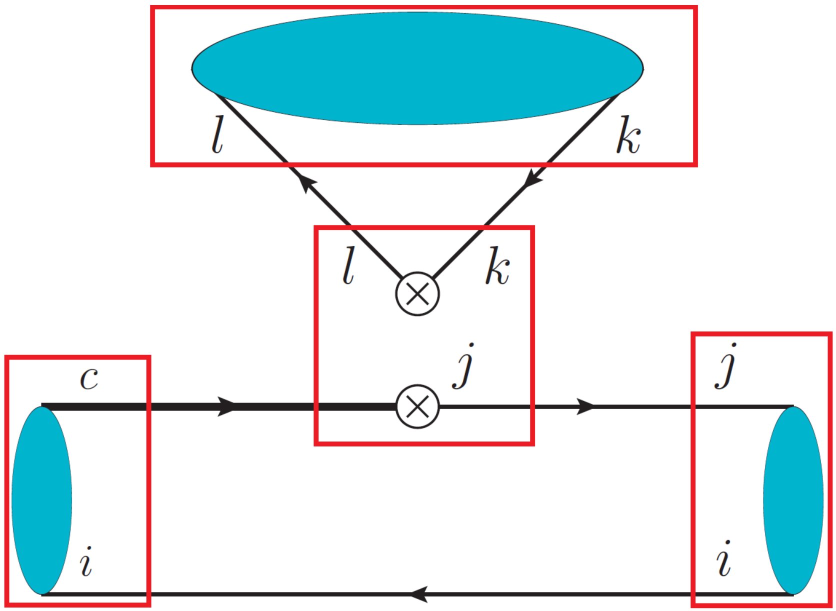
Topology $\leftrightharpoons$ Tensor contraction

- Four-quark Hamiltonian $\mathcal{H} = \mathcal{H}_{ij}^{k}(\bar{q}^i q_k)(\bar{q}^j c)$
- T diagram: $TD^i \mathcal{H}_{lj}^{k} (P)_i^j (P)_k^l$



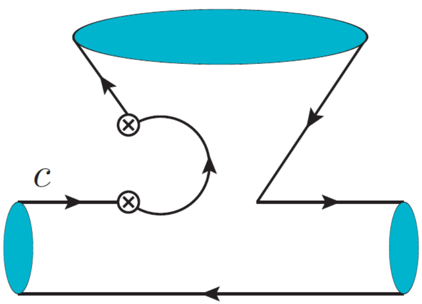
Topologies in $D \to PP$ decays

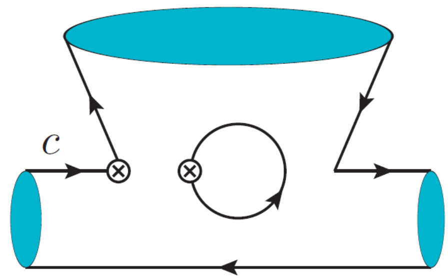
- The amplitude in $D \to PP$ decays can be written as

$$
\begin{aligned}
\mathcal{A}_{PP}^{TDA} = {} & TD^i\mathcal{H}^k_{lj}(P)^j_i(P)^l_k + CD^i\mathcal{H}^k_{jl}(P)^j_i(P)^l_k + ED^i\mathcal{H}^j_{il}(P)^k_j(P)^l_k \\
& + AD^i\mathcal{H}^j_{li}(P)^k_j(P)^l_k + T^{ES}D^i\mathcal{H}^l_{ij}(P)^j_l(P)^k_k + T^{AS}D^i\mathcal{H}^l_{ji}(P)^j_l(P)^k_k \\
& + T^{LP}D^i\mathcal{H}^l_{kl}(P)^j_i(P)^k_j + T^{LC}D^i\mathcal{H}^l_{jl}(P)^j_i(P)^k_k + T^{LA}D^i\mathcal{H}^l_{il}(P)^k_j(P)^j_k \\
& + T^{LS}D^i\mathcal{H}^l_{il}(P)^j_j(P)^k_k + T^{QP}D^i\mathcal{H}^l_{lk}(P)^j_i(P)^k_j + T^{QC}D^i\mathcal{H}^l_{lj}(P)^j_i(P)^k_k \\
& + T^{QA}D^i\mathcal{H}^l_{li}(P)^k_j(P)^j_k + T^{QS}D^i\mathcal{H}^l_{li}(P)^j_j(P)^k_k.
\end{aligned}
$$

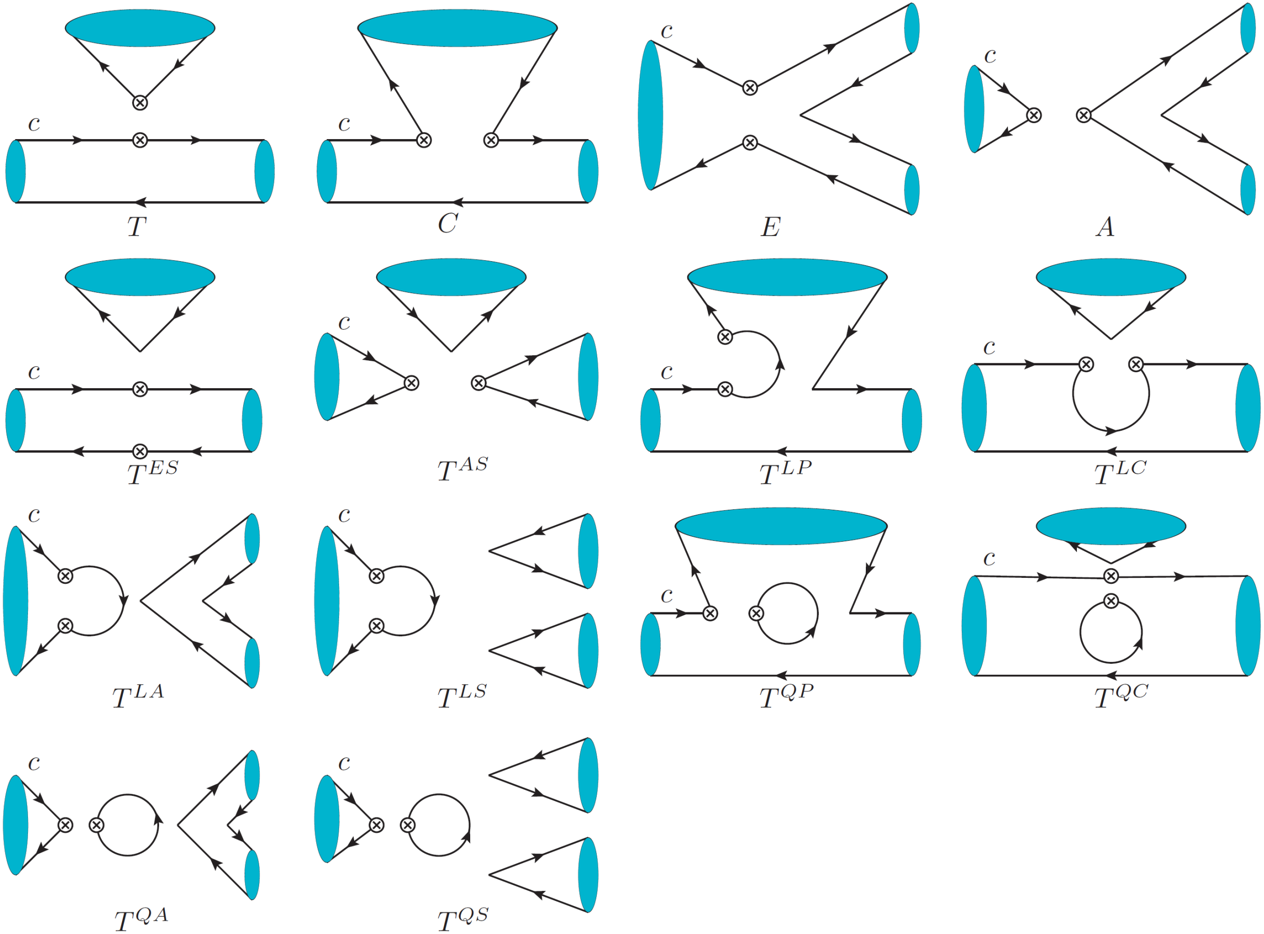
  - ☞ 14 terms. Each term presents one topological diagram.

- Quark loop





Topologies in $D \to PP$ decay



Completeness

- Each term in the amplitude is one topological amplitude.
- If all possible contractions are found, all possible topologies are also found completely.
- For $D \to PP$ decay, there are 4 covariant and 4 contravariant indices in each term of amplitude. The number of possible contractions is $4! = 24$.
- 10 diagrams: exchanging P_1 and $P_2 \Rightarrow$ two contractions; 4 diagrams: exchanging P_1 and $P_2 \Rightarrow$ one contraction.
- $2 \times 10 + 1 \times 4 = 24$.
- Simple in Meson decay, but complicated in baryon decay

$SU(3)$ decomposition

$SU(3)$ irreducible representation

$$\mathcal{H}_{ij}^{k} = \delta_j^k\Big(\frac{3}{8}\mathcal{H}(3_t)_i - \frac{1}{8}\mathcal{H}(3_p)_i\Big) + \delta_i^k\Big(\frac{3}{8}\mathcal{H}(3_p)_j - \frac{1}{8}\mathcal{H}(3_t)_j\Big) + \epsilon_{ijl}\mathcal{H}(\overline{6})^{lk} + \mathcal{H}(15)_{ij}^{k}.$$

3_p presentation:

$$\mathcal{H}(3_p)_1 = (\bar{u}u)(\bar{u}c) + (\bar{d}d)(\bar{u}c) + (\bar{s}s)(\bar{u}c),$$
$$\mathcal{H}(3_p)_2 = (\bar{u}u)(\bar{d}c) + (\bar{d}d)(\bar{d}c) + (\bar{s}s)(\bar{d}c),$$
$$\mathcal{H}(3_p)_3 = (\bar{u}u)(\bar{s}c) + (\bar{d}d)(\bar{s}c) + (\bar{s}s)(\bar{s}c).$$

3_t presentation:

$$\mathcal{H}(3_t)_1 = (\bar{u}u)(\bar{u}c) + (\bar{u}d)(\bar{d}c) + (\bar{u}s)(\bar{s}c),$$
$$\mathcal{H}(3_t)_2 = (\bar{d}u)(\bar{u}c) + (\bar{d}d)(\bar{d}c) + (\bar{d}s)(\bar{s}c),$$
$$\mathcal{H}(3_t)_3 = (\bar{s}u)(\bar{u}c) + (\bar{s}d)(\bar{d}c) + (\bar{s}s)(\bar{s}c).$$

$SU(3)$ decomposition

$SU(3)$ irreducible representation amplitude

$$
\begin{aligned}
\mathcal{A}^{IRA}_{PP} = {} & a_3^p D^i \mathcal{H}(3_p)_i (P)^j_k (P)^k_j + b_3^p D^i \mathcal{H}(3_p)_i (P)^k_k (P)^j_j + c_3^p D^i \mathcal{H}(3_p)_k (P)^k_i (P)^j_j \\
& + d_3^p D^i \mathcal{H}(3_p)_k (P)^j_i (P)^k_j + a_3^t D^i \mathcal{H}(3_t)_i (P)^j_k (P)^k_j + b_3^t D^i \mathcal{H}(3_t)_i (P)^k_k (P)^j_j \\
& + c_3^t D^i \mathcal{H}(3_t)_k (P)^k_i (P)^j_j + d_3^t D^i \mathcal{H}(3_t)_k (P)^j_i (P)^k_j \\
& + a_6 D^i \mathcal{H}(\overline{6})^k_{ij} (P)^j_l (P)^l_k + b_6 D^i \mathcal{H}(\overline{6})^k_{ij} (P)^j_k (P)^l_l + c_6 D^i \mathcal{H}(\overline{6})^k_{jl} (P)^j_i (P)^l_k \\
& + a_{15} D^i \mathcal{H}(15)^k_{ij} (P)^j_l (P)^l_k + b_{15} D^i \mathcal{H}(15)^k_{ij} (P)^j_k (P)^l_l \\
& + c_{15} D^i \mathcal{H}(15)^k_{jl} (P)^j_i (P)^l_k .
\end{aligned}
$$

- 14 terms
- Completeness

Equivalence

$$a_6 = E - A, \qquad b_6 = T^{ES} - T^{AS}, \qquad c_6 = -T + C,$$

$$a_{15} = E + A, \qquad b_{15} = T^{ES} + T^{AS}, \qquad c_{15} = T + C,$$

$$a_3^t = \frac{3}{8}E - \frac{1}{8}A + T^{LA}, \quad a_3^p = -\frac{1}{8}E + \frac{3}{8}A + T^{QA}, \quad b_3^t = \frac{3}{8}T^{ES} - \frac{1}{8}T^{AS} + T^{LS},$$

$$b_3^p = -\frac{1}{8}T^{ES} + \frac{3}{8}T^{AS} + T^{QS}, \quad c_3^t = -\frac{1}{8}T + \frac{3}{8}C - \frac{1}{8}T^{ES} + \frac{3}{8}T^{AS} + T^{LC},$$

$$c_3^p = \frac{3}{8}T - \frac{1}{8}C + \frac{3}{8}T^{ES} - \frac{1}{8}T^{AS} + T^{QC},$$

$$d_3^t = \frac{3}{8}T - \frac{1}{8}C - \frac{1}{8}E + \frac{3}{8}A + T^{LP}, \quad d_3^p = -\frac{1}{8}T + \frac{3}{8}C + \frac{3}{8}E - \frac{1}{8}A + T^{QP}.$$

- In fact, because the sole difference between these two methods is weather the tensor operator $\mathcal{H}_{ij}^k$ is decomposed into $SU(3)$ irreducible representation or not, the equivalence is unescapable, no matter weather the equivalent formula is derived or not.

Contents

Independence of topologies

- One of topologies in $D \to PP$ decay is not independent
  - X. G. He and W. Wang, Chin. Phys. C 42, 103108 (2018)
- $\mathcal{H}(\overline{6})^k_{ij}$ can be written as $\epsilon_{ijl}\mathcal{H}(\overline{6})^{lk}$ and two indices l, k are symmetric.

$$a_6 D^i \mathcal{H}(\overline{6})^k_{ij}(P)^j_l(P)^l_k = a_6 D^i \epsilon_{ijm}\mathcal{H}(\overline{6})^{km}(P)^j_l(P)^l_k = a_6 D_{[jm]}\mathcal{H}(\overline{6})^{km}(P)^j_l(P)^l_k,$$
$$b_6 D^i \mathcal{H}(\overline{6})^k_{ij}(P)^j_k(P)^l_l = b_6 D^i \epsilon_{ijm}\mathcal{H}(\overline{6})^{km}(P)^j_k(P)^l_l = b_6 D_{[jm]}\mathcal{H}(\overline{6})^{km}(P)^j_k(P)^l_l,$$
$$c_6 D^i \mathcal{H}(\overline{6})^k_{jl}(P)^j_i(P)^l_k = \frac{1}{2} c_6\, \epsilon^{pqi}\epsilon_{jlm} D_{[pq]}\mathcal{H}(\overline{6})^{km}(P)^j_i(P)^l_k$$
$$= c_6\left[D_{[jl]}\mathcal{H}(\overline{6})^{ki}(P)^j_i(P)^l_k - D_{[jm]}\mathcal{H}(\overline{6})^{km}(P)^j_l(P)^l_k + D_{[jm]}\mathcal{H}(\overline{6})^{km}(P)^j_k(P)^l_l\right],$$
$$= c_6\left[- D_{[jm]}\mathcal{H}(\overline{6})^{km}(P)^j_l(P)^l_k + D_{[jm]}\mathcal{H}(\overline{6})^{km}(P)^j_k(P)^l_l\right].$$

- $a'_6 = a_6 - c_6, \qquad b'_6 = b_6 + c_6$

Independence of topologies

- Model independent
- If quark-loop diagrams are dropped by approximation, there is still one not independent diagram in the remaining diagrams.
- If we drop the diagrams T^{ES} and T^{AS} but include those channels with η_1 in the analysis, all the diagrams T, C, E and A are independent.
  - ☞ $a_6' = a_6 - c_6, \qquad b_6' = b_6 + c_6$
- If those channels with η_1 are not included in the analysis, only three of T, C, E and A are independent.
  - ☞ $a_6 D^i \mathcal{H}(\overline{6})^k_{ij} (P)^j_l (P)^l_k = a_6 D_{[jm]} \mathcal{H}(\overline{6})^{km} (P)^j_l (P)^l_k,$
    $c_6 D^i \mathcal{H}(\overline{6})^k_{jl} (P)^j_i (P)^l_k = -c_6 D_{[jm]} \mathcal{H}(\overline{6})^{km} (P)^j_l (P)^l_k$
- $a_6' = a_6 - c_6$

Independence of topologies

- $\mathcal{H}(\overline{6})^k_{ij} = \epsilon_{ijl}\mathcal{H}(\overline{6})^{lk}$ and symmetric l, k in $\mathcal{H}(\overline{6})^{lk}$ are crucial. $\Rightarrow$ specialities of $SU(3)$ group
- The decomposition of $N \otimes N$ is written as

$$\square \otimes \square = \square\!\square \oplus \begin{array}{c}\square\\ \square\end{array},$$

- $\mathcal{H}_{ij} = \mathcal{H}_{\{ij\}} + \mathcal{H}_{[ij]}$
- The number of possible combination of antisymmetry i, j is $C_N^2 = N(N-1)/2$. And the number of possible combination of symmetry i, j is $N^2 - C_N^2 = N^2 - N(N-1)/2 = N(N+1)/2$.
- Example: $3 \otimes 3 = 6 \oplus \overline{3}$

Independence of topologies

- $\mathcal{H}_i^j$ can be decomposed as

$$\mathcal{H}_i^j = \Big\{\mathcal{H}_i^j - \delta_i^j\Big(\frac{1}{N}\sum_l \mathcal{H}_l^l\Big)\Big\} + \delta_i^j\Big(\frac{1}{N}\sum_c \mathcal{H}_l^l\Big).$$

- Example: $3 \otimes \overline{3} = 8 \oplus 1$
- $\mathcal{H}_{ij}^k$ can be decomposed as

$$\mathcal{H}_{ij}^k = \mathcal{H}_{\{ij\}}^k + \mathcal{H}_{[ij]}^k + \frac{1}{N^2-1}\Big\{\delta_i^k \sum_l (N\mathcal{H}_{lj}^l - \mathcal{H}_{jl}^l) + \delta_j^k \sum_l (N\mathcal{H}_{il}^l - \mathcal{H}_{li}^l)\Big\},$$

$$N \otimes N \otimes \overline{N} = N^2(N+1)/2 - N \oplus \overline{N^2(N-1)/2 - N} \oplus N \oplus N.$$

$$3 \otimes 3 \otimes \overline{3} = (6 \oplus \overline{3}) \otimes \overline{3} = (6 \otimes \overline{3}) \oplus (\overline{3} \otimes \overline{3}) = (15 \oplus 3) \oplus (\overline{6} \oplus 3).$$

- $\mathcal{H}(\overline{6})_{ij}^k = \epsilon_{ijl}\mathcal{H}(\overline{6})^{lk}$
- If $N = 4$

$$4 \otimes 4 \otimes \overline{4} = (10 \oplus \overline{6}) \otimes \overline{4} = (10 \otimes \overline{4}) \oplus (\overline{6} \otimes \overline{4}) = (36 \oplus 4) \oplus (\overline{20} \oplus 4).$$

Independence of topologies

Table 1: Irreducible representation of tensor $\mathcal{H}_{ij}^{k}$ if the indies of $\mathcal{H}_{ij}^{k}$ are transformed according to $SU(N)$ group.

$SU(N)$	$N^2(N+1)/2-N$	$\overline{N^2(N-1)/2-N}$	N	N
$SU(2)$	4	0	2	2
$SU(3)$	15	$\overline{6}$	3	3
$SU(4)$	36	$\overline{20}$	4	4
$SU(5)$	70	$\overline{45}$	5	5
...	..	...	...	...

$$N(N-1)/2 \leq N, \qquad N \geq 2 \;\&\; N \in Z.$$

$$N = 2 \;\&\; N = 3$$

Contents

Effective Hamiltonian in the SM

- Effective Hamiltonian in charm decay

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}}\left[\sum_{q=d,s} V^*_{cq}V_{uq}\left(\sum_{q=1}^{2} C_i(\mu)O_i(\mu)\right) - V^*_{cb}V_{ub}\left(\sum_{i=3}^{6} C_i(\mu)O_i(\mu) + C_{8g}(\mu)O_{8g}(\mu)\right)\right.$$

$$O_1 = (\bar{u}_\alpha q_{2\beta})_{V-A}(\bar{q}_{1\beta}c_\alpha)_{V-A}, \qquad O_2 = (\bar{u}_\alpha q_{2\alpha})_{V-A}(\bar{q}_{1\beta}c_\beta)_{V-A},$$

$$O_3 = \sum_{q'=u,d,s}(\bar{u}_\alpha c_\alpha)_{V-A}(\bar{q}'_\beta q'_\beta)_{V-A}, \quad O_4 = \sum_{q'=u,d,s}(\bar{u}_\alpha c_\beta)_{V-A}(\bar{q}'_\beta q'_\alpha)_{V-A},$$

$$O_5 = \sum_{q'=u,d,s}(\bar{u}_\alpha c_\alpha)_{V-A}(\bar{q}'_\beta q'_\beta)_{V+A}, \quad O_6 = \sum_{q'=u,d,s}(\bar{u}_\alpha c_\beta)_{V-A}(\bar{q}'_\beta q'_\alpha)_{V+A},$$

$$O_{8g} = \frac{g}{8\pi^2} m_c \bar{u}\sigma_{\mu\nu}(1+\gamma_5)T^a G^{a\mu\nu} c.$$

$$C_{3,5}(\mu) \to \; C_{3,5}(\mu) + \frac{\alpha_s(\mu)}{8\pi N_c}\frac{2m_c^2}{\langle l^2\rangle}C^{\text{eff}}_{8g}(\mu), \qquad C_{4,6}(\mu) \to \; C_{4,6}(\mu) - \frac{\alpha_s(\mu)}{8\pi}\frac{2m_c^2}{\langle l^2\rangle}C^{\text{eff}}_{8g}(\mu)$$

Quark loop

- In general, the quark-loop from the tree operators is absorbed into the Wilson coefficients of penguin operators

$$C_{3,5}(\mu) \to C_{3,5}(\mu) - \frac{\alpha_s(\mu)}{8\pi N_c} \sum_{q=d,s} \frac{\lambda_q}{\lambda_b} C^{(q)}(\mu, \langle l^2 \rangle),$$

$$C_{4,6}(\mu) \to C_{4,6}(\mu) - \frac{\alpha_s(\mu)}{8\pi} \sum_{q=d,s} \frac{\lambda_q}{\lambda_b} C^{(q)}(\mu, \langle l^2 \rangle),$$

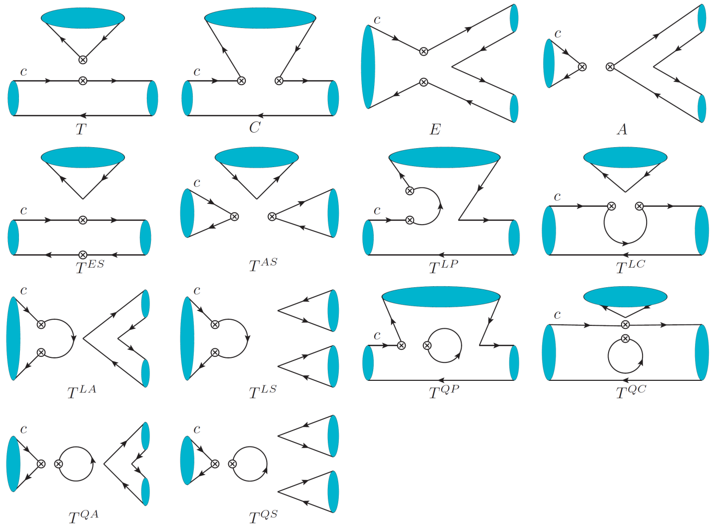
$$C^{(q)}(\mu, \langle l^2 \rangle) = \Big[-4 \int_0^1 dx\, x(1-x) \ln \frac{m_q^2 - x(1-x)\langle l^2 \rangle}{\mu^2} - \frac{2}{3} \Big] C_2(\mu).$$

  - M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B 606, 245 (2001)

- Tree diagram and penguin diagram.
- Redefine: ”tree operator-induced diagram” and ”penguin operator-induced diagram”

Topological diagrams

- If $O_{1,2}$ are inserted, the tree-operator-induced topologies are obtained. If O_{3-6} are inserted, the penguin-operator-induced topologies are obtained.



- In the SM: 10 tree-operator-induced diagrams, 14 penguin-operator-induced diagrams
- Being consistent with the results of QCD-factorization
  - M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B 606, 245 (2001)

Tensor operators

- Four-quark Hamiltonian $\mathcal{H} = \mathcal{H}^{k}_{ij}(\bar{q}^i q_k)(\bar{q}^j c)$
- Tree operators and their CKM matrix

$$V^*_{cs}V_{ud}\mathcal{H}^2_{13}, \quad V^*_{cd}V_{ud}\mathcal{H}^2_{12}, \quad V^*_{cs}V_{us}\mathcal{H}^3_{13}, \quad V^*_{cd}V_{us}\mathcal{H}^3_{12},$$

$$(V^*_{cd}V_{ud}+V^*_{cs}V_{us})\mathcal{H}(3_t)_1, \quad -\frac{1}{2}V^*_{cs}V_{ud}\mathcal{H}(\overline{6})^{22}, \quad \frac{1}{4}(V^*_{cd}V_{ud}-V^*_{cs}V_{us})\mathcal{H}(\overline{6})^{23},$$
$$\frac{1}{2}V^*_{cd}V_{us}\mathcal{H}(\overline{6})^{33}, \quad -\frac{1}{4}(V^*_{cd}V_{ud}+V^*_{cs}V_{us})\mathcal{H}(15)^1_{11}, \quad \frac{1}{2}V^*_{cs}V_{ud}\mathcal{H}(15)^2_{13},$$
$$\frac{1}{2}V^*_{cd}V_{us}\mathcal{H}(15)^3_{12}, \quad (\frac{3}{8}V^*_{cd}V_{ud}-\frac{1}{8}V^*_{cs}V_{us})\mathcal{H}(15)^2_{12}, \quad (\frac{3}{8}V^*_{cs}V_{us}-\frac{1}{8}V^*_{cd}V_{ud})\mathcal{H}(15)^3_{13}.$$

- Penguin operators and their CKM matrix

$$-V^*_{cb}V_{ub}\mathcal{H}^1_{11}, \quad -V^*_{cb}V_{ub}\mathcal{H}^2_{21}, \quad -V^*_{cb}V_{ub}\mathcal{H}^3_{31}.$$

$$-3V^*_{cb}V_{ub}\mathcal{H}(3_p)_1, \qquad -V^*_{cb}V_{ub}\mathcal{H}(3_t)_1.$$

Examples

Table 2: Tree-operator-induced and penguin-operator-induced amplitudes for Singly Cabibblo-suppressed $D \to PP$ decays.

channel	TDA	IRA
$D^0 \to \pi^+\pi^-$	$\lambda_d(T+E)+\lambda_+(T^{LP}+2T^{LA})$ $-\lambda_b(PC+PE+2PA+P^{LP}$ $+2P^{LA}+3P^{QP}+6P^{QA})$	$\lambda_+(2a_3^t+d_3^t-\frac{1}{4}a_{15})+\frac{1}{8}\lambda_1(a_{15}+c_{15})$ $+\frac{1}{4}\lambda_-(a_6-c_6)$ $-\lambda_b(6Pa_3^p+2Pa_3^t+3Pd_3^p+Pd_3^t)$
$D^0 \to \pi^0\pi^0$	$\frac{1}{\sqrt{2}}\lambda_d(E-C)+\frac{1}{\sqrt{2}}\lambda_+(T^{LP}+2T^{LA})$ $-\frac{1}{\sqrt{2}}\lambda_b(PC+PE+2PA+P^{LP}+2P^{LA}$ $+3P^{QP}+6P^{QA})$	$\frac{1}{\sqrt{2}}\lambda_+(2a_3^t+d_3^t-\frac{1}{4}(a_{15}+c_{15}))$ $+\frac{1}{8\sqrt{2}}\lambda_1(a_{15}-c_{15})+\frac{1}{4\sqrt{2}}\lambda_-(a_6-c_6)$ $-\frac{1}{\sqrt{2}}\lambda_b(6Pa_3^p+2Pa_3^t+3Pd_3^p+Pd_3^t)$
$D^0 \to K^+K^-$	$\lambda_s(T+E)+\lambda_+(T^{LP}+2T^{LA})$ $-\lambda_b(PC+PE+2PA+P^{LP}$ $+2P^{LA}+3P^{QP}+6P^{QA})$	$\lambda_+(2a_3^t+d_3^t-\frac{1}{4}a_{15})+\frac{1}{8}\lambda_2(a_{15}+c_{15})$ $+\frac{1}{4}\lambda_-(a_6-c_6)$ $-\lambda_b(2Pa_3^t+6Pa_3^p+Pd_3^t+3Pd_3^p)$
$D^0 \to K^0\overline{K}^0$	$\lambda_+(E+2T^{LA})-\lambda_b(2PA+2T^{LA}+6T^{QA})$	$\lambda_+(2a_3^t+\frac{1}{4}a_{15})-\lambda_b(2Pa_3^t+6Pa_3^p)$
$D^+ \to K^+\overline{K}^0$	$\lambda_dA+\lambda_sT+\lambda_+T^{LP}$ $-\lambda_b(PC+PE+P^{LP}+3P^{QP})$	$\lambda_+d_3^t+\frac{1}{8}\lambda_1A_{15}+\frac{1}{8}\lambda_2c_{15}-\frac{1}{4}\lambda_-(a_6-c_6)$ $-\lambda_b(Pd_3^t+3Pd_3^p)$
$D_s^+ \to \pi^+K^0$	$\lambda_dT+\lambda_sA+\lambda_+T^{LP}$ $-\lambda_b(PC+PE+P^{LP}+3P^{QP})$	$\lambda_+d_3^t+\frac{1}{8}\lambda_1c_{15}+\frac{1}{8}\lambda_2a_{15}+\frac{1}{4}\lambda_-(a_6-c_6)$ $-\lambda_b(Pd_3^t+3Pd_3^p)$
$D_s^+ \to \pi^0K^+$	$-\frac{1}{\sqrt{2}}(\lambda_dC-\lambda_sA-\lambda_+T^{LP})$ $-\frac{1}{\sqrt{2}}\lambda_b(PC+PE+P^{LP}+3P^{QP})$	$\frac{1}{\sqrt{2}}\lambda_+(d_3^t-\frac{1}{4}c_{15})-\frac{1}{8\sqrt{2}}\lambda_1c_{15}+\frac{1}{8\sqrt{2}}\lambda_2a_{15}$ $+\frac{1}{4}\lambda_-(a_6-c_6)-\frac{1}{\sqrt{2}}\lambda_b(Pd_3^t+3Pd_3^p)$

U-spin breaking

- Consider U-spin breaking, the amplitude of $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays can be written as

$$\mathcal{A}(D^0 \to K^+K^-) =$$
$$V_{cs}^* V_{us}(T_{KK} + E_{KK}) + V_{cd}^* V_{ud}(T_d^{LP} + 2T_d^{LA}) + V_{cs}^* V_{us}(T_s^{LP} + 2T_s^{LA})$$
$$\simeq \sin\theta_C(T_{KK} + E_{KK}) + \sin\theta_C(T_{\rm break}^{LP} + 2T_{\rm break}^{LA}),$$

$$\mathcal{A}(D^0 \to \pi^+\pi^-) \simeq -\sin\theta_C(T_{\pi\pi} + E_{\pi\pi}) + \sin\theta_C(T_{\rm break}^{LP} + 2T_{\rm break}^{LA}).$$

- $KK - \pi\pi$ puzzle, Glauber strong phase
  - H. n. Li and S. Mishima, Phys. Rev. D 83, 034023 (2011)
- Test in experiments: $D^+ \to K_S^0 K^{*+}$ and $D_s^+ \to K_S^0 \rho^+$

$$\mathcal{A}(D^+ \to K_S^0 K^{*+}) = \sin\theta_C\,(T_P^s + A_P^s + T_{P,\,\rm break}^{LP}),$$
$$\mathcal{A}(D_s^+ \to K_S^0 \rho^+) = -\sin\theta_C\,(T_P^d + A_P^d - T_{P,\,\rm break}^{LP}).$$

Other application

- Strangeless D decays and charmless B decays
  - B meson decay is more complicated
  - $\mathcal{H}_{ij}^{k}$ is not enough
  - Degeneration and splitting in topologies *vs* Energy level degeneration and splitting in atomic or nuclear physics

- Flavor $SU(N)$ breaking effect
  - Linear $SU(3)_F$ breaking in charm decay
    - S. Mjller, U. Nierste and S. Schacht, Phys. Rev. D 92, no. 1, 014004 (2015)
  - High order U-spin breaking
    - M. Gronau, Phys. Lett. B 730, 221 (2014)

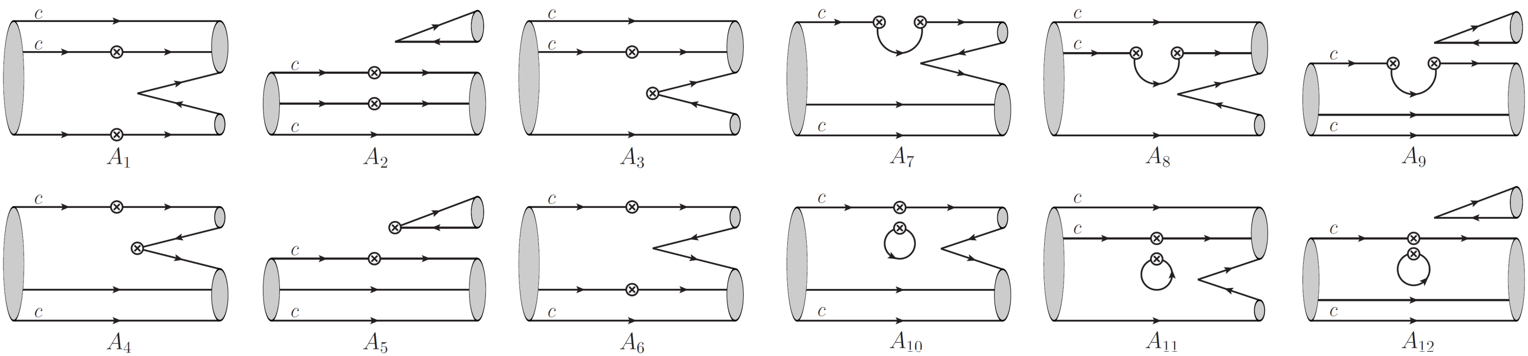
arXiv: 1812.XXXX

Contents

Charmed baryon decays

- Example $\mathcal{B}_{cc} \to \mathcal{B}_{c\bar{3}} M$

$$\mathcal{A}_{\rm eff}(\mathcal{B}_{cc} \to \mathcal{B}_{c\bar{3}} M) =$$
$$A_1(\mathcal{B}_{cc})_i \mathcal{H}^i_{jk} M^j_l \overline{\mathcal{B}}^{kl}_{c\bar{3}} + A_2(\mathcal{B}_{cc})_i \mathcal{H}^i_{jk} M^l_l \overline{\mathcal{B}}^{jk}_{c\bar{3}} + A_3(\mathcal{B}_{cc})_i \mathcal{H}^j_{lk} M^i_j \overline{\mathcal{B}}^{lk}_{c\bar{3}} + A_4(\mathcal{B}_{cc})_i \mathcal{H}^l_{jk} M^k_l \overline{\mathcal{B}}^{ij}_{c\bar{3}}$$
$$+ A_5(\mathcal{B}_{cc})_i \mathcal{H}^l_{kj} M^k_l \overline{\mathcal{B}}^{ij}_{c\bar{3}} + A_6(\mathcal{B}_{cc})_i \mathcal{H}^i_{kj} M^j_l \overline{\mathcal{B}}^{kl}_{c\bar{3}} + A_7(\mathcal{B}_{cc})_i \mathcal{H}^l_{jl} M^j_k \overline{\mathcal{B}}^{ik}_{c\bar{3}} + A_8(\mathcal{B}_{cc})_i \mathcal{H}^l_{jl} M^i_k \overline{\mathcal{B}}^{jk}_{c\bar{3}}$$
$$+ A_9(\mathcal{B}_{cc})_i \mathcal{H}^l_{jl} M^k_k \overline{\mathcal{B}}^{ij}_{c\bar{3}} + A_{10}(\mathcal{B}_{cc})_i \mathcal{H}^l_{lj} M^j_k \overline{\mathcal{B}}^{ik}_{c\bar{3}} + A_{11}(\mathcal{B}_{cc})_i \mathcal{H}^l_{lj} M^i_k \overline{\mathcal{B}}^{jk}_{c\bar{3}} + A_{12}(\mathcal{B}_{cc})_i \mathcal{H}^l_{lj} M^k_k \overline{\mathcal{B}}^{ij}_{c\bar{3}}$$



$\mathcal{B}_{cc} \to \mathcal{B}_{c\bar{3}}/\mathcal{B}_{c6}M$

- Wave function for a bound *cqq* state is $\Psi = \phi_{\rm flavour}\chi_{\rm spin}\xi_{\rm color}\eta_{\rm space}$. The overall wave function is required to be antisymmetric under the interchange of any two of the quarks. $\phi_{\rm flavour}\chi_{\rm spin}$ must be symmetric

$$A_{ij} = \frac{1}{\sqrt{2}}(q_iq_j - q_jq_i)c\,\chi_A, \qquad S_{ij} = \frac{1}{\sqrt{2}}(q_iq_j + q_jq_i)c\,\chi_S,$$

where

$$\chi_A^{1/2} = \frac{1}{\sqrt{2}}|(\uparrow\downarrow - \downarrow\uparrow)\uparrow\rangle, \qquad \chi_A^{-1/2} = \frac{1}{\sqrt{2}}|(\uparrow\downarrow - \downarrow\uparrow)\downarrow\rangle,$$

$$\chi_S^{1/2} = \frac{1}{\sqrt{3}}\Big| -\frac{(\uparrow\downarrow + \downarrow\uparrow)\uparrow}{\sqrt{2}} + \sqrt{2}\uparrow\uparrow\downarrow\Big\rangle, \qquad \chi_S^{-1/2} = \frac{1}{\sqrt{3}}\Big|\frac{(\uparrow\downarrow + \downarrow\uparrow)\downarrow}{\sqrt{2}} - \sqrt{2}\downarrow\downarrow\uparrow\Big\rangle.$$

- Topological diagrams in $\mathcal{B}_{cc} \to \mathcal{B}_{c\bar{3}}M$ and $\mathcal{B}_{cc} \to \mathcal{B}_{c6}M$ are two irrelevant sets.
  - ☞ octet and singlet mesons: topologies are in one set

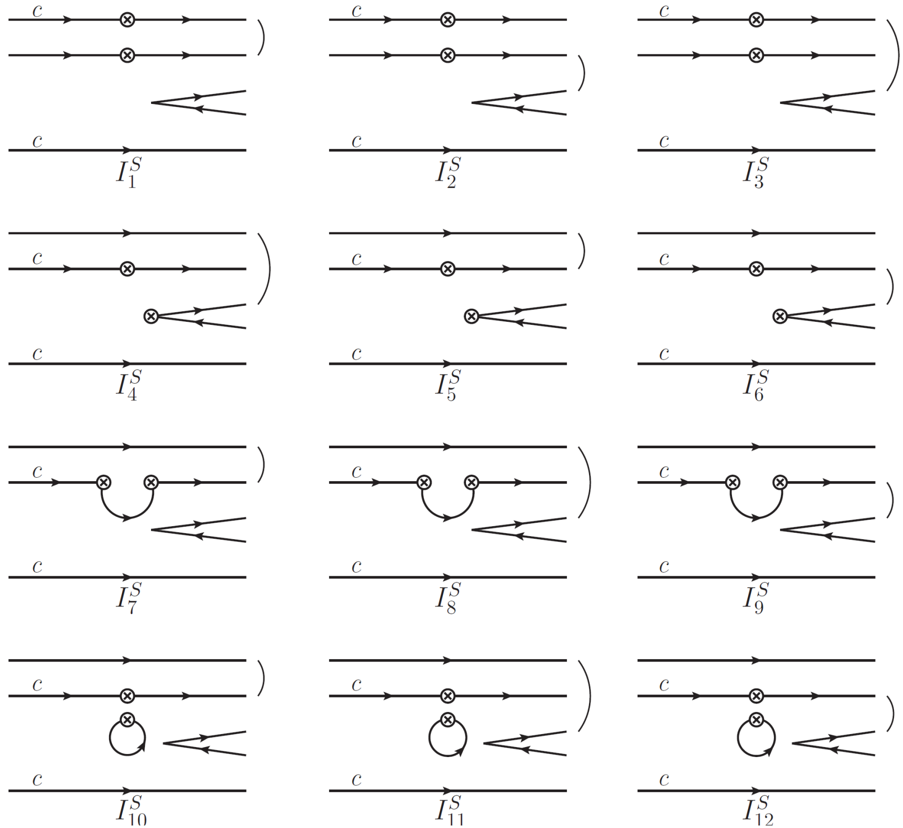
$\mathcal{B}_{cc} \to \mathcal{B}_8 D$

- There are two octets: symmetric and antisymmetric under 1 $\leftrightarrow$ 2

  $\Psi = \frac{1}{\sqrt{2}}(\phi_S\chi_S + \phi_A\chi_A)$

- Amplitude of $\mathcal{B}_{cc} \to \mathcal{B}_8 D$ is

$$\mathcal{A}_{\rm eff}(\mathcal{B}_{cc} \to \mathcal{B}_8 D) = \frac{1}{\sqrt{2}}\big[\mathcal{A}^S_{\rm eff}(\mathcal{B}_{cc} \to \mathcal{B}^S_8 D) + \mathcal{A}^A_{\rm eff}(\mathcal{B}_{cc} \to \mathcal{B}^A_8 D)\big].$$



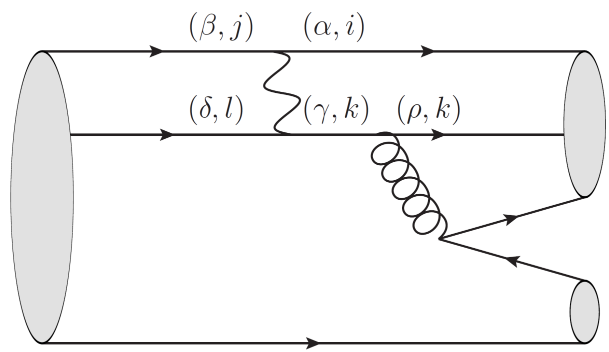
Pati-Woo theorem

The quark pair in a baryon produced by weak interactions is required to be antisymmetric in flavor.

J. C. Pati and C. H. Woo, Phys. Rev. D 3, 2920 (1971)

$$\{\overline{\psi}_{\alpha,i}(x)\gamma_\mu(1-\gamma_5)\psi_{\beta,j}(x)\}\{\overline{\psi}_{\gamma,k}(x)\gamma^\mu(1-\gamma_5)\psi_{\delta,l}(x)\}$$
$$=\{\overline{\psi}_{\gamma,k}(x)\gamma_\mu(1-\gamma_5)\psi_{\beta,j}(x)\}\{\overline{\psi}_{\alpha,i}(x)\gamma^\mu(1-\gamma_5)\psi_{\delta,l}(x)\}.$$

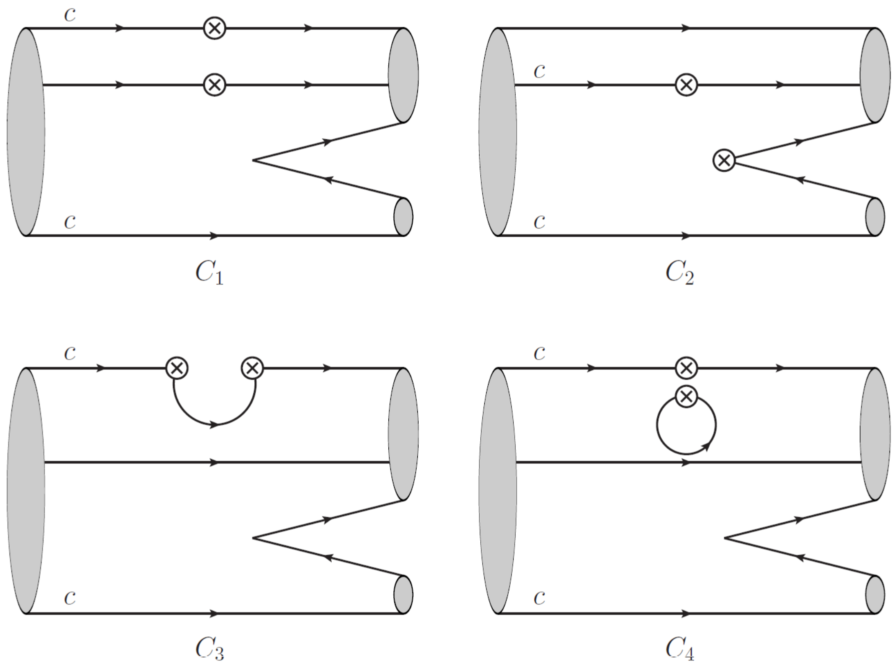
According to the Pati-Woo theorem, many diagrams in baryon decays, such as those with decuplet in final state.



The Pati-Woo theorem is invalid considering gluon exchange

$\mathcal{B}_{cc} \to \mathcal{B}_{10} D$

- 4 diagrams
- 2 diagrams without quark loop
- 18 decay channels



I, *U*, *V*-spin symmetry

- The degree of approximation of *I*, *U*, *V*-spin symmetries are different
  - Test in experiments
- If all amplitudes relations of three $SU(2)$ groups for one type of decay are found, all the amplitudes relations of $SU(3)$ group are found

$$I_\pm = T_1 \pm iT_2, \quad I_3 = T_3$$

$$U_\pm = T_6 \pm iT_7, \quad U_3 = \frac{\sqrt{3}}{2}T_8 - \frac{1}{2}T_3$$

$$V_\pm = T_4 \pm iT_5, \quad V_3 = \frac{\sqrt{3}}{2}T_8 + \frac{1}{2}T_3$$

U-spin symmetry

$$\sin^2\theta\mathcal{A}(\Lambda_c^+\to p\overline{K}^{*0})+\sin^2\theta\mathcal{A}(\Xi_c^+\to\Sigma^+\overline{K}^{*0})$$
$$+\sin\theta\mathcal{A}(\Lambda_c^+\to\Sigma^+K^{*0})=0$$

$$\mathcal{B}r(\Lambda_c^+\to p\overline{K}^{*0})=(1.98\pm0.28)\%,$$
$$\mathcal{B}r(\Lambda_c^+\to\Sigma^+K^{*0})=(0.36\pm0.10)\%.$$

$$(6.00\pm2.20)\%<\mathcal{B}r(\Xi_c^+\to\Sigma^+\overline{K}^{*0})<(39.1\pm6.2)\%$$

$$\mathcal{A}(\Lambda_c^+\to\Sigma^+K^{*0})-\mathcal{A}(\Xi_c^+\to p\overline{K}^{*0})=0$$

$$\mathcal{B}r(\Xi_c^+\to p\overline{K}^{*0})=(1.20\pm0.34)\%$$

CP violation sum rules

$$A_{CP}(\Lambda_c^+ \to \Delta^0\pi^+) + A_{CP}(\Xi_c^+ \to \Xi^{*0}K^+) = 0,$$
$$A_{CP}(\Lambda_c^+ \to \Sigma^{*+}K_S^0) + A_{CP}(\Xi_c^+ \to \Delta^+K_S^0) = 0,$$
$$A_{CP}(\Lambda_c^+ \to \Sigma^{*0}K^+) + A_{CP}(\Xi_c^+ \to \Sigma^{*0}K^+) = 0,$$
$$A_{CP}(\Lambda_c^+ \to \Delta^{++}\pi^-) + A_{CP}(\Xi_c^+ \to \Delta^{++}K^-) = 0,$$
$$A_{CP}(\Xi_c^0 \to \Sigma^{*-}\pi^+) + A_{CP}(\Xi_c^0 \to \Xi^{*-}K^+) = 0,$$
$$A_{CP}(\Xi_c^0 \to \Delta^0K_S^0) + A_{CP}(\Xi_c^0 \to \Xi^{*0}K_S^0) = 0,$$
$$A_{CP}(\Xi_c^0 \to \Sigma^{*+}\pi^-) + A_{CP}(\Xi_c^0 \to \Delta^+K^-) = 0.$$

$$A_{CP}(\Lambda_c^+ \to \Sigma^+K_S^0) + A_{CP}(\Xi_c^+ \to pK_S^0) = 0,$$
$$A_{CP}(\Lambda_c^+ \to n\pi^+) + A_{CP}(\Xi_c^+ \to \Xi^0K^+) = 0,$$
$$A_{CP}(\Xi_c^0 \to \Sigma^-\pi^+) + A_{CP}(\Xi_c^0 \to \Xi^-K^+) = 0,$$
$$A_{CP}(\Xi_c^0 \to nK_S^0) + A_{CP}(\Xi_c^0 \to \Xi^0K_S^0) = 0,$$
$$A_{CP}(\Xi_c^0 \to \Sigma^+\pi^-) + A_{CP}(\Xi_c^0 \to pK^-) = 0.$$

CP violation sum rules

$$A_{CP}(\Xi_{cc}^{++}\to\Lambda_c^+\pi^+)+A_{CP}(\Xi_{cc}^{++}\to\Xi_c^+K^+)=0,$$
$$A_{CP}(\Xi_{cc}^{+}\to\Xi_c^+K_S^0)+A_{CP}(\Omega_{cc}^{+}\to\Lambda_c^+K_S^0)=0,$$
$$A_{CP}(\Xi_{cc}^{+}\to\Xi_c^0K^+)+A_{CP}(\Omega_{cc}^{+}\to\Xi_c^0\pi^+)=0.$$

$$A_{CP}(\Xi_{cc}^{++}\to\Sigma_c^+\pi^+)+A_{CP}(\Xi_{cc}^{++}\to\Xi_c^{*+}K^+)=0,$$
$$A_{CP}(\Xi_{cc}^{+}\to\Sigma_c^{++}\pi^-)+A_{CP}(\Omega_{cc}^{+}\to\Sigma_c^{++}K^-)=0,$$
$$A_{CP}(\Xi_{cc}^{+}\to\Xi_c^{*0}\pi^+)+A_{CP}(\Omega_{cc}^{+}\to\Omega_c^0K^+)=0,$$
$$A_{CP}(\Xi_{cc}^{+}\to\Xi_c^{*+}K_S^0)+A_{CP}(\Omega_{cc}^{+}\to\Sigma_c^+K_S^0)=0,$$
$$A_{CP}(\Xi_{cc}^{+}\to\Xi_c^{*0}K^+)+A_{CP}(\Omega_{cc}^{+}\to\Xi_c^{*0}\pi^+)=0.$$

$$A_{CP}(\Xi_{cc}^{++}\to\Sigma^+D_s^+)+A_{CP}(\Xi_{cc}^{++}\to pD^+)=0,$$
$$A_{CP}(\Xi_{cc}^{+}\to pD^0)+A_{CP}(\Omega_{cc}^{+}\to\Sigma^+D^0)=0,$$
$$A_{CP}(\Xi_{cc}^{+}\to nD^+)+A_{CP}(\Omega_{cc}^{+}\to\Xi^0D_s^+)=0.$$

Summary

Summary

- The topologies can be formalized as tensor contractions between hadrons and four-fermion operators.
- The sole difference between TDA and IRA methods is weather the tensor operator is decomposed into $SU(3)$ irreducible representation or not.
- The fact that some topologies are not independent can be understood in group theory.
- The topologies are classified according to which operators(tree or penguin) being inserted into the effective weak vertex.

Summary

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- A self-consistent scheme of the topological amplitude and its $SU(N)$ decomposition is built. Application: strangeless charm decay, charmless beauty decay, heavy baryon decay, flavor $SU(N)$ breaking effect.

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- A self-consistent scheme of the topological amplitude and its $SU(N)$ decomposition is built. Application: strangeless charm decay, charmless beauty decay, heavy baryon decay, flavor $SU(N)$ breaking effect.

Thanks for your attention !

Backup: strangeless *D* decays

- For strange-less charm decay, the isospin symmetry is a good approximation. There are one D meson doublet $D^i = (D^0, D^+)$ and one light pseudoscalar quartet in the strange-less charm decay:

$$(P)^i_j = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 & \pi^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_q & 0 \\ 0 & \eta_q \end{pmatrix}.$$

- One probably write down the assemble of tensor operators $\mathcal{H}^k_{ij}$ as the effective Hamiltonian of strange-less charm decay. But $\mathcal{H}^k_{ij}$ is not enough. Tensor $\mathcal{H}^k_{ij}$ means that all the indices i, j and k must transform as the foundational or conjugate representation of $SU(2)$ group. So $\mathcal{H}^k_{ij}$ cannot contain s-quark loop.
- $\mathcal{H}^s_{is}$ and $\mathcal{H}^s_{si}$: only one index transforms as the foundational representation of $SU(2)$ group

Backup: strangeless D decays

With $\mathcal{H}_{ij}^{k}$, $\mathcal{H}_{is}^{s}$ and $\mathcal{H}_{si}^{s}$, the amplitude in strange-less charm decay can be written as

$$\begin{aligned}
\mathcal{A}_{s-less}^{TDA} = {} & TD^i\mathcal{H}_{lj}^{k}(P)_i^j(P)_k^l + CD^i\mathcal{H}_{jl}^{k}(P)_i^j(P)_k^l + ED^i\mathcal{H}_{il}^{j}(P)_j^k(P)_k^l \\
& + AD^i\mathcal{H}_{li}^{j}(P)_j^k(P)_k^l + T^{ES}D^i\mathcal{H}_{ij}^{l}(P)_l^j(P)_k^k + T^{AS}D^i\mathcal{H}_{ji}^{l}(P)_l^j(P)_k^k \\
& + T^{LP}D^i\mathcal{H}_{kl}^{l}(P)_i^j(P)_j^k + T^{LC}D^i\mathcal{H}_{jl}^{l}(P)_i^j(P)_k^k + T^{LA}D^i\mathcal{H}_{il}^{l}(P)_j^k(P)_k^j \\
& + T^{LS}D^i\mathcal{H}_{il}^{l}(P)_j^j(P)_k^k + T^{QP}D^i\mathcal{H}_{lk}^{l}(P)_i^j(P)_j^k + T^{QC}D^i\mathcal{H}_{lj}^{l}(P)_i^j(P)_k^k \\
& + T^{QA}D^i\mathcal{H}_{li}^{l}(P)_j^k(P)_k^j + T^{QS}D^i\mathcal{H}_{li}^{l}(P)_j^j(P)_k^k \\
& + T_s^{LP}D^i\mathcal{H}_{ks}^{s}(P)_i^j(P)_j^k + T_s^{LC}D^i\mathcal{H}_{js}^{s}(P)_i^j(P)_k^k + T_s^{LA}D^i\mathcal{H}_{is}^{s}(P)_j^k(P)_k^j \\
& + T_s^{LS}D^i\mathcal{H}_{is}^{s}(P)_j^j(P)_k^k + T_s^{QP}D^i\mathcal{H}_{sk}^{s}(P)_i^j(P)_j^k + T_s^{QC}D^i\mathcal{H}_{sj}^{s}(P)_i^j(P)_k^k \\
& + T_s^{QA}D^i\mathcal{H}_{si}^{s}(P)_j^k(P)_k^j + T_s^{QS}D^i\mathcal{H}_{si}^{s}(P)_j^j(P)_k^k.
\end{aligned}$$

Backup: strangeless D decays

- As an example, we write down the decay amplitude of $D^0 \to \pi^+\pi^-$:

$$\mathcal{A}(D^0 \to \pi^+\pi^-) = \lambda_d(T+E) + \lambda_d(T^{LP}+2T^{LA}) + \lambda_s(T_s^{LP}+2T_s^{LA})$$
$$-\lambda_b(PC+PE+2PA+P^{LP}+2P^{LA}+2P^{QP}+4P^{QA}+P_s^{QP}+2P_s^{QA})$$

  - ☞ The difference of s-quark loop and u/d-quark loop is ignored $\Rightarrow$ the result in the flavor $SU(3)$ symmetry.

- $SU(2)$ decomposition

$$\mathcal{H}_{ij}^k = \delta_j^k\Big(\frac{2}{3}\mathcal{H}(2_t)_i - \frac{1}{3}\mathcal{H}(2_p)_i\Big) + \delta_i^k\Big(\frac{2}{3}\mathcal{H}(2_p)_j - \frac{1}{3}\mathcal{H}(2_t)_j\Big) + \mathcal{H}(4)_{ij}^k.$$

  - ☞ The tensor operators $\mathcal{H}_{is}^s$ and $\mathcal{H}_{si}^s$ are irreducible representations of $SU(2)$ group themselves

Backup: strangeless D decays

The $SU(2)$ irreducible representation amplitude of the strange-less charm decay is expressed as

$$
\begin{aligned}
\mathcal{A}^{IRA}_{s-less} = {} & a^{p}_{2} D^{i}\mathcal{H}(2_p)_i(P)^j_k(P)^k_j + b^{p}_{2} D^{i}\mathcal{H}(2_p)_i(P)^k_k(P)^j_j + c^{p}_{2} D^{i}\mathcal{H}(2_p)_k(P)^k_i(P)^j_j \\
& + d^{p}_{2} D^{i}\mathcal{H}(2_p)_k(P)^j_i(P)^k_j + a^{t}_{2} D^{i}\mathcal{H}(2_t)_i(P)^j_k(P)^k_j + b^{t}_{2} D^{i}\mathcal{H}(2_t)_i(P)^k_k(P)^j_j \\
& + c^{t}_{2} D^{i}\mathcal{H}(2_t)_k(P)^k_i(P)^j_j + d^{t}_{2} D^{i}\mathcal{H}(2_t)_k(P)^j_i(P)^k_j \\
& + a_4 D^{i}\mathcal{H}(4)^k_{ij}(P)^j_l(P)^l_k + b_4 D^{i}\mathcal{H}(4)^k_{ij}(P)^j_k(P)^l_l + c_4 D^{i}\mathcal{H}(4)^k_{jl}(P)^j_i(P)^l_k \\
& + a'_2 D^{i}\mathcal{H}(2')_i(P)^j_k(P)^k_j + b'_2 D^{i}\mathcal{H}(2')_i(P)^k_k(P)^j_j + c'_2 D^{i}\mathcal{H}(2')_k(P)^k_i(P)^j_j \\
& + d'_2 D^{i}\mathcal{H}(2')_k(P)^j_i(P)^k_j + a''_2 D^{i}\mathcal{H}(2'')_i(P)^j_k(P)^k_j + b''_2 D^{i}\mathcal{H}(2'')_i(P)^k_k(P)^j_j \\
& + c''_2 D^{i}\mathcal{H}(2'')_k(P)^k_i(P)^j_j + d''_2 D^{i}\mathcal{H}(2'')_k(P)^j_i(P)^k_j .
\end{aligned}
$$

Backup: strangeless D decays

$$a_4 = E + A, \qquad b_4 = T^{ES} + T^{AS}, \qquad c_4 = T + C,$$

$$a_2^t = \frac{2}{3}E - \frac{1}{3}A + T^{LA}, \qquad a_2^p = -\frac{1}{3}E + \frac{2}{3}A + T^{QA},$$

$$b_2^t = \frac{2}{3}T^{ES} - \frac{1}{3}T^{AS} + T^{LS}, \qquad b_2^p = -\frac{1}{3}T^{ES} + \frac{2}{3}T^{AS} + T^{QS},$$

$$c_2^t = -\frac{1}{3}T + \frac{2}{3}C - \frac{1}{3}T^{ES} + \frac{2}{3}T^{AS} + T^{LC},$$

$$c_2^p = \frac{2}{3}T - \frac{1}{3}C + \frac{2}{3}T^{ES} - \frac{1}{3}T^{AS} + T^{QC},$$

$$d_2^t = \frac{2}{3}T - \frac{1}{3}C - \frac{1}{3}E + \frac{2}{3}A + T^{LP},$$

$$d_2^p = -\frac{1}{3}T + \frac{2}{3}C + \frac{2}{3}E - \frac{1}{3}A + T^{QP},$$

$$a_2' = T_s^{LA}, \qquad b_2' = T_s^{LS}, \qquad c_2' = T_s^{LC}, \qquad d_2' = T_s^{LP},$$

$$a_2'' = T_s^{QA}, \qquad b_2'' = T_s^{QS}, \qquad c_2'' = T_s^{QC}, \qquad d_2'' = T_s^{QP}.$$

Backup: charmless B decays

- The charmless B meson decay is quite similar to the strangeless D decay
- Amplitude of $\overline{B}^0 \to \pi^+\pi^-$ decay

$$\mathcal{A}(\overline{B}^0 \to \pi^+\pi^-) =$$
$$V_{ub}V_{ud}^*(T+E) + V_{ub}V_{ud}^*(T^{LP}+2T^{LA}) + V_{cb}V_{cd}^*(T_c^{LP}+2T_c^{LA}) -$$
$$V_{tb}V_{td}^*(PC+PE+2PA+P^{LP}+2P^{LA}+2P^{QP}+4P^{QA}+P_c^{QP}+2P_c^{QA})$$

- The difference of c-quark loop and $u/d/s$-quark loop is ignored, the result in the flavor $SU(4)$ symmetry:

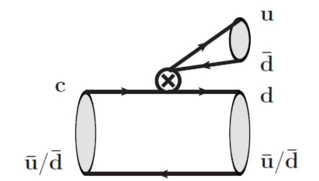
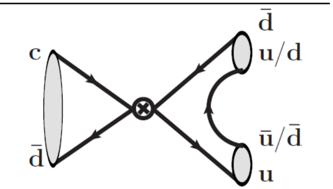
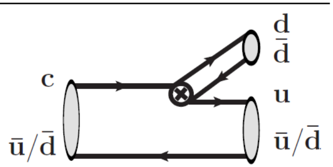
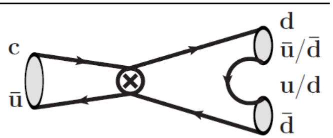
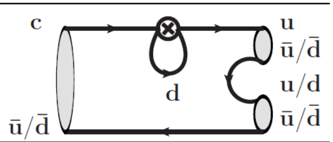
$$\mathcal{A}(\overline{B}^0 \to \pi^+\pi^-) = V_{ub}V_{ud}^*(T+E) + (V_{ub}V_{ud}^* + V_{cb}V_{cd}^*)(T^{LP}+2T^{LA})$$
$$- V_{tb}V_{td}^*(PC+PE+2PA+P^{LP}+2P^{LA}+3P^{QP}+6P^{QA}).$$

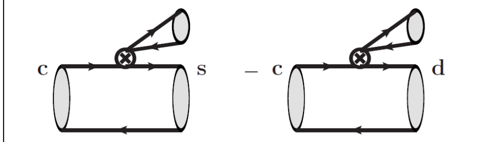
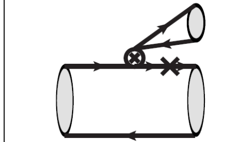
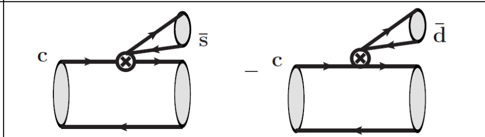
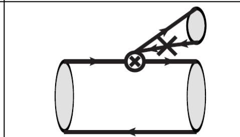
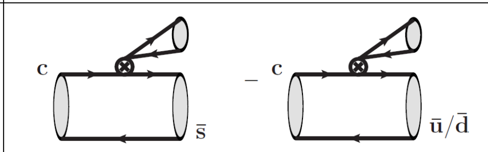
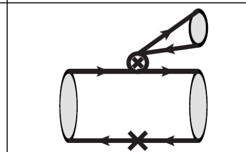
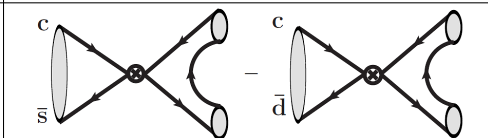
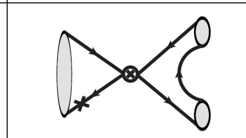
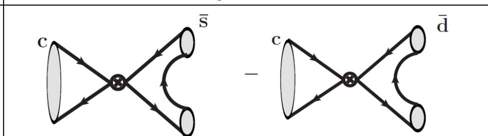
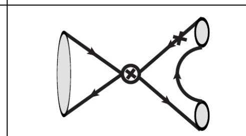
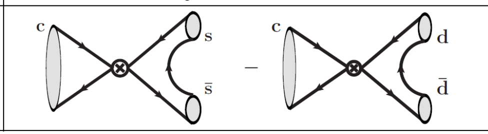
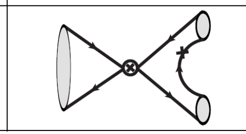
- Equivalence relation of TDA and IRA: $\mathcal{H}_{ij}^k$, $\mathcal{H}_{ci}^c$, $\mathcal{H}_{ic}^c$

Backup: degeneration and splitting

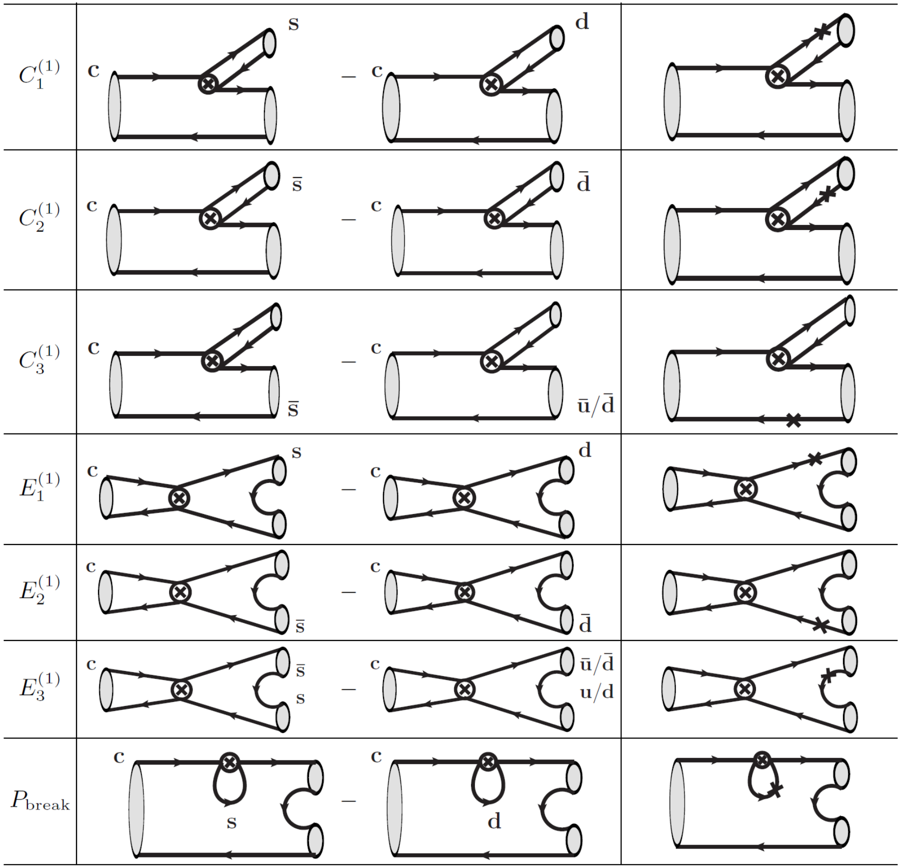
- Energy level degeneration and splitting in atomic or nuclear physics
- Strangeless charm decay:
  - In the flavor $SU(3)$ symmetry, the u, d-quark loops and s-quark loop are degenerate in charm decays.
  - If the flavor $SU(3)$ symmetry is broken into isospin $SU(2)$ symmetry, the identical u, d, s-quark loops turn into unequal u, d-quark loops and s-quark loop.
- Charmless B decay.
  - $SU(3) \to SU(4)$
  - u, d-quark loops $\to u, d, s$-quark loops; s-loop $\to c$-loop
  - $SU(2)$ symmetry in B decays: $SU(4) \to SU(3) \to SU(2)$
- K decay: flavor $SU(2)$ symmetry

Backup: linear $SU(3)_F$ breaking

Name	Diagrams
T	
A	
C	
E	
P_d	

Name	$s-d$ difference of topologies	denoted by Feynman rule
$T_1^{(1)}$		
$T_2^{(1)}$		
$T_3^{(1)}$		
$A_1^{(1)}$		
$A_2^{(1)}$		
$A_3^{(1)}$		

Backup: linear $SU(3)_F$ breaking



S. Mjller, U. Nierste and S. Schacht, Phys. Rev. D 92, no. 1, 014004 (2015)

Backup: linear $SU(3)_F$ breaking

$$
\begin{aligned}
p = & (T,\ T_1^{(1)},\ T_2^{(1)},\ T_3^{(1)},\ A,\ A_1^{(1)},\ A_2^{(1)},\ A_3^{(1)}, \\
& C,\ C_1^{(1)},\ C_2^{(1)},\ C_3^{(1)},\ E,\ E_1^{(1)},\ E_2^{(1)},\ E_3^{(1)},\ P_{\rm break})
\end{aligned}
$$

The amplitude of $D \to PP$ decay can be obtaned by summing all possible form that the index 3 is written explicitly and non-repetitive and giving a parameter for each term:

$$
\begin{aligned}
\mathcal{A}^{TDA}_{\not{SU(3)_F}} = & T D^i \mathcal{H}^k_{lj} (P)^j_i (P)^l_k + T_1^{(1)} D^i \mathcal{H}^k_{l3} (P)^3_i (P)^l_k + T_2^{(1)} D^i \mathcal{H}^3_{lj} (P)^j_i (P)^l_3 + T_3^{(1)} D^3 \mathcal{H}^k_{lj} (P)^j_l (P)^3_k \\
& + C D^i \mathcal{H}^k_{jl} (P)^j_i (P)^l_k + C_1^{(1)} D^i \mathcal{H}^k_{j3} (P)^j_i (P)^3_k + C_2^{(1)} D^i \mathcal{H}^3_{jl} (P)^j_i (P)^l_3 + C_3^{(1)} D^3 \mathcal{H}^k_{jl} (P)^j_3 (P)^l_k \\
& + E D^i \mathcal{H}^j_{il} (P)^k_j (P)^l_k + E_1^{(1)} D^i \mathcal{H}^j_{i3} (P)^k_j (P)^3_k + E_2^{(1)} D^i \mathcal{H}^3_{il} (P)^k_3 (P)^l_k + E_3^{(1)} D^i \mathcal{H}^j_{il} (P)^3_j (P)^l_3 \\
& + A D^i \mathcal{H}^j_{li} (P)^k_j (P)^l_k + A_1^{(1)} D^3 \mathcal{H}^j_{l3} (P)^k_j (P)^l_k + A_2^{(1)} D^i \mathcal{H}^3_{li} (P)^k_3 (P)^l_k + A_3^{(1)} D^i \mathcal{H}^j_{li} (P)^3_j (P)^l_3 \\
& \quad + T^{LP}_{\rm break} D^i \mathcal{H}^3_{k3} (P)^j_i (P)^k_j ,
\end{aligned}
$$

The terms are identical to the topological diagrams one by one.

S. Mjller, U. Nierste and S. Schacht, Phys. Rev. D 92, no. 1, 014004 (2015)

Backup: high order U-spin breaking

- A perturbative method to deal with U-spin breaking
  - M. Gronau, Phys. Lett. B 730, 221 (2014) Addendum: [Phys. Lett. B 735, 282 (2014)]
    - The corrections of arbitrary order to decay amplitude $\langle f|\mathcal{H}_{eff}|D^0\rangle$ are obtained by introducing in the Hamiltonian or in the final state powers of an $s-d$ spurion mass operator, $(m_{Ub})^s_s - (m_{Ub})^d_d = (m_{Ub})^2_2 - (m_{Ub})^1_1$, where $d=1$ and $s=2$.
    - Since The two indices of the $s-d$ spurion mass operator are transformed as the representation of U-spin $SU(2)$ group, we can write it as $(m_{Ub})^i_j$. Its non-zero components include $(m_{Ub})^1_1 = -1$ and $(m_{Ub})^2_2 = 1$.
    - $(m_{Ub})^i_j$ is a tensor operator

Backup: high order U-spin breaking

- Decay amplitude of D^0 decay

$$\mathcal{A}_{D^0}^{TDA} = A\, D^0 \mathcal{H}_{ui}^{j} (M_u)^i (M^u)_j + A^L\, D^0 \mathcal{H}_{ui}^{i} (M_u)^j (M^u)_j$$
$$+ A\, D^0 \Big[\sum_p (\mathcal{H}_u m_{Ub}^n)_{ia_1\ldots b_n}^{jb_1\ldots b_n}\, \varepsilon_p^{(n)} \Big] (M_u)^i (M^u)_j + A^L\, D^0 \Big[\sum_p (\mathcal{H}_u m_{Ub}^n)_{ia_1\ldots b_n}^{ib_1\ldots b_n}\, \varepsilon_p^{(n)} \Big] (M_u)^j (M^u)_j$$

- First order:

$$\mathcal{A}_{D^0}^{TDA} = A\, D^0 (\mathcal{H}_u)_i^j (M_u)^i (M^u)_j + A\, \varepsilon_1^{(1)} D^0 (\mathcal{H}_u)_i^k (m_{Ub})_k^j\, (M_u)^i (M^u)_j$$
$$+ A\, \varepsilon_2^{(1)} D^0 (\mathcal{H}_u)_k^j (m_{Ub})_j^k\, (M_u)^i (M^u)_j + A^L\, \varepsilon_3^{(1)} D^0 (\mathcal{H}_u)_i^k (m_{Ub})_k^i\, (M_u)^j (M^u)_j$$

- Examples:

$$\mathcal{A}(D^0 \to K^-\pi^+) = \cos^2\theta_C A(1 - \varepsilon_1^{(1)} + \varepsilon_2^{(1)}),$$
$$\mathcal{A}(D^0 \to K^+\pi^-) = -\sin^2\theta_C A(1 + \varepsilon_1^{(1)} - \varepsilon_2^{(1)}),$$
$$\mathcal{A}(D^0 \to K^+K^-) = \cos\theta_C \sin\theta_C A(1 + \varepsilon_1^{(1)} + \varepsilon_2^{(1)}) + 2\cos\theta_C \sin\theta_C A^L \varepsilon_3^{(1)},$$
$$\mathcal{A}(D^0 \to \pi^+\pi^-) = -\cos\theta_C \sin\theta_C A(1 - \varepsilon_1^{(1)} - \varepsilon_2^{(1)}) + 2\cos\theta_C \sin\theta_C A^L \varepsilon_3^{(1)}.$$

M. Gronau, Phys. Lett. B 730, 221 (2014)