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Form factors of double-heavy-baryon weak decays

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Outline

- Introduction
- Doubly heavy baryons weak decay in QCDSR
- Numerical results
- Doubly heavy baryons weak decay in LCSR
- Summary

Introduction

Selected for a Viewpoint in *Physics*

PRL **119,** 112001 (2017) PHYSICAL REVIEW LETTERS week ending 15 SEPTEMBER 2017

Observation of the Doubly Charmed Baryon Ξ_{cc}^{++}

R. Aaij *et al.**

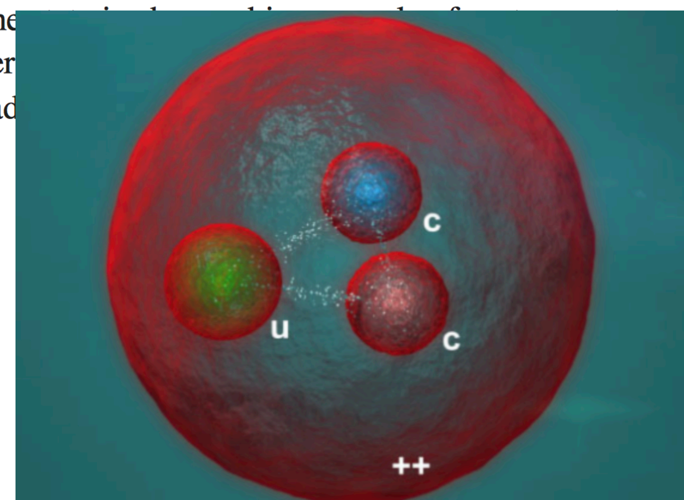
(LHCb Collaboration)

(Received 6 July 2017; revised manuscript received 2 August 2017; published 11 September 2017)

A highly significant structure is observed in the $\Lambda_c^+ K^- \pi^+ \pi^+$ mass spectrum, where the Λ_c^+ baryon is reconstructed in the decay mode $pK^-\pi^+$. The structure is consistent with originating from a weakly decaying particle, identified as the doubly charmed baryon Ξ_{cc}^{++}. The difference between the masses of the Ξ_{cc}^{++} and Λ_c^+ states is measured to be $1334.94 \pm 0.72(\text{stat.}) \pm 0.27(\text{syst.})$ MeV/c^2, and the Ξ_{cc}^{++} mass is then determined to be $3621.40 \pm 0.72(\text{stat.}) \pm 0.27(\text{syst.}) \pm 0.14(\Lambda_c^+)$ MeV/c^2, where the last uncertainty is due to the limited knowledge of the Λ_c^+ mass. The

collision data collected by the LHCb experiment at a center

integrated luminosity of 1.7 fb^{-1}, and confirmed in an ad



DOI: 10.1103/PhysRevLett.119.112001

Previous work on doubly heavy baryon

- LFQM:

  Weak decays of doubly heavy baryons: the 1/2 -> 1/2 case

  Weak decays of doubly heavy baryons: the 1/2 -> 3/2 case

  Weak decays of doubly heavy baryons: the FCNC processes

- SU(3) analysis

  Weak Decays of Doubly Heavy Baryons: SU(3) Analysis

  Weak Decays of Doubly Heavy Baryons: Multi-Body decay channels

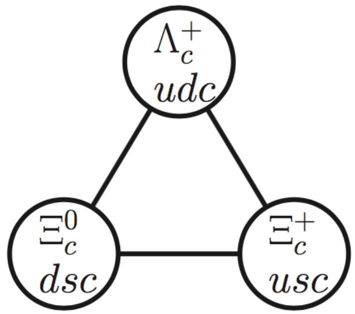
- QCDSR

  Weak decays of doubly heavy baryons: decay constants

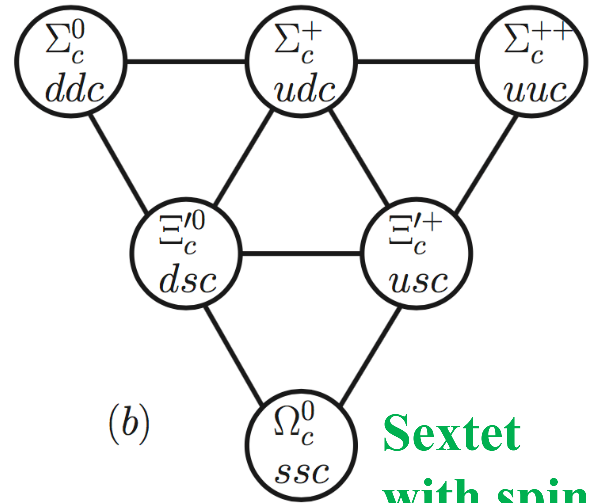
  Weak decays of doubly heavy baryons: the form factors

Doubly heavy baryon weak decays

Ξ_{cc}^{++} →



(a) **Antitriplet**



(b) **Sextet with spin 1/2**

The hadron transformation is parametrized by 6 form factors:

$$
\begin{aligned}
\langle p_h, s'|\bar{q}(0)\gamma_\mu(1-\gamma_5)Q(0)|p_H, s\rangle &= \bar{u}_h(p_h, s')(j_\mu^V - j_\mu^A)u_H(p_H, s) \\
&= \bar{u}_h(p_h, s')\left[f_1(q^2)\gamma_\mu + f_2(q^2)\frac{p_{h\mu}}{m_h} + f_3(q^2)\frac{p_{H\mu}}{m_H}\right]u_H(p_H, s) \\
&\quad -\bar{u}_h(p_h, s')\left[g_1(q^2)\gamma_\mu + g_2(q^2)\frac{p_{h\mu}}{m_h} + g_3(q^2)\frac{p_{H\mu}}{m_H}\right]\gamma_5 u_H(p_H, s)
\end{aligned}
$$

Form factors in QCDSR

Basic assumpsion: **Quark-Hadron Duality**

$$\Pi_\mu(p_1^2, p_2^2, q^2) = i^2 \int d^4x d^4y e^{ip_2\cdot x - ip_1\cdot y} \langle 0|T\{j_h(x) j_\mu(0) \bar{j}_H(y)\}|0\rangle$$

Hadron level **Quark level**

$$\frac{f_h f_H m_h m_H}{(p_2^2 - m_h^2)(p_1^2 - m_H^2)} f_1(q^2) + \int_{s_0^{(1)}} ds_1 \int_{s_0^{(2)}} ds_2 \frac{\rho^h(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)} = \int_{(M_1+M_2)^2} ds_1 \int_{M_2^2} ds_2 \frac{\rho^{QCD}(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)}$$

$$\frac{f_h f_H m_h m_H}{(p_2^2 - m_h^2)(p_1^2 - m_H^2)} f_1(q^2) = \int_{(M_1+M_2)^2}^{s_0^{(1)}} ds_1 \int_{M_2^2}^{s_0^{(2)}} ds_2 \frac{\rho^{QCD}(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)}$$

Hadron Level

$$\Pi_\mu(p_1^2,p_2^2,q^2)=i^2\int d^4x d^4y e^{ip_2\cdot x-ip_1\cdot y}\langle 0|T\{j_h(x)j_\mu(0)\bar{j}_H(y)\}|0\rangle$$

Insert identity operators 1 1

$$I = \sum_{s_2}\int\frac{d^4p_2}{(2\pi)^4}\frac{i}{p_2^2-M_2^2}|\Xi_{sQu}(p_2)\rangle\langle\Xi_{sQu}(p_2)|+\cdots$$

$$I = \sum_{s_1}\int\frac{d^4p_1}{(2\pi)^4}\frac{i}{p_1^2-M_1^2}|\Xi_{QQu}(p_1)\rangle\langle\Xi_{QQu}(p_1)|+\cdots$$

$$\begin{aligned}\Pi_\mu^{hadron}(p_1^2,p_2^2,q^2)_V &= \frac{f_hf_H}{(p_2^2-m_h^2)(p_1^2-m_H^2)}(\not{p}_2+m_h)[f(q^2)\gamma_\mu+f_2(q^2)p_{h\mu}+f_3(q^2)p_{H\mu}](\not{p}_1+m_H)\\ &= \frac{f_hf_H}{(p_2^2-m_h^2)(p_1^2-m_H^2)}[(m_hm_H\gamma_\mu+m_H\not{p}_2\gamma_\mu+m_h\gamma_\mu\not{p}_1+\not{p}_2\gamma_\mu\not{p}_1)f_1\\ &\quad+(m_hm_Hp_{2\mu}+m_Hp_{2\mu}\not{p}_2+m_hp_{2\mu}\not{p}_1+p_{2\mu}\not{p}_2\not{p}_1)f_2\\ &\quad+(m_hm_Hp_{1\mu}+m_Hp_{1\mu}\not{p}_2+m_hp_{1\mu}\not{p}_1+p_{1\mu}\not{p}_2\not{p}_1)f_3]\end{aligned}$$

Quark Level

Define the quark level currents:

$$\Pi_\mu(p_1^2,p_2^2,q^2)=i^2\int d^4x d^4y e^{ip_2\cdot x-ip_1\cdot y}\langle 0|T\{j_h(x)j_\mu(0)\bar{j}_H(y)\}|0\rangle$$

$$\begin{aligned} j_h(x) &= \epsilon_{abc}(q_{1a}^T(x)C\gamma^\alpha q_{2b}(x))\gamma_\alpha\gamma_5 Q_c(x) \\ j_\mu(0) &= \bar{q}_{1e}(0)\gamma_\mu(1-\gamma_5)Q_e(0) \\ \bar{j}_H(y) &= \epsilon_{a'b'c'}\bar{q}_{2c'}(y)\gamma_\beta\gamma_5(\bar{Q}_b(y)\gamma^\beta C\bar{Q}_a^T(y)) \end{aligned}$$

Use the approach of **OPE**

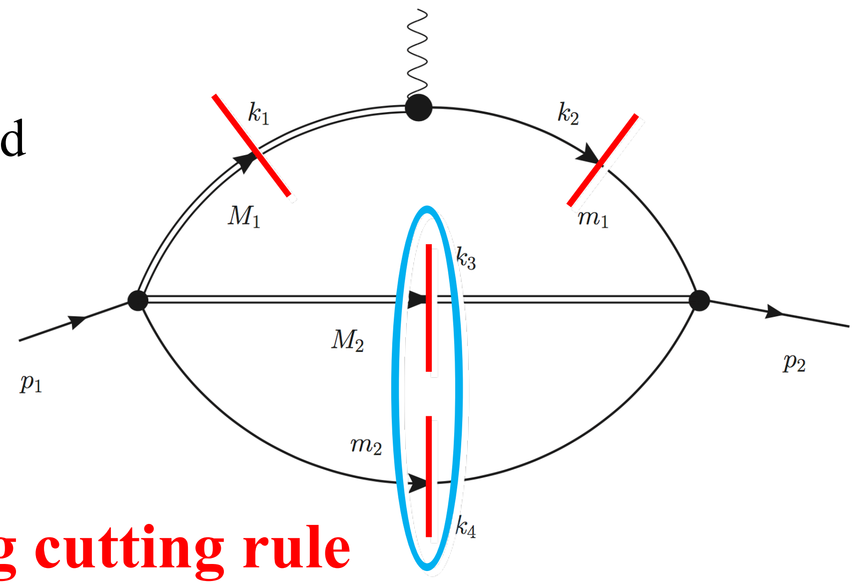
$$\Pi^{QCD}(s_1,s_2,q^2)=\Pi^{pert}(s_1,s_2,q^2)+\Pi^{\bar{q}q}(s_1,s_2,q^2)+\Pi^{GG}(s_1,s_2,q^2)+\Pi^{\bar{q}g_sGq}(s_1,s_2,q^2)$$

Each term should be expressed into doubly dispersion relation form:

$$\Pi_\mu^{pert}(p_1^2,p_2^2,q^2)=\int ds_1 ds_2\frac{\rho_\mu^{pert}(s_1,s_2,q^2)}{(s_1-p_1^2)(s_2-p_2^2)}$$

Perturbative Term

The discontinuity of the diagram is factorized into two independent parts:



$$\rho_\mu^{pert}(s_1, s_2, q^2) = 12 \int ds_{cq} \int d\Phi_\Delta \int d\Phi_2 \cdot N_\mu$$

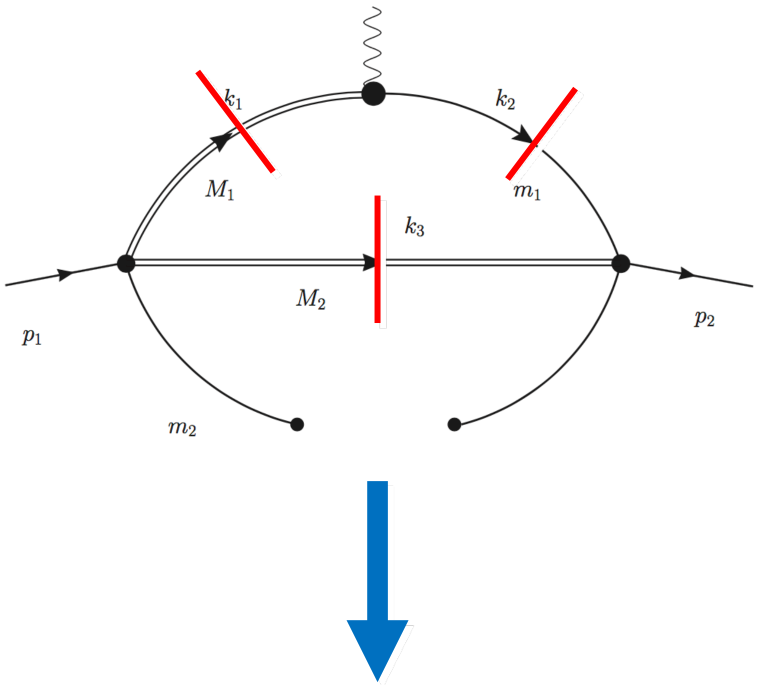
Using cutting rule

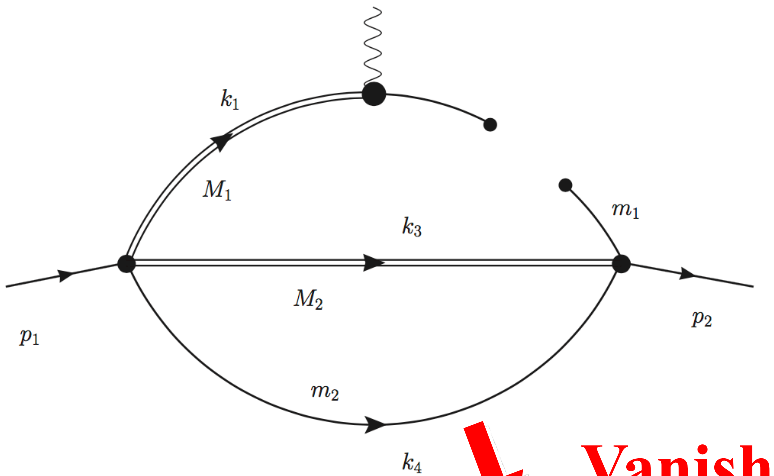
$$\int d\Phi_\Delta = \int d^4k_1 d^4k_2 d^4k_{34} \delta(k_1^2 - M_1^2)\delta(k_2^2 - m_1^2)\delta(k_{34}^2 - s_{cq})\delta^4(p_1 - k_{34} - k_1)\delta^4(p_2 - k_{34} - k_2)$$

$$\int d\Phi_2 = \int \frac{d^3k_3}{(2\pi)^3}\frac{1}{2E_3}\frac{d^3k_4}{(2\pi)^3}\frac{1}{2E_4}\delta^4(k_{34} - k_3 - k_4)$$

$$k_{34} = k_3 + k_4,\ \ s_{cq} = k_{34}^2.$$

qqbar Condensate





Vanish!

$$\sim \int \frac{d^4k_3}{(2\pi)^4} \frac{N_\mu^{\langle \bar{q}_1 q_1 \rangle}}{(q^2 - M_1^2)(k_3^2 - M_2^2)((p_2 - k_3)^2 - m_2^2)}$$

$$\rho_\mu^{\langle \bar{q}_2 q_2 \rangle}(p_1^2, p_2^2, q^2) = \langle \bar{q}q \rangle \frac{1}{2\pi} \int d\Phi_\Delta N_\mu^{\langle \bar{q}_2 q_2 \rangle}$$

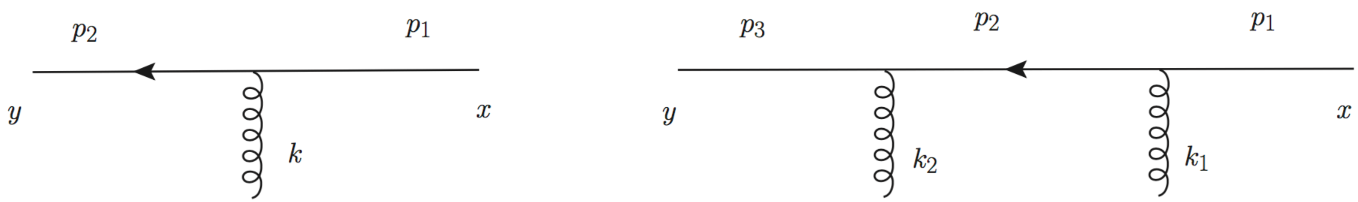
No discontinuity of p1

qqbar condensate:

$$\langle 0|\bar{q}_\alpha^a(x) q_\beta^b(y)|0\rangle = \delta^{ab}\langle \bar{q}q\rangle[\frac{1}{12}\delta_{\beta\alpha} + i\frac{m_q}{48}(\not{x} - \not{y})_{\beta\alpha} - \frac{m_q^2}{96}(x-y)^2\delta_{\beta\alpha}\cdots]$$

qqbarG Condensate

The quark propagators should be modified by the interaction with the background gluon fields.



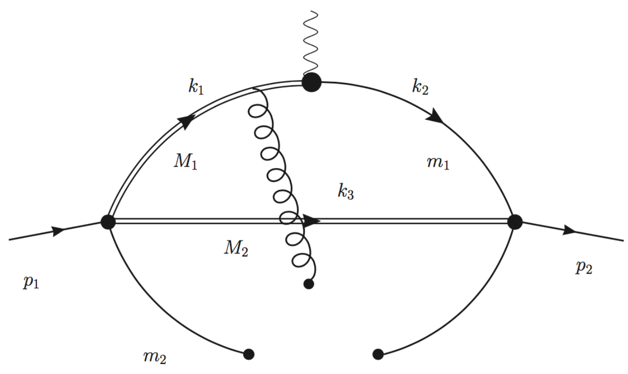
$$S^{ij}_{single}(x,\ y) = ig\int \frac{d^4k}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}\frac{i}{\not{p}_2 - m_Q}\gamma^\mu \frac{i}{\not{p}_2 - \not{k} - m_Q}\tilde{A}^{ij}_\mu(k)e^{-ip_2\cdot(x-y)}e^{-ik\cdot y}$$

$$S^{ij}_{double}(x,\ y) = -g^2\int \frac{d^4p_3}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\frac{d^4k_1}{(2\pi)^4}(\tilde{A}_\mu(k_2)\tilde{A}_\nu(k_1))^{ij}e^{-ip_3\cdot x}e^{i(p_3-k_2-k_1)\cdot y}$$
$$\times\frac{i}{\not{p}_3 - m_Q}\gamma^\mu\frac{i}{\not{p}_3 - \not{k}_2 - m_Q}\gamma^\mu\frac{i}{\not{p}_3 - \not{k}_2 - \not{k}_1 - m_Q}$$

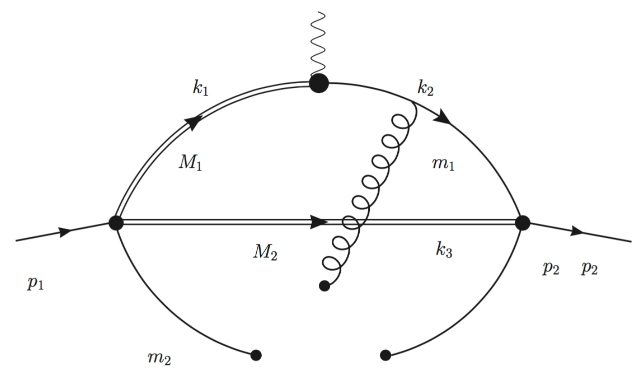
Using the fixed point gauge, the background gluon field is expanded up to leading order:

$$\tilde{A}^a_\rho(k) = -\frac{i}{2}(2\pi)^4 G^a_{\alpha\rho}\frac{\partial}{\partial k_\alpha}\delta^4(k)$$

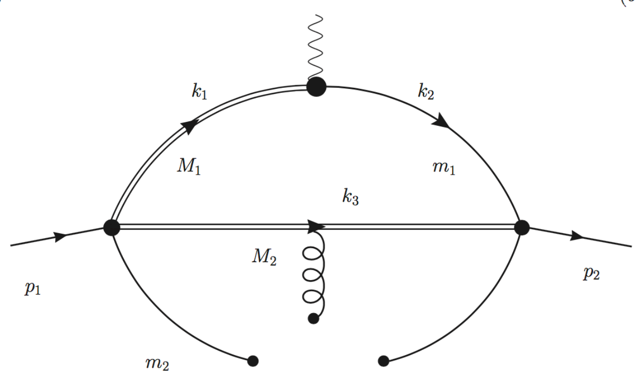
qqbarG Condensate



(a)



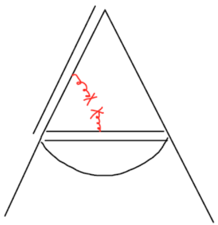
(b)



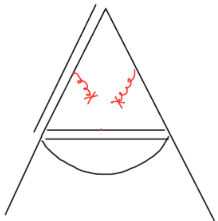
$$\rho_\mu^{\langle\bar q G_1 q\rangle}(p_1^2,p_2^2,q^2) = -\frac{1}{192}tr[T^aT^a]\langle\bar q g\sigma G q\rangle(-2\pi i)^3\frac{1}{(2\pi)^4}\frac{\partial^2}{(\partial M_{1s})^2}\int d\Phi_\Delta N_\mu(k_1^2\to M_{1s})$$

$$\langle q_a^i\bar q_b^j g_s G^c_{\mu\nu}\rangle = -\frac{1}{192}\langle\bar q g_s G q\rangle(\sigma_{\mu\nu})^{ij}(T^c)_{ab}$$

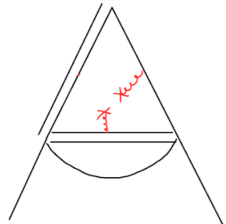
GG Condensate



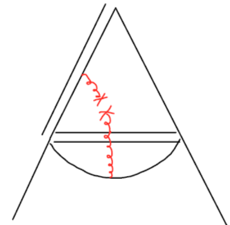
gg1



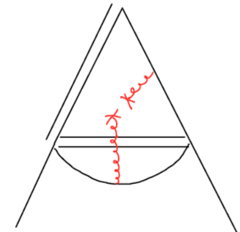
gg2



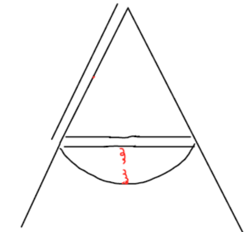
gg3



gg4



gg5



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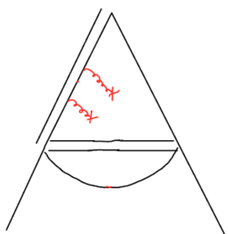
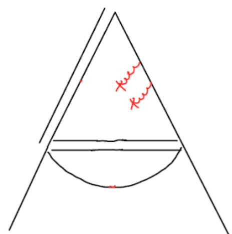
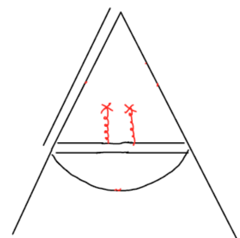


gg7



gg8



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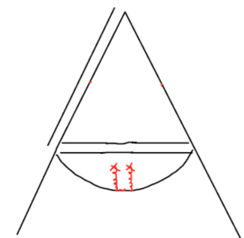

gg10

Form factors

$$\frac{f_h f_H m_h m_H}{(p_2^2-m_h^2)(p_1^2-m_H^2)} f_1(q^2) = \int_{(M_1+M_2)^2}^{s_0^{(1)}} ds_1 \int_{M_2^2}^{s_0^{(2)}} ds_2 \frac{\rho^{QCD}(s_1,s_2,q^2)}{(s_1-p_1^2)(s_2-p_2^2)}$$

Have been explicitly derived.

Conduct Borel transformation:

$$\hat{B}|_{p^2,M^2} \frac{1}{(s-p^2)^k} = \frac{1}{(k-1)!}\frac{1}{(M^2)^k} e^{-s/M^2}$$

$$f_i^{pert}(q^2) = \frac{12\sqrt{2}}{f_h f_H} exp(\frac{m_H^2}{M_1^2}) exp(\frac{m_h^2}{M_2^2}) \int s_{cq}^2 \int_{(M_1+M_2)^2}^{s_0^{(1)}} ds_1 \int_{M_2^2}^{s_0^{(2)}} ds_2 exp(-\frac{s_1}{M_1^2}) exp(-\frac{s_2}{M_2^2}) \left[\int d\Phi_\Delta \int d\Phi_2 \cdot N_{pert}^i\right]$$

$$f_i^{\bar{q}q}(q^2) = -\langle\bar{q}q\rangle \frac{1}{(2\pi)^3} \frac{\sqrt{2}}{f_h f_H} exp(\frac{m_H^2}{M_1^2}) exp(\frac{m_h^2}{M_2^2}) \int_{(M_1+M_2)^2}^{s_0^{(1)}} ds_1 \int_{M_2^2}^{s_0^{(2)}} ds_2 exp(-\frac{s_1}{M_1^2}) exp(-\frac{s_2}{M_2^2}) \int d\Phi_\Delta N_{\langle\bar{q}q\rangle}^i$$

$$f_i^{\bar{q}gGq}(q^2) = -\langle\bar{q}gGq\rangle \frac{1}{(2\pi)^3} \frac{4\sqrt{2}}{192 f_h f_H} exp(\frac{m_H^2}{M_1^2}) exp(\frac{m_h^2}{M_2^2}) \int_{(M_1+M_2)^2}^{s_0^{(1)}} ds_1 \int_{M_2^2}^{s_0^{(2)}} ds_2 exp(-\frac{s_1}{M_1^2}) exp(-\frac{s_2}{M_2^2}) \int d\Phi_\Delta N_{\langle\bar{q}gGq\rangle}^i$$

Numerical Results

$$\Xi_{cc}^{++} \to \Xi_c^{+}$$

$$\langle \mathcal{B}_2(p_2,s_2)|V_\mu|\mathcal{B}_1(p_1,s_1)\rangle = \bar{u}(p_2,s_2)[\gamma_\mu f_1(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{M_2}f_2(q^2) + \frac{q_\mu}{M}f_3(q^2)]u(p_1,s_1)$$

F	$F(0)$	$m_{\rm fit}$	δ	$F(0)$
f_1	1.002 ± 0.096	1.37 ± 0.16	-0.20 ± 0.16	0.914
f_2	0.107 ± 0.033	0.98 ± 0.35	-0.34 ± 0.10	0.012
f_3	-0.822 ± 0.147	0.69 ± 0.15	0.06 ± 0.09	- -

QCDSR **LFQM**

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$$F(q^2) = \frac{F(0)}{1 - \frac{q^2}{m_{\rm fit}^2} + \delta\left(\frac{q^2}{m_{\rm fit}^2}\right)^2}$$

Form factors in LCSR

$$\Pi_\mu(p_\Lambda, q) = i\int d^4x e^{iq\cdot x}\langle\underline{\Lambda_c(p_\Lambda)}|T\{j_{1\mu}(x)\bar{j}_2(0)\}|0\rangle$$

Not vacuum

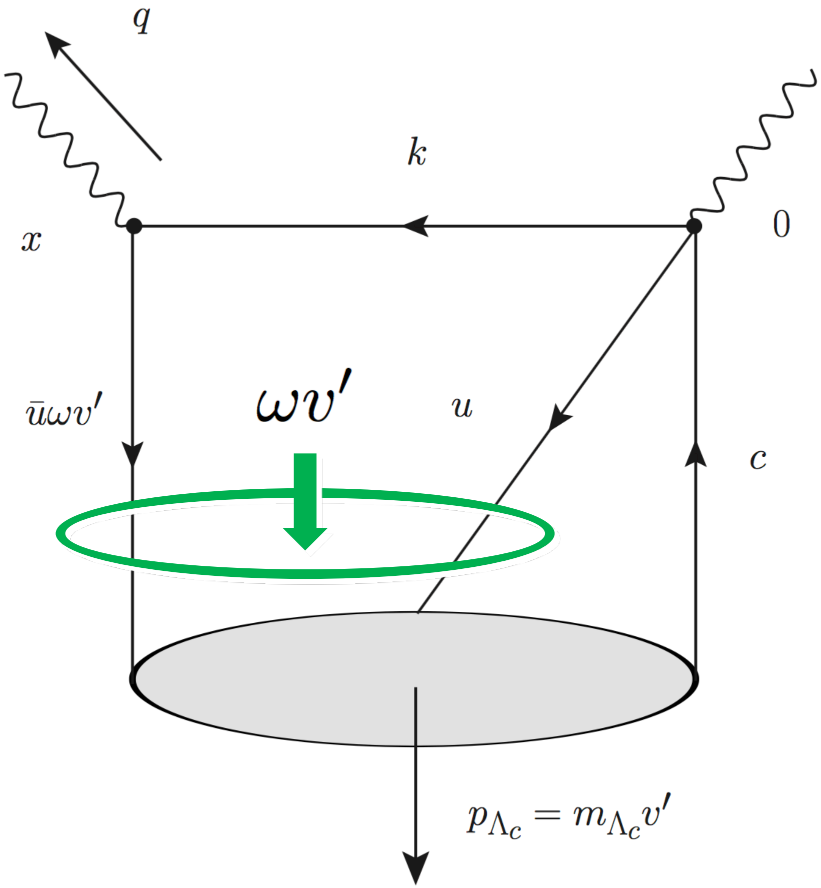
$$j_{1\mu}(x) = \bar{d}_e(x)\gamma_\mu(1-\gamma_5)Q_e(x) \qquad \bar{j}_2(0) = \epsilon_{abc}\bar{u}_c(0)\gamma_\mu\gamma_5(Q_b^T(0)\gamma_\mu C Q_a(0))$$

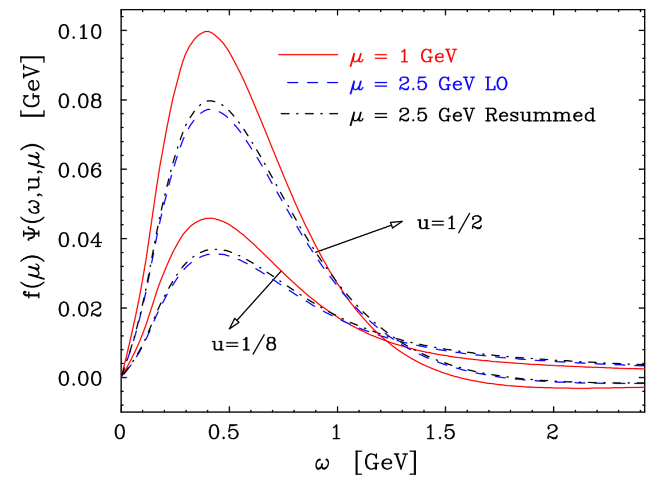
Instead of condensate, for LCSR we use LCDA to produce non-perturbative effects:

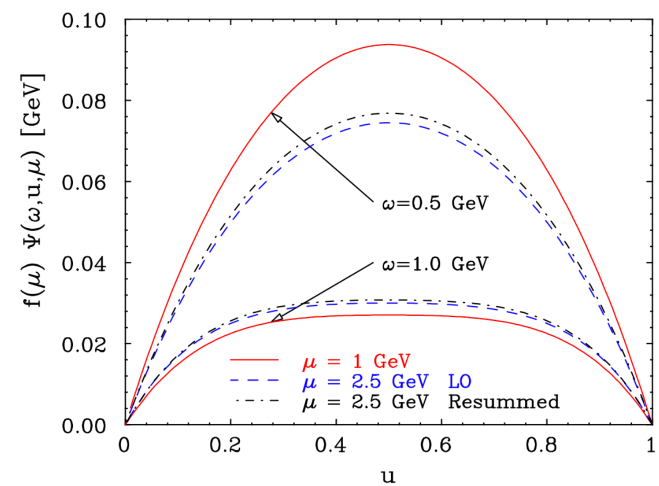
$$\begin{aligned}\epsilon_{abc}\langle\Lambda_c(v)|\bar{q}^a_{1k}(t_1)\bar{q}^b_{2i}(t_2)\bar{Q}^c_\gamma(0)|0\rangle = &\frac{1}{8}v_+\psi^{n*}(t_1,t_2)f^{(1)}\bar{u}_\gamma(C^{-1}\gamma_5\bar{\not{n}})_{ki} \\ &-\frac{1}{8}\psi^{n\bar{n}*}(t_1,t_2)f^{(2)}\bar{u}_\gamma(C^{-1}\gamma_5 i\sigma^{\mu\nu})_{ki}\bar{n}_\mu n_\nu \\ &+\frac{1}{4}\psi^{1*}(t_1,t_2)f^{(2)}\bar{u}_\gamma(C^{-1}\gamma_5)_{ki} \\ &+\frac{1}{8v_+}\psi^{\bar{n}*}(t_1,t_2)f^{(1)}\bar{u}_\gamma(C^{-1}\gamma_5\not{n})_{kl}\end{aligned}$$

LCDA of singly heavy baryon

$$\Psi(t_1,t_2) = \int_0^\infty d\omega_1 \int_0^\infty d\omega_2\, e^{-it_1\omega_1 - it_2\omega_2}\psi(\omega_1,\omega_2) = \int_0^\infty \omega\, d\omega \int_0^1 du\, e^{-i\omega(t_1 u + it_2\bar{u})}\widetilde{\psi}(\omega,u)$$







Summary

- Form factors of doubly heavy baryons weak decay in QCDSR
- The numerical results are consistent with previous works with LFQM
- LCSR is also applicable to doubly heavy baryon form factors

Thank you for your attention !