

Power corrections in the N-jettiness subtraction scheme

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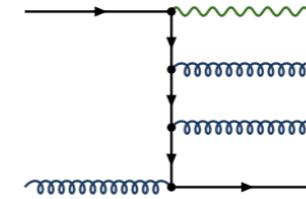
Outline

- Current status
- Power corrections
 - NLO
 - NNLO
- Conclusion

Current Status

Subtraction

$$\int dz \frac{f(z) - f(0)}{z} + \int dz z^{-1-a\epsilon} f(0)$$



$$= \int dz \frac{f(z)}{z^{1+a\epsilon}}$$

Slicing or non-local

$$\int \frac{f(z)}{z} \theta(z > z_0) - f(0) \frac{z_0^{-a\epsilon}}{a\epsilon} + \dots$$

- Construct counter terms point-wise in the phase space
 - Antenna subtraction 2 Gehrmann, Glover
 - STRIPPER + modifications Czakon + ...
 - ...

A physical observable (z_0) to regulate all related IR singularities

- qT-subtraction Catani, Grazzini
- Inclusive jet mass Gao, Li, Zhu
- N-jettiness subtraction Boughezal, Focke, XL, Petriello + ...

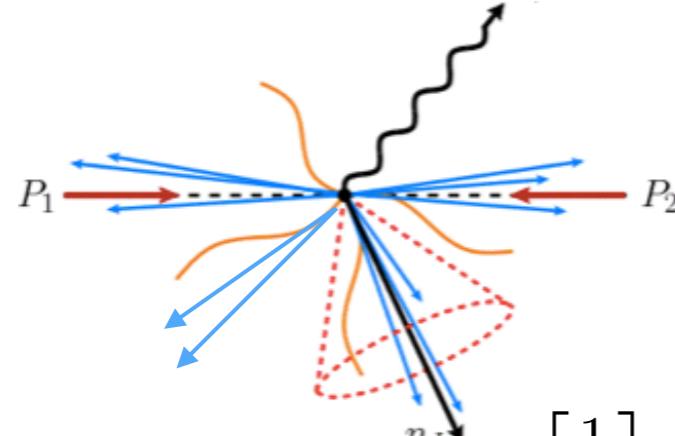
Evaluate counter terms —>
using current loop integral
techniques

Current Status

- Non-local schemes

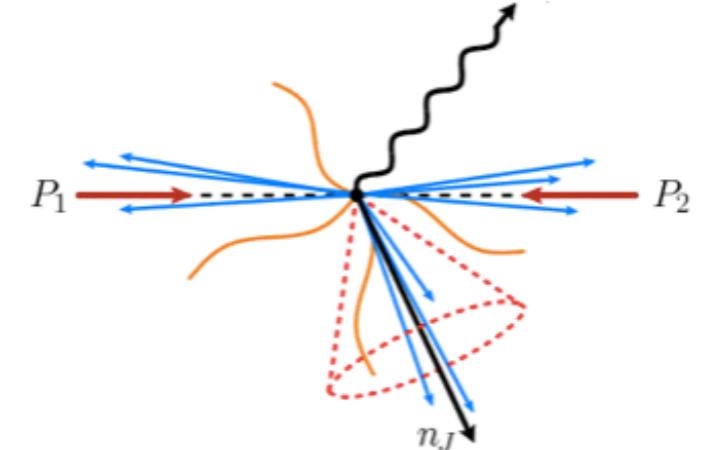
Catani, Grazzini; Gao, Li and Zhu; Boughezal, Focke, XL, Petriello; Gaunt, Stahlhofen, Tackmann, Walsh

At least $N+1$ hard
radiations



$$\left[\frac{1}{\tau} \right]_+ = \frac{1}{\tau} \theta(\tau - \tau_{cut}) + \delta(\tau) \log(\tau_{cut}) + \dots$$
$$\tau_{cut} \rightarrow 0$$

τ_{cut}



- a physical observable to set the boundary between NLO and NNLO
- ignorant of the NLO details
- NNLO using EFT based on LP F.T.
- universal building blocks
- conceptually appealing to implement

$$\text{Tr}[H \cdot S_N] \otimes B_a \otimes B_b \otimes J_i + \dots$$

Current Status

- Building Blocks
 - Beam function and jet functions => **NNLO splitting**, IBP, MI ...
 - Soft function => **NNLO eikonal approximation**, mapping UV to Collinear, Sector Decomposition for real emission

Current Status

- 0-jet
 - ggH/Drell-Yan/VH/di-photon

Gaunt, Stahlhofen, Tackmann, Walsh; Boughezal, et. al.; Campbell, Ellis, Williams; Campbell, Ellis, Li, Williams
 - MCFM 8

Boughezal, Campbell, Ellis, Focke, Giele, XL, Petriello, Williams;
- 1-jet
 - H/V + 1-jet

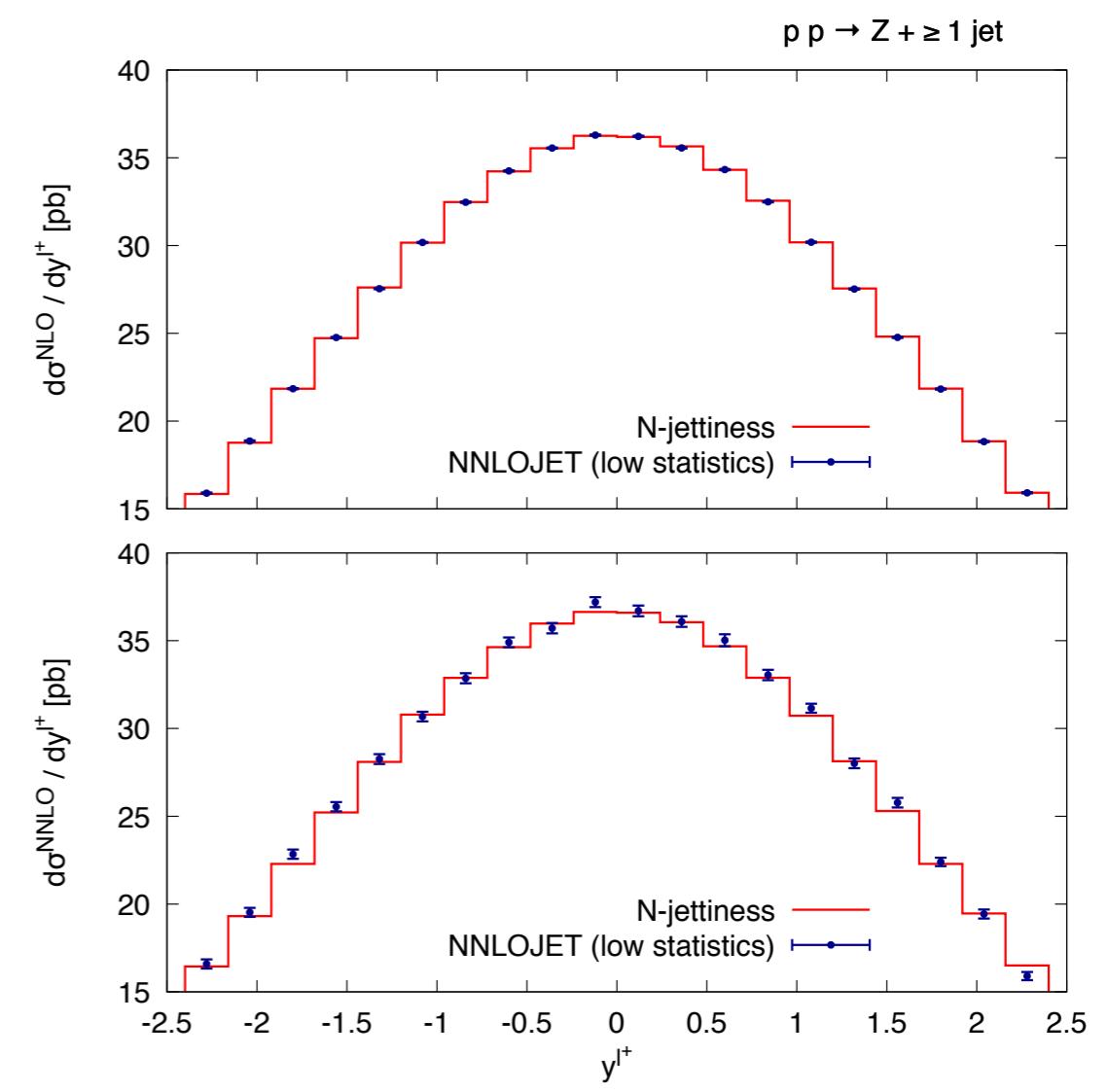
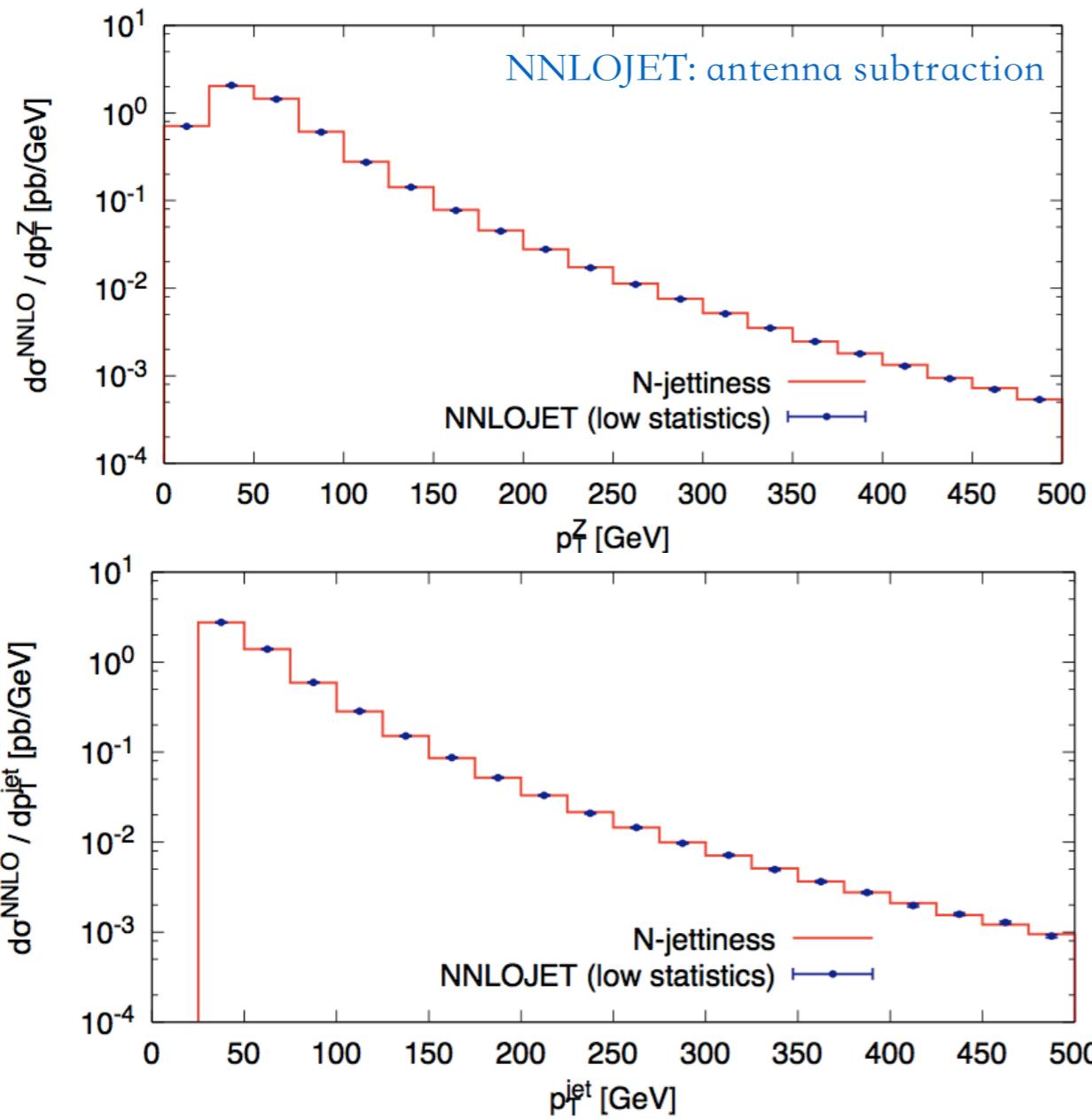
Boughezal, Focke, XL, Petriello; Boughezal, Focke, Giele, XL, Petriello;
Boughezal, Campbell, Ellis, Focke, Giele, XL, Petriello
 - DIS + 1-jet

Ablof, Boughezal, XL, Petriello

see Abelof's talk

Current Status

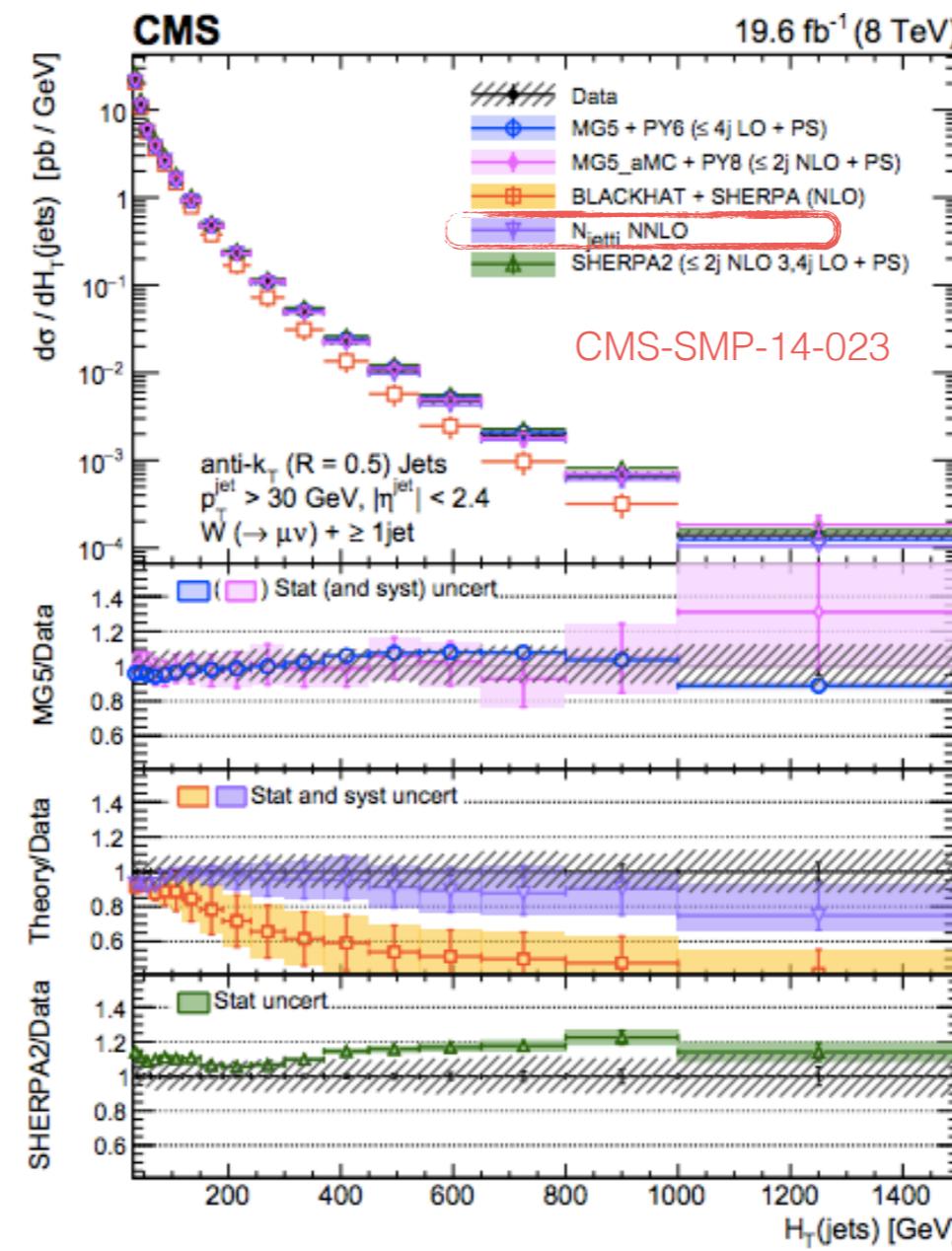
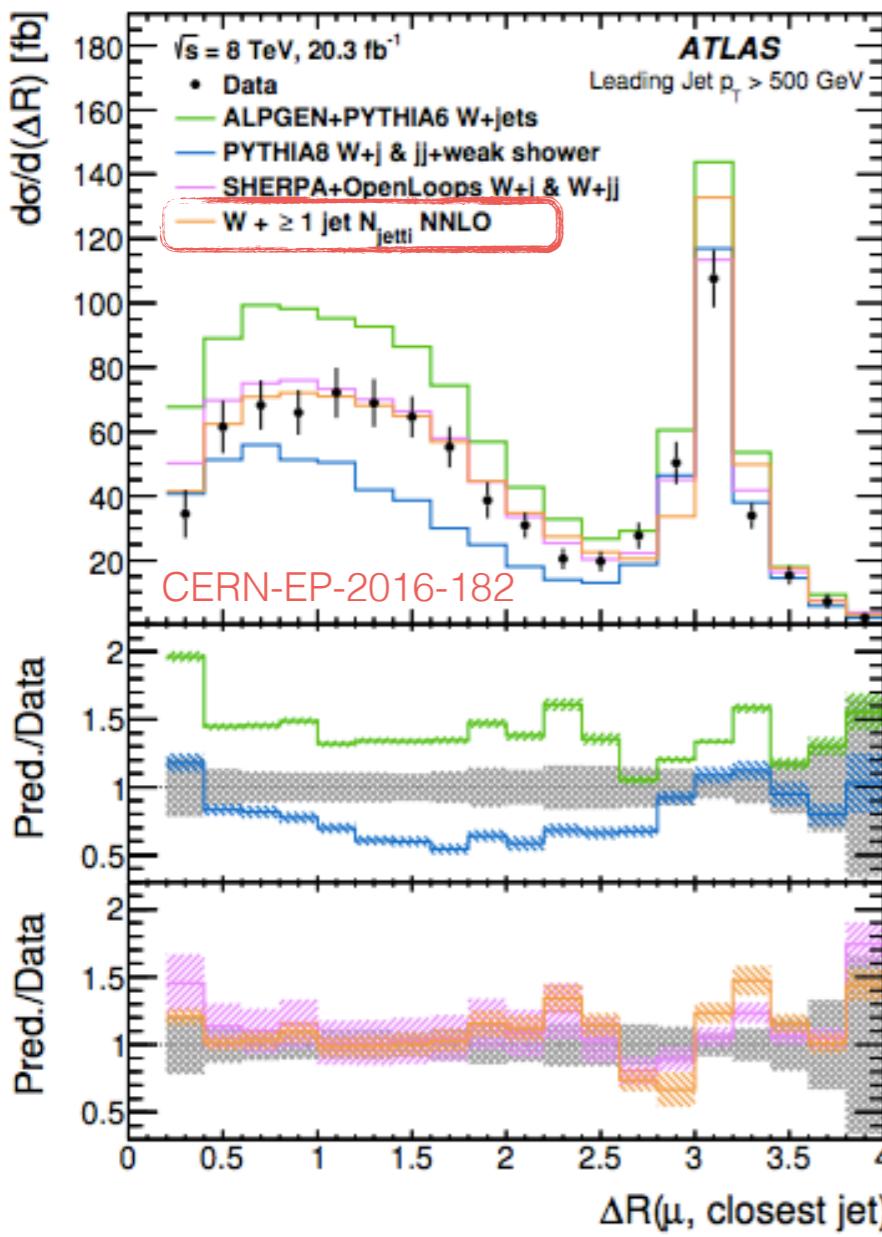
- Good agreement between independent calculations



Boughezal, Campbell, Ellis, Focke, Giele, XL, Petriello
Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan

Current Status

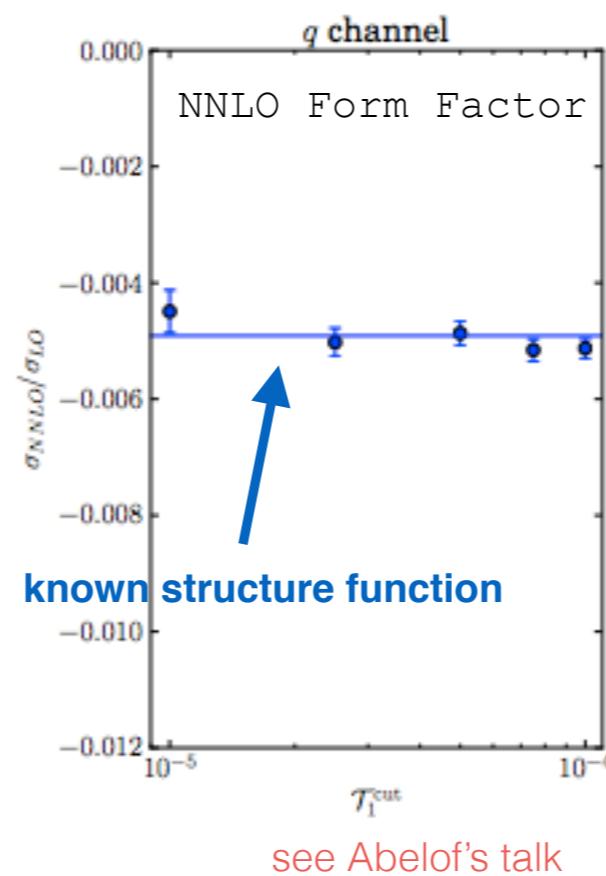
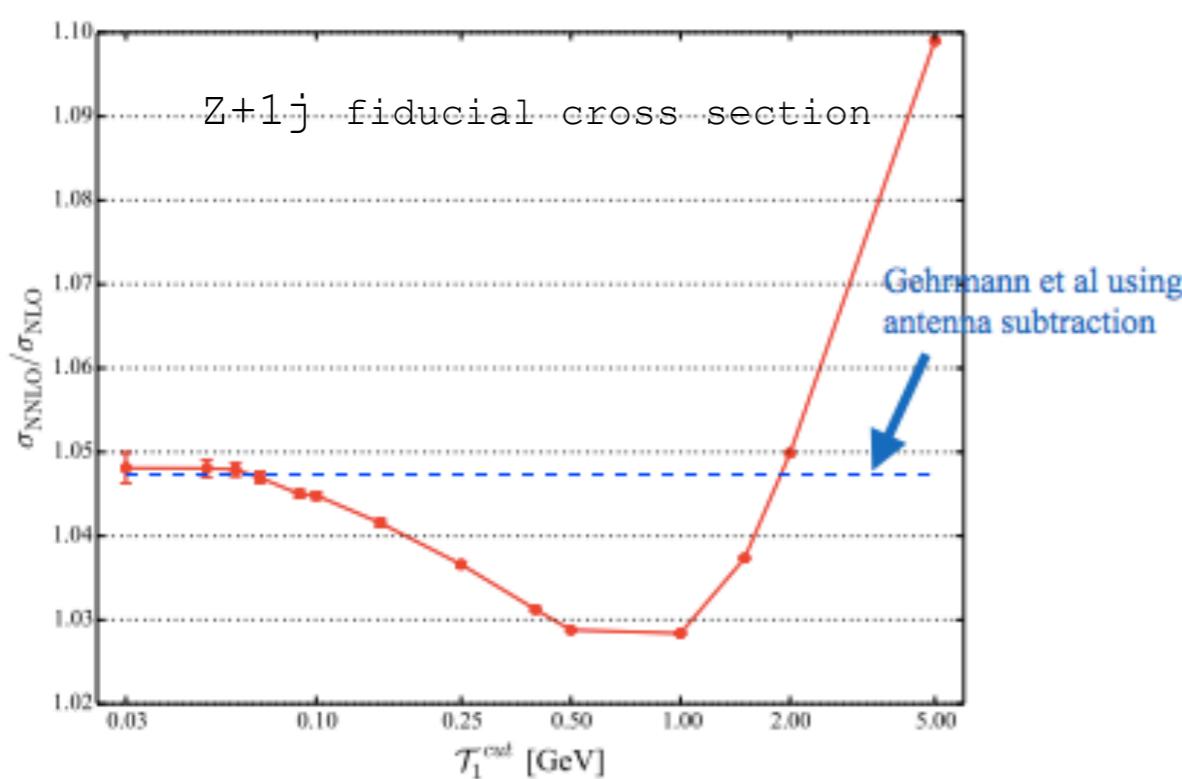
- Good agreement with data



- Improved agreements with all measured distributions

Current Status

- Require tiny tau-cut to suppress power corrections



$$\text{Tr}[H \cdot S_N] \otimes B_a \otimes B_b \otimes J_i \quad + \dots$$

• numerically challenging
for NNLO

ways to improve:

- benefit from analytic matrix elements
- design more efficient PS generators
- include power corrections

Power Corrections

- Logarithmic nature

$$\alpha_s^2 \tau_{cut} [C_{23} \log^3(\tau_{cut}) + C_{22} \log^2(\tau_{cut}) \dots]$$

- arise in the soft/collinear limits
- amenable to a direct FO calculation  this talk
- higher order can be predicted via lower order calculation in EFT

See Beneke, Wang, et. al

Moult, Stewart, Zhu, et.al

Power Corrections

- ggH/Drell-Yan @ NLO

$$\alpha_s C_{11} \tau_{cut} \log(\tau_{cut})$$

$$\tau_N = \sum_k \min \left[\frac{p_k \cdot n_a}{Q_a}, \frac{p_k \cdot n_b}{Q_b}, \frac{p_k \cdot n_1}{Q_1}, \dots, \frac{p_k \cdot n_N}{Q_N} \right]$$

Stewart, Tackmann, Waalewijn

ggH/Drell-Yan:

$$d\tau \ [\delta(k^+ - \tau) \Theta(k_- - k_+) + \delta(k^- - \tau) \Theta(k_+ - k_-)]$$

at NLO, for power corrections, we
only need to consider real emission.
The entire virtual information has
been included in the LP F.T.

$$\text{Tr}[H \cdot S_N] \otimes B_a \otimes B_b \otimes J_i + \dots$$

Power Corrections

- ggH/Drell-Yan @ NLO

$$\alpha_s C_{11} \tau_{cut} \log(\tau_{cut})$$

$$\int d\Phi |\mathcal{M}|_{ij}^2 f_i(x_a) f_j(x_b)$$

$$\frac{1}{4} \frac{\Omega_\perp}{(2\pi)^{d-2}} \frac{m_H^2}{s} \frac{e^{Y'}}{m_H} \frac{dz}{z^2} dY' d\tau dk_- (\tau k_-)^{-\epsilon} \delta \left(\frac{m_H e^{Y'} (1-z)}{z} - \frac{1}{z} e^{2Y'} \tau - k^- \right) \Theta(k^- - \tau)$$

expand everything to $\mathcal{O}(\tau)$

$$x_1 = \frac{m_H}{\sqrt{s}} z^{-1} e^{Y'}, \quad x_2 = \frac{m_H}{\sqrt{s}} e^{-Y'}$$

Power Corrections

- ggH/Drell-Yan @ NLO

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expand everything to $\mathcal{O}(\tau)$

$$x_1 = \frac{m_H}{\sqrt{s}} z^{-1} e^{Y'}, \quad x_2 = \frac{m_H}{\sqrt{s}} e^{-Y'}$$

$$\begin{aligned} |\mathcal{M}|^2 &= \frac{1}{\hat{t} \hat{u}} \times M(\hat{s}, \hat{t}, \hat{u}; \epsilon) = \frac{z}{m^2} \frac{1}{\tau k^-} \times M(\hat{s}, \hat{t}, \hat{u}; \epsilon) \\ &\sim \frac{1}{\tau (1-z)} H_0(0, z; \epsilon) + \frac{1}{(1-z)^2} \frac{e^{Y'}}{m} H_0(0, z; \epsilon) + \frac{1}{1-z} \partial_\tau H_0(0, z; \epsilon) + \mathcal{O}(\tau) \end{aligned}$$

power divergence means the ambiguity in defining the leading power matrix element

Power Corrections

- ggH/Drell-Yan @ NLO

$$\alpha_s C_{11} \tau_{cut} \log(\tau_{cut})$$

$$\int d\Phi |\mathcal{M}|_{ij}^2 f_i(x_a) f_j(x_b)$$

$$\frac{1}{4} \frac{\Omega_\perp}{(2\pi)^{d-2}} \frac{m_H^2}{s} \frac{e^{Y'}}{m_H} \frac{dz}{z^2} dY' d\tau dk_- (\tau k_-)^{-\epsilon} \delta \left(\frac{m_H e^{Y'} (1-z)}{z} - \frac{1}{z} e^{2Y'} \tau - k^- \right) \Theta(k^- - \tau)$$

$$x_1 = \frac{m_H}{\sqrt{s}} z^{-1} e^{Y'}, \quad x_2 = \frac{m_H}{\sqrt{s}} e^{-Y'}$$

expand everything to $\mathcal{O}(\tau)$

$$\begin{aligned} |\mathcal{M}|^2 &= \frac{1}{\hat{t} \hat{u}} \times M(\hat{s}, \hat{t}, \hat{u}; \epsilon) = \frac{z}{m^2} \frac{1}{\tau k^-} \times M(\hat{s}, \hat{t}, \hat{u}; \epsilon) \\ &\sim \frac{1}{\tau (1-z')} H_0(0, z'; \epsilon) + \frac{1}{1-z'} \partial_\tau H_0(0, z'; \epsilon) + \mathcal{O}(\tau) \end{aligned}$$

rescale z to avoid power divergence

$$z \rightarrow z' = \frac{z}{1 - \frac{\tau}{m} e^{Y'}}$$

Power Corrections

- ggH/Drell-Yan @ NLO

$$\alpha_s C_{11} \tau_{cut} \log(\tau_{cut})$$

$$\int d\Phi |\mathcal{M}|_{ij}^2 f_i(x_a) f_j(x_b)$$

$$\frac{1}{4} \frac{\Omega_\perp}{(2\pi)^{d-2}} \frac{m_H^2}{s} \frac{e^{Y'}}{m_H} \frac{dz}{z^2} dY' d\tau dk_- (\tau k_-)^{-\epsilon} \delta \left(\frac{m_H e^{Y'} (1-z)}{z} - \frac{1}{z} e^{2Y'} \tau - k^- \right) \Theta(k^- - \tau)$$

$$x_1 = \frac{m_H}{\sqrt{s}} z^{-1} e^{Y'}, \quad x_2 = \frac{m_H}{\sqrt{s}} e^{-Y'}$$

expand everything to $\mathcal{O}(\tau)$

$$|\mathcal{M}|^2 = \frac{1}{\hat{t} \hat{u}} \times M(\hat{s}, \hat{t}, \hat{u}; \epsilon) = \frac{z}{m^2} \frac{1}{\tau k^-} \times M(\hat{s}, \hat{t}, \hat{u}; \epsilon)$$

$$\sim \frac{1}{\tau (1-z')} H_0(0, 1; \epsilon) + \frac{1}{1-z'} \partial_\tau H_0(0, 1; \epsilon) + \mathcal{O}(\tau)$$

It has to be soft to generate the LLs in the power corrections

rescale z to avoid power divergence

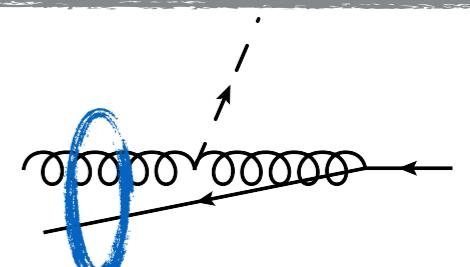
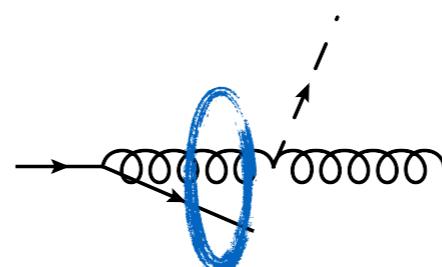
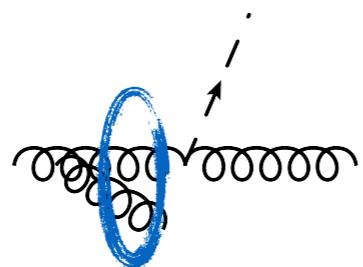
$$z \rightarrow z' = \frac{z}{1 - \frac{\tau}{m} e^{Y'}}$$

$$\int^{1-a\tau} dz' \frac{\mathcal{N}(1-z')}{1-z'} = \int^{1-a\tau} dz' \left[\frac{\mathcal{N}_0}{1-z'} + \mathcal{N}_1 + \dots \right]$$

Power Corrections

- ggH/Drell-Yan @ NLO

e.g. 1-real emission in ggH



$$|\mathcal{M}|^2 : \quad \frac{16\pi\alpha_s C_i}{\tau(1-z')} H_0 + \frac{1}{1-z'} \times 0$$

$$\frac{8\pi\alpha_s C_i}{\tau} H_0$$

$$\frac{8\pi\alpha_s C_i}{1-z'} \frac{e^{Y'}}{m} H_0$$

$$d\Phi : \quad \text{LP/SLP}$$

$$\text{LP}$$

$$\mathcal{L}_{ij} : \quad \mathcal{L}|_{z'=1}, \tau \times x_1 \partial_{x_1} \mathcal{L}|_{z'=1}$$

$$\mathcal{L}|_{z'=1}$$



$$\frac{\alpha_s C_A}{2\pi} L [2x_1 \partial_{x_1}] \mathcal{L}_{g_1 g_2} + \{x_1 \leftrightarrow x_2, Y' \leftrightarrow -Y'\}$$

$$L = \frac{e^{Y'}}{m} \log \left(\frac{\tau m e^{Y'}}{\tau^2} \right)$$

$$\frac{\alpha_s C_F}{2\pi} L \mathcal{L}_{g_1 q_2} + \{x_1 \leftrightarrow x_2, Y' \leftrightarrow -Y'\}$$

$$x_1 = \frac{m_H}{\sqrt{s}} z^{-1} e^{Y'}, \quad x_2 = \frac{m_H}{\sqrt{s}} e^{-Y'}$$

Power Corrections

- ggH/Drell-Yan @ NLO

$$\alpha_s C_{11} \tau_{cut} \log(\tau_{cut})$$

The results are free of divergence.

The form of the results depends on how to parameterize x_a and x_b , but one can check that different parameterizations lead to log-suppressed differences

$$|\mathcal{M}|^2 : \quad \frac{16\pi\alpha_s C_i}{\tau(1-z')} H_0 + \frac{1}{1-z'} \times 0$$

$$\frac{8\pi\alpha_s C_i}{\tau} H_0$$

$$\frac{8\pi\alpha_s C_i}{1-z'} \frac{e^{Y'}}{m} H_0$$

$$d\Phi :$$

$$LP/SLP$$

$$LP$$

$$\mathcal{L}_{ij} :$$

$$\mathcal{L}|_{z'=1}, \tau \times x_1 \partial_{x_1} \mathcal{L}|_{z'=1}$$

$$\mathcal{L}|_{z'=1}$$



$$\frac{\alpha_s C_A}{2\pi} L [2x_1 \partial_{x_1}] \mathcal{L}_{g_1 g_2} + \{x_1 \leftrightarrow x_2, Y' \leftrightarrow -Y'\}$$

$$L = \frac{e^{Y'}}{m} \log \left(\frac{\tau m e^{Y'}}{\tau^2} \right)$$

$$\frac{\alpha_s C_F}{2\pi} L \mathcal{L}_{g_1 q_2} + \{x_1 \leftrightarrow x_2, Y' \leftrightarrow -Y'\}$$

$$x_1 = \frac{m_H}{\sqrt{s}} z^{-1} e^{Y'}, \quad x_2 = \frac{m_H}{\sqrt{s}} e^{-Y'}$$

Power Corrections

- ggH/Drell-Yan @ NLO

$$\alpha_s C_{11} \tau_{cut} \log(\tau_{cut})$$

The MEs contribute to the leading log can be deduced from the leading power collinear kernel, if we relate $(1-z') \sim 0$ to the collinear limit ($k_- \sim 0$)

$$|\mathcal{M}|^2 : \frac{16\pi\alpha_s C_i}{\tau(1-z')} H_0 + \frac{1}{1-z'} \times 0$$

$$\frac{8\pi\alpha_s C_i}{\tau} H_0$$

$$\frac{8\pi\alpha_s C_i}{1-z'} \frac{e^{Y'}}{m} H_0$$

$$d\Phi :$$

LP/S]

$$\mathcal{L}_{ij} :$$

$\mathcal{L}|_{z'=1}, \tau \times$

$\propto \frac{1}{k^-} P_{qg}(x)$

$$\frac{\alpha_s C_A}{2\pi} L [2x_1 \partial_{x_1}] \mathcal{L}_{g_1 g_2} + \{ \dots \}$$

$$L = \frac{e^{Y'}}{m} \log \left(\frac{\tau m e^{Y'}}{\tau^2} \right)$$

offshellness $(1-z') \sim k_- \sim 0$

$1-x \sim k^+ \sim \tau$

$$\frac{C_F}{2\pi} L \mathcal{L}_{g_1 q_2} + \{ x_1 \leftrightarrow x_2, Y' \leftrightarrow -Y' \}$$

$$x_1 = \frac{m_H}{\sqrt{s}} z^{-1} e^{Y'}, \quad x_2 = \frac{m_H}{\sqrt{s}} e^{-Y'}$$

Power Corrections

- ggH/Drell-Yan @ NLO

$$\alpha_s C_{11} \tau_{cut} \log(\tau_{cut})$$

The one which does not contribute to the leading log in the leading power does not contribute to the leading log in the sub-leading power

$$|\mathcal{M}|^2 : \quad \frac{16\pi\alpha_s C_i}{\tau(1-z')} H_0 + \frac{1}{1-z'} \times 0$$

$$\boxed{\frac{8\pi\alpha_s C_i}{\tau} H_0}$$

$$\frac{8\pi\alpha_s C_i}{1-z'} \frac{e^{Y'}}{m} H_0$$

$$d\Phi : \quad \text{LP/SLP}$$

LP

$$\mathcal{L}_{ij} : \quad \mathcal{L}|_{z'=1}, \tau \times x_1 \partial_{x_1} \mathcal{L}|_{z'=1}$$

$$\mathcal{L}|_{z'=1}$$



$$\frac{\alpha_s C_A}{2\pi} L [2x_1 \partial_{x_1}] \mathcal{L}_{g_1 g_2} + \{x_1 \leftrightarrow x_2, Y' \leftrightarrow -Y'\}$$

$$\frac{\alpha_s C_F}{2\pi} L \mathcal{L}_{g_1 q_2} + \{x_1 \leftrightarrow x_2, Y' \leftrightarrow -Y'\}$$

$$L = \frac{e^{Y'}}{m} \log \left(\frac{\tau m e^{Y'}}{\tau^2} \right)$$

$$x_1 = \frac{m_H}{\sqrt{s}} z^{-1} e^{Y'}, \quad x_2 = \frac{m_H}{\sqrt{s}} e^{-Y'}$$

Power Corrections

- ggH/Drell-Yan @ NNLO

$$\alpha_s^2 \tau_{cut} C_{23} \log^3(\tau_{cut})$$

- poles from RV have to cancel against RR
- soft limit leads to the leading logs
- MEs deducible from collinear limits at least for q final state in RV and qg final state in RR
- MEs takes the strongly-order limit, for instance, in the case of the qg final state, $E_g \ll E_q$,
- qqbar does not contribute

Power Corrections

- ggH/Drell-Yan @ NNLO

$$\text{RV matrix} \quad \frac{\alpha_s C_F}{2\pi} \frac{-2}{\epsilon^2} \frac{8\pi\alpha_s \mu^{2\epsilon} e^Y}{m} C_F \frac{1}{k^-} \left(\frac{\mu^2}{m^2} \right)^\epsilon \quad \text{Hard virtual + coll.}$$

$$\frac{2}{16\pi^2} \times (4\pi\alpha_s)^2 \mu^{2\epsilon} C_F \times \frac{2}{-\hat{u}} \left[-\frac{1}{\epsilon^2} N_C + \frac{1}{\epsilon^2} \left(\left[\frac{\hat{t}}{\hat{s}} \right]^{-\epsilon} - 1 \right) \frac{1}{N_c} \right] \left(\frac{\hat{u}}{\mu^2} \right)^{-\epsilon} \text{coll virtual.}$$

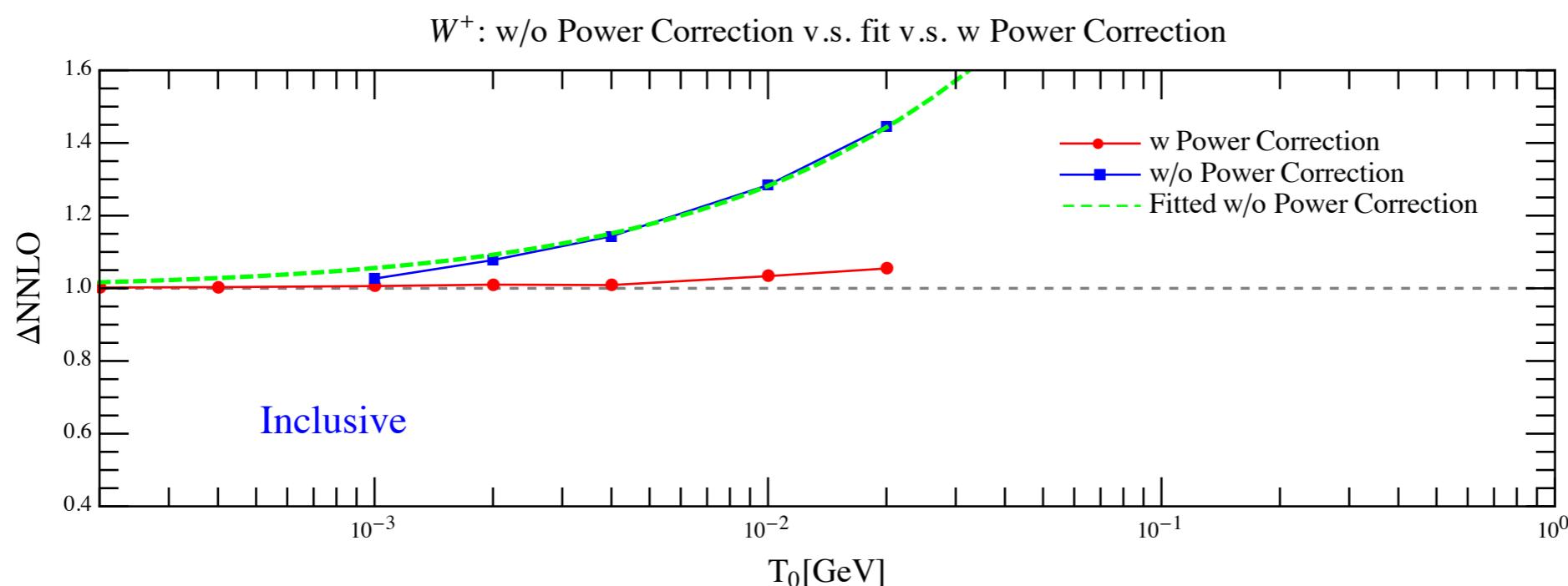
Power Corrections

- ggH/Drell-Yan @ NNLO

$$\begin{aligned}
\text{RR matrix} & \quad \left[(4\pi\alpha_s) \mu^{2\epsilon} 2C_F \frac{\hat{s}}{\hat{t}_2 \hat{u}_2} \right] \left[(8\pi\alpha_s) C_F \mu^{2\epsilon} \left(\frac{-1}{\hat{u}_1} - \frac{-1}{\hat{u}_1 + \hat{u}_2} \right) \right] \\
& \quad 2(4\pi\alpha_s)^2 C_F^2 \left(\frac{1}{-\hat{u}_1} \frac{\hat{t}_1}{\hat{t}_2 2k_1 \cdot k_2} \right) \\
2(4\pi\alpha_s)^2 C_F^2 \mu^{4\epsilon} & \left[\frac{3}{-\hat{u}_1} \frac{\hat{t}_1}{\hat{t}_2 2k_1 \cdot k_2} + 2 \frac{-u_2}{\hat{u}_1 + \hat{u}_2} \frac{1}{-\hat{u}_1} \frac{\hat{t}_1}{\hat{t}_2 2k_1 \cdot k_2} - 2 \frac{-\hat{u}_2}{(\hat{u}_1 + \hat{u}_2)^2} \frac{\hat{u}_1}{\hat{u}_2 2k_1 \cdot k_2} \right] \\
\left[2(4\pi\alpha_s) \mu^{2\epsilon} C_A \frac{\hat{s}}{\hat{t}_2 \hat{u}_2} \right] \left[(8\pi\alpha_s) \mu^{2\epsilon} C_F \left(\frac{-1}{\hat{u}_1} + \frac{-1}{2} \frac{-1}{\hat{u}_1} - \frac{-1}{2} \frac{-1}{\hat{u}_1 + \hat{u}_2} \right) \right] \\
2(4\pi\alpha_s)^2 \mu^{4\epsilon} C_F C_A & \left[\frac{\hat{u}_1}{\hat{u}_2 2k_1 \cdot k_2} \left(-\frac{1}{\hat{u}_1} - \frac{1}{\hat{u}_1 + \hat{u}_2} \right) - \left(\frac{1}{-\hat{u}_1} + \frac{1}{-\hat{u}_1 - \hat{u}_2} \right) \frac{\hat{t}_1}{\hat{t}_2 2k_1 \cdot k_2} \right]
\end{aligned}$$

Power Corrections

- Numerical consequence



- Including the power corrections helps improving the convergence of the N-jettiness subtraction scheme.
- allows us to relax the cut-off dramatically.
- improves the numerical stability

Conclusions

- A first step toward understanding the power corrections in the jettiness subtraction
- Calculate the leading logs in the power corrections
- Can increase tau-cut substantially and improve numerical stabilities
- Go beyond

thanks