QCD critical end point and droplet cold quark matter

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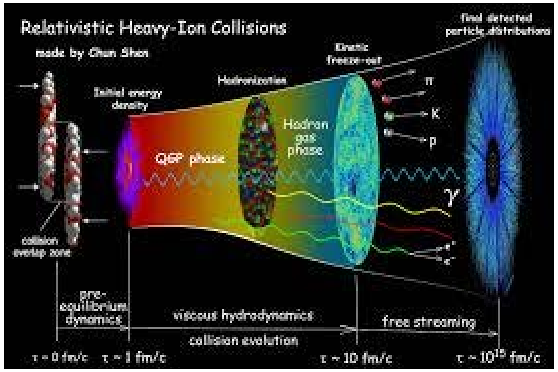
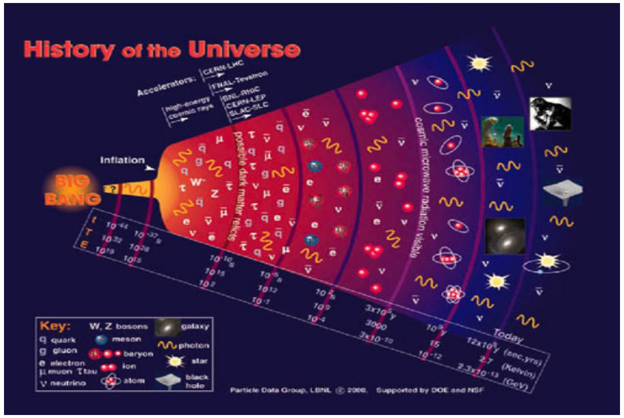
QCD matter under extreme conditions

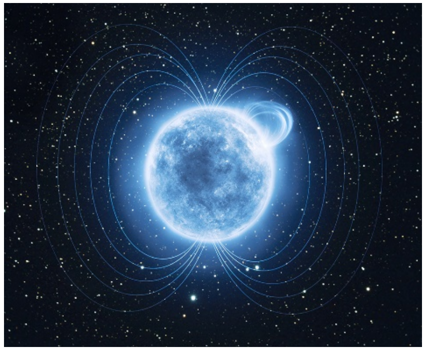
$$\mathrm{T}, \mu_B, \mathbf{B}, \mathbf{E}\cdot\mathbf{B}, \boldsymbol{\omega}, \mu_{\mathrm{I}}, \boldsymbol{L}$$



LHC,RHIC,FAIR,NICA,HIAF



Early universe

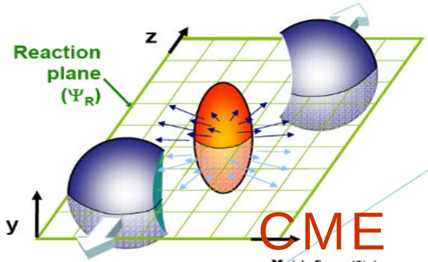


Neutron star

Neutron star merge→BH

Explored QCD phase diagram

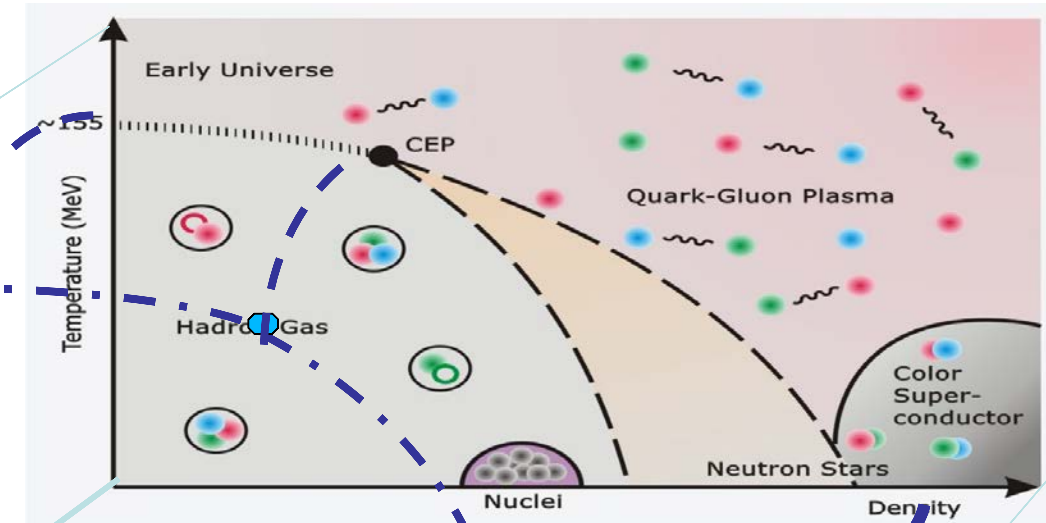
by theorists



CME

CVE

Inverse Magnetic Catalysis

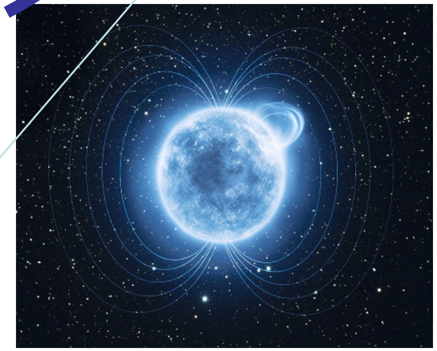


Charged rho condensation

Neutral/charged pion condensation

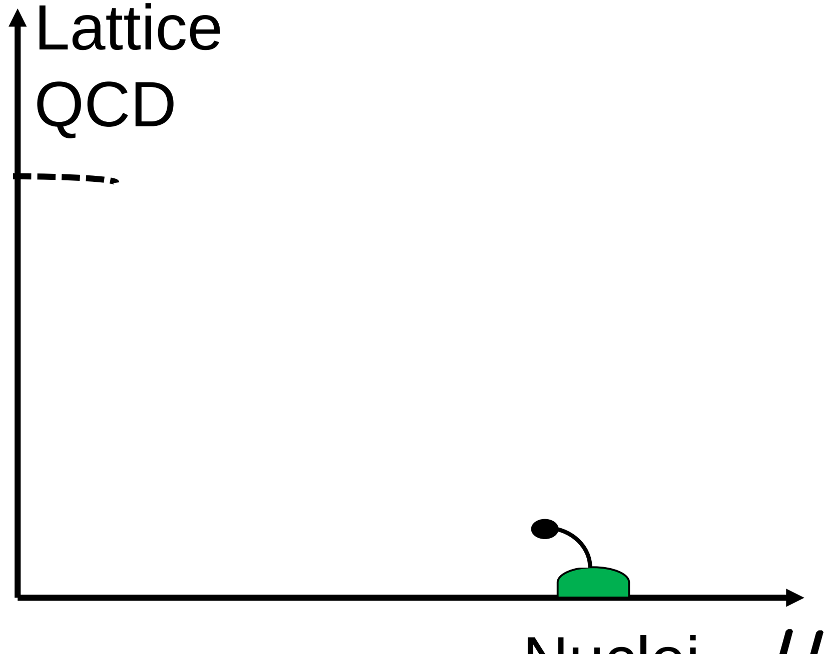
(Inverse) Magnetic Catalysis

Magnetar

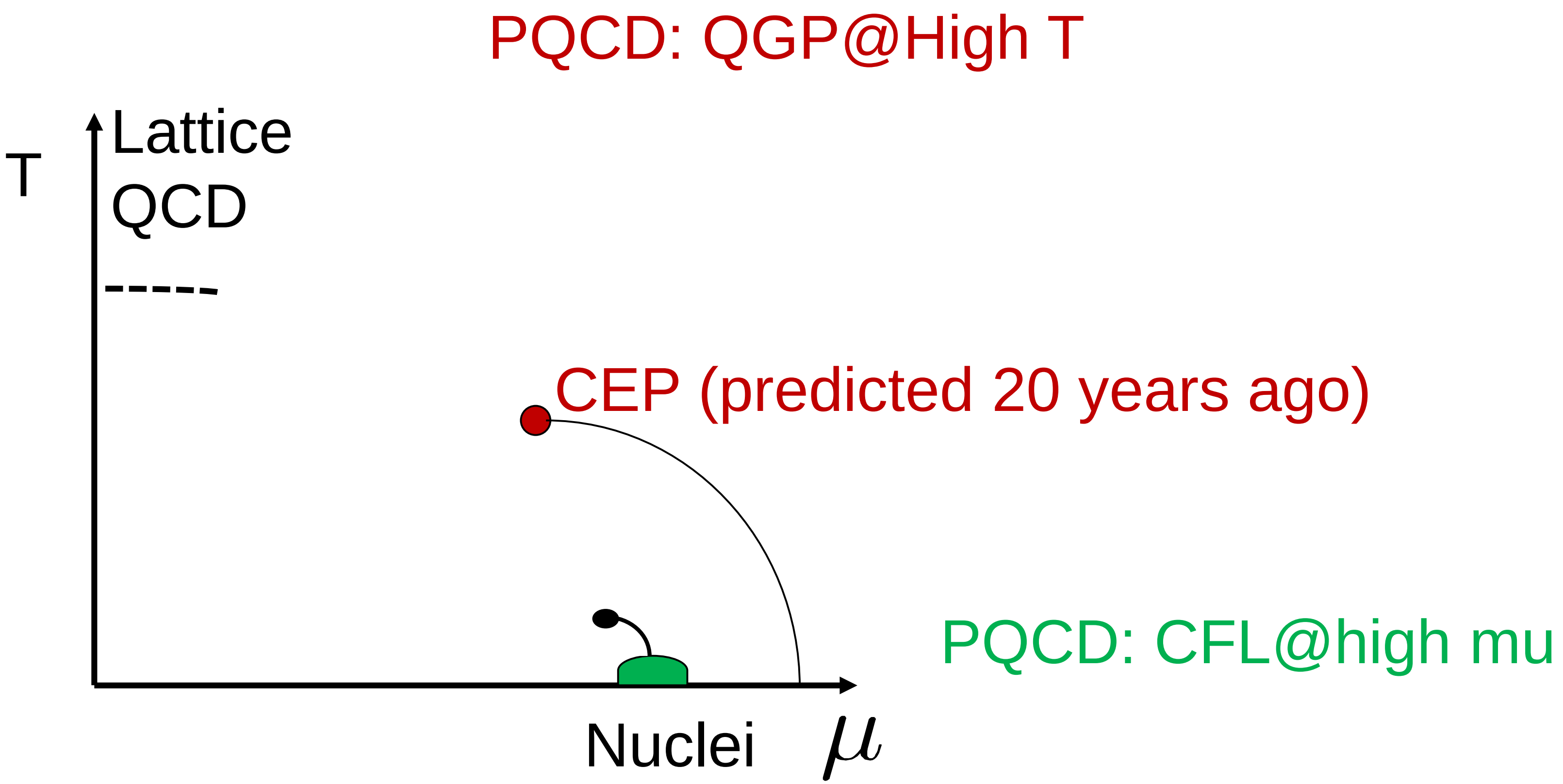


$$\mathbf{B}, \mathbf{E}\cdot\mathbf{B}, \boldsymbol{\omega}, \mu_I, \boldsymbol{L}$$

Confirmed QCD phase diagram

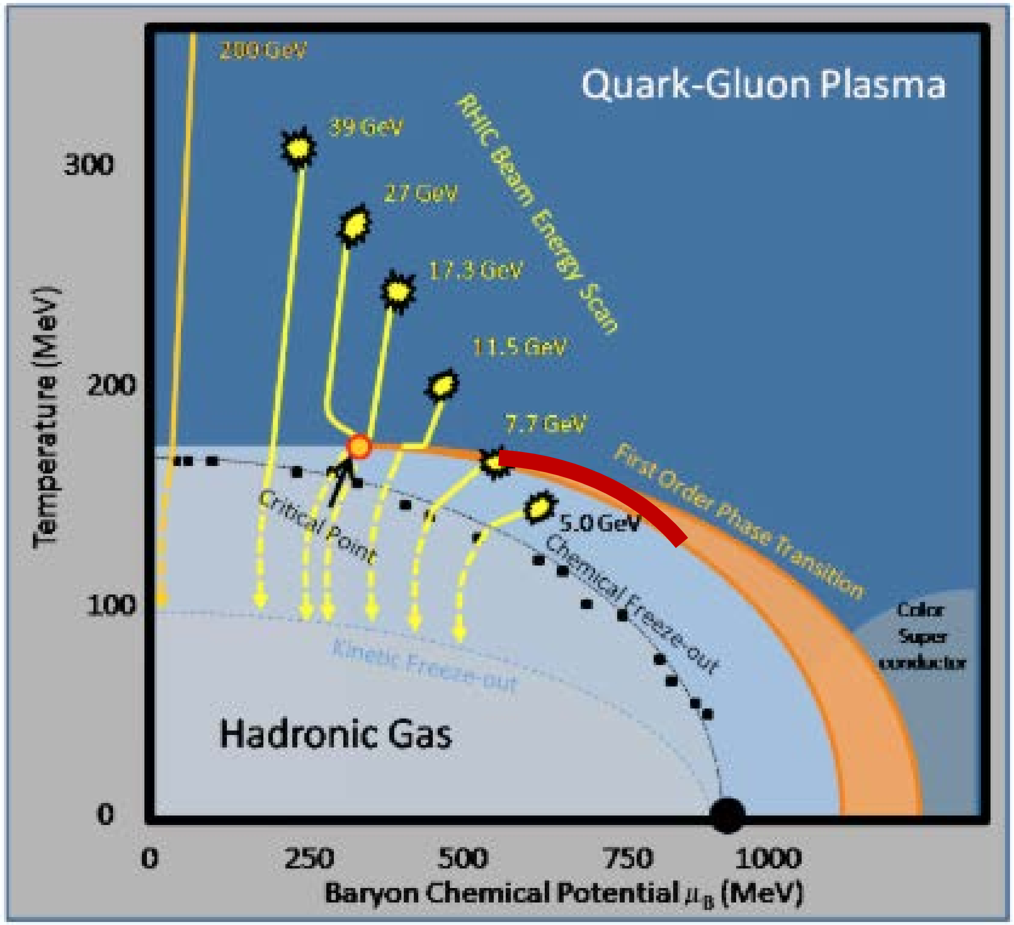


Searching for the QCD CEP



Locating CEP is essential for the QCD phase diagram!

Locating the QCD CEP



- ❑ **BES @ RHIC**
- ❑ **NICA @Dubna**
- ❑ **CBM@FAIR**
- ❑ **HIAF@IMP**

Chiral and deconfinement phase transitions

CEP is for chiral phase transition!

Chiral phase transition:

quark-antiquark condensate (for m=0)

Chiral symmetry breaking: $\langle\bar{\psi}\psi\rangle \neq 0$

Chiral symmetry restoration: $\langle\bar{\psi}\psi\rangle = 0$

Deconfinement phase transition:

referring to the "permanent confinement"

Polyakov loop (for m= infinity)

$$L(\vec{x}) = \frac{1}{N_c}\mathrm{tr}\,\mathcal{P}(\vec{x}) \text{ with } \mathcal{P}(\vec{x}) = \mathrm{P}\,\mathrm{e}^{ig\int_0^\beta dt\, A_0(t,\vec{x})}$$

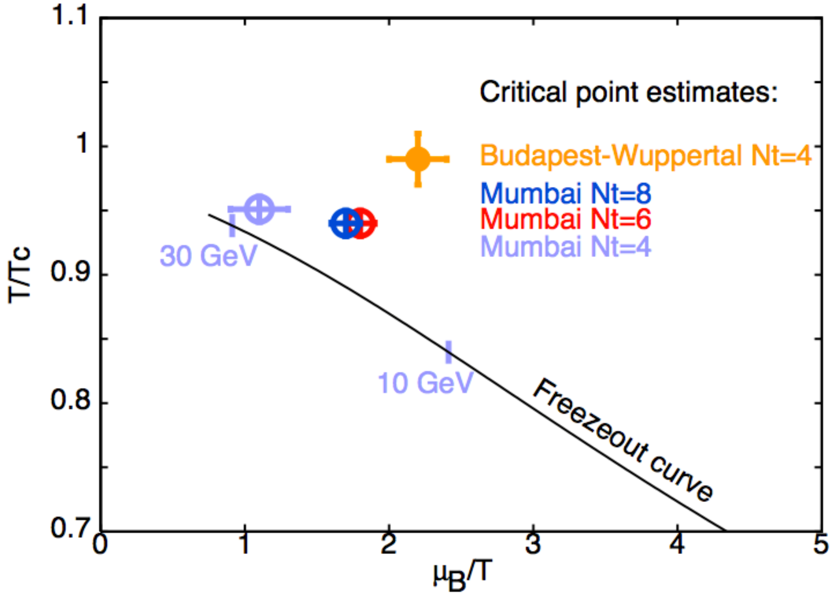
$$\langle L(\vec{x})\rangle \sim \exp(-\beta F_q)$$

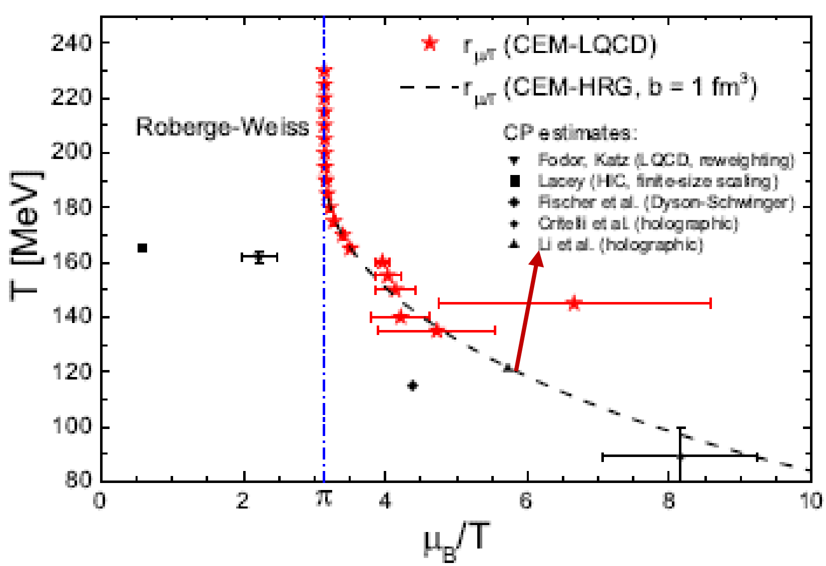
Confinement: center symmetric $\langle L\rangle = 0 \quad F_q \to \infty$

Deconfinement: center symmetry breaking $\langle L\rangle \neq 0 \quad F_q < \infty$

monopole (DI GIACOMO)

Location of CEP from Lattice QCD





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$(\mu^E_B, T_E) = (360, 162)$ MeV

2) Gavai&Gupta, NPA 904, 883c (2013)

$(\mu^E_B, T_E) = (279, 155)$ MeV

3) F. Karsch (CPOD2016)

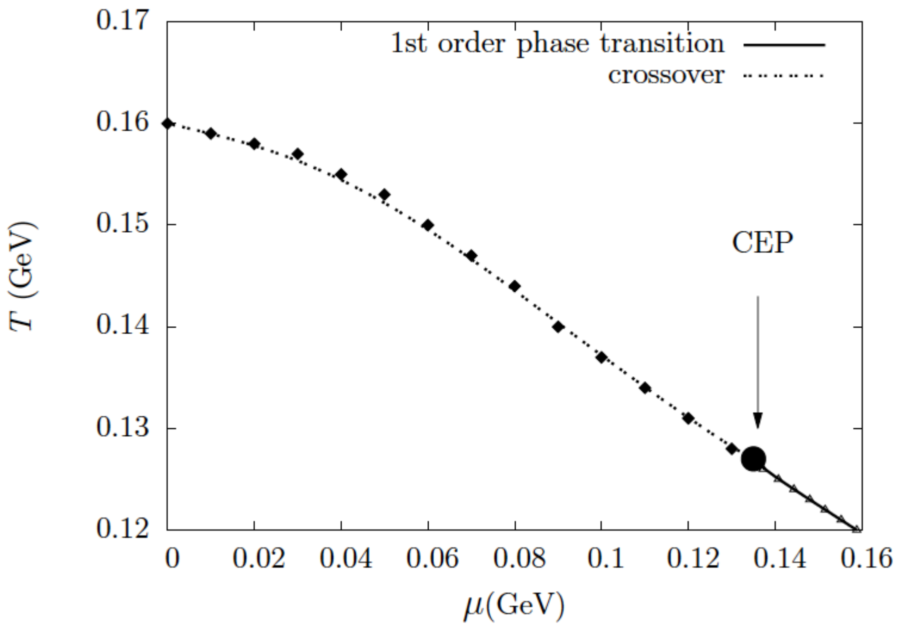
$\mu^E_B / T_E > 2$

4) V. Vovchenko, J. Steinheimer, O. Philipsen,H.Stoecker, arXiv:1711.01261

$$\mu_B^E / T_E > \pi$$

Latest lattice calculation shows that small baryon number density region for CEP is ruled out!

Location of CEP from DSE



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$(\mu^E_B, T^E) = (372, 129)$ MeV

2): Hong-shi Zong et al., JHEP 07, 014 (2014).

$(\mu^E_B, T_E) = (405, 127)$ MeV

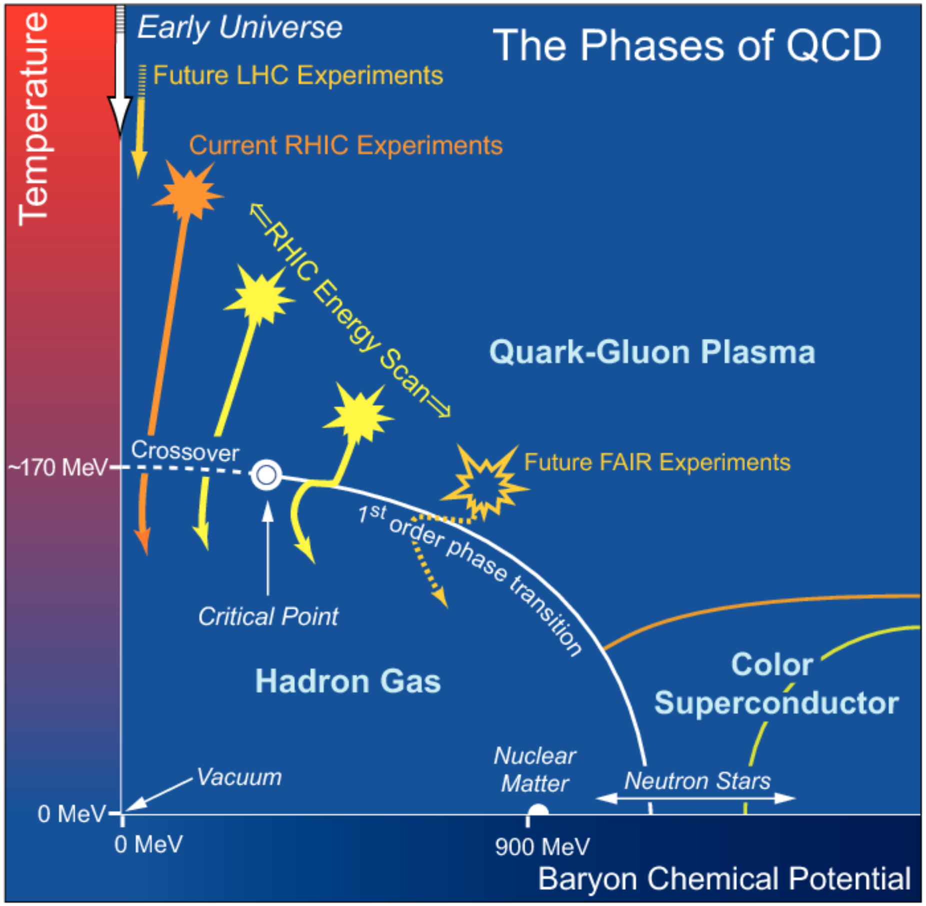
3): C. S. Fischer et al., PRD90, 034022 (2014).

$(\mu^E_B, T^E) = (504, 115)$ MeV

$$\mu_B = 3\mu_q$$

baryon number density region 300-500 MeV

Searching for the QCD CEP



BES Phase-I

$\sqrt{s_{NN}}$ (GeV)	Events (10^6)	Year	*μ_B (MeV)	*T_{CH} (MeV)
200	350	2010	25	166
62.4	67	2010	73	165
39	39	2010	112	164
27	70	2011	156	162
19.6	36	2011	206	160
14.5	20	2014	264	156
11.5	12	2010	316	152
7.7	4	2010	422	140

Higher Order Fluctuations of Conserved Quantities

$$\chi_n^B = \frac{\partial^n [P/T^4]}{\partial[\mu_B/T]^n}$$

$$B \rightarrow Q, s$$

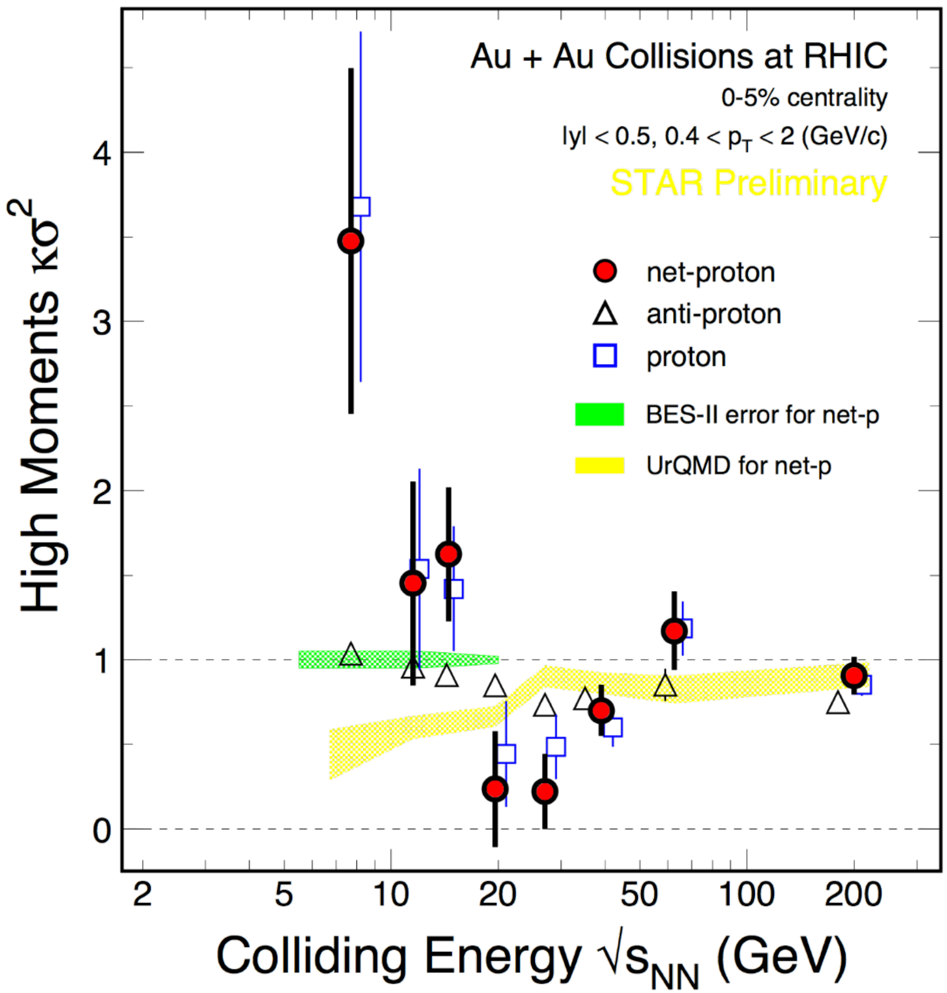
$$C_n^B = VT^3 \chi_n^B$$

$$\frac{\sigma^2}{M} = \frac{C_2^B}{C_1^B} = \frac{\chi_2^B}{\chi_1^B}, \quad S\sigma = \frac{C_3^B}{C_2^B} = \frac{\chi_3^B}{\chi_2^B},$$

$$\frac{S\sigma^3}{M} = \frac{C_3^B}{C_1^B} = \frac{\chi_3^B}{\chi_1^B}, \quad \boxed{\kappa\sigma^2 = \frac{C_4^B}{C_2^B} = \frac{\chi_4^B}{\chi_2^B}.}$$

S. Ejiri et al,Phys.Lett. B 633 (2006) 275. Cheng et al, PRD (2009) 074505. B. Friman et al., EPJC 71 (2011) 1694. F. Karsch and K. Redlich , PLB 695, 136 (2011).
S. Gupta, et al., Science, 332, 1525(2012). A. Bazavov et al., PRL109, 192302(12)
S. Borsanyi et al., PRL111, 062005(13), P. Alba et al., arXiv:1403.4903

Measurement of Higher Order Fluctuations of Conserved Quantities



Non-monotonic trend is observed for the 0-5% most central Au+Au collisions. Dip structure is observed around 19.6 GeV.

STAR: **PRL112**, 32302(14); **PRL113**,092301(14);
X.F.Luo, N.Xu, arXiv:1701.02105

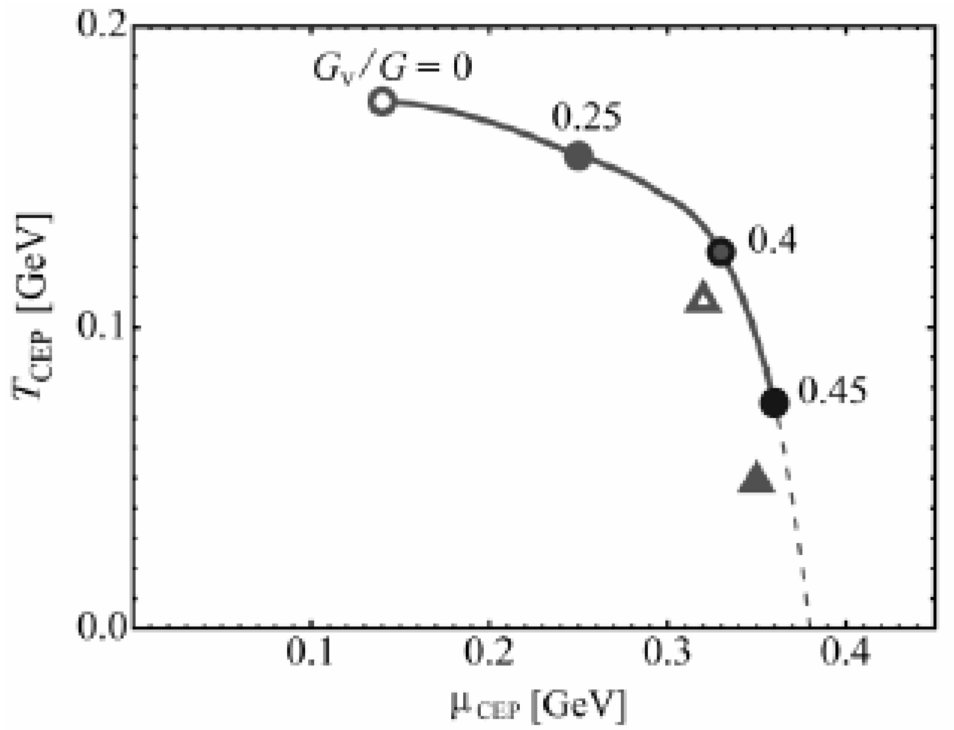
Is there a model can describe measurement well?

---- CEP from a realistic PNJL model

Z.B Li, K.Xu,X.Y.Wang, M.H,
arXiv:1801.09215,EPJC2019
arXiv:arXiv:1810.03524

Location of CEP: NJL

NJL, PNJL, Nonlocal NJL, ……



Hell, Kashiwa, Weise

Journal of Modern Physics, 2013, 4, 644-650

P.F Zhuang,M.Huang,
Y.X.Liu,W.J.Fu, Z.Zhang
H.S.Zong, X.Luo, G.Y.Shao…..
J.Deng, J.W.Chen,G.Q.Cao,
X.G.Huang……

Weise,
Klevansky,
Hatsuda,Kunihiro,
Fukushima,
Redlich,Sasaki,
Ratti,
……

$$\mu_B = 3\mu_q$$

from small to high baryon number density region ……

A realistic PNJL model

A. Bhattacharyya,S. K. Ghosh, S. Maity, S. Raha,
B. R. Ray, K. Saha and S. Upadhaya,arXiv:1609.07882.

NJL part:

$$\Omega = g_S \sum_f \sigma_f^2 - \frac{g_D}{2}\sigma_u \sigma_d \sigma_s + 3\frac{g_1}{2}(\sum_f \sigma_f^2)^2 + 3g_2 \sum_f \sigma_f^4 - 6\sum_f \int \frac{d^3p}{(2\pi)^3} E_f \Theta(\Lambda - |\vec{p}|)$$
$$- 2T \sum_f \int \frac{d^3p}{(2\pi)^3} \ln[1 + 3(\Phi + \bar{\Phi} e^{-(E_f-\mu_f)/T}) e^{-(E_f-\mu_f)/T} + e^{-3(E_f-\mu_f)/T}]$$
$$- 2T \sum_f \int \frac{d^3p}{(2\pi)^3} \ln[1 + 3(\Phi + \bar{\Phi} e^{-(E_f+\mu_f)/T}) e^{-(E_f+\mu_f)/T} + e^{-3(E_f+\mu_f)/T}]$$
$$+ U'(\Phi, \bar{\Phi}, T)$$

Polyakov Loop:

$$\frac{U'}{T^4} = \frac{U}{T^4} - \kappa \ln[J(\Phi, \bar{\Phi})] \qquad \frac{U}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}(\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4}(\Phi\bar{\Phi})^2$$

$$J = (\frac{27}{24\pi^2})(1 - 6\Phi\bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi\bar{\Phi})^2)$$

$$b_2(T) = a_0 + a_1 \frac{T_0}{T} \exp(-a_2 \frac{T}{T_0})$$

Parameters are fitted to lattice result at mu=0,

1) Tc=154 MeV;
2) EOS: p,e,s, trace anomaly;
3) Baryon number fluctuations

$m_{u,d}$(MeV)	m_s(MeV)	Λ(MeV)	$g_S\Lambda^2$	$g_D\Lambda^5$	g_1(MeV^{-8})	g_2(MeV^{-8})
5.5	183.468	637.720	2.914	75.968	2.193×10^{-21}	-5.890×10^{-22}

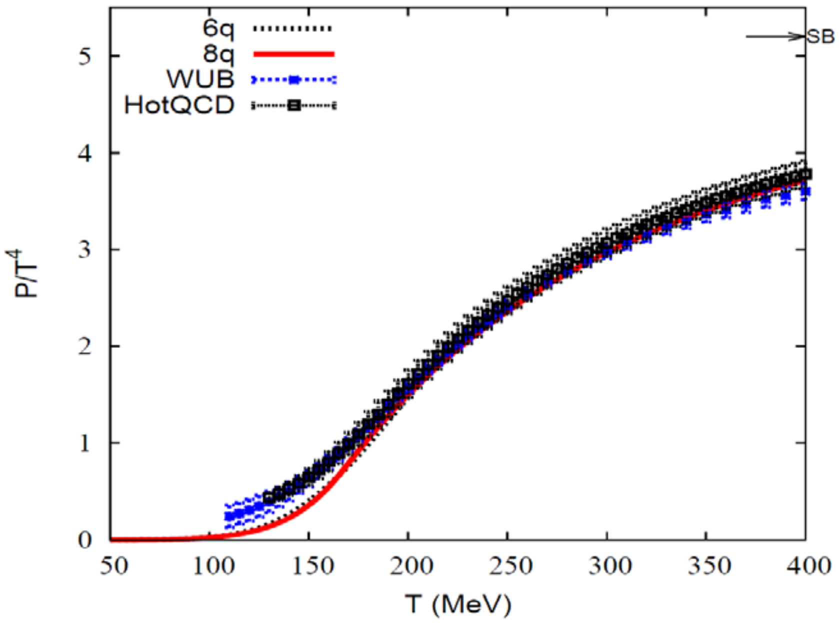
TABLE IV: Parameters for the NJL part in the realistic PNJL model.

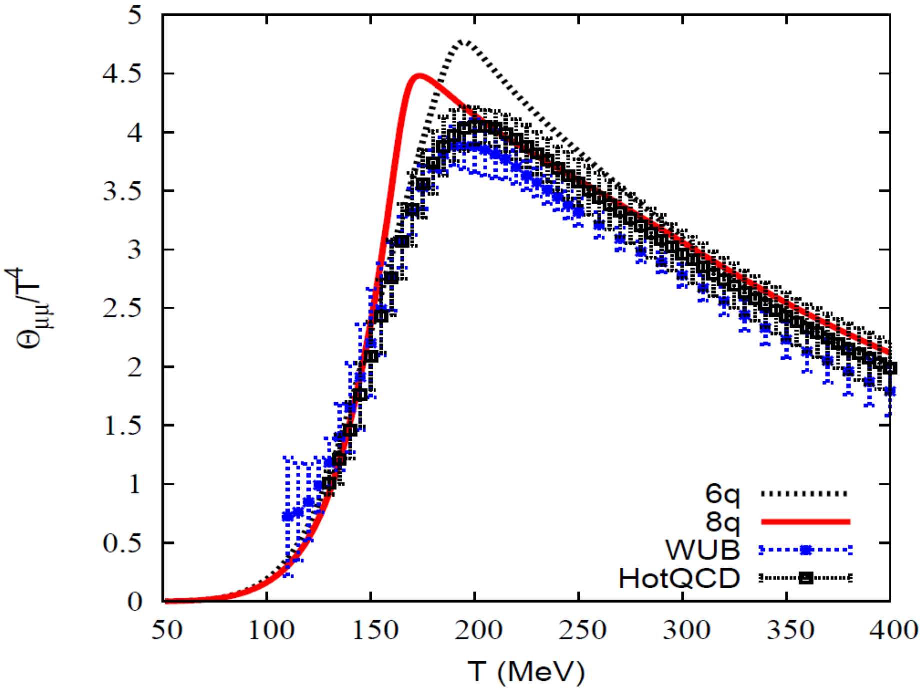
T_0 (MeV)	a_0	a_1	a_2	b_3	b_4	κ
175	6.75	-9.8	0.26	0.805	7.555	0.1

TABLE V: Parameters for the Polyakov loop part in the realistic PNJL model.

A. Bhattacharyya, S. K. Ghosh, S. Maity, S. Raha,
B. R. Ray, K. Saha and S. Upadhaya, arXiv:1609.07882.

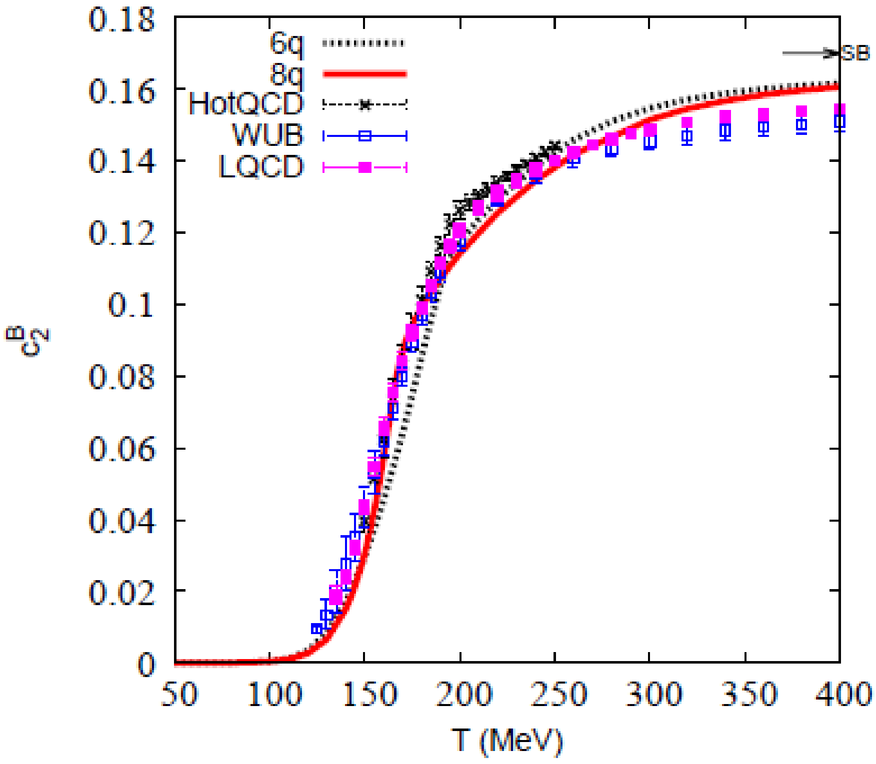
Equation of state at mu=0
model vs LQCD

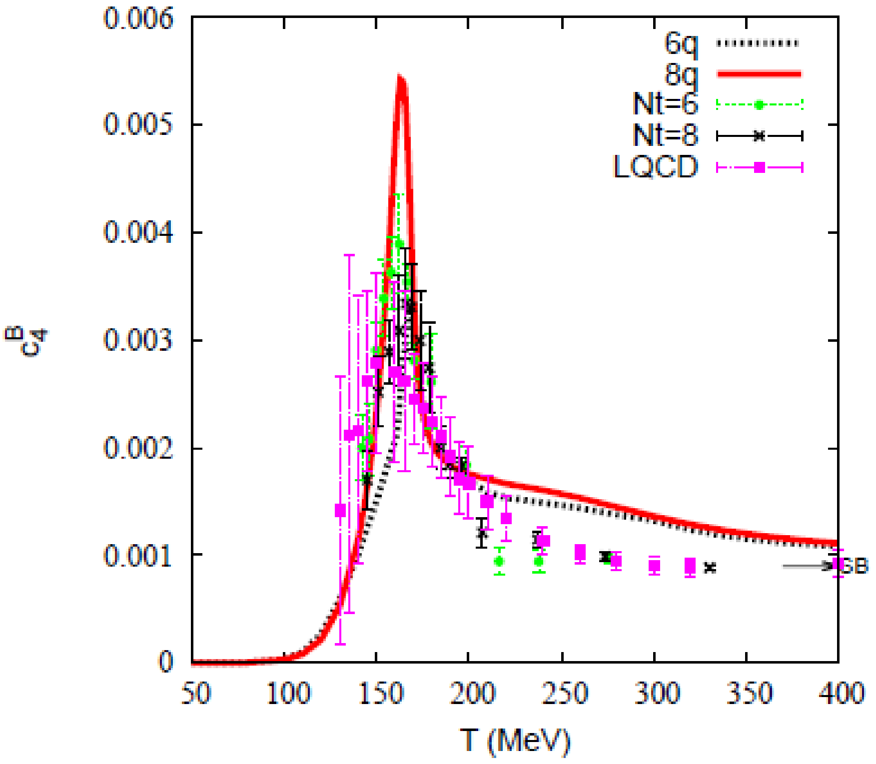




**A. Bhattacharyya,S. K. Ghosh, S. Maity, S. Raha,
B. R. Ray, K. Saha and S. Upadhaya,arXiv:1609.07882.**

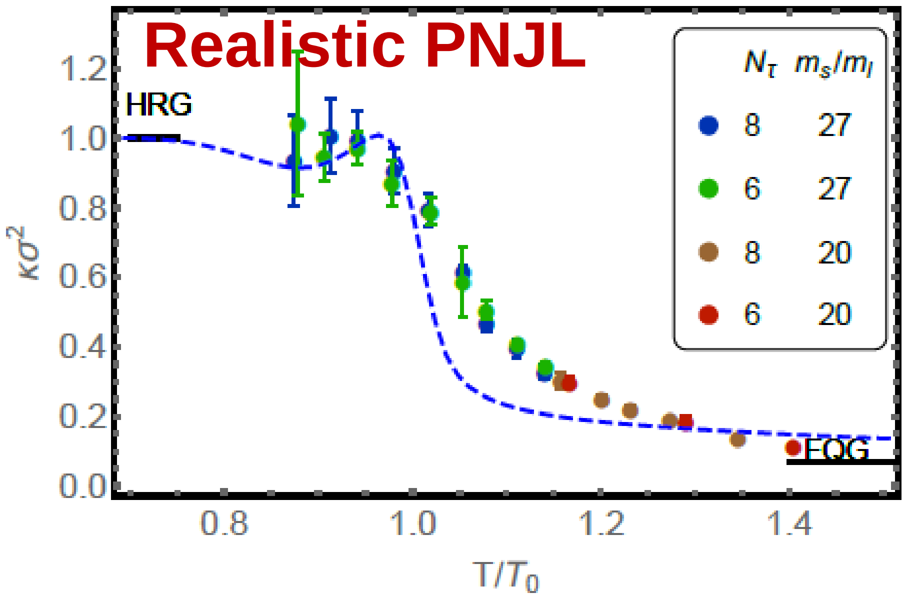
Baryon number fluctuation at mu=0
model vs LQCD

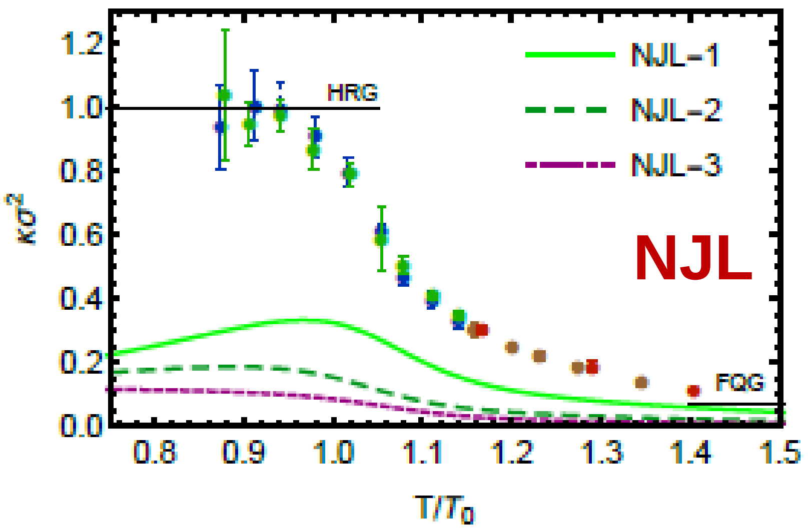




A. Bhattacharyya, S. K. Ghosh, S. Maity, S. Raha, B. R. Ray, K. Saha and S. Upadhaya, arXiv:1609.07882.

Kurtosis of baryon number fluctuation at mu=0





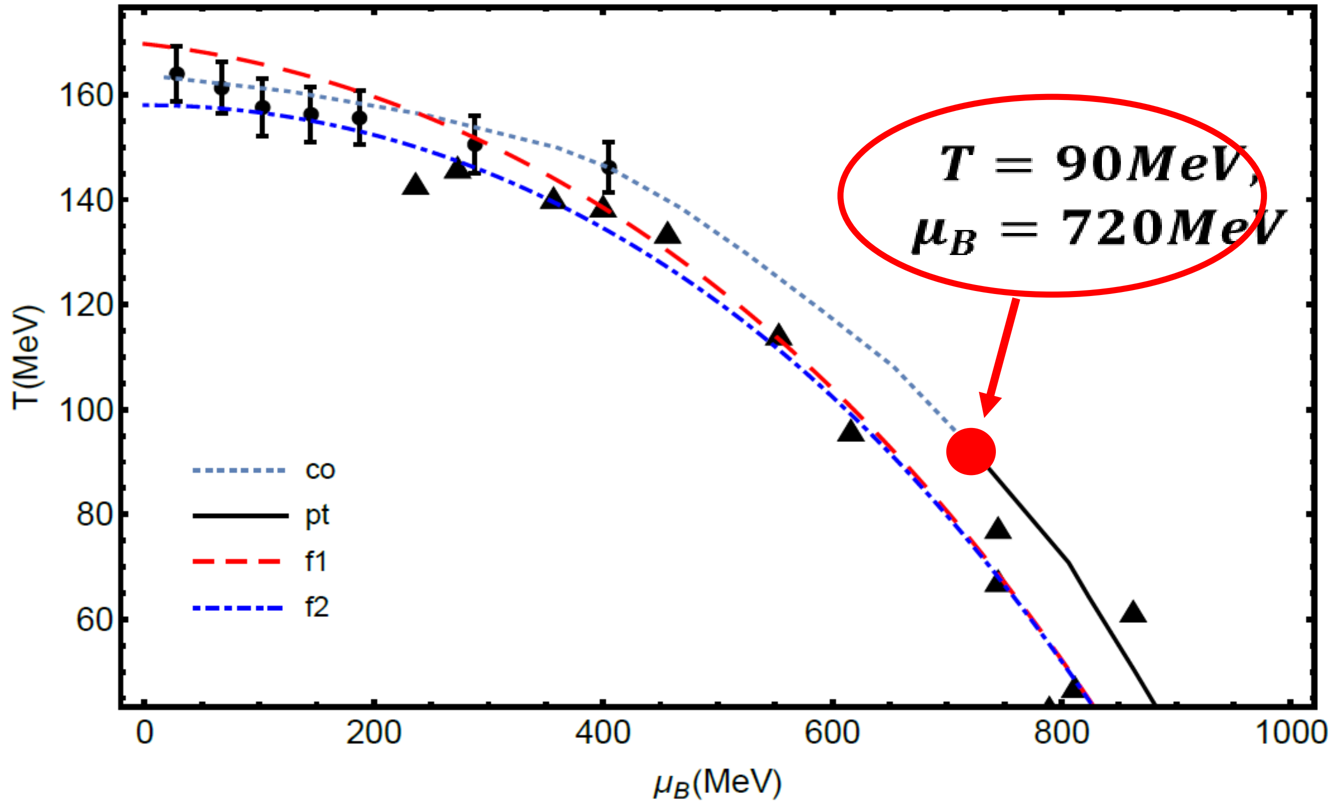
Gluodynamics is essential for C4/C2 !

Z.B Li, K.Xu,X.Y.Wang, M.Huang, arXiv:1801.09215

Predictions from the realistic PNJL model

Z.B Li, K.Xu,X.Y.Wang, M.Huang, arXiv:1801.09215

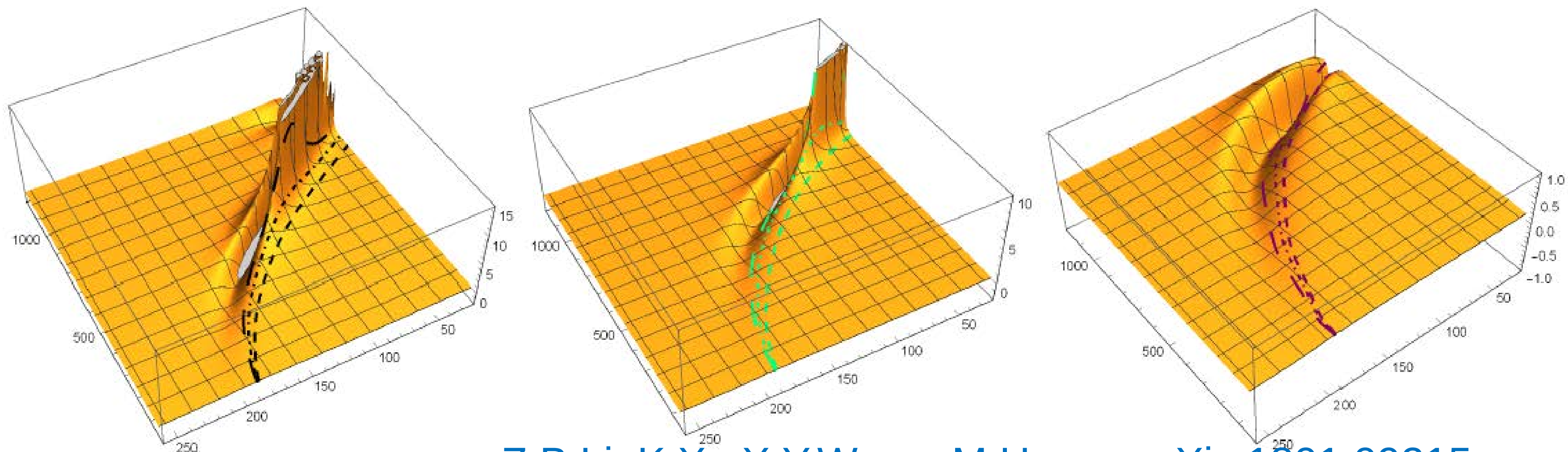
Phase boundary and CEP (mu_B^E=720 MeV, T^E=90 MeV)



Phase boundary is very close to the freeze-out data!!!

Z.B Li, K.Xu,X.Y.Wang, M.Huang,arXiv:1801.09215

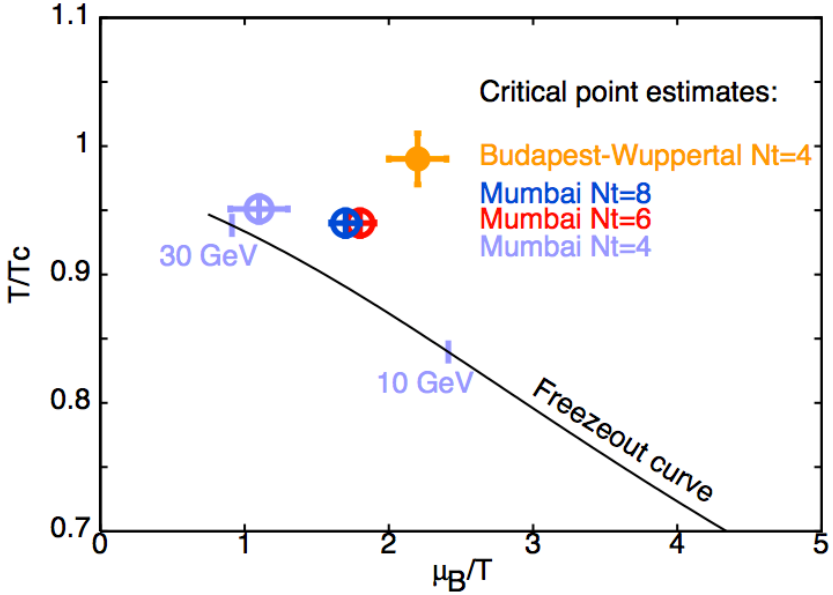
CEP location determines the location of the peak of kurtosis along the freeze-out line (close to the phase boundary) !

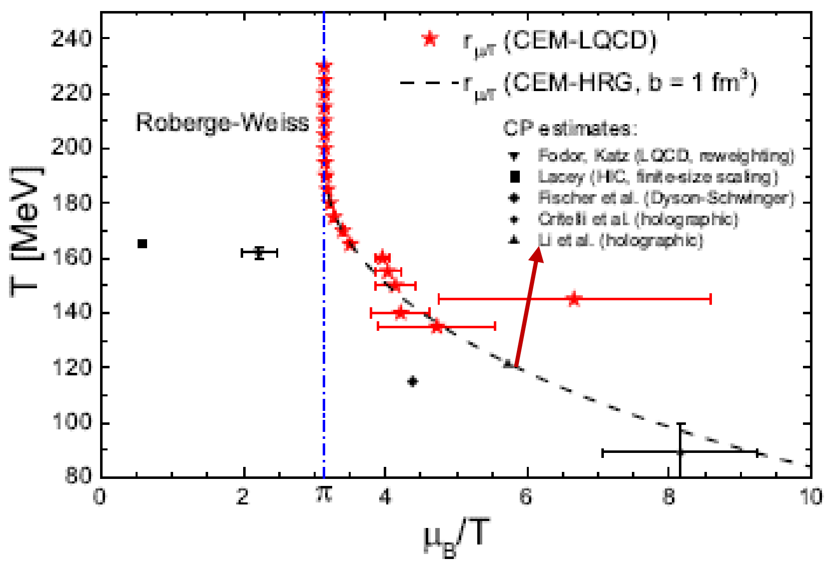


Z.B Li, K.Xu,X.Y.Wang, M.Huang,arXiv:1801.09215

BES-I measurement rules out the small baryon number density region for CEP!

Location of CEP from Lattice QCD





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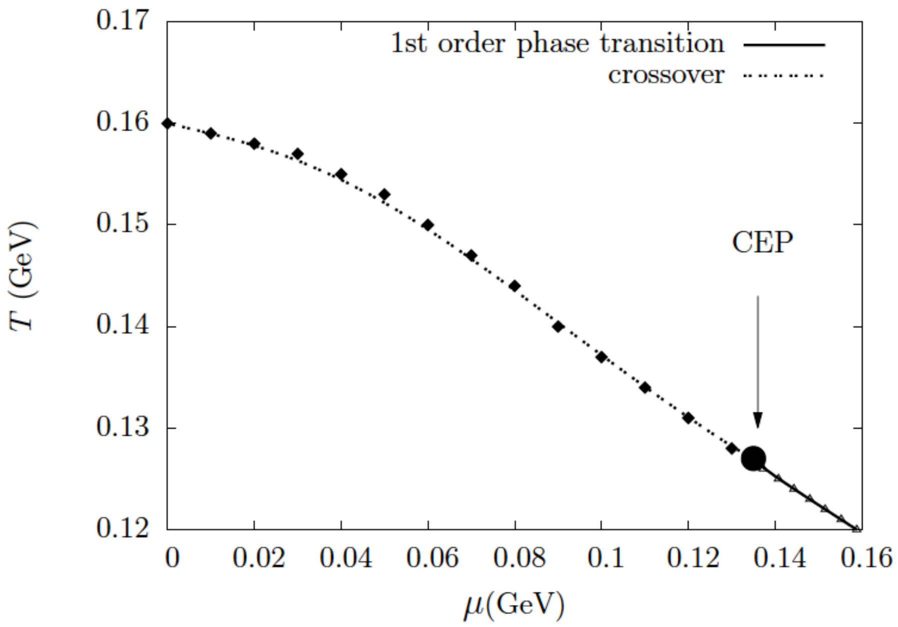
$$\mu_B^E / T_E > \pi$$

Latest lattice calculation shows that small baryon number density region for CEP is ruled out!

Location of CEP from DSE



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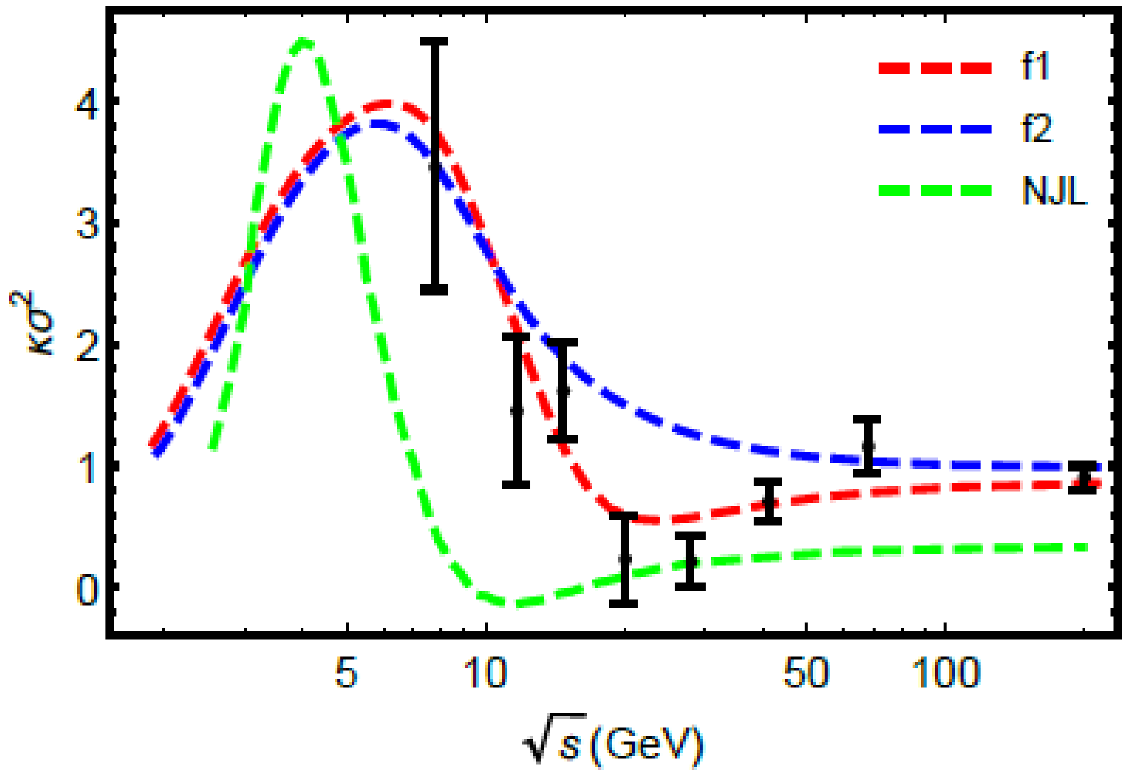
$(\mu^E_B, T^E) = (504, 115)$ MeV

$$\mu_B = 3\mu_q$$

baryon number density region 300-500 MeV

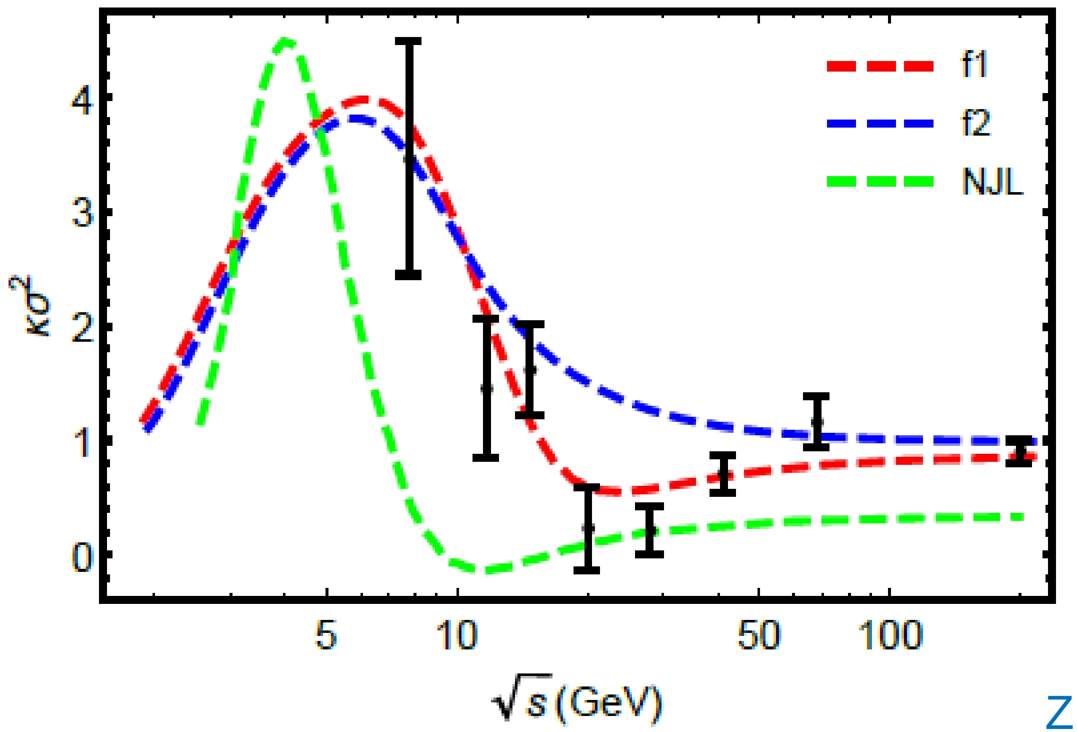
Kurtosis along experimental freeze-out lines

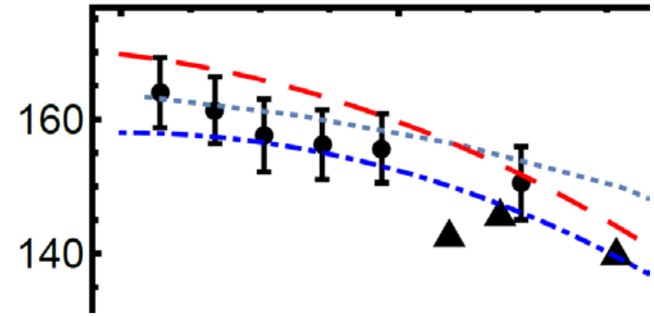


Realistic PNJL model results agree well with BES-I data! Equilibrium result can describe the experimental data!!!

Z.B Li, K.Xu,X.Y.Wang, M.Huang, arXiv:1801.09215

Dip structure



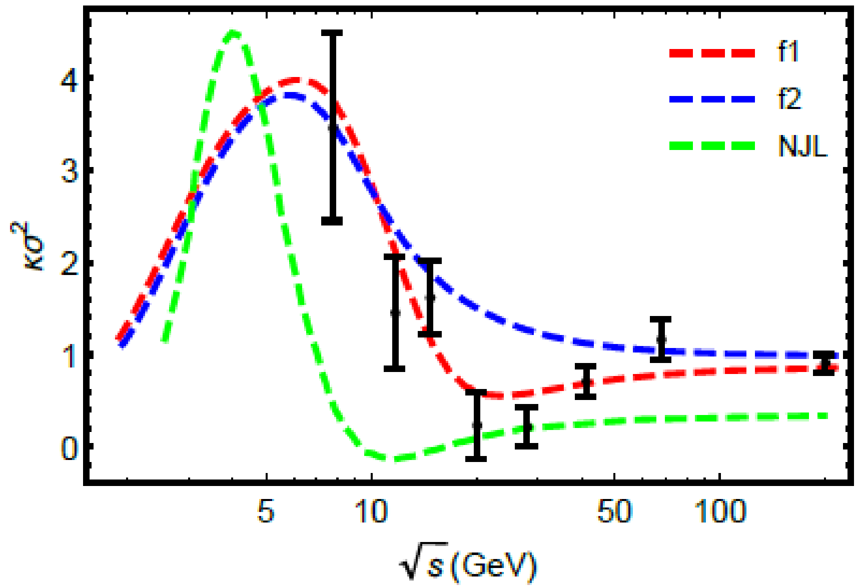


f1 cross the phase boundary while f2 not!

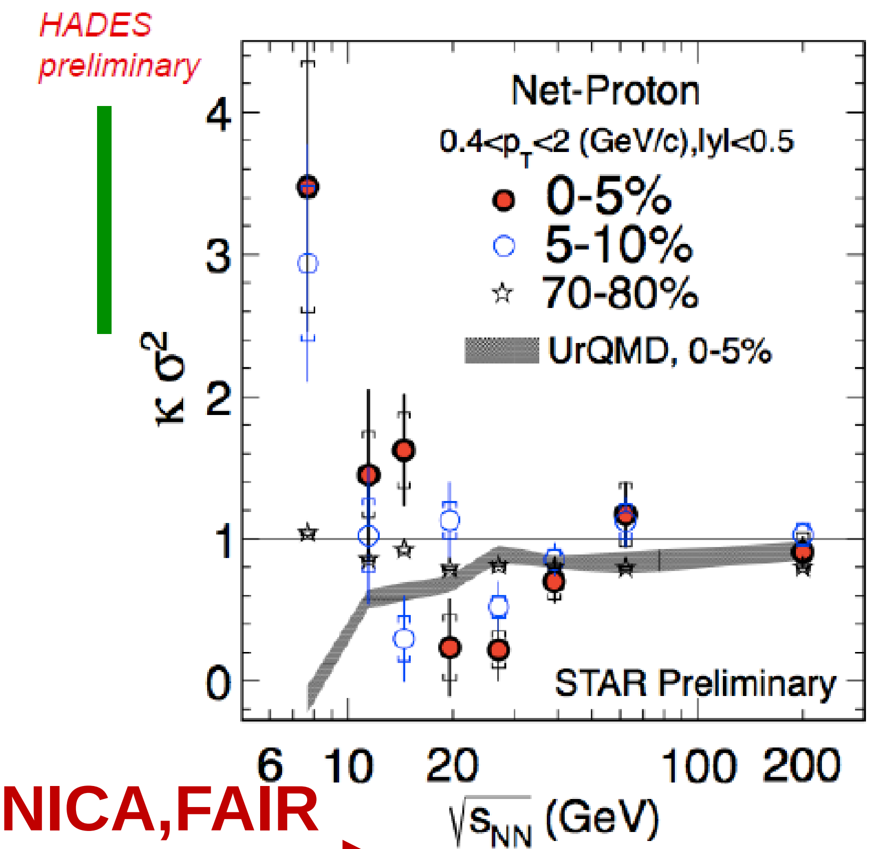
Z.B Li, K.Xu,X.Y.Wang, M.Huang
arXiv:1801.09215

The dip structure is sensitive to the relation between the freeze-out line and the phase boundary !

Peak structure is expected to show up in CBM and NICA



Z.B Li, K.Xu,X.Y.Wang, M.Huang
arXiv:1801.09215



NICA,FAIR

The peak structure along the freeze-out line is the residue of the divergence of CEP along phase boundary! Unique structure for CEP!

Cold droplet quark matter

Quantized 1st-order phase transition,

Two sets of CEP

Kun Xu, M.H., arXiv:1903.08416, 1904.1154

Finite size effect on phase transition and hadron physics

Four decades ago:

M.E. Fisher, in Critical phenomena, Proc. 51st Enrico Fermi Summer School, Varena, ed. M.S. Green (Academic Press, NY, 1972); M.E. Fisher and M.N. Barber, Phys. Rev. Lett. 28 (1972), 1516;
Y. Imry and D. Bergman, Phys. Rev. A3 (1971) 1416

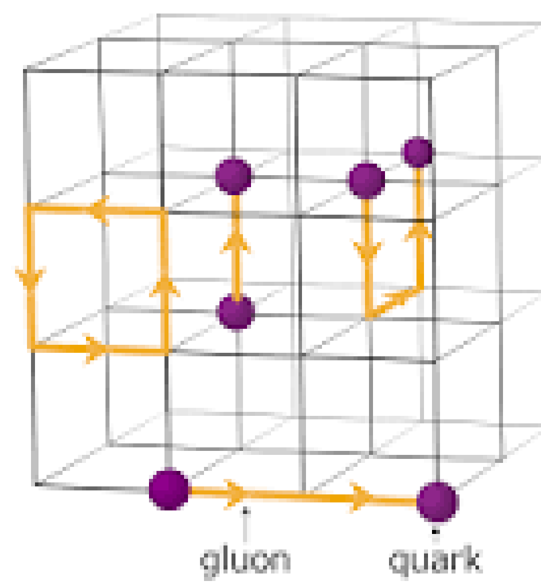
Barber, M.N.: Finite-size scaling. In: Phase transitions and critical phenomena. Vol. 8, Domb, C., Lebowitz, J.L. (ed.). London: Academic Press 1983
Brézin, E., Zinn-Justin, J.: Finite size effects in phase transitions. Nucl. Phys. B**257** [FS14], 867 (1985)

Volume Dependence of the Energy Spectrum in Massive Quantum Field Theories

M. Lüscher

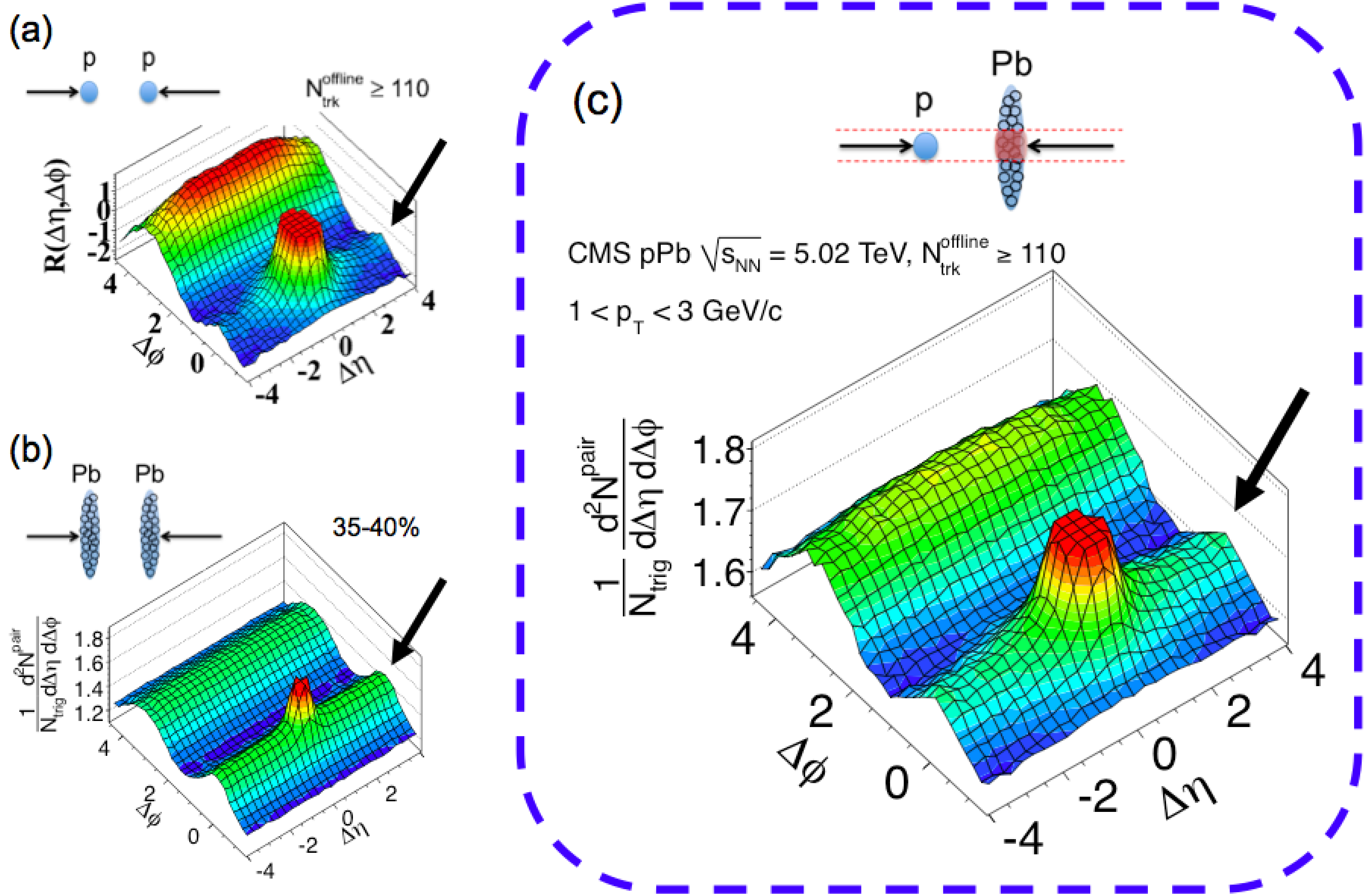
Commun. Math. Phys. 104, 177–206 (1986)

Commun. Math. Phys. 105, 153–188 (1986)



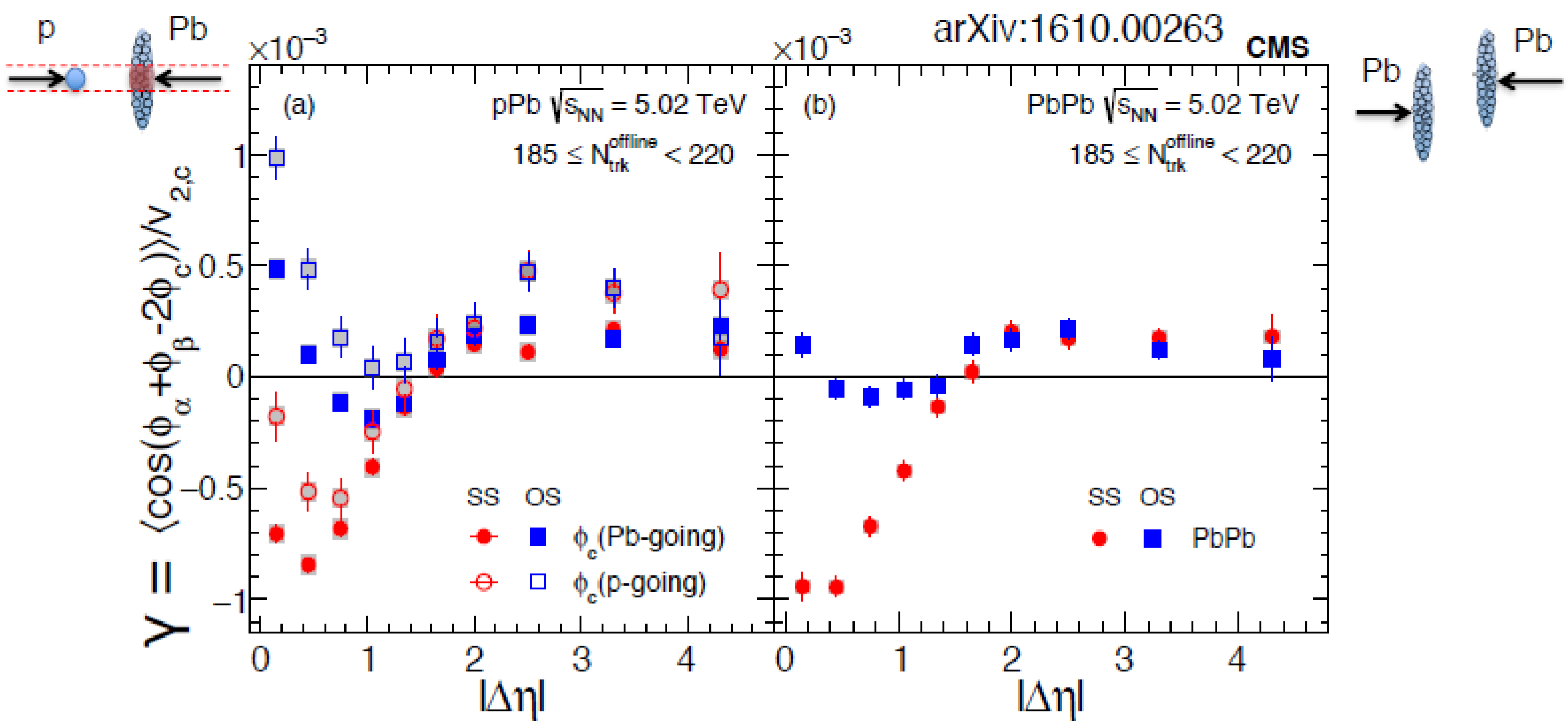
Yuxin Liu, Weijie Fu, Hongshi Zong, …
Ping Wang, Fengkun Guo, …

Small system from pp and pA collisions!



CME measurement in small system

|Δη| dependence of γ



Collective v_n well established for N_{trk}>185

Clear splitting of SS and OS in pPb, similar to PbPb

→ **NOT** in favor of CME interpretation?

Pion Compton length:

$$\lambda_\pi = \left(\frac{1}{140}\ \mathrm{MeV}^{-1}\right)(197.3\ \mathrm{MeV}\cdot\mathrm{fm}) = 1.41\ \mathrm{fm}.$$

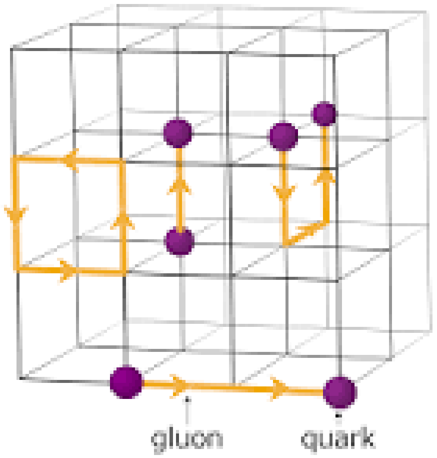
System size L >> pion Compton length: $L >> \lambda_\pi\,,$

Finite size scaling (FSS)

System size L comparable with pion Compton length:

This talk ! $L \sim \lambda_\pi$

Infinite Volume -> Finite Volume:



$$\int \frac{d^3p}{(2\pi)^3} \to \frac{1}{V}\sum_p$$

Periodic boundary condition (P-BC):

$$\vec{p}^2 = (\frac{2\pi}{L})^2 \sum_{i=x,y,z} n_i^2$$

Anti-periodic boundary condition(AP-BC):

$$\vec{p}^2 = (\frac{2\pi}{L})^2 \sum_{i=x,y,z} (n_i + \frac{1}{2})^2$$

Naturally, P-BC is applied for bosons.

Historically, both P-BC and AP-BC can be applied for fermions, normally, AP-BC is applied for fermions to keep the permutation symmetry with time direction.

System size L -> infinity:

P-BC and AP-BC equivalent

System size L >> pion Compton length: $L >> \lambda_{\pi},$

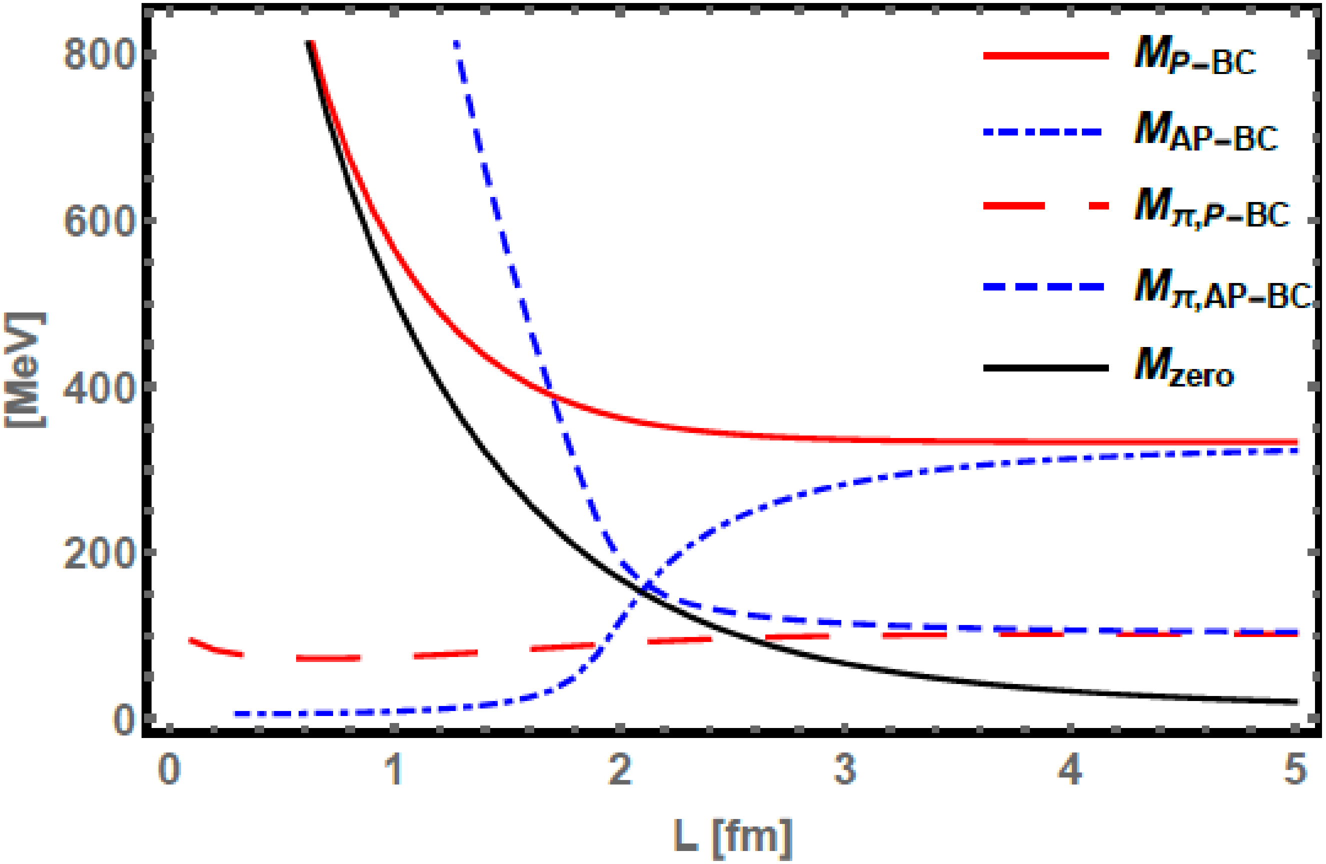
P-BC and AP-BC similar results

System size L comparable with pion Compton length:

$$L \sim \lambda_{\pi}$$

P-BC and AP-BC induce opposite results!

L>5fm: size effect can be neglected, P-BC and AP-BC the same; L<2fm: size effect is essential! P-BC and AP-BC induces opposite results. P-BC induces chiral symmetry restoration and heavy pion mass, AP-BC induces catalysis of chiral symmetry breaking and pion keeps as pseudo-Goldstone boson!



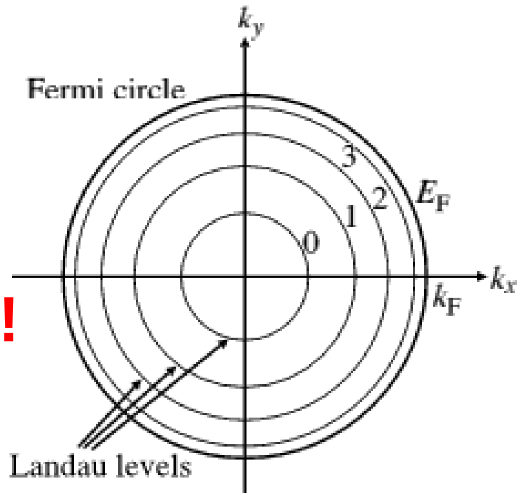
Why P-BC and AP-BC are so different in small size?

$$\vec{p}^2 = (\frac{2\pi}{L})^2 \sum_{i=x,y,z} n_i^2 \qquad \vec{p}^2 = (\frac{2\pi}{L})^2 \sum_{i=x,y,z} (n_i + \frac{1}{2})^2$$

Zero-momentum mode contribution dominates at small size!

Similar to strong magnetic field case, LLL is dominant!

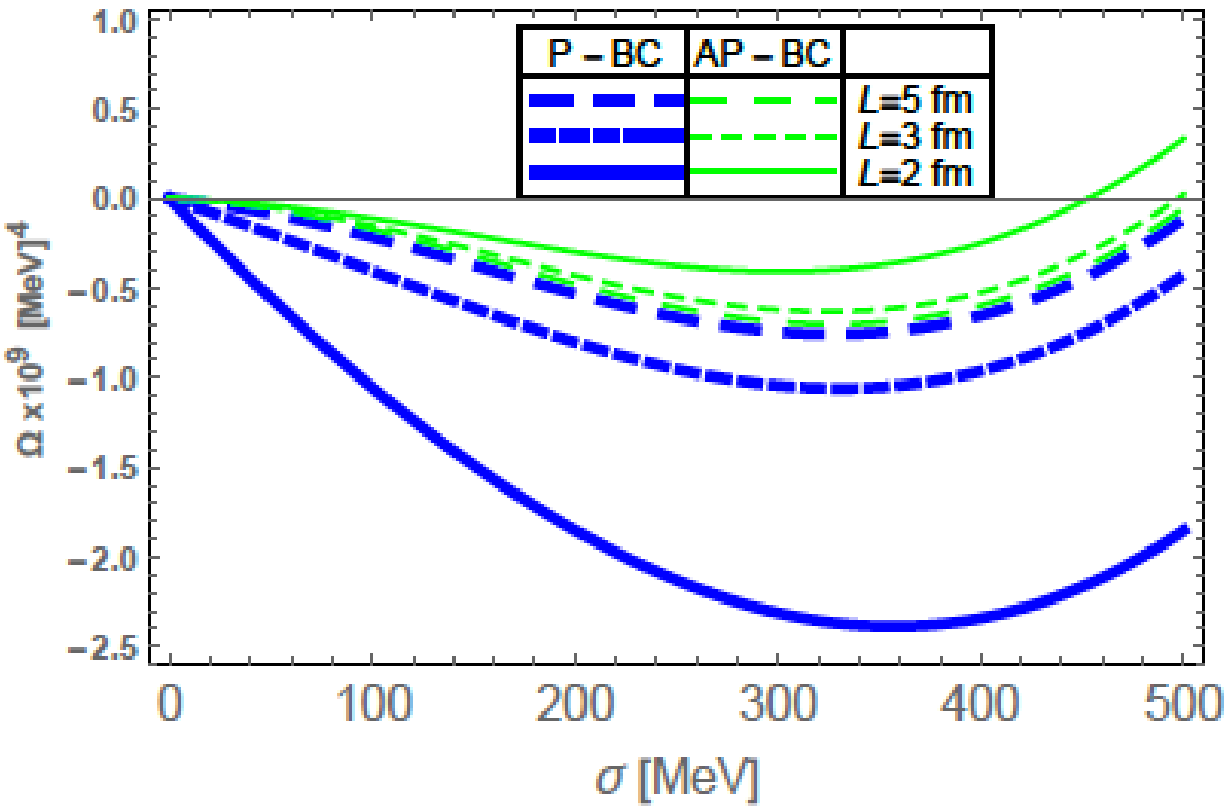
!!! For fermions, P-BC should be applied !!!



Zero-momentum mode cannot be neglected!
Pion keeps as Nambu-Goldstone boson!

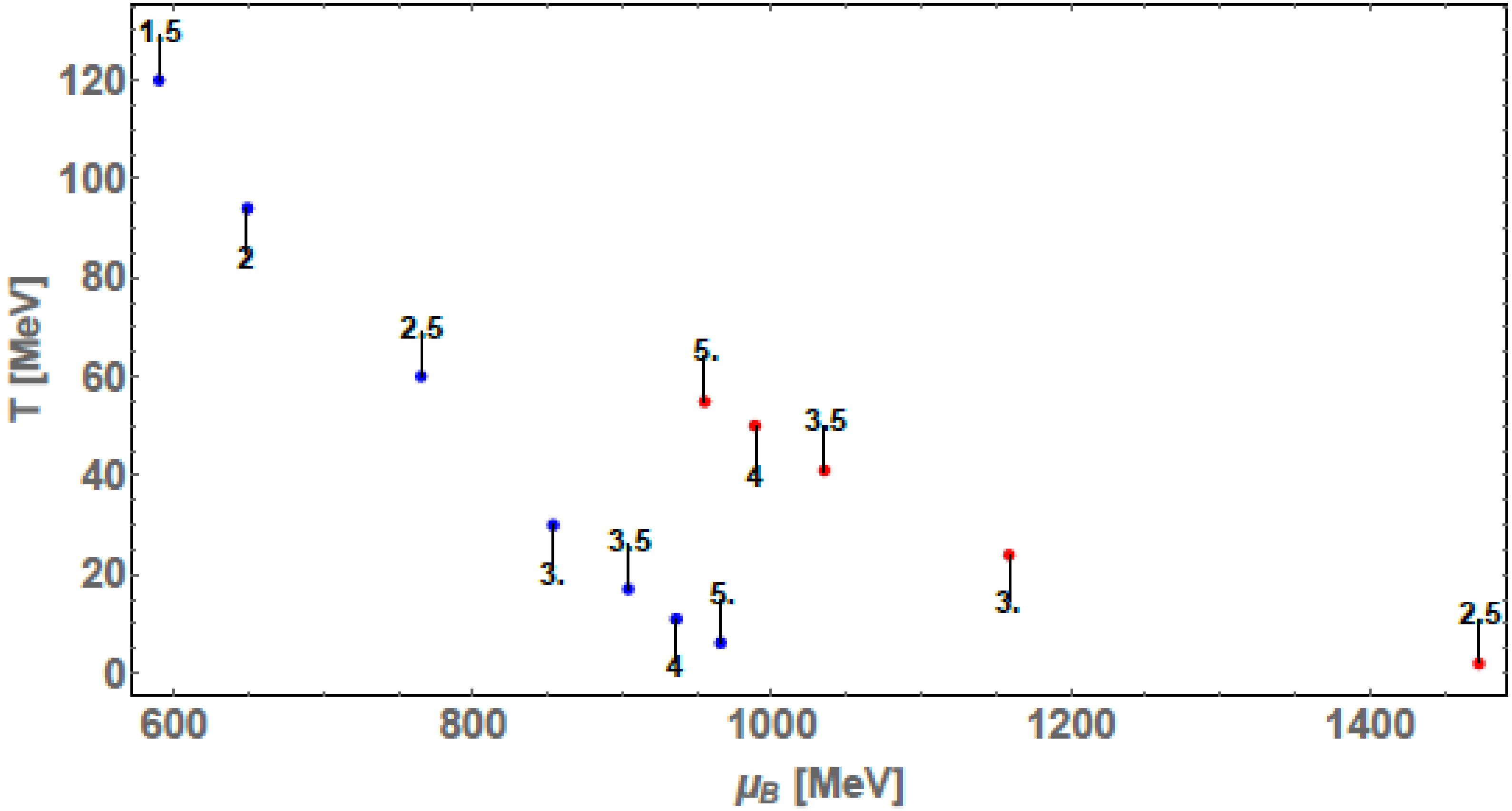
The ground state favors the P-BC!

$$\Omega = \frac{(M-m_0)^2}{4G} - \frac{2N_cN_f}{V}\sum_{\vec{p}}\left\{\sum_{j=0}^{3} c_j\sqrt{E^2+j\Lambda^2} + T\ln(1+e^{-\frac{E+\mu}{T}}) + T\ln(1+e^{-\frac{E-\mu}{T}})\right\}$$

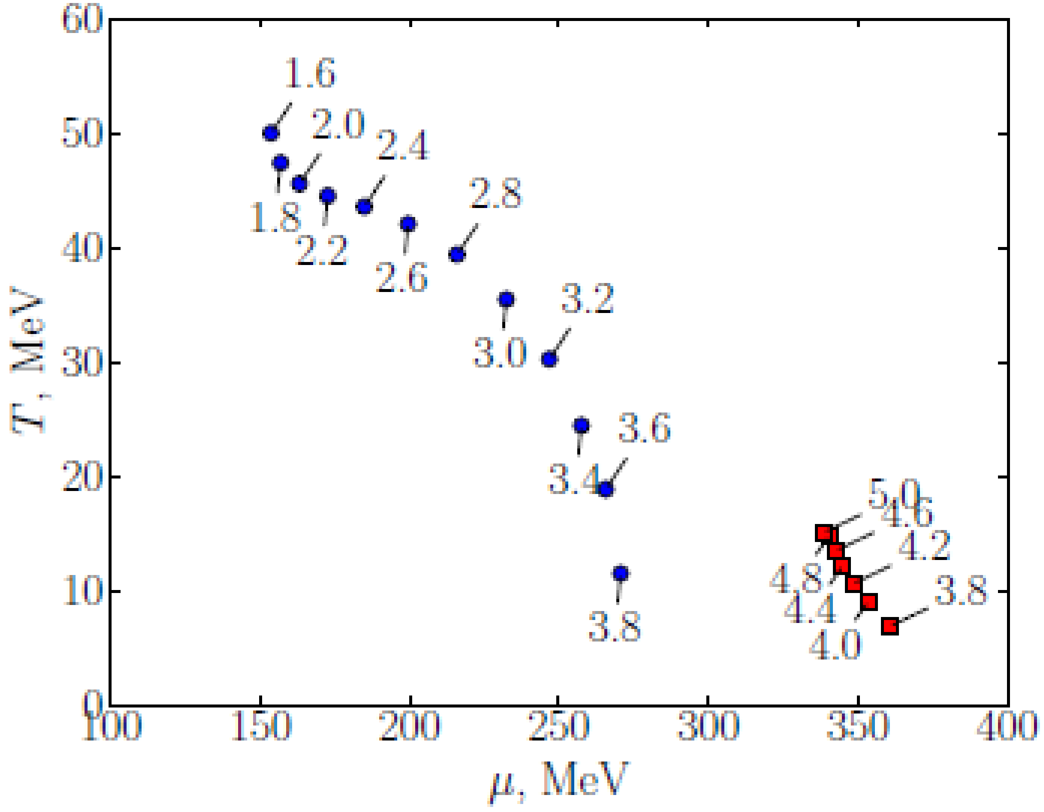


Kun Xu, M.H., arXiv:1903.08416, 1904.1154

In some small sizes, two branches of 1st-order phase transitions!

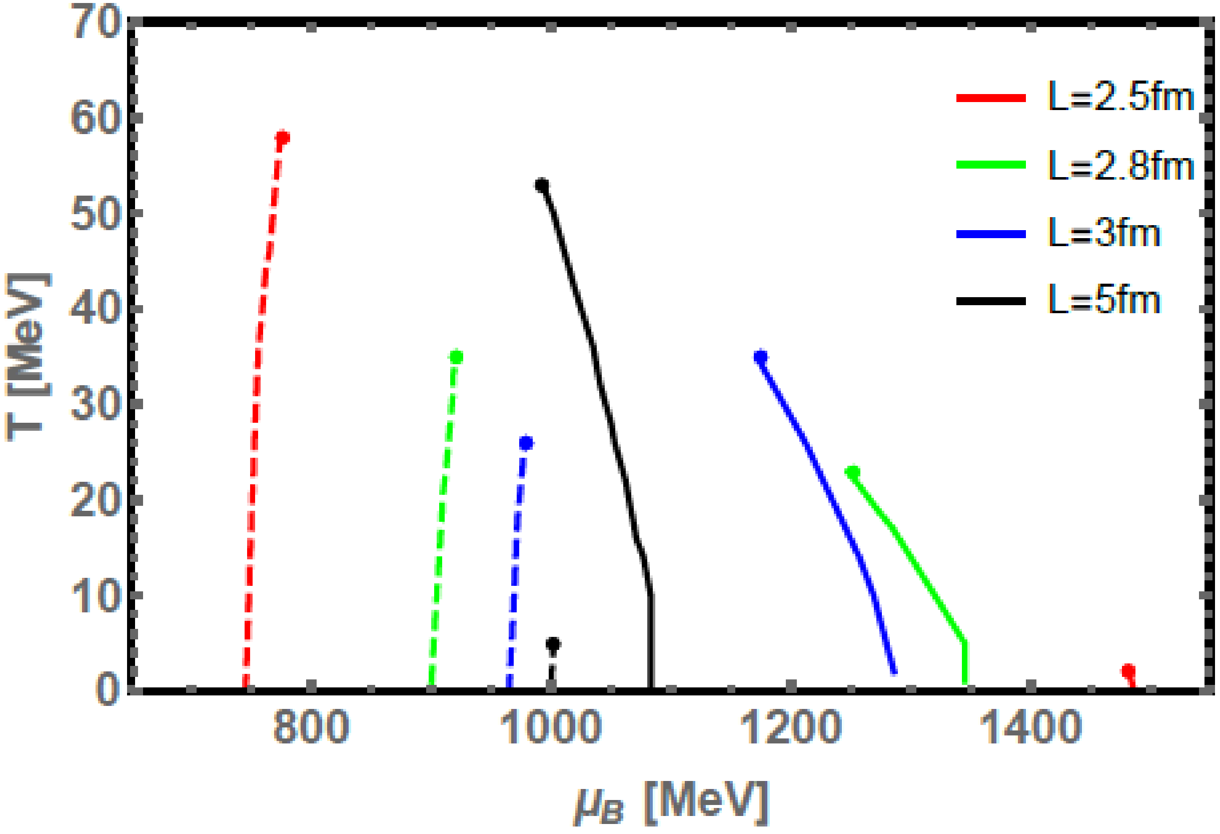


Similar to FRG result!



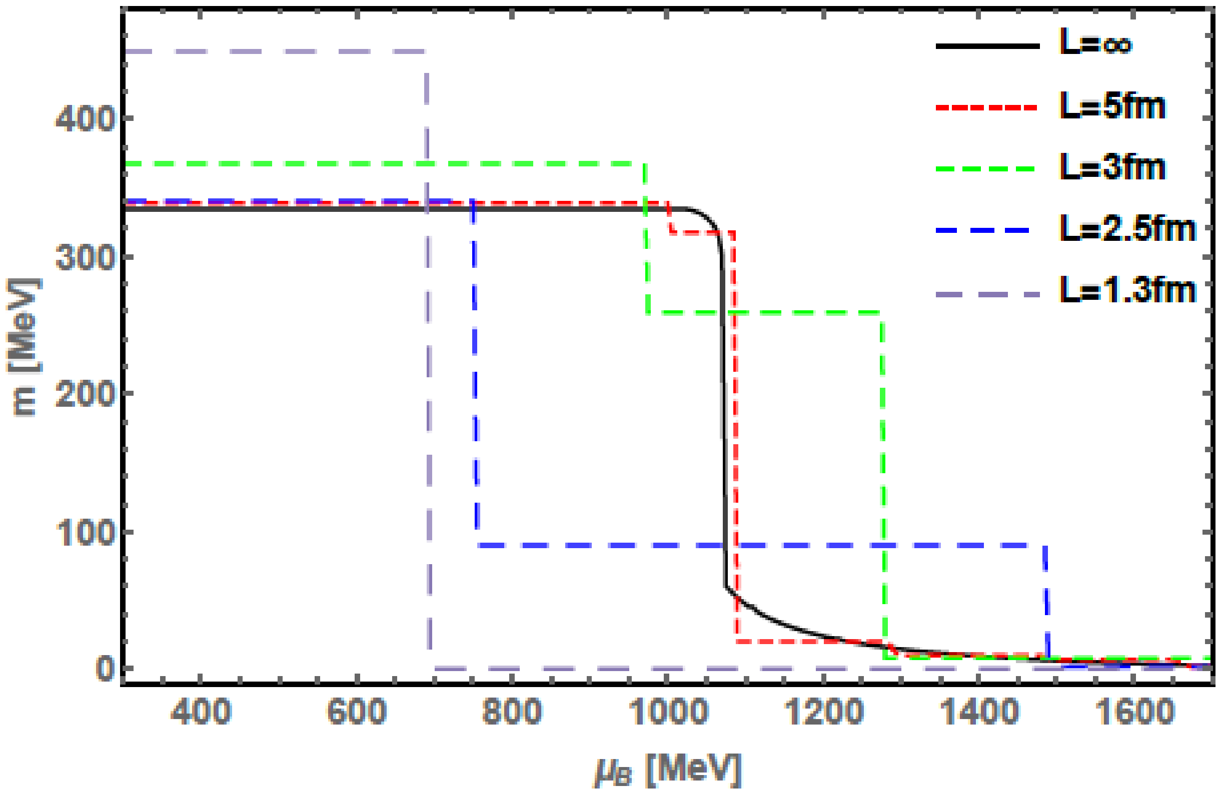
G.A.Almasi, R.Pisarski, V.Skokov, arXiv:1612.04416

In some small sizes, two branches of 1st-order phase transitions!



Kun Xu, M.H., arXiv:1903.08416, 1904.1154

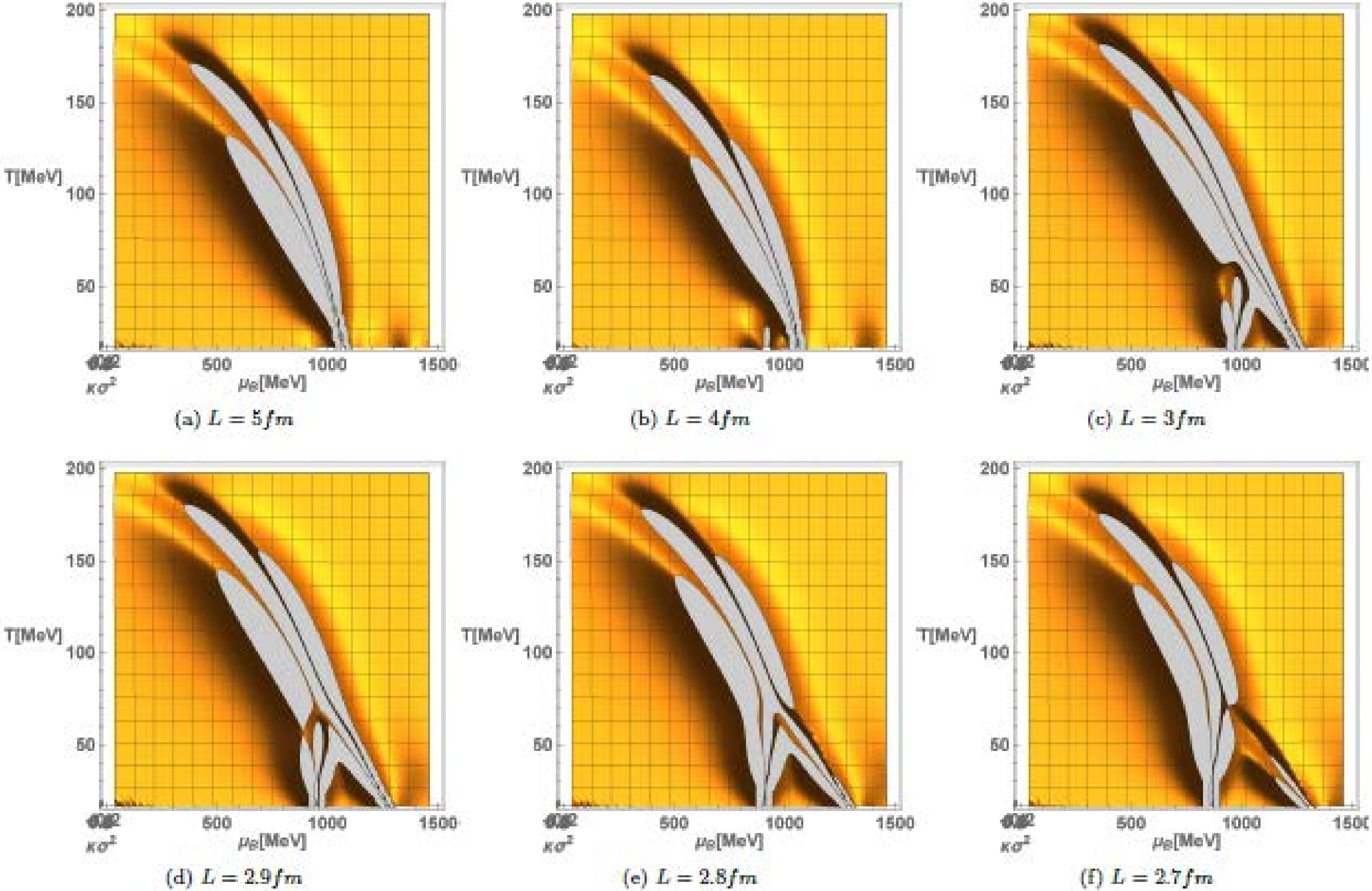
Why two branches?
Quantized 1st order phase transition!



Zero mode contribution
dominant at small size!

Kun Xu, M.H., arXiv:1903.08416, 1904.1154

Baryon number fluctuations in small system!

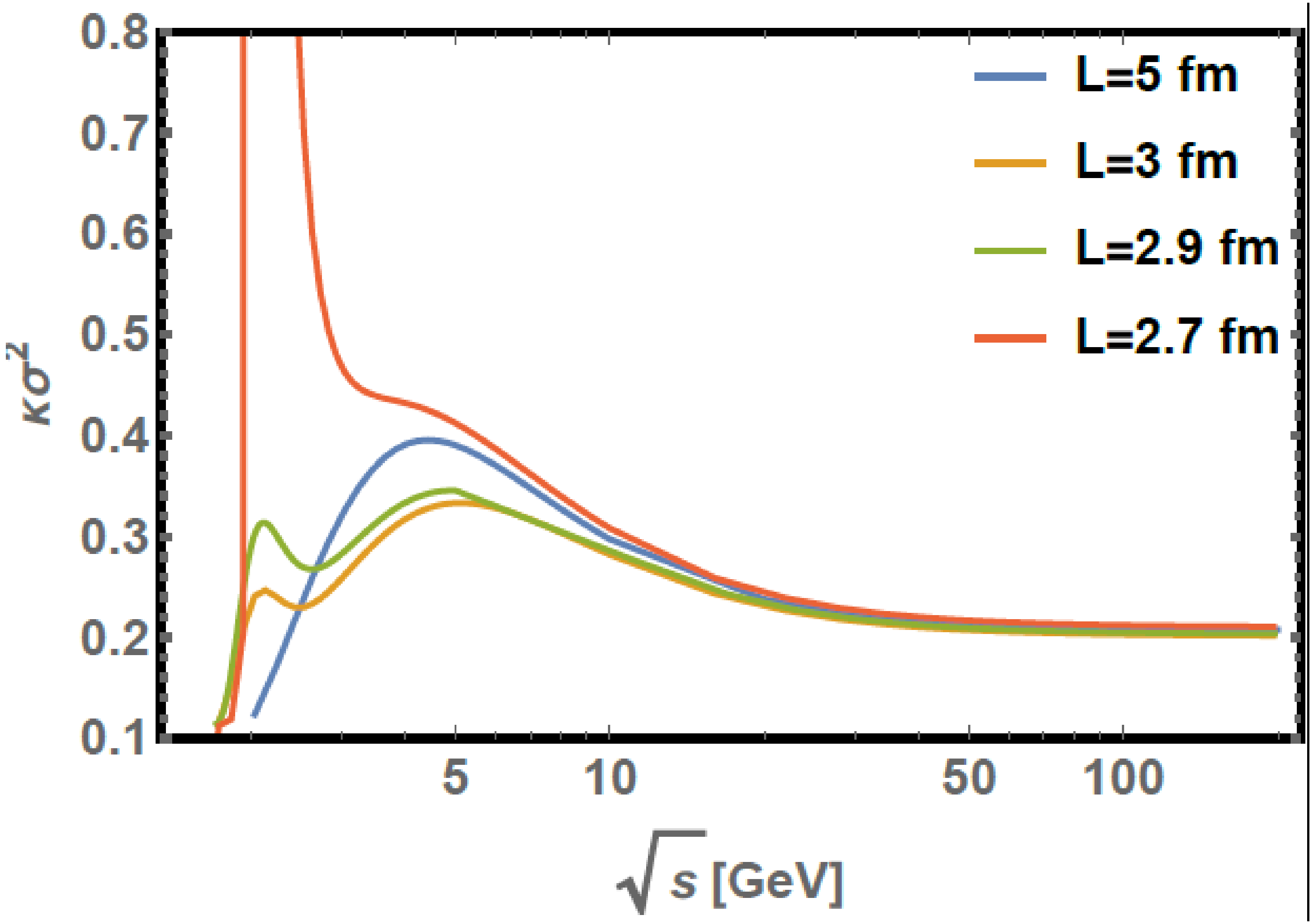


(a) $L = 5fm$ (b) $L = 4fm$ (c) $L = 3fm$

(d) $L = 2.9fm$ (e) $L = 2.8fm$ (f) $L = 2.7fm$

Kun Xu, M.H., to appear

Two bumps structure for baryon number fluctuations along the freeze-out line!



Kun Xu, M.H., arXiv:1903.08416, 1904.1154

Conclusion and Outlook

- BES-I measurement of baryon number fluctuation can be described well by a realistic PNJL mode!
- Peak structure along the freeze-out line is the residue of divergence of CEP along phase boundary, which is unique for CEP!

- Two branches of 1st-order phase transition in some small systems 1fm<L<5fm. Quantized 1st-order phase transition is a brand new phenomena!

Thanks for your attention!