Phase structure of dense QCD matter with magnetic field and Rotation

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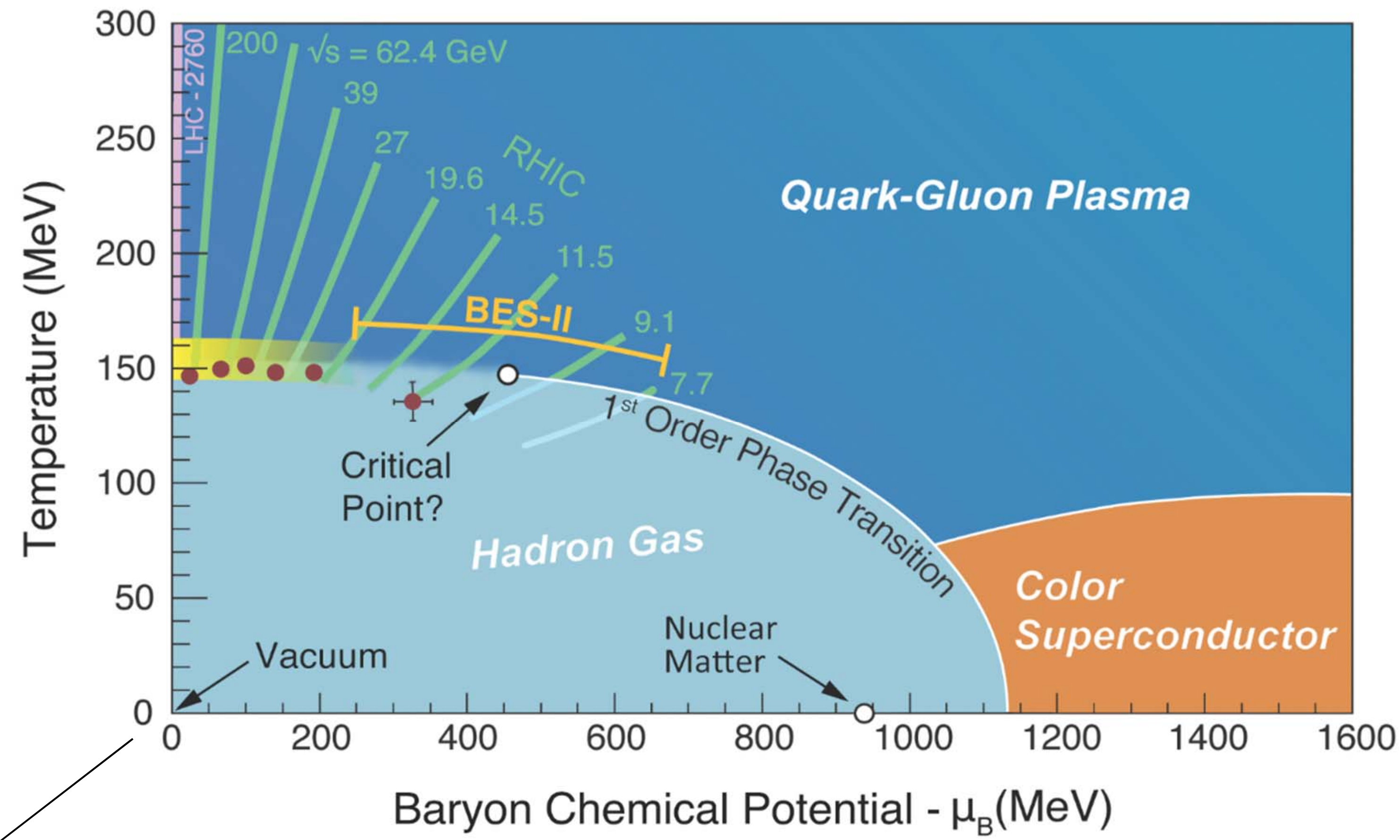
Outlines

- Motivations
- Phase structure with magnetic field (B)
- FRG study on xPT of dense QCD matter
- Rotation effects on phase structure
- Summary

H. Zhang, DF Hou, JF Liao, arxiv   1812.11787

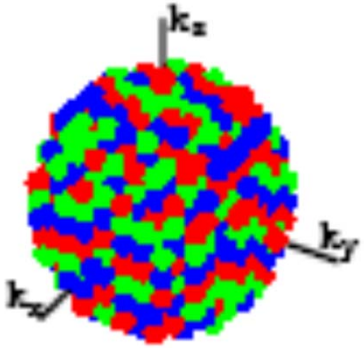


New dimension: Magnetic field, Rotation, etc. → change the phase diagram

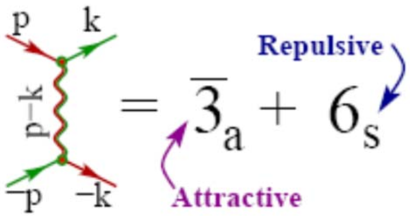
Dense quark matter

Ground state of dense quark matter is CSC

(i) Deconfined quarks($\mu >> \Lambda_{QCD}$)
(ii) Pauli principle(s=1/2)



(i) Effective models($\mu \geq \Lambda_{QCD}$)
(ii) One-gluon exchange($\mu >> \Lambda_{QCD}$)



Cooper instability

Color superconductivity

$$< (\overline{\psi}^C)_i^\alpha \gamma_5 \psi_j^\beta > \neq 0$$

B. Barrois, NPB 129, 390 (1977)
D. Bailin and A. Love, Phys. Rep. 107,325 (1984)
M. Alford et al., PLB 422, 247 (1998)
R. Rapp et al., PRL 81, 53 (1998)

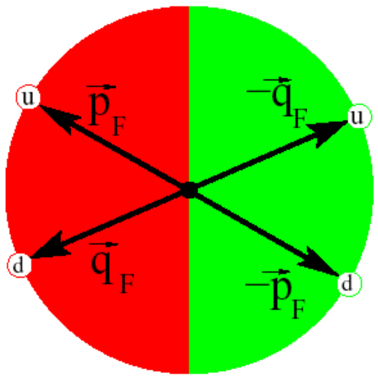
Phase structure in CSC

- BSC-like pairing

J=0: 2SC: u_r, d_r, u_g, d_g

CFL: all flavor and color

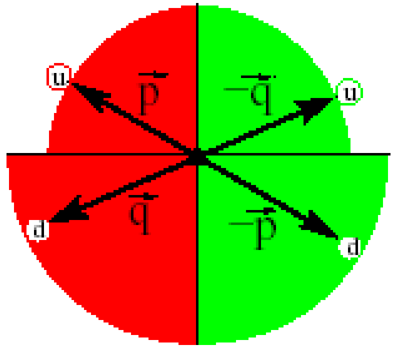
M. Alford, K. Rajagopal and F. Wilczek, NPB 537, 443 (1999)



J=1:

T. Schaefer, PRD 62, 094007 (2000)
A. Schmitt, PRD 71, 054016 (2005)

- Non-BCS pairing
  - gapless CSC
  - LOFF
  - ……

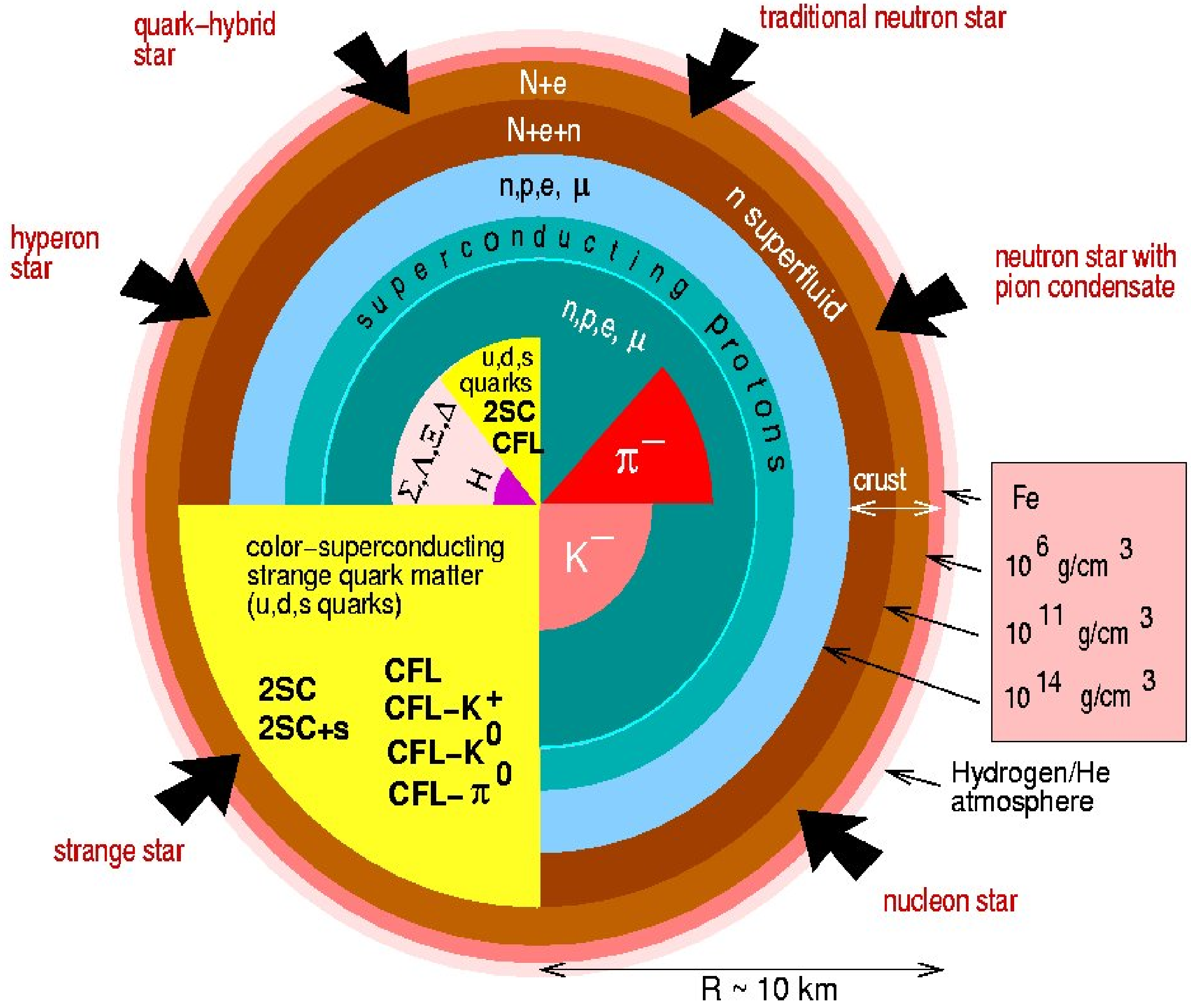


Shovkovy and M. Huang, PLB 546, 205 (2003)
M. Alford et al., PRL 92, 222001 (2004)
M. Alford et al., PRD 63, 074016 (2001)
…….

CSC in a compact star

Profile of neutron star

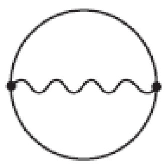
Webber, astro-ph/0407155



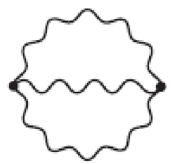
CJL effective action

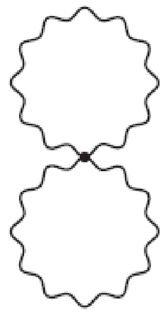
$$\Gamma[\bar{D},\bar{S}] = \frac{1}{2}\{Tr \ln \bar{D}^{-1} + Tr(D^{-1}\bar{D} - 1) - Tr \ln \bar{S}^{-1} - Tr(S^{-1}\bar{S}) - 2\Gamma_2[\bar{D},\bar{S}]$$

The two-loop approximation
to \gamma_2



Order of g^2mu^4





Powers of T

Stationary points

$$\left.\frac{\delta\Gamma}{\delta\bar{\mathcal{D}}}\right|_{\bar{\mathcal{D}}=\mathcal{D},\bar{\mathcal{S}}=\mathcal{S}} = 0\,, \qquad \left.\frac{\delta\Gamma}{\delta\bar{\mathcal{S}}}\right|_{\bar{\mathcal{D}}=\mathcal{D},\bar{\mathcal{S}}=\mathcal{S}} = 0$$

$$\Downarrow \qquad\qquad \Downarrow$$

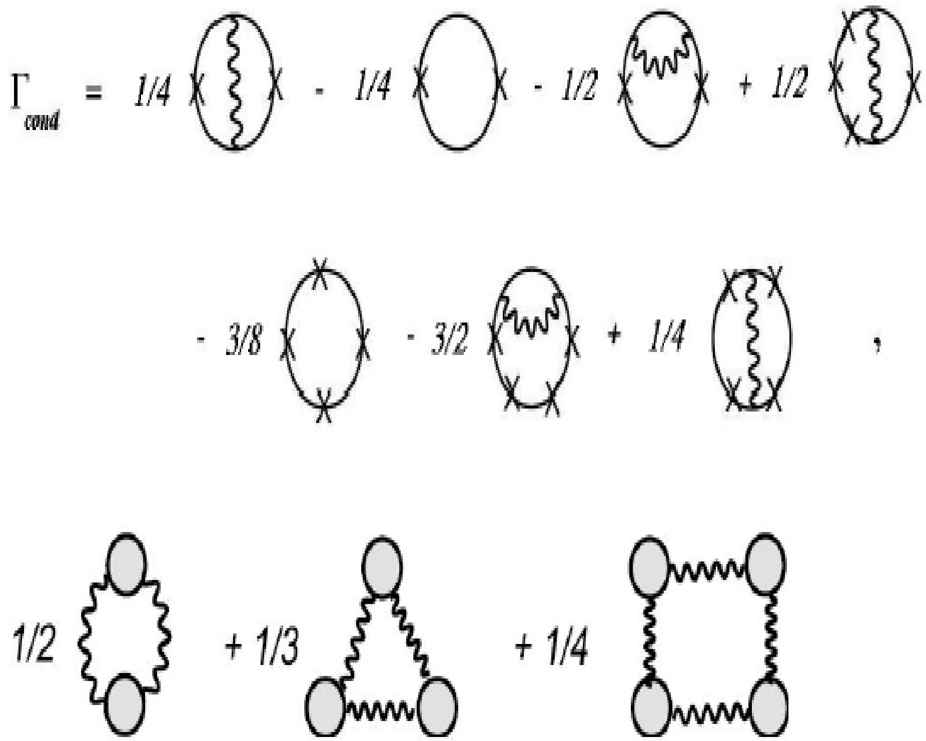
$$\mathcal{D}^{-1} = D^{-1} + \Pi[\mathcal{S}] \qquad \mathcal{S}^{-1} = \mathcal{S}_0^{-1} + \Sigma$$

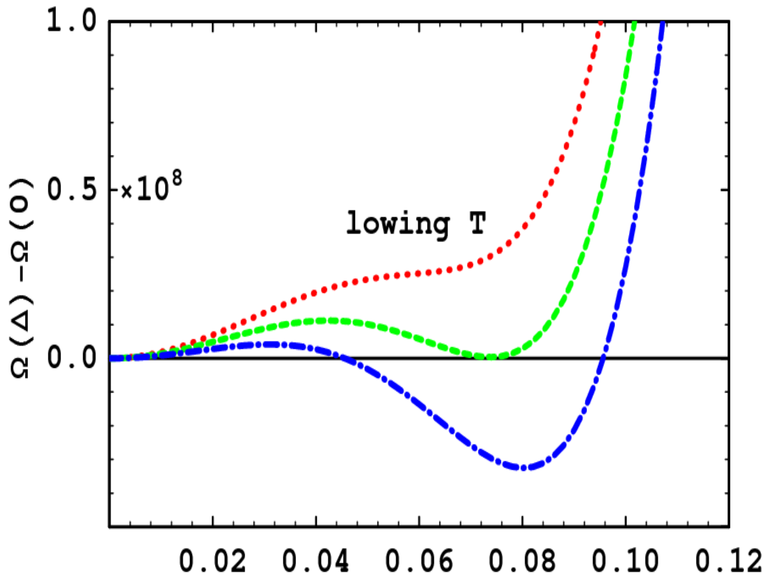
$$\Gamma_2[\bar{\mathcal{D}},\bar{\mathcal{S}}] = -\frac{1}{2}\,\mathrm{Tr}\{\bar{\mathcal{D}}\,\Pi[\bar{\mathcal{S}}]\}$$

D. Rischke Prog. Part. Nucl. Phys. 52 197 (2004)

Gauge field fluc. induce 1st order PT of CSC in dense QCD

Ginnakis, Hou, Ren, Rischke, PRL 93 (04) ; PRD73 (06)





Introduction of Δ^3 term in free energy by flucts.
Inducing 1st order PT in stead of 2nd order PT in MFA

Color Superconductor with B

Oscillation, decrease the gap at low B, and increase gap at high B

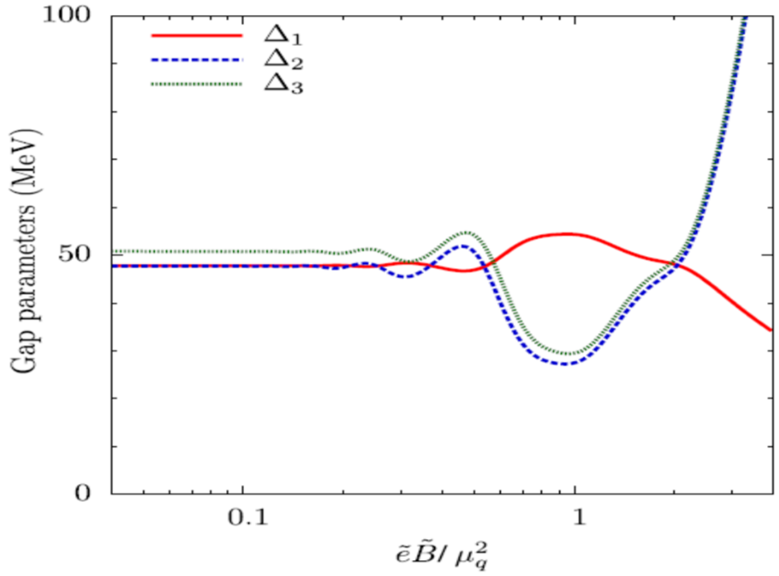


FIG. 1 (color online). Gap parameters as a function of $\tilde{e}\tilde{B}/\mu_q^2$ for $\mu_q = 500$ MeV and $M_s = 100$ MeV without neutrality.

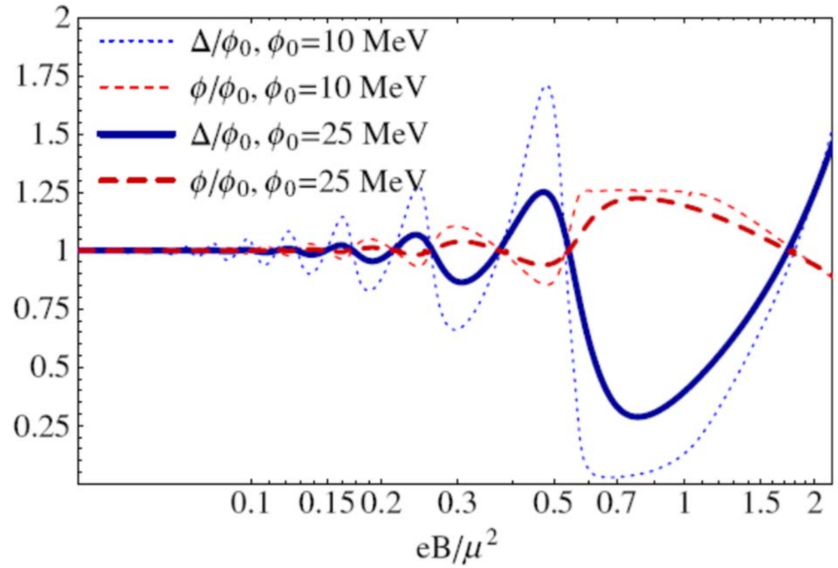


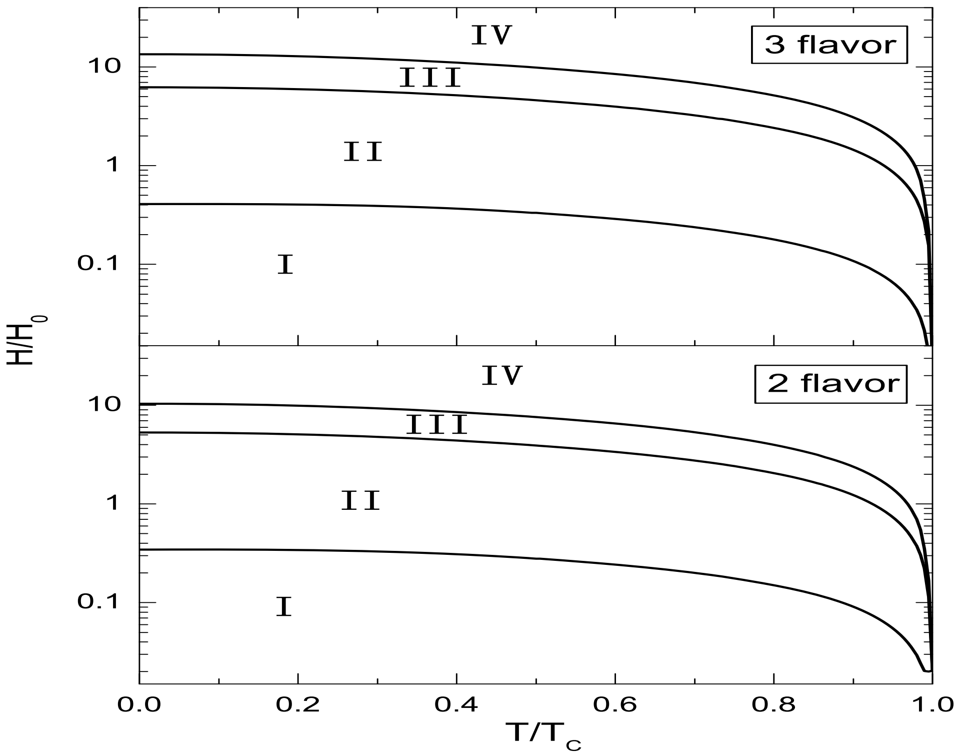
FIG. 1 (color online). Ratios Δ/ϕ_0 and ϕ/ϕ_0 versus eB/μ^2 for two sets of parameters that yield $\phi_0 = 10$ MeV and $\phi_0 = 25$ MeV.

K.Fukushima,etc.
PRL 100(2008)032007, CFL

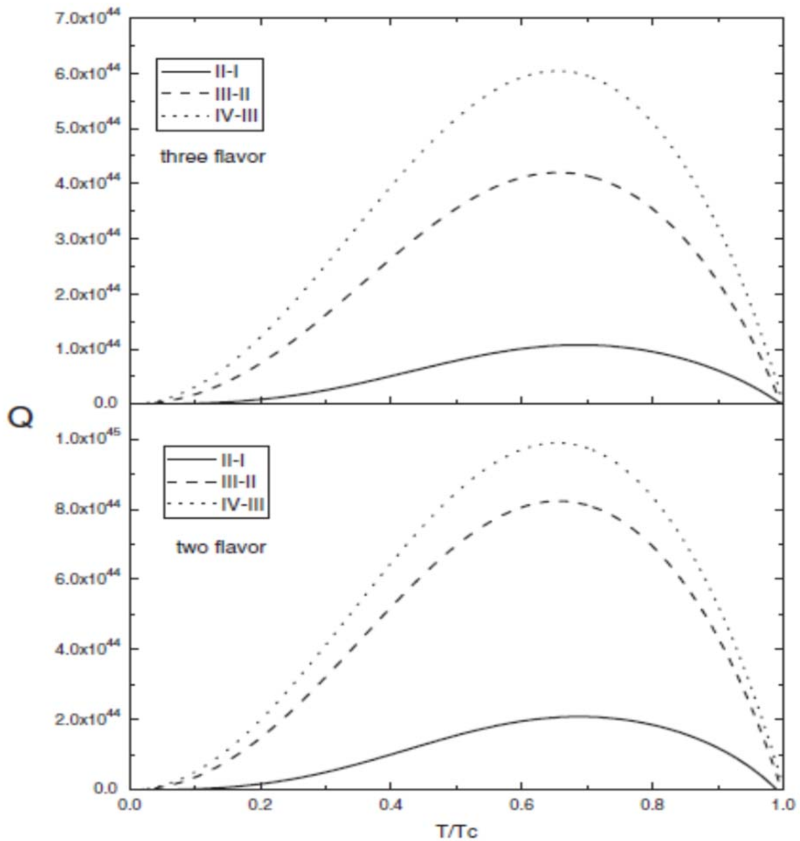
J.Noroha,etc. PRD 76(2007)105030, CFL

Nonspherical states in dense QCD with B

	I	II	III	IV	$T_C(10^{-1}$ MeV)
Two-flavor	CSL_u, CSL_d	$(polar)_u$, $(planar)_d$	$(normal)_u$, $(polar)_d$	$(normal)_u$, $(normal)_d$	1.35
Three-flavor	CSL_u, $CSL_{d,s}$	$(polar)_u$, $(planar)_{d,s}$	$(normal)_u$, $(polar)_{d,s}$	$(normal)_u$, $(normal)_{d,s}$	0.49



Feng, Hou, Ren,Wu, PRL 105(2010)



Wu, He, Hou, Ren, PRD84 (2011)

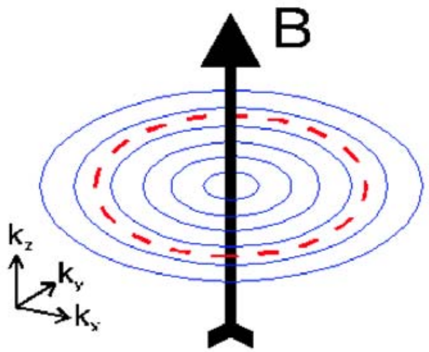
Chiral Magnetic Catalysis

- Chiral magnetic catalysis: Gusyin, Miransky & Shovkovy (1994)

$$<0|\bar{\psi}\psi|0> \overset{\lim\ m=0}{=} -\frac{|eB|}{2\pi}$$

Dynamical breakdown of chiral symmetry takes place at $m=0$ and $B\neq 0$ even without any additional interactions between fermions.

The essence of this effect is the dimensional reduction 3+1->1+1 in the dynamics of fermion pairing in a magnetic field.



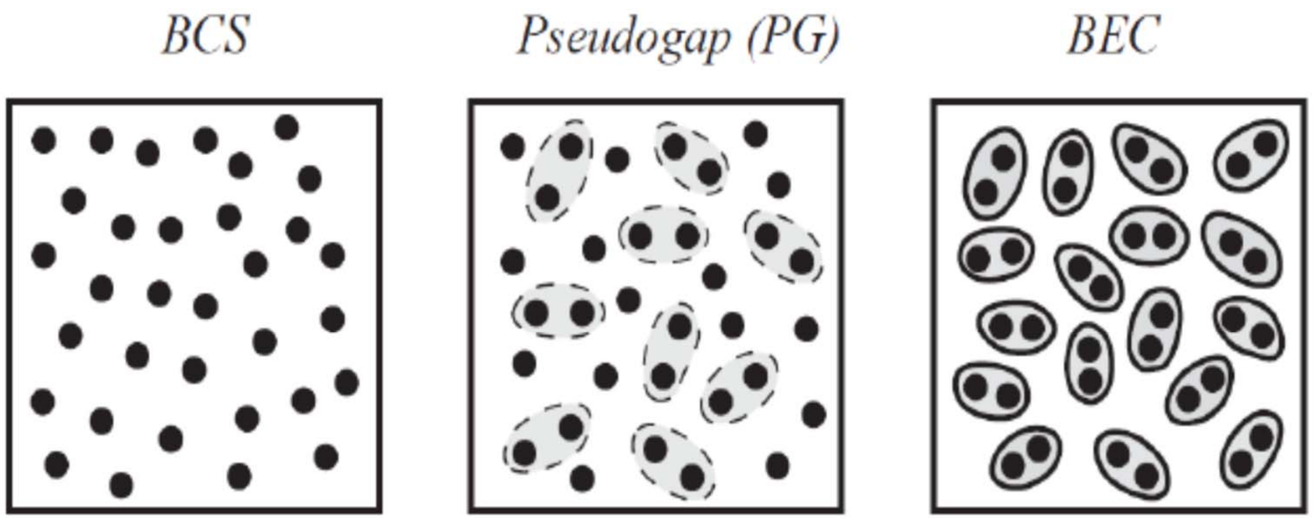
$$E_n(p_3) = \pm\sqrt{m_{dyn}^2 + 2|eB|n + p_3^2}, \quad n = 0,1,2......$$

In a strong magnetic field, all charged fermions will be restricted in lowest Landau level only, thus effectively reduce the dimension of the system.

$$m_{dyn} \approx C\sqrt{eB}\exp\left[-\left(\frac{\pi}{\alpha}\right)^{1/2}\right]$$

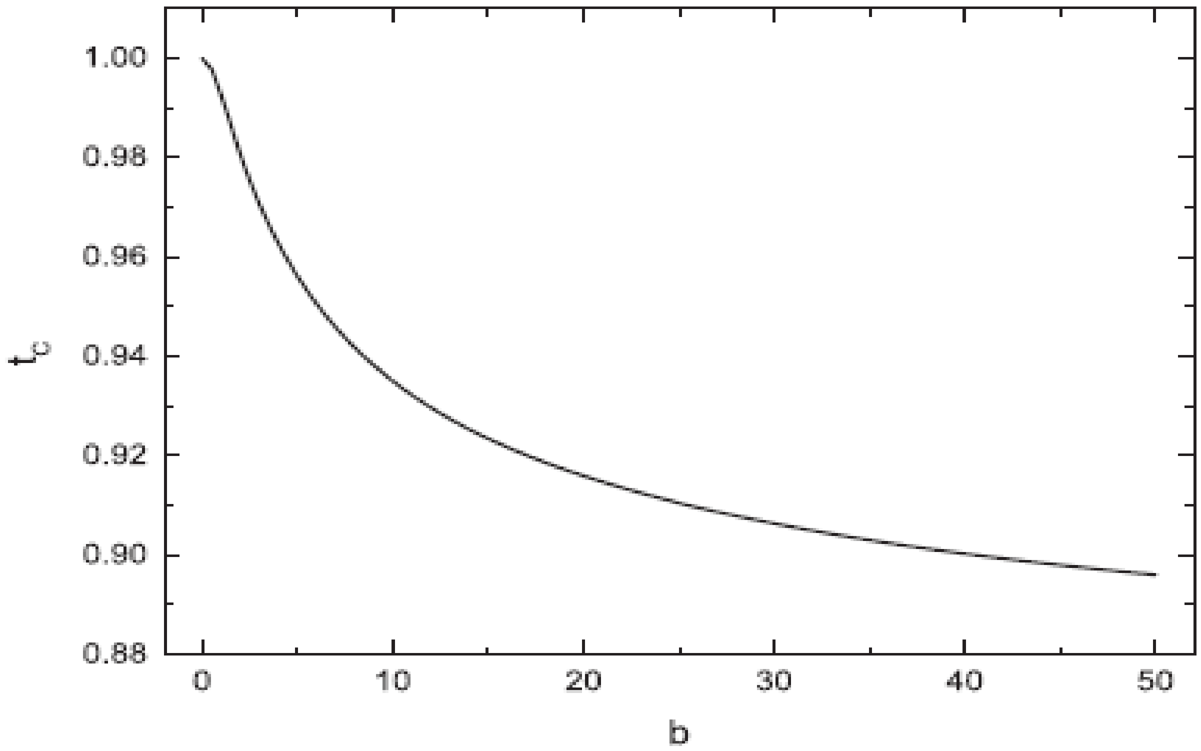
BEC with Magnetic field B

1. BEC shares the same physics as chiral condensate.
2. Chiral condensates correspond to the BEC limit in BCS/BEC crossover.



BEC with Magnetic field B

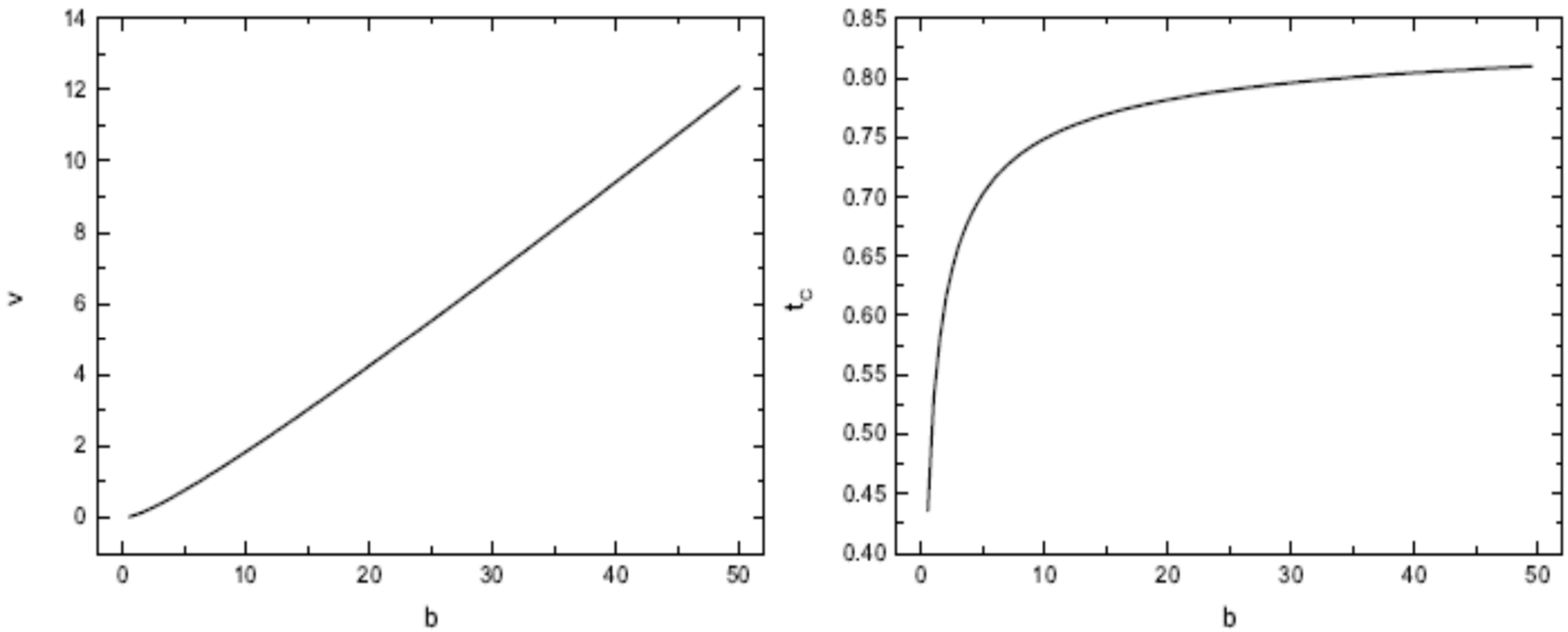
Inverse chiral catalysis in strong coupling



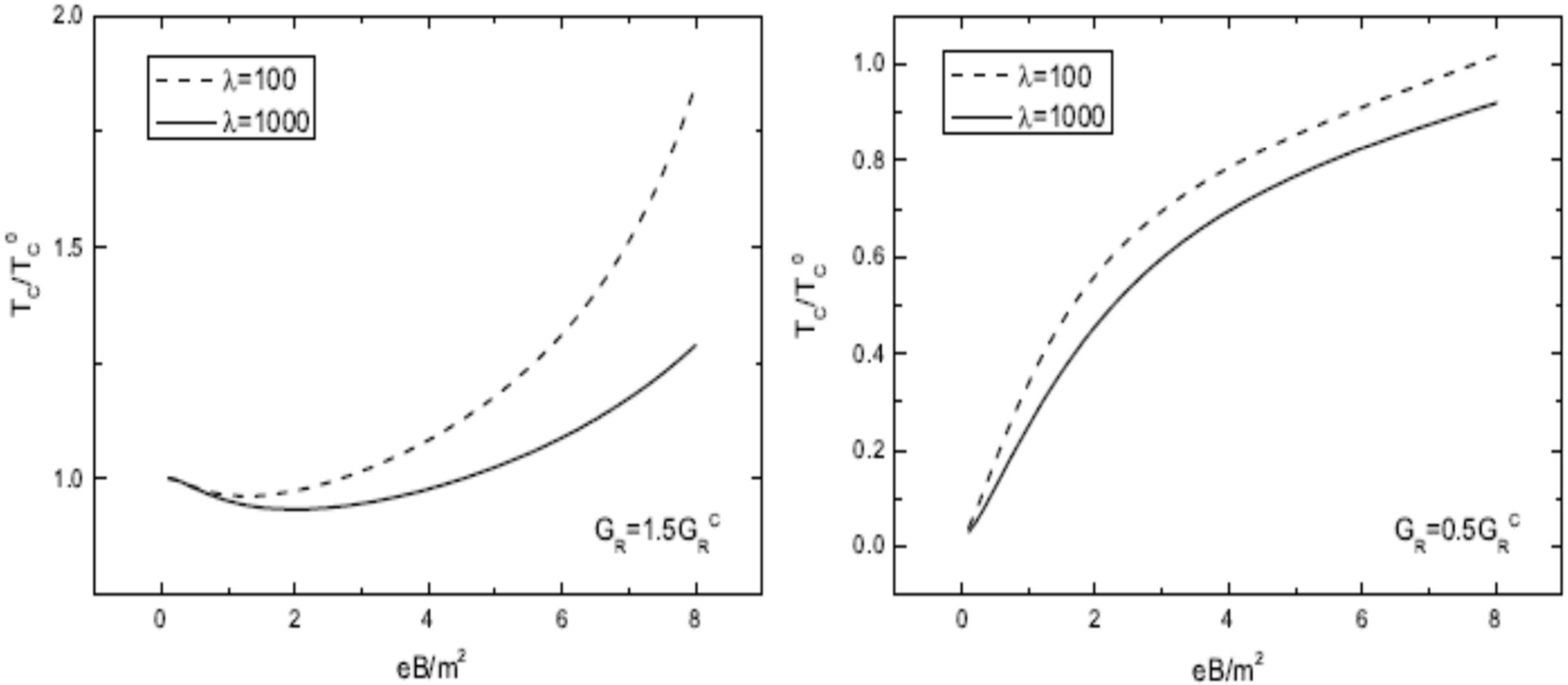
The ratio of BEC temperature tc versus b

Feng, Hou, Ren PRD 92(2015)

Chiral catalysis in weak coupling



Feng, Hou, Ren PRD 92(2015)



Condensation temperature versus the dimensionless magnetic field

Feng, Hou, Ren , Wu, _**PRD 93 (2016)085019**

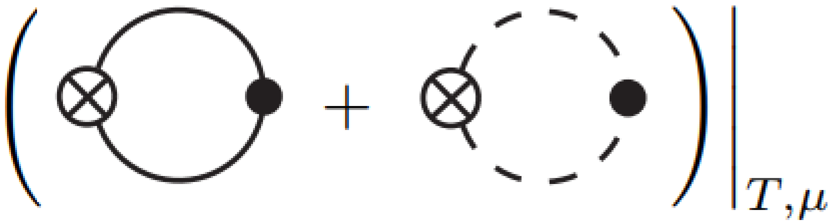
FRG and phase structute

FRG flow equation

- For continuum field theory
- Non-perturbative
- (known) microscopic laws → complex macroscopic phenomena
- Flow from classical action S[φ] to effective action Γ[φ]
- Scale dependent effective action $\Gamma_k[\varphi]$

Wetterich, PLB301, 90 (1993).

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{Tr} \left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right]$$



FRG study of phase structure at finite density

Zhang, Hou, Kojo , Qin Phys.Rev. D96,114029 (2017)

$$\mathcal{L} = \bar{\psi}\Big[i\gamma_\mu\partial^\mu - g_s(\sigma + i\gamma_5\,\boldsymbol{\tau}\cdot\boldsymbol{\pi}) - g_v\gamma_\mu\omega^\mu - \gamma_0\mu\Big]\psi$$
$$+ \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \tfrac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$- U(\sigma, \boldsymbol{\pi}, \omega)$$
$$F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu \qquad \psi = (u, d)^T$$

The potential for σ, π, and ω is

$$U(\sigma,\boldsymbol{\pi},\omega) = \frac{\lambda}{4}(\sigma^2 + \boldsymbol{\pi}^2 - f_\pi^2)^2 - \frac{m_v^2}{2}\omega_\mu\omega^\mu,$$ For chiral limit

$$U(\sigma,\boldsymbol{\pi},\omega) = \frac{\lambda}{4}(\sigma^2 + \boldsymbol{\pi}^2 - \nu^2)^2 + H\sigma - \frac{m_v^2}{2}\omega_\mu\omega^\mu,$$ For explicit SB

Mean-Field

For T=0 & μ=0, the MF potential is

$$U_{\rm MF}(\sigma,\omega_0) = \frac{\lambda}{4}(\sigma^2 - f_\pi^2)^2 - \frac{m_v^2}{2}\omega_0^2,$$

For chiral limit

$$U_{\rm MF}(\sigma,\omega_0) = \frac{\lambda}{4}(\sigma^2 - \nu^2)^2 + H\sigma - \frac{m_v^2}{2}\omega_0^2,$$

For explicit Chiral SB.

For T≠0 & μ≠0, the MF potential is

$$\Omega_{\rm MF} = \Omega_{\bar\psi\psi} + U_{\rm MF}(\sigma,\omega_0)$$

$$\Omega_{\bar\psi\psi} = -\nu_q T \int \frac{{\rm d}^3\boldsymbol{p}}{(2\pi)^3}\left\{ \ln[1+e^{-(E_q-\mu_{\rm eff})/T}] + \ln[1+e^{-(E_q+\mu_{\rm eff})/T}] \right\}$$

$$m_{\rm eff} = g_s\sigma, \qquad \mu_{\rm eff} = \mu - g_v\omega_0.$$

f_π=93MeV, m_π=138MeV, λ=20, g_s=3.3 O. Scavenius, A. Mocsy, I. N. Mishustin & D. H. Rischke, Phys. Rev. C 64, 045202 (2001)

FRG flow equation

3d-analogue of the optimized regulator

$$R_{k,B}(\boldsymbol{p}) = (k^2 - \boldsymbol{p}^2)\theta(k^2 - \boldsymbol{p}^2),$$

$$R_{k,F}(\boldsymbol{p}) = -\boldsymbol{p}\cdot\boldsymbol{\gamma}\left(\sqrt{\frac{k^2}{\boldsymbol{p}^2}} - 1\right)\theta(k^2 - \boldsymbol{p}^2),$$

the flow equation for the potential U_k^ϕ can be obtained as Schaefer & Wambach NPA 2005

$$\partial_k U_k^\phi(T,\mu) = \frac{k^4}{12\pi^2}\left\{\frac{3[1+2n_{\mathrm{B}}(E_\pi)]}{E_\pi} + \frac{1+2n_{\mathrm{B}}(E_\sigma)}{E_\sigma} - \frac{2\nu_q\left[1 - n_{\mathrm{F}}(E_q, \mu_{\mathrm{eff}}^k) - n_{\mathrm{F}}(E_q, -\mu_{\mathrm{eff}}^k)\right]}{E_q}\right\}$$

with single-particle energies are

$$E_\pi = \sqrt{k^2 + 2U_k'}\,,$$

$$E_\sigma = \sqrt{k^2 + 2U_k' + 4\phi^2 U_k''}\,,$$

$$E_q = \sqrt{k^2 + g_s^2\phi^2}$$

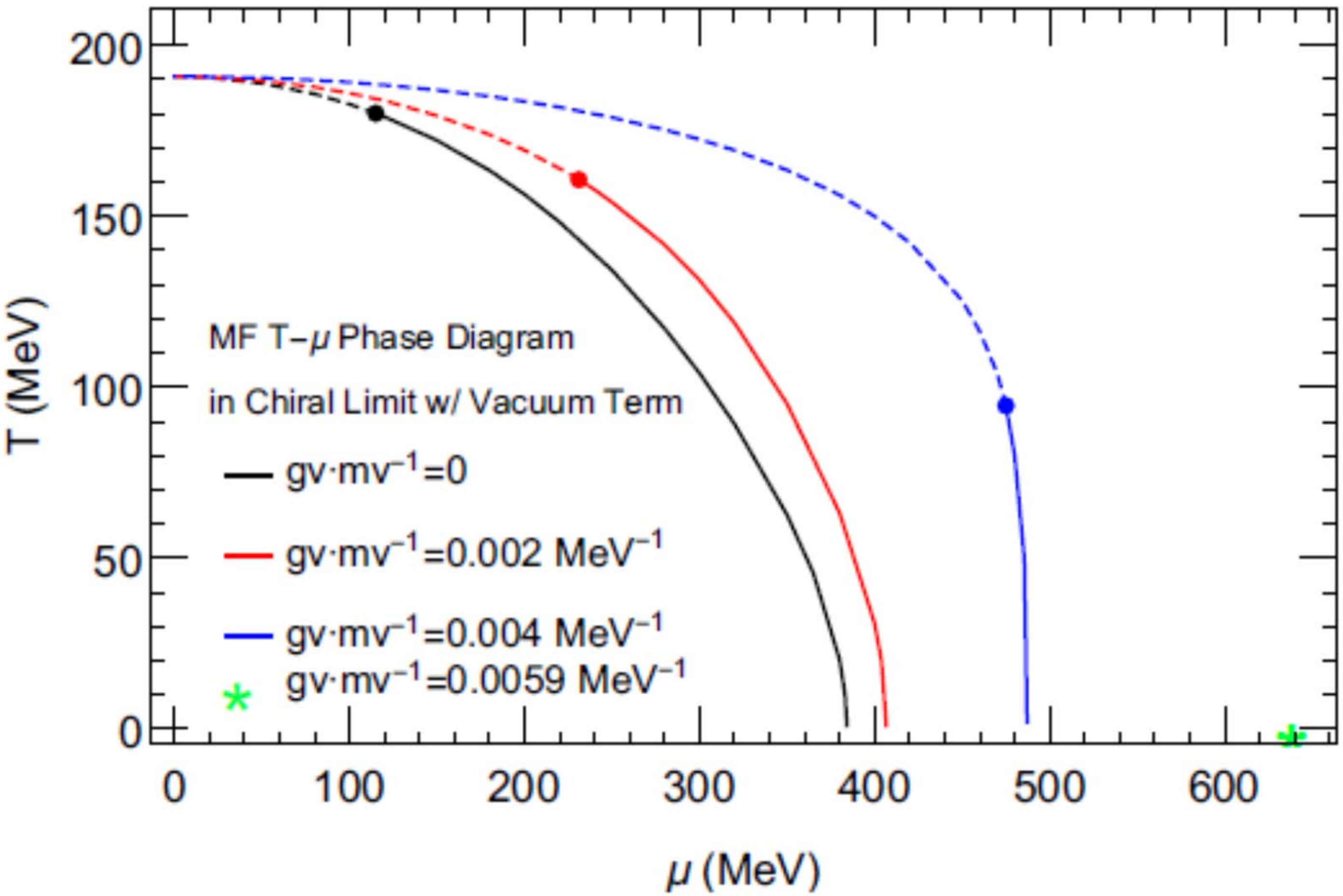
The boson and fermion occupation numbers are

$$n_B(E) = \frac{1}{e^{\beta E} - 1}$$

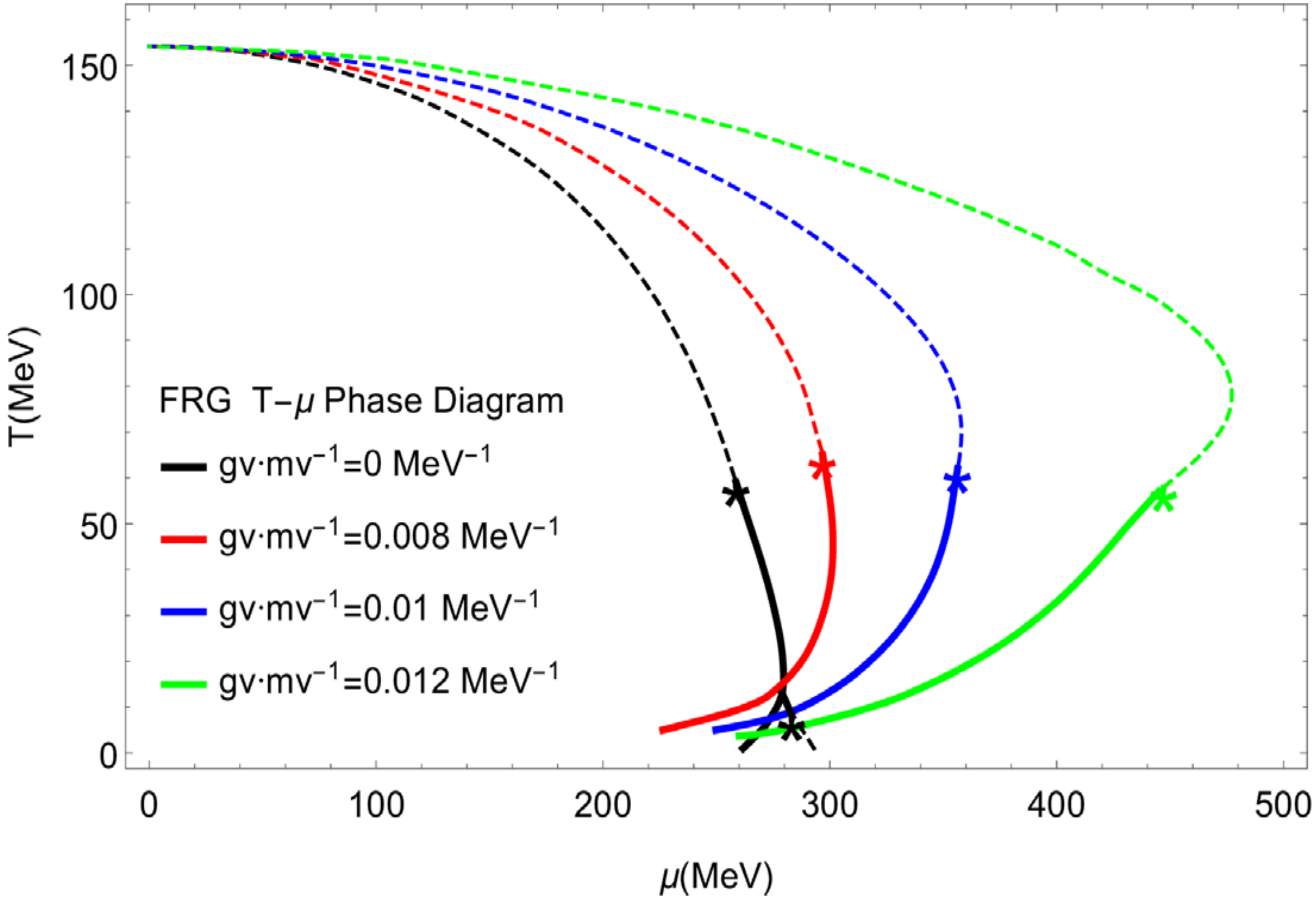
$$n_F(E,\mu) = \frac{1}{e^{\beta(E-\mu)} + 1}$$

Phase structure

Zhang, Hou, Kojo , Qin Phys.Rev. D96,114029 (2017)

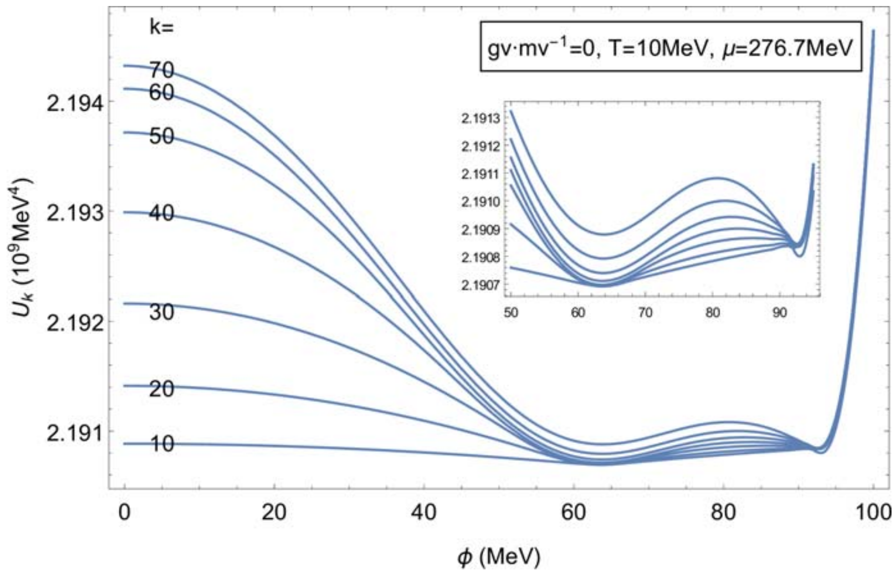


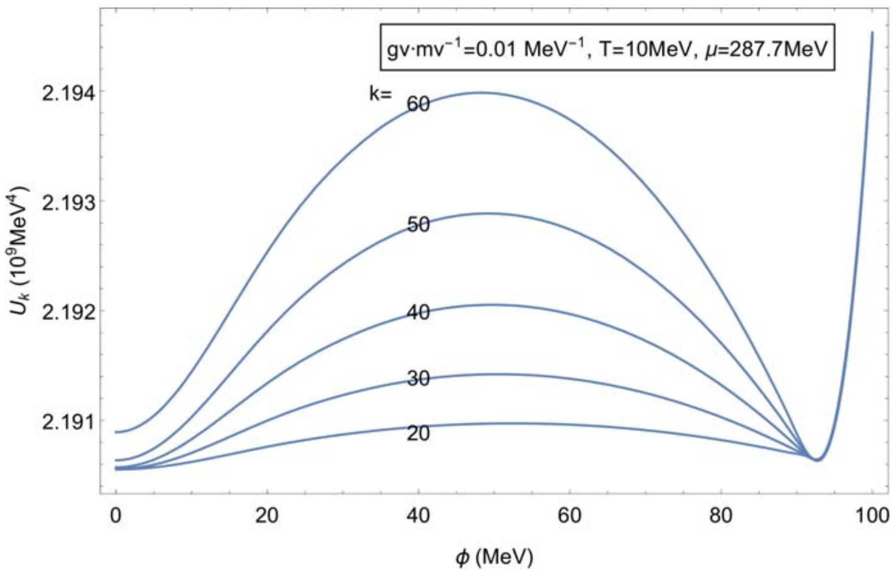
Zhang, Hou, Kojo , Qin Phys.Rev. D96,114029 (2017)



(i) At high T the fluctuations turn the 1st order line in the MF into 2nd order, yielding the TCP

(ii) While the critical μ of the TCP is sensitive to the vector coupling, its critical T is similar





The scale evolution of the EP $\Gamma_k(\varphi)$ at low T. (left) g_v/m_v=0, T=10 MeV & μ=276.7 MeV; (right) g_v/m_v=0.01 MeV^−1, T=10 MeV & μ=287.7 MeV.

The fluctuations erase the barrier between two local minima in the MF potential.

At finite vector coupling, the essential features remain the same as the g_v=0 case; the flucts. do not modify the potential around φ≃93 MeV, the potential around φ≃0 is strongly affected.

Order parameter and baryon density

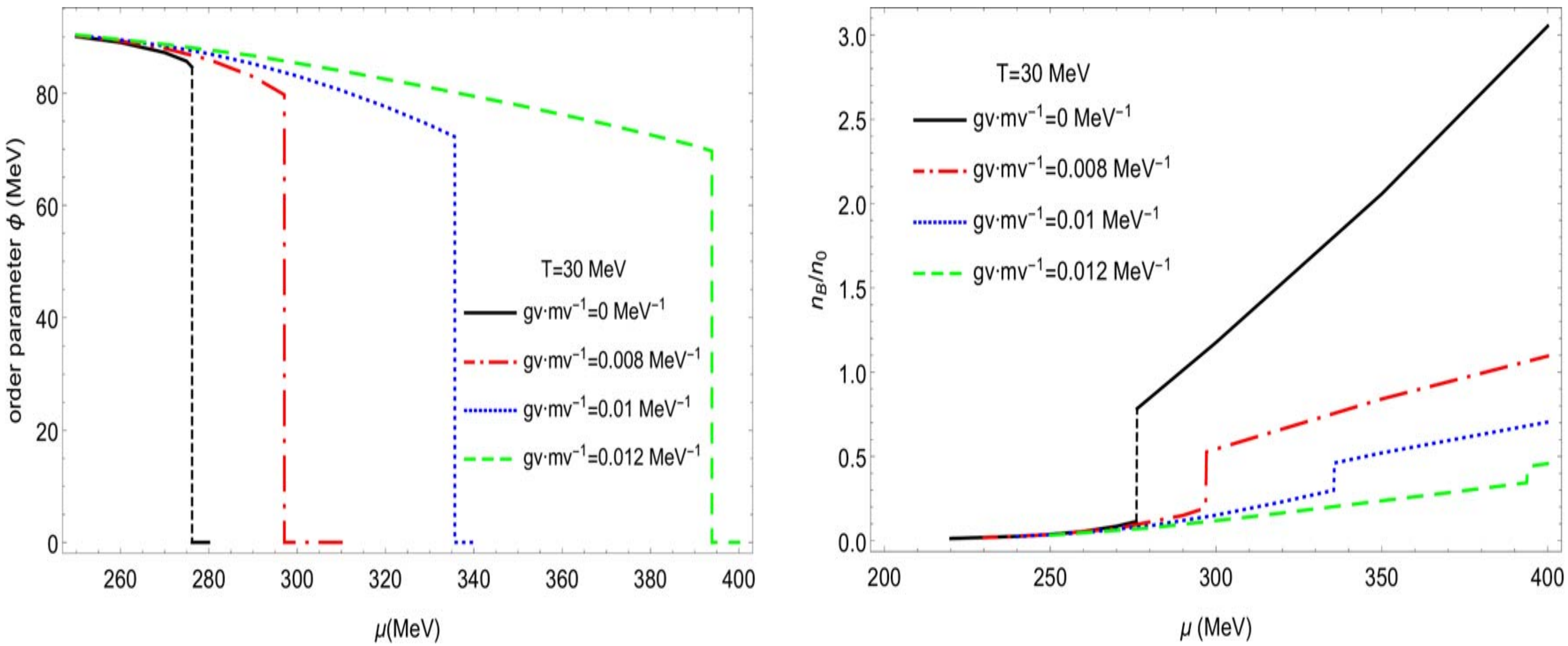


Figure: The μ-dependence of the baryon density considerably deviates from $\sim \mu^3$ behavior expected from the single particle contributions.
$n_B = n_B^{\text{single}} + n_B^{\text{fluct}}$

Several other checks

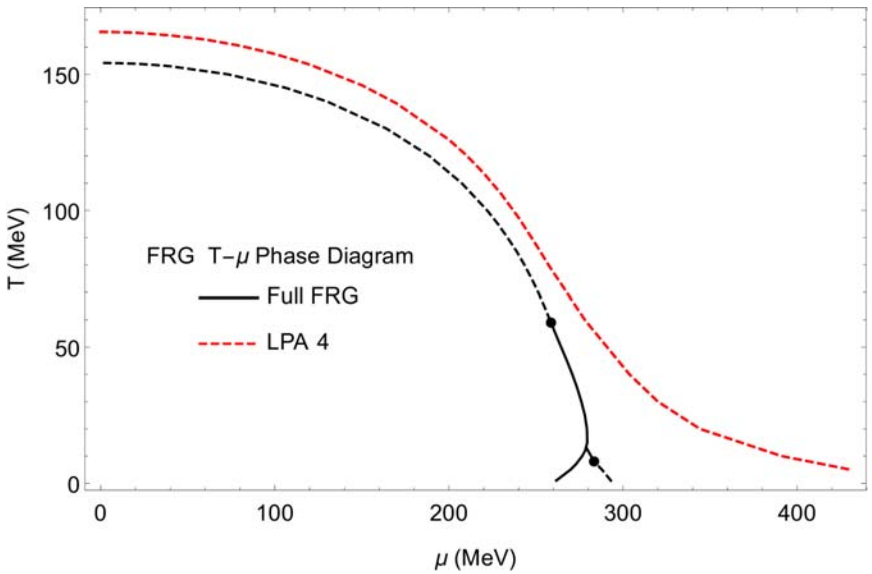
LPA 4 $U_k(\phi) = \frac{\lambda_k}{4}(\phi^2 - a_k)^2$

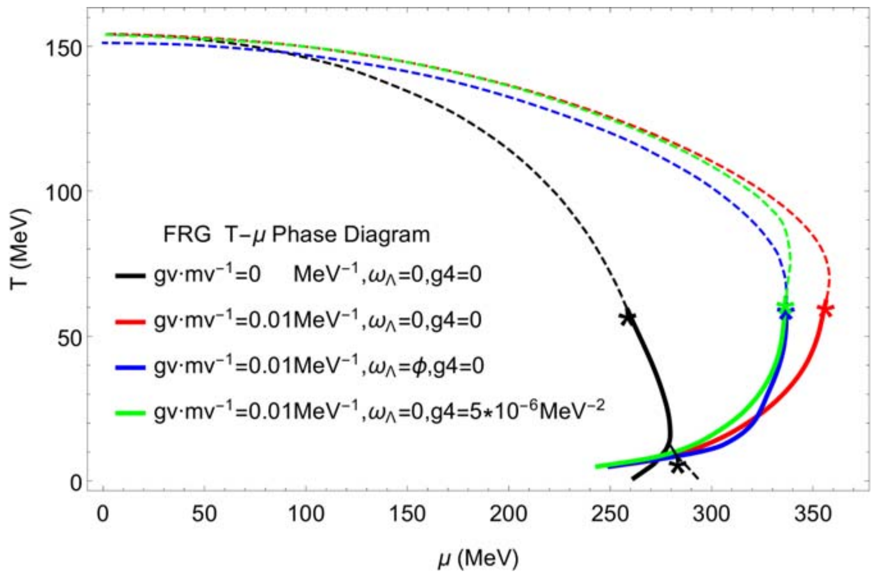
With g_4 & initial condition for ω

$$\omega_\Lambda = \phi$$

$$U_k(\omega) = -\frac{1}{2}m_v^2\omega_{0,k}^2 + \frac{1}{12}g_4 \cdot (g_v^2 m_v^2) \cdot \omega_{0,k}^4$$

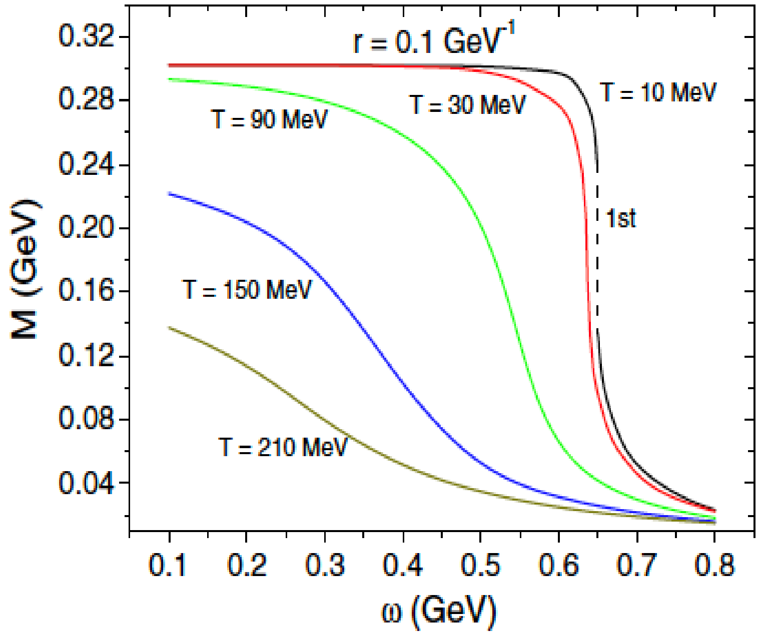
With the quartic term, the overall structure such as the back bending behavior is not significantly affected.

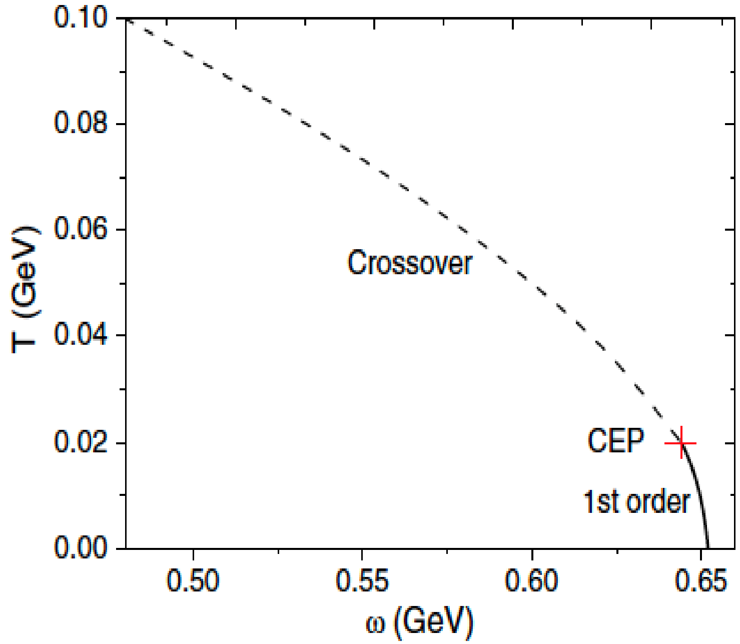




Phase structure under rotation

Jiang,Liao: PRL117(2016)192303





Mesonic superfluidity under rotation

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu\partial^\mu - m_0 + \frac{\mu_I}{2}\gamma_0\tau_3)\psi \quad + G_s\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi\right)^2\right]$$
$$- G_v\left[\left(\bar{\psi}\gamma_\mu\boldsymbol{\tau}\psi\right)^2 + \left(\bar{\psi}\gamma_\mu\gamma_5\boldsymbol{\tau}\psi\right)^2\right]$$

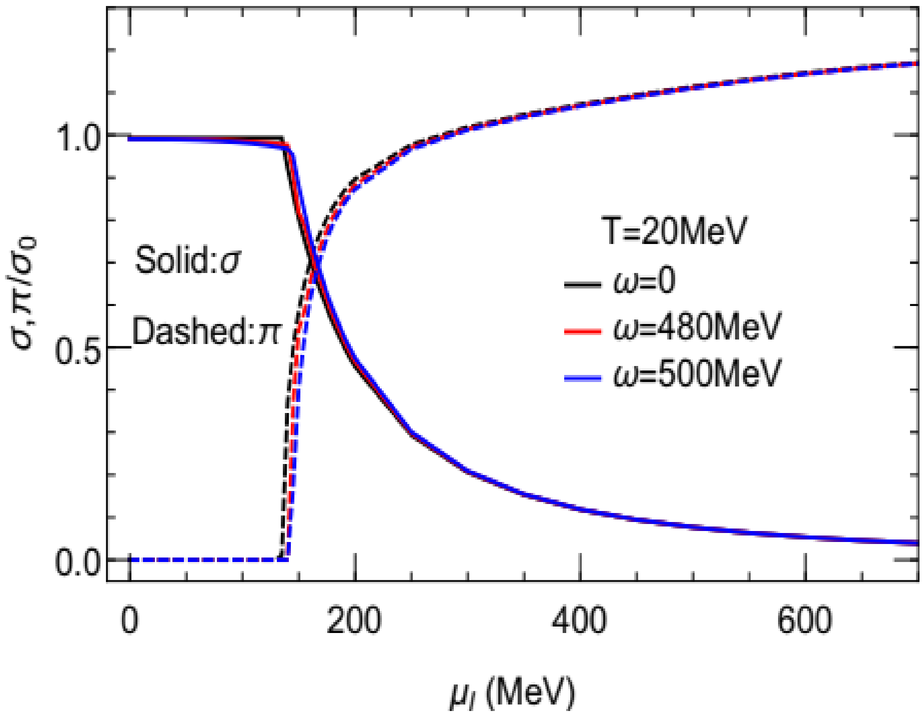
MF approximation: $\sigma = \langle\bar{\psi}\psi\rangle,\ \pi = \langle\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi\rangle,\ \rho = \langle\bar{\psi}i\gamma_0\gamma_5\tau_3\psi\rangle$

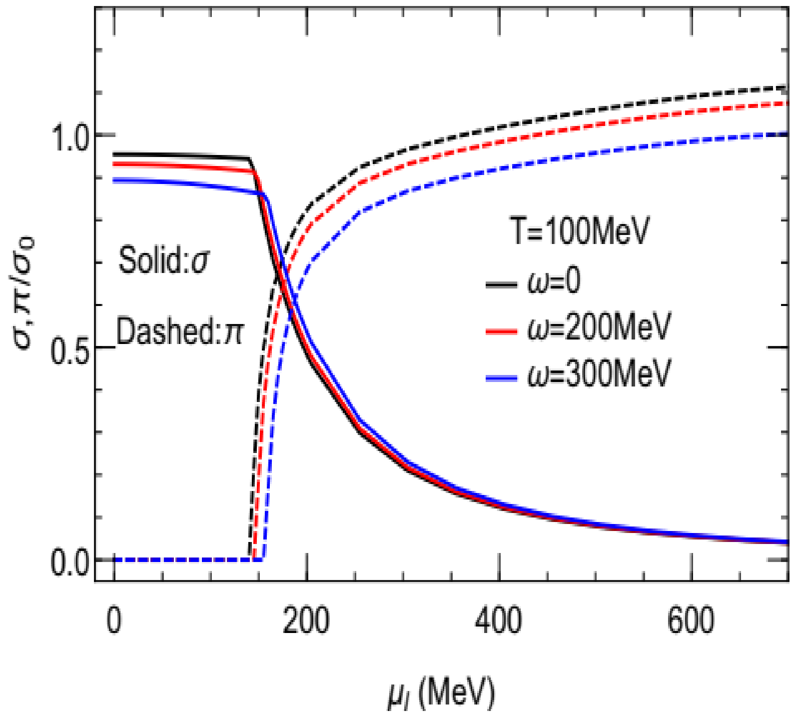
$$\begin{aligned}\Omega \ &= \ G(\sigma^2+\pi^2) - G\rho^2 - \frac{\mathrm{N_c N_f}}{16\pi^2}\sum_n \int \mathrm{d}k_t^2 \int \mathrm{d}k_z [J_{n+1}(k_t r)^2 + J_n(k_t r)^2] \\ &\times\ T\Big[\ln\Big(1+\exp(-\frac{\omega^+ - (n+\frac{1}{2})\omega}{T})\Big) + \ln\Big(1+\exp(\frac{\omega^+ - (n+\frac{1}{2})\omega}{T})\Big) \\ &\quad + \ln\Big(1+\exp(-\frac{\omega^- - (n+\frac{1}{2})\omega}{T})\Big) + \ln\Big(1+\exp(\frac{\omega^- - (n+\frac{1}{2})\omega}{T})\Big)\Big]\end{aligned}$$

Rotational suppression of Pion superfluid

Rotation weaken spin 0 condensate, inverse catalysis effect

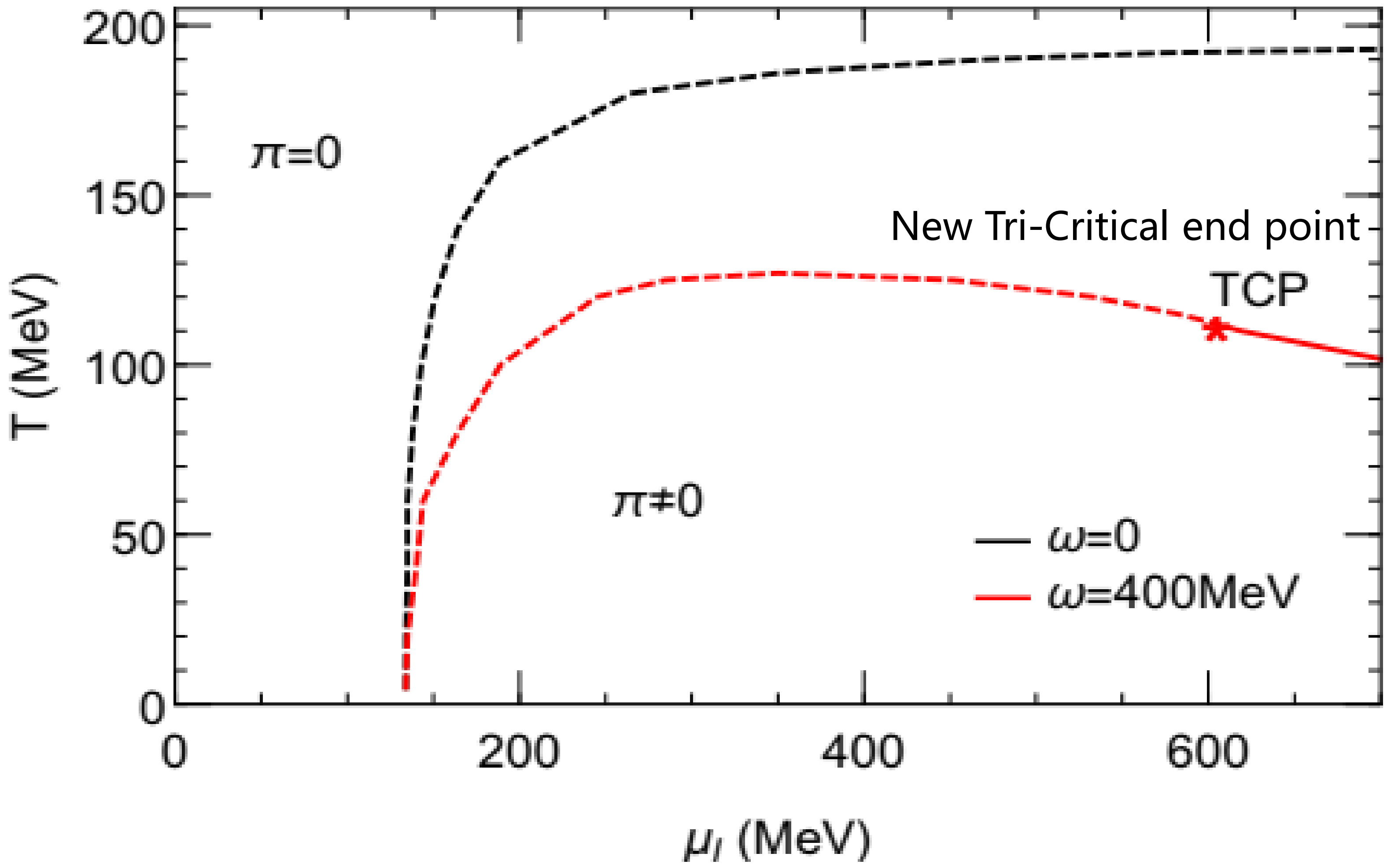
H. Zhang, DF Hou, JF Liao, arxiv 1812.11787





He, M. Jin and P. Zhuang, Phys. Rev. D 71,116001 (2005);
L. He and P. Zhuang, Phys. Lett. B 615, 93 (2005)

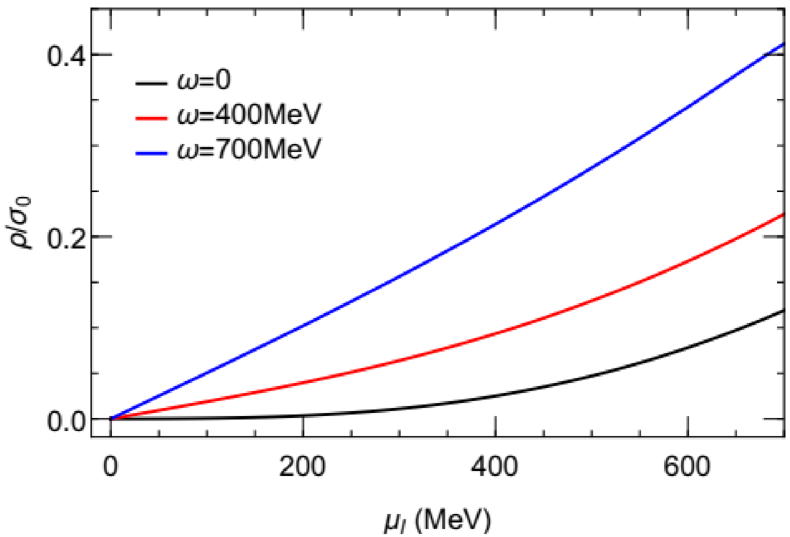
Pion superfluidity phase diagram in T-μI

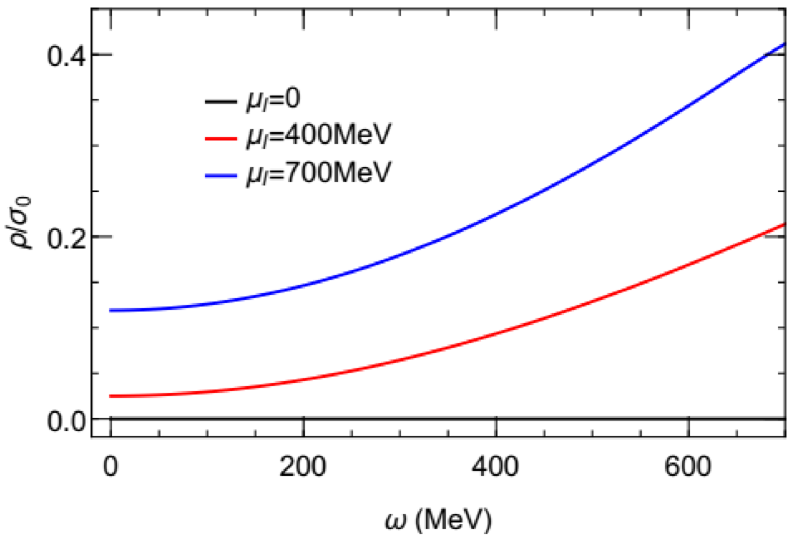


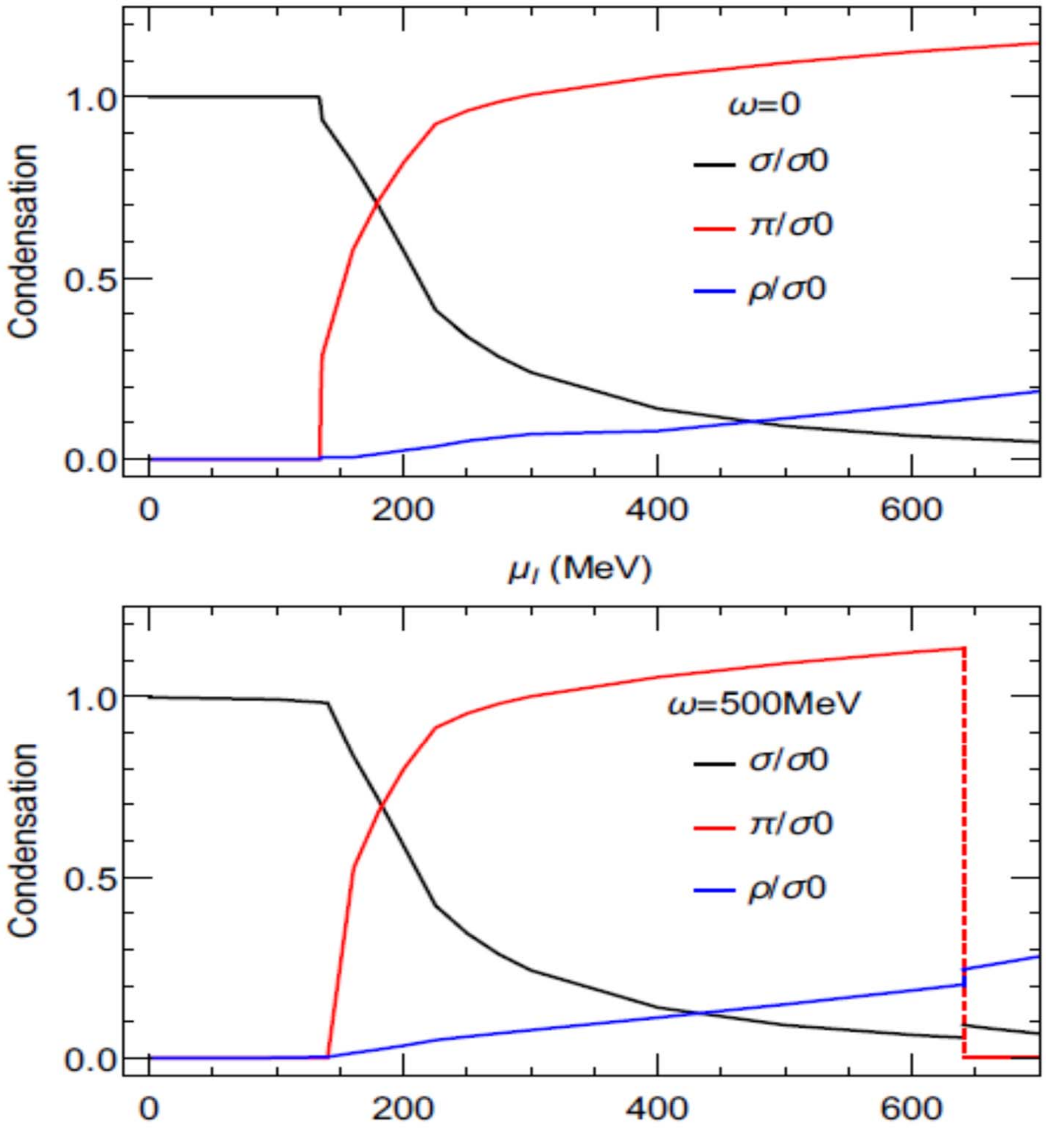
Enhenced Rho Superfluid under rotation

Rotation enhances spin 1 condensate ρ channel

Rho condensate at T=mu=0 with none zero isospin chemical potential under rotation

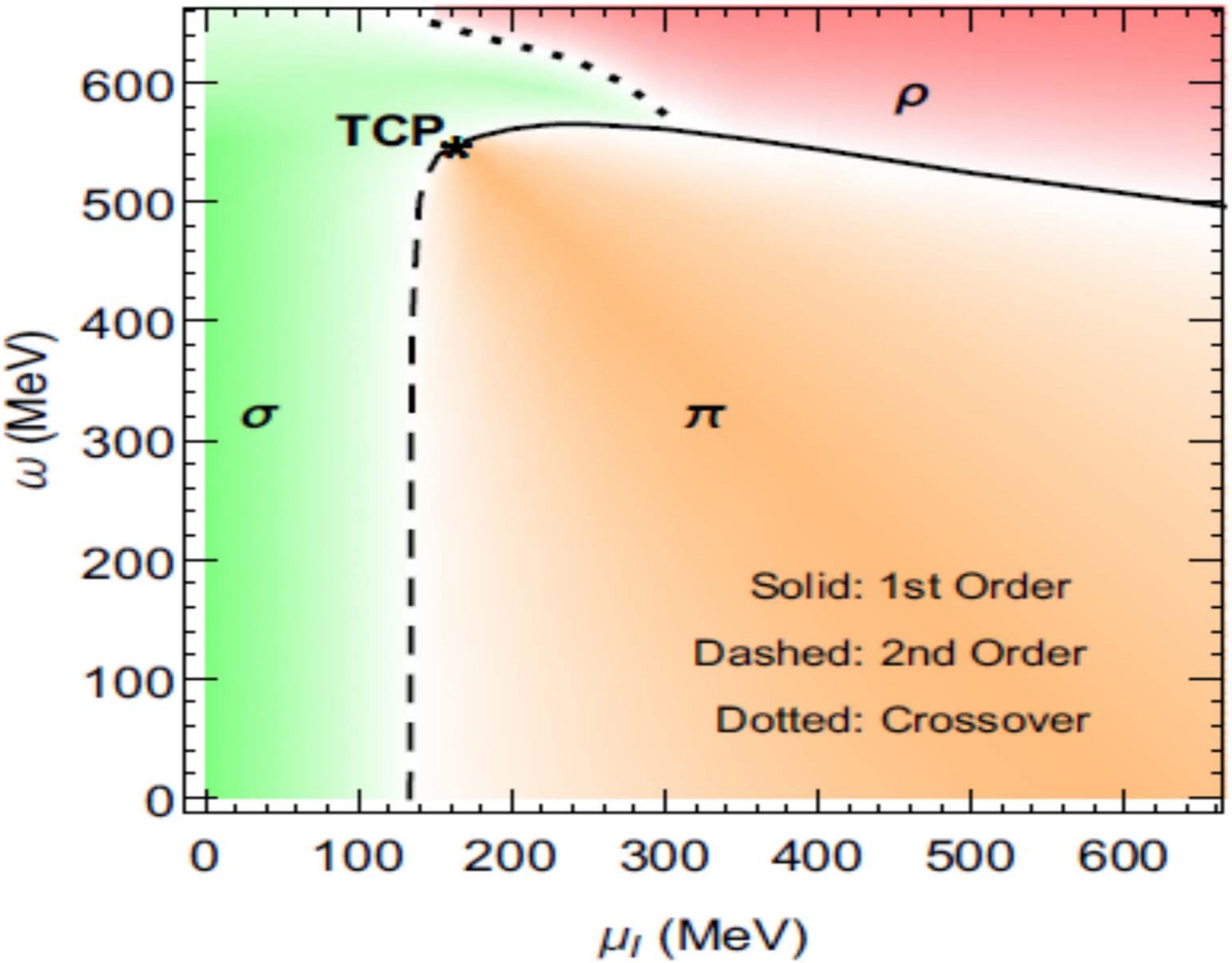






ω=0
σ/σ0
π/σ0
ρ/σ0
Condensation
1.0
0.5
0.0
0
200
400
600
μ_I (MeV)
ω=500MeV
σ/σ0
π/σ0
ρ/σ0
Condensation
1.0
0.5
0.0
0
200
400
600

New mesonic superfluid phase diagram



H. Zhang, DF Hou, JF Liao, arxiv 1812.11787

Summary and outlook

- Dense QCD matter has very rich phase structures.

- Fluctuations are important for TCP of QCD, FRG provides an useful tool

- Magnetic field has nontrivial effects on phase structure
- (Magnetic Catalyse & inverse Magetic catalyse)

- Rotation suppresses spin 0 condensate , enhances nonzero spin ones

- A new phase diagram for isospin matter under rotation with a new TCP

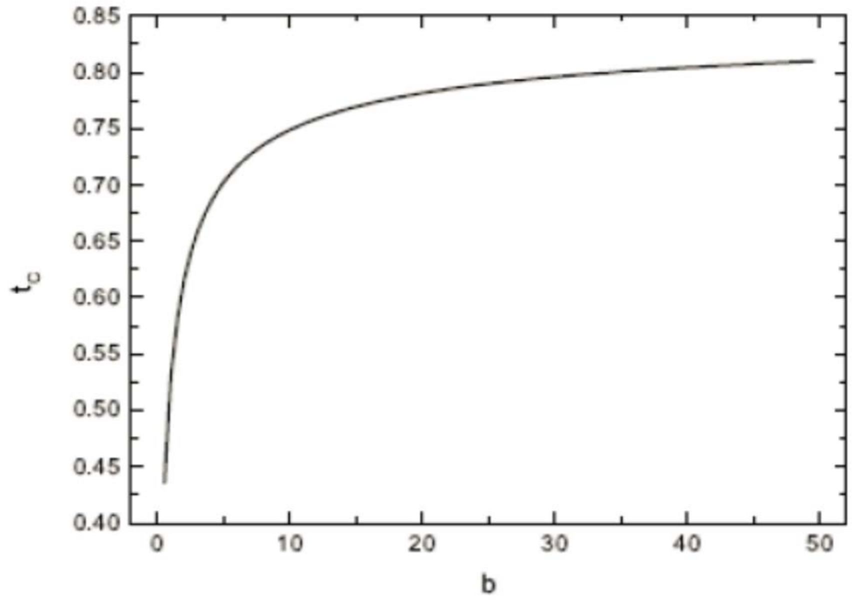
Thank you very much for your attention!

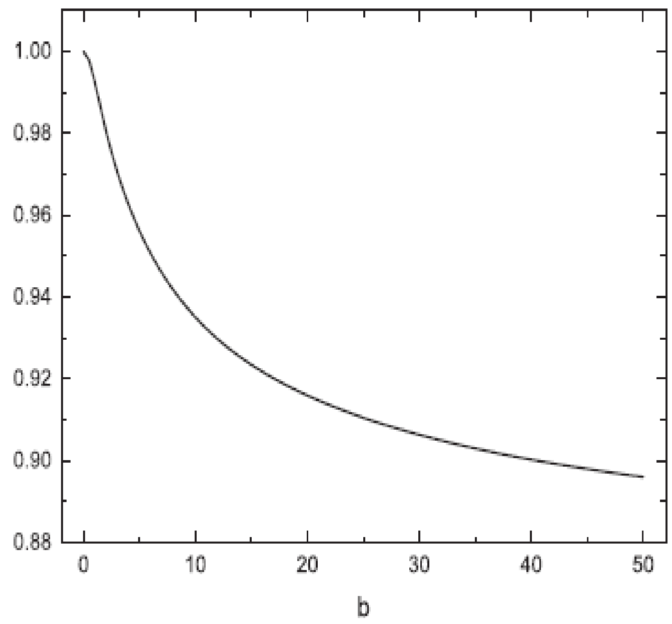
Summary for BEC under B

- We point out that magnetic field has two effects: dimension reduction and enhancement of fluctuations.
- We elaborate this mechnism via a simpe example: BEC of neutral composite bosons.
- We find that in NR case the fluctuations play a signifcant role and inverse magnetic catalysis arises in strong coupling domain.
- In relativistic case, the fluctuations are *NOT* as signifcant as that in NR. The inverse magnetic catalysis found in Lattice QCD may due to the complex in the dynamics in QCD.

Magnetic (Inverse)chiral catalysis at weak (strong) coupling

Feng, Hou, Ren PRD 92(2015)

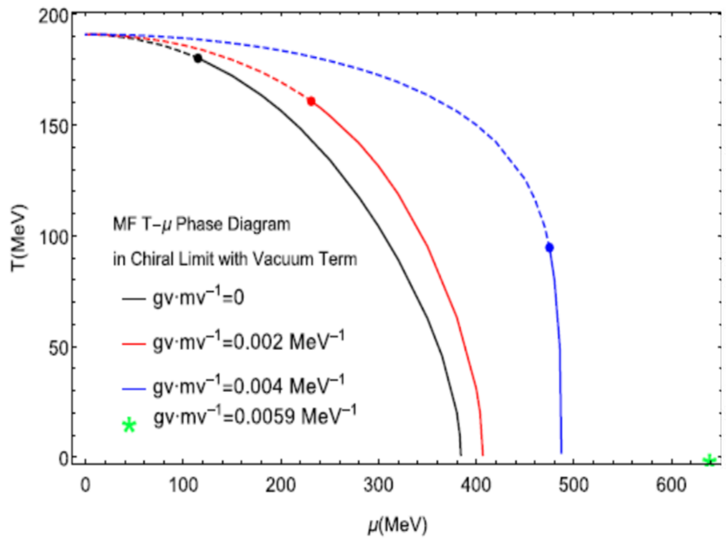


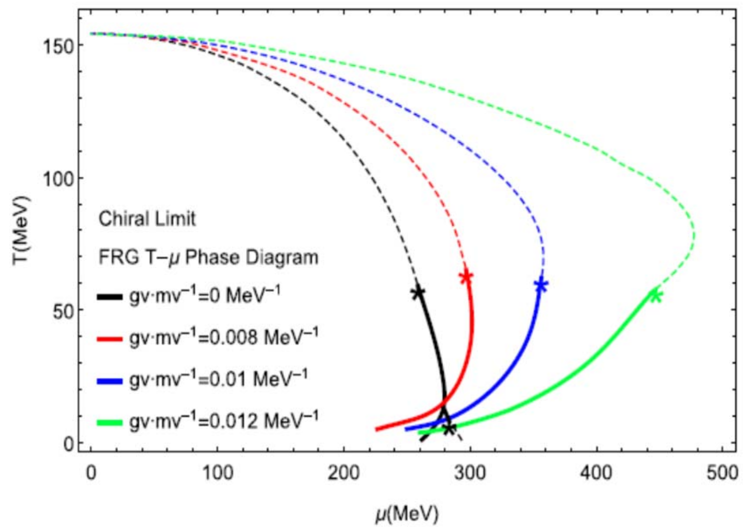


The ratio of critical temperature tc versus magnetic field b at weak and strong coupling

FRGE study of phase diagram: Flucts on CEP

Zhang, Hou , Kojo, Qin, PRD96 (2017)



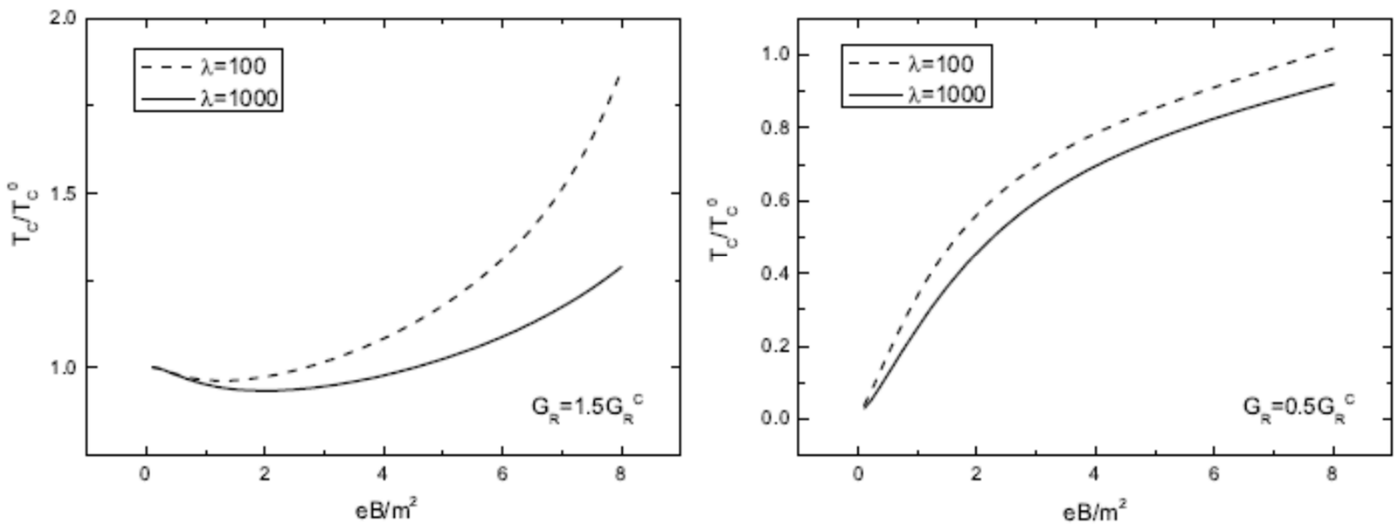


Nematic Isotropic (NI) Puzzle with FRGE

Qin, Hou, Huang, Zhang, PRB98, (2018)

Magnetic (Inverse)chiral catalysis at weak (strong) coupling

Feng, Hou, Ren , Wu, _**PRD 93 (2016)085019**



Condensation temperature versus the dimensionless magnetic field