$K^- \to \pi^+\mu^-\mu^-$ from the perspective of effective field theory

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Outline

1. Why $K^- \to \pi^+\mu^-\mu^-$?
2. Backgrounds of different EFTs
3. Back to $K^- \to \pi^+\mu^-\mu^-$
4. Conclusion

1 Why $K^- \to \pi^+\mu^-\mu^-$?

2 Backgrounds of different EFTs

3 Back to $K^- \to \pi^+\mu^-\mu^-$

4 Conclusion

Why we explore lepton number violation(LNV)?

- Neutrino oscillation $\Rightarrow$ non-vanishing neutrino mass $\Leftarrow$ Majorana neutrino mass;
- The asymmetry of matter and anti-matter $\Leftarrow$ baryogengesis $\Leftarrow$ leptogenesis;
- The nature of dark matter;
- Anomalies/bumps from terrestrial experiments: $g_\mu - 2$, $R_K(R_K^*), \cdots$ (?!)
- LNV processes are definite signal for **new physics(NP)**;
- $\cdots$

LNV processes

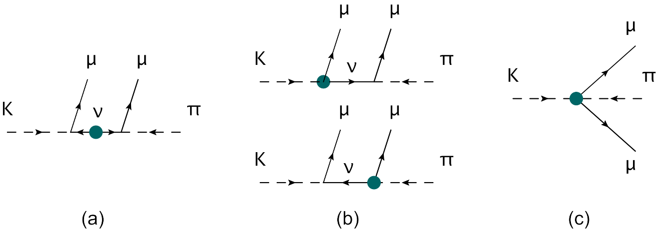
- Neutrinoless double beta decay: $X \to X' + 2p + 2e^-$;
- Nuclear muon-positron/anti-muon conversion: $\mu^- X \to e^+(\mu^+)X'$;
- Trimuon production from neutrino-proton collision: $\nu p \to \mu^-\mu^+\mu^+$;
- Production of Majorana neutrino from electron-proton collison: $e^+ p \to \overline{\nu} l_1^+ l_2^+ X$ from HERA;
- The tau lepton decaying: $\tau^- \to M_1^- M_2^- \mu^+$;
- Kaon neutrinoless double-muon decay: $K^- \to \pi^+\mu^-\mu^-$ ✓;
- Also for the lepton number violating decays from other mesons like D and B [Belle, LHCb, ...].

Why $K^- \to \pi^+\mu^-\mu^-$?

- The negative result from $0\nu\beta\beta$;
- Easily formulated from theory;
- The unprecedented precision for the decay: $\mathrm{Br}(K^- \to \pi^+\mu^-\mu^-) < 4.2 \times 10^{-11}$[1905.07770];
- It can provide complementary constraints on NP parameters together with other LNV processes, like $0\nu\beta\beta$;

How to study $K^- \to \pi^+\mu^-\mu^-$?

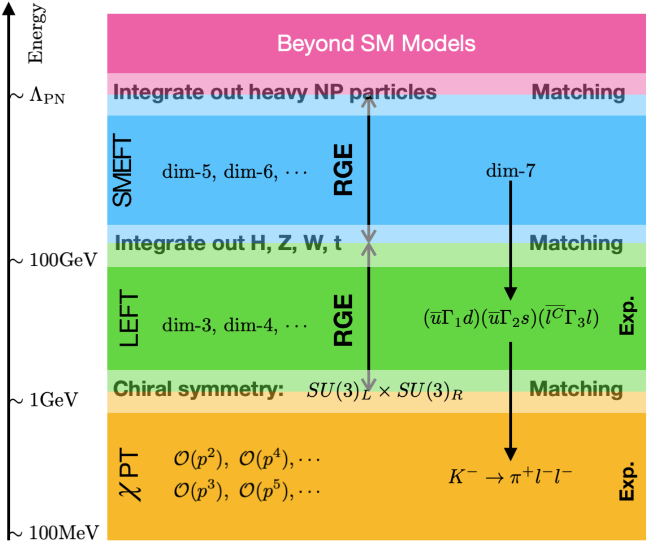
EFT: NP $\Rightarrow$ **SMEFT** $\Rightarrow$ **LEFT** $\Rightarrow$ χ**PT** $\Rightarrow$ $K^- \to \pi^+\mu^-\mu^-$



(a) mass mechanism; (b) long-range interaction; (c) short-range interaction.

Here we assume the process is **dominated by (c)**, i.e., the local dim-9 operators in LEFT, and only focus on it!

A general picture of EFT calculation for $K^- \to \pi^+\mu^-\mu^-$



1 Why $K^- \to \pi^+\mu^-\mu^-$?

2 Backgrounds of different EFTs

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SMEFT 1

- **Assumption**: NP beyond SM exist, with $\Lambda_{\rm NP} \gg v$;
- **Ingredients**: SM fields $+$ SM gauge symmetry;
- SMEFT$=$ all possible local, SM gauge invariant operators built from SM fields ordered by the inverse power of $\Lambda_{\rm NP}$, i.e.,

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda_{\rm NP}^{D-4}} \sum_{D\geq 5} C_i^D \mathcal{O}_i^D,$$

  Wilson coefficients C_i^D encode the contribution from unknown NP.
- Merit: play physics in a model-independent way;
- **Warning**: not all operators need to be considered $\leftarrow$ *the equivalence theorem* $\Rightarrow$ focus on a minimal basis in each dimension! **[1901.10302]**

SMEFT 2

- dim-5: $1(\not{L} \cap B)$ [Weinberg 79] $\to$ dim-$D \in$ odd [Liao 10]

$$\mathcal{O}_{LH}^{D} = \left[(L^{\mathsf{T}} \epsilon H) C (L^{\mathsf{T}} \epsilon H)^{\mathsf{T}}\right] (H^{\dagger} H)^{(D-5)/2}$$

- dim-6: $59(L \cap B) + 4(\not{L} \cap \not{B})$ [Buchmuller *et al* 86; Grzadkowski *et al* 10]
- dim-7: $12(\not{L} \cap B) + 6(\not{L} \cap \not{B})$ [Lehman 14; **1607.07309**]
- dim-8, 9, $\cdots$ [Lehman *et al* 15; Henning *et al* 15, 17[1706.08520]]
- $D \in$ even(odd) if $|B - L|/2$ is even(odd) for SMEFT;
- $D = 6: \ |B - L| = 0$ vs $D = 7: \ |B - L| = 2$;
- $D \in$ odd: L is violated;
- Also the νSMEFT at dim-6 and dim-7 [**1612.04527**]

SMEFT 3: dim-7 operators [Lehman 14; 1607.07309]

	$\psi^2 H^4$ + h.c.		$\psi^2 H^3 D$ + h.c.
$\mathcal{O}_{LH}$	$\epsilon_{ij}\epsilon_{mn}(L^i C L^m)H^j H^n (H^\dagger H)$	$\mathcal{O}_{LeHD}$	$\epsilon_{ij}\epsilon_{mn}(L^i C\gamma_\mu e)H^j H^m iD^\mu H^n$
	$\psi^2 H^2 D^2$ + h.c.		$\psi^2 H^2 X$ + h.c.
$\mathcal{O}_{LHD1}$ $\mathcal{O}_{LHD2}$	$\epsilon_{ij}\epsilon_{mn}(L^i C D^\mu L^j)H^m(D_\mu H^n)$ $\epsilon_{im}\epsilon_{jn}(L^i C D^\mu L^j)H^m(D_\mu H^n)$	$\mathcal{O}_{LHB}$ $\mathcal{O}_{LHW}$	$g_1\epsilon_{ij}\epsilon_{mn}(L^i C\sigma_{\mu\nu}L^m)H^j H^n B^{\mu\nu}$ $g_2\epsilon_{ij}(\epsilon\tau^I)_{mn}(L^i C\sigma_{\mu\nu}L^m)H^j H^n W^{I\mu\nu}$
	$\psi^4 D$ + h.c.		$\psi^4 H$ + h.c.
$\mathcal{O}_{\bar{d}uLLD}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^i C iD^\mu L^j)$	$\mathcal{O}_{\bar{e}LLLH}$ $\mathcal{O}_{\bar{d}LQLH1}$ $\mathcal{O}_{\bar{d}LQLH2}$ $\mathcal{O}_{\bar{d}LueH}$ $\mathcal{O}_{\bar{Q}uLLH}$	$\epsilon_{ij}\epsilon_{mn}(\bar{e}L^i)(L^j C L^m)H^n$ $\epsilon_{ij}\epsilon_{mn}(\bar{d}L^i)(Q^j C L^m)H^n$ $\epsilon_{im}\epsilon_{jn}(\bar{d}L^i)(Q^j C L^m)H^n$ $\epsilon_{ij}(\bar{d}L^i)(uCe)H^j$ $\epsilon_{ij}(\bar{Q}u)(LCL^i)H^j$
$\mathcal{O}_{\bar{L}QddD}$ $\mathcal{O}_{\bar{e}dddD}$	$(\bar{L}\gamma_\mu Q)(dCiD^\mu d)$ $(\bar{e}\gamma_\mu d)(dCiD^\mu d)$	$\mathcal{O}_{\bar{L}dud\tilde{H}}$ $\mathcal{O}_{\bar{L}dddH}$ $\mathcal{O}_{\bar{e}Qdd\tilde{H}}$ $\mathcal{O}_{\bar{L}dQQ\tilde{H}}$	$(\bar{L}d)(uCd)\tilde{H}$ $(\bar{L}d)(dCd)H$ $\epsilon_{ij}(\bar{e}Q^i)(dCd)\tilde{H}^j$ $\epsilon_{ij}(\bar{L}d)(QCQ^i)\tilde{H}^j$

Operators contributing to $K^- \to \pi^+\mu^-\mu^-$

- Mass mechanism: $\mathcal{O}^{5\dagger}_{LH}$, $\mathcal{O}^\dagger_{LH}$
- Long-range interaction: $\mathcal{O}^\dagger_{LeHD}$, $\mathcal{O}^\dagger_{\bar{d}LQLH1}$, $\mathcal{O}^\dagger_{\bar{d}LQLH2}$, $\mathcal{O}^\dagger_{\bar{d}LueH}$, $\mathcal{O}^\dagger_{\bar{Q}uLLH}$;
- **Short-range interaction**: $\mathcal{O}^\dagger_{LHD1}$, $\mathcal{O}^\dagger_{LHW}$, $\mathcal{O}^\dagger_{\bar{d}uLLD}$(Our starting point!)

LEFT

- **Worked scale**: $m_p < \Lambda < m_W$;
- **Fields**: $u,\ d,\ s,\ c,\ b,\ e,\ \nu_e, \mu,\ \nu_\mu,\ \tau,\ \nu_\tau, \gamma, g$;
- **Symmetry**: $U(1)_{\text{EM}} \times SU(3)_C$;
- LEFT=all possible local, $U(1)_{\text{EM}} \times SU(3)_C$ invariant operators constructed from the above fields ordered by the inverse power of vev v, i.e.,

$$\mathcal{L}_{\text{LEFT}} = \mathcal{L}_{\leq 4} + \frac{1}{v^{D-4}} \sum_{D \geq 5} L_i^D \mathcal{Q}_i^D,$$

- dim-5: $70(\Delta L = \Delta B = 0)$ & dim-6: $3631(\Delta L = \Delta B = 0)$ [Manohar *et al* 17, 18];
- dim-7: $3168(\Delta L = \Delta B = 0) + 750(\Delta L = \pm 2,\ \Delta B = 0) + 588(\Delta L = -\Delta B = \pm 1) + 612(\Delta L = \Delta B = \pm 1)$; (**In preparation)**
- dim-9: 5832 six-fermion, $(\Delta L,\ \Delta B) = (2,\ 0)$ operators.

Basis of dim-9 $(\Delta L,\ \Delta B) = (2,\ 0)$ operators LEFT

Operator	Specific form	Operator	Specific form
$\mathcal{O}^{LLLL,\,S/P}_{prst}$	$(\overline{u^p_L}\gamma^\mu d^r_L)[\overline{u^s_L}\gamma_\mu d^t_L](j^{\alpha\beta}/j_5^{\alpha\beta})$	$\mathcal{O}^{RRRR,\,S/P}_{prst}$	$(\overline{u^p_R}\gamma^\mu d^r_R)[\overline{u^s_R}\gamma_\mu d^t_R](j^{\alpha\beta}/j_5^{\alpha\beta})$
$\mathcal{O}^{LLLL,\,T}_{prst}$	$(\overline{u^p_L}\gamma^\mu d^r_L)[\overline{u^s_L}\gamma^\nu d^t_L](j^{\alpha\beta}_{\mu\nu})$	$\mathcal{O}^{RRRR,\,T}_{prst}$	$(\overline{u^p_R}\gamma^\mu d^r_R)[\overline{u^s_R}\gamma^\nu d^t_R](j^{\alpha\beta}_{\mu\nu})$
$\tilde{\mathcal{O}}^{LLLL,\,T}_{prst}$	$(\overline{u^p_L}\gamma^\mu d^r_L][\overline{u^s_L}\gamma^\nu d^t_L)(j^{\alpha\beta}_{\mu\nu})$	$\tilde{\mathcal{O}}^{RRRR,\,T}_{prst}$	$(\overline{u^p_R}\gamma^\mu d^r_R][\overline{u^s_R}\gamma^\nu d^t_R)(j^{\alpha\beta}_{\mu\nu})$
$\mathcal{O}^{LRLR,\,S/P}_{prst}$	$(\overline{u^p_L}d^r_R)[\overline{u^s_L}d^t_R](j^{\alpha\beta}/j_5^{\alpha\beta})$	$\mathcal{O}^{RLRL,\,S/P}_{prst}$	$(\overline{u^p_R}d^r_L)[\overline{u^s_R}d^t_L](j^{\alpha\beta}/j_5^{\alpha\beta})$
$\tilde{\mathcal{O}}^{LRLR,\,S/P}_{prst}$	$(\overline{u^p_L}d^r_R][\overline{u^s_L}d^t_R)(j^{\alpha\beta}/j_5^{\alpha\beta})$	$\tilde{\mathcal{O}}^{RLRL,\,S/P}_{prst}$	$(\overline{u^p_R}d^r_L][\overline{u^s_R}d^t_L)(j^{\alpha\beta}/j_5^{\alpha\beta})$
$\mathcal{O}^{LRLR,\,T}_{prst}$	$(\overline{u^p_L}\sigma^{\mu\nu}d^r_R)[\overline{u^s_L}d^t_R](j^{\alpha\beta}_{\mu\nu})$	$\mathcal{O}^{RLRL,\,T}_{prst}$	$(\overline{u^p_R}\sigma^{\mu\nu}d^r_L)[\overline{u^s_R}d^t_L](j^{\alpha\beta}_{\mu\nu})$
$\tilde{\mathcal{O}}^{LRLR,\,T}_{prst}$	$(\overline{u^p_L}\sigma^{\mu\rho}d^r_R)[\overline{u^s_L}\sigma^{\nu}{}_{\rho}d^t_R](j^{\alpha\beta}_{\mu\nu})$	$\tilde{\mathcal{O}}^{RLRL,\,T}_{prst}$	$(\overline{u^p_R}\sigma^{\mu\rho}d^r_L)[\overline{u^s_R}\sigma^{\nu}{}_{\rho}d^t_L](j^{\alpha\beta}_{\mu\nu})$
$\mathcal{O}^{LRLL,\,V/A}_{prst}$	$(\overline{u^p_L}d^r_R)[\overline{u^s_L}\gamma^\mu d^t_L](j^{\alpha\beta}_\mu/j^{\alpha\beta}_{5,\,\mu})$	$\mathcal{O}^{RLRR,\,V/A}_{prst}$	$(\overline{u^p_R}d^r_L)[\overline{u^s_R}\gamma^\mu d^t_R](j^{\alpha\beta}_\mu/j^{\alpha\beta}_{5,\,\mu})$
$\tilde{\mathcal{O}}^{LRLL,\,V/A}_{prst}$	$(\overline{u^p_L}d^r_R][\overline{u^s_L}\gamma^\mu d^t_L)(j^{\alpha\beta}_\mu/j^{\alpha\beta}_{5,\,\mu})$	$\tilde{\mathcal{O}}^{RLRR,\,V/A}_{prst}$	$(\overline{u^p_R}d^r_L][\overline{u^s_R}\gamma^\mu d^t_R)(j^{\alpha\beta}_\mu/j^{\alpha\beta}_{5,\,\mu})$
$\mathcal{O}^{LRRR,\,V/A}_{prst}$	$(\overline{u^p_L}d^r_R)[\overline{u^s_R}\gamma^\mu d^t_R](j^{\alpha\beta}_\mu/j^{\alpha\beta}_{5,\,\mu})$	$\mathcal{O}^{RLLL,\,V/A}_{prst}$	$(\overline{u^p_R}d^r_L)[\overline{u^s_L}\gamma^\mu d^t_L](j^{\alpha\beta}_\mu/j^{\alpha\beta}_{5,\,\mu})$
$\tilde{\mathcal{O}}^{LRRR,\,V/A}_{prst}$	$(\overline{u^p_L}d^r_R][\overline{u^s_R}\gamma^\mu d^t_R)(j_\mu/j^{\alpha\beta}_{5,\,\mu})$	$\tilde{\mathcal{O}}^{RLLL,\,V/A}_{prst}$	$(\overline{u^p_R}d^r_L][\overline{u^s_L}\gamma^\mu d^t_L)(j_\mu/j^{\alpha\beta}_{5,\,\mu})$
$\mathcal{O}^{LRRL,\,S/P}_{prst}$	$(\overline{u^p_L}d^r_R)[\overline{u^s_R}d^t_L](j^{\alpha\beta}/j_5^{\alpha\beta})$	$\tilde{\mathcal{O}}^{LRRL,\,S/P}_{prst}$	$(\overline{u^p_L}d^r_R][\overline{u^s_R}d^t_L)(j^{\alpha\beta}/j_5^{\alpha\beta})$
$\mathcal{O}^{LRRL,\,T1}_{prst}$	$(\overline{u^p_L}\sigma^{\mu\nu}d^r_R)[\overline{u^s_R}d^t_L](j^{\alpha\beta}_{\mu\nu})$	$\mathcal{O}^{LRRL,\,T2}_{prst}$	$(\overline{u^p_L}d^r_R)[\overline{u^s_R}\sigma^{\mu\nu}d^t_L](j^{\alpha\beta}_{\mu\nu})$
$\tilde{\mathcal{O}}^{LRRL,\,T1}_{prst}$	$(\overline{u^p_L}\sigma^{\mu\nu}d^r_R][\overline{u^s_R}d^t_L)(j^{\alpha\beta}_{\mu\nu})$	$\tilde{\mathcal{O}}^{LRRL,\,T2}_{prst}$	$(\overline{u^p_L}d^r_R][\overline{u^s_R}\sigma^{\mu\nu}d^t_L)(j^{\alpha\beta}_{\mu\nu})$

- $j^{\alpha\beta} = (\overline{l_\alpha} l^C_\beta),\ j_5^{\alpha\beta} = (\overline{l_\alpha}\gamma_5 l^C_\beta),\ j^{\alpha\beta}_{5,\,\mu} = (\overline{l_\alpha}\gamma_\mu\gamma_5 l^C_\beta)$ (symmetric)
- $j^{\alpha\beta}_\mu = (\overline{l_\alpha}\gamma_\mu l^C_\beta),\ j^{\alpha\beta}_{\mu\nu} = (\overline{l_\alpha}\sigma_{\mu\nu} l^C_\beta),\ j^{\alpha\beta}_{5,\,\mu\nu} = (\overline{l_\alpha}\sigma_{\mu\nu}\gamma_5 l^C_\beta)$ (anti-symmetric)

χPT 1: basics

- Here we only focus on mesonic part;
- **Origin**: QCD has an approximate $G = SU(3)_L \times SU(3)_R$ flavor symmetry for $u,\ d,\ s$ quarks;
- **Characteristic scale**: $\Lambda_{\rm QCD}$;
- **Fields**: Pseudo Nambu-Goldstone bosons, i.e., light mesons, represented as $\xi = \sqrt{\Sigma} = \exp[i\pi^a\lambda^a/2F_0]$; and possibly, the external sources;
- Under G: $\xi \to L\xi U^\dagger = U\xi R^\dagger$;
- Mesonic χPT=all possible local, G invariant operators constructed via D_μ and $\xi(\Sigma)$, and ordered by number of derivatives $\mathcal{O}(p^n)$, for instance, $\mathcal{L}_{p^2} = \frac{F_0^2}{4}{\rm Tr}[\partial_\mu\Sigma\partial^\mu\Sigma^\dagger] + \frac{F_0^2}{4}(2B_0){\rm Tr}[M\Sigma^\dagger + \Sigma M^\dagger]$
- The state of art: $\mathcal{L}_{p^4}$ [Gasser & Leutwyler], $\mathcal{L}_{p^6}$[9408346, 9902437], $\mathcal{L}_{p^8}$[1810.06834]

χPT 2: How to construct hadronic operators from quark level operators?

The method of spurion analysis

- Step 1: Take the quark level operator(irrep. under G) as

$$\mathcal{O} = T^{ab}_{cd} (\overline{q_{X1}}^c \Gamma_1 q_{Y1,a})(\overline{q_{X2}}^d \Gamma_2 q_{Y2,b}) \sim T^{ab}_{cd} \overline{q_{X1}}^c \overline{q_{X2}}^d q_{Y1,a} q_{Y2,b}$$

  $X_i(Y_i)$ are chiral projectors P_L/P_R. Under chiral group G, the quark fields transform as

$$q_{L,a} \to L_a^p q_{L,p},\ \overline{q_R}^b \to \overline{q_R}^p (R^\dagger)_p^b,\ q_{R,a} \to R_a^p q_{R,p},\ \overline{q_L}^b \to \overline{q_L}^p (L^\dagger)_p^b$$

  Require $\mathcal{O}$ to be invariant under chiral group, which means treat T^{ab}_{cd} as a spurion field with a proper transformation law.
- Step 2: Construct the corresponding hadronic operators by T^{ab}_{cd} together with the Nambu-Goldstone matrix ξ, ..., and require the resulting operators to be invariant under G;
- Step 3: For each independent operator, accompany an unknown low energy constant.

χPT 3: How to construct hadronic operators from quark level operators?

We find the LO matching is the simple replacement:

$$q_{L,a} \to \xi_a^{\ \alpha},\ \overline{q_L}^a \to \xi_\alpha^{\dagger\ a},\ q_{R,a} \to \xi_a^{\dagger\ \alpha},\ \overline{q_R}^a \to \xi_\alpha^{\ a}$$

with the Greek letters contracted with each other. And for the NLO matching,

$$q_{L,a} \to ((D_\mu\xi^\dagger)^\dagger)_a^{\ \alpha},\ \overline{q_L}^a \to (D_\mu\xi^\dagger)_\alpha^{\ a},\ q_{R,a} \to (D_\mu\xi)_a^{\ \alpha},\ \overline{q_R}^a \to (D_\mu\xi)_\alpha^{\dagger a}$$

with $D_\mu = \partial_\mu + (\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger)/2$.

- **Remark 1**: Using the above replacement, we reproduced the results by Savage 99, Graesser 17, Cirigliano et al 17;
- **Remark 2**: We see the matching is **irrelevant** with the color contraction of quark level operators, which means the operators with **different** color contractions will match onto the **same** hadronic operators but with **different** low energy constants.
- The low energy constants can be determined by chiral symmetry, LQCD,...

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Operator basis for $K^- \to \pi^+\mu^-\mu^-$ in LEFT

Operator	Specific form	Chiral $SU(3)_L \times SU(3)_R$ representation
$\mathcal{O}_{udus}^{LLLL,\ S/P}$	$(\overline{u_L}\gamma^\mu d_L)[\overline{u_L}\gamma_\mu s_L](j/j_5)$	$\mathbf{27}_L \times \mathbf{1}_R$
$\mathcal{O}_{udus}^{LRLR,\ S/P}$	$(\overline{u_L}d_R)[\overline{u_L}s_R](j/j_5)$	$\bar{\mathbf{6}}_L \times \mathbf{6}_R$
$\tilde{\mathcal{O}}_{udus}^{LRLR,\ S/P}$	$(\overline{u_L}d_R][\overline{u_L}s_R)(j/j_5)$	$\bar{\mathbf{6}}_L \times \mathbf{6}_R$
$\mathcal{O}_{uiuj}^{LRLL,\ A}$	$(\overline{u_L}i_R)[\overline{u_L}\gamma^\mu j_L]j_{5,\ \mu}$	$\bar{\mathbf{15}}_L \times \mathbf{3}_R$
$\tilde{\mathcal{O}}_{uiuj}^{LRLL,\ A}$	$(\overline{u_L}i_R][\overline{u_L}\gamma^\mu j_L)j_{5,\ \mu}$	$\bar{\mathbf{15}}_L \times \mathbf{3}_R$
$\mathcal{O}_{u(dus)}^{LRRR,\ A}$	$\frac{1}{2}\left[(\overline{u_L}d_R)[\overline{u_R}\gamma^\mu s_R] + d \leftrightarrow s\right]j_{5,\ \mu}$	$\bar{\mathbf{3}}_L \times \mathbf{15}_R$
$\mathcal{O}_{u[dus]}^{LRRR,\ A}$	$\frac{1}{2}\left[(\overline{u_L}d_R)[\overline{u_R}\gamma^\mu s_R] - d \leftrightarrow s\right]j_{5,\ \mu}$	$\bar{\mathbf{3}}_L \times \bar{\mathbf{6}}_R$
$\tilde{\mathcal{O}}_{u(dus)}^{LRRR,\ A}$	$\frac{1}{2}\left[(\overline{u_L}d_R][\overline{u_R}\gamma^\mu s_R) + d \leftrightarrow s\right]j_{5,\ \mu}$	$\bar{\mathbf{3}}_L \times \mathbf{15}_R$
$\tilde{\mathcal{O}}_{u[dus]}^{LRRR,\ A}$	$\frac{1}{2}\left[(\overline{u_L}d_R][\overline{u_R}\gamma^\mu s_R) - d \leftrightarrow s\right]j_{5,\ \mu}$	$\bar{\mathbf{3}}_L \times \bar{\mathbf{6}}_R$
$\mathcal{O}_{uiuj}^{LRRL,\ S/P}$	$(\overline{u_L}i_R)[\overline{u_R}j_L](j/j_5)$	$\mathbf{8}_L \times \mathbf{8}_R$
$\tilde{\mathcal{O}}_{uiuj}^{LRRL,\ S/P}$	$(\overline{u_L}i_R][\overline{u_R}j_L)(j/j_5)$	$\mathbf{8}_L \times \mathbf{8}_R$

and also the parity partner operators with $L \leftrightarrow R$. The lepton current $j = (\overline{\mu}\mu^C),\ j_5 = (\overline{\mu}\gamma_5\mu^C),\ j_{5,\ \mu} = (\overline{\mu}\gamma_\mu\gamma_5\mu^C)$, and $(i,\ j) = (d,\ s)$ or $(i,\ j) = (s,\ d)$, respectively.

- There are totally 36 operators for the process;
- They take 11 different irreducible representations of chiral group $SU(3)_L \times SU(3)_R$;
- Different operators corresponding to the same chiral representation will match onto the same hadronic operators but different coefficients.

Matching between LEFT and SMEFT at m_W

- If we assume the local dim-9 LEFT operators stemmed from SMEFT dim-7 operators, therefore, the matching results are

$$\begin{aligned}
C_{udus}^{LLLL,\ S/P}(m_W) &= -2\sqrt{2}G_F V_{ud} V_{us}\left(C_{LHD1}^{22\dagger}(m_W) + 4C_{LHW}^{22\dagger}(m_W)\right),\\
\tilde{C}_{udus}^{LRRL\ S/P}(m_W) &= 2\sqrt{2}G_F V_{ud} V_{us} C_{\bar{d}uLLD}^{1122\dagger}(m_W),\\
\tilde{C}_{usud}^{LRRL\ S/P}(m_W) &= 2\sqrt{2}G_F V_{ud} V_{us} C_{\bar{d}uLLD}^{2122\dagger}(m_W),
\end{aligned}$$

and all others vanish.

- QCD running effect:

$$\begin{aligned}
\mu\frac{d}{d\mu}C_{udus}^{LLLL,\ S/P} &= -\frac{\alpha_s}{2\pi}\left(\frac{3}{N}-3\right)C_{udus}^{LLLL,\ S/P},\\
\mu\frac{d}{d\mu}\begin{pmatrix} C_{uiuj}^{LRRL,\ S/P} \\ \tilde{C}_{uiuj}^{LRRL,\ S/P}\end{pmatrix} &= -\frac{\alpha_s}{2\pi}\begin{pmatrix} 6C_F & 3 \\ 0 & -\frac{3}{N}\end{pmatrix}\begin{pmatrix} C_{uiuj}^{LRRL,\ S/P} \\ \tilde{C}_{uiuj}^{LRRL,\ S/P}\end{pmatrix}.
\end{aligned}$$

which gives

$$\begin{aligned}
C_{udus}^{LLLL,\ S/P}(m_K) &= 0.82 C_{udus}^{LLLL,\ S/P}(m_W),\\
\tilde{C}_{uiuj}^{LRRL,\ S/P}(m_K) &= 0.9 \tilde{C}_{uiuj}^{LRRL,\ S/P}(m_W),\\
C_{uiuj}^{LRRL,\ S/P}(m_K) &= 0.45 \tilde{C}_{uiuj}^{LRRL,\ S/P}(m_W).
\end{aligned}$$

Matching between LEFT and χPT at m_K

From the SMEFT perspective, here we only need to consider the hadronic partners of ten LEFT operators:

- $\mathcal{O}_{udus}^{LLLL,\ S/P}$

$$\mathcal{Q}_{27\times 1} = 2g_{27\times 1}^{K\pi} F_0^4 (\xi i D_\mu \xi^\dagger)_2{}^1 (\xi i D^\mu \xi^\dagger)_3{}^1 (j/j_5) = g_{27\times 1}^{K\pi} F_0^2 \partial_\mu K^- \partial^\mu \pi^- (j/j_5) + \cdots ,$$

- $\mathcal{O}_{uiuj}^{LRRL,\ S/P}$ and $\tilde{\mathcal{O}}_{uiuj}^{LRRL,\ S/P}$($(i,\ j) = (d,\ s)$ or $(i, j) = (s,\ d)$):

$$\mathcal{Q}_{8\times 8}^1 = g_{8\times 8}^{K\pi} \frac{F_0^2}{4} (\xi^{\dagger 2})_i{}^1 (\xi^2)_j{}^1 (j/j_5) = \frac{1}{2} g_{8\times 8}^{K\pi} F_0^2 K^- \pi^- (j/j_5) + \cdots ,$$

$$\mathcal{Q}_{8\times 8}^2 = g_{8\times 8}^{K\pi\prime} \frac{F_0^2}{4} (\xi^{\dagger 2})_i{}^1 (\xi^2)_j{}^1 (j/j_5) = \frac{1}{2} g_{8\times 8}^{K\pi\prime} F_0^2 K^- \pi^- (j/j_5) + \cdots ,$$

- Chiral symmetry, experimental data, and LQCD $\Rightarrow$ LECs: $g_{27\times 1}^{K\pi},\ g_{8\times 8}^{K\pi\prime},\ g_{8\times 8}^{K\pi\prime}$ [1805.02634, 1806.02780]:

$$g_{27\times 1}^{K\pi} = \frac{5}{6} g_1^{\pi\pi} = 0.3,\ \ g_{8\times 8}^{K\pi} = -\frac{1}{2} g_5^{\pi\pi} = 4\text{GeV}^2.\ g_{8\times 8}^{K\pi\prime} = -\frac{1}{2} g_4^{\pi\pi} = 1\text{GeV}^2$$

Effective Lagrangian for $K^- \to \pi^+\mu^-\mu^-$

$$\mathcal{L}_{K^-\to\pi^+\mu^-\mu^-} = \frac{1}{2}K^-\pi^-(c_1 j + c_2 j_5) + \frac{1}{2}\partial^\mu K^- \partial_\mu \pi^-(c_3 j + c_4 j_5)$$

with

$$j = (\overline{\mu}\mu^C),\ j_5 = (\overline{\mu}\gamma_5\mu^C)$$

and

$$\begin{aligned}
c_1 = c_2 \quad &= \left(C_{udus}^{LRRL,\ S/P}(m_K) + C_{usud}^{LRRL,\ S/P}(m_K)\right) g_{8\times 8}^{K\pi} F_0^2 \\
&\quad + \left(\tilde{C}_{udus}^{LRRL,\ S/P}(m_K) + \tilde{C}_{usud}^{LRRL,\ S/P}(m_K)\right) g_{8\times 8}^{K\pi\prime} F_0^2 \\
&= \left(\tilde{C}_{udus}^{LRRL,\ S/P}(m_W) + \tilde{C}_{usud}^{LRRL,\ S/P}(m_W)\right)\left(0.45 g_{8\times 8}^{K\pi} + 0.9 g_{8\times 8}^{K\pi\prime}\right) F_0^2 \\
&= 2\sqrt{2}G_F V_{ud} V_{us}\left(C_{\bar{d}uLLD}^{1122\dagger}(m_W) + C_{\bar{d}uLLD}^{2122\dagger}(m_W)\right)\left(0.45 g_{8\times 8}^{K\pi} + 0.9 g_{8\times 8}^{K\pi\prime}\right) F_0^2, \\
c_3 = c_4 \quad &= 2 C_{udus}^{LLLL,\ S/P}(m_K) g_{27\times 1}^{K\pi} F_0^2 \\
&= 1.62 C_{udus}^{LLLL,\ S/P}(m_W) g_{27\times 1}^{K\pi} F_0^2 \\
&= -2\sqrt{2}G_F V_{ud} V_{us}\left(C_{LHD1}^{22\dagger}(m_W) + 4 C_{LHW}^{22\dagger}(m_W)\right)(1.62 g_{27\times 1}^{K\pi}) F_0^2,
\end{aligned}$$

Effective Lagrangian for $K^- \to \pi^+\mu^-\mu^-$

The decay width calculated to be

$$\Gamma_{K^- \to \pi^+\mu^-\mu^-} = \frac{1}{2!}\frac{1}{2m_K}\frac{1}{128\pi^3 m_K^2}\int ds \int dt |\mathcal{M}|^2,$$

with

$$\sum |\mathcal{M}_{\mathrm{SMEFT}}|^2 = \left|2c_1 + c_3\left(m_K^2 + m_\pi^2 - s\right)\right|^2 \left(s - 2m_\mu^2\right)$$

Experimentally,

$$\mathrm{Br}(K^- \to \pi^+\mu^-\mu^-) < 4.2 \times 10^{-11} [1905.07770]$$

$$\Rightarrow \Lambda_{\mathrm{NP}} \sim \left(\frac{1}{C_{\bar{d}uLLD}^{1122\dagger}}\right)^{\frac{1}{3}} > \mathcal{O}(0.1)\mathrm{GeV}$$

- **Remark 1**: all calculation is done in tree level for estimation;
- **Remark 2**: the constraint for NP scale is rather loose than $0\nu\beta\beta$: $\mathcal{O}(1)$TeV;

1. Why $K^- \to \pi^+\mu^-\mu^-$?
2. Backgrounds of different EFTs
3. Back to $K^- \to \pi^+\mu^-\mu^-$
4. Conclusion

Conclusion

- $K^- \to \pi^+\mu^-\mu^-$ is studied in the framework of EFTs;
- Matching and running is done between different EFTs;
- The experimental constraint for the process is rather loose, but it opens new way to detect LNVs and NP signals.

Next project

- Estimate the uncertainties from χPT calculation via chiral logarithms;
- Extend to long-range case and other decay channels;
- Extend to the LNV decays of other heavier mesons.

THE END

THANK YOU FOR YOUR ATTENTION!