NLP corrections to $B \to \pi$, K form factors with higher-twist corrections

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※ Motivation & introduction

※ $B \to \pi$, K form factors with higher-twist corrections

- Form factors in LCSR
- Radiative correction at leading power
- Higher-twist B-meson LCDA corrections (NLP)

※ Numerical analysis

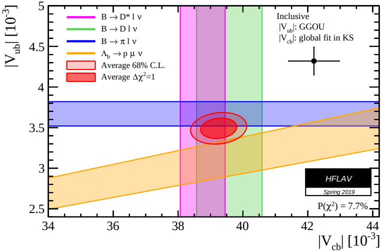
- Numerical effects of LP@NLL and NLP@LO
- Flavor SU(3) symmetry breaking effects
- Phenomenological applications

※ Summary & conclusion

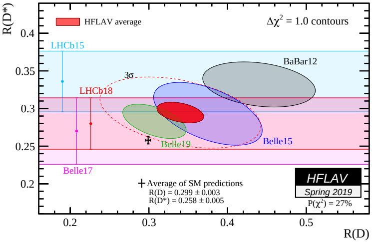
Motivation

Definition of B-meson transition form factors

$$\langle P(p)|\bar{q}\,\sigma_{\mu\nu}\,q^{\nu}\,b|\bar{B}(p+q)\rangle = i\,\frac{f^{T}_{B\to P}(q^2)}{m_B+m_P}\left[q^2\,(2p+q)_{\mu} - (m_B^2 - m_P^2)\,q_{\mu}\right]$$



- ⋇ Input parameters of B-meson decays $|V_{ub}|$, $|V_{cb}|$, $R_{D^{(*)}}$, $R_{K^{(*)}}$, CPV $\cdots \to$ NP.
- ⋇ Factorization properties of QCD.



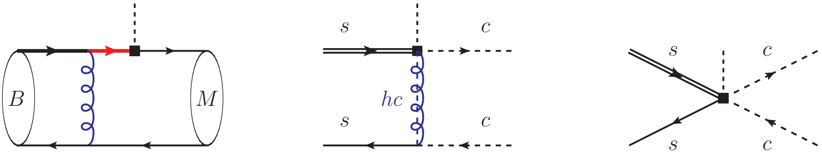
Calculation method

- ⋇ At large q^2 region: LQCD, HQET $\cdots$
- ⋇ At small q^2 region: SCET, PQCD, **LCSR** $\cdots$

Form factors in SCET

Since there are two large scales, we need two-step matching

$$\mathrm{QCD} \xrightarrow{\mathcal{O}(m_b)} \mathrm{SCET_I} \xrightarrow{\mathcal{O}(\sqrt{\Lambda m_b})} \mathrm{SCET_{II}}$$



QCD SCET$_{\mathrm{I}}$ SCET$_{\mathrm{II}}$

From QCD to SCET$_{\mathrm{I}}$: hard function

$$f^i_{B\to M}(E) = C_i(E)\,\xi_a(E) + \int d\tau\, C_i^{(B1)}(E,\tau)\,\Xi_a(\tau,E)$$

From SCET$_{\mathrm{I}}$ to SCET$_{\mathrm{II}}$: jet function

$$\Xi_a \propto J_a \otimes \phi_M \otimes \phi_B$$

Subleading power correction

Factorization formula valid at each power in Λ/m_b

$$\begin{array}{llllllll} \text{LP}: & 1 & + & \mathcal{O}(\alpha_s) & + & \mathcal{O}(\alpha_s^2) & + & \cdots \\ \textbf{NLP}: & 1 & + & \mathcal{O}(\alpha_s) & + & \mathcal{O}(\alpha_s^2) & + & \cdots \\ \vdots & & & & & & & \end{array}$$

Numerically, the NLP@LO contribution could be as large as LP@NLO contribution

$$\alpha_s(m_b)/\pi \qquad \sim \qquad \Lambda/m_b$$

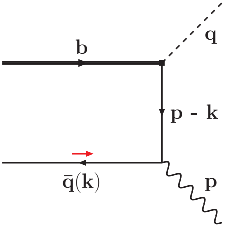
At NLP: power suppressed SCET operators and Lagrangian

End-point singularity will appear at NLP in $B \to \gamma \ell \nu$

Form factors in LCSR

To avoid the end-point singularity

- ⋇ PQCD approach, H.N. Li, Y.L. Shen and Y.M. Wang, 12'
- ⋇ LCSR approach
  - light-meson LCSR, A. Khodjamirian and A.V. Rusov, 17'
  - ***B*-meson LCSR**, Y.M. Wang and Y.L. Shen, 15', Y.L. Shen, **YBW** and C.D. Lü, 16'



B-meson LCSR, start from two-point correction function

$$\Pi_\mu(n\cdot p,\bar{n}\cdot p)=\int d^4x\; e^{ip\cdot x}\langle 0|\,T\Big\{\bar{d}(x)\,\not{n}\,\gamma_5\,q(x)\,,\bar{q}(0)\,\Gamma_\mu\,b(0)\Big\}|\bar{B}(p+q)\rangle$$

$$n\cdot p\sim\mathcal{O}(m_b)\,,\quad |\,\bar{n}\cdot p\,|\sim\mathcal{O}(\Lambda)\,,\quad p^2<0$$

Light-cone OPE: $x^2\to 0$

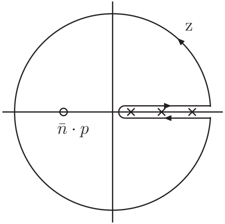
Form factors in LCSR

$$\Pi_\mu(n\cdot p, \bar{n}\cdot p) = \int d^4x\, e^{ip\cdot x} \langle 0| T\left\{\bar{d}(x)\, \not{n}\,\gamma_5\, q(x),\, \bar{q}(0)\,\Gamma_\mu\, b(0)\right\} |\bar{B}(p+q)\rangle$$

$$\Uparrow$$

$$|\pi\rangle\langle\pi|$$

- Hadronic level: insert complete set $\sum_n |\, n\rangle\langle n\,|$
- Partonic level: factorization formula

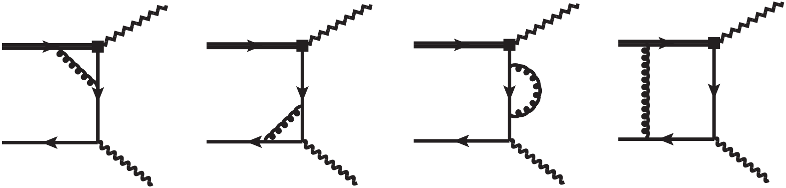


Standard QCD sum rules technique

- **Dispersion relation**: $\Pi(p^2) = \frac{1}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\Pi(s)}{s-p^2}$
- Parton-hadron duality ansatz
- Borel transformation

$$f^+_{B\to P}(q^2) = \frac{\tilde{f}_B(\mu)\, m_B}{n\cdot p\, f_P}\, e^{m_P^2/(n\cdot p\,\omega_M)} \int_0^{\omega_s} d\omega\, \phi_B^-(\omega)\, e^{-\omega/\omega_M}$$

NLO correction at leading power



※ **Method of regions**: M. Beneke and V.A. Smirnov, 97'
hard, hard-collinear and soft regions have leading power contribution
soft region $= \phi^{(1)} \otimes T^{(0)}$

※ Factorization: hard scale $[\mathcal{O}(m_b)]$, hard-collinear scale $[\mathcal{O}(\sqrt{m_b \Lambda})]$ and soft scale $[\mathcal{O}(\Lambda)]$

$$\Pi = \tilde{f}_B(\mu)\, m_B\, C(n\cdot p, \mu) \int_0^\infty \frac{d\omega}{\omega - \bar{n}\cdot p}\, J\left(\frac{\mu^2}{n\cdot p\,\omega}, \frac{\omega}{\bar{n}\cdot p}\right) \phi_B(\omega, \mu)$$

※ Resummation: factorization scale $\mu \sim \mathcal{O}(\sqrt{m_b \Lambda})$, sum logs in C

Subleading power correction

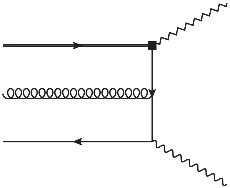
Subleading power

$$\text{NLP@LO} \quad \sim \quad \text{LP@NLO}$$

Higher-twist B-meson LCDA contribution

⋇ Three-particle LCDA up to twist-6: quark in background field method

$$\begin{array}{ccc} \text{NLO} & \to & \text{LP} \\ \text{LO} & \to & \text{NLP} \end{array}$$



⋇ Two-particle higher-twist LCDA: off light-cone

The LCDAs satisfy the EOM constraint

$$f_{B\to P}(q^2) = \underbrace{f_{B\to P}^{\rm 2PNLL}(q^2)}_{\text{LP@NLL}} + \underbrace{f_{B\to P}^{\rm 2PHT}(q^2) + f_{B\to P}^{\rm 3PHT}(q^2)}_{\text{NLP@LO}}$$

B-meson LCDAs

B-meson LCDAs are the nonperturbative inputs: up to twist-6

The LCDAs are not independent: **EOM** at tree level

$$-\omega \frac{d}{d\omega}\phi_B^-(\omega) = \phi_B^+(\omega) + 2\int_0^\omega \frac{d\omega_2}{\omega_2}\left[\left(\frac{d}{d\omega}+\frac{1}{\omega_2}\right)\Phi_3(\omega-\omega_2,\omega_2) - \frac{1}{\omega_2}\Phi_3(\omega)\right]$$

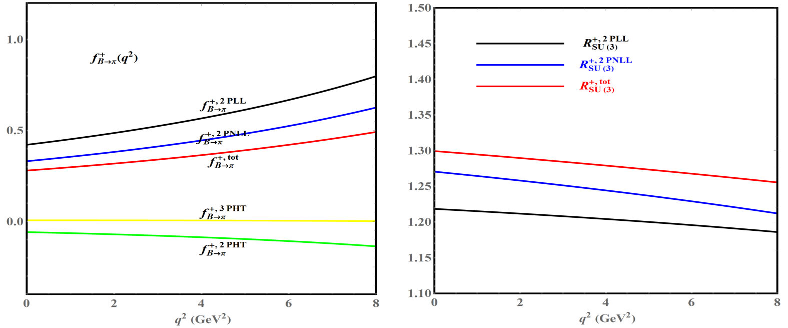
To reduce uncertainty: two sets of LCDA models

- ⋇ Exponential model: $\phi_B^{+,\,\exp}(\omega,\mu) = \frac{\omega}{\omega_0^2}\, e^{-\omega/\omega_0} \quad \overset{\omega\to 0}{\sim} \quad \omega$
- ⋇ Local Duality model: $\phi_B^{+,\mathrm{LD}}(\omega,\mu) = \frac{5}{8\,\omega_0^5}\,\omega(2\,\omega_0-\omega)^3\,\theta(2\,\omega_0-\omega)$

Local Duality model for twist-5 and 6 LCDAs: QCD sum rule in local-duality limit

$$\int d^4y\, e^{-i\,\omega\, y}\, \langle 0|\, T\{\bar{q}(z_1)\, G(z_2)\, \Gamma_1\, h_v(0)\, ,\bar{h}_v(y)\, G(y)\, \Gamma_2\, q(y)\}|0\rangle$$

Numerical results



LP@NLL (20%) $\sim$ NLP@LO (15%) $\Leftrightarrow$ $\alpha_s/\pi \sim \Lambda/m_b$

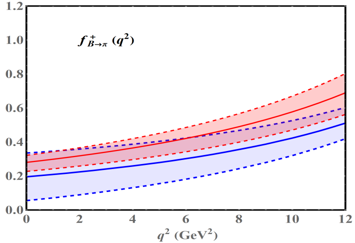
Flavor SU(3) symmetry breaking: m_s, m_P, f_P, Borel parameter $\cdots$

$$R^i_{\mathrm{SU(3)}}(q^2) = \frac{f^i_{B\to K}(q^2)}{f^i_{B\to\pi}(q^2)} : \qquad \text{LP@NLL} \sim \text{NLP@LO}$$

Phenomenology

z-series expansion: $|z(q^2,\ t_0)| \leq 1$

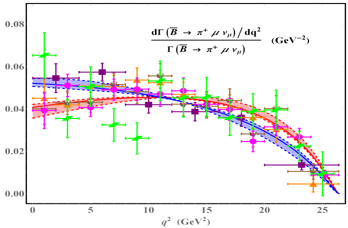
$$f^{+,T}_{B\to P}(q^2) = \frac{f^{+,T}_{B\to P}(0)}{1-q^2/m^2_{B^*_{(s)}}}\left\{1 + \sum_{k=1}^{N-1} b^{+,T}_{k,P}\left[z(q^2)^k - z(0)^k - (-1)^{N-k}\frac{k}{N}\left[z(q^2)^N - z(0)^N\right]\right]\right\}$$



Red: B-meson LCSR
Blue: LQCD (up) light-meson LCSR (low)

$$|V_{ub}|_{\rm exc.} = \left(3.23\,{}^{+0.66}_{-0.48}\Big|_{\rm th.}\,{}^{+0.11}_{-0.11}\Big|_{\rm exp.}\right) \times 10^{-3}$$

$$|V_{ub}|_{\rm inc.} = \left(4.49 \pm 0.15\,{}^{+0.16}_{-0.17} \pm 0.17\right) \times 10^{-3}$$



$$R_{K\pi}(q_1^2, q_2^2) = \frac{\int_{q_1^2}^{q_2^2} dq^2 d\Gamma(B\to K\nu\nu)/dq^2}{\int_{q_1^2}^{q_2^2} dq^2 d\Gamma(B\to \pi\mu\nu_\mu)/dq^2}$$

$$R_{K\pi}(0, \max) = 4.06^{+0.39}_{-0.30} \times 10^{-2}$$

$$\Gamma(B \to K\nu\nu) = 6.02^{+1.68}_{-1.76} \times 10^{-6}$$

Summary

$B \to \pi\,, K$ form factors within B-meson LCSR

- ⋇ **NLP**: Higher-twist B-meson LCDA correction up to twist-6
- ⋇ Flavor SU(3) symmetry breaking
- ⋇ NLL resummation at leading power
  - Method of regions $\to$ factorization formula of correlation function
  - RGE $\to$ NLL resummation
- ⋇ z-series expansion: $B \to \pi\ell\nu$ and $B \to K\nu\nu$

Outlook and future improvements

- ⋇ Perturbative corrections to the higher-twist contributions
- ⋇ EOM of LCDAs at NLO, OPE constraint on LCDAs

Thank you!

z-series expansion

$$f^{+,T}_{B\to P}(q^2) = \frac{f^{+,T}_{B\to P}(0)}{1 - q^2/m^2_{B^*_{(s)}}} \left\{ 1 + \sum_{k=1}^{N-1} b^{+,T}_{k,P} \left(z(q^2,\ t_0)^k - z(0,\ t_0)^k \right.\right.$$
$$\left.\left. - (-1)^{N-k} \frac{k}{N} \left[z(q^2,\ t_0)^N - z(0,\ t_0)^N \right] \right) \right\}$$
$$f^0_{B\to P}(q^2) = f^0_{B\to P}(0) \left\{ 1 + \sum_{k=1}^{N} b^0_{k,P} \left(z(q^2,\ t_0)^k - z(0,\ t_0)^k \right) \right\}$$

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$
$$t_+ = (m_B + m_P)^2$$
$$t_0 = (m_B + m_P)\left(\sqrt{m_B} + \sqrt{m_P}\right)^2$$