重味介子遍举产生的HQET因子化理论

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Talk based on S. Ishaq, 贾宇，熊小努，杨德山，
arXiv:1903.12627[hep-ph] and arXiv: 1905.06930

2019年全国重味物理与CP破坏研讨会，呼和浩特，7/29-8/1

报告内容

- 回顾粲味强子产生的不对称性疑难, 及重夸克重组合机制
- 重味介子遍举产生的标准理论框架 — 光锥（共线）因子化
- 重味介子遍举产生的新理论框架 — HQET因子化
- 关于重味介子光锥分布振幅(LCDA)的新的因子化定理
- 总结

重味物理前沿: 主要聚焦于重味强子遍举衰变

Trace the origin of the CP violation

Precisely pin down the CKM matrix

Hunt the possible footprints of BSM Physics

Exclusive B meson decays

QCD factorization, pQCD, SCET, ...

Very rich phenomenology

Amplitude → Short-distance part ⊗ Long distance part

重味强子产生的一个著名历史疑难

领头粒子效应：若粲强子所含的轻味价夸克与beam强子所含的价夸克一致，则其产率在向前方向会被增强。 此效应被多个费米实验室固定靶实验所观测

例如，E791实验是500 GeV π^-(ubar d) 轰击原子核靶，观测到在π^- beam方向产生的D^-(cbar d) 要比D^+(c dbar)多很多！

$$\alpha[D^+] = \frac{d\sigma[D^-] - d\sigma[D^+]}{d\sigma[D^-] + d\sigma[D^+]},$$

But, why? How to understand this peculiar correlation pattern?

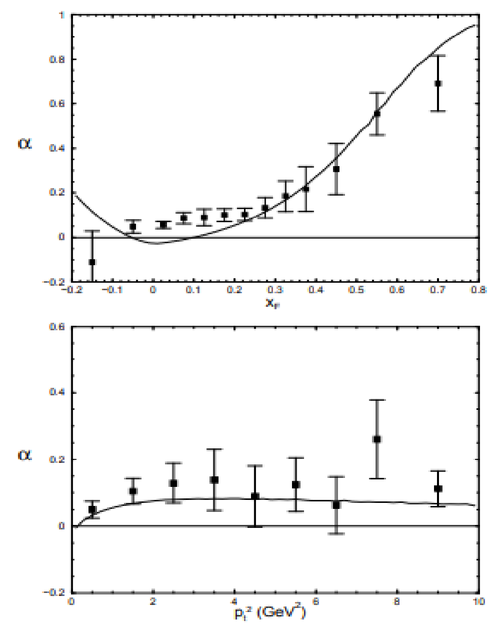


FIG. 2. The asymmetry variable $\alpha[D^+]$ vs. x_F and $p_\perp^2$. The data points are the measurements of E791 and the solid line is our prediction with $\rho_1^{sim} = 0.06$.

关于$D^{+/-}$介子产生的非对称性疑难

QCD因子化定理：

$$d\sigma[hh' \to D + X] = \sum_{i,j} f_{i/h} \otimes f_{j/h'} \otimes d\hat{\sigma}(ij \to c\bar{c} + X) \otimes D_{c\to D}$$

在$1/p_t$的领头阶, 粲强子通过碎裂机制产生。部分子硬散射过程gg, qqbar-> c+cbar以及c->D碎裂函数关于电荷共轭对称。不可能诱导任何非对称性！

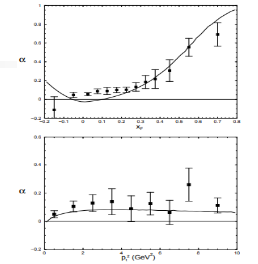
因此，领头粒子效应一定源于higher twist贡献（power correction）

为了解释此效应，文献中建议了各种可能的非微扰机制：

- Beam drag effect — Lund string model, PYTHIA 2001
- Intrinsic charm — Vogt and Brodsky, NPB 1996
- Heavy quark recombination — Braaten, Jia, Mehen, PRL 2002

最简单、最经济的机制

重夸克重组合机制 (Heavy quark recombination mechanism, or HQR)



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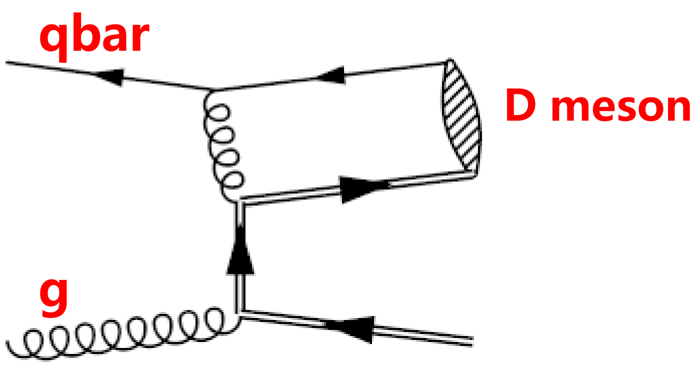


HSR因子化公式：

$$d\hat{\sigma}[D] = d\hat{\sigma}[\bar{q}g \to (c\bar{q})^n + \bar{c}]\,\rho[(c\bar{q})^n \to D]\,.$$

Recombination几率：正比于D介子LCDA的inverse moment的模平方

$$\left.\frac{d\hat{\sigma}[\bar{q}g \to (c\bar{q})({}^1S_0^{(1)}, {}^3S_1^{(1)}) + \bar{c}]}{d\hat{\sigma}[gg \to \bar{c}c]}\right|_{\theta=0} \approx \frac{256\,\pi}{81}\alpha_s\,.$$

HSR机制背后的物理图像：经过硬散射后，如果轻夸克在重夸克的静止系中动量很软(soft), 则它们有一定的几率重组合为D介子 / HQET蕴含的图像

类似于NRQCD因子化之于重夸克偶素产生

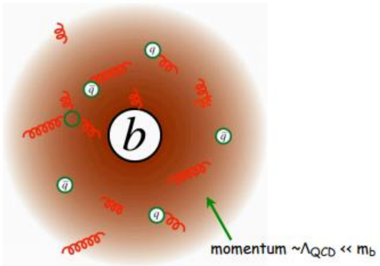
最初起的名字：双部分子碎裂机制

领头粒子效应变得易于理解：因为π^-中的d夸克成分远大于dbar, 因此D^- 产率要比D^+大！

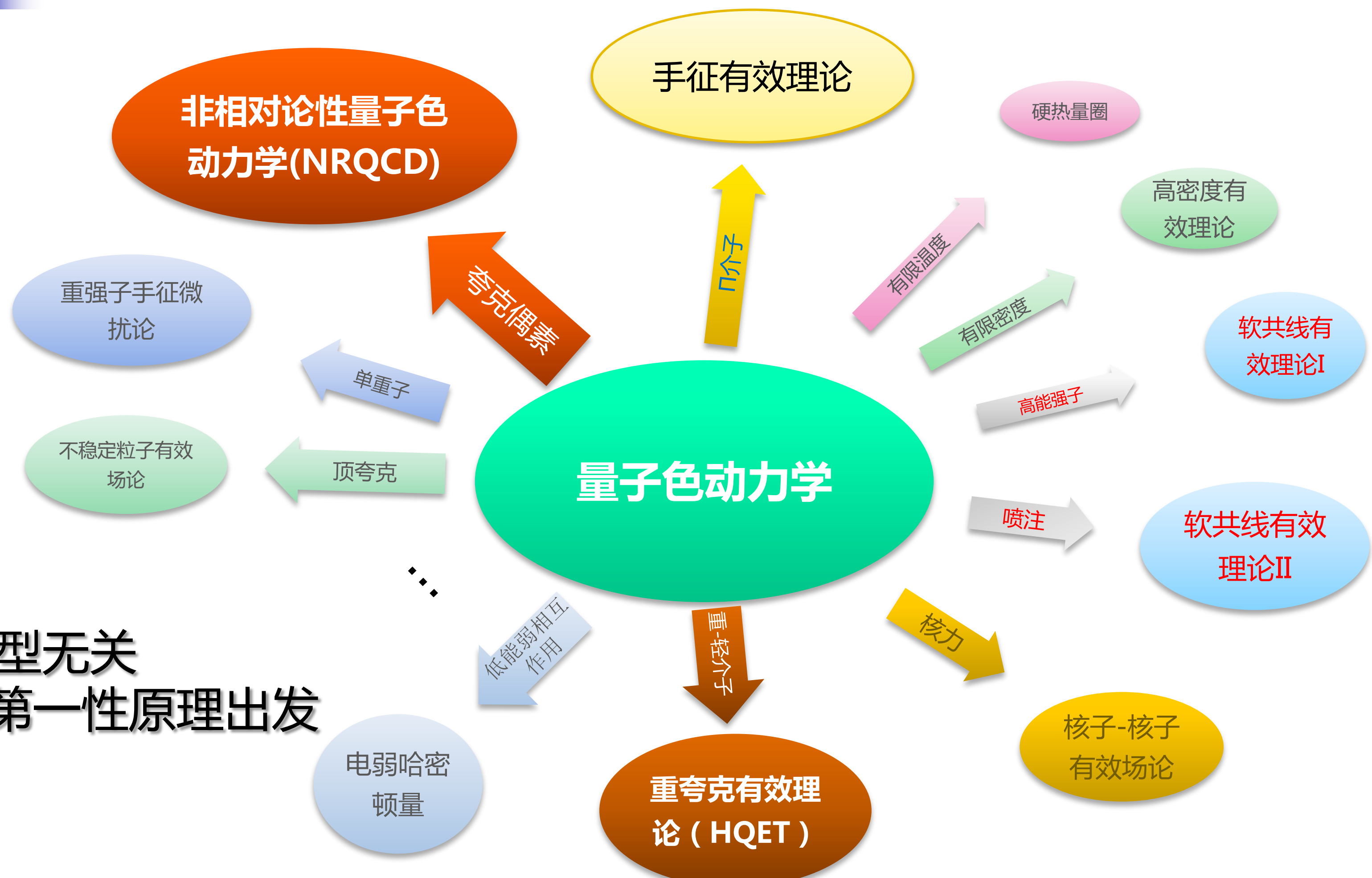
如何将HSR推广到α_s次领头阶

- HSR能够成功解释领头粒子效应/证明其捕捉住了正确的物理
- 如何描述重味强子遍举产生的α_s次领头阶辐射修正？
- 例如，如何有效描述$W\to D_s+\gamma$，$W\to B+\gamma$？
- 如何将HSR机制在有效场论(EFT)理论框架下formulate?
- HSR → HQET因子化框架

Ishaq, Jia, Xiong, Yang, 2019



QCD的各种有效场论



- 模型无关
  从第一性原理出发

重味介子遍举产生的传统理论框架：共线因子化(或光锥因子化)

Grossman, Koenig, Neubert, JHEP 2015

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Exclusive radiative decays of W and Z bosons in QCD factorization

Yuval Grossman,[a] Matthias König[b] and Matthias Neubert[a,b]

Decay mode	Branching ratio	asymptotic	LO
$Z^0 \to \pi^0\gamma$	$(9.80\,^{+0.09}_{-0.14}\,{}_{\mu} \pm 0.03_f \pm 0.61_{a_2} \pm 0.82_{a_4}) \cdot 10^{-12}$	7.71	14.67
$Z^0 \to \rho^0\gamma$	$(4.19\,^{+0.04}_{-0.06}\,{}_{\mu} \pm 0.16_f \pm 0.24_{a_2} \pm 0.37_{a_4}) \cdot 10^{-9}$	3.63	5.68
$Z^0 \to \omega\gamma$	$(2.82\,^{+0.03}_{-0.04}\,{}_{\mu} \pm 0.15_f \pm 0.28_{a_2} \pm 0.25_{a_4}) \cdot 10^{-8}$	2.48	3.76
$Z^0 \to \phi\gamma$	$(1.04\,^{+0.01}_{-0.02}\,{}_{\mu} \pm 0.05_f \pm 0.07_{a_2} \pm 0.09_{a_4}) \cdot 10^{-8}$	0.86	1.49
$Z^0 \to J/\psi\,\gamma$	$(8.02\,^{+0.14}_{-0.15}\,{}_{\mu} \pm 0.20_f\,^{+0.39}_{-0.36}\,{}_{\sigma}) \cdot 10^{-8}$	10.48	6.55
$Z^0 \to \Upsilon(1S)\,\gamma$	$(5.39\,^{+0.10}_{-0.10}\,{}_{\mu} \pm 0.08_f\,^{+0.11}_{-0.08}\,{}_{\sigma}) \cdot 10^{-8}$	7.55	4.11
$Z^0 \to \Upsilon(4S)\,\gamma$	$(1.22\,^{+0.02}_{-0.02}\,{}_{\mu} \pm 0.13_f\,^{+0.02}_{-0.02}\,{}_{\sigma}) \cdot 10^{-8}$	1.71	0.93
$Z^0 \to \Upsilon(nS)\,\gamma$	$(9.96\,^{+0.18}_{-0.19}\,{}_{\mu} \pm 0.09_f\,^{+0.20}_{-0.15}\,{}_{\sigma}) \cdot 10^{-8}$	13.96	7.59

Decay mode	Branching ratio	asymptotic	LO
$W^\pm \to \pi^\pm\gamma$	$(4.00\,^{+0.06}_{-0.11}\,{}_{\mu} \pm 0.01_f \pm 0.49_{a_2} \pm 0.66_{a_4}) \cdot 10^{-9}$	2.45	8.09
$W^\pm \to \rho^\pm\gamma$	$(8.74\,^{+0.17}_{-0.26}\,{}_{\mu} \pm 0.33_f \pm 1.02_{a_2} \pm 1.57_{a_4}) \cdot 10^{-9}$	6.48	15.12
$W^\pm \to K^\pm\gamma$	$(3.25\,^{+0.05}_{-0.09}\,{}_{\mu} \pm 0.03_f \pm 0.24_{a_1} \pm 0.38_{a_2} \pm 0.51_{a_4}) \cdot 10^{-10}$	1.88	6.38
$W^\pm \to K^{*\pm}\gamma$	$(4.78\,^{+0.09}_{-0.14}\,{}_{\mu} \pm 0.28_f \pm 0.39_{a_1} \pm 0.66_{a_2} \pm 0.80_{a_4}) \cdot 10^{-10}$	3.18	8.47
$W^\pm \to D_s\gamma$	$(3.66\,^{+0.02}_{-0.07}\,{}_{\mu} \pm 0.12_{\rm CKM} \pm 0.13_f\,^{+1.47}_{-0.82}\,{}_{\sigma}) \cdot 10^{-8}$	0.98	8.59
$W^\pm \to D^\pm\gamma$	$(1.38\,^{+0.01}_{-0.02}\,{}_{\mu} \pm 0.10_{\rm CKM} \pm 0.07_f\,^{+0.50}_{-0.30}\,{}_{\sigma}) \cdot 10^{-9}$	0.32	3.42
$W^\pm \to B^\pm\gamma$	$(1.55\,^{+0.00}_{-0.03}\,{}_{\mu} \pm 0.37_{\rm CKM} \pm 0.15_f\,^{+0.68}_{-0.45}\,{}_{\sigma}) \cdot 10^{-12}$	0.09	6.44

利用条件$M_W >> m_B$，因子化定理将振幅表示成硬散射核卷积B介子光锥分布振幅(LCDA)的形式

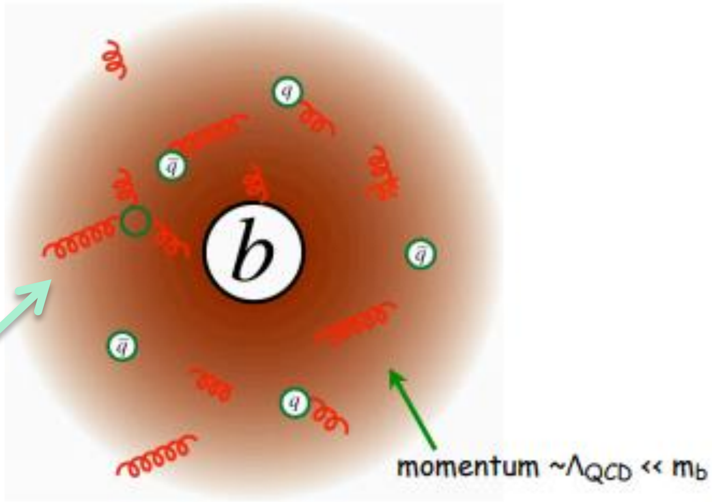
然而，B介子的QCD LCDA人们知道的并不清楚，阻碍了预言能力；另外，B的LCDA依然含有微扰能标m_b, 需要剥离

Heavy Quark Effective Theory (HQET)

$$L_{HQET} = \bar{h}_v iv \cdot D h_v - \bar{h}_v \frac{D_\perp^2}{2m_Q} h_v - g\bar{h}_v \frac{\sigma_{\mu\nu} G^{\mu\nu}}{4m_Q} h_v + O\left(1/m_Q^2\right)$$

E. Eichten, PLB, 1990;
H. Georgi, PLB, 1990M

Leading term in $1/m_b$, Possesses both heavy quark flavor and spin symmetries



N. Isgur refers it to brown muck

HQET factorization: standard tools in dealing with B decay

$B \to M_1 M_2$

QCDF:
Beneke, Buchalla, Neubert and Sachrajda (BBNS) ; D.-S. Du, Y.-D. Yang, M.-Z. Yang, D.-S. Yang, G.-H. Zhu, X.-Q. Li, Q. Chang, …

kt factorization:
H.-n Li, C.-D. Lu, Z.-J. Xiao, …

Please refer to Yu-Ming Wang’s comprehensive overview of the status in nonleptonic B decay

$B \to \gamma l \nu$

G. P. Korchemsky, D. Pirjol, T.-M. Yan (2002); S. Descotes-Genon and C.T. Sachrajda(2003); E. Lunghi, D. Pirjol, D. Wyler (2003);

For 1/mb correction, see Y. M. Wang 2017; M. Beneke and Y.B. Wei 2017

Kinematics in W->B+γ

$$\mathcal{M}\left(W^+ \to B^+\gamma\right) = \frac{e_u e^2 V_{ub}}{4\sqrt{2}\sin\theta_W}\left(\epsilon_{\mu\nu\alpha\beta}\frac{P^\mu q^\nu \varepsilon_W^\alpha \varepsilon_\gamma^{*\beta}}{P\cdot q}F_V + i\varepsilon_W\cdot\varepsilon_\gamma^* F_A\right)$$

$$\Gamma\left(W^+ \to B^+\gamma\right) = \frac{e_u^2\pi\alpha^2}{48\sin^2\theta_W m_W^3}\left|V_{ub}\right|^2\left(m_W^2 - m_B^2\right)\left(\left|F_V\right|^2 + \left|F_A\right|^2\right),$$

$$P^\mu\Big|_{W\text{ rest frame}} = (P^+, P^-, \boldsymbol{P}_\perp) = \frac{1}{\sqrt{2}}\left(m_W, \frac{m_B^2}{m_W}, \boldsymbol{0}_\perp\right),$$
$$q^\mu\Big|_{W\text{ rest frame}} = (q^+, q^-, \boldsymbol{q}_\perp) = \frac{1}{\sqrt{2}}\left(0, \frac{m_W^2 - m_B^2}{m_W}, \boldsymbol{0}_\perp\right),$$

$$P^\mu\Big|_{B\text{ rest frame}} = (P^+, P^-, \boldsymbol{P}_\perp) = \frac{1}{\sqrt{2}}\left(m_B, m_B, \boldsymbol{0}_\perp\right),$$
$$q^\mu\Big|_{B\text{ rest frame}} = (q^+, q^-, \boldsymbol{q}_\perp) = \frac{1}{\sqrt{2}}\left(0, \frac{m_W^2 - m_B^2}{m_B}, \boldsymbol{0}_\perp\right).$$

三个分离很大的scales: $m_w >> m_b >> \Lambda_{QCD}$

如果假设 $m_w >> m_b \sim \Lambda_{QCD}$　　Light-cone factorization

如果假设 $m_w \sim m_b >> \Lambda_{QCD}$　　HQET factorization

HQET factorization for exclusive production of B meson

$$\mathcal{M}\left(W^+ \to B^+\gamma\right) = \frac{e_u e^2 V_{ub}}{4\sqrt{2}\sin\theta_W}\left(\epsilon_{\mu\nu\alpha\beta}\frac{P^\mu q^\nu \varepsilon_W^\alpha \varepsilon_\gamma^{*\beta}}{P\cdot q}F_V + i\varepsilon_W\cdot\varepsilon_\gamma^* F_A\right)$$

Lorentz invariant form-factors

With P and q as B-meson and photon momenta

$$\mathrm{M}\left(W^+ \to B^+\gamma\right) = \hat{f}_B(\mu_F)\int_0^\infty d\omega T(\omega,\mu_F)\phi_B^+(\omega,\mu_F) + \mathrm{O}(m_b^{-1})$$

和B衰变的因子化公式一样！

B meson decay constant in HQET

$$f_B = \hat{f}_B(\mu_F)\left[1 - \frac{\alpha_s C_F}{4\pi}\left(3\ln\frac{\mu_F}{m_b} + 2\right)\right] + \mathcal{O}\left(\alpha_s^2\right).$$

E. Eichten and B. R. Hill (1990)

Hard-scattering kernel insensitive to long-distance effect, can be calculated via perturbative matching procedure

B^+ replaced by fictitious state
a free pair of $\bar{b}u$

HQET factorization: perturbative matching

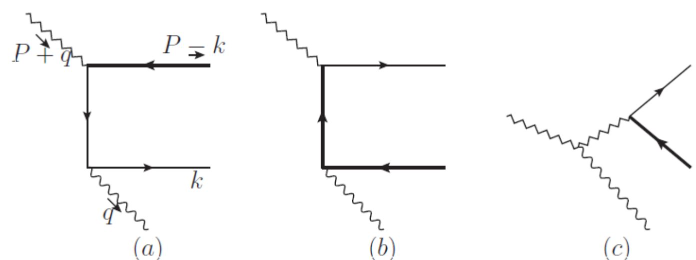
$$\mathcal{M} = \mathcal{M}^{(0)} + \mathcal{M}^{(1)} + \mathcal{O}(\alpha_s^2),$$

$$\mathcal{M}^{(0)} = \Phi^{+(0)}_{[\bar{b}u]} \otimes T^{(0)},$$
$$\mathcal{M}^{(1)} = \Phi^{+(0)}_{[\bar{b}u]} \otimes T^{(1)} + \Phi^{+(1)}_{[\bar{b}u]} \otimes T^{(0)},$$

$$\Phi^{\pm}_{[\bar{b}u]}(\omega) = \frac{1}{v^{\pm}} \int \frac{dt}{2\pi}\, e^{i\omega t} \langle [\bar{b}u](P)|\bar{u}(z)[z,0]\not{n}_{\mp}\gamma_5 h_v(0)|0\rangle\Big|_{z^+, z^\perp = 0}.$$

LO prediction in HQET factorization

$$\mathcal{M}^{(0)}(W^+ \to [\bar{b}u] + \gamma) = \frac{eV_{ub}}{2\sqrt{2}\sin\theta_W} \langle [\bar{b}u](P)\gamma(q,\varepsilon_\gamma)\, |\bar{u}\not{\varepsilon}_W(1-\gamma_5)b|\, 0\rangle$$
$$\approx \frac{e_u e^2 V_{ub}}{4\sqrt{2}\sin\theta_W q^- k^+} \mathrm{Tr}\left[\frac{1-\not{v}}{4}\gamma_5 \not{\varepsilon}^*_\gamma \not{q} \not{\varepsilon}_W (1-\gamma_5)\right]$$
$$= \frac{e_u e^2 V_{ub}}{4\sqrt{2}\sin\theta_W} \left(-i\frac{\epsilon_{\mu\nu\alpha\beta} v^\mu n_-^\nu \varepsilon_W^\alpha \varepsilon_\gamma^{*\beta}}{v^+} + \varepsilon_W \cdot \varepsilon_\gamma^*\right) \int_0^\infty \frac{d\omega}{\omega} \delta\left(k^+/v^+ - \omega\right).$$



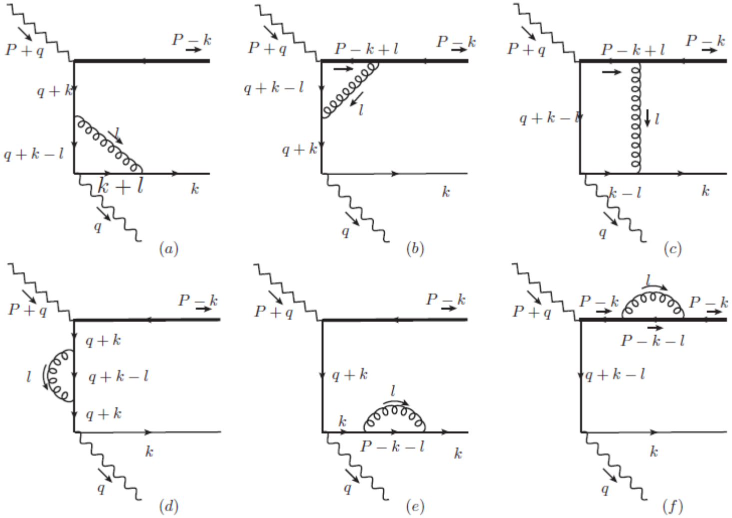
$$\Phi^{\pm(0)}_{[\bar{b}u]}(\omega) = \frac{1}{v^\pm}\delta\left(k^+/v^+ - \omega\right) \mathrm{Tr}\left[\frac{1-\not{v}}{4}\gamma_5 \not{n}_\mp \gamma_5\right] = \delta\left(k^+/v^+ - \omega\right).$$

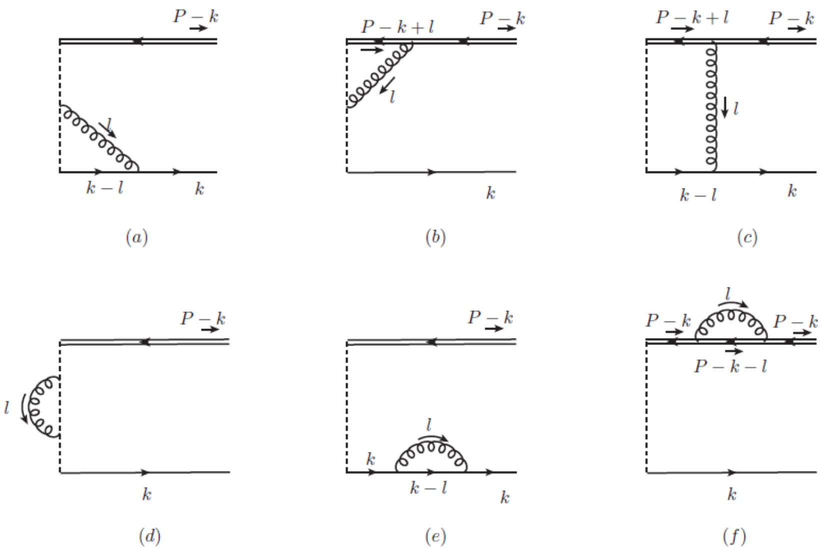
$$T^{(0)}(\omega) = \frac{e_u e^2 V_{ub}}{4\sqrt{2}\sin\theta_W} \left(-i\frac{\epsilon_{\mu\nu\alpha\beta} P^\mu q^\nu \varepsilon_W^\alpha \varepsilon_\gamma^{*\beta}}{P \cdot q} + \varepsilon_W \cdot \varepsilon_\gamma^*\right) \frac{1}{\omega}.$$

$$F_V^{(0)} = F_A^{(0)} = \hat{f}_B m_B \int_0^\infty \frac{d\omega}{\omega} \phi_B^+(\omega) = \frac{\hat{f}_B m_B}{\lambda_B}.$$

$$\lambda_B^{-1}(\mu) \equiv \int_0^\infty \frac{d\omega}{\omega} \phi_B^+(\omega, \mu).$$

HQET factorization: NLO perturbative correction





One-loop correction to QCD amplitude

One-loop correction to HQET LCDA

$$\Phi^{(0)} \otimes T^{(1)} = \mathcal{M}^{(1)} - \Phi^{(1)} \otimes T^{(0)},$$

Determine the order-alpha_s hard-scattering kernel through this equation

HQET factorization: NLO perturbative correction

Explicit calculation has verified that the hard-scattering kernel is indeed IR finite. Therefore HQET factorization holds at least to one-loop level

$$
\begin{aligned}
T^{(1)}(\omega, m_b, \mu_F) = & \frac{\alpha_s C_F}{4\pi}\left\{ \ln^2 \frac{2q^- v^+ \omega}{\mu_F^2} - 2\ln^2 \frac{m_b}{\mu_F} + \left(5 - 4\ln\frac{1-r}{r}\right)\ln\frac{m_b}{\mu_F} \right. \\
& + 2\mathrm{Li}_2(r) + \ln^2 r - \left(2\ln\frac{1-r}{r} - 3 + r\right)\ln\frac{1-r}{r} + \frac{\pi^2}{12} - 7 \\
& \left. - i\pi\left[2\ln\frac{2q^- v^+ \omega}{\mu_F^2} - 4\ln\frac{m_b}{\mu_F} - r - 4\ln(1-r) + 2\ln r + 3\right]\right\} T^{(0)}(\omega).
\end{aligned}
\tag{37}
$$

$r \equiv m_b^2/m_W^2$.

Dependence of factorization scale muF, compatible with Lange-Neubert evolution equation:

$$
\mu_F \frac{d}{d\mu_F} T^{(1)}(\omega, \mu_F) = -\frac{\alpha_s C_F}{4\pi}\left(4\ln\frac{\omega}{\mu_F} + 5\right) T^{(0)}(\omega) + \mathcal{O}\left(\alpha_s^2\right).
$$

Shortcoming of HQET factorization: large collinear logarithm inevitably appearing in fixed-order hard-scattering kernel

$$T^{(1)}(\omega, m_b, \mu_F)\Big|_{\rm expd} = \frac{\alpha_s C_F}{4\pi}\left[\ln^2\frac{\omega}{\mu_F} - \ln^2\frac{m_b}{\mu_F} + \ln\frac{m_b}{\mu_F}\left(5 + 2\ln\frac{\omega}{\mu_F}\right) + \ln\frac{m_W^2}{m_b^2}\left(3 + 2\ln\frac{\omega}{m_b}\right) + \frac{\pi^2}{12} - 7 - i\pi\left(3 + 2\ln\frac{\omega}{m_b}\right)\right] T^{(0)}(\omega) + \mathcal{O}(r). \quad (39)$$

这些大对数项原则上损害了预言的微扰收敛性。原则上可以通过ERBL方程重求和

一个有趣的解决方案是通过重因子化B介子QCD LCDA；见报告的第二部分

完整的NLO预言

$$F_V^{(1)} = F_A^{(1)} = F_{V/A}^{(0)} \int_0^\infty \frac{d\omega}{\omega} \frac{T^{(1)}(\omega)}{T^{(0)}(\omega)} \phi_B^+(\omega)$$

$$= F_{V/A}^{(0)} \frac{\alpha_s C_F}{4\pi} \Bigg\{ -\ln^2 \frac{m_b}{\mu_F} - \ln \frac{m_b}{\mu_F} \left(2\ln \frac{1-r}{r} - 2 \right) + 2\mathrm{Li}_2(r) - \ln^2(1-r)$$

$$+ 2\ln r \ln(1-r) + (3-r)\ln \frac{1-r}{r} + \frac{\pi^2}{12} - 5 - 2\sigma_{B,1} \left(\ln \frac{1-r}{r} + \ln \frac{m_b}{\mu_F} \right)$$

$$- \sigma_{B,2} + i\pi \left[2\ln \frac{m_b}{\mu_F} - 3 + r + 2\ln(1-r) + 2\sigma_{B,1} \right] \Bigg\},$$

Logarithmic inverse moments

$$\lambda_B^{-1}\sigma_{B,n}(\mu) \equiv -\int_0^\infty \frac{d\omega}{\omega} \ln^n \frac{\omega}{\mu} \phi_B^+(\omega,\mu), \qquad n = 1,2$$

Heavy-quark spin symmetry?

$$F_{V/A}^{(1)}\Big|_{\mathrm{expd}} = F_{V/A}^{(0)} \frac{\alpha_s C_F}{4\pi} \Bigg[\ln \frac{m_b^2}{m_W^2} \left(2\ln \frac{m_b}{\mu_F} + 2\sigma_{B,1} - 3 \right) - \ln^2 \frac{m_b}{\mu_F} + 2(1-\sigma_{B,1}) \ln \frac{m_b}{\mu_F}$$

$$- \sigma_{B,2} + \frac{\pi^2}{12} - 5 + i\pi \left(2\ln \frac{m_b}{\mu_F} + 2\sigma_{B,1} - 3 \right) \Bigg] + \mathcal{O}(r).$$

和Feng, Jia, Sang, 2019对于W->Bc+γ的NRQCD短程系数展开行为几乎一致

唯象预言

$$
\begin{aligned}
&\sin\theta_W = 0.481, && \alpha\,(m_W/2) = 1/130, && m_W = 80.379\ \text{GeV}, && f_B = 0.187\ \text{GeV},\\
&|V_{ub}| = 3.65\times 10^{-3}, && m_b = 4.6\ \text{GeV}, && m_B = 5.279\ \text{GeV}, && f_D = 0.249\ \text{GeV},\\
&|V_{cs}| = 0.997, && m_c = 1.4\ \text{GeV}, && m_D = 1.968\ \text{GeV}.
\end{aligned}
$$

$$
\phi_M^+(\omega) = \frac{\omega}{\lambda_M^2}\exp\left(-\frac{\omega}{\lambda_M}\right),
$$

$$
\frac{d}{d\ln\mu}\phi_B^+(\omega,\mu) = -\frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega'\left\{\left(4\ln\frac{\mu}{\omega}-2\right)\delta\left(\omega-\omega'\right) - 4\omega\left[\frac{\theta\left(\omega'-\omega\right)}{\omega'\left(\omega'-\omega\right)} + \frac{\theta\left(\omega-\omega'\right)}{\omega\left(\omega-\omega'\right)}\right]_+\right\}\phi_B^+\left(\omega',\mu\right),
$$

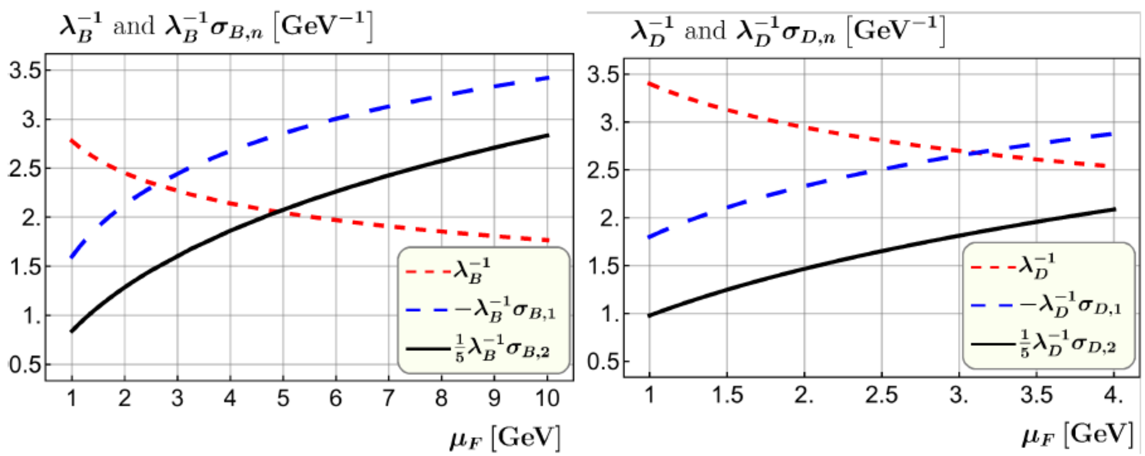


Fig. 5. Scale dependence of the first inverse moments λ_M^{-1} and the logarithmic inverse moments $\lambda_M^{-1}\sigma_{M,1/2}$ for $M = B^+, D_s^+$. The renormalization scale ranges from 1 GeV to twice meson mass.

唯象预言

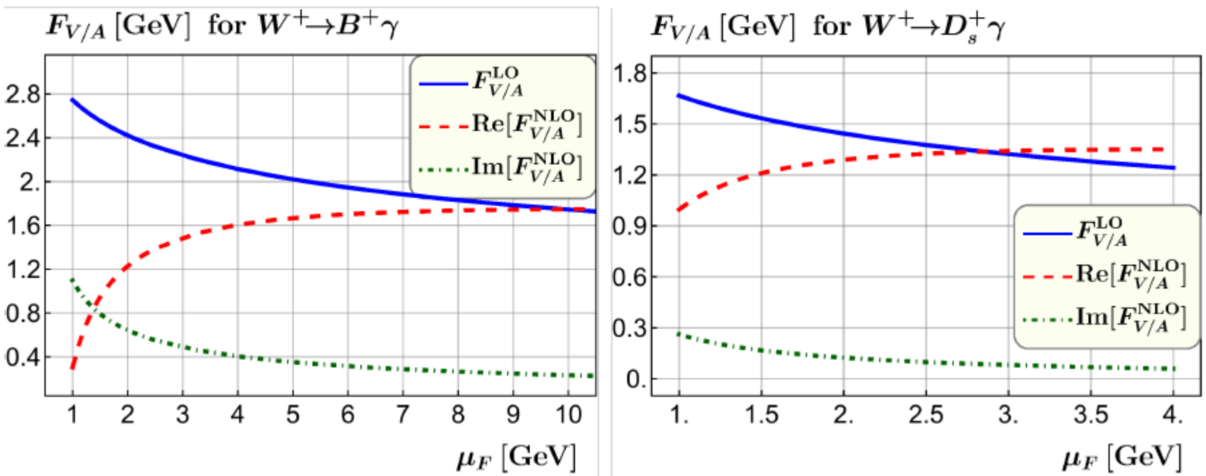


Fig. 6. Factorization scale dependence of the vector/axial-vector form factors $F_{V/A}$ at LO and NLO in α_s, for the process $W^+ \to B^+\gamma$ and $W^+ \to D_s^+ + \gamma$, respectively. The range of μ_F lies between 1 GeV to twice meson mass.

	$\Gamma^{\rm LO}$ (GeV)	$\Gamma^{\rm NLO}$ (GeV)	$\mathrm{Br}^{\rm NLO}$
$W^+ \to B^+\gamma$	$(0.75 \sim 1.9) \times 10^{-11}$	$(3.1 \sim 7.7) \times 10^{-12}$	$(1.5 \sim 3.7) \times 10^{-12}$
$W^+ \to D_s^+\gamma$	$(0.72 \sim 1.3) \times 10^{-7}$	$(4.9 \sim 8.4) \times 10^{-8}$	$(2.3 \sim 4.0) \times 10^{-8}$

唯象预言

我们的预言对比于Grossman基于light-cone因子化的预言

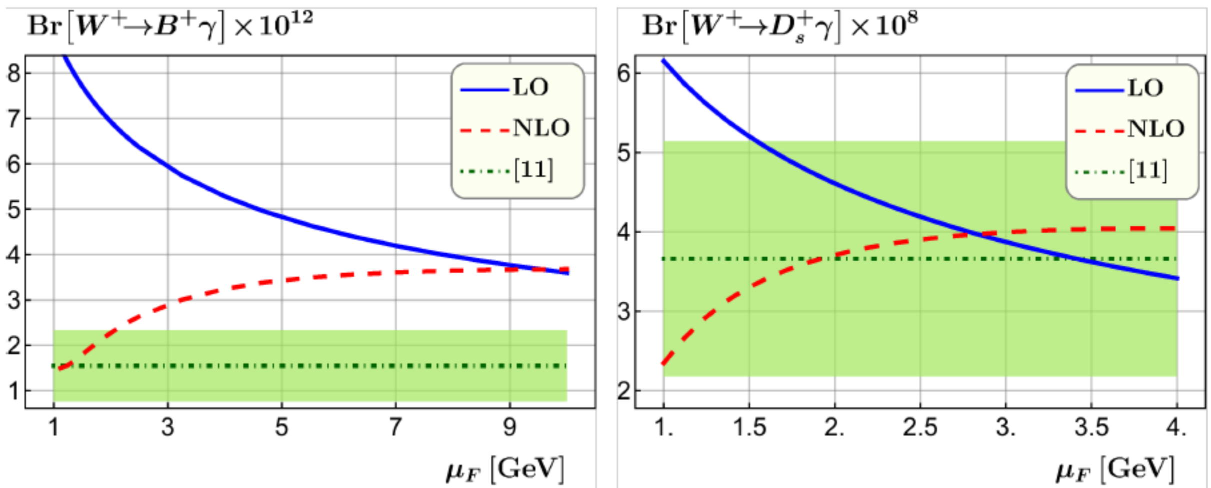


Fig. 8. Branching fractions of $W^+ \to B^+\gamma$ and $W^+ \to D_s^+\gamma$ as a function of μ_F, which ranges from 1 GeV to twice meson mass. Our predictions are juxtaposed with the existing ones obtained from the collinear factorization [11], which are represented by the green bands.

the branching fraction for $W^+ \to D_s^+\gamma$ is predicted to be within $(2.3-4.0)\times 10^{-8}$,

LHC积分亮度3000 fb^{-1}，对应10^{11}W/Z玻色子，应该可以测量到W辐射衰变到Ds

Refactorizing B meson QCD LCDA

对于重味介子遍举产生，有两种基于第一性原理的因子化方案

光锥因子化

$$\mathcal{M} = \int_0^1 dx\, T(x;\mu_Q)\, \Phi^{\rm QCD}(x;\mu_Q) + \mathcal{O}(1/Q),$$

$$\mu_Q \frac{d}{d\mu_Q} \Phi^{\rm QCD}(x;\mu_Q) = \frac{\alpha_s C_F}{\pi} \int_0^1 dy\, V_0(x,y)\, \Phi^{\rm QCD}(y;\mu_Q),$$

$$V_0(x,y) = \left[\frac{x}{y} \left(1 + \frac{1}{y-x} \right) \theta(y-x) + \left(\begin{matrix} x \to \bar{x} \\ y \to \bar{y} \end{matrix} \right) \right]_+ ,$$

$m_W >> m_b \sim \Lambda_{QCD}$

Resum $\log(m_W/m_b)$

HQET因子化

$$\mathcal{M} = \int_0^\infty d\omega\, \mathcal{T}(\omega, Q, m_b; \mu_H)\, \Phi_+^{\rm HQET}(\omega;\mu_H) + \mathcal{O}(1/m_b),$$

$$\mu_H \frac{d}{d\mu_H} \phi_+^{\rm HQET}(\omega;\mu_H) = -\frac{\alpha_s C_F}{4\pi} \int_0^\infty d\omega'\, \gamma_+(\omega,\omega';\mu_H)\, \phi_+^{\rm HQET}(\omega';\mu_H),$$

$$\gamma_+(\omega,\omega';\mu_H) = \left(\Gamma_{\rm cusp}^{(1)} \ln\frac{\mu_H}{\omega} - 2 \right) \delta(\omega-\omega') - \Gamma_{\rm cusp}^{(1)} \omega \left[\frac{\theta(\omega'-\omega)}{\omega'(\omega'-\omega)} + \frac{\theta(\omega-\omega')}{\omega(\omega-\omega')} \right]_+ ,$$

$m_W \sim m_b >> \Lambda_{QCD}$

Resum $\log(m_b/\Lambda_{QCD})$

如何将两种因子化方案有效联合，从而给出最优化预言？

$$\Phi^{\rm QCD}(x,\mu_Q) \equiv f_B\phi^{\rm QCD}(x,\mu_Q) = -i\int\frac{dz^-}{2\pi}\,e^{ixP^+z^-}\langle 0|\bar{q}(z)[z,0]\not{n}\gamma_5 b(0)|\overline{B}(P)\rangle,$$

$$\Phi_+^{\rm HQET}(\omega,\mu_H) \equiv \hat{f}_B\phi_+^{\rm HQET}(\omega,\mu_H) = \frac{-i}{m_B v^+}\int\frac{dt}{2\pi}\,e^{i\omega t}\langle 0|\bar{q}(z)[z,0]\not{n}\gamma_5 h_v(0)|\overline{B}(v)\rangle,$$

核心insight: 这两种LCDAs具有完全同样的IR行为，它们的区别仅仅在于UV 行为(k> m_b)。因此，可以通过一个因子化定理联系起来

$$\Phi^{\rm QCD}(x,\mu_Q) = \int_0^\infty d\omega\, Z(x,\omega,m_b;\mu_Q,\mu_H)\,\Phi_+^{\rm HQET}(\omega,\mu_H),$$

QCD的渐近自由性质使得短程函数Z factor可以微扰计算！

$$Z(x,\omega,m_b;\mu_Q,\mu_H) = Z^{(0)}(x,\omega,m_b) + \frac{\alpha_s C_F}{4\pi}Z^{(1)}(x,\omega,m_b;\mu_Q,\mu_H) + \mathcal{O}(\alpha_s^2).$$

Refactorizing B meson QCD LCDA

LO

$$\phi^{\text{QCD}\,(0)}(x) = \delta(x - x_0), \qquad \phi_+^{\text{HQET}\,(0)}(\omega) = \delta(\omega - m_q),$$

$$Z^{(0)}(x, \omega, m_b) = \delta\left(x - \frac{\omega}{m_b + \omega}\right).$$

NLO

$$Z^{(1)}(x, \omega, m_b; \mu_Q, \mu_H) = \phi^{\text{QCD}\,(1)}(x, \mu_Q)\Big|_{m_q \to \omega} - \frac{m_b}{(1-x)^2}\phi_+^{\text{HQET}\,(1)}\left(\frac{m_b x}{1-x}, \mu_H\right)\Big|_{m_q \to \omega} - \left(3\ln\frac{\mu_H}{m_b} + 2\right) Z^{(0)}(x, \omega, m_b). \qquad (14)$$

$$x_\omega \equiv \frac{\omega}{m_b + \omega}, \qquad \omega_x = \frac{m_b x}{1-x}.$$

$$\begin{aligned} &Z^{(1)}(x, \omega, m_b; \mu_Q, \mu_H) \qquad (15) \\ &= 2\left\{\left(\ln\frac{\mu_Q^2}{(m_b+\omega)^2 (x_\omega - x)^2} - 1\right)\left[\left(1 + \frac{1}{x_\omega - x}\right)\frac{x}{x_\omega}\theta(x_\omega - x) + \left(\begin{array}{c} x \leftrightarrow \bar{x} \\ x_\omega \leftrightarrow \bar{x}_\omega \end{array}\right)\right]\right\}_{[x]+} \\ &+ 4\left\{\frac{x(1-x)}{(x - x_\omega)^2}\right\}_{[x]++} + 2\delta'(x - x_\omega)\left(2x_\omega(1-x_\omega)\ln\frac{x_\omega}{1-x_\omega} + 2x_\omega - 1\right) \\ &- \omega_x\frac{d\omega_x}{dx}\left\{2\left[\left(\ln\left[\frac{\mu_H^2}{(\omega_x - \omega)^2}\right] - 1\right)\left(\frac{\theta(\omega - \omega_x)}{\omega(\omega - \omega_x)} + \frac{\theta(\omega_x - \omega)}{\omega_x(\omega_x - \omega)}\right)\right]_{[\omega]+} + \frac{4\theta(\omega_x - 2\omega)}{(\omega_x - \omega)^2}\right. \\ &\left. + 4\left[\frac{\theta(2\omega - \omega_x)}{(\omega_x - \omega)^2}\right]_{[\omega]++} - \frac{\delta(\omega_x - \omega)}{\omega}\left(\frac{1}{2}\ln^2\frac{\mu_H^2}{\omega^2} - \ln\frac{\mu_H^2}{\omega^2} + \frac{3\pi^2}{4} + 2\right)\right\} - \left(3\ln\frac{\mu_H}{m_b} + 2\right)\delta(x - x_\omega), \end{aligned}$$

Bell , Feldmann, JHEP 2008已经完成了所有必备的单圈计算，我们仅需要做组装！

Z函数满足both ERBL方程and LN方程!

重求和大对数项变得容易

$$\mathcal{M} = \int_0^\infty d\omega\, \mathcal{T}^{\rm expd}(\omega, Q/m_b; \mu_H)\, \Phi_+^{\rm HQET}(\omega, \mu_H) + \mathcal{O}(m_b/Q, 1/m_b),$$

$$\mathcal{T}^{\rm expd}(\omega, Q/m_b; \mu_H) = \int_0^1 dx\, T(x, \mu_Q)\, Z(x, \omega, m_b; \mu_Q, \mu_H).$$

$$\mathcal{T}_{\rm LL}^{\rm expd}(\omega, Q/m_b; \mu_H) = \int_0^1 dx\, T(x, Q)\, Z(x, \omega, m_b; Q, \mu_H)$$

$$= \int_0^1 dx\, T^{(0)}(x)\, \exp\left[\kappa C_F V_0 *\right] Z^{(0)}(x, \omega, m_b),$$

$$\kappa \equiv \frac{2}{\beta_0} \ln \frac{\alpha_s(m_b^2)}{\alpha_s(Q^2)} \approx \frac{\alpha_s(Q^2)}{2\pi} \ln \frac{Q^2}{m_b^2} + \beta_0^2 \frac{\alpha_s^2(Q^2)}{(4\pi)^2} \ln \frac{Q^2}{m_b^2} + \cdots,$$

Following the method outlined in Jia and Yang, NPB 2009

$$\mathcal{T}_{\rm LL}^{\rm expd}(\omega, Q/m_b; \mu_H) = \int_0^1 dx\, T^{(0)}(x) Z^{(0)}(x, \omega, m_b)$$
$$+ \kappa C_F \int_0^1 dx \int_0^1 dy\, T^{(0)}(x) V_0(x, y) Z^{(0)}(y, \omega, m_b)$$
$$+ \frac{\kappa^2 C_F^2}{2} \int_0^1 dx \int_0^1 dy \int_0^1 dz\, T^{(0)}(x) V_0(x, y) V_0(y, z) Z^{(0)}(z, \omega, m_b) + \cdots.$$

$$\mathcal{T}_{\rm LL}^{\rm expd}(\omega) \propto \frac{m_b}{\omega} \left\{ 1 + \frac{\alpha_s C_F}{4\pi} \ln \frac{Q^2}{m_b^2} \left(3 + 2 \ln \frac{\omega}{m_b}\right) + \left(\frac{\alpha_s C_F}{4\pi}\right)^2 \ln^2 \frac{Q^2}{m_b^2} \left[\frac{C_F}{2}\left(3 + 2\ln\frac{\omega}{m_b}\right)^2 + \frac{\beta_0}{2}\left(3 + 2\ln\frac{\omega}{m_b}\right)\right] + \cdots \right\}.$$

和之前我们HQET因子化下NLO计算结果一样！

Heartening remark from one of referees

HQET appeared in 1990; the first paper contained the (1-loop) matching formulas for the local heavy-light quark currents in QCD and HQET. The B-meson HQET distribution amplitude was introduced in 1997; it is the Fourier transform of the matrix element of the bilocal heavy-light current (with a light-like splitting). Now, after this article has appeared in arXiv, the idea to generalize the matching calculation from local currents to (Fourier transforms of) bilocal ones seems very natural.

My first thought after I saw this article in arXiv was: why haven't I thought about this idea long ago? But nobody has done so from 1997 to 2019.

Heartening remark from one of referees

The main result of this paper (15) is not only nice (it satisfies both the ERBL evolution equation in x and the Lange-Neubert evolution equation in \omega) but also useful.

The authors obtained their main result without difficult calculations: the necessary calculations have already been done 10 years ago in [27]. However, the idea to combine these results into the matching formula (9) is entirely new. The paper is well written. In fact, more than a half of the text is a long and highly understandable introduction, which reviews the current status and naturally leads to the idea of matching the QCD and HQET distribution amplitudes.

总结

- 将重夸克重组合机制推广到NLO in α_s
- 发展了重味介子遍举产生的新理论框架 — HQET因子化
  单圈显示计算证明此因子化成立
- 发现了重味介子光锥分布振幅(LCDA)的新的因子化定理
- 提供了一个最优化的描述重味介子遍举产生的理论框架

Thanks for your attention!