Two-photon processes $\gamma\gamma \to \pi^+\pi^-,\ K^+K^-$ in k_T factorization

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OUTLINE

THE EXPERIMENTAL SIDE

TPC/Two Gamma [1]: 1.3 GeV $< Q <$ 3.5 GeV

ALEPH [2]: 2.0 GeV $< Q <$ 6.0 GeV

BELLE [3]: 2.4 GeV $< Q <$ 4.1 GeV

H. Aihara *et al.* (TPC/Two Gamma)
Pion and Kaon Pair Production in Photon-Photon Collisions
Phys. Rev. Lett. 57, 404 (1986)

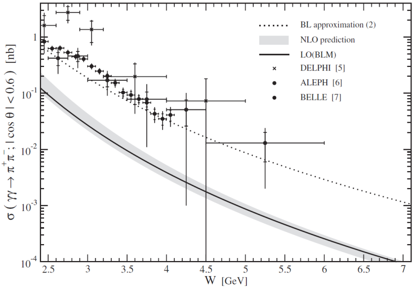
A. Heister *et al.* (ALEPH)
Exclusive production of pion and kaon meson pairs in two photon collisions at LEP
Phys. Lett. B569, 140 (2003)

H. Nakazawa, *et al.* (BELLE)
Measurement of the $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to K^+K^-$ processes at energies of 2.4 $-$ 4.1 GeV
Phys. Lett. B615, 39 (2005)

High precision measurements at BELLE-II...

COLLINEAR FACTORIZATION

At twist-2 level



Solid line: LO [1]

Grey band: NLO [2]

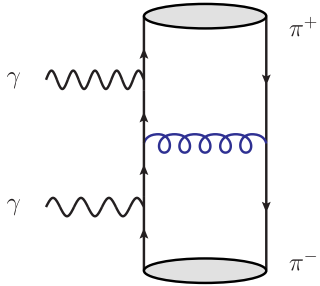
Dotted line:
BL approximation [1]

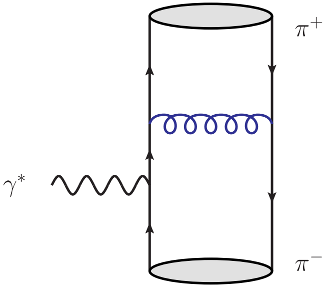
[1] S. J. Brodsky and G. P. Lepage
Large Angle Two Photon Exclusive Channels in Quantum Chromodynamics
Phys. Rev. D24, 1808 (1981)

[2] G. Duplančić and B. Nižić
NLO perturbative QCD predictions for $\gamma\gamma \to M^+M^-$ $(M = \pi,\ K)$.
Phys. Rev. Lett. 97, 142003 (2006)

BL APPROXIMATION

$\gamma\gamma \to \pi^+\pi^-$ $\gamma^* \to \pi^+\pi^-$





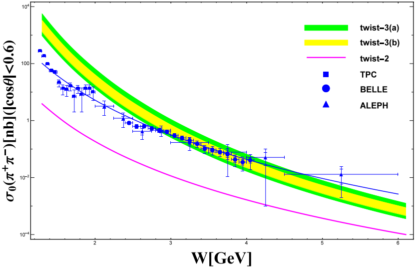
At twist-2 level:

$$\frac{\mathrm{d}\sigma(\gamma\gamma \to M^+M^-)}{\mathrm{d}|\cos\theta|} \approx \frac{8\pi\alpha^2}{Q^2}\frac{|F_M(Q^2)|^2}{\sin^4\theta}$$

with $F_M(Q^2) \approx 0.4\ \mathrm{GeV}^2/Q^2$. *a larger FF suggested by CLEO*

COLLINEAR FACTORIZATION

At twist-3 level



BHL prescription:

Magenta line: twist-2 result

The twist-2 result [1] is almost an order of magnitude smaller than the experiment data.

Bands: twist-3 result

The twist-3 results [1, 2] are several times greater than the experiment data.

[1] C. Wang, M.-Z. zhou, and H. Chen
$\gamma\gamma \to M^+M^-$ $(M = \pi, K)$ processes with twist-3 corrections in QCD.
Eur. Phys. J. C77, 219 (2017)

[2] A. S. Gorsky
The power correction to the asymptotics of the cross section for the process $\gamma\gamma \to \pi^+\pi^-$ in QCD
Sov. J. Nucl. Phys. 50, 708 (1989)

k_T FACTORIZATION: TWIST-2 LEVEL

The twist-2 result is much smaller than the experiment data [1]
$\longrightarrow$ The Handbag model [2]

The differential cross sections [1, 3]: QCD sum rule vs pQCD
$\longrightarrow$ The Transition of nonperturbative to perturbative QCD occurs at $Q \approx 2$ GeV and $\theta \approx 40^\circ$

[1] C. Vogt
Gamma gamma to π^+, π^- in the modified perturbative approach
arXiv:hep-ph/9911253

[2] M. Diehl, P. Kroll, and C. Vogt
The Handbag contribution to $\gamma\gamma \to \pi^+\pi^-, K^+K^-$
Phys. Lett. B532, 99 (2002)

[3] C. Corianò and H.-N. Li
The Transition to perturbative QCD in Compton scattering
Nucl. Phys. B434, 535 (1995)

[4] R.-C. Hsieh and H.-n. Li
Transition to perturbative QCD in two photon collisions
Phys. Rev. D70, 056002 (2004)

k_T FACTORIZATION: TWIST-3 LEVEL (1905.07008)

Helicity amplitude

$$\mathcal{M}_{\lambda_1\lambda_2}(Q,\theta) = \int_0^1 \mathrm{d}x\mathrm{d}y \int \frac{\mathrm{d}^2\mathbf{b}}{(4\pi)^2} S_t(x)S_t(y)\exp[-S(x,y,Q,b,t)] \sum_{i,j=\pi,p,\sigma,\sigma'} \hat{\Psi}^i_\pi(x,\mathbf{b},\frac{1}{b})\hat{T}^{ij}_{\lambda_1\lambda_2}(x,y,Q,\theta,\mathbf{b},t)\hat{\Psi}^j_\pi(y,\mathbf{b},\frac{1}{b})$$

with

$$\hat{\Psi}^{\sigma'}_\pi(x,\mathbf{b}_1,\mu_F) = \frac{\partial\hat{\Psi}^\sigma_\pi(x,\mathbf{b}_1,\mu_F)}{\partial x}, \qquad \hat{\Psi}^{\sigma'}_\pi(y,\mathbf{b}_2,\mu_F) = \frac{\partial\hat{\Psi}^\sigma_\pi(y,\mathbf{b}_2,\mu_F)}{\partial y}.$$

The nonperturbative contributions from the end-point region are suppressed effectively by

$$S_t(x) \propto [x(1-x)]^c, \qquad \exp[-S] \propto \exp\left[-const. \times \ln(xQ/\Lambda)\ln\frac{\ln(xQ/\Lambda)}{\ln 1/(b\Lambda)}\right].$$

SUPPRESSION FACTORS

Leading order contributions : twist-2×twist-2

Two parton twist-3 contributions : Suppressed by μ_π^2/Q^2

Three parton twist-3 contributions : Suppressed by $f_{3\pi}/(f_\pi Q)$

Two parton twist-4 contributions : Suppressed by m_π^2/Q^2

Soft contributions : suppressed by M^2/Q^2 $(M \sim 1\ \text{GeV})$.

In a few GeV energy region:

$$\frac{\mu_\pi^2}{Q^2} \sim \mathcal{O}(1), \qquad \frac{f_{3\pi}}{f_\pi Q} \sim 10^{-2}, \qquad \frac{m_\pi^2}{Q^2} \sim 10^{-2}$$

[1] A. V. Radyushkin
Hadronic form-factors: Perturbative QCD versus QCD sum rules
Nucl. Phys. A532, 141 (1991)

LIGHT-CONE WAVE FUNCTION

$$\Psi(x, \mathbf{k}_\perp, \mu) = \phi(x, \mu)\Sigma(x, \mathbf{k}_\perp).$$

Twist-2 DAs:

$$\phi_\pi^\pi(x, \mu) = 6x(1-x)\left[1 + a_2^\pi(\mu^2)C_2^{3/2}(2x-1) + a_4^\pi(\mu^2)C_4^{3/2}(2x-1)\right],$$

$$\phi_K^K(x, \mu) = 6x(1-x)\left[1 + a_1^K(\mu^2)C_1^{3/2}(2x-1) + a_2^K(\mu^2)C_2^{3/2}(2x-1)\right],$$

The $k_\perp$-dependence function:

$$\Sigma(x, \mathbf{k}_\perp) = \frac{16\pi^2\beta^2}{x(1-x)}\exp[-\frac{\beta^2\mathbf{k}_\perp^2}{x(1-x)}].$$

Main uncertainties in our calculations come from:

- ⇓ Gegenbauer momentums
- ⇓ Chiral enhancement parameters

GEGENBAUER MOMENTS

QCD sum rule:

Table: Gegenbauer moments of the DAs at the scale $\mu_0 = 1\,\text{GeV}$

π	[1, 2]	K	[1]
a_2^π	0.17 ± 0.08	a_1^K	0.10 ± 0.04
a_4^π	0.06 ± 0.10	a_2^K	0.25 ± 0.15

[1] A. Khodjamirian, C. Klein, T. Mannel, and N. Offen
Semileptonic charm decays $D \to \pi l \nu_l$ and $D \to K l \nu_l$ from QCD Light-Cone Sum Rules
Phys. Rev. D80, 114005 (2009)

[2] A. Khodjamirian, T. Mannel, N. Offen, and Y. M. Wang
$B \to \pi \ell \nu_l$ Width and $|V_{ub}|$ from QCD Light-Cone Sum Rules
Phys. Rev. D83, 094031 (2011)

$\longrightarrow$ Five sample models of the DAs

Five sample models of the DAs

Table: Gegenbauer moments of five sample models of the DAs

	$\gamma\gamma \to \pi^+\pi^-$		$\gamma\gamma \to K^+K^-$	
	a_2^π	a_4^π	a_1^K	a_2^K
Model I	0.17	0.06	0.10	0.25
Model II	0.09	−0.04	0.06	0.10
Model III	0.25	0.16	0.14	0.40
Model IV	0.09	0.16	0.06	0.40
Model V	0.25	−0.04	0.14	0.10

Gegenbauer moments are chosen in the region, which can cover the ones given in the recent lattice determinations [1]:

$$a_2^\pi(2\ \mathrm{GeV}) = 0.099^{+24}_{-24}, \qquad a_1^K(2\ \mathrm{GeV}) = 0.0533^{+34}_{-35}, \qquad a_2^K(2\ \mathrm{GeV}) = 0.089^{+19}_{-20}$$

[1] G. S. Bali, *et al.*
Light-cone distribution amplitudes of pseudoscalar mesons from lattice QCD
arXiv:1903.08038 [hep-lat].

CHIRAL ENHANCEMENT PARAMETERS

$$\longrightarrow \quad \mu_{\pi(K)} = \frac{m^2_{\pi(K)}}{m_u + m_{d(s)}}$$

Table: Nonperturbative parameters of twist-3 DAs

π	$\mu_0 = 2\,\mathrm{GeV}$	K	$\mu_0 = 2\,\mathrm{GeV}$	units/Refs.
μ_π	2.50 ± 0.30	μ_K	2.49 ± 0.26	GeV, [1]
$f_{3\pi}$	0.0031	f_{3K}	0.0033	GeV^2, [2]
$\omega_{3\pi}$	-1.1	ω_{3K}	-0.9	[2]
$\lambda_{3\pi}$	0	λ_{3K}	1.45	[2]

In a few GeV energy region, $\frac{\mu_M}{Q} \sim \mathcal{O}(1)$ and $\frac{f_{3M}}{f_M Q} \sim 10^{-2}$

[1] A. Khodjamirian and A. V. Rusov
$B_s \to K\ell\nu_\ell$ and $B_{(s)} \to \pi(K)\ell^+\ell^-$ decays at large recoil and CKM matrix elements
JHEP 08, 112 (2017)

[1] P. Ball, V. M. Braun, and A. Lenz
Higher-twist distribution amplitudes of the K meson in QCD
JHEP 05, 004 (2006)

CROSS SECTIONS

The differential cross sections: $M = \pi,\ K$

$$\frac{d\sigma(\gamma\gamma \to M^+M^-)}{d|\cos\theta|} = \frac{1}{32\pi Q^2}\frac{1}{4}\sum_{\lambda_1,\lambda_2=\pm1} |\ \mathcal{M}_{\lambda_1\lambda_2}\ |^2 .$$

The cross sections $\sigma_0(\pi^+\pi^-)$ and $\sigma_0(K^+K^-)$ for $|\cos\theta| < 0.6$

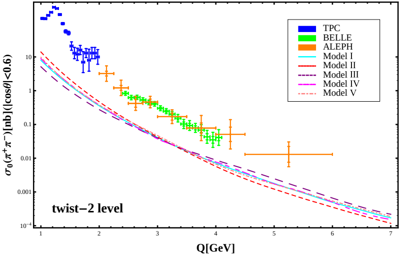
- ⋆ Gegenbauer momentums
- ⋆ Chiral enhancement parameters

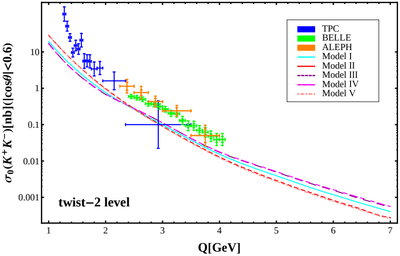
The normalized differential cross sections $\sigma_0^{-1} d\sigma/d|\cos\theta|$

- ⋆ Uncertainties from input parameters cancel to a large extent

TWIST-2 RESULTS IN k_T FACTORIZATION

With uncertainties from Gegenbauer momentums





Parametrization:

$$\sigma_0(M^+M^-) \propto Q^{-n_M}$$

Twist-2 result:

n_π depend on $a_2^\pi + a_4^\pi$

n_K depend only on a_2^K

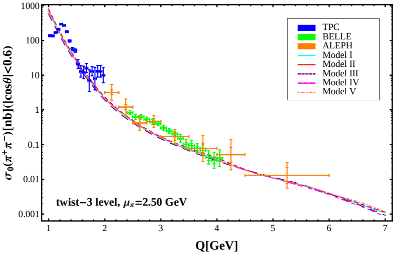
$$n_\pi = 5.2 \sim 6.9$$
$$n_K = 6.4 \sim 6.7$$

BELLE result:

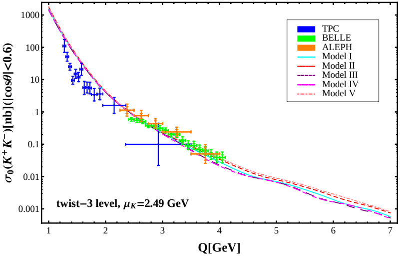
$$n_\pi = 7.9 \pm 0.4 \pm 1.5$$
$$n_K = 7.3 \pm 0.3 \pm 1.5$$

Twist-3 results in k_T factorization

With uncertainties from Gegenbauer momentums



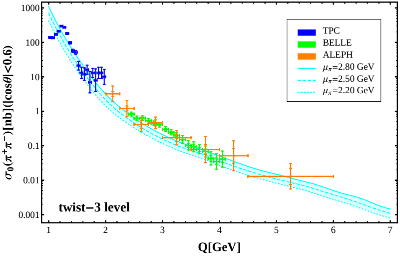
$$\mu_\pi(2\text{ GeV}) = 2.80\text{ GeV}$$

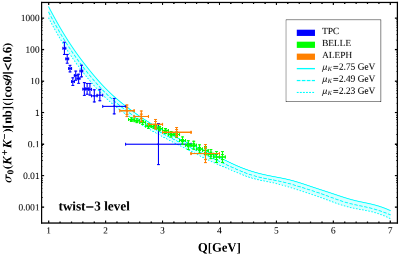


$$\mu_K(2\text{ GeV}) = 2.75\text{ GeV}$$

TWIST-3 RESULTS IN k_T FACTORIZATION

With uncertainties from chiral enhancement parameters





Gegenbauer momentums:

the model I of DAs

Chiral enhancement parameters:

$$\mu_\pi(2\ \text{GeV}) = 2.50 \pm 0.30\ \text{GeV}$$

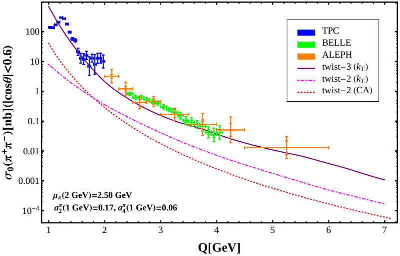
$$\mu_K(2\ \text{GeV}) = 2.49 \pm 0.26\ \text{GeV}$$

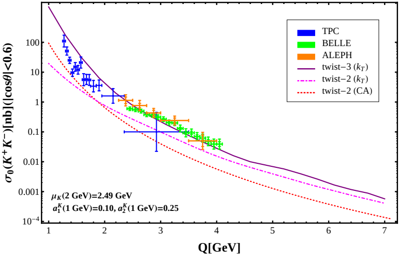
Twist-3 results:

agree with the BELLE and ALEPH measurements [2, 3].

TWIST-2 VS TWIST-3

Comparison between the twist-2 and twist-3 results



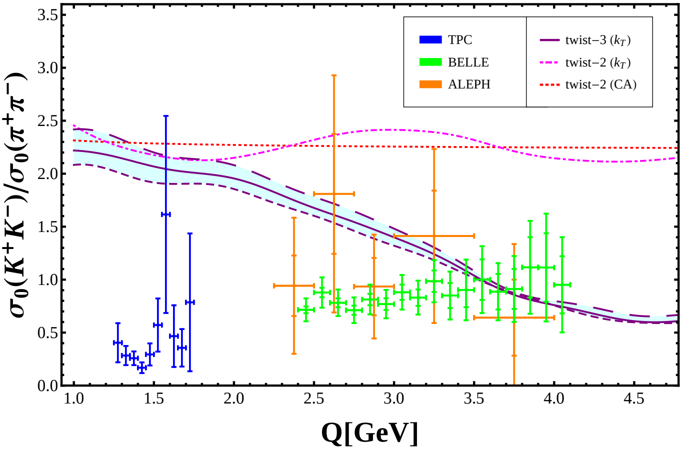


⋆ The corrections from transverse momentum play a significant role in the intermediate energy region.

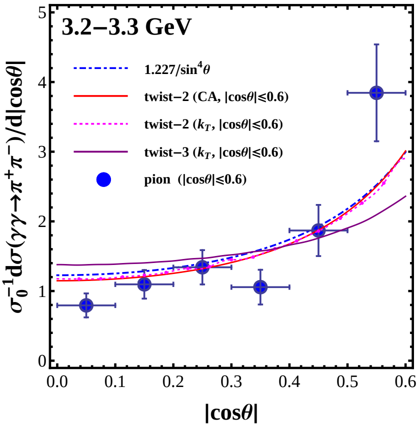
⋆ In a few GeV region, the twist-3 corrections dominate in the two-photon process.

CROSS SECTION RATIO: $\frac{\sigma_0(K^+K^-)}{\sigma_0(\pi^+\pi^-)}$

Twist-2 results: $\frac{\sigma_0(K^+K^-)}{\sigma_0(\pi^+\pi^-)} \approx \left(\frac{f_K}{f_\pi}\right)^4$



THE DIFFERENTIAL CROSS SECTIONS: $\frac{1}{\sigma_0}\frac{d\sigma(\gamma\gamma\to\pi^+\pi^-)}{d|\cos\theta|}$



$\frac{1}{\sigma_0}\frac{d\sigma(\gamma\gamma\to\pi^+\pi^-)}{d|\cos\theta|}$

insensitive to the input parameters

The blue line: BLA

corresponding to $\frac{1.227}{\sin^4\theta}$

SUMMARY

- At twist-2 level, our results of the cross sections are much smaller than the experimental data.
- The cross sections are found to be improved significantly by the corrections from $\mathbf{k}_\perp$ and the two-parton twist-3 effects in the intermediate energy region $1.0\ \mathrm{GeV} < Q < 7.0\ \mathrm{GeV}$.
- At twist-3 level, the cross sections $\sigma_0(\pi^+\pi^-)$ and $\sigma_0(K^+K^-)$ agree with AlEPH and BELLE measurements, but disagree with the experimental measurements in the relatively low energy region $1.0\ \mathrm{GeV} < Q < 2.0\ \mathrm{GeV}$.
- The predicted pion and kaon angular distributions are still different from the BELLE measurements.

Thanks for your attention!

CONCLUSION

$$\textit{nonperturbative QCD} \longrightarrow \textit{perturbative QCD}$$

The transition from nonperturbative to perturbative QCD in the large angle scattering processes $\gamma\gamma \to \pi^+\pi^-, K^+K^-$ maybe at $Q \approx 2$ GeV and $\theta \approx 40^\circ$ [1, 2, 3]

[1] C. Corianò and H.-N. Li
The Transition to perturbative QCD in Compton scattering
Nucl. Phys. B434, 535 (1995)

[2] C. Corianò, H.-n. Li, and C. Savkli
Exclusive processes at intermediate-energy, quark-hadron duality and the transition to perturbative QCD
JHEP 07, 008 (1998)

[3] R.-C. Hsieh and H.-n. Li
Transition to perturbative QCD in two photon collisions
Phys. Rev. D70, 056002 (2004)