Precision QCD Calculations for Heavy Quark Decays

Yu-Ming Wang

Nankai University

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Why precision calculations?

- Understanding the general properties of power expansion in EFTs (HQET, SCET, NRQCD).

- Interesting to understand the strong interaction dynamics of heavy quark decays.
  - Factorization properties of the subleading-power amplitudes.
  - Renormalization and asymptotic properties of the higher-twist B-meson DAs.
  - Interplay of different QCD techniques.

- Precision determinations of the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$.
  Power corrections, QED corrections, BSM physics.

- Crucial to understand the CP violation in B-meson decays.
  Strong phase of $\mathscr{A}(B \to M_1 M_2)$ @ m_b scale in the leading power.

- Indispensable for understanding the flavour puzzles (see EPS-HEP 2019 for updates).
  - P_5' and $R_{K^{(*)}}$ anomalies in $B \to K^{(*)} \ell^+ \ell^-$.
  - $R_{D^{(*)}}$ anomalies in $B \to D^{(*)} \ell \bar{\nu}_\ell$.
  - Color suppressed hadronic B-meson decays.
  - Polarization fractions of penguin dominated $B_{(s)} \to VV$ decays.

Theory tools for precision flavor physics

New Physics: $\mathscr{L}_{NP}$

$\downarrow$

EW scale (m_W): $\mathscr{L}_{SM} + \mathscr{L}_{D>4}$

$\downarrow$

Heavy-quark scale (m_b): $\mathscr{L}_{eff} = -\frac{G_F}{\sqrt{2}} \sum_i C_i Q_i + \mathscr{L}_{eff,D>6}$

$\downarrow$

QCD scale ($\Lambda_{\rm QCD}$)

- Aim: $\langle f | Q_i | \bar{B} \rangle = ?$
- QCD factorization [Diagrammatic approach].
- SCET factorization [Operator formalism].
- TMD factorization.
- (Light-cone) QCD sum rules.
- Lattice QCD.

- Key concepts: Factorization, Resummation, Evolution.

Factorization in Classical Physics

- Galileo's Leaning Tower of Pisa Experiment:

$$\mathscr{L}_{\rm eff} = \frac{1}{2} m \dot{h}^2 - m g h \,.$$

Symmetry of the effective Lagrangian: $h \to h + a$.
Dynamical interpretation: The force acting on the ball, $F = m g$, independent of h.

- Newton's Gravity Theory:

$$V_{\rm full}(h) = G \frac{M m}{r} = G \frac{M m}{R+h} \,.$$

  - Power expansion of the full potential energy:

$$V_{\rm eff}(h) = C_1(R) \, m \, (h/R) + C_2(R) \, m \, (h/R)^2 + \ldots \,.$$

    The general form of the effective potential can be written without knowing $V_{\rm full}$.
  - Matching the full theory and the effective theory:

$$C_1(R) = -C_2(R) = \frac{G M}{R} \,, \qquad V_{\rm eff}(h) = m g h - \frac{m g}{R} h^2 + \ldots \,.$$

- Symmetry of the effective Lagrangian broken in the full theory.

$$g(r) = \frac{G M}{r^2} \,, \qquad r \frac{\partial}{\partial r} g(r) = \gamma_g \, g(r) \,.$$

This differential equation is actually a renormalization group equation.

Factorization in Quantum Physics

- Wigner-Eckart theorem:

$$\langle \tau' j' m' | T^k_q | \tau j m \rangle = \langle j k j' | m q m' \rangle \, \langle \tau' j' || T^k || \tau j \rangle \, .$$

Separation of geometry and dynamics.

- Generalized Wigner-Eckart theorem in Lie algebra.
An example from SU(3): u, v and W are all 8s.

$$\langle u | W | v \rangle = \lambda_1 \, \mathrm{Tr}[\bar{u} \, W \, v] + \lambda_2 \, \mathrm{Tr}[\bar{u} \, v \, W] \, .$$

Notice that $8^3 = 512$ matrix elements expressed in terms of only two parameters.

- Factorization for strong interaction physics.
An example from $B \to \gamma \ell \nu_\ell$:

$$F_{V,\mathrm{LP}}(n \cdot p) = \frac{Q_u \, m_B}{n \cdot p} \tilde{f}_B(\mu) \, C_\perp(n \cdot p, \mu) \int_0^\infty d\omega \, \frac{\phi_B^+(\omega, \mu)}{\omega} \, J_\perp(n \cdot p, \omega, \mu) \, .$$

  - Separation of hard, hard-collinear and soft fluctuations.
  - Key input: B-meson light-cone distribution amplitude $\phi_B^+(\omega, \mu)$.

Story I: *B*-meson distribution amplitudes

- The light-ray HQET matrix element [Grozin, Neubert, 1997]:

$$\langle 0|\bar{q}_\beta(z)\,[z,0]\,h_{v\alpha}(0)|\bar{B}(v)\rangle = -\frac{i\tilde{f}_B m_B}{4}\left[\frac{1+\not{v}}{2}\left\{2\,\tilde{\phi}_B^+(t,\mu) + \frac{\tilde{\phi}_B^-(t,\mu)-\tilde{\phi}_B^+(t,\mu)}{t}\,\not{z}\right\}\gamma_5\right]_{\alpha\beta}.$$

- Evolution equation at one loop [Lange, Neubert, 2003]:

$$\frac{d\phi_B^+(\omega,\mu)}{d\ln\mu} = -\left[\Gamma_{\rm cusp}(\alpha_s)\,\ln\frac{\mu}{\omega}+\gamma_+(\alpha_s)\right]\phi_B^+(\omega,\mu) - \omega\int_0^\infty d\eta\,\Gamma_+(\omega,\eta,\alpha_s)\,\phi_B^+(\eta,\mu)\,.$$

This is an integral-differential equation!

- (Relatively) complicated solution [Lee, Neubert, 2005]:

$$\begin{aligned}\phi_B^+(\omega,\mu) &= e^{V-2\gamma_E g}\,\frac{\Gamma(2-g)}{\Gamma(g)}\int_0^\infty\frac{d\eta}{\eta}\,\phi_B^+(\eta,\mu_0)\left(\frac{\max(\omega,\eta)}{\mu_0}\right)^g \\ &\quad\times\frac{\min(\omega,\eta)}{\max(\omega,\eta)}\,{}_2F_1\left(1-g,2-g,2,\frac{\min(\omega,\eta)}{\max(\omega,\eta)}\right), \\ g(\mu,\mu_0) &= \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} d\alpha\,\frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)} \approx \frac{2C_F}{\beta_0}\,\ln\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\,.\end{aligned}$$

Making the QCD resummation complicated!

Story I: B-meson distribution amplitudes

- Renormalization of $[\bar{q}_s(t\bar{n})\,\Gamma\, b_v(0)]$ does not commute with the short-distance expansion [Braun, Ivanov, Korchemsky, 2004].

$$[(\bar{q}_s\, Y_s)(t\,\bar{n})\not{\bar{n}}\,\Gamma\,(Y_s^{\dagger}\, b_v)(0)]_R \neq \sum_{p=0} \frac{t^p}{p!}\,\Big[\bar{q}_s(0)\,(n\cdot \overleftarrow{D})^p\, \not{\bar{n}}\,\Gamma\, b_v(0)\Big]_R\,.$$

  Many other examples in QCD physics (e.g., Light-cone projection and renormalization)!

- Integral transformation [Bell, Feldmann, Wang and Yip, 2013]:

$$\begin{aligned}
\phi_B^+(\omega,\mu) &= \int_0^\infty \frac{d\omega'}{\omega'}\,\sqrt{\frac{\omega}{\omega'}}\,J_1\left(2\sqrt{\frac{\omega}{\omega'}}\right)\rho_B^+(\omega',\mu)\,,\\
\rho_B^+(\omega',\mu) &= \int_0^\infty \frac{d\omega}{\omega}\,\sqrt{\frac{\omega}{\omega'}}\,J_1\left(2\sqrt{\frac{\omega}{\omega'}}\right)\phi_B^+(\omega,\mu)\,.
\end{aligned}$$

  Eigenfunction of the Lange-Neubert kernel at one-loop is the Bessel function!

- Linear differential equation:

$$\frac{d\rho_B^+(\omega',\mu)}{d\ln\mu} = -\left[\Gamma_{\rm cusp}(\alpha_s)\,\ln\frac{\mu}{\hat{\omega}'} + \gamma_+(\alpha_s)\right]\rho_B^+(\omega',\mu)\,.$$

  Local evolution in the dual space!

Story I: B-meson distribution amplitudes

- Solution to the RGE in dual space [Bell, Feldmann, Wang, Yip, 2013]:

$$\rho_B^+(\omega', \mu) \quad = e^V \left(\frac{\mu_0}{\hat{\omega}'}\right)^{-g} \rho_B^+(\omega', \mu_0) \,.$$

Very compact expression in a full analytical form!

- Solution to the RGE in momentum space:

$$\phi_B^+(\omega, \mu) = e^V \int_0^\infty \frac{d\omega'}{\omega'} \sqrt{\frac{\omega}{\omega'}} J_1\left(2\sqrt{\frac{\omega}{\omega'}}\right) \left(\frac{\mu_0}{\hat{\omega}'}\right)^{-g} \rho_B^+(\omega', \mu_0) \,.$$

Still a beautiful expression!

- Key difference between the Lange-Neubert and the Brodsy-Lepage evolutions:
  Continuous spectrum for the former, while discrete spectrum for the latter!
  - Implication I: Asymptotic behaviour indicates the ill-defined positive moments.
  - Implication II: All the logarithmic-inverse moments needed conceptually due to the non-trivial mixing under renormalization.
  - For mathematicians: Spectrum for the Hamiltonian system with given boundary conditions.

Story I: B-meson distribution amplitudes

- Collinear conformal symmetry for the Lange-Neubert kernel [Braun, Manashov, 2014]:

$$\left(\frac{d}{d\ln\mu} + \mathscr{H}_{\rm LN}\right) O_+(z,\mu) = 0\,.$$

$\mathscr{H}_{\rm LN}$ is almost determined by the commutation relations completely [Knoedlseder, Offen, 2011]

$$[S_+, \mathscr{H}_{\rm LN}] = 0, \qquad [S_0, \mathscr{H}_{\rm LN}] = 1\,.$$

The beautiful solution in terms of S_+:

$$\mathscr{H}_{\rm LN} = \ln(i\mu S^+) - \psi(1) - \frac{5}{4}\,.$$

- Generators of the collinear conformal group:

$$S_+ = z^2\partial_z + 2jz\,, \qquad S_0 = z\partial_z + j\,, \qquad S_- = -\partial_z\,.$$

Eigenfunctions of S_+ [Braun, Manashov, 2014]:

$$\begin{aligned} iS_+ Q_s(z) &= s\,Q_s(z)\,, \qquad Q_s(z) = -\frac{1}{z^2}\,e^{is/z}\,. \\ \langle e^{-i\omega z}|Q_s(z)\rangle &= \frac{1}{\sqrt{\omega s}}\,J_1(2\sqrt{\omega s})\,. \end{aligned}$$

Wide applications of the conformal symmetry in higher energy physics!

Story I: B-meson distribution amplitudes

- A 16-year dream: What is the two-loop Lange-Neubert kernel? [Braun, Ji, Manashov, 2019]

$$\mathscr{H}_{\rm LN}\, O_+(z,\mu) = \Gamma_{\rm cusp}(a) \bigg\{ \ln(i\tilde{\mu} z)\, O_+(z,\mu) + \int_0^1 du\, \frac{\bar{u}}{u}\, [1 + a\, h(u)] \times [O_+(z,\mu) - O_+(\bar{u} z,\mu)] \bigg\} + \gamma_+(a)\, O_+(z,\mu)\,.$$

The resulting two-loop kernels:

$$h(u) = \ln\bar{u} \left[\beta_0 + 2\, C_F \left(\ln\bar{u} - \frac{1+\bar{u}}{\bar{u}} \ln u - \frac{3}{2} \right) \right],$$

$$\gamma_+(a) = -a\, C_F + a^2\, C_F \left\{ 4\, C_F \left[\frac{8}{21} + \frac{\pi^2}{3} - 6\,\zeta_3 \right] + C_A \left[\frac{83}{9} - \frac{2\,\pi^2}{3} - 6\,\zeta_3 \right] + \beta_0 \left[\frac{35}{18} - \frac{\pi^2}{6} \right] \right\}.$$

- An interesting question: What are the eigenfunctions for the two-loop Lange-Neubert kernel? Exact expression and the corresponding large β_0 approximation for the coffee time!

Story I: B-meson distribution amplitudes at higher twist

- Three-particle B-meson distribution amplitudes [Kawamura, Kodaira, Qiao, Tanaka, 2000]:

$$\langle 0|\bar{u}_\alpha(x)\, G_{\lambda\rho}(ux)\, b_{v\beta}(0)|B^-(v)\rangle\Big|_{x^2=0}$$
$$= \frac{\tilde{f}_B(\mu)\, m_B}{4} \int_0^\infty d\omega \int_0^\infty d\xi\, e^{-i(\omega+u\xi)\, v\cdot x} \Big[(1+\not{v}) \Big\{ (v_\lambda \gamma_\rho - v_\rho \gamma_\lambda) \left[\Psi_A(\omega,\xi) - \Psi_V(\omega,\xi)\right]$$
$$- i\sigma_{\lambda\rho}\, \Psi_V(\omega,\xi) - \frac{x_\lambda v_\rho - x_\rho v_\lambda}{v\cdot x} X_A(\omega,\xi) + \frac{x_\lambda \gamma_\rho - x_\rho \gamma_\lambda}{v\cdot x} Y_A(\omega,\xi) \Big\} \gamma_5 \Big]_{\beta\alpha}.$$

Four independent distribution functions!

- QCD equations of motion at the classical level [Kawamura, Kodaira, Qiao and Tanaka, 2000]:

$$\omega\, \phi_B^-(\omega) - m\, \phi_B^+(\omega) + \frac{D-2}{2} \int_0^\omega d\eta\, \left[\phi_B^+(\eta) - \phi_B^-(\eta)\right]$$
$$= (D-2) \int_0^\omega d\eta \int_{\omega-\eta}^\infty \frac{d\xi}{\xi} \frac{\partial}{\partial \xi} \left[\Psi_A(\eta,\xi) - \Psi_V(\eta,\xi)\right], \quad \text{[correction to the WW relation!]}$$

$$(\omega + m)\, \phi_B^-(\omega) + (\omega - 2\bar{\Lambda} - m)\, \phi_B^+(\omega)$$
$$\stackrel{?}{=} -2 \frac{d}{d\omega} \int_0^\omega d\eta \int_{\omega-\eta}^\infty \frac{d\xi}{\xi} \left[\Psi_A(\eta,\xi) + X_A(\eta,\xi)\right] - 2(D-2) \int_0^\omega d\eta \int_{\omega-\eta}^\infty \frac{d\xi}{\xi} \frac{\partial \Psi_V(\eta,\xi)}{\partial \xi}.$$

More discussions on the EOM constraints: [Huang, Wu, Zhou, 2005; Huang, Qiao, Wu, 2006].

Story I: *B*-meson distribution amplitudes at higher twist

- A complete decomposition [Braun, Ji, Manashov, 2017]:

$$
\begin{aligned}
&\langle 0|\bar{u}_\alpha(z_1\bar{n})\, G_{\lambda\rho}(z_2\bar{n})\, b_{v\beta}(0)|B^-(v)\rangle\big| \\
&= \frac{\tilde{f}_B(\mu)\, m_B}{4}\bigg[(1+\not{v})\Big\{(v_\mu\gamma_\nu - v_\nu\gamma_\mu)\,[\Psi_A(z_1,z_2,\mu)-\Psi_V(z_1,z_2,\mu)] - i\,\sigma_{\mu\nu}\,\Psi_V(z_1,z_2,\mu) \\
&-(\bar{n}_\mu v_\nu - \bar{n}_\nu v_\mu) X_A(z_1,z_2,\mu) + (\bar{n}_\mu\gamma_\nu - \bar{n}_\nu\gamma_\mu)\,[W(z_1,z_2,\mu)+Y_A(z_1,z_2,\mu)] \\
&+ i\,\varepsilon_{\mu\nu\alpha\beta}\,\bar{n}^\alpha v^\beta\,\gamma_5\,\tilde{X}_A(z_1,z_2,\mu) - i\,\varepsilon_{\mu\nu\alpha\beta}\,\bar{n}^\alpha\gamma^\beta\,\gamma_5\,\tilde{Y}_A(z_1,z_2,\mu) \\
&-(\bar{n}_\mu v_\nu - \bar{n}_\nu v_\mu)\not{\bar{n}}\, W(z_1,z_2,\mu) + (\bar{n}_\mu\gamma_\nu - \bar{n}_\nu\gamma_\mu)\not{\bar{n}}\, Z(z_1,z_2,\mu)\Big\}\gamma_5\bigg]_{\beta\alpha}.
\end{aligned}
$$

  Eight independent distribution functions up to the twist-six accuracy!

- Classical equations of motion [Braun, Ji, Manashov, 2017; Lü, Shen, Wang, Wei, 2019]:

$$
\begin{aligned}
-2\,\frac{d^2}{d\omega^2}\, g_B^+(\omega,\mu) &= \left[\frac{3}{2} + (\omega-\bar{\Lambda})\frac{d}{d\omega}\right]\phi_B^+(\omega,\mu) - \frac{1}{2}\,\phi_B^-(\omega,\mu) + \int_0^\infty \frac{d\omega_2}{\omega_2}\,\frac{d}{d\omega}\,\Psi_4(\omega,\omega_2,\mu) \\
&\quad - \int_0^\infty \frac{d\omega_2}{\omega_2^2}\,\Psi_4(\omega,\omega_2,\mu) + \int_0^\omega \frac{d\omega_2}{\omega_2^2}\,\Psi_4(\omega-\omega_2,\omega_2,\mu), \\
-2\,\frac{d^2}{d\omega^2}\, g_B^-(\omega,\mu) &= \left[\frac{3}{2} + (\omega-\bar{\Lambda})\frac{d}{d\omega}\right]\phi_B^-(\omega,\mu) - \frac{1}{2}\,\phi_B^+(\omega,\mu) + \int_0^\infty \frac{d\omega_2}{\omega_2}\,\frac{d}{d\omega}\,\Psi_5(\omega,\omega_2,\mu) \\
&\quad - \int_0^\infty \frac{d\omega_2}{\omega_2^2}\,\Psi_5(\omega,\omega_2,\mu) + \int_0^\omega \frac{d\omega_2}{\omega_2^2}\,\Psi_5(\omega-\omega_2,\omega_2,\mu).
\end{aligned}
$$

  Truncation at the twist-four accuracy already violates the classical EOM!

Story I: B-meson distribution amplitudes at higher twist

- One-loop renormalization of the three-particle DA $\tilde{\Psi}_3(z_1,z_2)$ [Braun, Manashov, Offen, 2015]:

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(g_s)\frac{\partial}{\partial g_s} + \frac{\alpha_s}{2\pi}\mathscr{H}\right] F_{\rm stat}(\mu)\,\tilde{\Psi}_3(z_1,z_2,\mu) = 0\,,$$

$$\tilde{\Psi}_3(z_1,z_2) \equiv \Psi_A(z_1,z_2) - \Psi_V(z_1,z_2)\,, \qquad \mathscr{H} = N_c\,H_0 + N_c^{-1}\,\delta H\,.$$

An additional "hidden" symmetry for H_0: $[\hat{Q}_1,\hat{Q}_2] = [\hat{Q}_1,H_0] = [\hat{Q}_2,H_0] = 0$.

- Eigenfunctions:

$$H_0\,Y_{s,x}(z_1,z_2) = E(s,x)\,Y_{s,x}(z_1,z_2)\,, \qquad Y_{s,i/2}(z_1,z_2) = \frac{i\,s^2}{z_1^2\,z_2^3}\int_0^1 du\,u\,\bar{u}\,e^{is(u/z_1+\bar{u}/z_2)}\,,$$

$$\Delta E = \underbrace{E(s,0)}_{\text{continuous spectrum}} - \underbrace{E(s,i/2)}_{\text{ground state}} = 2\,\psi(3/2) - \psi(2) - \psi(1)\,.$$

- Expansion, "asymptotics" and RGE of ϕ_B^-:

$$\tilde{\Psi}_3(z_1,z_2,\mu) = \int_0^\infty ds\left[\underbrace{\eta_0(s,\mu)\,Y_{s,i/2}(z_1,z_2)}_{\text{"asymptotical" behaviour}} + \frac{1}{2}\int_{-\infty}^{+\infty} dx\,\eta(s,x,\mu)\,Y_{s,x}(z_1,z_2)\right].$$

$$\Psi_3^{\rm asy}(\omega_1,\omega_2,\mu) = \frac{\omega_1\,\omega_2}{\omega_1+\omega_2}\left[f_1(\omega_1+\omega_2) - f_0(\omega_1+\omega_2)\right] + \omega_1\left[f_1(\omega_1+\omega_2) - f_1(\omega_1)\right].$$

$$\phi_B^-(\omega,\mu) = \int_0^\infty ds\left[\hat{\phi}_B^+(s,\mu) + \underbrace{\eta_0(s,\mu)}\right] J_0(2\sqrt{\omega\,s})\,.$$

continuous spectrum of $\tilde{\Psi}_3(z_1,z_2,\mu)$ irrelevant at large N_c

Story I: B-meson distribution amplitudes [OPE constraints]

- Perturbative constraints in momentum space [Lee, Neubert, 2005]:
  - Step 1: Perturbative calculations of the regularized moments:

$$M_N(\Lambda_{\rm UV},\mu) = \int_0^{\Lambda_{\rm UV}} d\omega\, \omega^N\, \phi_B^+(\omega,\mu)\,.$$

  - Step 2: The two-component ansatz:

$$\phi_B^+(\omega,\mu) = N\, \frac{\omega}{{\omega_0}^2}\, e^{-\omega/\omega_0} + \theta(\omega-\omega_t)\, \frac{\alpha_s\, C_F}{\pi\,\omega} \left[\left(\frac{1}{2} - \ln\frac{\omega}{\mu} \right) + \frac{4\,\bar{\Lambda}_{\rm DA}}{3\,\omega} \left(2 - \ln\frac{\omega}{\mu} \right) \right].$$

  - Step 3: Fixing ω_t from the continuous condition, determining N and ω_0 from perturbative constraints of M_0 and M_1.

- Disadvantages:
  - No explicit implementation of the RG evolution.
  - Non-trivial resummation in momentum space due to the non-local renormalization kernels.

Story I: B-meson distribution amplitudes [OPE constraints]

- Perturbative constraints in dual space [Feldmann, Lange, Wang, 2014]:
  - Step 1: Perturbative calculation of the dual function at large ω':

$$\rho_B^+(\omega') = \frac{1}{\omega'} \int_0^\infty \frac{d\Lambda_{\rm UV}}{\Lambda_{\rm UV}} \left\{ M_0 \Big|_{\bar{\Lambda}\to 0} J_2\left(2\sqrt{\frac{\Lambda_{\rm UV}}{\omega'}}\right) + \frac{\partial}{\partial \bar{\Lambda}} \left(2M_0 - \frac{3M_1}{\Lambda_{\rm UV}}\right)_{\bar{\Lambda}\to 0} \bar{\Lambda} J_4\left(2\sqrt{\frac{\Lambda_{\rm UV}}{\omega'}}\right) + \ldots \right\}.$$

  - Step 2: Improved construction for $\rho_B^+(\omega', \mu)$:

$$\rho_B^+(\omega',\mu) = U_{\omega'}(\mu, \mu_{\omega'}(\mu_0)) \left[\rho^{\rm model}(\omega') - \sum_{n=0}^{N} D_n^{\rm model}\, p_n(\omega')\right] + U_{\omega'}(\mu, \mu_{\omega'}(\mu)) \sum_{n=0}^{N} D_n^{\rm pert}\left(\ln \frac{\mu_{\omega'}(\mu)}{\hat{\omega}'}, \mu_{\omega'}(\mu)\right) p_n(\omega').$$

  - Step 3: Choose the basis functions $p_n(\omega')$ to vanish quickly at small ω' and to approach $1/(\omega')^{n+1}$ at large ω'.

Story I: B-meson distribution amplitudes: $B \to \gamma \ell \nu$

- Factorization properties at leading power [Korchemsky, Pirjol, Yan, 2000; Descotes-Genon, Sachrajda, 2002; Lunghi, Pirjol, Wyler, 2003; Bosch, Hill, Lange, Neubert, 2003].

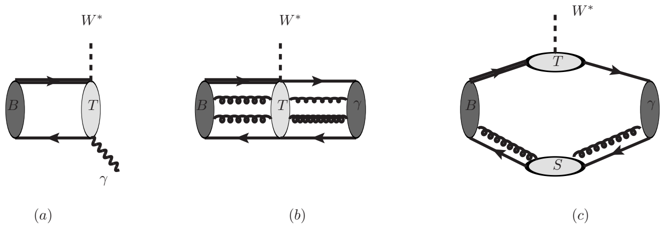
- Leading power contributions at NLL and (partial)-subleading power corrections at tree level [Beneke, Rohrwild, 2011].

- Subleading power corrections from the dispersion technique:
  - Soft two-particle correction at tree level [Braun, Khodjamirian, 2013].
  - Soft two-particle correction at one loop [Wang, 2016].
  - Three-particle B-meson DA's contribution at tree level [Wang, 2016; Beneke et al, 2018].
  - Subleading effective current and twist-5 and 6 corrections at tree level. [Beneke et al, 2018].

- Subleading power corrections from the direct QCD approach:
  - Hadronic photon corrections at tree level up to the twist-4 accuracy [Khodjamirian, Stoll, Wyler, 1995; Ali, Braun, 1995; Eilam, Halperin, Mendel, 1995].
  - Hadronic photon corrections of twist-two at one loop and of higher-twist at tree level [Ball, Kou, 2003; Wang, Shen, 2018].

Story I: B-meson distribution amplitudes: $B \to \gamma \ell \nu$

- Schematic structure of the distinct mechanisms:



A: hard subgraph that includes both photon and W^* vertices $\quad \left(\frac{\Lambda}{m_b}\right)^{1/2} + \left(\frac{\Lambda}{m_b}\right)^{3/2} + \ldots$

B: real photon emission at large distances $\quad \left(\frac{\Lambda}{m_b}\right)^{3/2} + \ldots$

C: Feynman mechanism: soft quark spectator $\quad \left(\frac{\Lambda}{m_b}\right)^{3/2} + \ldots$

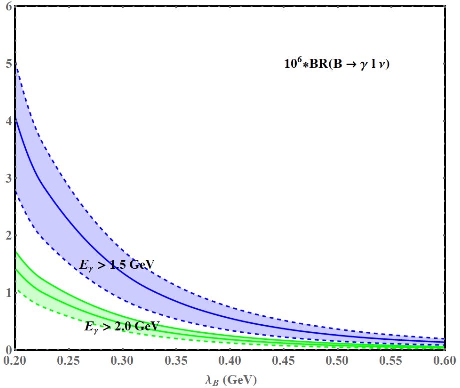
- Operator definitions of different terms needed for an unambiguous classification.

Theory predictions for $B \to \gamma \ell \nu$

- Integrated decay rate $\Delta BR(E_{\rm cut})$:

$$\Delta BR(E_{\rm cut}) = \tau_B \int_{E_{\rm cut}}^{m_B/2} dE_\gamma \, \frac{d\Gamma}{dE_\gamma} (B \to \gamma \ell \nu) \, .$$

- $\lambda_B(\mu_0)$ dependence of $\Delta BR(E_{\rm cut})$ [Wang, Shen, 2018]:



- Belle 2015 data: $\Delta BR(1\,{\rm GeV}) < 3.5 \times 10^{-6}$.
- Belle 2018 data [arXiv:1810.12976]: $\Delta BR(1\,{\rm GeV}) = (1.4 \pm 1.0 \pm 0.4) \times 10^{-6}$.
- Expected statistical error for $\Delta BR(1\,{\rm GeV})$ with $50\,{\rm ab}^{-1}$ of Belle-II data: ${}^{+0.18}_{-0.17} \times 10^{-6}$.
- The photon-energy cut not sufficiently large. Power corrections numerically important for $E_\gamma < 1.5\,{\rm GeV}$.

Story II: QCD factorization approach

- Factorization formulae for semeleptonic B-meson decays [BBNS, BPRS, and many others].

$$F_i^{B\to M}(E) = C_i^{(\mathrm{A0})}(E)\,\xi_a(E) + \int_0^\infty \frac{d\omega}{\omega} \int_0^1 dv\, \underbrace{T_i(E;\ln\omega, v)}_{C_i^{(\mathrm{B1})} * J_i}\, \phi_B^+(\omega)\phi_M(v)\,.$$

Need the hadronic matrix elements of both the LP and NLP SCET currents!

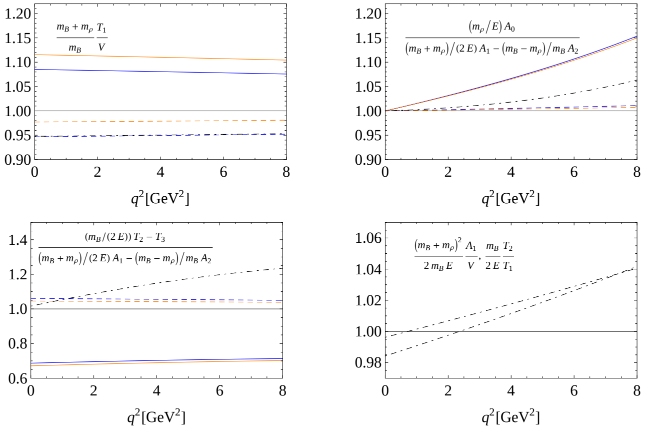
- Diagrammatic factorization for heavy-to-light form factors at one loop [Beneke, Feldmann, 2001].

- Perturbative calculations of the hard matching coefficients $C_i^{(\mathrm{A0})}(E)$:
  - One-loop SCET computations in [Bauer, Fleming, Pirjol, Stewart, 2001; Beneke, Kiyo, Yang, 2004].
  - Two-loop SCET computations in [Bonciani, Ferroglia, 2008; Asatrian, Greub, Pecjak, 2008; Beneke, Huber, Li, 2009; Bell, 2009; Bell, Beneke, Huber, Li, 2011].

- Perturbative calculations of the hard matching coefficients $C_i^{(\mathrm{B1})}$:
  - Infrared subtractions complicated by the appearance of evanescent operators and the D-dimensional Fierz transformation.
  - One-loop SCET computations in [Becher, Hill, 2004; Hill, Becher, Lee, Neubert, 2004; Beneke, Yang, 2006].

Story II: QCD factorization approach

- Long-standing puzzles for 18 years [Bell, Beneke, Huber, Li, 2011]:



Discrepancies between the SCET predictions (blue solid) and the LCSR results (black dashed-dotted).

- Identify the dominant QCD mechanisms for such discrepancies in [Gao, Lü, Shen, Wang, Wei, 2019].

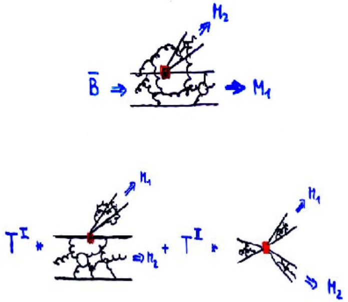
Story II: QCD factorization approach

- QCD factorization for charmless hadronic B-meson decays [BBNS, BPRS, Chay, Kim, and many others].

Heavy quark limit: $m_b \gg \Lambda_{\rm QCD}$
Large-energy limit: $E_M \approx m_b/2 \gg \Lambda_{\rm QCD}$
Scales: m_b, $\sqrt{m_b \Lambda_{\rm QCD}}$, $\Lambda_{\rm QCD}$, $(M_{\rm EW}, \Lambda_{\rm NP})$

$\mu \approx m_b$

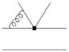
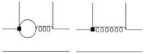
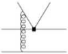
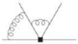
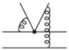
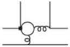
$\mu \approx 1\,\mathrm{GeV}$



- Reduces $\langle M_1 M_2|\mathcal{O}|B\rangle$ to simpler $\langle M|\mathcal{O}|B\rangle$ (form factors), $\langle 0|\mathcal{O}|B\rangle$, $\langle M|\mathcal{O}|0\rangle$ (decay constants and distribution amplitudes).
- Calculation from first principles, but limited accuracy by $\Lambda_{\rm QCD}/m_b$ corrections.

Status of NNLO QCD factorization calculations

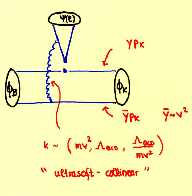
$$\langle M_1 M_2 | C_i O_i | \bar{B} \rangle_{\mathcal{L}_{\rm eff}} = \sum_{\rm terms} C(\mu_h) \times \Big\{ F_{B \to M_1} \times \underbrace{T^{\rm I}(\mu_h, \mu_s)}_{1+\alpha_s+\ldots} \star f_{M_2} \Phi_{M_2}(\mu_s)$$
$$+ f_B \Phi_B(\mu_s) \star \Big[\underbrace{T^{\rm II}(\mu_h, \mu_I)}_{1+\ldots} \star \underbrace{J^{\rm II}(\mu_I, \mu_s)}_{\alpha_s+\ldots} \Big] \star f_{M_1} \Phi_{M_1}(\mu_s) \star f_{M_2} \Phi_{M_2}(\mu_s) \Big\}$$

	T^I, tree	T^I, penguin	T^{II}, tree	T^{II}, penguin
LO: $\mathcal{O}(1)$				
NLO: $\mathcal{O}(\alpha_s)$ BBNS '99-'04				
NNLO: $\mathcal{O}(\alpha_s^2)$	 Bell '07,'09 Beneke, Huber, Li '09	 Kim, Yoon '11, Bell Beneke, Huber, Li '15	 Beneke, Jager '05 Kivel '06, Pilipp '07	 Beneke, Jager '06 Jain, Rothstein, Stewart '07

- Two-loop tree and penguin topologies for the insertions of Q_{3-6} and Q_{8g} are already computed by Bell, Beneke, Huber, Li. Comprehensive phenomenologies studies are in progress.
- More QCDF calculations for $B \to M_1 M_2$ at NLO [Talk by Beneke @ the MITP-2019 Workshop].

Story II: Sketch of spectator-scattering $B \to \chi_{cJ} K$

Sketch of spectator-scattering $B \to \chi_{cJ} K$ [MB, Vernazza, 2008]



$$A(B\to HK)_{spect} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \cdot 2C_1 \cdot \frac{\pi \alpha_s C_F}{N_c^2} \langle 0|[n]|H[n]\rangle \cdot \frac{1}{\lambda_B} \cdot M_B$$

$$\int_0^1 dy\, f_K \phi_K(y) \left\{ \left[\frac{e^{B_{fin}}[n]}{\bar{y}} + \frac{B[n]}{\bar{y}^2} \right] \theta(1-\mu-y) \right.$$

hard / P-wave colour singlet

$$\left. + \; B[n] \frac{1}{\left(b + \sqrt{-(\bar{y}+a)}\right)^4} \theta(y-(1-\mu)) \right\}$$

ultrasoft / S-wave colour octet

$b = \frac{\gamma}{m_b \sqrt{1-z}}$, $a = \frac{4 m_c E_B}{m_b^2 (1-z)}$

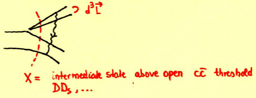
Endpoint div. in hard spectator-scattering (Song et al., 2002; Meng et al., 2005)

NEW

$$\int_0^1 dy\, \phi_K(y) \{\;\} = e^{B_{fin}}[n] \int_0^1 dy \frac{\phi_K(y)}{y} + B[n] \int_0^1 dy \frac{\phi_K(y) + \bar{y}\phi_K'(1)}{\bar{y}^2} + B[n]\, \phi_K'(1) \ln \mu$$

$$- B[n]\, \phi_K'(1) \left\{ \ln \mu + \ln \frac{m_b^2 (1-z)}{\gamma^2} - i\pi - 2 \ln(1+A) + 1 + \frac{2}{3} \frac{4+A}{(1+A)^2} \right\}$$

"large log" $\ln \frac{m_b^2}{m_c^2 v^2}$: endpoint log $A = \sqrt{-\frac{4(E_B+E)}{\gamma^2/m_c}} = \mathcal{O}(1)$



Large rescattering phase from endpoint contribution, none from hard scattering.

- The first field-theoretical treatment of rapidity divergences in the heavy-quark limit $m_c v^2 \gg \Lambda_{QCD}$.

Story II: Theory challenges for QCD factorization

- Precision determination of the inverse moment λ_B in high demand.
- Determine the NNLO correction to the NLP scalar penguin amplitude a_6 to complete the short-distance prediction [Beneke, Jager, Wang, 2019+].
- General treatment of the rapidity divergences in exclusive B-meson decays.
- Systematic approach for the subleading power corrections (even for $B \to \gamma \ell \nu_\ell$).
- SCET factorization theorems for QED corrections to heavy hadron decays (even for $B_s \to \ell\ell$).
- Glauber gluons and factorization violations [Rothstein, Stewart, 2016].

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mitp Mainz Institute for Theoretical Physics

Factorization Violation and Glauber Gluons

12-23 August 2019
Mainz Institute for Theoretical Physics, Johannes Gutenberg University
Europe/Berlin timezone

Story II: PQCD factorization approach

- Pioneer works on the hard exclusive reactions:
  - Hard scattering approach formulated in [Lepage, Brodsky, 1979, 1980].
  - The hadronic wave function in QCD [Brodsky, Lepage, Huang, 1980].
  - Sudakov effects in hadron-hadron elastic (Landshoff) scattering [Botts, Sterman, 1989].
  - Sudakov resummation for the pion electromagnetic form factor [Li, Sterman, 1992].
    Key observation: Perturbative QCD formalism to hard exclusive processes applicable for $\sqrt{Q^2} \sim 20\,\Lambda_{\rm QCD}$.
    Saving us from the strong doubts raised in [Isgur, Llewellyn Smith, 1988; Radyushkin, 1984].
  - More references can be found in the Bible by John Collins.

Important pieces of work accomplished in China:

- Applicability of PQCD factorization for the pion electromagnetic form factor [Huang, Shen, 1990].
- Sudakov suppression for hard exclusive reactions [PhD thesis by Jun Cao].
- Many other interesting papers to be discussed for the coffee time.

微扰量子色动力学应用到遍举过程中的几个问题

曹　俊

指导教师姓名 ：　黄　涛　　吴慧芳
职称 ：　研究员　　研究员
单位 ：　中国科学院高能物理所

Story II: PQCD factorization approach

- Birth of the modern PQCD approach for charmless hadronic B-meson decays:
  - Penguin enhancement for $B \to \pi K$ [Keum, Li, Sanda, 2001].
  - Tree-dominated processes $B \to \pi\pi$ [Lü, Ukai, Yang, 2001].

- Hundreds of papers on the tree-level PQCD calculations [Groups led by Li, Lü, Xiao, etc].

- PQCD calculations at NLO complicated by the appearance of multi-scales:
  - The pion-photon form factor at large momentum transfer [Nandi, Li, 2007].
  - The pion electromagnetic form factor at large Q^2 [Li, Shen, Wang, Zou, 2011].
  - $B \to \pi$ form factors at large recoil [Li, Shen, Wang, 2012].
  - Increasing complexities mainly due to the infrared subtractions!

- Intensive NLO PQCD calculations subsequently:
  - NLO twist-3 correction to the pion electromagnetic form factor [Cheng, Fan, Xiao, 2014].
  - NLO twist-3 correction to $B \to \pi$ form factors [Cheng, Fan, Yu, Lü, Xiao, 2014].
  - NLO correction to $B \to \rho$ form factors [Hua, Zhang, Xiao, 2018].
  - (Partial)-NLO correction to $B \to \pi\pi$ [Cheng, Xiao, Zhang, 2014].
  - (Partial)-NLO correction to $B_s \to PP$ [Yan, Liu, Xiao, 2019].
  - Many more papers on the NLO PQCD calculations.

Story II: PQCD factorization approach

- Formal developments of the PQCD approach:
  - Rapidity resummation for the TMD B-meson wavefunction in Mellin space [Li, Shen, Wang, 2013].
  - Joint resummation for the threshold and Sudakov logarithms for the pion-photon form factor [Li, Shen, Wang, 2014].
  - Factorization-compatible definitions of the TMD pion wavefunction [Li, Wang, 2015]

- Naïve definition of the TMD pion wavefunction:

$$\phi_\pi^{\rm naive}(x, \vec{k}_T, \mu) \stackrel{?}{=} \int \frac{dz_-}{2\pi} \int \frac{d^2 z_T}{(2\pi)^2} \, e^{i(x p_+ z_- - \vec{k}_T \cdot \vec{z}_T)}$$
$$\times \langle 0 | \bar{q}(0) W_{n_-}^\dagger(+\infty, 0) \, \not{n}_- \, \gamma_5 \, [\text{tr. link}] \, W_{n_-}(+\infty, z) \, q(z) | \pi^+(p) \rangle \, .$$

- Rapidity divergence in the infrared subtraction:

$$\phi_\pi^{(1)} \otimes H^{(0)} \supset \int [dl] \frac{1}{[(k+l)^2 + i0][l_+ + i0][l^2 + i0]}$$
$$\times \left[H^{(0)}(x + l_+/p_+, \vec{k}_T + \vec{l}_T) - H^{(0)}(x, \vec{k}_T) \right] .$$

  - Rapidity divergence due to the eikonal propagator.
  - Key difference: both the longitudinal and transverse components of the partonic momentum changed in TMD factorization!

Light-cone singularity

- Regularization of the rapidity divergence [Collins, 2003].
  - Rotating the gauge links away from the light-cone ($u = (u_+, u_-, \vec{0}_T)$):

$$\phi_\pi(x, \vec{k}_T, y_u, \mu) \stackrel{?}{=} \int \frac{dz_-}{2\pi} \int \frac{d^2 z_T}{(2\pi)^2}\, e^{i(x p_+ z_- - \vec{k}_T \cdot \vec{z}_T)}$$

$$\times \langle 0 | \bar{q}(0) W_u^\dagger(+\infty, 0)\, \not{n}_-\, \gamma_5\, [\text{tr. link}]\, W_u(+\infty, z)\, q(z) | \pi^+(p) \rangle\,.$$

  - Introducing soft subtractions:

$$\phi_\pi(x, \vec{k}_T, y_u, \mu) \stackrel{?}{=} \int \frac{dz_-}{2\pi} \int \frac{d^2 z_T}{(2\pi)^2}\, e^{i(x p_+ z_- - \vec{k}_T \cdot \vec{z}_T)}$$

$$\times \frac{\langle 0 | \bar{q}(0) W_{n_-}^\dagger(+\infty, 0)\, \not{n}_-\, \gamma_5\, [\text{tr. link}]\, W_{n_-}(+\infty, z)\, q(z) | \pi^+(p) \rangle}{\langle 0 | W_{n_-}^\dagger(+\infty, 0) W_u(+\infty, 0)\, [\text{tr. link}]\, W_{n_-}(+\infty, z)\, W_u^\dagger(+\infty, z) | 0 \rangle}\,.$$

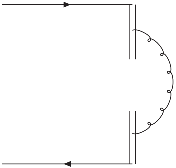
- Are these definitions compatible with the factorization theorems?

- Can employ multiple non-light-like Wilson lines at the price of introducing the soft function in the factorization formulae and using another parameter ρ beyond the scale parameters of CSS [Ji, Ma, Yuan, 2004].

Pinch singularity

- Singularity from Wilson-line self energies [Bacchetta, Boer, Diehl, Mulders, 2008].
  - Pinch singularity only appears in a TMD parton density with $u^2 < 0$.
  - Pinch singularity appears in the TMD wave functions for any off-light-cone u.

$$\phi_\pi \supset \int [dl] \frac{u^2}{[l+i0)][u \cdot l + i0][u \cdot l - i0]} \times \delta(x' - x + l_+/p_+) \delta^{(2)}(\vec{k}'_T - \vec{k}_T + \vec{l}_T) .$$



  - Pinch singularity corresponds to the linear divergence in the length of the Wilson line in the coordinate space.

- Off-light-cone Wilson lines regularize rapidity divergence, at the price of introducing unwanted pinch singularity.

- How to achieve factorization-compatible definitions of TMD wavefunctions?

Collins modification

- New definition without pinch singularity [Collins, 2011]:

$$\phi_\pi^{C}(x,\vec{k}_T,y_2,\mu) = \lim_{\substack{y_1\to+\infty \\ y_u\to-\infty}} \int \frac{dz_-}{2\pi} \int \frac{d^2 z_T}{(2\pi)^2} e^{i(xp_+ z_- - \vec{k}_T\cdot\vec{z}_T)}$$
$$\times \langle 0|\bar{q}(0) W_u^\dagger(+\infty,0)\, \not{n}_-\, \gamma_5\, [\text{tr. link}]\, W_u(+\infty,z)\, q(z)|\pi^+(p)\rangle$$
$$\times \sqrt{\frac{S(z_T;y_1,y_2)}{S(z_T;y_1,y_u)\, S(z_T;y_2,y_u)}}\,.$$

$\uparrow$

rapidity of the gauge vector $n_2 = (e^{y_2}, e^{-y_2}, \vec{0}_T)$

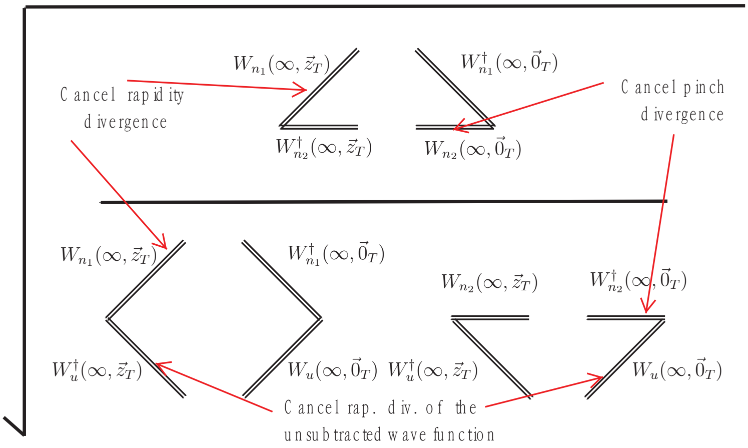
  - Soft function:

$$S(z_T;y_A,y_B) = \frac{1}{N_c}\langle 0|W_{n_B}^\dagger(\infty,\vec{z}_T)_{ca}\, W_{n_A}(\infty,\vec{z}_T)_{ad}\, W_{n_B}(\infty,0)_{bc}\, W_{n_A}^\dagger(\infty,0)_{db}|0\rangle\,.$$

- General properties of the new definition:
  - The unsubtracted wave function only involves light-cone Wilson lines.
  - Each soft factor has at most one off-light-cone Wilson line.
  - No rapidity divergences and no pinch singularities.
  - A detailed comparison with many other definitions [Collins, arXiv:1409.5408].

Why the new definition works?

- Cancellation mechanism ($y_1 \to +\infty$, $y_u \to -\infty$):



Simplified definitions of TMDs possible?

- Treatment of rapidity and pinch singularities:

Light-cone Wilson lines

$\downarrow$ TMD hard function

Rapidity divergence

$\downarrow$

Off-light-cone Wilson lines

$\downarrow$ Dipolar structure

Pinch singularity

$\swarrow$ $\searrow$

Nontrivial soft subtraction | Non-dipolar Wilson lines

$\downarrow$

Simper soft subtraction

TMDs with non-dipolar Wilson lines

- Orthogonal Wilson lines ($n_2 \cdot v = 0$) [Li, Wang, 2015]:

$$\phi_\pi^{\rm I}(x, \vec{k}_T, y_2, \mu) = \int \frac{dz_-}{2\pi} \int \frac{d^2 z_T}{(2\pi)^2} e^{i(xp_+ z_- - \vec{k}_T \cdot \vec{z}_T)} \times \langle 0 | \bar{q}(0) W_{n_2}^\dagger(+\infty, 0) \not{n}_- \gamma_5 \, [\text{links}@\infty] \, W_v(+\infty, z) \, q(z) | \pi^+(p) \rangle .$$

$$\uparrow \qquad\qquad\qquad \uparrow$$

$$n_2 = (e^{y_2}, e^{-y_2}, \vec{0}_T), \qquad v = (-e^{y_2}, e^{-y_2}, \vec{0}_T) .$$

  - Wilson-line self energies vanish in Feynman gauge.
    $\Rightarrow$ Soft subtraction not needed.
  - $\phi_\pi^{\rm I} \otimes H^{(0)}$ reproduces the collinear logarithm of QCD diagrams:

$$\phi_\pi^{\rm I} \otimes H^{(0)} = -\frac{\alpha_s C_F}{4\pi} \left[2 \ln x + 3 \right] \ln \left(\frac{k_T^2}{Q^2} \right) H^{(0)}(x, k_T) + \cdots .$$

- Antiparallel Wilson lines [Li, Wang, 2015]:

$$\phi_\pi^{\rm II}(x, \vec{k}_T, y_2, \mu) = \int \frac{dz_-}{2\pi} \int \frac{d^2 z_T}{(2\pi)^2} e^{i(xp_+ z_- - \vec{k}_T \cdot \vec{z}_T)} \times \frac{\langle 0 | \bar{q}(0) W_{n_2}^\dagger(+\infty, 0) \not{n}_- \gamma_5 \, [\text{links}@\infty] \, W_{n_2}(-\infty, z) \, q(z) | \pi^+(p) \rangle}{[\text{color}] \, \langle 0 | W_{n_2}^\dagger(+\infty, 0) \, [\text{links}@\infty] \, W_{n_2}(-\infty, 0) | 0 \rangle} .$$

Story II: Theory challenges for PQCD factorization

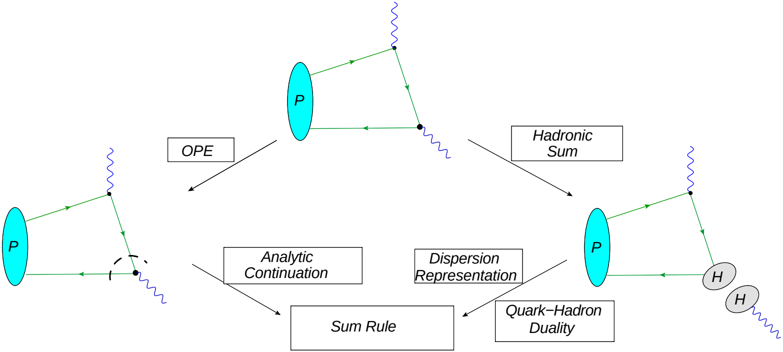
- Rigorous definitions of TMD B-meson wavefunctions in HQET.
- Systematic NLO PQCD calculations for the higher-twist contributions beyond the leading-Fock-state approximation.
- A complete NLL resummation for heavy-ro-light form factors with TMD factorization.
- A complete NLO PQCD calculation for the radiative and electroweak penguin B-meson decays. Improving PQCD calculation for $B_{(s)} \to V\gamma$ and $B_{(s)} \to A\gamma$ presented in [Wei Wang, Li, 2007].
- A complete NLO PQCD calculation for charmless hadronic B-meson decays. Technically demanding and conceptually challenging!
- Systematic power counting scheme, including k_T, in the presence of the Sudakov mechanism.
- Subleading power corrections to exclusive B-meson decays with TMD factorization. SCET formulation for the diagrammatic PQCD formalism.

Story II: Light-cone sum rules in QCD/SCET

- LCSR for $B \to P$ form factors [Balitsky, Braun, Kolesnichenko, 1989; Chernyak, Zhitnitsky, 1990]:

$$\langle P(p')|\bar{q}\,\gamma^{\mu} b|\bar{B}(p)\rangle = f_{+}(q^2)\left[p^{\mu}+p'^{\mu}-\frac{M^2-m_P^2}{q^2}\,q^{\mu}\right]+f_0(q^2)\,\frac{M^2-m_P^2}{q^2}\,q^{\mu}\,.$$

- General procedure for the sum-rule construction:



Story II: Different versions of light-cone sum rules

- Light-cone QCD sum rules with the light-meson distributions [Ball, Braun, Khodjamirian, etc]:
  - Interpolating the heavy B-meson by a local QCD current.
  - Diagrammatical factorization for the vacuum-to-light-meson correction functions.
  - Disadvantage: different non-perturbative inputs for different decay observables.
- Light-cone QCD sum rules with the B-meson distributions [Khodjamirian, Lü, Shen, Wang, etc]:
  - Interpolating the light energetic meson by a local QCD current.
  - Diagrammatical factorization for the vacuum-to-B-meson correction functions.
  - Advantage: universal non-perturbative inputs for different decay observables.
- Light-cone SCET sum rules with the B-meson distributions [Feldmann, Lü, Shen, Wang, etc]:
  - Interpolating the light energetic meson by a local SCET current.
  - SCET factorization for the vacuum-to-B-meson correction functions.
  - Advantage I: Computation of the short-distance functions much easier.
  - Advantage II: Systematic resummation of enhanced logarithms beyond the LL accuracy.
- Light-cone QCD sum rules with the chiral current for the light meson [Huang, Li, Sun, Zhi-Gang Wang, Wu, Zuo, etc]:
  - Advantage: Twist-three light-meson LCDAs do not contribute (at least) at NLO.
  - Heavy hadrons of both positive and negative parities enter the hadronic dispersion relation.
- Still more versions to be discussed for the coffee time.

Light-cone QCD sum rules with light-hadron DAs

- Semileptonic $B \to P$ form factors:
  - LO QCD calculations of $B \to P$ form factors [Belyaev, Khodjamirian, Rückl, 1993].
  - NLO QCD calculations of $B \to P$ form factors at twist-2 accuracy [Khodjamirian, Rückl, Weinzierl, Yakovlev, 1997].
  - NLO QCD calculations of $B \to P$ form factors at twist-3 accuracy [Ball, Zwicky, 2005; Duplancic, Khodjamirian, Mannel, Melic, Offen, 2008].
  - Improved NLO QCD calculations of $B \to P$ form factors at twist-3 accuracy [Khodjamirian, Mannel, Offen, Wang, 2011].
  - (Partial) NNLO QCD calculations of $B \to P$ form factors at twist-2 accuracy [Bharucha, 2012].
  - NLO QCD calculations of $B \to P$ form factors with the chiral interpolating current [Li, Si, Ying Wang, Zhu, 2015].

- Semileptonic $B \to V(\to P_1 P_2)$ form factors:
  - NLO QCD calculations of $B \to V$ form factors at twist-3 accuracy [Ball, Zwicky, 2005].
  - Updated NLO QCD calculations of $B \to V$ form factors at twist-3 accuracy [Bharucha, Straub, Zwicky, 2015].
  - LO QCD calculations of $B \to \pi K$ form factors [Meißner, Wei Wang, 2014]
  - LO QCD calculations of $B \to \pi\pi$ form factors [Hambrock, Khodjamirian, 2016]

- Semileptonic Λ_b-baryon form factors:
  - LO QCD calculation of $\Lambda_b \to \Lambda$ form factors [Wang, Li, Lü, 2008].
  - LO QCD calculation of $\Lambda_b \to p$ form factors [Khodjamirian, Mannel, Klein, Wang, 2011].

- Many other interesting extensions in different directions.

Light-cone QCD sum rules with heavy-hadron DAs

- LO QCD calculations of $B \to M$ form factors [Khodjamirian, Offen, Mannel, 2006].
- LO QCD calculations of the non-factorizable charm loops in $B \to K^{(*)} \ell\ell$ [Khodjamirian, Mannel, Pivovarov, Wang, 2010].
- NLO QCD calculations of $B \to \pi$ form factors at leading-twist accuracy [Wang, Shen, 2015].
- NLO QCD calculations of $B \to D$ form factors at leading-twist accuracy and LO QCD calculations of higher-twsit corrections up to the twist-six accuracy [Wang, Wei, Shen, Lü, 2017].
  - NLO leading-twist jet function complicated by two distinct hard-collinear variables.
  - Power-enhanced charm-quark mass effect.
- LO QCD calculations of $B \to P$ and $B \to V$ form factors at twist-four accuracy [Gubernari, Kokulu, van Dyk, 2018].
  - Violation of the QCD equations of motion at tree level.
  - Sizable theory uncertainties for phenomenological applications.
- Subleading power soft corrections to $B \to \gamma \ell \nu$ [Wang, 2016; Beneke, Braun, Ji, Wei, 2018].
- NLO QCD calculations of $\Lambda_b \to \Lambda$ form factors at twist-four accuracy [Wang, Shen, 2016].
  - Non-trivial demonstration of the factorization-scale independence.
  - Immediately confirmed by the Lattice QCD calculations [Detmold, Meinel, 2016].

Light-cone SCET sum rules with heavy-hadron DAs

- NLO QCD calculations of $B \to M$ form factors at leading-twist accuracy [De Fazio, Feldmann, Hurth, 2006; 2008].

- NLO QCD calculations of $B \to V$ form factors at leading-twist accuracy and LO QCD calculation of higher-twsit corrections up to the twist-six accuracy [Gao, Lü, Shen, Wang, Wei, 2019].
  - Rigorous perturbative matching with the evanescent-operator approach.
  - First SCET computation of the SU(3)-symmetry breaking effects.
  - Identify mechanisms responsible for the discrepancies between QCDF and LCSR:

$$\mathscr{R}_{1,\mathrm{LCSR}} = 1 + (-0.049)\big|_{C_i^{(\mathrm{A0})}} + (+0.054)\big|_{C_i^{(\mathrm{B1})}} + (-3.5\times 10^{-5})\big|_{\mathrm{3PHT}},$$

$$\mathscr{R}_{1,\mathrm{QCDF}} = 1 + (-0.023)\big|_{C_i^{(\mathrm{A0})}} + (+0.086)\,[1+\mathcal{O}(\alpha_s)]\big|_{C_i^{(\mathrm{B1})}}.$$

(a) No perturbative expansion for the A0-type $\mathrm{SCET_I}$ form factor in QCDF.

(b) Almost a factor of two smaller prediction of the B1-type $\mathrm{SCET_I}$ form factor in LCSR.

- Interesting extensions to many other exclusive B-meson decays in progress.

Story II: Theory challenges for light-cone sum rules

- NLO QCD calculations of the higher-twist corrections to $B \to M$ form factors.
  - Complicated by the infrared subtractions in the presence of the non-trivial renormalization mixing (2-, 3-, 4-particle LCDAs).
  - Consistent implementation of the QCD equations of motion at one loop.

- Complete NNLO QCD calculations of the leading-twist corrections to $B \to M$ form factors.
  - Complicated by the non-trivial jet functions due to two hard-collinear variables.
  - Modern techniques of multiloop calculations in demand.
  - NNLL resummation calls for the three-loop Lange-Neubert kernel.

- Nonfactorizable charm-loop effects in $B \to V\ell\ell$ and $B \to V\gamma$ at $\mathcal{O}(\alpha_s^2)$.
  - Non-trivial factorization theorems dependent on the power-counting schemes.
  - Two-loop four-point functions with three different scales in QCD.
  - RGEs of three-particle B-meson LCDAs for the NLL resummation.

- Improving the duality ansatz implemented in the sum-rule construction.
  - Intensive model-dependent investigations [Blok, Shifman, Zhang, 1998; González-Alonso, Pich, Prades, 2010, Jamin, 2011; etc].
  - Better understanding essential to the sum-rule improvement.

Story II: General aspects of $\Lambda_b \to p$ form factors

- Traditional parameterizations [Manohar, Wise $\oplus$ many others]:

$$\langle N(P')|\bar{u}\,\gamma_\mu\, b|\Lambda_b(P)\rangle = \bar{N}(P')\left\{f_1(q^2)\,\gamma_\mu + i\,\frac{f_2(q^2)}{m_{\Lambda_b}}\,\sigma_{\mu\nu}q^\nu + \frac{f_3(q^2)}{m_{\Lambda_b}}\,q_\mu\right\}\Lambda_b(P)\,,$$

$$\langle N(P')|\bar{u}\,\gamma_\mu\gamma_5\, b|\Lambda_b(P)\rangle = \bar{N}(P')\left\{g_1(q^2)\,\gamma_\mu + i\,\frac{g_2(q^2)}{m_{\Lambda_b}}\,\sigma_{\mu\nu}q^\nu + \frac{g_3(q^2)}{m_{\Lambda_b}}\,q_\mu\right\}\gamma_5\Lambda_b(P)\,.$$

  Axial-vector matrix element does not vanish, different from $B \to \pi$ transition.

- Helicity-based parameterizations [Feldmann, Yip, 2011]:

$$\langle N(P')|\bar{u}\,\gamma_\mu\, b|\Lambda_b(P)\rangle = \bar{N}(P')\left\{f_+(q^2)\,\frac{m_{\Lambda_b}+m_N}{s_+}\left(P_\mu + P'_\mu - \frac{q_\mu}{q^2}\,(m_{\Lambda_b}^2 - m_N^2)\right)\right.$$
$$\left. + f_\perp(q^2)\left(\gamma_\mu - \frac{2\,m_N}{s_+}\,P_\mu - \frac{2\,m_{\Lambda_b}}{s_+}\,P'_\mu\right) + f_0(q^2)\,(m_{\Lambda_b} - m_N)\,\frac{q_\mu}{q^2}\right\}\Lambda_b(P)\,.$$

  Simper expressions for angular distributions and for unitary bounds.

- Symmetry-based parameterizations [Feldmann, Yip, 2011]:

$$\langle N(P')|\bar{u}\,\Gamma\, b|\Lambda_b(P)\rangle = \xi_{ij}^{(\pm)}(v\cdot P')\,\bar{N}(P')\left\{\Gamma_i\,\frac{\not{n}_\pm \not{n}_\mp}{4}\,\Gamma\,\Gamma_j\right\}\Lambda_b(P)\,.$$

  $\xi_{ij}^{(-)}(v\cdot P')$ suppressed in both the HQET and SCET limits.

Symmetry relations of $\Lambda_b \to p$ form factors

- Form factors in the HQET limit [Manohar, Wise 2000]:

$$\langle N(P')|\bar{u}\,\Gamma\, b|\Lambda_b(P)\rangle = \bar{N}(P')\left[F_1(v\cdot P') + F_2(v\cdot P')\,\not{v}\right]\Gamma\Lambda_b(P)\,,$$

implying the relations

$$f_1 = g_1 = F_1 + \frac{m_N}{m_{\Lambda_b}}\,F_2\,, \qquad f_2 = f_3 = g_2 = g_3 = F_2\,,$$
$$f_0 = g_+ = g_\perp = F_1 + F_2\,, \qquad g_0 = f_+ = f_\perp = F_1 - F_2\,.$$

  - Only two form factors in the HQET limit (10 independent form factors in total).

- (Soft)-form factors in the SCET limit [Mannel, Wang, 2011; Feldmann, Yip, 2011]:

$$\langle N(P')|\bar{u}\,\Gamma\, b|\Lambda_b(P)\rangle = F(n_+\cdot P')\,\bar{N}(P')\,\Gamma\,\Lambda_b(P)\,.$$

  - Only a single (soft) form factor in the large recoil limit.
  - Symmetry relations still hold including the leading-power hard spectator interaction.
  - Symmetry breaking effects induced by the perturbative and power corrections.

SCET factorization for $\Lambda_b \to p$ form factors

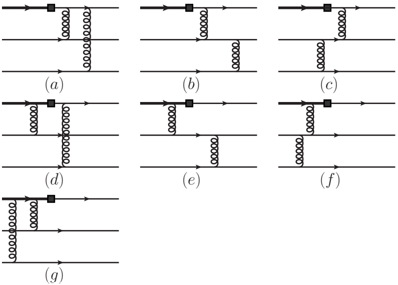
- QCD factorization at leading power in Λ/m_b [Wei Wang, 2011]:

$$F(n_+ \cdot P') = \Phi_{\Lambda_b}(\omega_i) \otimes H(\omega_i, x_i) \otimes \Phi_N(x_i) + \mathcal{O}(\Lambda_{QCD}/E)\,.$$

  - Leading power contribution due to the exchanges of two hard-collinear gluons.
  - Leading power contribution completely calculable in QCD factorization.
  - The scaling behaviour (different from the soft contribution):

$$F(n_+ \cdot P') \sim \mathcal{O}(\Lambda_{QCD}^2/(n_+ \cdot P')^2)\,.$$

- The LO QCD diagrams:
  - Leading power contributions from the diagrams (a), (b), (f), but (b) + (f) vanishes.
  - For the diagram (a), both two gluons are transverse polarized.
  - Need the light-cone projectors of both the Λ_b-baryon and nucleon for a complete calculation even at tree level [BFWY, 2013; BFMS, 2000].



Theoretical wishlist

- Systematic understanding of the (high-twist) B-meson distribution amplitudes.
  - Renormalization properties beyond the one-loop approximation [conformal symmetry].
  - Perturbative constraints at large ω_i [OPE technique].
  - Renormalon analysis and the renormalization-scheme dependence.
  - Precision determinations of the inverse moment λ_B.
- QCD factorization for the subleading power corrections.
  - SCET analysis for the pion-photon form factor as the first step [operator structures, symmetry constraints, etc].
  - General treatment of the rapidity divergences in the (naïve)-factorization formulae.
  - Rigorous factorization proof taking into account the Glauber gluons.
  - Novel resummation techniques for enhanced logarithms [symmetry, geometry].
- Technical issues for future improvements.
  - Factorization techniques for electromagnetic corrections.
  - NNLO QCD computations for $B \to V\ell\ell$ and $B \to V\gamma$.
  - QCD factorization for the radiative and electroweak penguin decays of the Λ_b-baryon.
  - Improved understanding of the parton-hadron duality violation.
- Very promising future for QCD aspects of heavy-quark physics!