The radiative decays $h_c \to \gamma\eta^{(\prime)}$ with relativistic corrections

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Outline

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- $h_c \to \gamma\eta^{(\prime)}$ with relativistic corrections
- Numerical Results
- Conclusion

Motivation

Experimental aspect

$$\mathcal{B}(h_c \to \gamma\eta) = (4.7 \pm 1.5 \pm 1.4) \times 10^{-4}$$

$$\mathcal{B}(h_c \to \gamma\eta') = (1.52 \pm 0.27 \pm 0.29) \times 10^{-3}$$

$$R_{h_c} = \frac{\mathcal{B}(h_c \to \gamma\eta)}{\mathcal{B}(h_c \to \gamma\eta')} = (30.7 \pm 11.3 \pm 8.7)\%$$

Theoretical aspect

	$\mathcal{B}(h_c \to \gamma\eta)$	$\mathcal{B}(h_c \to \gamma\eta')$	R_{h_c}
Zhu	1.30×10^{-4}	1.94×10^{-3}	6.7%
Fan	1.1×10^{-4}	0.37×10^{-3}	30.1%

$$|\eta_8\rangle = |u\bar{u} + d\bar{d} - 2s\bar{s}\rangle/\sqrt{6}$$
$$|\eta_0\rangle = |u\bar{u} + d\bar{d} + s\bar{s}\rangle/\sqrt{3}$$

- ♣ M. Ablikim et al. (BESIII Collaboration) Phys. Rev. Lett. 116, 251802 (2016).
- ♣ R.-L. Zhu and J.-P. Dai, Phys. Rev. D94, 094034 (2016). NRQCD
- ♣ Q. Wu, G. Li, and Y. Zhang, Eur. Phys. J. C77, 336 (2017). Meson loops model
- ♣ C.-J. Fan and J.-K. He (2019), arXiv:1906.07353. pQCD

Bethe-Salpeter equation

For a quank-antiquark bound state:

$$(\not{f} - \hat{m}_c)\Psi(K,q)(\bar{\not{f}} + \hat{m}_c) = i\int \frac{\mathrm{d}^4 q'}{(2\pi)^4}\mathcal{K}(K,q,q')\Psi(K,q')$$

quark and antiquark momenta:

$$f = \frac{K}{2} + q, \qquad \bar{f} = \frac{K}{2} - q$$

In the rest frame of the bound state and CIA:

$$\mathcal{K}(K,q,q') = \mathcal{K}(\hat{q},\hat{q}') \qquad \begin{cases} \hat{q}^{\mu} \equiv q^{\mu} - \frac{q_{\parallel}}{M}K^{\mu} \Rightarrow (0,\mathbf{q}) \\ q_{\parallel} \equiv \frac{q\cdot K}{M} \qquad\qquad \Rightarrow q^0 \end{cases}$$

The Salpeter wave function:

$$\psi(\hat{q}) = \frac{i}{2\pi}\int \mathrm{d}q_{\parallel}\Psi(K,q)$$

Salpeter wave function

$$\psi(\hat{q}) = \hat{q} \cdot \epsilon \left[1 + \frac{\not{K}}{M} + \frac{\hat{\not{q}}\not{K}}{\hat{m}_c M} \right] \gamma^5 f(\hat{q}^2)$$

$$f(\hat{q}^2) \quad = \quad N_A \left(\frac{2}{3} \right)^{\frac{1}{2}} \frac{1}{\pi^{\frac{3}{4}} \beta_{h_c}^{\frac{5}{2}}} \left| \hat{\mathbf{q}} \right| e^{-\frac{\hat{\mathbf{q}}^2}{2\beta_{h_c}^2}}$$

normalization constant: N_A harmonic oscillator parameter: β_{h_c}

- G.-L. Wang, Phys. Lett. B650, 15 (2007).
- S. Bhatnagar and L. Alemu, Phys. Rev. D97, 034021 (2018).

The typical Feynman diagrams

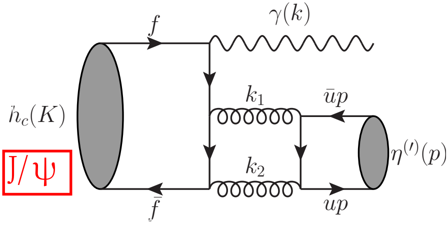

Figure: quark-untiquark contributions.

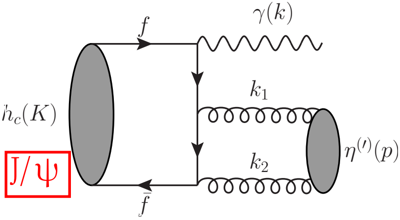

Figure: gluonic contributions.

♣ J.-K. He and Y.-D. Yang, Nucl. Phys. B943, 114627 (2019).

Amplitude (quark-untiquark contributions)

$$T^q_{\alpha\beta} E^\alpha(K)\epsilon^{*\beta}(k) = \frac{1}{2}\int \frac{\mathrm{d}^4 k_1}{(2\pi)^4} \mathcal{M}_{\alpha\beta\mu\nu}\mathcal{A}^{\mu\nu} \frac{i}{k_1^2 + i\epsilon}\frac{i}{k_2^2 + i\epsilon} E^\alpha(K)\epsilon^{*\beta}(k)$$

The coupling of $h_c \to \gamma g^* g^*$:

$$\mathcal{M}^{\alpha\beta\mu\nu} E_\alpha(K)\epsilon^*_\beta(k)\epsilon^*_\mu(k_1)\epsilon^*_\nu(k_2) = \sqrt{3}\int \frac{\mathrm{d}^4 q}{(2\pi)^4} \mathrm{Tr}\left[\Psi(K,q)\mathcal{O}(q)\right] \simeq -i\sqrt{3}\int \frac{\mathrm{d}^3 \hat{q}}{(2\pi)^3} \mathrm{Tr}\left[\psi(\hat{q})\mathcal{O}(\hat{q})\right]$$

The coupling of $g^* g^* - \eta^{(\prime)}$:

$$\mathcal{A}^{\mu\nu} = -i(4\pi\alpha_s)\delta_{ab}\epsilon^{\mu\nu\rho\sigma} k_{1\rho}k_{2\sigma} \sum_{q=u,d,s} \frac{f^q_{\eta^{(\prime)}}}{6}\int_0^1 du\phi^q(u)\Big(\frac{1}{\bar{u}k_1^2 + uk_2^2 - u\bar{u}p^2 - m_q^2} + (u \leftrightarrow \bar{u})\Big)$$

The light-cone DA:

$$\phi^q(u) \quad = \quad 6u(1-u)\Big[1 + \sum_{n=2,4\cdots} c^q_n(\mu) C^{\frac{3}{2}}_n(2u-1)\Big]$$

- ♣ A. Ali and A. Ya. Parkhomenko, Phys. Rev. D65, 074020 (2002).
- ♣ S. S. Agaev et al., Phys. Rev. D90, 074019 (2014).

Helicity amplitude (quark-untiquark contributions)

Lorentz invariance, parity conservation, and gauge invariance

$$T^q_{\alpha\beta} \propto h_{\alpha\beta}, \qquad h_{\alpha\beta} = -g_{\alpha\beta} + \frac{k_\alpha K_\beta}{K\cdot k}$$

Helicity projector:

$$\mathbb{P}^{\alpha\beta} = \frac{1}{2} h_{\alpha'\beta'}\Big(-g^{\alpha\alpha'} + \frac{K^\alpha K^{\alpha'}}{M^2}\Big)\Big(-g^{\beta\beta'}\Big) = \frac{1}{2}\Big(-g^{\alpha\beta} + \frac{k^\alpha K^\beta}{k\cdot K}\Big)$$

Helicity amplitude:

$$H^q_{QCD} = T^{\alpha\beta}\mathbb{P}_{\alpha\beta} = \frac{2Q_c}{3\sqrt{3}}\sqrt{4\pi\alpha}(4\pi\alpha_s)^2 \sum_{q=u,d,s} f^q_{\eta^{(\prime)}} H_q$$

♣ J. G. Körner, J. H. Kühn, M. Krammer, and H. Schneider, Nucl. Phys. B229, 115 (1983).

Helicity amplitude (gluonic contributions)

The matrix elements of $\eta^{(\prime)}$ over two-gluon fields:

$$\langle \eta^{(\prime)}(p)|A_\alpha^a(x)A_\beta^b(y)|0\rangle = \frac{1}{4}\epsilon_{\alpha\beta\mu\nu}\frac{k^\mu p^\nu}{p\cdot k}\frac{C_F}{\sqrt{3}}\frac{\delta^{ab}}{8}f^1_{\eta^{(\prime)}}\int \mathrm{d}u e^{i(up\cdot x+\bar{u}p\cdot y)}\frac{\phi^g(u)}{u(1-u)}$$

Gluonic twist-2 DA:

$$\phi^g(u) = 30u^2(1-u)^2 \sum_{n=2,4\cdots} c_n^g(\mu)C_{n-1}^{\frac{5}{2}}(2u-1)$$

Effective decay constant: $f^1_{\eta^{(\prime)}} = \frac{1}{\sqrt{3}}(f^u_{\eta^{(\prime)}} + f^d_{\eta^{(\prime)}} + f^s_{\eta^{(\prime)}})$

Helicity amplitude:

$$H^g_{QCD} = \frac{2Q_c}{9}\sqrt{4\pi\alpha}(4\pi\alpha_s)f^1_{\eta^{(\prime)}}H_g$$

- P. Ball and G. W. Jones, JHEP 08, 025 (2007).
- S. S. Agaev et al., Phys. Rev. D90, 074019 (2014).

Decay widths and light-cone DAs

Decay widths:

$$\Gamma(h_c \to \gamma\eta^{(\prime)}) = \frac{2}{3}\frac{M^2 - m^2}{16\pi M^3}\left|H^q_{QCD} + H^g_{QCD}\right|^2$$

Light-cone DAs:

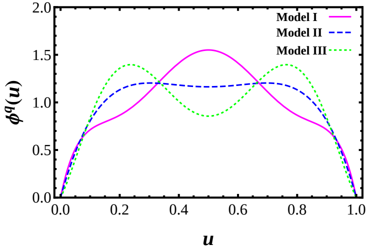

Figure: The shapes of DAs with the scale $\mu = m_c$.

Table: Gegenbauer coefficients at the scale $\mu_0 = 1\,\mathrm{GeV}$

Model	$c_2^q(\mu_0)$	$c_4^q(\mu_0)$	$c_2^g(\mu_0)$
I	0.10	0.10	−0.26
II	0.20	0.0	−0.31
III	0.25	−0.10	−0.25

♣ S. S. Agaev et al., Phys. Rev. D90, 074019 (2014).

Phenomenological parameters

FKS scheme (quark flavor basis):

$$f_{\eta}^{u(d)} = \frac{f_q}{\sqrt{2}}\cos\phi \qquad f_{\eta}^{s} = -f_s \sin\phi$$
$$f_{\eta'}^{u(d)} = \frac{f_q}{\sqrt{2}}\sin\phi \qquad f_{\eta'}^{s} = f_s \cos\phi$$

Table: The values of ϕ, f_q and f_s with three phenomenological approaches

	ϕ°	f_q/f_π	f_s/f_π
LEPs [1]	40.6 ± 0.9	1.10 ± 0.03	1.66 ± 0.06
ηTFF [2]	40.3 ± 1.8	1.06 ± 0.01	1.56 ± 0.24
η'TFF [2]	33.5 ± 0.9	1.09 ± 0.02	0.96 ± 0.04

♣ [1] R. Escribano and J.-M. Frère, JHEP 06, 029 (2005).
♣ [2] R. Escribano, P. Masjuan, and P. Sanchez-Puertas, Phys. Rev. D89, 034014 (2014).

Our results

Table: The quark-antiquark contributions

	LEPs	ηTFF	η'TFF	Exp
$\mathcal{B}(h_c \to \gamma\eta)$	3.4×10^{-6}	6.0×10^{-6}	1.9×10^{-4}	$(4.7 \pm 1.5 \pm 1.4) \times 10^{-4}$
$\mathcal{B}(h_c \to \gamma\eta')$	1.13×10^{-3}	1.02×10^{-3}	0.60×10^{-3}	$(1.52 \pm 0.27 \pm 0.29) \times 10^{-3}$
R_{h_c}	0.3%	0.6%	31.7%	$(30.7 \pm 11.3 \pm 8.7)$%

Table: The gluonic contributions

	LEPs	ηTFF	η'TFF	Exp
$\mathcal{B}(h_c \to \gamma\eta)$	1.3×10^{-6}	2.3×10^{-6}	0.7×10^{-4}	$(4.7 \pm 1.5 \pm 1.4) \times 10^{-4}$
$\mathcal{B}(h_c \to \gamma\eta')$	0.58×10^{-3}	0.53×10^{-3}	0.31×10^{-3}	$(1.52 \pm 0.27 \pm 0.29) \times 10^{-3}$
R_{h_c}	0.2%	0.4%	23.4%	$(30.7 \pm 11.3 \pm 8.7)$%

Table: Both quark-antiquark and gluonic contributions

	LEPs	ηTFF	η'TFF	Exp
$\mathcal{B}(h_c \to \gamma\eta)$	8.5×10^{-6}	1.5×10^{-5}	4.7×10^{-4}	$(4.7 \pm 1.5 \pm 1.4) \times 10^{-4}$
$\mathcal{B}(h_c \to \gamma\eta')$	2.97×10^{-3}	2.68×10^{-3}	1.57×10^{-3}	$(1.52 \pm 0.27 \pm 0.29) \times 10^{-3}$
R_{h_c}	0.3%	0.6%	30.3%	$(30.7 \pm 11.3 \pm 8.7)$%

Showing the contributions of relativistic corrections

Table: The quark-untiquark contributions

	zero-binding approximation	B-S equation	Exp
$\mathcal{B}(h_c \to \gamma\eta)$	0.7×10^{-4}	1.9×10^{-4}	$(4.7 \pm 1.5 \pm 1.4) \times 10^{-4}$
$\mathcal{B}(h_c \to \gamma\eta')$	0.26×10^{-3}	0.6×10^{-3}	$(1.52 \pm 0.27 \pm 0.29) \times 10^{-3}$
R_{h_c}	27.5%	31.7%	$(30.7 \pm 11.3 \pm 8.7)$%

Table: The gluonic contributions

	zero-binding approximation	B-S equation	Exp
$\mathcal{B}(h_c \to \gamma\eta)$	0.4×10^{-4}	0.7×10^{-4}	$(4.7 \pm 1.5 \pm 1.4) \times 10^{-4}$
$\mathcal{B}(h_c \to \gamma\eta')$	0.19×10^{-3}	0.31×10^{-3}	$(1.52 \pm 0.27 \pm 0.29) \times 10^{-3}$
R_{h_c}	23.8%	23.4%	$(30.7 \pm 11.3 \pm 8.7)$%

Table: Both quark-untiquark and gluonic contributions

	zero-binding approximation	B-S equation	Exp
$\mathcal{B}(h_c \to \gamma\eta)$	1.9×10^{-4}	4.7×10^{-4}	$(4.7 \pm 1.5 \pm 1.4) \times 10^{-4}$
$\mathcal{B}(h_c \to \gamma\eta')$	0.63×10^{-3}	1.57×10^{-3}	$(1.52 \pm 0.27 \pm 0.29) \times 10^{-3}$
R_{h_c}	30.2%	30.3%	$(30.7 \pm 11.3 \pm 8.7)$%

♣ C.-J. Fan and J.-K. He (2019), arXiv:1906.07353.

The prediction of the mixing angle ϕ

Using our calculation

$$R_{h_c} = \frac{M^2 - m_\eta^2}{M^2 - m_{\eta'}^2} \frac{|H^q_{QCD} + H^g_{QCD}|^2_{m=m_\eta}}{|H^q_{QCD} + H^g_{QCD}|^2_{m=m_{\eta'}}}$$

and the ratio

$$\frac{\Gamma(\eta \to \gamma\gamma)}{\Gamma(\eta' \to \gamma\gamma)} = \frac{m_\eta^3}{m_{\eta'}^3} \left(\frac{5\sqrt{2}\frac{f_s}{f_q} - 2\tan\phi}{5\sqrt{2}\frac{f_s}{f_q}\tan\phi + 2} \right)^2$$

Comparing the experimental values

$$R_{h_c}^{exp} = (30.7 \pm 11.3 \pm 8.7)\%$$
$$\Gamma^{exp}(\eta' \to \gamma\gamma) = 4.36(14)\ \mathrm{KeV}$$
$$\Gamma^{exp}(\eta \to \gamma\gamma) = 0.516(18)\ \mathrm{KeV}$$

$$\Rightarrow \quad \phi = 33.8^\circ \pm 2.5^\circ$$

The dependence of R_{h_c} on the mixing angle ϕ

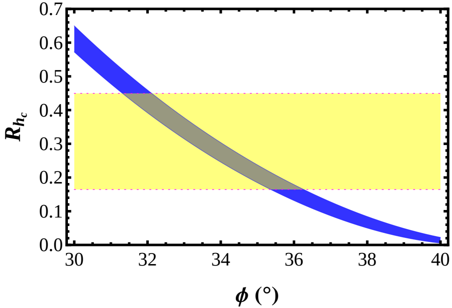

Figure: The dependence of the ratio R_{h_c} on the mixing angle ϕ. The blue band is our calculated results. The yellow band denotes the experimental value of R_{h_c} with 1σ uncertainty.

UKQCD collaboration: $\phi = 34^\circ \pm 3^\circ$ ETM collaboration: $\phi = 38.8^\circ \pm 3.3^\circ$

- E. B. Gregory, A. C. Irving, C. M. Richards, and C. McNeile (UKQCD), Phys. Rev. D86, 014504 (2012).
- K. Ottnad and C. Urbach (ETM), Phys. Rev. D97, 054508 (2018).

Conclusion

- ✓ The helicity amplitude from the quark-antiquark contributions is insensitive to the light quark masses and the shapes of the $\eta^{(\prime)}$ DAs.
- ✓ Both the quark-antiquark content contributions and the gluonic content contributions are important in the decays $h_c \to \gamma\eta^{(\prime)}$.
- ✓ The contributions from the relativistic corrections are actually rather significant and meaningful in the exclusive P-wave decays $h_c \to \gamma\eta^{(\prime)}$, although there is no IR divergence.

Thanks for your attention!

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