D meson mixing via dispersion relation

Hsiang-nan Li
Academia Sinica/Jinan U
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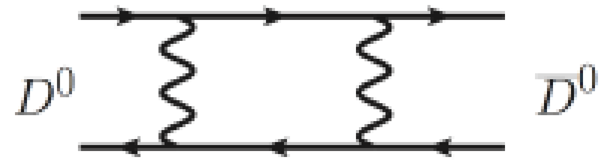
In collaboration with H. Umeeda (梅枝), F. Xu, F. Yu

Goal

- Try to resolve long-standing (2-decade) challenge---how to understand large D meson mixing?
- All theories predicted mixing parameters x, y < 10E-5, but data show x, y > 10E-3

Year	Exper.	D^0 final state(s)	$y(\%)$
2007	BABAR [31]	$K^+K^-, \pi^+\pi^-$	$1.03 \pm 0.33 \pm 0.19$
2007	Belle [30]	$K^+K^-, \pi^+\pi^-$	$1.31 \pm 0.32 \pm 0.25$
2001	CLEO [32]	$K^+K^-, \pi^+\pi^-$	$-1.2 \pm 2.5 \pm 1.4$
2001	Belle [33]	K^+K^-	$-0.5 \pm 1.0^{+0.7}_{-0.8}$
2000	FOCUS [34]	K^+K^-	$3.42 \pm 1.39 \pm 0.74$
1999	E791 [35]	K^+K^-	$0.8 \pm 2.9 \pm 1.0$

$D^0 - \overline{D}^0$ Mixing



- The time evolution

2X2 matrices

$$i\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix}$$

virtual real contribution

- Mass eigenstates in terms of weak eigenstates

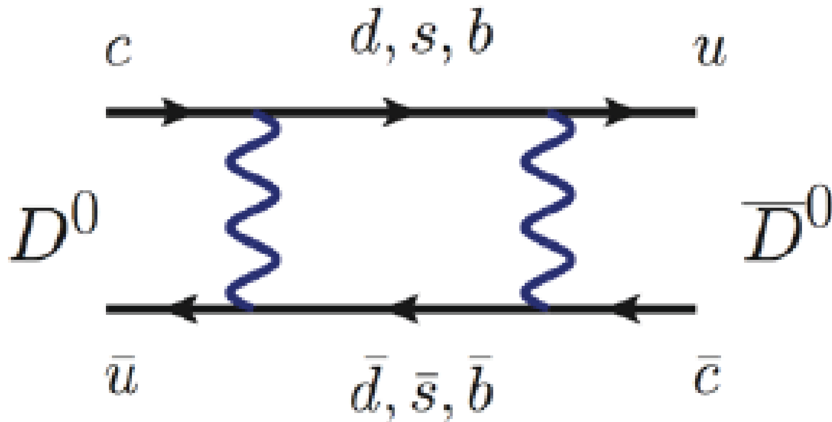
$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle$$

- Mass difference and Width difference

$$x \equiv \frac{\Delta m}{\Gamma} = \frac{m_1 - m_2}{\Gamma}$$

$$y \equiv \frac{\Delta\Gamma}{2\Gamma} = \frac{\Gamma_1 - \Gamma_2}{2\Gamma}$$

SU(3) breaking



third generation can be neglected

$$V_{cd}^* V_{ud} + V_{cs}^* V_{us} + V_{cb}^* V_{ub} = 0$$

$-\lambda * 1$ $\quad$ $1 * \lambda$ $\quad$ $\lambda^2 * \lambda^3$

$\lambda \sim 0.2$

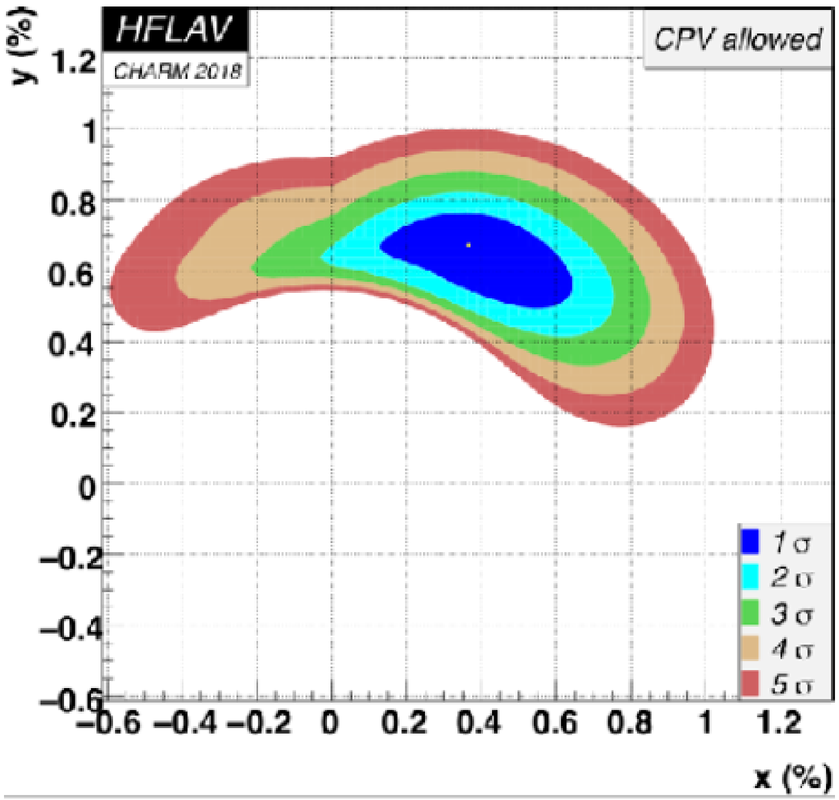
Wolfenstein Parameter; Cabibbo angle

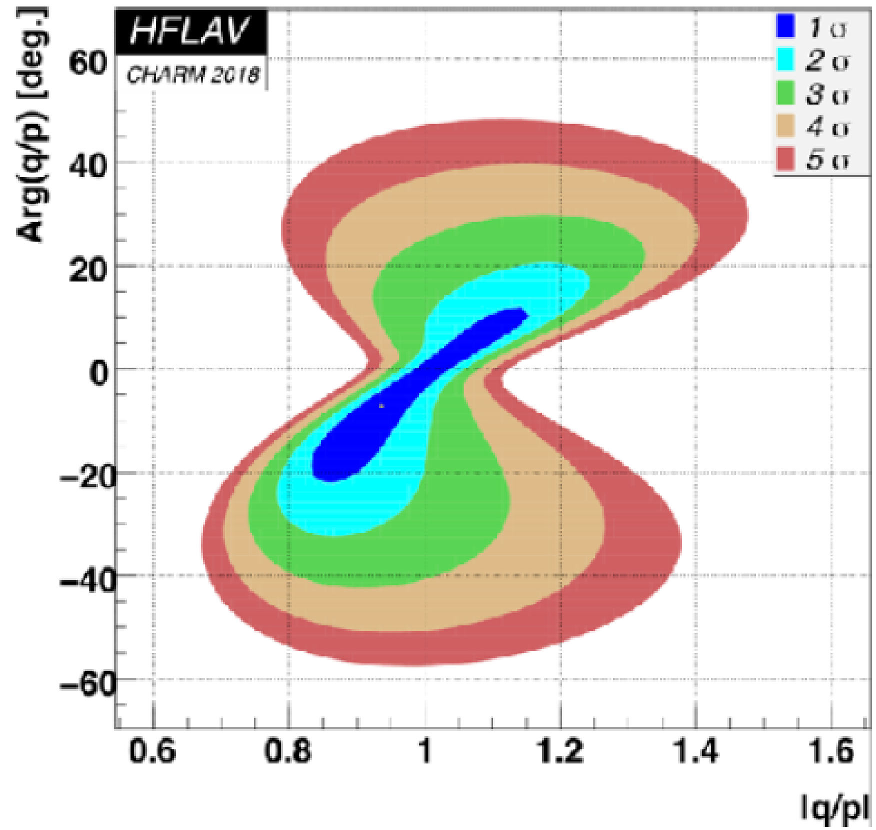
GIM: x~y~0 in the SU(3) limit

Non-zero mixing from SU(3) breaking effects

more precisely, U-spin symmetry breaking

$D^0 - \bar{D}^0$ mixing: data





$(x, y) = (0, 0)$ is excluded by 11.5σ

➡ Mixing is confirmed

$(|q/p|, \arg) = (1, 0)$ is allowed

➡ No evidence of CP violation

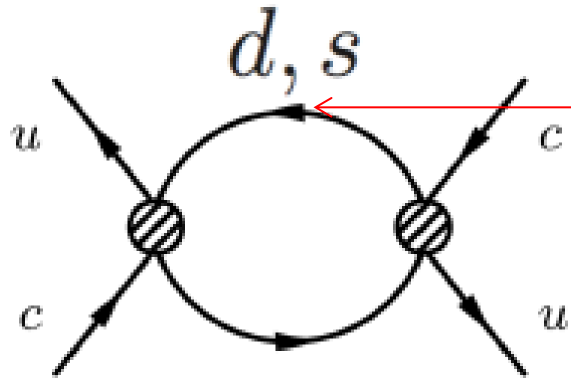
Challenges

- Charm not heavy enough to apply HQET, PQCD, NRQCD… ; large $1/m_C, \alpha_S(m_C)$ corrections
- Charm not light enough to apply chiral perturbation, and to allow finite number of decay channels
- GIM mechanism, charm mixing due to SU(3) breaking, difficult to estimate
- Multi-particle channels give significant contribution to y, difficult to calculate

Two major approaches: inclusive and exclusive

Inclusive approach:

quark level **Heavy Quark Expansion**



based on quark-hadron duality

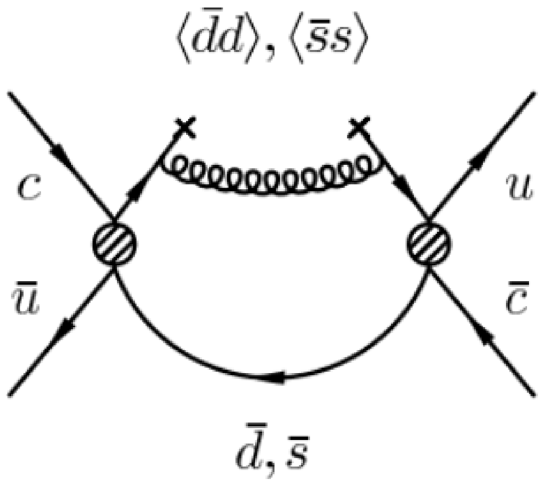
small SU(3) breaking

$x \sim (m_s/m_c)^4$ $y \sim (m_s/m_c)^6$

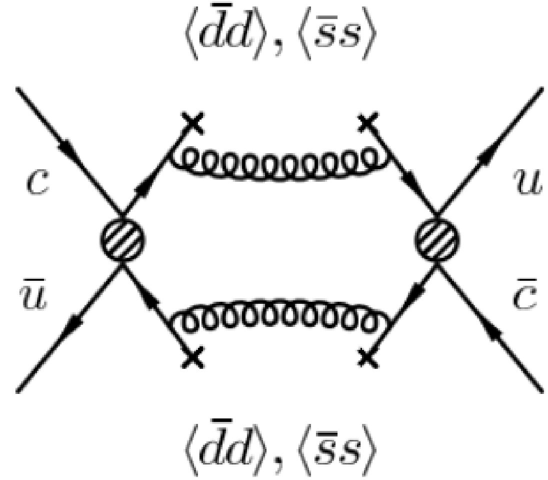
$x \sim 10^{-6}$ $y \lesssim 0.9 \times 10^{-5}$ [Bobrowski, Lenz, 09'] [Lenz,et al, 10']

Short-distance contributions are small

Higer-dimensional operators [1002.4794]



D=9



D=12

$\mathcal{O}(\alpha_s (4\pi) \langle \bar{q}q \rangle / m_c^3)$

$\mathcal{O}(\alpha_s^2 (4\pi)^2 \langle \bar{q}q \rangle^2 / m_c^6)$

y	no GIM	with GIM
$D = 6, 7$	$2 \cdot 10^{-2}$	$5 \cdot 10^{-7}$
$D = 9$	$5 \cdot 10^{-4}$	?
$D = 12$	$2 \cdot 10^{-5}$	?

do not work

Exclusive Approach

$$y \approx \frac{\Gamma_+ - \Gamma_-}{2\Gamma} = \frac{1}{2}\sum_n (Br(D_+ \to n) - Br(D_- \to n))$$

$$= \frac{1}{2\Gamma}\sum_n \rho_n \left(|\langle D_+|H_w|n\rangle|^2 - |\langle D_-|H_w|n\rangle|^2\right),$$

phase space factor

$CP|n\rangle = \eta_{CP}|\bar{n}\rangle$ $\quad \mathcal{A}(\overline{D}^0 \to n) = \mathcal{A}(D^0 \to \bar{n})$ $\quad$ No CPV

$$y = \frac{1}{2\Gamma}\sum_n \rho_n \eta_{CP}(n)(\langle D^0|H_w|n\rangle\langle\bar{n}|H_w|D^0\rangle + \langle D^0|H_w|\bar{n}\rangle\langle n|H_w|D^0\rangle)$$

$$= \sum_n \eta_{CKM}(n)\eta_{CP}(n)\cos\delta_n\sqrt{Br(D^0 \to n)Br(D^0 \to \bar{n})},$$

$(-1)^{n_S}$

number of s quarks

sum up all the intermediate states

$$y_{PP+VP} = (0.36 \pm 0.26)\% \text{ or } (0.24 \pm 0.22)\%$$

[Cheng, Chiang, 2010]

Factorization-assisted topological amplitudes

Li et al. 2012, 2017

- With higher precision in global fit

$$y_{PP+PV} = (0.21 \pm 0.07)\%,$$
$$y_{VV} = (0.28 \pm 0.47) \times 10^{-3}$$

$\chi^2 = 6.9$ per d.o.f. reduced from 87 by Cheng, Chiang

- PP, PV, VV (amount up to 50% Br of D decays) cannot explain y
- Other 2-body and multi-body modes relevant
- Not applicable to evaluation of x; exclusive approach not practical

possibilities discussed in the literature

HQE is not valid for charm? likely

-testable in lifetime ratio?

Lenz and Rauh, 2013

Higer dimensional operator (D=9, 12)?

-able to avoid severe GIM cancellation? unlikely

Bigi and Uraltsev, 2001

Beyond the standard model?

Golowich and Pakvasa and Petrov, 2007 last resort

20% breakdown of quark-hadron duality?

Jubb, Kirk, Lenz and Tetlalmatzi-Xolocotzi, 2017 who knows?

Way out of desperate situation?

OPE result for short-distance

D meson	B_s meson	B_d meson
Golowich and Petrov 2005	Artuso, Borissov and Lenz, 2016	Artuso, Borissov and Lenz, 2016
SM $\begin{cases} x_D \simeq 6 \times 10^{-7} \\ y_D \simeq 6 \times 10^{-7} \end{cases}$	SM $\begin{cases} \Delta M_s = (18.3 \pm 2.7)\mathrm{ps}^{-1} \\ \Delta\Gamma_s = (0.088 \pm 0.020)\mathrm{ps}^{-1} \end{cases}$	SM $\begin{cases} \Delta M_d = (0.528 \pm 0.078)\ \mathrm{ps}^{-1} \\ \Delta\Gamma_d = (2.61 \pm 0.59) \cdot 10^{-3}\ \mathrm{ps}^{-1} \end{cases}$
much below data Suppressed by GIM		
HFLAV at CHARM18	HFAG	HFAG
Exp. $\begin{cases} x_D = 3.6^{+2.1}_{-1.6} \times 10^{-3} \\ y_D = 6.7^{+0.6}_{-1.3} \times 10^{-3} \end{cases}$	Exp. $\begin{cases} \Delta M_s = (17.757 \pm 0.021)\ \mathrm{ps}^{-1} \\ \Delta\Gamma_s = (0.082 \pm 0.006)\ \mathrm{ps}^{-1} \end{cases}$	Exp. $\begin{cases} \Delta M_d = (0.5055 \pm 0.0020)\ \mathrm{ps}^{-1} \\ \Delta\Gamma_d = 0.66(1 \pm 10) \cdot 10^{-3}\ \mathrm{ps}^{-1} \end{cases}$

- Order of magnitude is not reproduced for D meson.
- The SM is in agreement with data for B(d, s) meson.

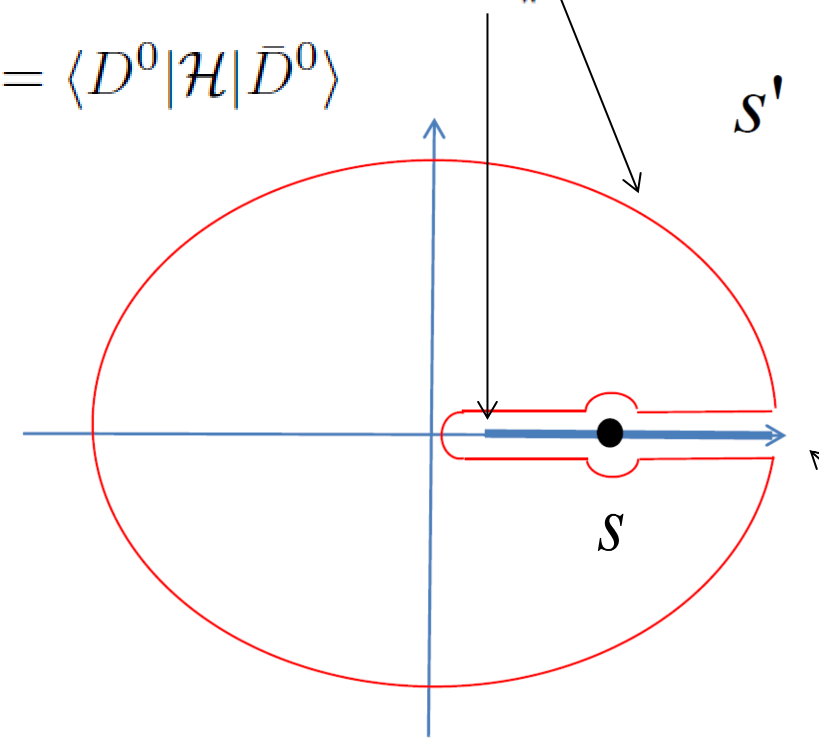
Idea:
connection between high and low mass regions, via which HQE constrains D mixing

Dispersion relation

- Consider "fictitious D meson" of mass s

real part $M_{12}(s) = -\frac{P}{2\pi}\int_{4m_\pi^2}^{R} ds' \frac{\Gamma_{12}(s')}{s'-s}$ imaginary part

$$M_{12}(s) - \frac{i}{2}\Gamma_{12}(s) = \langle D^0|\mathcal{H}|\bar{D}^0\rangle$$



R large enough of order $\sim m_W^2$

branch cut caused by log function

Implementation

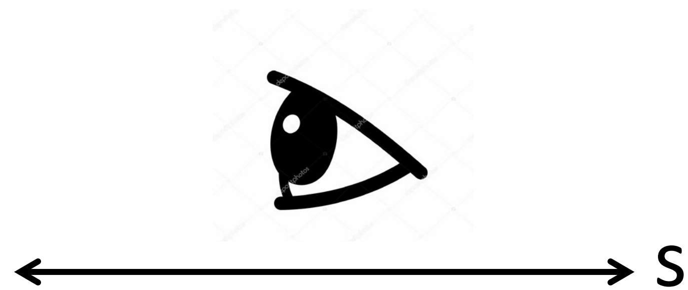
- $s > \Lambda^2$: known heavy-quark input
- $s < \Lambda^2$: unknown to be solved
- Divide both sides by measured total width

$$\int_{4m_\pi^2}^{\Lambda^2} ds' \frac{y(s')}{s-s'} = \pi x(s) + \int_{\Lambda^2}^{m_W^2/2} ds' \frac{y(s')}{s'-s} \equiv \omega(s)$$

↑ where y vanishes to avoid end-point singularity

“charge distribution” at low s

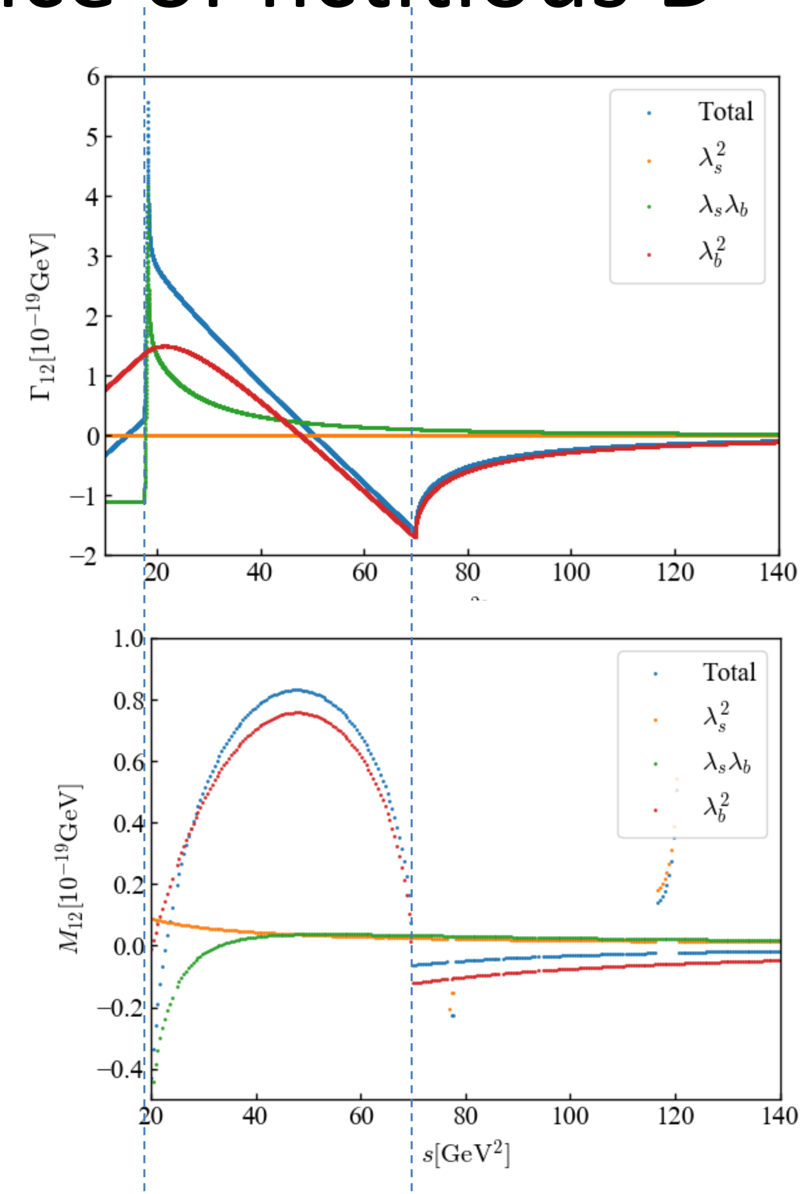
“potential” at high s



Width, mass difference of fictitious D

- Fictitious D meson can be heavier than b quark so b quark information is consistent on two sides of relation
- Γ_{12} and M_{12} both show thresholds of single b quark and b quark pair

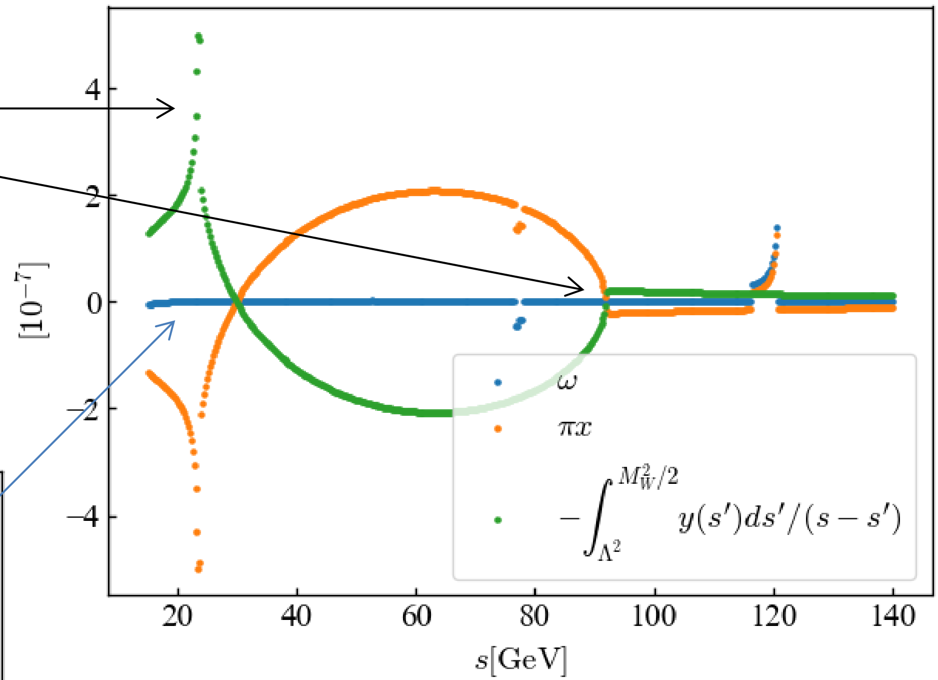
formulas referred to
Buras, Slominski, Steger, 1984

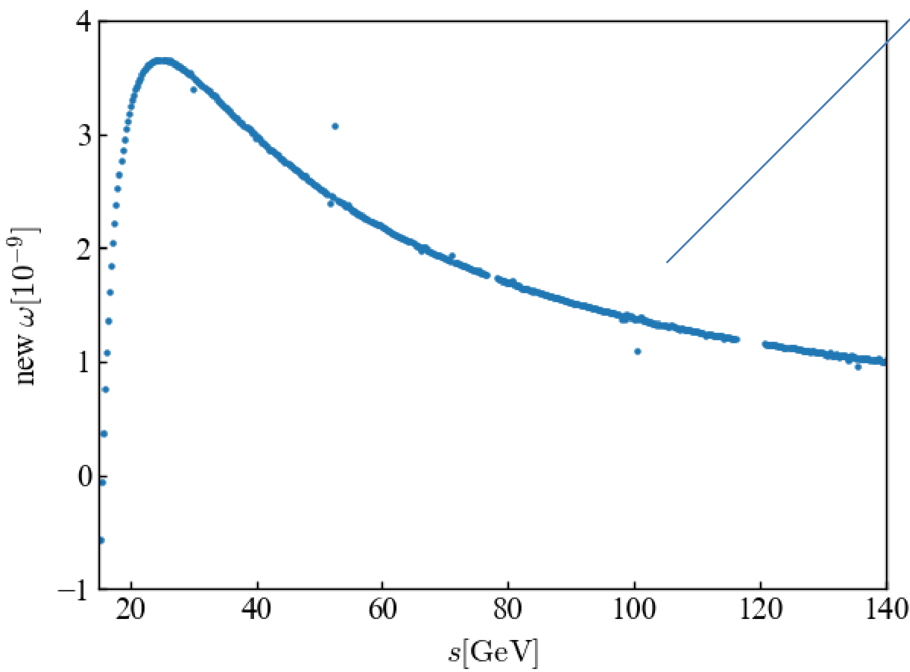


Known input at large s

cusps cancel

low s contribution
should not know
b quark threshold





use this input at high s
to constrain y at low s

Highly nontrivial to solve dispersion relation, took almost one year effort

One of many failed attempts

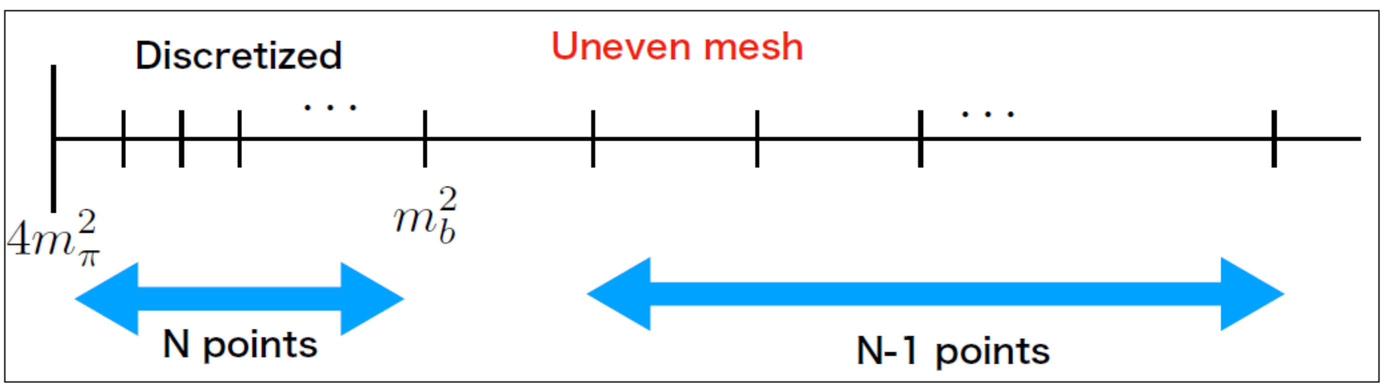
- Discretize integral equation

$$\frac{1}{\pi}\sum_{j=2}^{N} M_{ij}y_j = x(s_{N'+i}) + \frac{1}{\pi}P\int_{m_b^2}^{M_W^2}\frac{y(s)ds}{s-s_{N'+i}}, \quad (\text{for } i=1,2,\cdots,N-1)$$

unknowns   input

$$M_{ij} = \begin{cases} 1/(i-j), & i \neq j \\ 0, & i=j \end{cases}$$

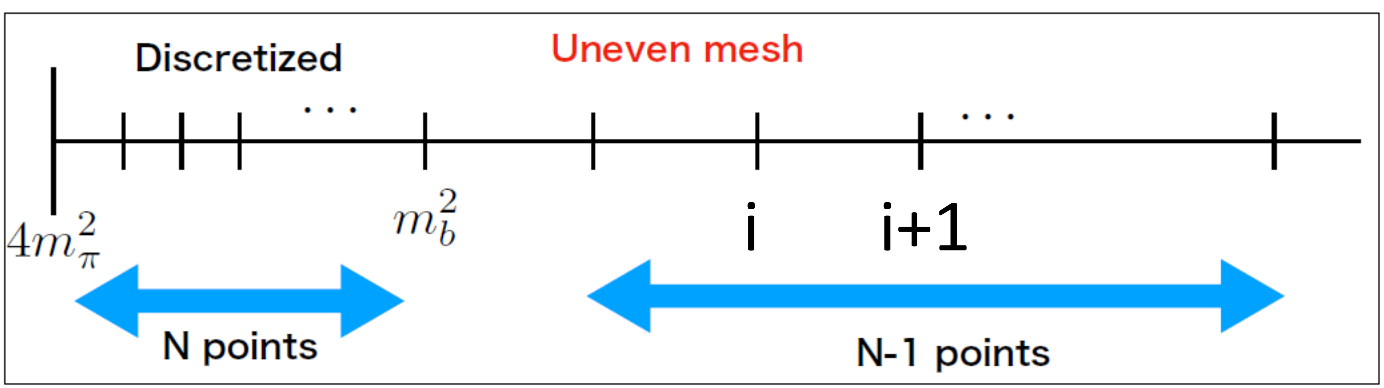
- initial condition $y_1 = y(4m_\pi^2) = 0$
- Inverse matrix to get $y(m_c^2)$, and then $x(m_c^2)$



Singular matrix

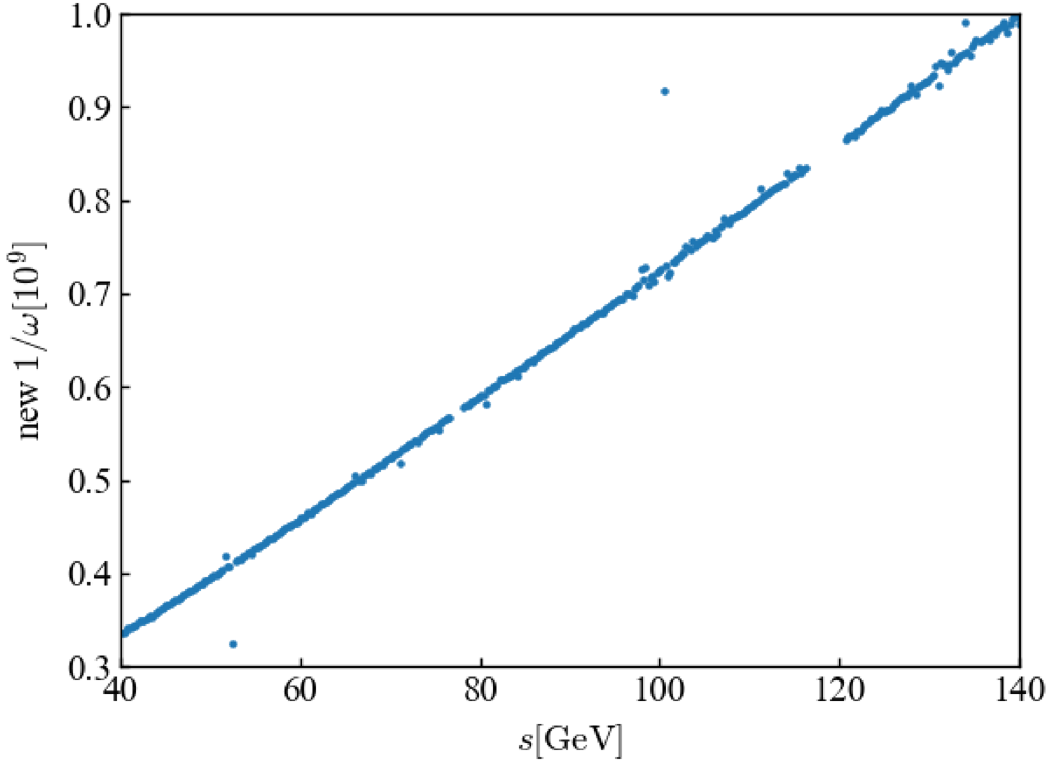
- It is called non-singular Fredholm integral equation: domains of s and s' do not overlap
- Matrix M becomes singular quickly for fine meshes, and solution diverges

the rows M_{ij} and $M_{(i+1)j}$ become almost identical, when mesh gets finer



After getting solution, we realized why M is singular. If it is not singular, there will be unique solution, which is perturbative solution, and small.

Deep frustration…until observing that $\omega(s)$ becomes 1/s quickly

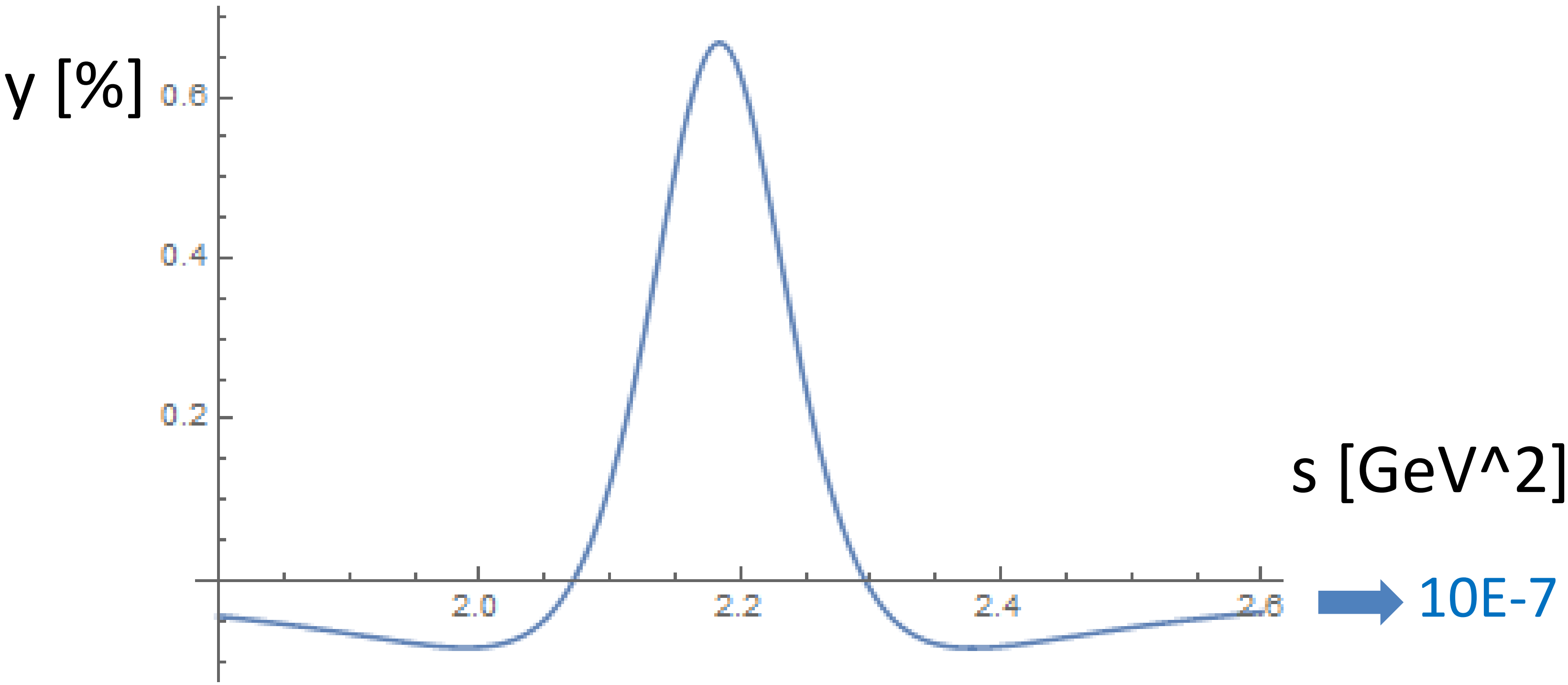


Qualitative behavior

- The shape of lhs is close to $1/(s-m^2)$
  ↑
  surprise! of order charm mass squared
- If exact $1/(s-m^2)$, solution is $y(s') \sim \delta(s'-m^2)$
- Recall pole term (δ -function) in spectral density for QCD sum rules…
- Deviation implies smearing of δ -function
- It hints that y shows substantial distribution only around the scale m_c^2 within narrow interval. Magnitude of y can be enhanced.

Preliminary solution y(s)

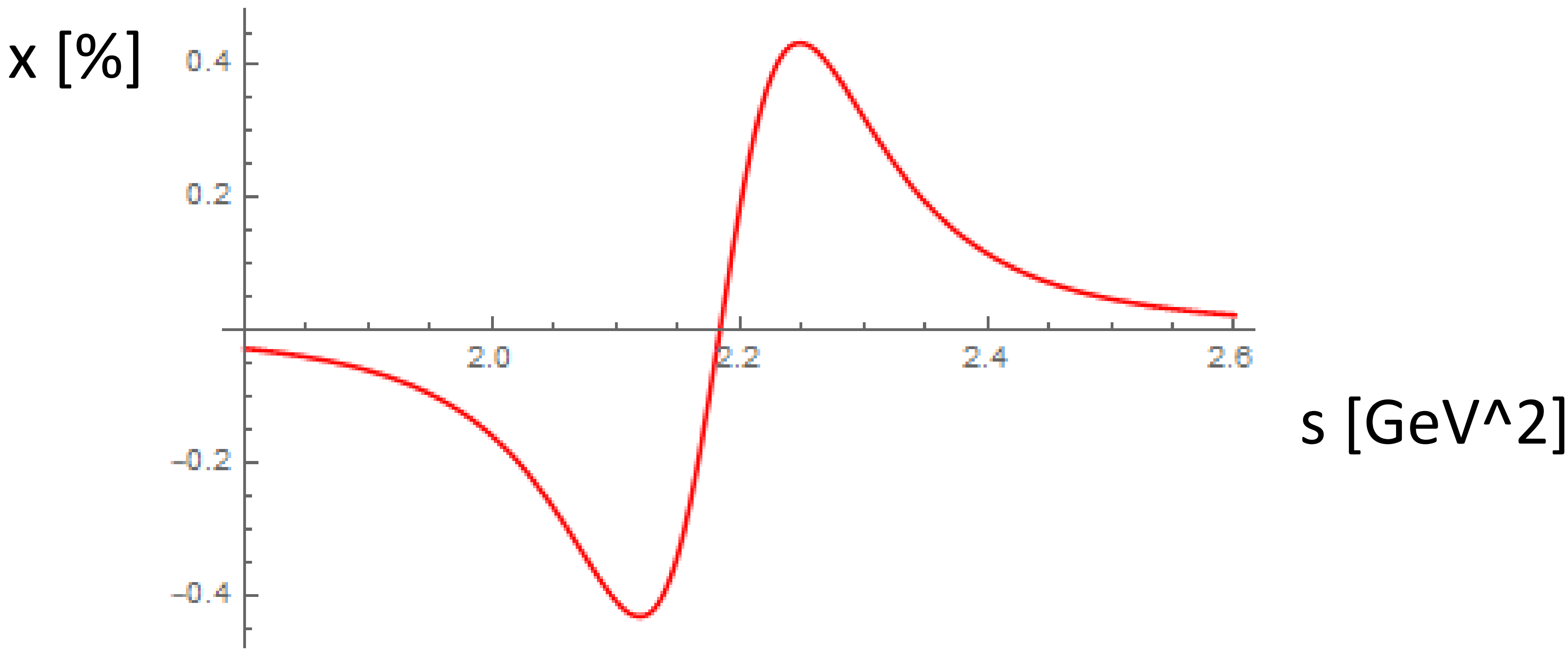
- Assume CP symmetry
- Input b quark mass 4.8 GeV



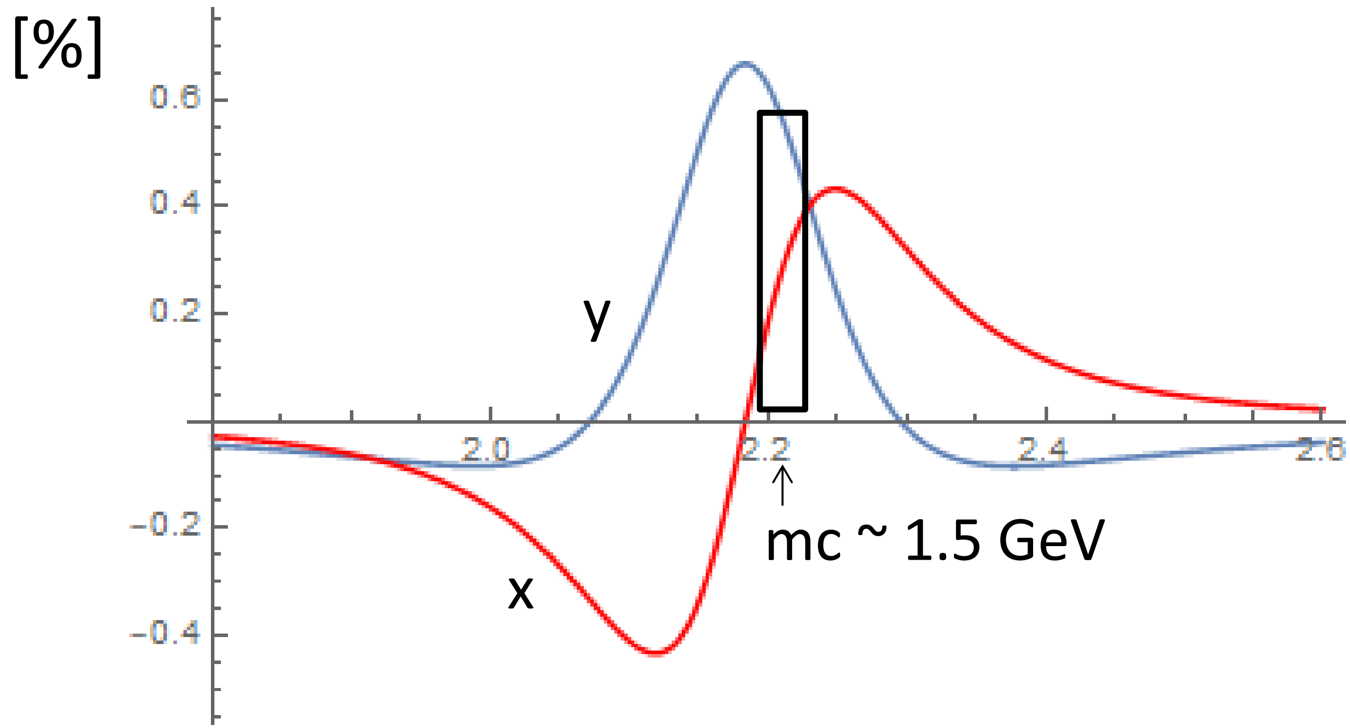
about 30-40%
theoretical uncertainty

Preliminary solution x(s)

- Substitute y(s’) into dispersion relation, we get x(s)



Comparison with data



our result
x ~ 0.3%
y ~ 0.5%

Parameter	No CP Violation	CP Violation Allowed	95% C.L. Interval CPV Allowed
$x(\%)$	$0.46^{+0.14}_{-0.15}$	0.32 ± 0.14	$[0.04, 0.62]$
$y(\%)$	0.62 ± 0.08	$0.69^{+0.06}_{-0.07}$	$[0.50, 0.80]$

PDG 2019

Summary

- Promising to understand D mixing in SM via dispersion relation (1st successful approach)
- Solve for x, y at low mass from inputs at high mass; $\lambda_s\lambda_b, \lambda_b^2$ contributions dominate
- Naïve estimate agrees with data in order of magnitude
- Need to implement (systematic) HQET, include QCD correction, bag parameters, allow CPV, analyze theoretical uncertainty,…
- Applicable to kaon mixing?

Back-up slides

Buras' formulas for B mixing

virtual contribution

$$M_{12} = \frac{G_F^2 f_p^2 m_p M_W^2 B_p}{12\pi^2}\left(\lambda_c^2 U_{cc}^{(d)} + \lambda_t^2 U_{tt}^{(d)} + 2\lambda_c\lambda_t U_{ct}^{(d)}\right)$$

real contribution

$$\Gamma_{12} = \frac{G_F^2 f_p^2 m_p M_W^2 B_p}{12\pi^2}\left(\lambda_c^2 U_{cc}^{(a)} + \lambda_t^2 U_{tt}^{(a)} + 2\lambda_c\lambda_t U_{ct}^{(a)}\right)$$

$$U_{ij}^{(x)} = A_{uu}^{(x)} + A_{ij}^{(x)} - A_{ui}^{(x)} - A_{uj}^{(x)} \qquad x = a, d$$

$$x_i = \frac{m_i^2}{M_W^2}$$

source of SU(3) breaking

$$A_{ij}^{(a)} = -\frac{\pi}{2x_h^2}\frac{1}{(1-x_i)(1-x_j)}\sqrt{(x_i - x_j)^2 + x_h^2 - 2x_h(x_i + x_j)}$$

$$\times\{(1+\tfrac{1}{4}x_i x_j)[3x_h^2 - x_h(x_i + x_j) - 2(x_i - x_j)^2] + 2x_h(x_i + x_j)(x_i + x_j - x_h)\}$$

Linear rise in width difference

$$\Gamma_{12}(\bar{B}_p^0 \to B_p^0) = -\frac{G_F^2 B_p f_B^2 m_B^{(p)}}{8\pi}[\lambda_c^{(p)^2} U_4 + \lambda_t^{(p)^2} U_5 + 2\lambda_t^{(p)} \lambda_c^{(p)} U_6]$$

$$U_4 = m_b^2 \eta_4^{(B)}[\sqrt{1-4z}(1+2z) - 1 + 6z^2 - 4z^3]$$

$$-\tfrac{8}{3} m_c^2 \eta_5^{(B)}[\sqrt{1-4z} - (1-z)^2],$$

$$U_5 = m_b^2 \eta_4^{(B)},$$

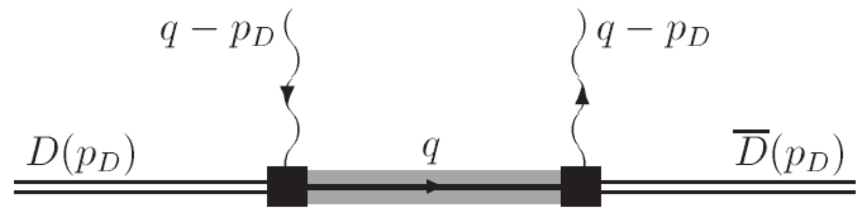
$$U_6 = m_b^2 \eta_4^{(B)}(3z^2 - 2z^3) + \tfrac{4}{3} m_c^2 \eta_5^{(B)}(1-z)^2,$$

no top contribution
linear rise renders
integral not convergent

$$M_{12}(s) = -\frac{P}{2\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\Gamma_{12}(s')}{s'-s}$$

Difference from literature

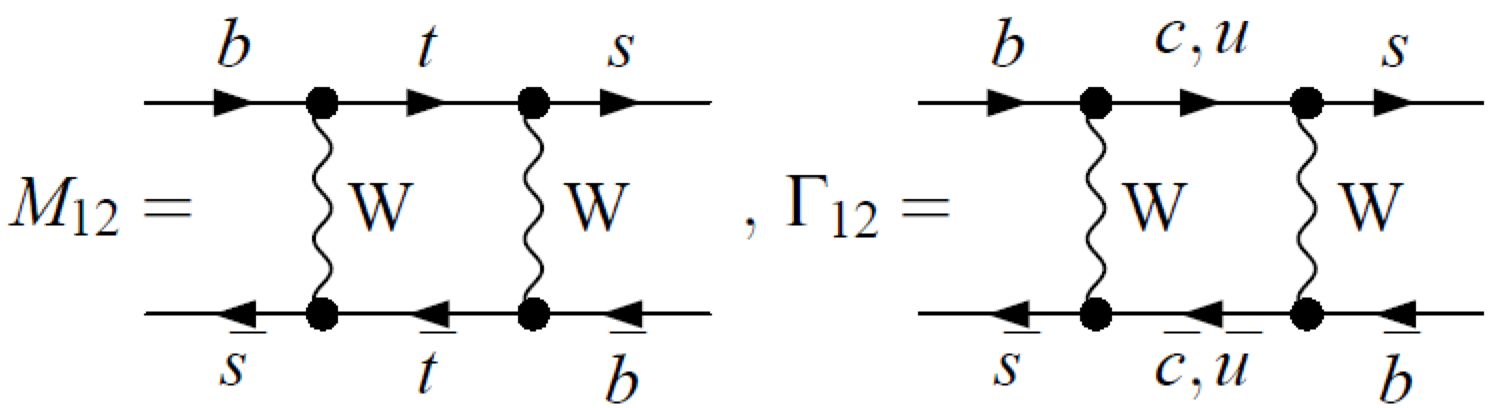
- In 0402204 (Falk et al), physical D meson was considered, with external momentum being injected to vary its mass



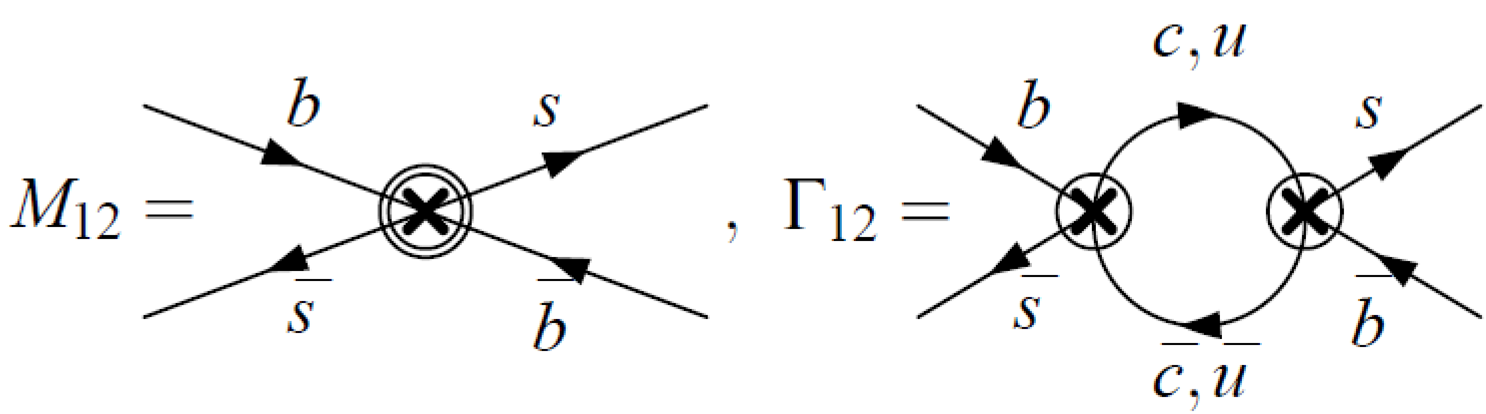
$$\Delta m = -\frac{1}{2\pi} \mathrm{P} \int_{2m_\pi}^{\infty} \mathrm{d}E \left[\frac{\Delta\Gamma(E)}{E - m_D} + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{E} \right) \right]$$

- Apply HQET to remove m_D dependence, and get dispersion relation with external energy E
- Model shape of $\Delta\Gamma(E)$, and then compute x/y using dispersion relation

Bs mixing



- Integrate out W boson



- Delta B = 2

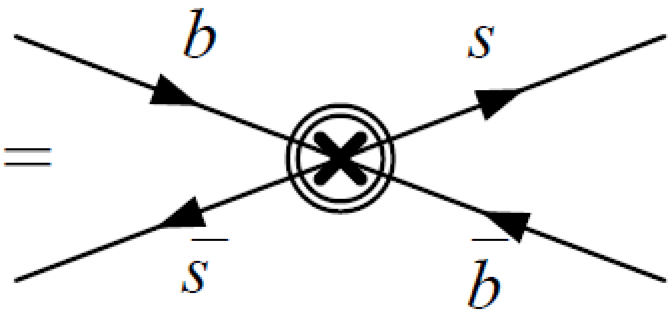
Delta B = 1

HQET

hep-ph 9605259

- Width difference expanded

$\Gamma_{12} =$



Wilson coefficient

$$\sum_n \frac{C_n}{m_b^n} \mathcal{O}_n^{\Delta B=2}(0)$$

- 4-fermion Delta B = 2 operators give bag parameters

$$Q = (\bar{b}_i s_i)_{V-A}(\bar{b}_j s_j)_{V-A}$$
$$Q_S = (\bar{b}_i s_i)_{S-P}(\bar{b}_j s_j)_{S-P}$$

- Like those in Delta m

$$M_{12,q} = \frac{G_F^2}{12\pi^2}(V_{tq}^* V_{tb})^2 M_W^2 S_0(x_t) B_{B_q} f_{B_q}^2 M_{B_q} \hat{\eta}_B$$

Failed method 2

- Expand both unknown and input in terms of orthogonal functions
- Solve for coefficients of orthogonal functions using dispersion relation
- Expansion of delta function in terms of Legendre polynomials Pl
- Coefficients al grow with l, no convergence
- Expected, because large coefficients needed to express infinity