Study of semileptonic decays $B_{(s)} \to D^{(*)}_{(s)}\, l\, \nu$ in the "PQCD + Lattice" approach

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Outline

- *Motivation*
- *Framework*
- *Contents*
- *Results*
- *Summary*

Motivation

$B_{(s)}$ meson semileptonic decay

- Determination of CKM

(For example: $|V_{cb}|$ in $B \to Dl\nu$)

- Examination of SM

(Testing the lepton flavor universality)
(QCDSR,LCSR,LFQM,NRQCD etc.)

- Hints of new physics

(2HDM, Leptoquark mode etc.)

$R(D^{(*)})$ Anomaly

$$R(D) = \frac{Br(B_0 \to D_+ \tau\nu) + Br(B_+ \to D_0 \tau\nu)}{Br(B_0 \to D_+ l\nu) + Br(B_+ \to D_0 l\nu)}$$

$l=e,\mu$

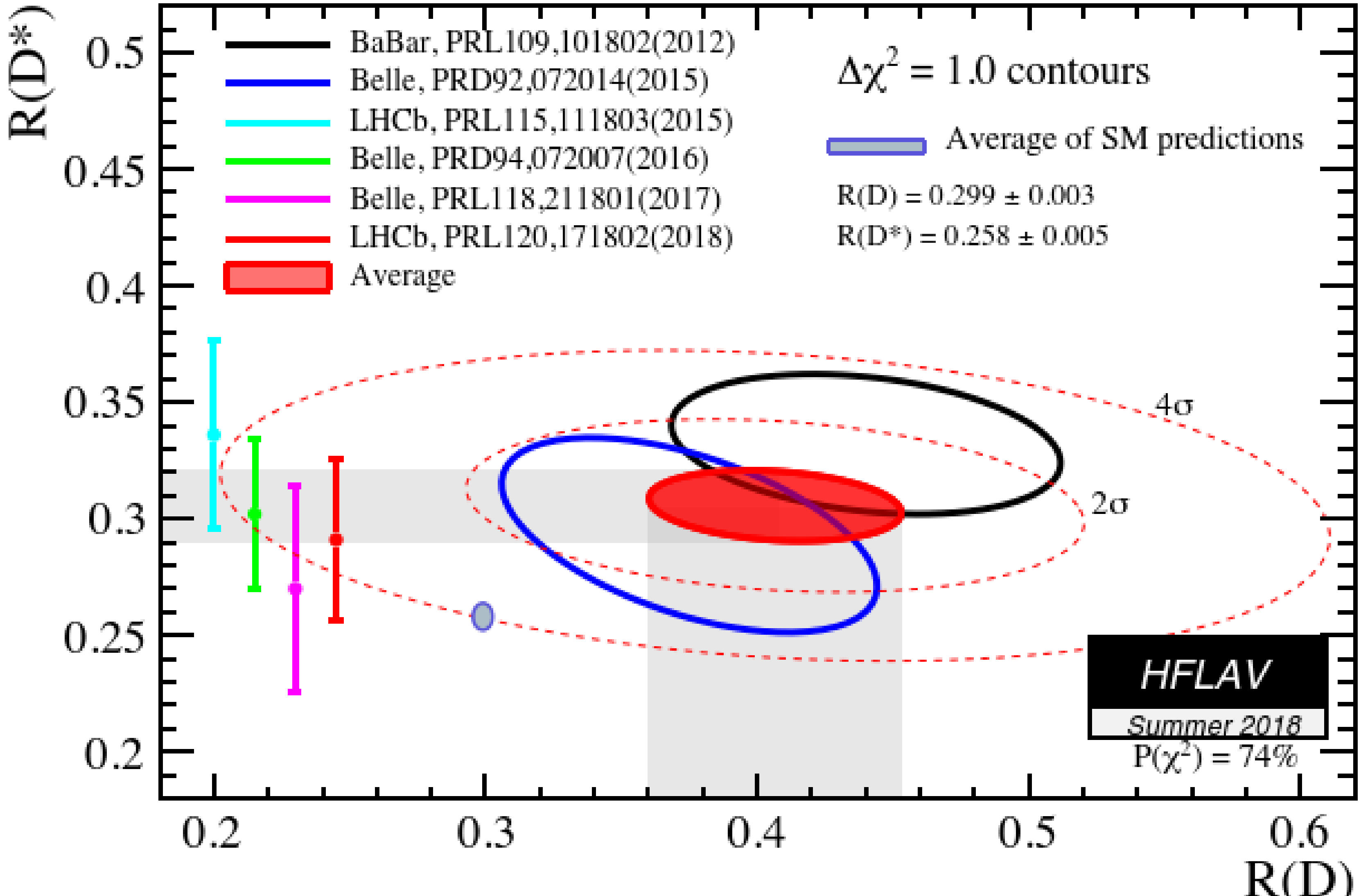
$$R(D^*) = \frac{Br(B_0 \to D_+{}^* \tau\nu) + Br(B_+ \to D_0{}^* \tau\nu)}{Br(B_0 \to D_+{}^* l\nu) + Br(B_+ \to D_0{}^* l\nu)}$$

1. The theoretical uncertainties of each branching ratio are large.

2. It mainly comes from the Form Factors and V_{cb}.

3. The theoretical uncertainties in $R(D^{(*)})$ are significantly reduced.

$R(D^{(*)})$ Anomaly



BaBar, PRL109,101802(2012)
Belle, PRD92,072014(2015)
LHCb, PRL115,111803(2015)
Belle, PRD94,072007(2016)
Belle, PRL118,211801(2017)
LHCb, PRL120,171802(2018)
Average
Δχ² = 1.0 contours
Average of SM predictions
R(D) = 0.299 ± 0.003
R(D*) = 0.258 ± 0.005
4σ
2σ
HFLAV
Summer 2018
P(χ²) = 74%
R(D*)
R(D)
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.2
0.3
0.4
0.5
0.6

$R(D^{(*)})$ Anomaly

Recent experiment results:

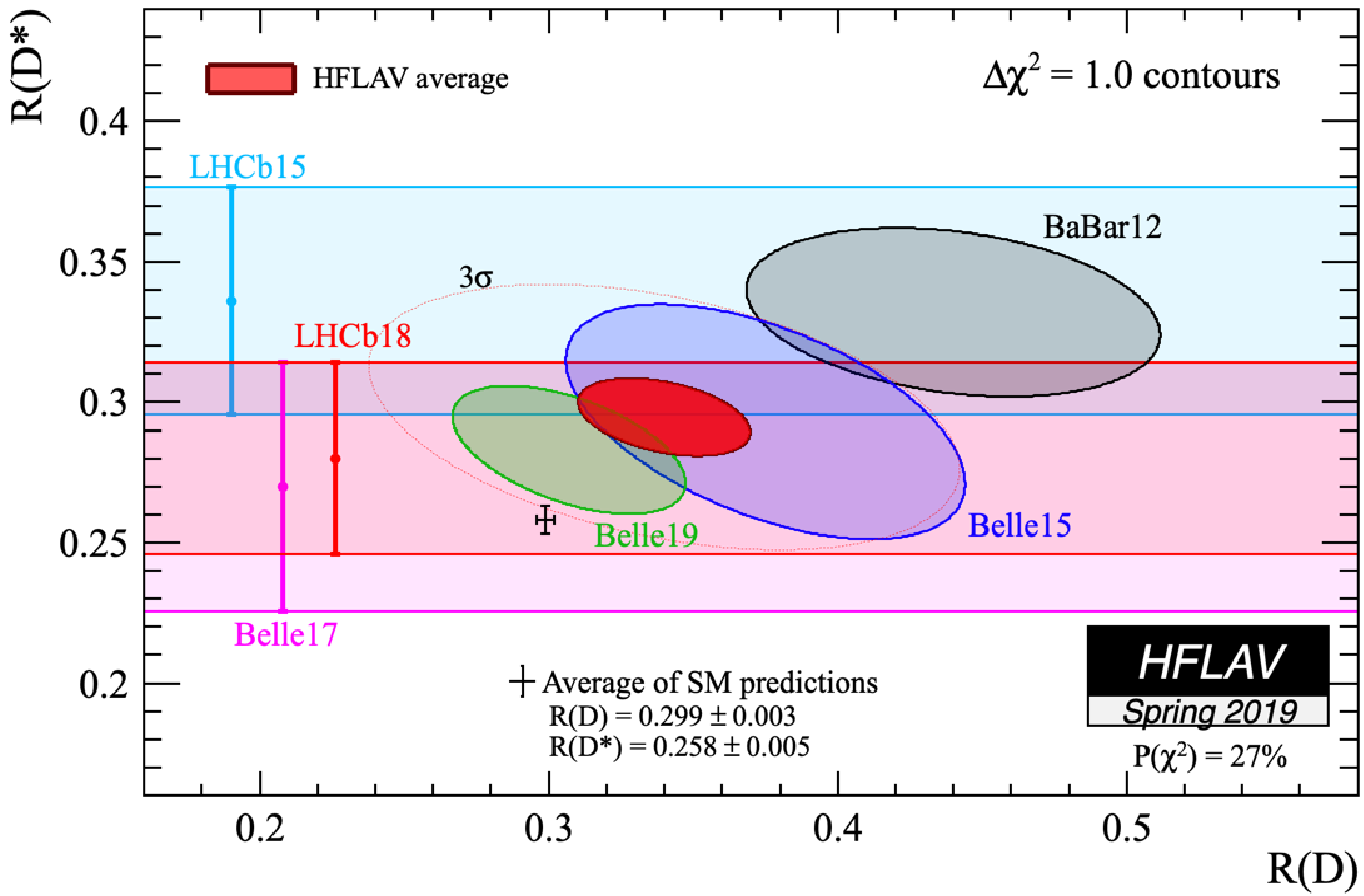
$$R(D) = \begin{cases} 0.440 \pm 0.058 \pm 0.042 \\ 0.307 \pm 0.037 \pm 0.016 \\ 0.340 \pm 0.027 \pm 0.013 \end{cases}$$

Babar PRD 88, 072012 (2013)
Belle arXiv:1904.08794
HFAG EPJC 77, 895 (2017)

$$R(D^*) = \begin{cases} 0.280 \pm 0.018 \pm 0.029 \\ 0.283 \pm 0.018 \pm 0.014 \\ 0.295 \pm 0.011 \pm 0.008 \end{cases}$$

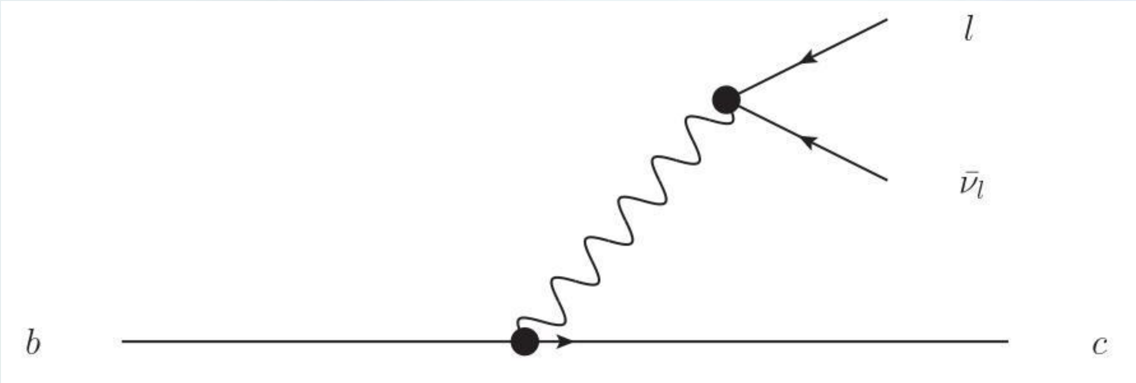
LHCb PRD 97, 072013 (2018)
Belle arXiv:1904.08794
HFAG EPJC 77, 895 (2017)

$R(D^{(*)})$ Anomaly

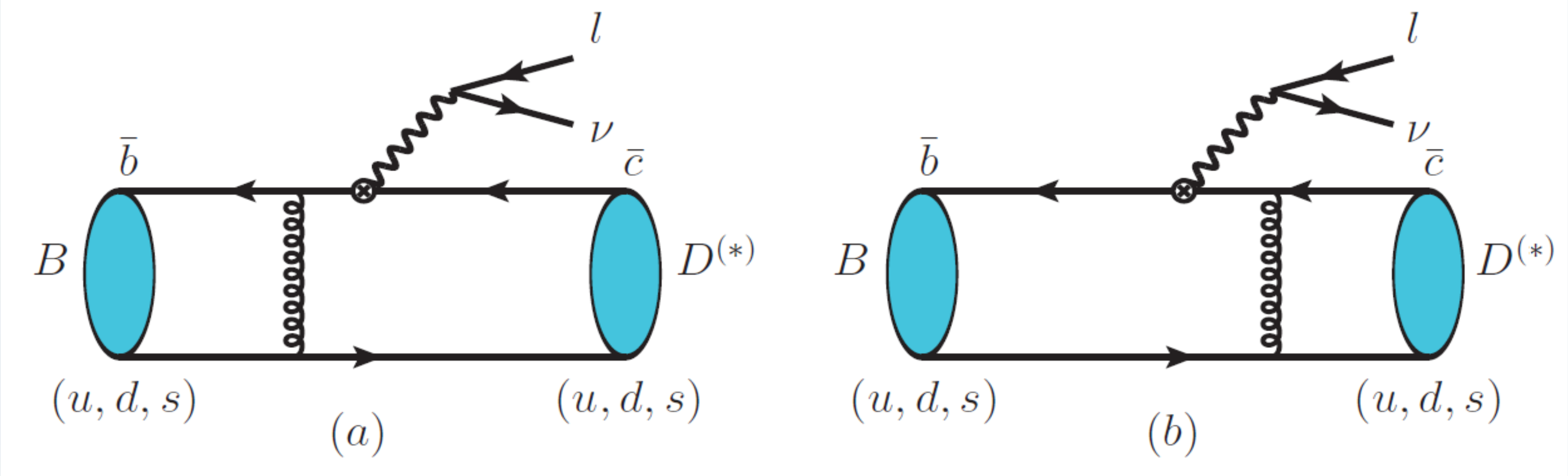

HFLAV average
Δχ² = 1.0 contours
LHCb15
BaBar12
3σ
LHCb18
Belle19
Belle15
Belle17
Average of SM predictions
R(D) = 0.299 ± 0.003
R(D*) = 0.258 ± 0.005
HFLAV
Spring 2019
P(χ²) = 27%
R(D*)
R(D)
0.2
0.25
0.3
0.35
0.4
0.2
0.3
0.4
0.5

Framework

$$B_{(s)} \to \left(D_{(s)}, {D^*}_{(s)}\right) l\nu \qquad l = e\,,\mu\,,\tau$$



Quark level

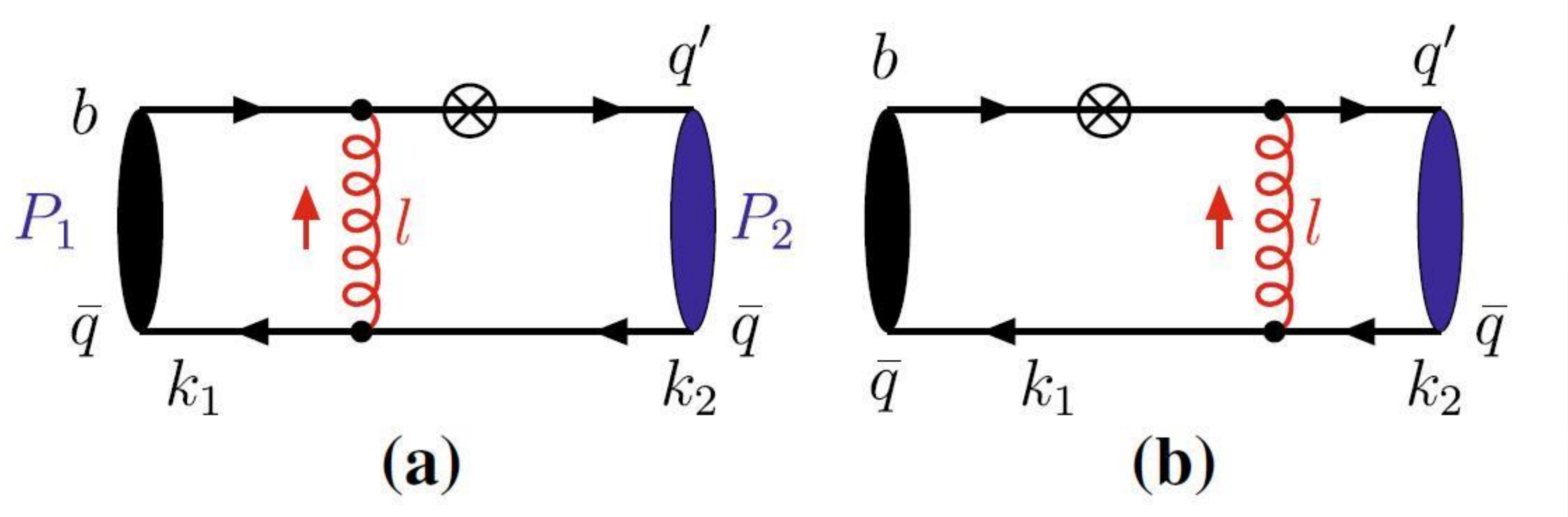


LO diagrams

Framework

$$\mathcal{H}_{eff}(b \to c l \bar{\nu}_l) = \frac{G_F}{\sqrt{2}} V_{cb}\, \bar{c}\gamma_\mu(1-\gamma_5)b \cdot \bar{l}\gamma^\mu(1-\gamma_5)\nu_l$$

Effective Hamiltonian



Effective Diagrams

Form Factors

Definition of Form Factors:

For pseudo-scalar meson $D_{(s)}$

$$\langle D_{(u,d,s)}(p_2)|\bar{c}(0)\gamma_\mu b(0)|B_{(u,d,s)}(p_1)\rangle = \left[(p_1+p_2)_\mu - \frac{m^2_{B_{(u,d,s)}} - m^2_{D_{(u,d,s)}}}{q^2} q_\mu\right] F_+(q^2) + \left[\frac{m^2_{B_{(u,d,s)}} - m^2_{D_{(u,d,s)}}}{q^2} q_\mu\right] F_0(q^2)$$

For vector meson $D^*{}_{(s)}$

$$\langle D^*_{(u,d,s)}(p_2)|\bar{c}(0)\gamma_\mu b(0)|B_{(u,d,s)}(p_1)\rangle = \frac{2iV(q^2)}{m_{B_{(u,d,s)}} + m_{D^*_{(u,d,s)}}} \epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu} p_1^\alpha p_2^\beta,$$

$$\langle D^*_{(u,d,s)}(p_2)|\bar{c}(0)\gamma_\mu\gamma_5 b(0)|B_{(u,d,s)}(p_1)\rangle = 2m_{D^*_{(u,d,s)}} A_0(q^2)\frac{\epsilon^*\cdot q}{q^2} q_\mu + (m_{B_{(u,d,s)}} + m_{D^*_{(u,d,s)}}) A_1(q^2)\left(\epsilon^*_\mu - \frac{\epsilon^*\cdot q}{q^2} q_\mu\right) - A_2(q^2)\frac{\epsilon^*\cdot q}{m_{B_{(u,d,s)}} + m_{D^*_{(u,d,s)}}}\left[(p_1+p_2)_\mu - \frac{m^2_{B_{(u,d,s)}} - m^2_{D^*_{(u,d,s)}}}{q^2} q_\mu\right].$$

Branching Ratio

$q = p_1 - p_2$ is the lepton-pair momentum.

For $B_{(s)} \to D_{(s)} l \nu$

$$\frac{d\Gamma(B_{(s)} \to D_{(s)} l\nu)}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{192\pi^3 m_{B_{(s)}}^3} \left(1 - \frac{m_l^2}{q^2}\right)^2 \frac{\lambda^{1/2}(q^2)}{2q^2} \cdot \left\{3m_l^2 \left(m_{B_{(s)}}^2 - m^2\right)^2 |F_0(q^2)|^2 + \left(m_l^2 + 2q^2\right) \lambda(q^2) |F_+(q^2)|^2\right\}, \quad ($$

$\lambda(q^2) = (m_{B_{(s)}}^2 + m_{D^*_{(s)}}^2 - q^2)^2 - 4m_{B_{(s)}}^2 m_{D^*_{(s)}}^2$ is the phase space factor.

For $B_{(s)} \to D^*{}_{(s)} l \nu$

Branching Ratio

Longitude part:

$$\frac{d\Gamma_L(B_{(s)} \to D^*_{(s)} l\nu)}{dq^2} = \frac{G_F^2|V_{cb}|^2}{192\pi^3 m^3_{B_{(s)}}}\left(1-\frac{m_l^2}{q^2}\right)^2 \frac{\lambda^{1/2}(q^2)}{2q^2}\cdot\left\{3m_l^2\lambda(q^2)A_0^2(q^2) + \frac{m_l^2+2q^2}{4m^2}\cdot\left[(m^2_{B_{(s)}}-m^2-q^2)(m_{B_{(s)}}+m)A_1(q^2)-\frac{\lambda(q^2)}{m_{B_{(s)}}+m}A_2(q^2)\right]^2\right\}$$

Transverse part:

$$\frac{d\Gamma_\pm(B_{(s)} \to D^*_{(s)} l\nu)}{dq^2} = \frac{G_F^2|V_{cb}|^2}{192\pi^3 m^3_{B_{(s)}}}\left(1-\frac{m_l^2}{q^2}\right)^2 \frac{\lambda^{3/2}(q^2)}{2} \times\left\{(m_l^2+2q^2)\left[\frac{V(q^2)}{m_{B_{(s)}}+m}\mp\frac{(m_{B_{(s)}}+m)A_1(q^2)}{\sqrt{\lambda(q^2)}}\right]^2\right\}$$

Total differential decay widths:

$$\frac{d\Gamma_T}{dq^2} = \frac{d\Gamma^+}{dq^2}+\frac{d\Gamma^-}{dq^2}, \qquad \frac{d\Gamma}{dq^2} = \frac{d\Gamma_L}{dq^2}+\frac{d\Gamma_T}{dq^2}.$$

Contents

Transition matrix elements:

$$=> \quad < D^{(*)}_{(u,d,s)} \, | J_{(A/V)} | \, B_{(u,d,s)} >$$

In PQCD factorization approach:

$$A \sim \int dx_1\, dx_2 b_1 db_1 b_2 db_2$$

$$\times Tr[\Phi_B(x_1, b_1)\Phi_D(x_2, b_2)H_i(t_i)]$$

$$H_i(t_i) = h_i(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_i) \cdot \exp[-S_{ab}(t_i)]$$

Contents

Explicit expressions of relevant form factors in PQCD factorization approach.

$$f_1(q^2) = 8\pi m_{B_{(s)}}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_{(s)}}(x_1, b_1)\phi_{D_{(s)}}(x_2, b_2)$$
$$\times\Big\{[2r\,(1 - rx_2)] \cdot H_1(t_1)$$
$$+ \left[2r(2r_c - r) + x_1 r(-2 + 2\eta + \sqrt{\eta^2 - 1} - \frac{2\eta}{\sqrt{\eta^2-1}} + \frac{\eta^2}{\sqrt{\eta^2-1}})\right] \cdot H_2(t_2)\Big\},$$

$$f_2(q^2) = 8\pi m_{B_{(s)}}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_{(s)}}(x_1, b_1)\phi_{D_{(s)}}(x_2, b_2)$$
$$\times \Big\{[2 - 4x_2 r(1 - \eta)] \cdot H_1(t_1)$$
$$+ \left[4r - 2r_c - x_1 + \frac{x_1}{\sqrt{\eta^2-1}}(2 - \eta)\right] \cdot H_2(t_2)\Big\},$$

$$V(q^2) = 8\pi m_{B_{(s)}}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_{(s)}}(x_1, b_1)\phi_{D^*_{(s)}}^T(x_2, b_2)\cdot(1+r)$$

$$\times\left\{[1 - r x_2]\cdot H_1(t_1) + \left[r + \frac{x_1}{2\sqrt{\eta^2-1}}\right]\cdot H_2(t_2)\right\},$$

$$A_0(q^2) = 8\pi m_{B_{(s)}}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_{(s)}}(x_1, b_1)\phi_{D^*_{(s)}}^L(x_2, b_2)$$

$$\times\left\{[1 + r - r x_2(2 + r - 2\eta)]\cdot H_1(t_1)\right.$$

$$\left. + \left[r^2 + r_c + \frac{x_1}{2} + \frac{\eta x_1}{2\sqrt{\eta^2-1}} + \frac{r x_1}{2\sqrt{\eta^2-1}}(1 - 2\eta(\eta + \sqrt{\eta^2-1}))\right]\cdot H_2(t_2)\right\},$$

$$A_1(q^2) = 8\pi m_{B_{(s)}}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_{(s)}}(x_1, b_1)\phi_{D^*_{(s)}}^T(x_2, b_2)\cdot\frac{r}{1+r}$$

$$\times\left\{2[1 + \eta - 2 r x_2 + r\eta x_2]\cdot H_1(t_1) + [2r_c + 2\eta r - x_1]\cdot H_2(t_2)\right\},$$

$$A_2(q^2) = \frac{(1+r)^2(\eta - r)}{2r(\eta^2-1)}\cdot A_1(q^2) - 8\pi m_{B_{(s)}}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_{(s)}}(x_1, b_1)$$

$$\cdot\phi_{D^*_{(s)}}^L(x_2, b_2)\cdot\frac{1+r}{\eta^2-1}\times\left\{[(1+\eta)(1-r) - r x_2(1 - 2r + \eta(2 + r - 2\eta))]\cdot H_1(t_1)\right.$$

$$\left. + \left[r + r_c(\eta - r) - \eta r^2 + r x_1 \eta^2 - \frac{x_1}{2}(\eta + r) + x_1(\eta r - \frac{1}{2})\sqrt{\eta^2-1}\right]\cdot H_2(t_2)\right\},$$

Contents

1. PQCD predictions for the considered form factors are much more reliable at low q^2 region

2. The calculations of the form factors become rather difficult at high q^2 region, one has to make an extrapolation

3. The traditional method is calculate the form factors by PQCD at low q^2 region, then make extrapolation by using the pole model:

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_B^2) + b\,(q^2/m_B^2)^2}$$

Lattice QCD

1.The lattice QCD has its own advantage to obtain the relevant form factors at large q^2 region.

2.At zero recoil ($q^2 = q^2_{max}$) region, the lattice QCD predictions are much reliable.

3.Use the lattice QCD results at the endpoint as the additional inputs in the fitting process.

4. Compare the difference between the two methods: the traditional PQCD approach and the "PQCD+Lattice QCD" approach.

Lattice QCD

Lattice inputs:

$$\mathcal{G}^{B\to D}(1) = 1.033 \pm 0.095, \quad \mathcal{F}^{B\to D^*}(1) = 0.895 \pm 0.010 \pm 0.024,$$
$$\mathcal{G}^{B_s\to D_s}(1) = 1.052 \pm 0.046, \quad \mathcal{F}^{B_s\to D_s^*}(1) = 0.883 \pm 0.012 \pm 0.028,$$

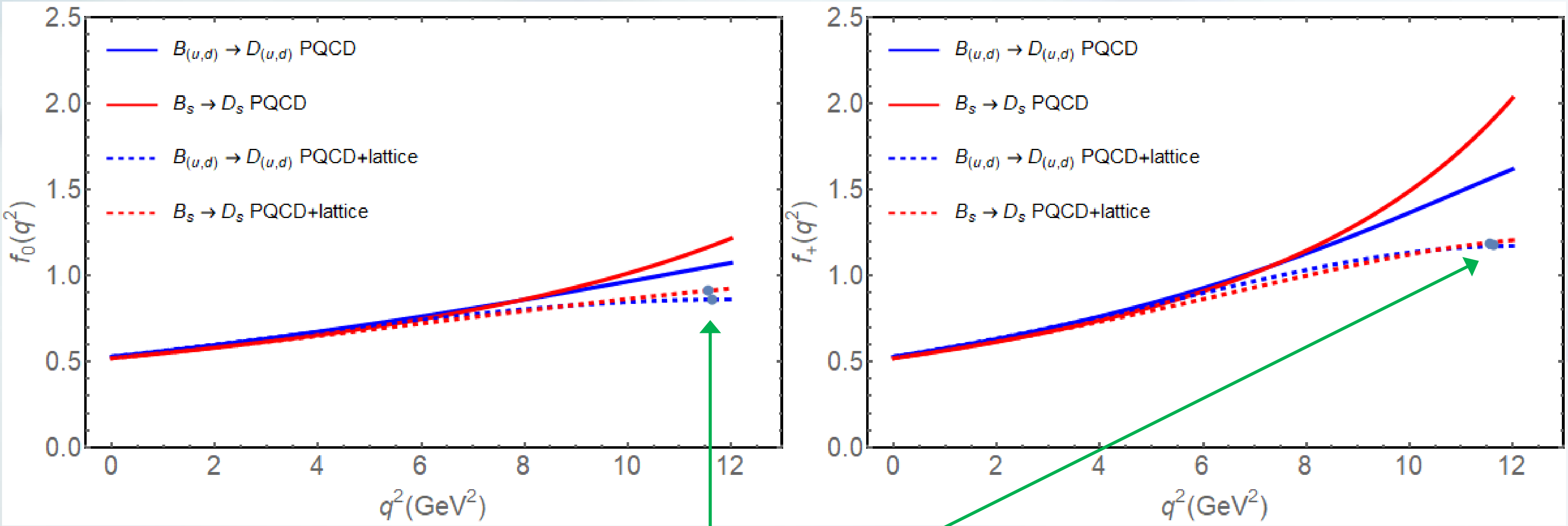
HPQCD PRD 97, 054502 (2018) *M. Atoui et al, EPJC 74, 2861 (2014)*

Transformation relation:

$$F_+(q^2_{max}) = \frac{1+r}{2\sqrt{r}}\mathcal{G}(1),$$
$$V(q^2_{max}) = A_0(q^2_{max}) = A_2(q^2_{max}) = \frac{1}{A_1(q^2_{max})} = \frac{1+r}{2\sqrt{r}}\mathcal{F}(1),$$
$$F_0^{B\to D}/F_+^{B\to D} = 0.73 \pm 0.04, \quad F_0^{B_s\to D_s}/F_+^{B_s\to D_s} = 0.77 \pm 0.02,$$

Fitting

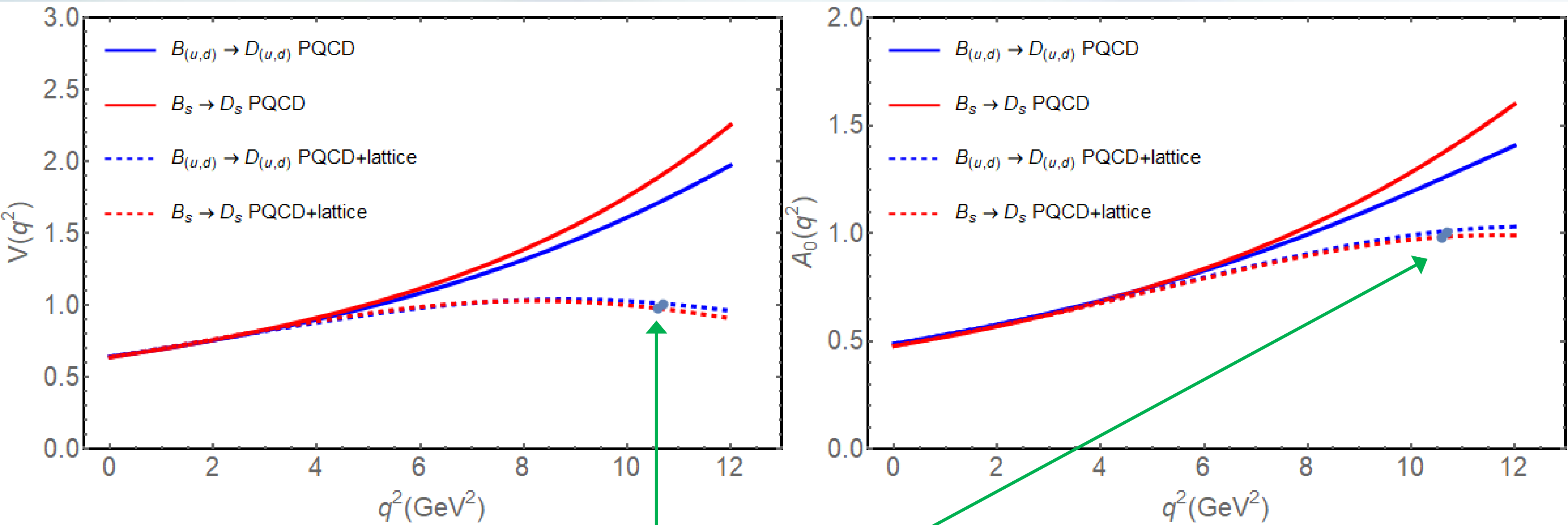
For $B_{(s)} \to D_{(s)} l \nu$



Lattice inputs at the endpoint

Fitting

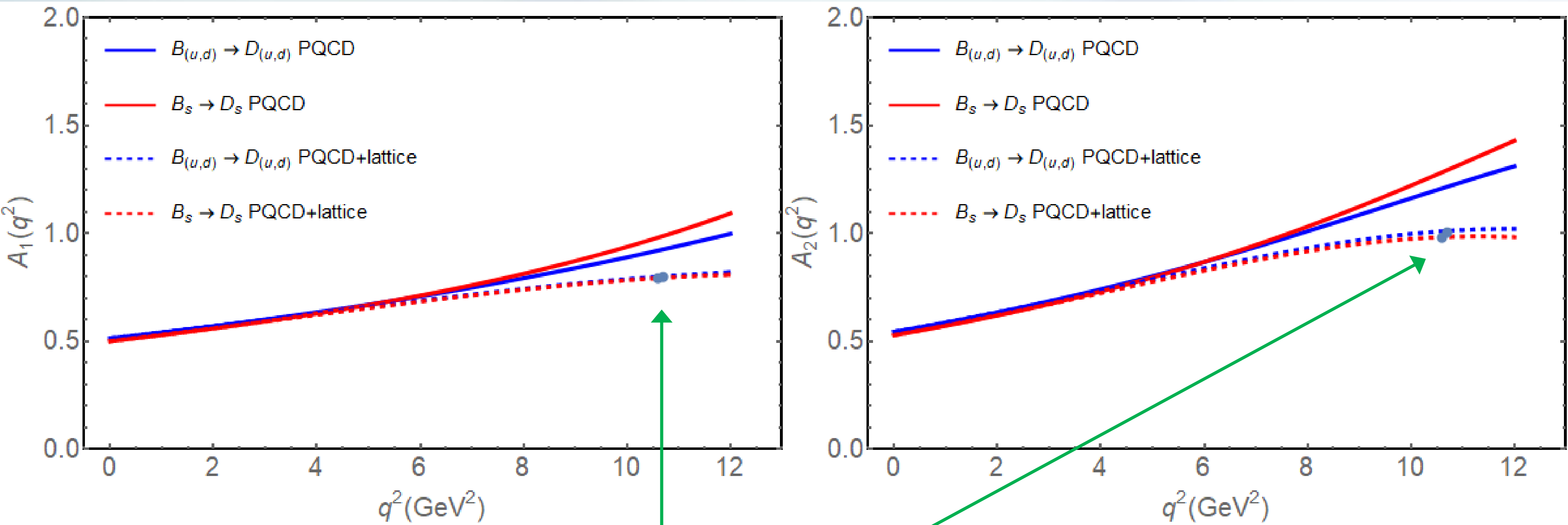
For $B_{(s)} \rightarrow D^{*}{}_{(s)} l\, \nu$



Lattice inputs at the endpoint

Fitting

For $B_{(s)} \rightarrow D^*{}_{(s)} l \nu$



Lattice inputs at the endpoint

Results

For $B \to D^{(*)} l \nu$:

Results ($\times 10^{-2}$)	PQCD	PQCD+Lattice	HFAG	PDG
$\mathcal{B}(B_+ \to D_0 l \bar{\nu}_l)$	$2.36^{+0.88}_{-0.68}$	$2.24^{+0.88}_{-0.63}$	2.35 ± 0.09	2.20 ± 0.10
$\mathcal{B}(B_+ \to D_0 \tau \bar{\nu}_\tau)$	$0.89^{+0.29}_{-0.22}$	$0.77^{+0.19}_{-0.15}$	$-$	0.77 ± 0.25
$\mathcal{B}(B_+ \to D_0^* l \bar{\nu}_l)$	$5.61^{+1.12}_{-1.06}$	$5.14^{+1.02}_{-0.96}$	5.66 ± 0.22	4.88 ± 0.10
$\mathcal{B}(B_+ \to D_0^* \tau \bar{\nu}_\tau)$	$1.64^{+0.28}_{-0.29}$	$1.42^{+0.19}_{-0.20}$	$-$	1.88 ± 0.20
$\mathcal{B}(B_0 \to D_+ l \bar{\nu}_l)$	$2.30^{+0.89}_{-0.67}$	$2.14^{+0.83}_{-0.60}$	2.31 ± 0.10	2.20 ± 0.10
$\mathcal{B}(B_0 \to D_+ \tau \bar{\nu}_\tau)$	$0.87^{+0.27}_{-0.22}$	$0.72^{+0.17}_{-0.15}$	$-$	1.03 ± 0.22
$\mathcal{B}(B_0 \to D_+^* l \bar{\nu}_l)$	$5.40^{+1.11}_{-1.03}$	$4.89^{+1.07}_{-0.91}$	5.05 ± 0.14	4.88 ± 0.10
$\mathcal{B}(B_0 \to D_+^* \tau \bar{\nu}_\tau)$	$1.57^{+0.26}_{-0.28}$	$1.33^{+0.19}_{-0.18}$	$-$	1.67 ± 0.13

Results

$$R(D) = \frac{Br(B_0 \to D_+ \tau\nu) + Br(B_+ \to D_0 \tau\nu)}{Br(B_0 \to D_+ l\nu) + Br(B_+ \to D_0 l\nu)}$$

$$R(D^*) = \frac{Br(B_0 \to D_+{}^* \tau\nu) + Br(B_+ \to D_0{}^* \tau\nu)}{Br(B_0 \to D_+{}^* l\nu) + Br(B_+ \to D_0{}^* l\nu)}$$

Results	PQCD	PQCD+Lattice	BaBar	Belle	LHCb	HFAG
$R(D)$	$0.377^{+0.021}_{-0.023}$	$0.340^{+0.032}_{-0.034}$	0.440 ± 0.072	0.307 ± 0.041	$-$	0.340 ± 0.030
$R(D^*)$	$0.291^{+0.012}_{-0.015}$	$0.274^{+0.016}_{-0.017}$	0.332 ± 0.030	0.283 ± 0.023	0.291 ± 0.035	0.295 ± 0.014

The theoretical uncertainties can be significantly reduced in the predictions for the ratio of branching ratios .

Results

For $B_s \to D_s^{(*)} l\,\nu$:

Results ($\times 10^{-2}$)	$\mathcal{B}(B_s \to D_s l\bar{\nu}_l)$	$\mathcal{B}(B_s \to D_s \tau\bar{\nu}_\tau)$	$\mathcal{B}(B_s \to D_s^* l\bar{\nu}_l)$	$\mathcal{B}(B_s \to D_s^* \tau\bar{\nu}_\tau)$
PQCD	$2.14^{+0.89}_{-0.68}$	$0.84^{+0.31}_{-0.25}$	$5.27^{+1.14}_{-1.21}$	$1.56^{+0.33}_{-0.31}$
PQCD+Lattice	$1.95^{+0.85}_{-0.59}$	$0.68^{+0.19}_{-0.16}$	$4.64^{+1.12}_{-0.98}$	$1.27^{+0.22}_{-0.21}$
IAMF	$1.4 - 1.7$	$0.47 - 0.55$	$5.1 - 5.8$	$1.2 - 1.3$
RQM	2.1 ± 0.2	0.62 ± 0.05	5.3 ± 0.5	1.3 ± 0.1
LCSR	$1.0{+}0.4_{-0.3}$	$0.33^{+0.14}_{-0.11}$	–	–
LFQM	–	–	5.2 ± 0.6	$1.3^{+0.2}_{-0.1}$
CQM	$2.73 - 3.00$	–	$7.49 - 7.66$	–
QCDSR	$2.8 - 3.8$	–	$1.89 - 6.61$	–
Lattice	$2.013 - 2.469$	$0.619 - 0.724$	–	–

Results

$$R(D_s) = \frac{Br(B_s \to D_s \tau \nu)}{Br(B_s \to D_s l \nu)}$$

$$R(D_s^*) = \frac{Br(B_s \to D_s{}^* \tau \nu)}{Br(B_s \to D_s{}^* l \nu)}$$

Results	PQCD	PQCD+Lattice	RQM	LCSR	LFQM	Lattice
$R(D_s)$	$0.390^{+0.015}_{-0.023}$	$0.348^{+0.036}_{-0.038}$	0.295	0.33	–	0.299
$R(D_s^*)$	$0.295^{+0.010}_{-0.013}$	$0.273^{+0.018}_{-0.020}$	0.245	–	0.25	–

Our predictions are slightly larger than other theories but still agree with them considering the errors.

Additional physical observables

Besides the decay rates and the ratios ***R(D^(*))***, the longitudinal polarization of the ***τ*** lepton ***P (τ)*** , the fraction of $\boldsymbol{D^*}$ longitudinal polarization $\boldsymbol{F_L(D^*)}$ and the forward-backward asymmetry of the tau lepton $\boldsymbol{A_{FB}(\tau)}$ are also the additional physical observables which are very important because of their sensitivity to some kinds of new physics.

The first measurement of ***P (τ)*** and $\boldsymbol{F_L(D^*)}$ have been reported very recently by Belle Collaboration

$$P_\tau(D^*) = -0.38 \pm 0.51(stat.)^{+0.21}_{-0.16}(syst.),$$
$$F_L(D^*) = 0.60 \pm 0.08(stat.) \pm 0.04(syst.).$$

Phys. Rev. D 97, 012004 (2018)

arXiv:1903. 03102

Results

Our predictions for $\boldsymbol{P(\tau)}$, $\boldsymbol{F_L(D^*)}$ and $\boldsymbol{A_{FB}(\tau)}$ via both the traditional PQCD approach and the "PQCD+Lattice" approach

Observable	$P(\tau)$		$F_L(D^*)$		$A_{FB}(\tau)$	
Approach	PQCD	PQCD+Lattice	PQCD	PQCD+Lattice	PQCD	PQCD+Lattice
$B_+ \to D_0 l\bar{\nu}_l$	$0.269(6)$	$0.275(11)$	–	–	$0.357(3)$	$0.361(6)$
$B_0 \to D_+ l\bar{\nu}_l$	$0.273(6)$	$0.278(10)$	–	–	$0.357(3)$	$0.362(5)$
$B_s \to D_s l\bar{\nu}_l$	$0.266(6)$	$0.297(11)$	–	–	$0.356(3)$	$0.359(6)$
$B_+ \to D_0^* l\bar{\nu}_l$	$-0.532(7)$	$-0.525(10)$	$0.422(4)$	$0.432(5)$	$-0.093(7)$	$-0.063(4)$
$B_0 \to D_+^* l\bar{\nu}_l$	$-0.532(8)$	$-0.523(10)$	$0.422(4)$	$0.433(5)$	$-0.092(6)$	$-0.062(3)$
$B_s \to D_s^* l\bar{\nu}_l$	$-0.533(8)$	$-0.523(11)$	$0.421(3)$	$0.433(5)$	$-0.092(5)$	$-0.058(4)$

Consistent with the experiment results considering the errors.

Summary

1. The PQCD predictions for the branching ratios ***Brs*** and the ratios of Brs ***R(X)*** both become smaller by a degree of (5−15)% when the Lattice QCD results are taken into account in the extrapolation of the relevant form factors.

2. The theoretical predictions for $\boldsymbol{P(\tau)}$, $\boldsymbol{F_L(D^*)}$ and $\boldsymbol{A_{FB}(\tau)}$ are consistent with each other in both traditional PQCD approach and the "PQCD+Lattice" approach.

3. Both the traditional PQCD approach and the"PQCD+Lattice" approach support the SU(3) flavor symmetry in the semi-leptonic $B_{(s)} \to D^*_{(s)} l \nu$ decays, while the breaking of SU(3) flavor symmetry is less than 10% due to the mass effect.

Summary

4. Our results for the branching ratios ***Brs*** and the ratios of Brs ***R(X)*** generally agree well with previous experiment results. Our predictions for the additional physical observables $\boldsymbol{P(\tau)}$, $\boldsymbol{F_L(D^*)}$ and $\boldsymbol{A_{FB}(\tau)}$ are consistent with other theories in SM but a little deviation from the central values measured in the recent experiments. However, it is still acceptable since the large errors both in theory and experiment.

5. We are look forward to that those physical observables could be measured in high precision at the future LHCb and Belle experiments and it can help us to test the theoretical models or approaches.

Thank you