Sub-leading power corrections to $D^* \to D\gamma$ and $B^* \to B\gamma$ with LCSR

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1. Motivation

2. Framework

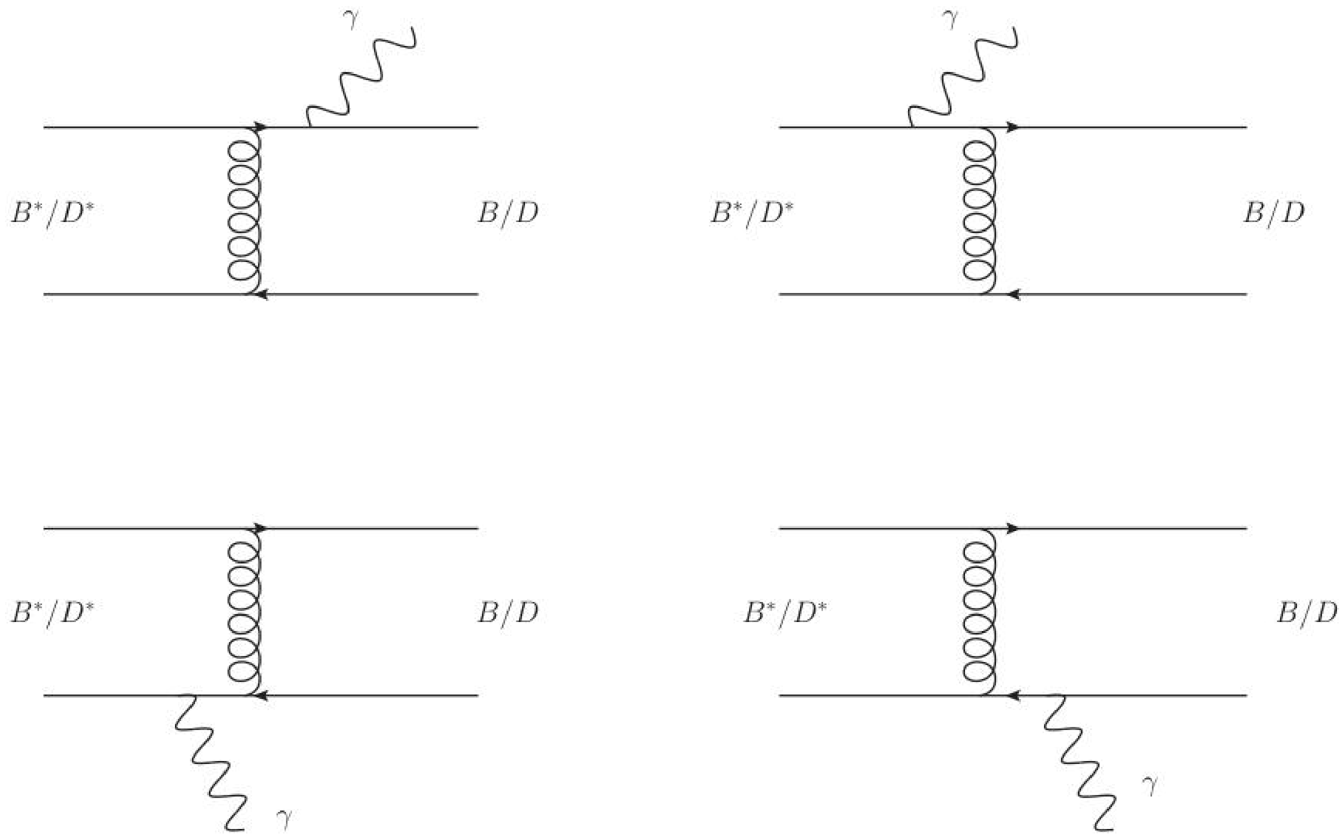
3. Tree level higher twist corrections

4. NLO corrections on leading twist

5. Theoretical results and numerical analysis

6. Summary

Quark level diagrams for radiative decay processes



Motivation

1. Coupling constants $g_{B^*B\gamma}$, $g_{D^*D\gamma}$ are the most important parameters in the heavy meson electromagnetic decay processes.

2. Values of $g_{D^*D\gamma}$ from experiments are more precise than theoretical calculations, inclusion of sub-leading power corrections is necessory .

3. Calculation of Sub-leading power corrections is an important way to understand QCD.

Numerical values for coupling constants(GeV^{-1})

Coupling constants	Experiment	Lattice
$g_{D^{*+}D^{+}\gamma}$	$-(0.47 \pm 0.06)$	-0.2 ± 0.3
$g_{D^{*0}D^{0}\gamma}$	< 11	2.0 ± 0.6
$g_{B^{*+}B^{+}\gamma}$	\	\
$g_{B^{*0}B^{0}\gamma}$	\	\

Radiative B*B γ and D*D γ decays in light-cone QCD sum rules. (PhysRevD.54.857)

Calculation of them up to tree level high twist, by using an old photon DA which is outmoded. (PhysRevD.51.6177)

Framework

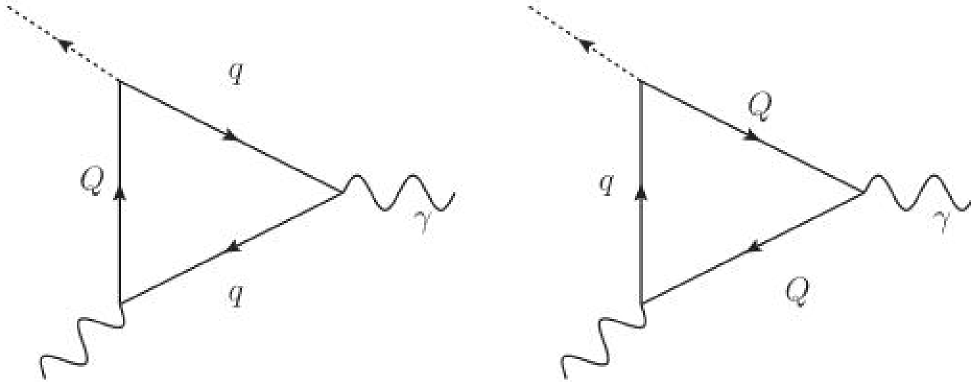
For the process D* $\longrightarrow$D γ , what we can directly get from matrix element is coupling constant: $g_{D^*D\gamma}$

$$\langle D(p+q) \mid j_{em}^{\mu} \mid D^*(p)\rangle = -\epsilon^{\mu\nu\alpha\beta} p_\nu q_\alpha \epsilon^*_\beta g_{D^*D\gamma}$$

To proceed LCSR calculations, we need to construct the vacuum to photon correlation function with an interpolating current for D^* and D meson:

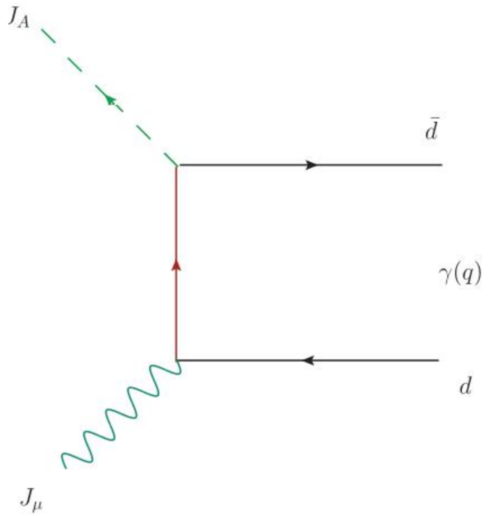
$$\Pi^\mu(p,q) = \int d^4x e^{ip\cdot x} \langle \gamma(q) \mid T\{\bar{d}(x) i\gamma_5 c(x), \bar{c}(0)\gamma^\mu d(0)\} \mid 0\rangle$$

Leading power perturbative diagrams:



Framework

Sub-leading power tree diagram:

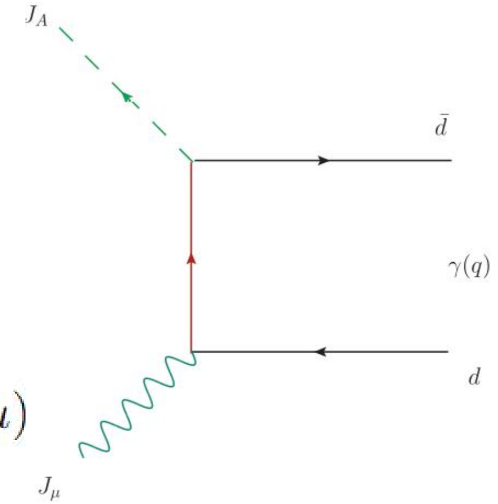


For the 4-point QCD amplitude:

$$T^\mu(p,q) = \int d^4x e^{ip\cdot x} \langle q(zq)\bar{q}(\bar{z}q) \mid T\{\bar{d}(x)i\gamma_5 c(x), \bar{c}(0)\gamma^\mu d(0)\} \mid 0\rangle$$

The amplitude reads: $$T^{\mu(0)}(p,q) = \frac{i}{2}\frac{\bar{n}\cdot p}{z(p+q)^2 + \bar{z}p^2 - m^2}\bar{d}(zq)\gamma^\mu \not{n}\gamma^5 d(\bar{z}q)$$

Factorization formula



Leading twist Photon distribution amplitudes :

$$\langle\gamma(p)|\bar{q}(x)\,W_c(x,0)\,\sigma_{\alpha\beta}\,q(0)|0\rangle$$
$$= i\,g_{\rm em}\,Q_q\,\langle\bar{q}q\rangle(\mu)\,(p_\beta\,\epsilon^*_\alpha - p_\alpha\,\epsilon^*_\beta)\int_0^1 dz\,e^{i\,z\,p\cdot x}\quad\chi(\mu)\,\phi_\gamma(z,\mu)$$

Tree level formula:

$$T^{\mu(0)}(p,q) = \frac{i}{2}\frac{\bar{n}\cdot p}{z(p+q)^2+\bar{z}p^2-m^2}\bar{d}(zq)\gamma^\mu \not{n}\gamma^5 d(\bar{z}q)$$

⬇

$$\Pi^\mu(p,q) = \frac{1}{2}e_d\chi(\mu)\langle\bar{q}q\rangle\epsilon^{\mu\nu\alpha\beta}n_\nu v_\alpha\epsilon_\beta\int_0^1 du\phi_\gamma(u,\mu)\frac{(n\cdot q)(\bar{n}\cdot p)}{u(p+q)^2+\bar{u}p^2-m^2}+\mathcal{O}(\alpha_s)$$

Lightcone sum rules

Operator product expa-nsion (OPE) on CF we get perturbative quantities and nonperturbative quantities (quark condensate or DA)

Matching ⟷

Dispersion relation on CF we get the physical quantities which can be measured like masses of rensonance states, decay constans.

Dispersion formula for correlation function

1. In complex plain, we use Cauchy formula:

$$\Pi(q^2) = \frac{1}{2\pi i}\int_C ds \frac{\Pi(s)}{s-q^2} = \frac{1}{\pi}\int_{s_{min}}^{\infty} ds \frac{Im\Pi(s)}{s-q^2}$$

2. Interpolating hadron states in the correlation function:

$$\begin{aligned}
\Pi(q^2) &= i\int d^4x e^{iq\cdot x}\langle\Omega \mid T J_\Gamma(x) J_\Gamma^\dagger(0) \mid \Omega\rangle \\
&= i\int d^4x\, e^{iq\cdot x}\frac{d^4p}{(2\pi)^4}\sum_\lambda \frac{i}{p^2-m_\lambda^2}\langle\Omega|J_\Gamma(x)|\lambda_p\rangle\langle\lambda_p|J_\Gamma^\dagger(0)|\Omega\rangle \\
&= i\sum_\lambda \frac{i}{q^2-m_\lambda^2}|\langle\Omega|J_\Gamma(0)|\lambda_0\rangle|^2 \\
&= \int_{s_{min}}^{\infty} ds \sum_\lambda \frac{\delta(s-m_\lambda^2)}{s-q^2}|\langle\Omega|J_\Gamma(0)|\lambda_0\rangle|^2 \\
&= \int_{s_{min}}^{\infty} ds \frac{\rho(s)}{s-q^2}.
\end{aligned}$$

$\rho(s)$ → spectral density

$$\langle 0 \mid \bar{d}\gamma^\mu c \mid D^*\rangle = m_{D^*} f_{D^*}\epsilon^\mu$$

$$\langle 0 \mid \bar{d} i\gamma^5 c \mid D\rangle = \frac{m_D^2}{m_c} f_D.$$

$$\Pi^\mu(p,q) = -\frac{g_{D^*D\gamma} f_{D^*} f_D m_D^2 m_{D^*}}{(p^2 - m_{D^*}^2)((p+q)^2 - m_D^2) m_c} \epsilon^{\mu\nu\alpha\beta} p_\nu q_\alpha \epsilon_\beta + \int_{s_0}^{\infty} ds \int_{s_0'}^{\infty} ds' \frac{\rho(s,s')}{(s-q^2)(s'-(p+q)^2)}$$

Quark-hadron duality, Borel transformation, continuum subtraction

$$\Pi^\mu(p,q) = \frac{1}{2} e_d \chi(\mu) \langle \bar{q} q \rangle \epsilon^{\mu\nu\alpha\beta} n_\nu v_\alpha \epsilon_\beta \int_0^1 du \phi_\gamma(u,\mu) \frac{(n\cdot q)(\bar{n}\cdot p)}{u(p+q)^2 + \bar{u} p^2 - m^2}$$

⇓

$$f_H\, f_{H^*}\, g_{H^* H \gamma} = -\frac{m_Q}{m_H^2\, m_{H^*}} e^{\frac{m_H^2 + m_{H^*}^2}{M^2}} e_q \langle \bar{q} q \rangle(\mu)\, \chi(\mu)\, \mathcal{F}^{LO}_{tw-2}\left(\frac{s_0 - m_Q^2}{M^2}\right)$$

with

$$\begin{aligned}
\mathcal{F}^{LO}_{tw-2}(x) &= \sum_k b_k(a_i) \int_0^{s_0} ds \int_0^{s_0} ds' e^{-\frac{s+s'}{M^2}} \rho^{2P}_{1,k}(s,s') \\
&= \frac{3M^2}{32} e^{-\frac{2m_Q^2}{M^2}} \Big\{ 8\, a_0(\mu) \left[1 - e^{-2x}(1 + 2x - 2x^2)\right] \\
&\quad -4\, a_2(\mu) \left[3 - e^{-2x}(3 + 6x + 54x^2 - 60x^3 + 10x^4)\right] + a_4(\mu) \\
&\quad \times \left[15 - e^{-2x}(15 + 30x - 450x^2 + 2100x^3 - 1750x^4 + 420x^5 - 28x^6)\right] \Big\}
\end{aligned}$$

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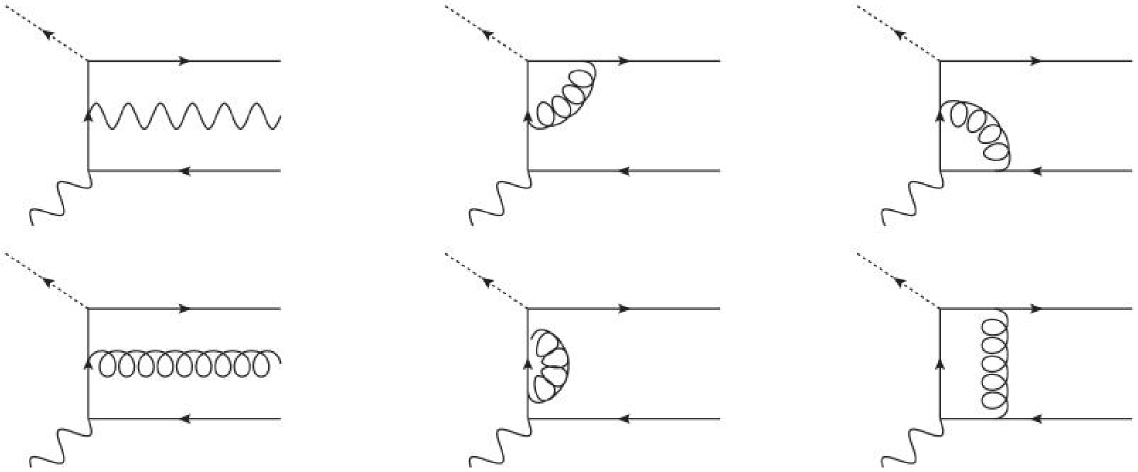
Subleading power corrections result from the use of photon DA, we work on two aspects in the next:

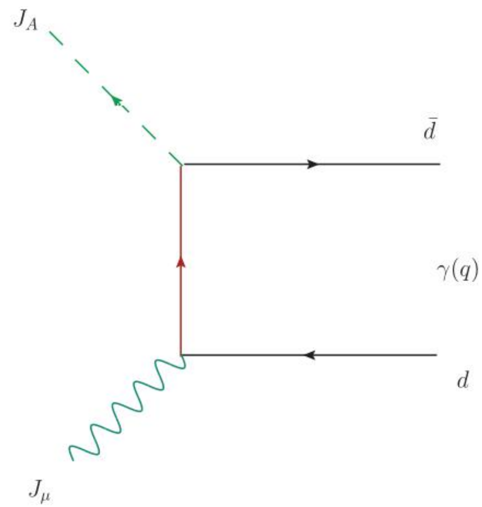
1> LO calculations at higher twist(up to twist 4) photon DA

2> NLO QCD corrections at leading twist photon DA.



Tree level higer twist 2-particle corrections

$$\langle\gamma(p)|\bar q(x)\,W_c(x,0)\,\sigma_{\alpha\beta}\,q(0)|0\rangle$$
$$= i\,g_{\rm em}\,Q_q\,\langle\bar q q\rangle(\mu)\,(p_\beta\,\epsilon^*_\alpha - p_\alpha\,\epsilon^*_\beta)\int_0^1 dz\,e^{izp\cdot x}\left[\chi(\mu)\,\phi_\gamma(z,\mu)+\frac{x^2}{16}\,\mathbb{A}(z,\mu)\right]$$
$$+\frac{i}{2}\,g_{\rm em}\,Q_q\,\frac{\langle\bar q q\rangle(\mu)}{q\cdot x}\,(x_\beta\,\epsilon^*_\alpha - x_\alpha\,\epsilon^*_\beta)\int_0^1 dz\,e^{izp\cdot x}\,h_\gamma(z,\mu)\,.$$



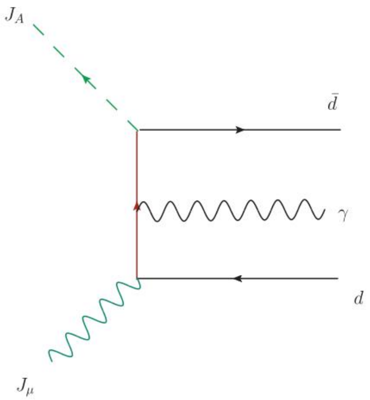
$$\langle\gamma(p)|\bar q(x)\,W_c(x,0)\,\gamma_\alpha\,\gamma_5\,q(0)|0\rangle$$
$$=\frac{g_{\rm em}}{4}\,Q_q\,f_{3\gamma}(\mu)\,\varepsilon_{\alpha\beta\rho\tau}\,p^\rho\,x^\tau\,\epsilon^{*\,\beta}\int_0^1 dz\,e^{izp\cdot x}\,\psi^{(a)}(z,\mu)$$

$$\Pi_\mu(p,q)\supset\frac{1}{4}g_{em}Q_d(p\cdot q)\int_0^1 dz\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}n^\alpha v^\beta\left\{\frac{\rho_{V,2}((p+q)^2,q^2,z)}{[(zp+q)^2-m_c^2]^2}+\frac{\rho_{V,3}((p+q)^2,q^2,z)}{[(zp+q)^2-m_c^2]^3}\right\}$$

$$\rho_{V,2}((p+q)^2,q^2,z)=2m_cf_{3\gamma}\psi^{(a)}(z)+\langle\bar q q\rangle A(z),$$
$$\rho_{V,3}((p+q)^2,q^2,z)=-2m_c^2\langle\bar q q\rangle A(z).$$

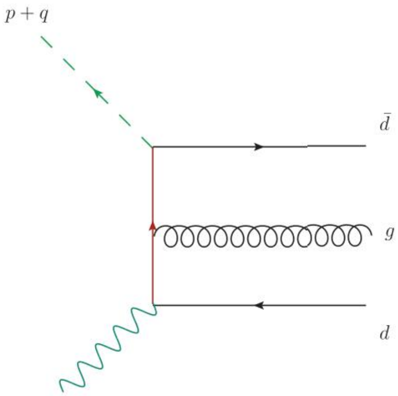
Tree level higer twist 3-particle corrections

$$\langle 0 \mid T\{\bar{c}(x), c(0)\} \mid 0\rangle = ig\int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x}\int_0^1 du[\frac{ux_\mu}{k^2-m^2}G^{\mu\nu}(ux)\gamma_\nu - \frac{\not{k}+m}{2(k^2-m^2)^2}G^{\mu\nu}(ux)\sigma_{\mu\nu}]$$
$$+ie_c\int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x}\int_0^1 du[\frac{ux_\mu}{k^2-m^2}F^{\mu\nu}(ux)\gamma_\nu - \frac{\not{k}+m}{2(k^2-m^2)^2}F^{\mu\nu}(ux)\sigma_{\mu\nu}].$$



$$\Pi^{(\gamma)}_{\mu,3P} = g_{em}Q_c\langle\bar{q}q\rangle(p\cdot q)\int_0^1 dv\int[d\alpha]\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}n^\alpha v^\beta\cdot$$
$$\left\{\frac{\rho^{3P}_{V,2}((p+q)^2,q^2,\alpha_i,v)}{[(q+(\alpha_q+v\alpha_g)p)^2-m_c^2]^2}+\frac{\rho^{3P}_{V,3}((p+q)^2,q^2,\alpha_i,v)}{[(q+(\alpha_q+v\alpha_g)p)^2-m_c^2]^3}\right\}$$

$$\rho^{3P}_{V,2} = -S_\gamma(\alpha_i)$$
$$\rho^{3P}_{V,3} = 2[(p+q)^2-q^2](2v-1)\left[-\bar{T}^\gamma_4(\alpha_i)\right]$$



$$\Pi^{(g)}_{\mu,3P} = g_{em}Q_d\langle\bar{q}q\rangle(p\cdot q)\int_0^1 dv\int[d\alpha]\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}n^\alpha v^\beta\cdot$$
$$\left\{\frac{\rho^{3P}_{V,2}((p+q)^2,q^2,\alpha_i,v)}{[(q+(\alpha_q+v\alpha_g)p)^2-m_c^2]^2}+\frac{\rho^{3P}_{V,3}((p+q)^2,q^2,\alpha_i,v)}{[(q+(\alpha_q+v\alpha_g)p)^2-m_c^2]^3}\right\}$$

$$\rho^{3P}_{V,2} = -S(\alpha_i)+(1-2v)\tilde{S}(\alpha_i)-\left[T_1(\alpha_i)-T_2(\alpha_i)\right]$$
$$\rho^{3P}_{V,3} = 2[(p+q)^2-q^2](2v-1)\left[\bar{T}_3(\alpha_i)-\bar{T}_4(\alpha_i)\right]$$

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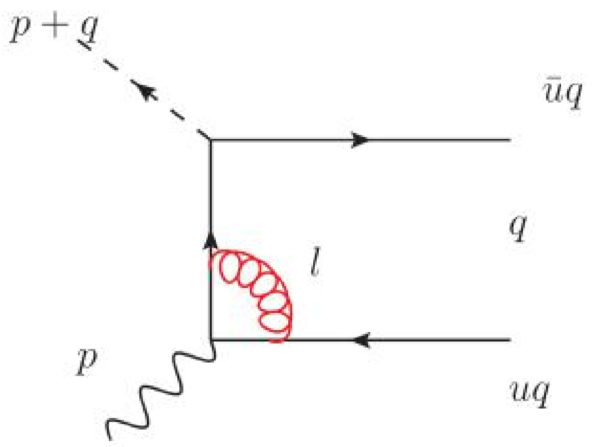


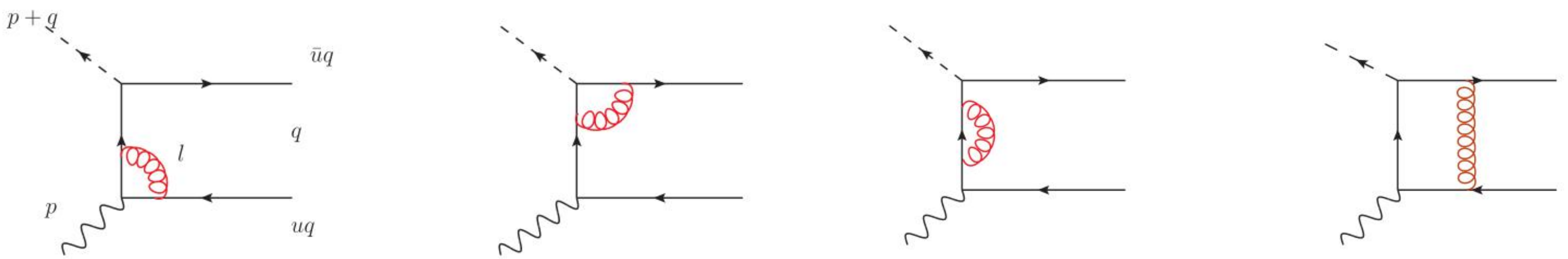
From kinematics, we only need to consider hard region and collinear region, and from power conunting, hard region is the only one left to work on.

$$p^\mu = (\frac{\bar{n}\cdot p}{2}n^\mu, \frac{n\cdot p}{2}\bar{n}^\mu, 0) \sim Q(1,1,0), \quad q^\mu = (0, \frac{n\cdot q}{2}\bar{n}^\mu, 0)$$

Hard **Collinear**

$$\frac{\alpha_s C_F}{4\pi}\left\{\left[\frac{2(1-r_2)}{r_2-r_1}\ln\frac{1-r_1}{1-r_2}-1\right]\left[\frac{1}{\epsilon}+\ln\frac{\mu^2}{m_b^2}-\ln[(1-r_1)(1-r_2)]\right]-\ln[(1-r_1)(1-r_2)]\right.$$
$$+\frac{1}{r_1-r_2}\left[2(1-r_2)\left(\mathrm{Li}_2(r_1)-\mathrm{Li}_2(r_2)\right)+\frac{(1-r_1)(1-r_1-2r_2)}{r_1}\ln(1-r_1)\right.$$
$$\left.\left.-\frac{(1-r_2)(1-3r_2)}{r_2}\ln(1-r_2)\right]-4\right\}F_\mu^{(0)},$$

$$r_1=(p+zq)^2/m^2, r_2=p^2/m^2, r_3=(p+q)^2/m^2$$



Collecting everything together, we get the final result of NLO corrections:

$$
\begin{aligned}
T^{\mu(1)} =& \frac{\alpha_s C_F}{4\pi}\Big\{\Big(\frac{2(1-r_2)}{r_2-r_1} ln\frac{1-r_1}{1-r_2} + \frac{2(1-r_3)}{r_3-r_1} ln\frac{1-r_1}{1-r_3} - \frac{6}{1-r_1}\Big)\big(\frac{1}{\epsilon} + ln\frac{\mu^2}{m^2}\big) \\
&+ 2\Big[\big(\frac{1-r_2}{r_1-r_2} + \frac{1-r_3}{r_1-r_3}\big)Li_2(r_1) - \frac{1-r_2}{r_1-r_2}Li_2(r_2) - \frac{1-r_3}{r_1-r_3}Li_2(r_3) + (Li_2(r_i) \leftrightarrow ln^2(1-r_i))\Big] \\
&+ \Big[\frac{1-r_1}{r_1}\big(\frac{1-r_1-2r_2}{r_1-r_2} - 2\frac{1-r_1+r_3}{r_1-r_3}\big) + \frac{1}{1-r_1}\big(\frac{1-7r_1}{r_1^2} + 6\big) + 1\Big] ln(1-r_1) \\
&- \frac{(1-r_2)(1-3r_2)}{r_2(r_1-r_2)} ln(1-r_2) + \frac{2(1-r_3)}{r_3(r_1-r_3)} ln(1-r_3) + \frac{1-9r_1}{r_1(1-r_1)} - 3\Big\} T^{\mu(0)}
\end{aligned}
$$

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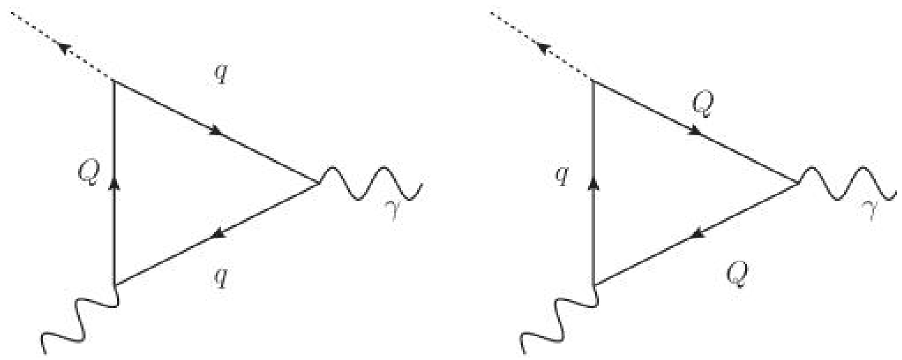
Coupling constant from leading power

With the aid of quark-hadron duality approximation, Borel transformation, continuum subtraction:

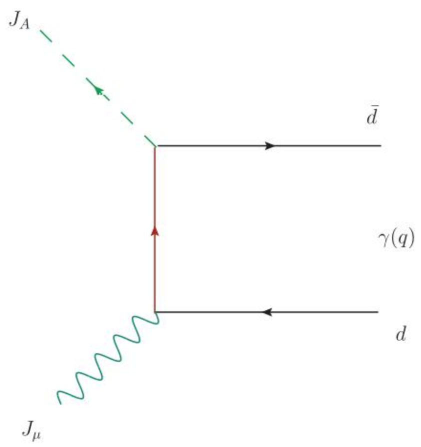


$$f_H\, f_{H^*}\, g_{H^* H\gamma} = -\frac{N_c}{8\,\pi^2}\frac{m_Q^2}{m_H^2\, m_{H^*}} e^{\frac{m_H^2+m_{H^*}^2}{M^2}} \left\{ \left(e^{-2\,r_m} - e^{-\frac{2\,s_0}{M^2}} \right) M^2\,(e_Q - e_q) - e^{-\frac{2\,s_0}{M^2}} M^2\, e_Q \ln\left(\frac{m_Q^2}{s_0}\right) + M^2\,[e_Q + 2\,(e_Q - e_q)\, r_m] \left[\mathrm{Ei}(-2\,r_m) - \mathrm{Ei}\left(-\frac{2\,s_0}{M^2}\right)\right]\right\}$$

where

$r_m = \frac{m_Q^2}{M^2}$, the exponential integral $\mathrm{Ei}(x)$ is defined as $\mathrm{Ei}(x) = -\int_{-x}^{\infty} dt\, \frac{e^{-t}}{t}$.

Tree level higher-twist corrections for coupling constant



2-particle higher twist:

$$f_H\, f_{H^*}\, g_{H^*H\gamma} = \frac{m_Q}{m_H^2\, m_{H^*}} e^{\frac{m_H^2+m_{H^*}^2}{M^2}} e_q \left[\mathcal{F}_2^{2P}\left(\frac{s_0-m_Q^2}{M^2}\right) + \mathcal{F}_3^{2P}\left(\frac{s_0-m_Q^2}{M^2}\right)\right]$$

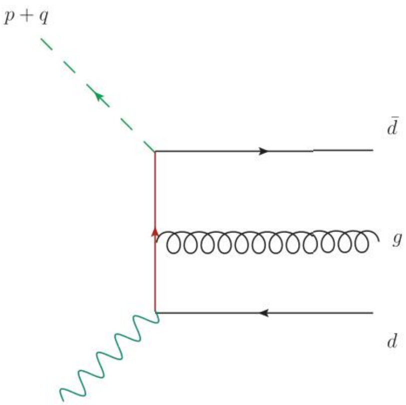
with

$$\mathcal{F}_2^{2P}(x) = -\frac{5}{12} m_Q\, f_{3\gamma}(\mu)\, e^{\frac{-2m_Q^2}{M^2}} \left(1+\frac{9}{16}\omega_\gamma^V(\mu) - \frac{3}{16}\omega_\gamma^A(\mu)\right)\left[3-e^{-2x}(3+54x-90x^2+20x^3)\right]$$
$$+\frac{1}{24}\langle\bar{q}q\rangle(\mu)\, e^{\frac{-2m_Q^2}{M^2}} \left\{5(3\kappa(\mu)-\kappa^+(\mu)+1)\left[3-e^{-2x}(3+6x-18x^2+4x^3)\right]\right.$$
$$\left.+\left(\zeta_2^+(\mu)-3\zeta_2(\mu)\right)\left[-33-e^{-2x}(63+30x-234x^2+52x^3)\right]\right\},$$

$$\mathcal{F}_3^{2P}(x) = \frac{m_Q^2}{4M^2}\langle\bar{q}q\rangle(\mu)\, e^{\frac{-2m_Q^2}{M^2}} \left\{5(3\kappa(\mu)-\kappa^+(\mu)+1)\left[1-e^{-2x}(1-6x+2x^2)\right]\right.$$
$$\left.-\left(\zeta_2^+(\mu)-3\zeta_2(\mu)\right)\left[43+e^{-2x}(5-78x+26x^2)\right]\right\},$$

Tree level higher-twist corrections for coupling constant

3-particle with gluon:



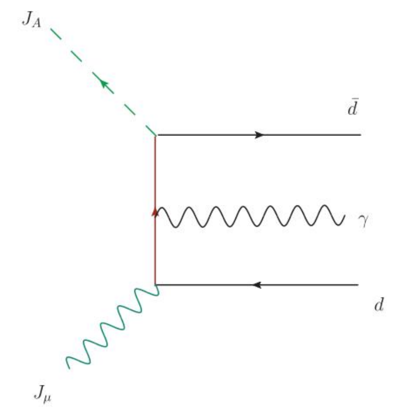
$$f_H\, f_{H^*}\, g_{H^*H\gamma} = \frac{m_Q}{m_H^2\, m_{H^*}} e^{\frac{m_H^2+m_{H^*}^2}{M^2}}\, e_q\, \langle \bar{q}q\rangle(\mu)\, \left[\mathcal{F}^{3P}_{2,G}\left(\frac{s_0-m_Q^2}{M^2}\right) + \mathcal{F}^{3P}_{3,G}\left(\frac{s_0-m_Q^2}{M^2}\right)\right]$$

with

$$\mathcal{F}^{3P}_{2,G}(x) = \frac{5}{48} e^{\frac{-2m_Q^2}{M^2}} \Big\{\zeta_2(\mu)\left[3+e^{-2x}(-3-54x+90x^2-20x^3)\right]$$
$$-(\kappa(\mu)+\kappa^+(\mu))\left[15+e^{-2x}(-15+18x+18x^2-4x^3)\right]\Big\},$$

$$\mathcal{F}^{3P}_{3,G}(x) = \frac{1}{8} e^{\frac{-2m_Q^2}{M^2}} \Big\{(3\zeta_2(\mu)-\zeta_2^+(\mu))\left[\frac{123}{5} - 32\ln 2 + e^{-2x}(-27-6x+90x^2-20x^3)\right]$$
$$-\frac{5}{6}(\zeta_2(\mu)+\kappa(\mu)+\kappa^+(\mu))\left[3-e^{-2x}(3+54x-90x^2+20x^3)\right]\Big\}.$$

3-particle with photon:



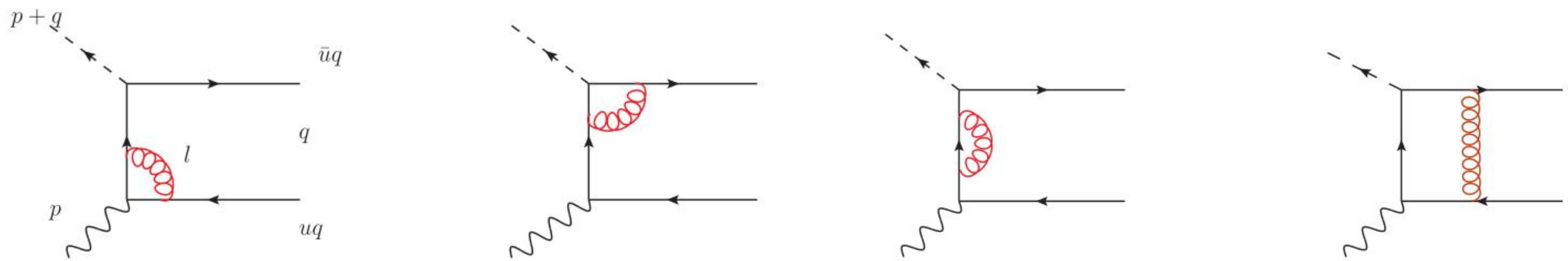
$$f_H\, f_{H^*}\, g_{H^*H\gamma} = \frac{m_Q}{m_H^2\, m_{H^*}} e^{\frac{m_H^2+m_{H^*}^2}{M^2}}\, e_Q\, \langle \bar{q}q\rangle(\mu)\, \left[\mathcal{F}^{3P}_{2,\gamma}\left(\frac{s_0-m_Q^2}{M^2}\right) + \mathcal{F}^{3P}_{3,\gamma}\left(\frac{s_0-m_Q^2}{M^2}\right)\right]$$

with

$$\mathcal{F}^{3P}_{2,\gamma}(x) = \frac{5}{48} e^{\frac{-2m_Q^2}{M^2}} \left[-15+e^{-2x}(15-18x-18x^2+4x^3)\right],$$

$$\mathcal{F}^{3P}_{3,\gamma}(x) = -\frac{5}{48} e^{\frac{-2m_Q^2}{M^2}} \left[-3+e^{-2x}(3-54x-90x^2+20x^3)\right].$$

NLO corrections for coupling constant



$$f_H\, f_{H^*}\, g_{H^* H\gamma} = \frac{\alpha_s C_F}{4\pi} \frac{m_Q}{m_H^2\, m_{H^*}} e^{\frac{m_H^2+m_{H^*}^2}{M^2}}\, e_q\, \langle \bar{q}q \rangle(\mu)\, \chi(\mu) \int_{2m_Q^2}^{2s_0} d\tilde{s}\, \mathcal{F}_{tw-2}^{\rm NLO}\left(\frac{\tilde{s}}{m_Q^2}-2\right) e^{-\frac{\tilde{s}}{M^2}}$$

with

$$\begin{aligned}
\mathcal{F}_{tw-2}^{\rm NLO}(x) &= \int_0^\infty dr\, \frac{x}{(r+1)^2}\, \rho^{\rm NLO}(r,x) \\
&= -6{\rm Li}_2\left(-\frac{x}{2}\right) + 3{\rm Li}_2(-x-1) + 3{\rm Li}_2(-x) - 3\ln\frac{x}{2}\ln\left(1+\frac{x}{2}\right) + 3\ln(x+1)\ln(x+2) \\
&\quad -3\ln\frac{\mu^2}{m_Q^2} + \frac{3(7x^3+50x^2+100x+64)}{4(x+2)^3}\ln\frac{x}{2} + \frac{3}{2}\ln\frac{x+2}{2} - \frac{6(x+1)^2}{(x+2)^3}\ln(x+1) \\
&\quad + \frac{3(9x^2+20x+16)}{8(x+2)^2} - \frac{\pi^2}{4}.
\end{aligned}$$

Numerical analysis

Inputs:

$\langle \bar{q}q \rangle(\mu_0)$ (MeV^3)	$\chi(\mu_0)$ (GeV^{-2})	$a_2(\mu_0)$	$a_4(\mu_0)$	$f_{3\gamma}(\mu_0)$ (GeV^2)	$\omega_\gamma^V(\mu_0)$
$-(266 \pm 21)^3$ [24]	3.15 ± 0.3 [25]	0.07 ± 0.07 [25]	0	$-(4 \pm 2) \times 10^{-3}$	3.8 ± 1.8
$\omega_\gamma^A(\mu_0)$	$\kappa(\mu_0)$	$\kappa^+(\mu_0)$	$\zeta_1(\mu_0)$	$\zeta_1^+(\mu_0)$	$\zeta_2^+(\mu_0)$
-2.1 ± 1.0	0.2 ± 0.2	0	0.4 ± 0.4	0	0

$$\begin{cases} s_0 = 7.0 \pm 0.5 \text{ GeV}^2, \\ M^2 = 4.5 \pm 1.0 \text{ GeV}^2, \end{cases} \text{for } D^* D\gamma, \qquad \begin{cases} s_0 = 35 \pm 2 \text{ GeV}^2, \\ M^2 = 18 \pm 3 \text{ GeV}^2, \end{cases} \text{for } B^* B\gamma$$

Numerical analysis

Method	$g_{D^{*+}D^{+}\gamma}$ (GeV^{-1})	$g_{D^{*0}D^{0}\gamma}$ (GeV^{-1})	$\Gamma(D^{*+} \to D^{+}\gamma)$ (keV)	$\Gamma(D^{*0} \to D^{0}\gamma)$ (keV)
Exp. [27] 2018	$-(0.47 \pm 0.06)$	< 11	1.3 ± 0.3	< 741
Lattice [32] 2011	-0.2 ± 0.3	2.0 ± 0.6	0.2 ± 0.7	25 ± 15
this work	$-0.39^{+0.12}_{-0.13}$	$2.09^{+0.35}_{-0.33}$	$0.90^{+0.56}_{-0.61}$	$27.3^{+9.1}_{-8.6}$
LCSR [17] 1996	-0.48 ± 0.12	1.47 ± 0.24	1.50	14.40
HQET+VMD [7] 1993	-0.29 ± 0.05	1.60 ± 0.37	0.51 ± 0.18	16.0 ± 7.5
RQM [12][a] 2001	-0.38 ± 0.01	2.04 ± 0.04	0.90 ± 0.02	26 ± 1
LFQM [13][b] 2007	-0.384	1.738	0.90 ± 0.02	20.0 ± 0.3
covariant model [14] 2014	$-0.38^{+0.06}_{-0.04}$	1.9 ± 0.1	$0.9^{+0.3}_{-0.2}$	$22.7^{+2.1}_{-2.2}$

Numerical analysis

Method	$g_{B^{*+}B^{+}\gamma}$ (GeV^{-1})	$g_{B^{*0}B^{0}\gamma}$ (GeV^{-1})	$\Gamma(B^{*+} \to B^{+}\gamma)$ (keV)	$\Gamma(B^{*0} \to B^{0}\gamma)$ (keV)
this work	$2.10^{+0.38}_{-0.35}$	$-1.23^{+0.19}_{-0.21}$	$0.98^{+0.36}_{-0.33}$	$0.33^{+0.10}_{-0.11}$
LCSR [17]	1.69 ± 0.17	-0.85 ± 0.17	0.63	0.16
HQET+VMD [7]	0.99 ± 0.20	-0.58 ± 0.10	0.22 ± 0.09	0.075 ± 0.027
RQM [12][a]	1.60 ± 0.10	-0.90 ± 0.05	0.57 ± 0.07	0.18 ± 0.02
LFQM [13][b]	1.311	-0.749	0.40 ± 0.03	0.13 ± 0.01
covariant model [14]	$1.45^{+0.11}_{-0.12}$	-0.81 ± 0.05	$0.468^{+0.073}_{-0.075}$	0.148 ± 0.020

Summary

1. We calculate the subleading power corrections to $B^* \to B\gamma$ and $D^* \to D\gamma$ based on LCSR for the first time, and with the most accurate subtraction schemes.

2. Our calculation is consistent with previous results in a model independent way.

3. We expect more precise results from experiments, lattice, and theoretical calculations in the future.