Partonic model simulations of vorticity and spin polarization in heavy-ion collisions

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Motivation of the talk

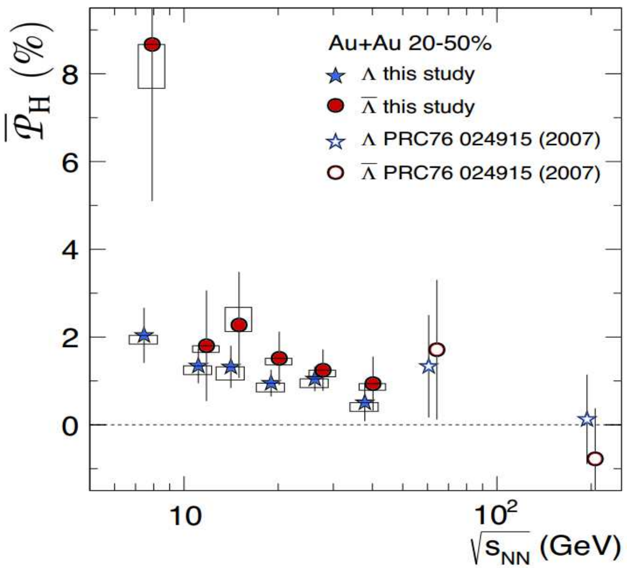
LETTER

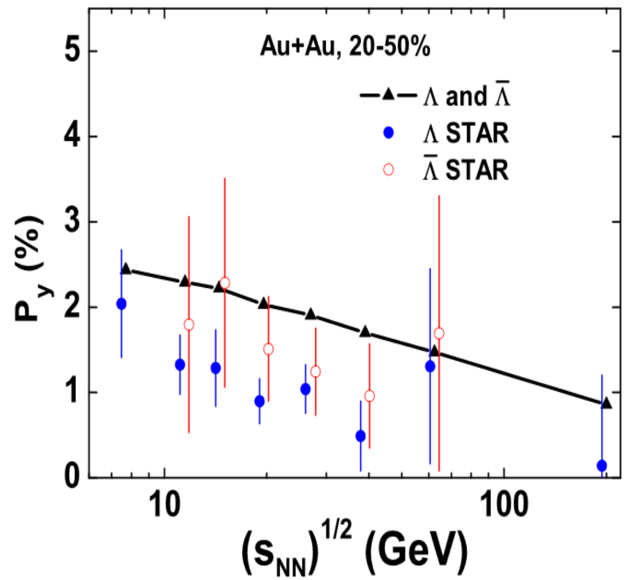
Quark-gluon plasma: "The most vortical fluid"

doi:10.1038/nature23004

Global Λ hyperon polarization in nuclear collisions

The STAR Collaboration*





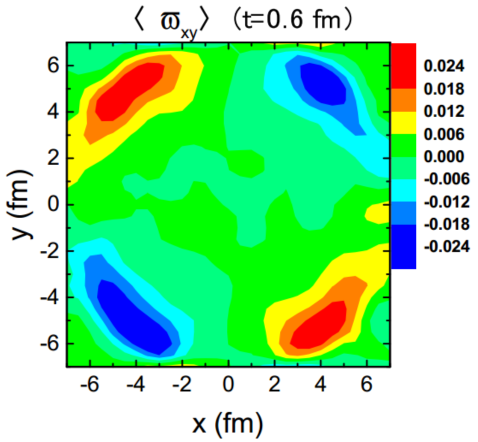
Sun-Ko 2017

Experiment = Theory

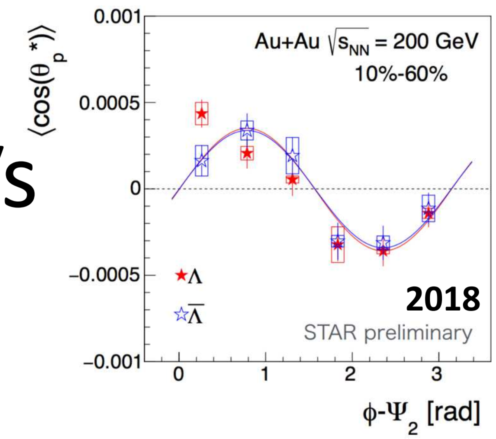
Motivation of the talk

- **But: discrepancies between theory and experiments**

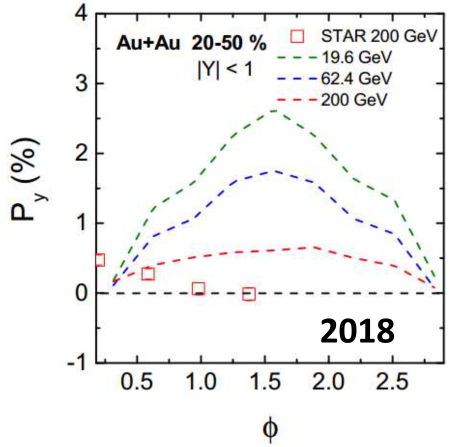
1) longitudinal polarization vs ϕ



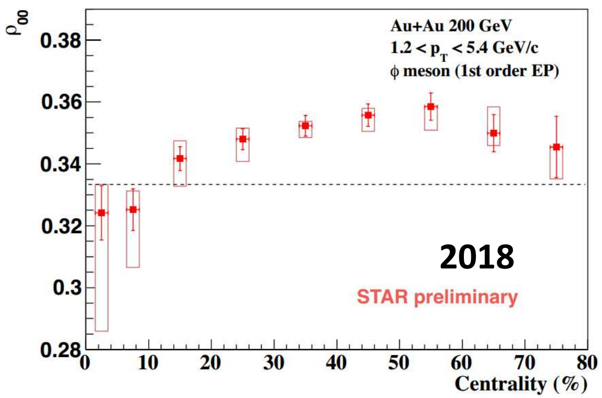
Vs



2) Transverse polarization vs ϕ



3) Vector meson spin alignment



Experiment Refs:

STAR Collaboration, arXiv:1805.04400

Niida, Quark matter 2018

C. Zhou, Quark matter 2018

B. Tu, Quark matter 2018

Singh, Chirality 2019

Motivation of the talk

- **To resolve the puzzle, from the theory side, we need to:**

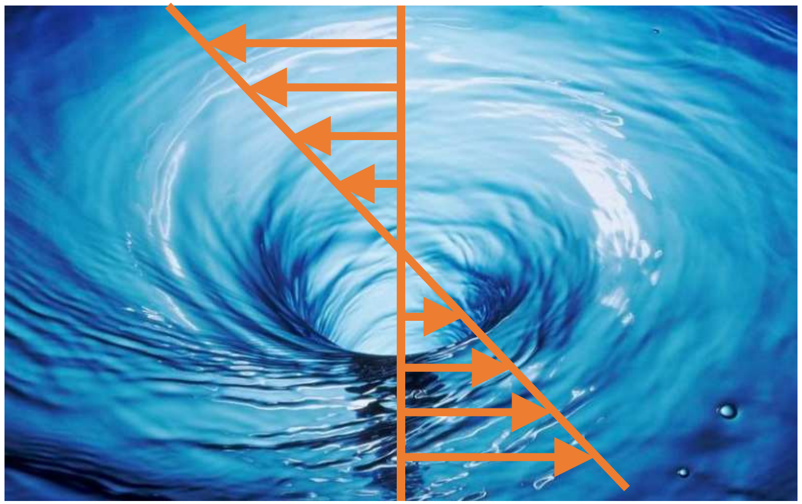
  - Understand the properties of fluid vorticity
  - Understand the magnetic field contribution, the feed-down contribution, … …
  - Find other observables which are always helpful: spin-alignment at central collisions, the chiral vorticity effects **(Sun-Ko 2018), … …**
  - Understand how vorticity polarizes spin and how the spin polarization evolve: spin kinetic theory or spin hydrodynamics

Vorticity in heavy-ion collisions

Deng-XGH, arXiv: 1603.06117 (HIJING)

Wei-Deng-XGH, arXiv: 1810.00151 (AMPT)

Fluid vorticity



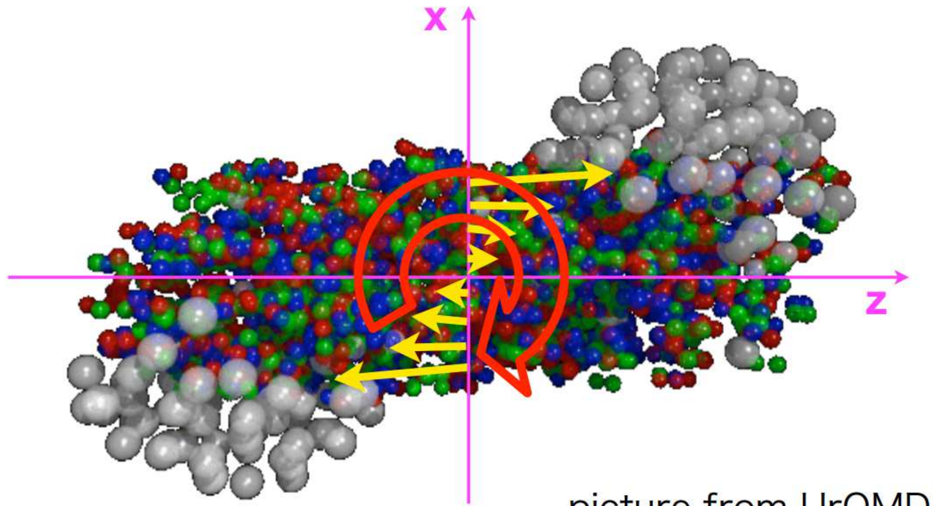
Nonrelativistic: $\boldsymbol{\omega} = \boldsymbol{\nabla} \times \boldsymbol{v}$ $\sim$ local angular momentum

Relativistic:

$$\omega^{\mu} = \epsilon^{\mu\nu\rho\sigma} u_{\nu} \partial_{\rho} u_{\sigma}$$ $\sim$ local angular momentum + local boost

$$\varpi_{\mu\nu} = \frac{1}{2}[\partial_{\nu}(u_{\mu}/T) - \partial_{\mu}(u_{\nu}/T)]$$ $\sim$ thermal vorticity

Angular momentum in HIC



picture from UrQMD

Global angular momentum

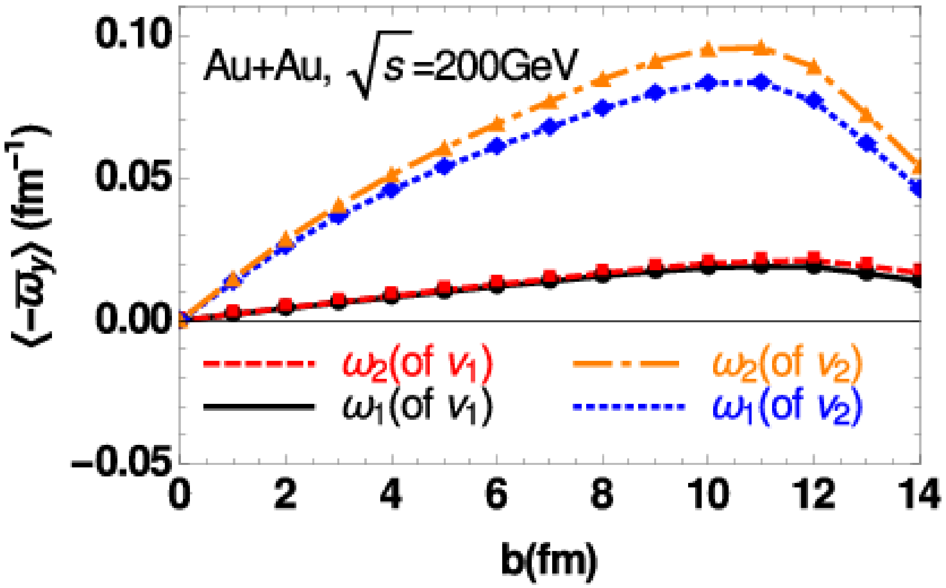
$$J_0 \sim \frac{Ab\sqrt{s}}{2} \sim 10^6 \hbar$$

(RHIC Au+Au 200 GeV, b=10 fm)

Local vorticity

$$\omega \sim ?$$

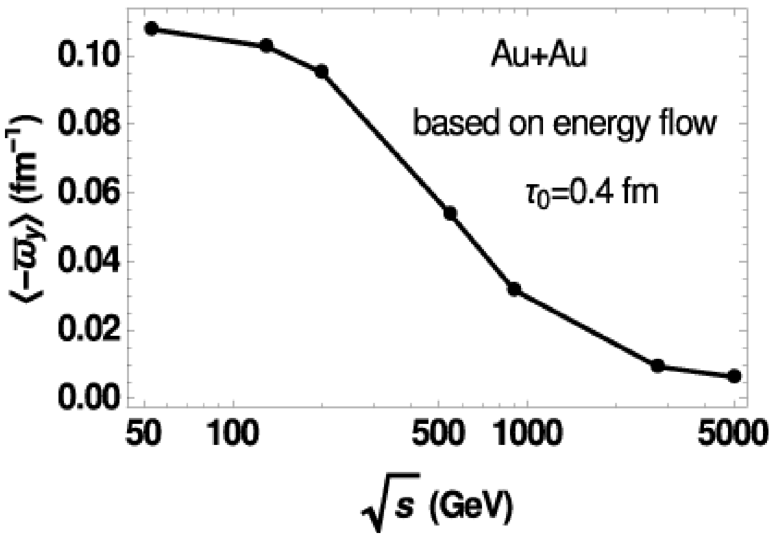
Vorticity by global AM

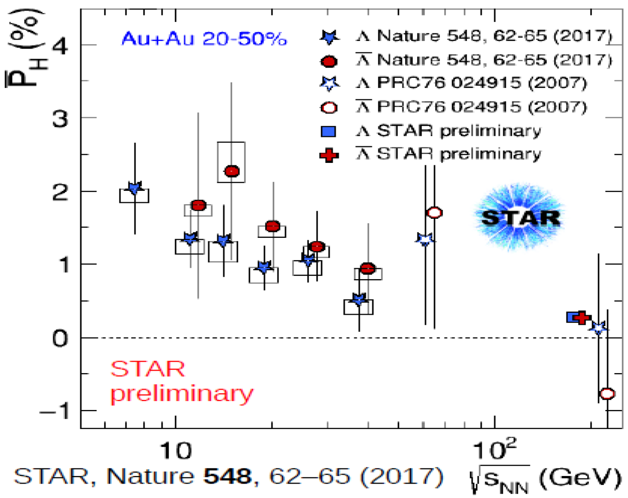


Vorticity in Au+Au@RHIC at $b = 10$ fm is $10^{20} - 10^{21} s^{-1}$

See also: Becattini etal 2015,2016; Jiang-Lin-Liao 2016;Pang-Petersen-Wang-Wang 2016; Xia-Li-Wang 2017,2018; Sun-Ko 2017; … …

Vorticity by global AM

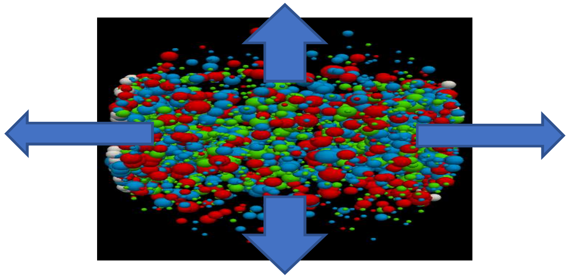


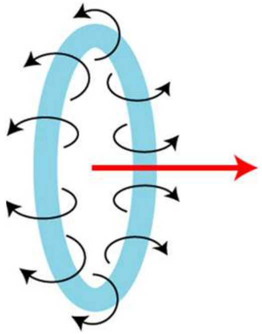


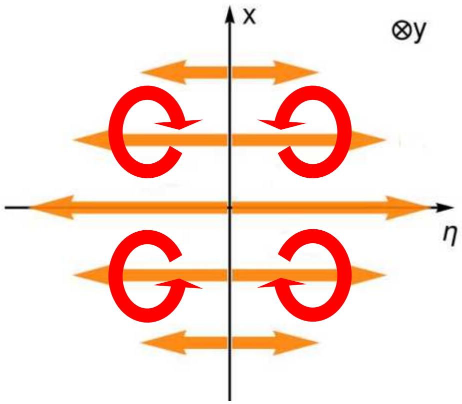
Vorticity at mid-rapidity decreases with increasing $\sqrt{S}$

See also: Jiang-Lin-Liao 2016, Xia-Li-Wang 2017, … …

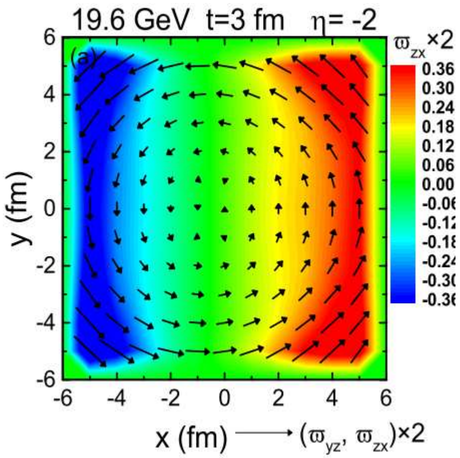
Vorticity due to expansion

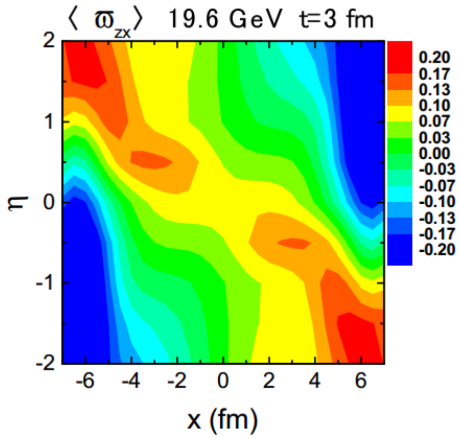




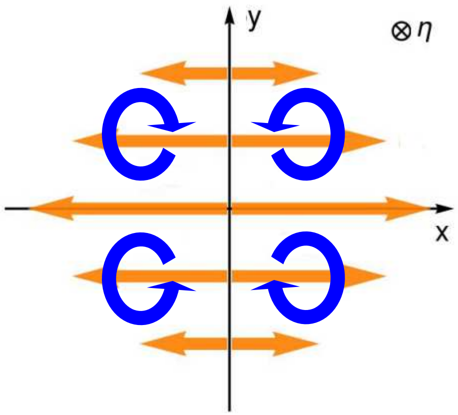


Transverse

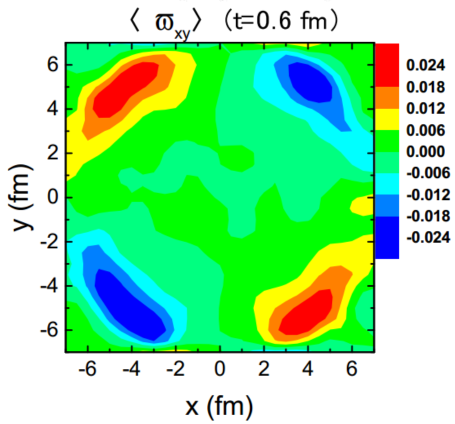




Thermal vorticity



Longitudinal

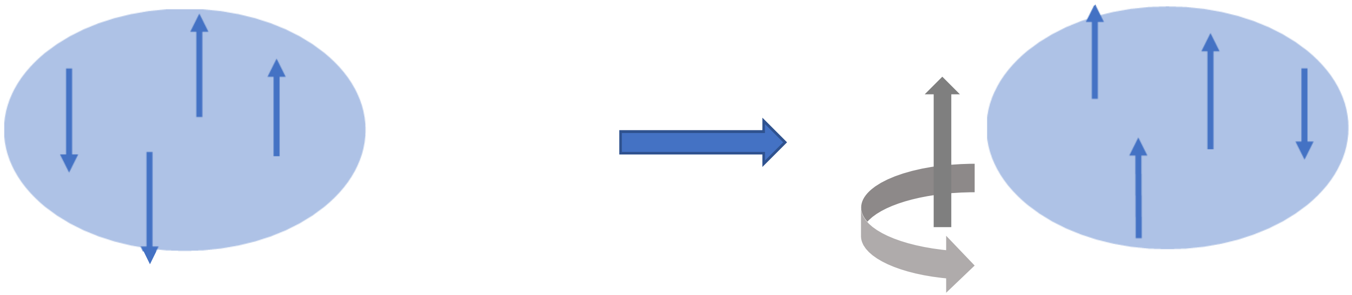


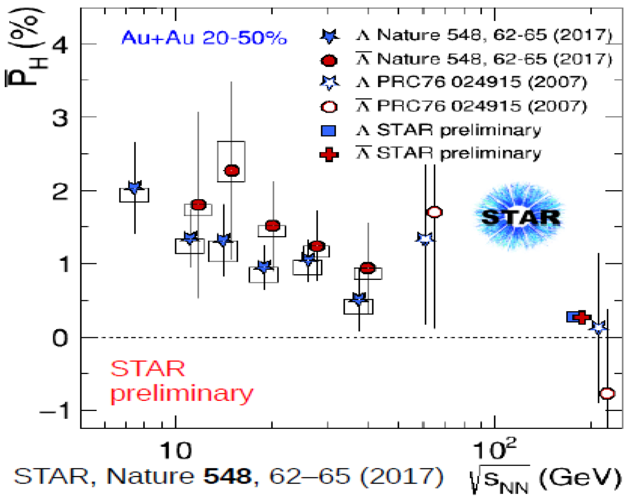
(see also: Becattini etal 2017; Jiang-Lin-Liao 2016; Xia-Li-Wang 2017; Teryaev-Usubov 2015, …)

Spin-vorticity coupling

Early consideration: Liang-Wang 2004; Voloshin 2004

$$H = H_0 - \boldsymbol{\omega} \cdot \boldsymbol{J} \quad \Rightarrow \quad \frac{dN}{d\boldsymbol{p}} \sim e^{-(H_0 - \boldsymbol{\omega}\cdot\boldsymbol{J})/T}$$





$$\mathrm{P} = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} \sim \frac{\langle \omega \rangle}{T}$$

Possible magnetic-field contribution. A way to measure B?

$$H = H_0 - \boldsymbol{\omega} \cdot \boldsymbol{J} - \boldsymbol{m} \cdot \boldsymbol{B}$$

Spin-vorticity coupling

More careful examination: Becattini-Chandra-Grossi 2013; Fang-Pang-Wang-Wang 2016

$$S^{\mu}(x,p) = -\frac{s(s+1)}{6m}(1-n_F)\epsilon^{\mu\nu\rho\sigma}p_{\nu}\varpi_{\rho\sigma}(x) + O(\varpi)^2$$

where $n_F(p_0)$ is the Fermi-Dirac distribution function and $p_0 = \sqrt{\boldsymbol{p}^2 + m^2}$

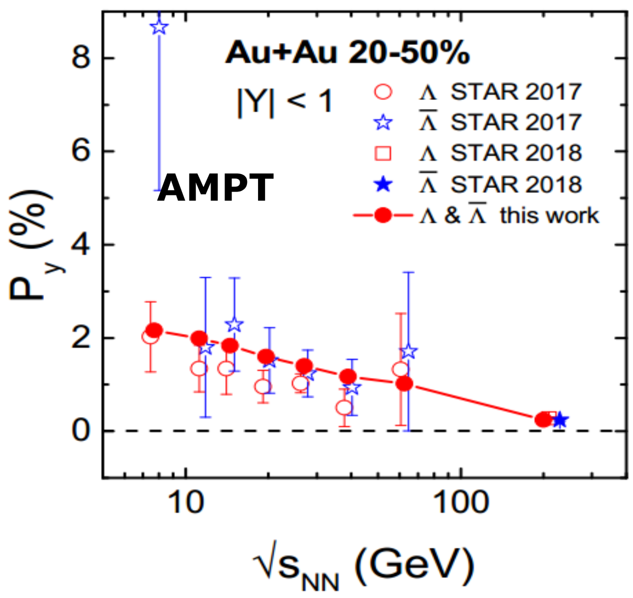
Rest frame of particle: $$\boldsymbol{S}^* = \boldsymbol{S} - \frac{\boldsymbol{p}\cdot\boldsymbol{S}}{p_0(p_0+m)}\boldsymbol{p}$$

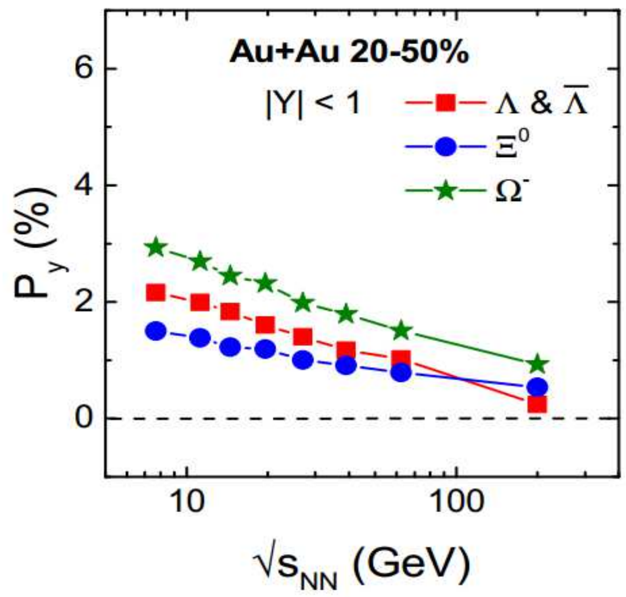
Polarization in direction n: $$P_n = \frac{1}{s}\boldsymbol{S}^*\cdot\boldsymbol{n}$$

Assumption used: thermal equilibrium. Is spin degree of freedom thermalized in HICs? Open question.

Hyperon polarization

- Global spin polarization



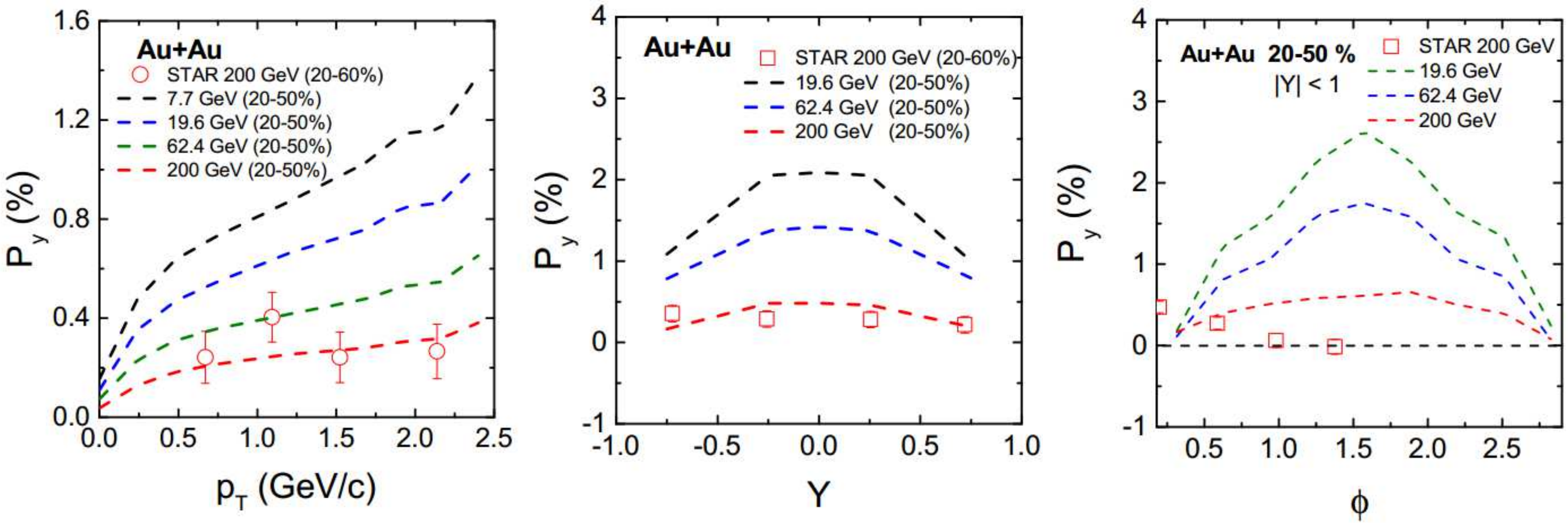


- Mass ordering among $\Omega^-(sss)$, $\Xi^0(uss)$, and $\Lambda(uds)$.
- Magnetic moments $\mu_\Omega : \mu_\Xi : \mu_\Lambda = 3:2:1$. Test magnetic contribution.

D.X.Wei-W.T.Deng-XGH, 1810.00151

The sign problem

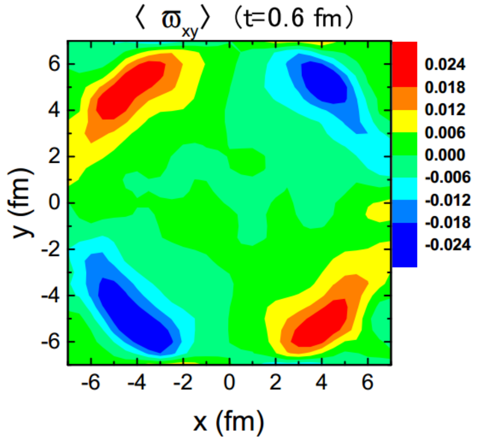
- p_T, rapidity, and azimuthal dependence, theory vs expts.



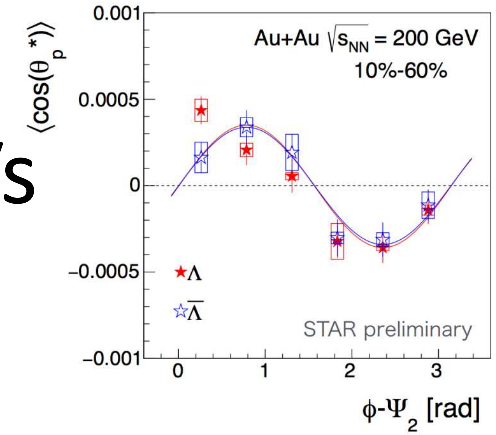
- Theory consistent with experiments in p_T and rapidity dependence.
- Puzzle: opposite ϕ dependence in theory and experiment.

The sign problem

- **Longitudinal sign problem:**

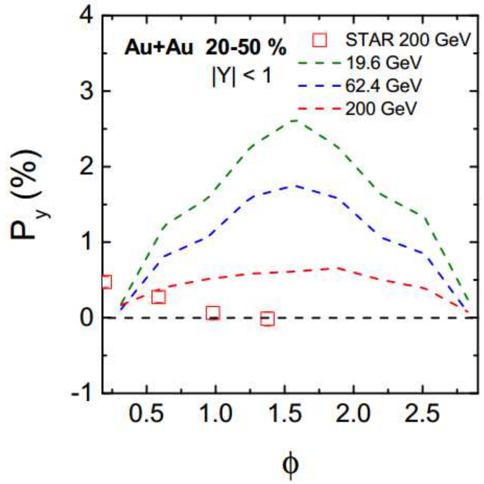


Vs



- **Transverse sign problem:**

Data: STAR Collaboration
Calculation: Wei-Deng-XGH 2018

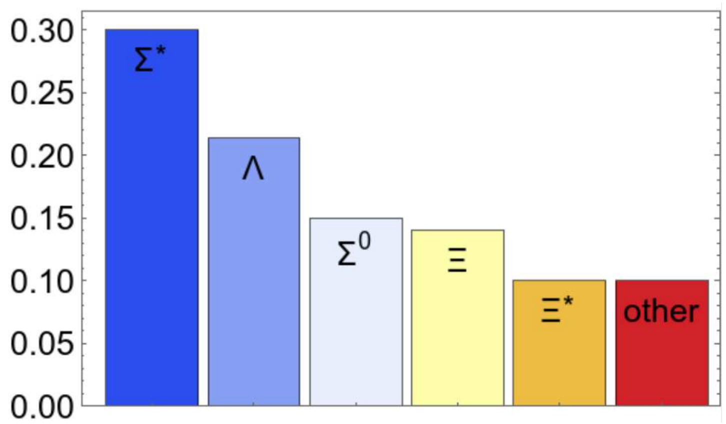


Feed-down effect

Xia-Li-XGH-Huang, arXiv: 1905.03120

Motivations

(1) A large fraction of the Λ hyperon comes from decays of higher-lying hyperons



Cf. Hui Li

(2) The feed-down effect may provide a resolution to the "polarization sign problem". For example, EM decay, if Σ is polarization along the vorticity, its daughter Λ must be polarized opposite to the vorticity

$$\Sigma^0 \to \Lambda + \gamma \qquad \left(\frac{1}{2}\right)^+ \to \left(\frac{1}{2}\right)^+ 1^-$$

Spin transfer

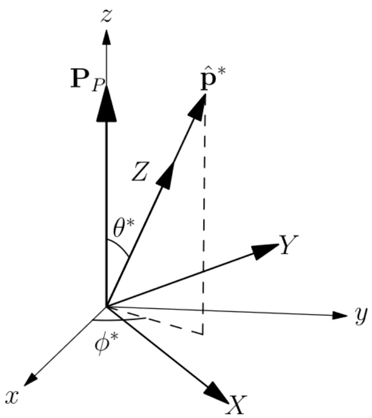
TABLE I. Daughter angular distribution and polarization vector $\mathbf{P}_D$ in different decay channels

	spin and parity	$(1/N)dN/d\Omega^*$	$\mathbf{P}_D$	$\langle \mathbf{P}_D \rangle / \mathbf{P}_P$
strong decay	$1/2^+ \to 1/2^+ 0^-$	$1/(4\pi)$	$2(\mathbf{P}_P \cdot \hat{\mathbf{p}}^*)\hat{\mathbf{p}}^* - \mathbf{P}_P$	-1/3
strong decay	$1/2^- \to 1/2^+ 0^-$	$1/(4\pi)$	$\mathbf{P}_P$	1
strong decay	$3/2^+ \to 1/2^+ 0^-$	$3\left[1 - 2\Delta/3 - (1-2\Delta)\cos^2\theta^*\right]/(8\pi)$	Eq. (40)	1
strong decay	$3/2^- \to 1/2^+ 0^-$	$3\left[1 - 2\Delta/3 - (1-2\Delta)\cos^2\theta^*\right]/(8\pi)$	Eq. (41)	-3/5
weak decay	$1/2 \quad \to 1/2 \quad 0$	$(1 + \alpha P_P \cos\theta^*)/(4\pi)$	Eq. (28)	$(2\gamma + 1)/3$
EM decay	$1/2^+ \to 1/2^+ 1^-$	$1/(4\pi)$	$-(\mathbf{P}_P \cdot \hat{\mathbf{p}}^*)\hat{\mathbf{p}}^*$	-1/3

$$\mathbf{P}_D = \frac{-4\delta(\mathbf{P}_P \cdot \hat{\mathbf{p}}^*)\hat{\mathbf{p}}^* + \left[1 - 2\delta - (1-10\delta)\left(\hat{\mathbf{P}}_P \cdot \hat{\mathbf{p}}^*\right)^2\right]\mathbf{P}_P}{1 - 2\Delta/3 - (1-2\Delta)\left(\hat{\mathbf{P}}_P \cdot \hat{\mathbf{p}}^*\right)^2}, \tag{40}$$

and

$$\frac{2\left[1 - 4\delta - (1-10\delta)\left(\hat{\mathbf{P}}_P \cdot \hat{\mathbf{p}}^*\right)^2\right](\mathbf{P}_P \cdot \hat{\mathbf{p}}^*)\hat{\mathbf{p}}^* - \left[1 - 2\delta - (1-10\delta)\left(\hat{\mathbf{P}}_P \cdot \hat{\mathbf{p}}^*\right)^2\right]\mathbf{P}_P}{1 - 2\Delta/3 - (1-2\Delta)\left(\hat{\mathbf{P}}_P \cdot \hat{\mathbf{p}}^*\right)^2}. \tag{41}$$



$$\mathbf{P}_D = \frac{(\alpha + \mathbf{P}_P \cdot \hat{\mathbf{p}}^*)\hat{\mathbf{p}}^* + \beta(\mathbf{P}_P \times \hat{\mathbf{p}}^*) + \gamma\hat{\mathbf{p}}^* \times (\mathbf{P}_P \times \hat{\mathbf{p}}^*)}{1 + \alpha\mathbf{P}_P \cdot \hat{\mathbf{p}}^*}. \tag{28}$$

Spin transfer

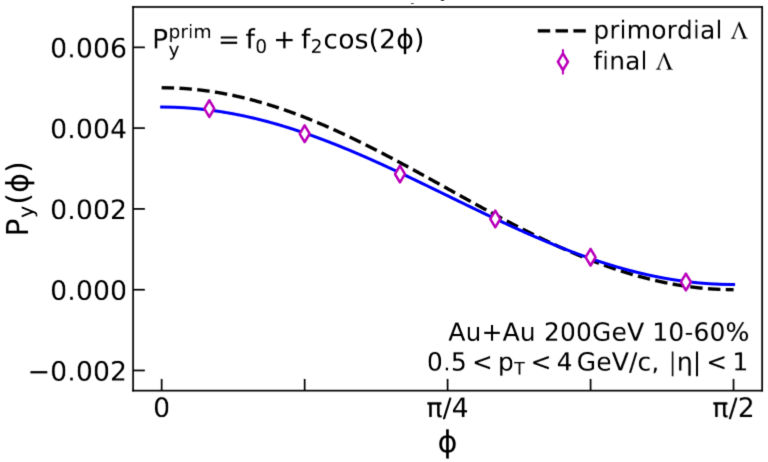
TABLE II. The primordial yield ratio N_i/N_Λ, spin, parity, and decay channels of strange particles

	N_i/N_Λ	spin and parity	decay channel
Λ	1	$1/2^+$	-
$\Lambda(1405)$	0.236	$1/2^-$	$\Sigma^0\pi$
$\Lambda(1520)$	0.265	$3/2^-$	$\Sigma^0\pi$
$\Lambda(1600)$	0.098	$1/2^+$	$\Sigma^0\pi$
$\Lambda(1670)$	0.061	$1/2^-$	$\Sigma^0\pi$, $\Lambda\eta$
$\Lambda(1690)$	0.112	$3/2^-$	$\Sigma^0\pi$
Σ^0	0.686	$1/2^+$	$\Lambda\gamma$
Σ^{*0}	0.533	$3/2^+$	$\Lambda\pi$
Σ^{*+}	0.535	$3/2^+$	$\Lambda\pi$, $\Sigma^0\pi$
Σ^{*-}	0.524	$3/2^+$	$\Lambda\pi$, $\Sigma^0\pi$
$\Sigma(1660)$	0.068	$1/2^+$	$\Lambda\pi$, $\Sigma^0\pi$
$\Sigma(1670)$	0.125	$3/2^-$	$\Lambda\pi$, $\Sigma^0\pi$
Ξ^0	0.343	$1/2^+$	$\Lambda\pi$
Ξ^-	0.332	$1/2^+$	$\Lambda\pi$
Ξ^{*0}	0.228	$3/2^+$	$\Xi\pi$
Ξ^{*-}	0.224	$3/2^+$	$\Xi\pi$

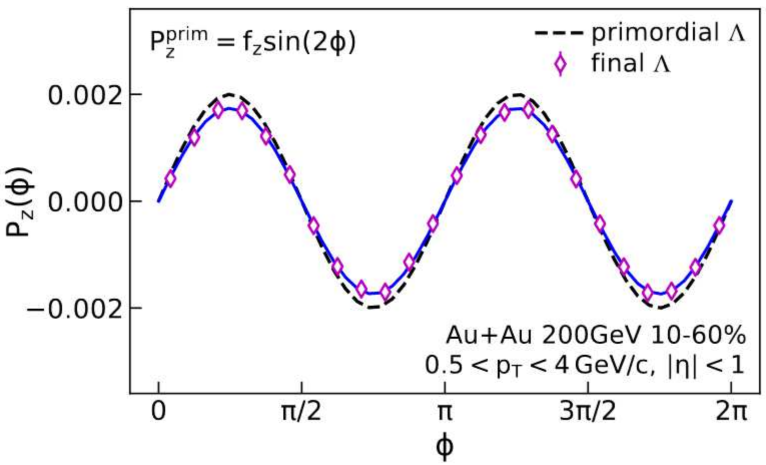
Decay contribution

- **Assuming the primordial particles are polarized the same :**

Transverse polarization



Longitudinal polarization



Conclusion:

Feed-down decays suppress 10% the primordial polarization, but it does not solve the sign problem

Sign problem is still there.

Any suggestions, comments, are welcome.

See also: Becattini-Cao-Speranza, arXiv:1905.03123

Dissipative spin hydrodynamics

Hattori-Hongo-XGH-Mameda-Matsuo-Taya, arXiv:1901.06615

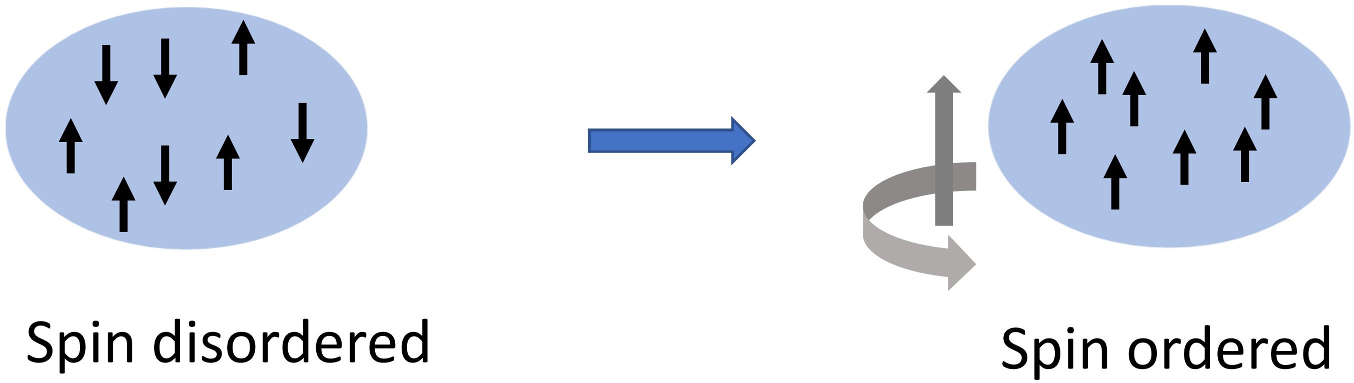
Spin hydrodynamics

- **Ideal spin hydro:** **(Florkowski etal 2017)**

$$\partial_\mu T^{\mu\nu} = 0 \qquad T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu}$$

$$\partial_\lambda S^{\lambda,\mu\nu} = 0 \qquad S^{\lambda,\mu\nu} = \frac{w u^\lambda}{4\zeta}\omega^{\mu\nu}$$

- **Why dissipation is important?**



Spin configuration entropy decrease: The polarization process must be dissipative so that the total entropy increase.

Spin hydrodynamics

- Go beyond the naïve picture of spin polarization by vorticity
- Consider collective dynamics of spin: spin hydrodynamics

Energy-momentum conservation: $\partial_\mu \Theta^{\mu\nu} = 0$

Angular-momentum conservation: $\partial_\mu J^{\mu\alpha\beta} = 0$

$$J^{\mu\alpha\beta} = \underbrace{\left(x^\alpha \Theta^{\mu\beta} - x^\beta \Theta^{\mu\alpha}\right)}_{\text{Orbital}} + \underbrace{\Sigma^{\mu\alpha\beta}}_{\text{Spin}}$$

$$\Theta^{\mu\nu} = \Theta_s^{\mu\nu} + \Theta_a^{\mu\nu}$$

$$\Rightarrow \quad \partial_\mu \Sigma^{\mu\alpha\beta} = -2\Theta_{(a)}^{\alpha\beta}$$

Identify the hydrodynamic variable: T and u^μ (4 for translation), $\omega^{\mu\nu}$(3 for rotation, 3 for boost)

Express $\Theta^{\mu\nu}$ and $J^{\mu\varrho\sigma}$ in terms of hydro variables and make derivative expansion

Spin hydrodynamics

- We have

$$\Theta^{\mu\nu} = eu^\mu u^\nu + p\Delta^{\mu\nu} + \Theta^{\mu\nu}_{(1)} \qquad \Theta^{\mu\nu}_{(1s)} = 2h^{(\mu}u^{\nu)} + \tau^{\mu\nu}$$

$$\Sigma^{\mu\alpha\beta} = u^\mu S^{\alpha\beta} + \Sigma^{\mu\alpha\beta}_{(1)} \qquad \Theta^{\mu\nu}_{(1a)} = 2q^{[\mu}u^{\nu]} + \phi^{\mu\nu}$$

- Apply the 2nd law of thermodynamics can give the constitutive relations at $O(\partial)$:

$$h^\mu = -\kappa(Du^\mu + \beta\partial_\perp^\mu T) \qquad q^\mu = -\lambda\left(-Du^\mu + \beta\partial_\perp^\mu T - 4\omega^{\mu\nu}u_\nu\right)$$

$$\tau^{\mu\nu} = -2\eta\partial_\perp^{\langle\mu}u^{\nu\rangle} - \zeta\theta\Delta^{\mu\nu} \qquad \phi^{\mu\nu} = -2\gamma\left(\partial_\perp^{[\mu}u^{\nu]} - 2\Delta^\mu_\rho\Delta^\nu_\lambda\omega^{\rho\lambda}\right)$$

Transport coefficients: thermal conductivity κ, viscosities η, ζ, and new transport coefficients: boost heat conductivity λ and rotational viscosity γ. They are all semipositive.

- This completes the construction of spin hydro at $O(\partial)$

Spin hydrodynamics

- Possible consequences: (1) New collective modes

$$\omega = -2iD_s,$$ ⟵ Longitudinal spin damping

$$\omega = -2iD_b,$$ ⟵ Longitudinal boost damping

$$\omega = \begin{cases} -2iD_s - i\gamma' k_z^2 + \mathcal{O}(k_z^4), \\ -i\gamma_\perp k_z^2 + O(k_z^4),, \end{cases}$$

⟵ Transverse spin damping

⟵ Shear viscous damping

$$\omega = \begin{cases} \pm c_s k_z - i\frac{\gamma_\parallel}{2} k_z^2 + \mathcal{O}(k_z^3), \\ -2iD_b - 2ic_s^2\lambda' k_z^2 + \mathcal{O}(k_z^4). \end{cases}$$

⟵ Sound and bulk viscous damping

⟵ Transverse boost damping

- (2) Partonic simulation of spin transport coefficients

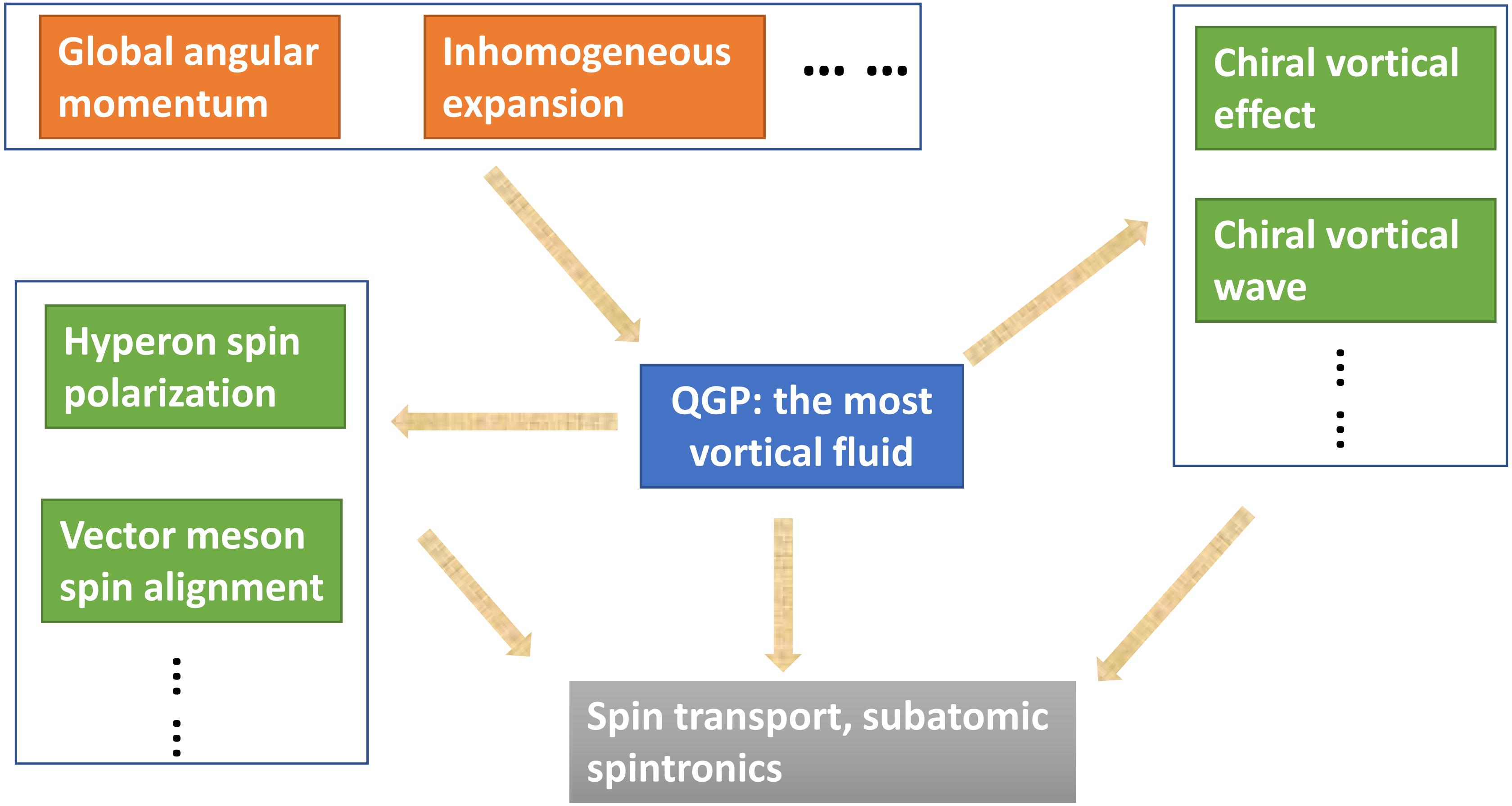
boost heat conductivity

$$\lambda \sim \lim_{\omega\to 0}\lim_{p\to 0}\frac{\partial}{\partial\omega} G_R^{T^{[0i]}T^{[0i]}}(\omega, p)$$

rotational viscosity

$$\gamma \sim \lim_{\omega\to 0}\lim_{p\to 0}\frac{\partial}{\partial\omega} G_R^{T^{[ij]}T^{[ij]}}(\omega, p)$$

} New insight to QCD matter!



Thank you and congratulations to Prof. Che Ming Ko for the 50-year scientific career!

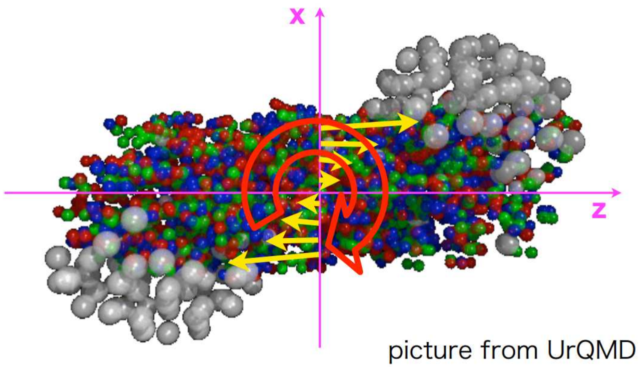
Spin hydrodynamics

- **Discussion**

1) Can we formulate spin hydrodynamics with a symmetric energy momentum tensor?
2) To form a causal and numerically stable set of equations, we need to consider the second order spin hydrodynamics
3) Calculation of the new transport coefficients of QCD: rotational viscosity and boost heat conductivity
4) Derive spin hydrodynamics from kinetic theory, Wigner function, etc (early trials: Becattni etal 2018, Florkowski etal 2018)
5) Applications: Numerical spin hydrodynamics

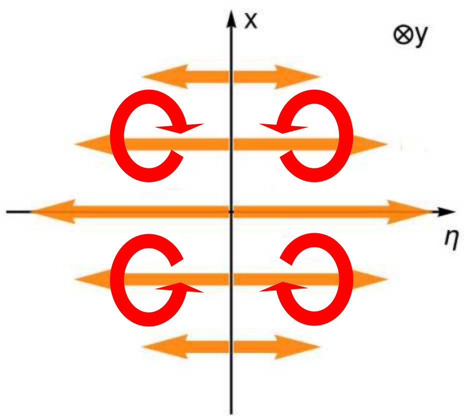
Main message:

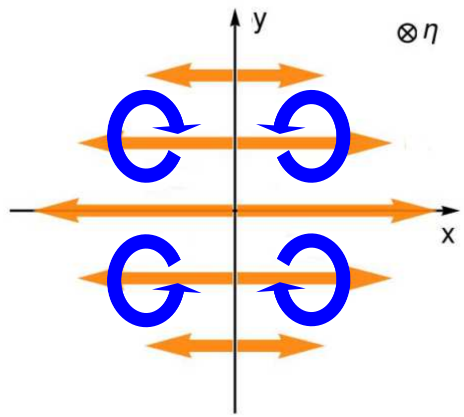
1. Global AM induces strong vorticity in HICs



$: \omega \approx 10^{21} - 10^{22}\ s^{-1}$

2. Inhomogeneous expansion: quadrupoles in both xy and xz planes





Velocity field in partonic model

- To calculate the vorticity, we need to know the velocity

Definition of velocity field in HIJING or AMPT model

$$v_1^a(x) = \frac{1}{\sum_i \Phi(x,x_i)} \sum_i \frac{p_i^a}{p_i^0}\Phi(x,x_i) = \frac{J^a}{J^0}$$ **~ Particle flow velocity**

$$v_2^a(x) = \frac{\sum_i p_i^a \Phi(x,x_i)}{\sum_i [p_i^0 + (p_i^a)^2/p_i^0]\Phi(x,x_i)} = \frac{T^{0a}}{T^{00}+T^{aa}}$$ **~ Energy flow velocity**

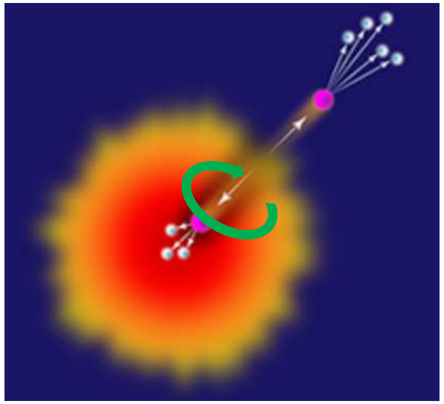
Smearing function Phi

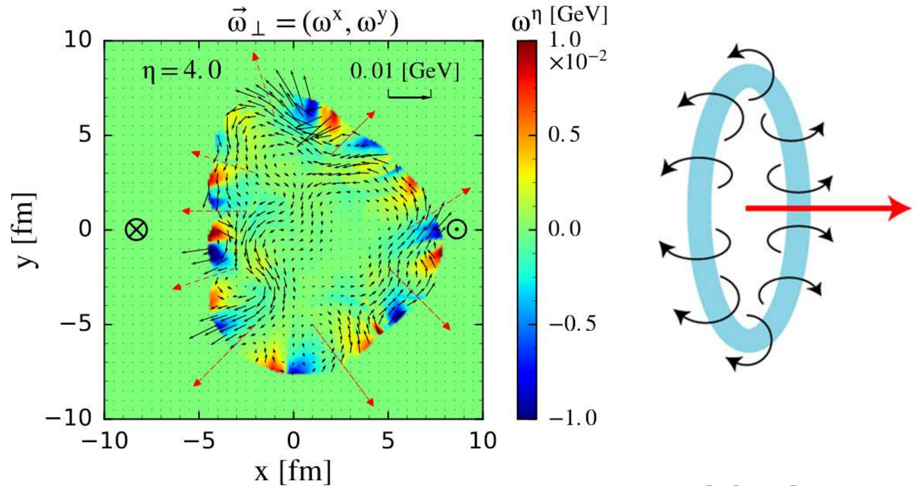
$$\Phi_G(x,x_i) = \frac{K}{\tau_0\sqrt{2\pi\sigma_\eta^2 2\pi\sigma_r^2}} \exp\left[-\frac{(x-x_i)^2+(y-y_i)^2}{2\sigma_r^2} - \frac{(\eta-\eta_i)^2}{2\sigma_\eta^2}\right]$$

Parameters are so chosen that with hydro, it is consistent with elliptic data (Pang-Wang-Wang 2012)

Other sources of vorticity

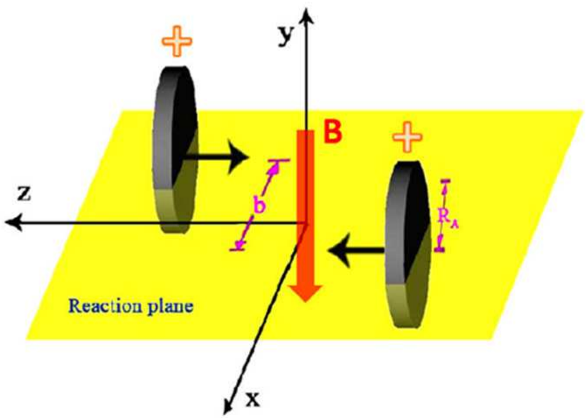
1) Jet

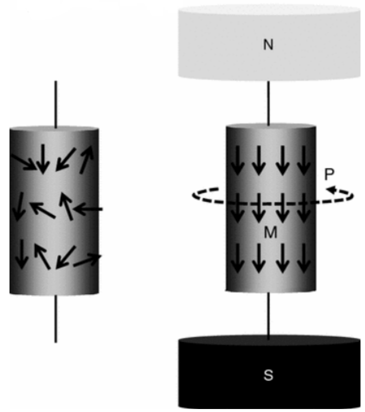




Pang-Peterson-Wang-Wang 2016

2) Magnetic field





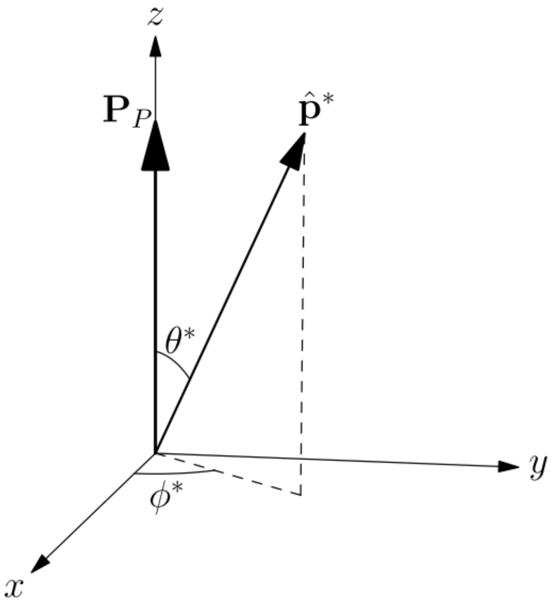
Einstein-de-Haas effect

Spin transfer

- Consider the decay process

$$P \to D + X$$

- The parent P is spin-polarized along z, the daughter D moves along p* in P's rest frame



Density matrix

$$\rho^f_{\lambda_D\lambda_X;\lambda'_D\lambda'_X}(\theta^*,\phi^*) = \sum_{M_P,M'_P} H_{\lambda_D\lambda_X;M_P}\rho^i_{M_P;M'_P}H^\dagger_{M'_P;\lambda'_D\lambda'_X}$$

$$|f\rangle = |\theta^*\phi^*\lambda_D\lambda_X\rangle \quad \Leftarrow \quad |i\rangle = |S_P M_P\rangle$$

The spin polarization of D:

$$\mathbf{P}_D = \mathrm{tr}_D\left(\widehat{\mathbf{P}}\rho^D_{\lambda_D;\lambda'_D}\right)/\mathrm{tr}_D\left(\rho^D_{\lambda_D;\lambda'_D}\right)$$

$$\rho^D_{\lambda_D;\lambda'_D} = \mathrm{tr}_X\left(\rho^f_{\lambda_D\lambda_X;\lambda'_D\lambda'_X}\right)$$

Spin transfer

- **For example, consider the EM decay $1/2^+ \to 1/2^+\, 1^-$:**

Initial density matrix:
$$\rho^i_{M_P;M'_P} = \mathrm{diag}\left(\frac{1+P_P}{2}, \frac{1-P_P}{2}\right)$$

$$\Rightarrow \quad \rho^f_{\lambda_D\lambda_X;\lambda'_D\lambda'_X} = \frac{1}{8\pi}\begin{pmatrix} 1+P_P\cos\theta^* & 0 & 0 & -P_P\sin\theta^* \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -P_P\sin\theta^* & 0 & 0 & 1-P_P\cos\theta^* \end{pmatrix}$$

$$\Rightarrow \quad \rho^D_{\lambda_D;\lambda'_D} = \frac{1}{8\pi}\begin{pmatrix} 1+P_P\cos\theta^* & 0 \\ 0 & 1-P_P\cos\theta^* \end{pmatrix}$$

$$\Rightarrow \quad \mathbf{P}_D = -(\mathbf{P}_P\cdot\hat{\mathbf{p}}^*)\,\hat{\mathbf{p}}^*$$

First derived by Gatto 1958

Main decay channels

- **Main decay channels contributing to Λ spin polarization:**

$$\text{Strong decay}: \Sigma^{*0} \to \Lambda + \pi^0,\ \Sigma^{*+} \to \Lambda + \pi^+,\ \left(\frac{3}{2}\right)^+ \to \left(\frac{1}{2}\right)^+ 0^-$$

$$\text{EM decay}: \Sigma^0 \to \Lambda + \gamma,\ \left(\frac{1}{2}\right)^+ \to \left(\frac{1}{2}\right)^+ 1^-$$

$$\text{Weak decay}: \Xi^0 \to \Lambda + \pi^0,\ \Xi^- \to \Lambda + \pi^-,\ \left(\frac{1}{2}\right)^+ \to \left(\frac{1}{2}\right)^+ 0^-$$

Summary

- **Most vortical fluid created in HICs.**
- **Global polarization can be understood: vorticity induced by global AM**
- **Inhomogeneous expansion leads to quadrupolar vortical structure in transverse plane and reaction plane**
- **Sign problem in the azimuthal-angle dependence of both transverse and longitudinal polarizations**
- **Resonance decays don’t solve sign problem**
- **New observables: rapidity dependent spin harmonic flows, spin alignment in central collisions**